The rayleigh: interpretation of the unit in terms of column emission rate or apparent radiance expressed in SI units

Doran J. Baker and Gerald J. Romick

The rayleigh, originally defined as a unit to express the total column light emission rate [10^{10} photons $\sec^{-1}$ (m^2column)$^{-1}$] can equivalently be defined as a unit for apparent photon radiance ($1/4\pi$ 10^{10} photons $\sec^{-1}$ m^{-2} sr^{-1}). The selection of the appropriate definition will depend upon the physical situation and the interests of the user. The applicability of the unit for expressing the quantitative measurement of all extended light sources, including optically thick media, is both handy and valid.

Introduction

Historically, the rayleigh was introduced by Hunten *et al.*[1] as a unit of measure for the total column light emission rate from optically thin gas species in the upper atmosphere. The use of this unit was further elucidated and suggested as a unit for surface brightness by Chamberlain[2] in his benchmark work *Physics of the Aurora and Airglow*. Since that publication the rayleigh has been almost universally adopted by experimentalists in aeronomy. With the recent centennial[3] of the birth of the Fourth Baron Rayleigh, Robert John Strutt (1875–1947), an airglow session of the Fall Meeting of the American Geophysical Union at San Francisco in December 1975 was dedicated in honor of Lord Rayleigh.[4] This paper, discussing the use of the unit named after him, is a product of the memorial session.

Baker[5] discussed the applicability of the rayleigh as a unit for the general concept of the apparent radiance of extended light sources and gave his formula for conversion of quasi-monochromatic radiance L in W cm^{-2} sr^{-1} to rayleighs, namely,

$$R = 2.103617\pi\lambda L \times 10^{13}$$
$$\cong 2\pi\lambda L \times 10^{13}. \tag{1}$$

The wavelength λ is expressed in μm, and the numerical values of Planck's constant and the speed of light in a vacuum are taken from Cohen and Taylor[6] as suggested by Van Tassel and Paulsen[7] in making the conversion back and forth between watts and photon/second:

$$1 \text{ photon/sec} \equiv 1.986475(1/\lambda) \times 10^{-19}\ \text{W}. \tag{2}$$

The present paper attempts to clarify the rayleigh unit and show that it can be defined and unambiguously applied both for column emission rate and for apparent radiance. This latter entity is called photon sterance by Nicodemus.[8] The relationships between the foregoing entities are shown and extended to volume emission rates for the case of isotropic radiators and optically thin media. In keeping with the International System of Units (SI) we illustrate and recommend the conversion of the rayleigh into its component units as expressed in the International System using the terms available in standard references on radiation terminology.[8–11]

Historical Definition of the Rayleigh

Historically, the rayleigh unit was defined as an emission rate of 1 million photons/sec from an extended column of 1-cm cross section. In SI units this is

$$\boxed{1 \text{ rayleigh} \triangleq 10^{10} \text{ photons sec}^{-1}\ (\text{m}^2\text{-col})^{-1}.} \tag{3}$$

The entity called total photon column emission rate is defined by

$$\eta \triangleq \int_{\text{col}} \phi(z)dz, \tag{4}$$

where

$$\phi(z) = \lim^{\Delta V \to 0} \Delta\varphi/\Delta V$$

is the spatially dependent volume emission rate (radiant flux density). For isotropic radiation in an optically thin medium, the column emission rate η is equivalent to the total emission rate (radiant flux) φ in that column of radiating gas:

$$\varphi \triangleq \lim_{\Delta V \to \text{col}}^{1} \Delta\phi(z)\Delta V. \tag{5}$$

The incremental radiant flux $\Delta\varphi$ can be expressed in photons/second, megaphotons/second, watts, or even

D. J. Baker is with Utah State University, Logan, Utah; G. J. Romick is with University of Alaska, Geophysical Institute, Fairbanks, Alaska 99701.

Received 8 March 1976.

lumens to suit the user; the incremental volume ΔV has usually been expressed in cubic centimeters (cubic meters in SI units). Consequently, with the total column emission rate η expressed in units of rayleigh [10^{10} photons sec^{-1} $(m^2\text{-col})^{-1}$], the volume emission rate $\phi(z)$ is in 10^{10} photons sec^{-1} m^{-3}. If the distribution $\phi(z)$ is ascertained as a function of altitude, latitude, and longitude, η can be calculated from the line integral of Eq. (4).

One is next led to the question: what is the utility of the rayleigh unit in those cases where radiative transfer complications arise, as is the case with major atmospheric emission species including O_2, O_3, and CO_2 as well as optically thick chemical releases such as Li, Na, and Ba.[2] Can the unit be used with another interpretation that is physically meaningful for these other types of sources?

Generalized Definition of Rayleigh

In order to generalize the concept of the rayleigh unit it is interesting to investigate the procedure of an airglow measurement such as the one pioneered by the Fourth Baron Rayleigh. We will calculate the apparent radiance due to a given column emission rate and vice versa.

Lord Rayleigh[12] achieved an absolute measurement of the intensity of the 5577-Å night airglow auroral line using a filter photometer. He calibrated his photometer against the light from a standard tungsten lamp of known candlepower reflected off of a magnesium oxide Lambertian screen that completely filled the field of view of the photometer. His airglow photometer was thereby calibrated in candles per square meter. Since 1 candle $\equiv$ lm/sr, the calibration was in lm m^{-2} sr^{-1}. The intensity of the oxygen green line at $\lambda5577$ was thus in units of luminance. He reported his mean observed brightness, corrected to the zenith, to be (in SI units)

$$L_{5577} = 3.2 \times 10^{-5}\ \text{lm m}^{-2}\ \text{sr}^{-1}. \tag{6}$$

Using the mechanical equivalent of light, 0.0016 W/lm at $\lambda5560$, Rayleigh's $\lambda5577$ luminance is

$$L_{5577} = 5.1 \times 10^{-8}\ \text{W m}^{-2}\ \text{sr}^{-1}. \tag{7}$$

In terms of photon flux with λ in micrometers this is

$$L_{5577} = 5.1 \times 10^{-8} \frac{\text{W}}{\text{m}^2\ \text{sr}} \frac{\text{photon/sec}}{(2/\lambda) \times 10^{-19}\ \text{W}}$$
$$= 14 \times 10^{10}\ \text{photons sec}^{-1}\ \text{m}^{-2}\ \text{sr}^{-1}. \tag{8}$$

Lord Rayleigh concluded that a Lambertian screen of L candles/m^2 emits L lm/m^2, but that the sky layer emits $4\pi L$ lm/m^2. He assumed that for the $\lambda5577$ line each volume emits isotropically as a spherically uniform source having the same candlepower in all directions with negligible absorption and reemission. Thus, he calculated a total emission amounting to

$$\eta_{5577} = 4\pi \times 3.2 \times 10^{-5} = 4.0 \times 10^{-4}\ \text{lm/m}^2, \tag{9}$$

which in photon flux amounts to

$$\eta_{5577} = 176 \times 10^{10}\ \text{photons sec}^{-1}\ \text{m}^{-2}. \tag{10}$$

In Rayleigh's development, the total volume emission from a square meter was derived from a surface brightness calibration, i.e., he simply multiplied L by 4π. Thus,

$$\eta_{5577} = 4\pi L_{5577} = 176 \times 10^{10}\ \text{photons sec}^{-1}\ (\text{m}^2\text{-col})^{-1}. \tag{11}$$

By the column emission rate definition of the unit rayleigh, Lord Rayleigh's interpreted measurement is equivalent to

$$\eta_{5577} = 176\ \text{rayleighs}. \tag{12}$$

By following the steps used by Rayleigh to obtain absolute measurements of the airglow, it can be seen that the use of the unit named in his honor might equally well be used to describe absolutely the apparent surface brightness of the observed radiation regardless of its source. This could be accomplished if the L_{5577} of Eq. (8) can itself be presented directly in rayleighs. This can be accomplished consistently with Eq. (11) if the rayleigh is defined as

$$\boxed{1\ \text{rayleigh} \triangleq (1/4\pi) \times 10^{10}\ \text{photons sec}^{-1}\ \text{m}^{-2}\ \text{sr}^{-1}.} \tag{13}$$

Thus, Lord Rayleigh's measurement gives a photon radiance or photon sterance of

$$L_{5577} = 176\ \text{rayleighs}. \tag{14}$$

The airglow intensity can be interpreted in terms of apparent surface brightness by using definition (13) or in terms of a column emission rate using definition (3). From the latter we can proceed for an isotropic source as follows.

If the $\lambda5577$ airglow layer is approximated as 10 km thick, the mean volume emission rate (radiant flux density) in the zenith is

$$\bar{\phi} \cong \frac{\eta}{h} = \frac{176\ 10^{10}\ \text{photons sec}^{-1}\ (\text{m}^2\text{-col})^{-1}}{10^4\ \text{m/col}}$$
$$= 1.8 \times 10^8\ \text{photons sec}^{-1}\ \text{m}^{-3} \tag{15}$$
$$= 1.8 \times 10^8 \frac{\text{photons}}{\text{sec}} \left(\frac{2 \times 10^{-19}\ \text{W}}{0.5577\ \text{photon/sec}}\right) \text{m}^{-3} = 6.5 \times 10^{-11}\ \text{W m}^{-3}.$$

The corresponding mean directional radiant flux density is

$$\bar{f} = \frac{\bar{\phi}}{4\pi} = \frac{1.8 \times 10^8}{4\pi} = 1.4 \times 10^7\ \text{photons sec}^{-1}\ \text{m}^{-3}\ \text{sr}^{-1}$$
$$= 5.1 \times 10^{-12}\ \text{W m}^{-3}\ \text{sr}^{-1}. \tag{16}$$

Conclusions and Summary

The foregoing example illustrates that the rayleigh unit of measure defined in the International System of Units as either 1×10^{10} photons sec^{-1} $(m^2\text{-col})^{-1}$ for use with the column emission rate or as $(1/4\pi) \times 10^{10}$ photons sec^{-1} m^{-2} sr^{-1} for use with photon radiance is self-consistent and can be used to suit the needs of the user. The latter gives a generalization of the rayleigh unit without an *a priori* commitment having to be made as to details of the nature of the radiative transport process actually taking place in the medium of the source. The various radiative entities, as used with the

Table I. Airglow Radiant Entities and Their Units of Measure

Conditions of use		Isotropic radiators—nonabsorbing media		
Entity	All aeronomic emission sources Apparent surface brightness L	Column emission rate η	Volume emission rate distribution ϕ	Directional volume emission rate distribution f
Technical name	Radiance (photon sterance)	Column radiant density (total volume emission rate)	Radiant flux density	Directional radiant flux density
Definition	$L \hat{=} \lim \dfrac{\Delta\Phi}{\Delta s \Delta\Omega}$	$\eta \hat{=} \int_{\mathrm{col}} \phi(z) dz$	$\phi \hat{=} \lim \dfrac{\Delta\Phi}{\Delta V}$	$f \hat{=} \lim \dfrac{\Delta\Phi}{\Delta\Omega}$
Quantum definition unit	$\dfrac{10^{10}}{4\pi}$ photons $\mathrm{sec}^{-1}\ \mathrm{m}^{-2}\ \mathrm{sr}^{-1}$	10^{10} photons sec^{-1} $(\mathrm{m}^2\text{-col})^{-1}$	10^{10} photons $\mathrm{sec}^{-1}\ \mathrm{m}^{-3}$	$\dfrac{10^{10}}{4\pi}$ photons $\mathrm{sec}^{-1}\ \mathrm{m}^{-3}\ \mathrm{sr}^{-1}$
Power definition unit	$\sim \dfrac{10^{-9}}{2\pi\lambda}\ \mathrm{W\ m^{-2}\ sr^{-1}}$	$\sim \dfrac{2 \times 10^{-9}}{\lambda}\ \mathrm{W}\ (\mathrm{m}^2\text{-col})^{-1}$	$\sim \dfrac{2 \times 10^{-9}}{\lambda}\ \mathrm{W\ m^{-3}}$	$\sim \dfrac{2 \times 10^{-9}}{\lambda}\ \mathrm{W\ m^{-3}\ sr^{-1}}$
Rayleigh unit	rayleigh	rayleigh	rayleigh/m	rayleigh/m
Simple zenith observation of isotropic, optically thin layer of thickness h	$L = fh$ $= \eta/4\pi$ $= \phi h/4\pi$	$\eta = \phi h$ $= 4\pi L$ $= 4\pi f h$	$\phi = \eta/h$ $= 4\pi f$ $= 4\pi L/h$	$f = L/h$ $= \phi/4\pi$ $= \eta/4\pi h$

ϕ is flux in photons/sec, W or lm.
h is layer thickness in m.
λ is wavelength in micrometers (μm).
1 photon/sec $\cong 2 \times 10^{-19}/\lambda$ W.

International System of Units, are summarized in Table I. In this table both the column emission rate η and the radiance (photon sterance) L are expressed in rayleighs. The appropriate expressions for volume emission rate (radiant flux density) ϕ and the directional radiant flux density f in the case of isotropic radiators and optically thin, nonabsorbing media are likewise expressed in terms of this unit.

The authors acknowledge the helpful comments and suggestions of Kay Baker, John Howard, Charles Deehr, and Kjell Henriksen. Special recognition is given to the Fourth Baron Rayleigh (Robert John Strutt) of Terling Place, Essex, England, who pioneered in the measurements of the light from the sky and to whose memory this paper is dedicated.

Partial support for this research was through grants ATM 75-02185 A01 and GA-41923 to the University of Alaska and GA-38794 to the Utah State University, from the National Science Foundation, Atmospheric Sciences Section.

References

1. D. M. Hunten, F. E. Roach, and J. W. Chamberlain, J. Atmos. Terr. Phys. 8, 345 (1956).
2. J. W. Chamberlain, *Physics of the Aurora and Airglow* (Academic, New York, 1961).
3. J. N. Howard, Appl. Opt. 14, 2827 (1975).
4. EOS, Am. Geophys. Union 57, 1031 (1975).
5. D. J. Baker, Appl. Opt. 13, 2160 (1974).
6. E. R. Cohen and B. N. Taylor, J. Phys. Chem. Ref. Data 2, 715 (1973).
7. R. A. Van Tassel and D. E. Paulsen, Appl. Opt. 14, 559 (1975).
8. F. E. Nicodemus, NBS, private communication (1975).
9. F. J. Gardner, *RCA Electro-Optics Handbook EOH-10,* RCA Corp. of America, Commercial Engineering, Harrison, N.J. 07029 (1968).
10. J. R. Meyer-Arendt, Appl. Opt. 7, 2081 (1968).
11. D. G. Fink and J. M. Carroll, Eds., *Standard Handbook for Electrical Engineers* (McGraw-Hill, New York, 1969).
12. Lord Rayleigh, Proc. R. Soc. London, Ser. A: 129, 458 (1930).