M.Sc. Entrance Examination

Department of Economic Sciences

Indian Institute of Technology, Kanpur

June 7, 2024

Instructions:

This question paper has a total of 50 questions. Each question is followed by 4 choices. You must select the most appropriate choice. Please indicate the most appropriate choice clearly on the answer booklet provided. Each correct answer will earn you 2 points. For each wrong answer, you will lose 0.50 points. For each question you leave unanswered, you earn 0 points.

You have 2 hours to complete the examination. You are required to answer the questions on your own with no external assistance or equipment other than a pen. You may not communicate with anyone inside or outside the examination hall while the examination is in progress, except the exam invigilators. Furthermore, you are not allowed to use mobile phones, calculators or AI-based tools. Attempting to access the internet in any manner is strictly prohibited. Any violation will lead to cancellation of your candidacy in this round and permanent disqualification from appearing on future iterations of this examination.

Questions:

1. $\lim_{x\to 1} \frac{2x\sqrt{x}-3\sqrt{x}-2x+3}{\sqrt{x^3+x^2-5x+3}}$ is
   a. 0.25
   b. 0
   c. -0.25
   d. Does not exist.

2. Let $f:\mathbb{R}\to\mathbb{R}, f(x)=|ax^2+b|x|-c|$ for some $a,b,c>0$. Let m denote the number of points where f is not continuous, and n, the number of points where f is not differentiable. Then the value of $m+n$ is:
   a. 1
   b. 2
   c. 3
   d. 4

3. $\int \left(\frac{1-\log_e x}{1+(\log_e x)^2}\right)^2 dx$ is
   a. $\frac{2x}{1+(\log_e x)^2} + C$
   b. $\frac{x/2}{1+(\log_e x)^2} + C$
   c. $\frac{x^2}{1+(\log_e x)^2} + C$
   d. None of the above

4. $\int_0^1 (2x^3 - 3x^2 - x + 1)^{\frac{1}{3}} dx$ is
   a. 0
   b. 1
   c. -1
   d. None of the above

5. Let $f: \mathbb{R}^2 \to \mathbb{R}, f(x, y) = \frac{|x-y|+1}{|x+y|+1}$. Then the maximum of f is
   a. 0
   b. 1
   c. 2
   d. Does not exist.

6. Let $f: \mathbb{R}^2 \to \mathbb{R}, f(x, y) = x^3 + y^3$. We wish to compute the maximum and minimum of f when (x, y) is constrained to satisfy $x + y = 1$. Which of the following is true regarding the constrained optimization problem?
   a. Minimum exists but maximum does not
   b. Maximum exists but minimum does not
   c. Both maximum and minimum exists
   d. Neither maximum nor minimum exists

7. If $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{pmatrix}$, I is the 3×3 identity matrix, and $A^{-1} = \frac{1}{6}(A^2 + cA + dI)$, then the value of (c, d) is
   a. (1, 4)
   b. (1, -4)
   c. (-6, 11)
   d. None of the above

8. The value of the determinant $\begin{vmatrix} b^2 + c^2 & a^2 & a^2 \\ b^2 & c^2 + a^2 & b^2 \\ c^2 & c^2 & a^2 + b^2 \end{vmatrix}$ is
   a. $a^2b^2c^2$
   b. $4a^2b^2c^2$
   c. $-a^2b^2c^2$
   d. $-4a^2b^2c^2$

9. The number of values of k for which the system of equations $(k+1)x+8y=4k$ and $kx+(k+3)y=(3k-1)$ has infinitely many solutions is
   a. 0
   b. 1
   c. 2
   d. Infinitely many

10. Let Q_n denote the number of quadrilaterals which can be formed using the vertices of a regular polygon having n sides. If $Q_{n+1}-Q_n=20$, then $n=$
   a. 9
   b. 8
   c. 6
   d. 7

11. Let two positive numbers α and β have arithmetic mean of 3 and geometric mean of 2. Then α and β are the roots of which of the following quadratic equation?
   a. $x^2-6x-4=0$
   b. $x^2-3x+2=0$
   c. $x^2+6x+4=0$
   d. $x^2-6x+4=0$

12. The number of subsets of the set $\{1,2,3,4,5\}$ that contain at least one even number is:
   a. 24
   b. 22
   c. 23
   d. 25

13. If the matrix $\begin{pmatrix} 1 & -1 & x \\ 1 & x & -1 \\ -1 & 1 & 1 \end{pmatrix}$ has no inverse, then the value/values of x is/are:
   a. 0
   b. -1
   c. -1,1
   d. 0,1

14. Let $\beta \neq 0$ be a real number. The coefficient of the middle term in the binomial expansion (in powers of x) of $(\beta+x)^4$ and $(\beta-x)^6$ is the same if β equals the following:
   a. $\frac{-5}{3}$
   b. $\frac{3}{20}$
   c. $\frac{-3}{10}$
   d. $\frac{10}{3}$

15. If the series of number $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}$ form an arithmetic progression such that the difference between any two consecutive terms of this series is $d > 0$, then the value of the determinant $\begin{vmatrix} a_1 + a_2 & a_4 + a_5 & a_7 + a_8 \\ a_2 + a_3 & a_5 + a_6 & a_8 + a_9 \\ a_3 + a_4 & a_6 + a_7 & a_9 + a_{10} \end{vmatrix}$ is
    a. $6d$
    b. 0
    c. -1
    d. $12d$

16. Let $4x + 6y + 2z = 0$, $3x + 6y + 2z = 0$ and $3x + 2y + 4z = 0$ be a system of equations. Then, which of the following option is correct?
    a. $x = y = z = 0$ is a solution to the system of equations
    b. The above system of equations can be reduced to a single equation and so a unique non-trivial solution does not exist.
    c. Both (a) and (b) are true
    d. Neither (a) nor (b) are true

17. A student is appearing in an exam which has 6 questions. In order to pass the exam, the student must answer all the 6 questions correctly. The number of ways in which the student can fail the exam is:
    a. 64
    b. 31
    c. 65
    d. 63

18. If the coefficients of second, third and fourth terms in the binomial expansion of $(1 + x)^n$ form an arithmetic progression, then $n =$
    a. 7
    b. 6
    c. 9
    d. 8

19. If the positive numbers $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in arithmetic progression, then $log\ (x + z) + log\ (x - 2y + z)$ equals
    a. $3\log |x - z|$
    b. $2 \log |x - z|$
    c. $4 \log |x - z|$
    d. $\log |x - z|$

20. Suppose X has a normal distribution with mean 0 and variance σ^2. Let Y be an independent random variable taking values -1 and 1 with equal probability. If $Z = XY + X/Y$ then
    a. $Var(Z) = \sigma^2$
    b. $Var(Z) < \sigma^2$
    c. $Var(Z) > \sigma^2$
    d. $2Var(Z) < \sigma^2$

21. A particular airline has 6 $A.M.$ flights from Delhi to Mumbai, Kolkata, and Chennai. Let A denote the event that the flight from Delhi to Mumbai is full and define events B and C analogously for the other two flights. Suppose $P(A) = 0.6$, $P(B) = 0.5$, $P(C) = 0.4$ and the three events are independent. What is the probability that exactly one of the three flights is full?
    a. 0.18
    b. 0.08
    c. 0.88
    d. 0.38

22. A satellite can fail for two possible reasons: the failure of computer and the failure of engine. For a given mission, it is known that the probability of engine failure is 0.008 and that of computer failure is 0.001. It is also known that the probability of satellite failure given that the engine has failed is 0.98 and the probability of satellite failure given that the computer has failed is 0.45. Given that the satellite failed, find the probability that the failure was due to engine failure.
    a. 0.00829
    b. 0.00784
    c. 0.9457
    d. 0.0543

23. A production process that manufactures transistors operates on the average, at 2% fraction defective. A random sample of size 50 is taken from the process. If the sample contains more than two defectives, the process must be stopped. What is the probability that the process will be stopped?
    a. $1 - \frac{5}{2}e^{-1}$
    b. $2e^{-1}$
    c. $\frac{e^{-1}}{2!}$
    d. e^{-1}

24. Let $P(X = i) = p$ for all i and $P(Y = j) = q$ for all j, where $i, j = 1, 2, 3, \ldots, n$. If X and Y are two mutually independent random variables, then $E(XY)$ is
    a. n^2
    b. $\frac{(n+1)^2}{4}$
    c. $\frac{n^2}{2}$
    d. n

25. Let T denote the lifetime of a light bulb (the number of hours a light bulb will last before burning out). Suppose that $X = \log_e T$ has a normal distribution with mean $\mu = 8$ hours and standard deviation $\sigma = 4$ hours. What is the probability that the bulb will fail in less than 900 hours? Given that $\log_e 900 \approx 6.8, and \int_{-\infty}^{0.3} \frac{1}{\sqrt{2\pi}} e^{\frac{-t^2}{2}} dt = 0.6179.$
    a. 0.6179
    b. 0.0120
    c. 0.0352
    d. 0.3821

26. Suppose the inverse demand function for a commodity is given by the equation: p = 50/q, where p and q denote price and quantity, respectively. For this demand function, the absolute value of the elasticity of demand:
    a. Varies across price levels
    b. Is constant across price levels and equals 50
    c. Is constant across price levels and equals 1
    d. Depends on the elasticity of the supply curve

27. Talking about inferior goods and Giffen goods,
    a. All inferior goods are Giffen goods
    b. All Giffen goods are inferior goods
    c. Both of the above
    d. None of the above

28. Thinking about the average and marginal product curves associated with a standard neo-classical production function, which of the following options is most appropriate?
    a. If average product is rising, average product is greater than marginal product
    b. If average product is falling, average product is greater than marginal product
    c. Both of the above
    d. None of the above

29. In a perfectly competitive market, a firm continues to remain in business in the short-run if and only if the following holds:
    a. Total revenue equals or exceeds total cost
    b. Average revenue equals or exceeds average total cost
    c. Average revenue equals or exceeds average fixed cost
    d. Average revenue equals or exceeds average variable cost

30. Due to unseasonal rainfall, a large part of the wheat crop got destroyed last year. The harvest was bad, and the price of wheat shot up. This scenario is best thought of as:
    a. A leftward shift in the supply curve
    b. A rightward shift in the demand curve
    c. A movement along the supply curve from bottom left towards top right
    d. A movement along the demand curve from bottom right towards top left

31. Suppose you purchased a car for Rs. 200,000 in 2020 and after two years in 2023, you sold it for Rs. 100,000 through a second-hand car selling company which charged Rs. 15000 as commission to sell the car. Which of the following will be added to the GDP of 2023?
    a. Rs. 100,000
    b. Rs. 115,000
    c. Rs. 15000
    d. Rs. 315,000

32. Suppose the personal income of the household is Rs. 900, and personal income taxes are Rs. 180, consumption is Rs. 430, personal interest payments are Rs. 10, and personal savings are Rs. 40, which of the following is the appropriate personal disposable income of this individual
    a. Rs. 800
    b. Rs. 720
    c. Rs. 710
    d. Rs. 830

33. Suppose we have collected the following market basket data to calculate CPI.

Year	Milk	Price Per litre	Cereals	Price per kgs	Footwear	Price per pair	Spices	Price Per kg
2023	5	40	5	120	5	140	5	200
2024	6	90	9	190	6	160	10	250

Using 2023 as the base year which of the following shows the appropriate inflation rate in 2024?

a. 34.6 percent
b. 38.0 percent
c. 28.4 percent
d. 39.7 percent

34. Suppose GDP at market price (GDP_{MP}) is Rs 1100 crores and Net Factor Income from Abroad (NFIA) is Rs 1000 crores. The value of depreciation is Rs. 150 crores. National income (NNP_{FC}) is Rs 850 crores. Which of the following shows the appropriate value of net indirect taxes?

a. Rs. 1100
b. Rs. 1950
c. Rs. 1250
d. Rs. 1000

35. Suppose there are three firms A, B, and C. Firms A and B produce steel worth Rs. 3000 each, and both sell steel to firm C, which uses it to produce bicycles. Firm C sells bicycles to consumers for Rs. 10,000. Which of the following shows the true value of GDP using the value-added method?

a. 16000
b. 13000
c. 4000
d. 10000

36. Photographic evidence suggests that liquid water once existed in great quantity on the surface of Mars. Two types of flow features are seen: runoff channels and outflow channels. Runoff channels are found in the southern highlands. These flow features are extensive systems—sometimes hundreds of kilometres in total length— of interconnecting, twisting channels that seem to merge into larger, wider channels. They bear a strong resemblance to river systems on Earth, and geologists think that they are dried-up beds of long-gone rivers that once carried rainfall on Mars from the mountains down into the valleys. Runoff channels on Mars speak of a time 4 billion years ago (the age of the Martian highlands), when the atmosphere was thicker, the surface warmer, and liquid water widespread.

The word "merge" in the passage is closest in meaning to

a. expand

b. separate

c. straighten out

d. combine

37. Assume that some chians are pohs, some weis are tian and all shans are weis. Then, it makes sense that:
    a. all pohs are shans
    b. some chians are shans
    c. all shans are tians
    d. some shans may also be tians

38. Pointing to a photograph, a man said, "I have no brother or sister but that man's father is my father's son." Whose photograph was it?
    a. His own
    b. His Son
    c. His Father
    d. His Grandfather

39. If CALM is written as ZDIP, YEAR may be written as

a. ZXHV

b. HVXZ

c. VHXU

d. VHWZ

40. A cube has sides of length 6 units. It is painted red on all faces, then cut into 1-unit cubes. How many 1-unit cubes have exactly one face painted?
    a. 96
    b. 64
    c. 16
    d. 48

41. In a certain language, BOOK is written as 43, MATE is written as 39. Then 16 will represent which word out of given alternatives?
    a. GOOD
    b. WOOD
    c. CRAB
    d. CAGE

42. P, Q, R, S, T, U, and V are sitting around a circular table facing the center. R is immediately to the left of U, and V is second to the left of R. P is sitting third to the left of T. Q is between S and T. Which of the following pairs is correct: the first person in the pair sitting to the immediate left of the second person in the pair?
    a. QT
    b. RP
    c. SV
    d. VS

43. If South-East becomes North, North-East becomes West and so on. What will West become?
    a. North-East
    b. North-West
    c. South-East
    d. South-West

44. A person starts from point A and travels 3 km eastwards to B and then turns left and travels thrice that distance to reach C. He again turns left and travels five times the distance he covered between A and B and reaches his destination D. The shortest distance between the starting point and the destination is
    a. 12
    b. 15
    c. 16
    d. 18

45. Obesity and uncontrolled diet_____together to_____the risk of liver disease in both men and women.
    a. result; aggravate
    b. act; increase
    c. taken; arrest
    d. put; heighten

46. A highly cohesive work group is a prerequisite for high team performance. Sociologists point out that the association between success and group cohesion owes to the support individual team members give to one another and their acceptance of the group's activities and goals.

Each of the following, if true, either supports or cannot weaken the sociologists' assumption about the relationship between success and cohesion EXCEPT

a. A group of Japanese researchers found that the successful work teams were led by dominant leaders

b. University researchers found that there was a significant correlation between team productivity and the extent to which team members understood and complied with the group's objectives

c. American researchers found that successful team members tended to rate their fellow members more favourably

d. Industrial Psychologists of UK found that work groups who tended to participate in after-hours social activities were more productive

47. A passage consists of six sentences. The first and sixth sentence are given in the beginning and the end. The middle four sentences have been removed and jumbled up. These are labelled as P, Q, R and S. Find out the proper order for the four sentences. S1: Calcutta unlike other cities kept its trams. P: As a result, there is horrendous congestion. Q: It was going to be the first in South Asia. R: They run down the center of the road. S: To ease in, the city decided to build an underground railway line. S6: The foundation stone was laid in 1972.

The proper sequence should be:

a. PRSQ

b. PSQR

c. SQRP

d. RPSQ

48. Choose the odd combination of letters from the given list.

a. PQT
b. DEH
c. UVY
d. IJN

49. Choose the one which can be substituted for the given phrase: That which cannot be corrected
    a. Unintelligible
    b. Indelible
    c. Illegible
    d. Incorrigible

50. The question consists of five statements labelled A, B, C, D and E which when logically ordered form a coherent passage. Choose the option that represents the most logical order.

    A. The US market will continue to be the dominant one in the foreseeable future. The rupee could become even stronger.

    B. A greater recourse to hedging as well as striving for multi-currency revenue streams automatically suggests itself.

    C. Already one company, TCS, by resorting to these methods extensively has turned in an above - average performance during the first quarter.

    D. Most IT companies have been grappling with more mundane problems such as a high level of attrition amidst rising wage costs and inability to secure the right type and number of American visas.

    E. The BPO industry and many medium-sized software exporters are reportedly operating on thin margins.

    a. BCADE

    b. ABCDE

    c. DCBAE

    d. EDABC

**********The End**********