From chemical reaction networks to Descartes' rule of signs

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Abstract:
In the context of chemical reaction networks with mass-action and other rational kinetics, a major question is to preclude or to guarantee multiple positive steady states. I will explain this motivation and will present necessary and sufficient conditions in terms of sign vectors for the injectivity of families of polynomials maps with arbitrary real exponents defined on the positive orthant. These conditions extend existing injectivity conditions expressed in terms of Jacobian matrices and determinants, obtained by several authors. In the context of real algebraic geometry, this approach can be seen as the first partial multivariate generalization of the classical Descartes' rule, which bounds the number of positive real roots of a univariate real polynomial in terms of the number of sign variations of its coefficients. This is joint work with Stefan M\"uller, Elisenda Feliu, Georg Regensburger, Anne Shiu and Carsten Conradi. Our results can be applied to a general class of biochemical networks involving enzymatic reactions defined in collaboration with Mercedes P\'erez Mill\'an, that I will briefly describe. I will also present some further advances in this multivariate generalization obtained in collaboration with Fr\'ed\'eric Bihan.