Overlap gap property: An algorithmic barrier to optimization in random structures and statistical physics

David Gamarnik[1]

[1] (MIT)

Abstract: Many optimization problems over random structures and statistical physics exhibit a gap between the existential and algorithmically achievable values, dubbed as statistics-to-computation gap. Examples include the problem of finding a largest independent set in a random graph, the problem of finding a near ground state in a spin glass model, the problem of finding a satisfying assignment in a random constraint satisfaction problem, and many more. At the same time, no formal computational hardness of these problems exists which explains this persistent algorithmic gap.

In the talk we describe a recent approach for establishing algorithmic intractability for these problems based on the so-called the overlap gap property. Originating in statistical physics, and specifically in the theory of spin glasses, this is a simple to describe property which a) emerges in most models known to exhibit an evidence of algorithmic hardness; b) is consistent with the hardness/tractability phase transition for many models analyzed to the day; and, importantly, c) allows to mathematically rigorously rule out a large class of algorithms as potential contenders, specifically the algorithms which exhibit noise insensitivity.

We will illustrate how to use this property to obtain tight algorithmic bounds for two algorithmic problems: finding large independent sets in sparse random graphs, and finding low energy states in spin glasses.