SAR POLARIMETRY

Basics Concepts, Advanced Concepts and Applications

→ 4th ADVANCED COURSE ON RADAR POLARIMETRY

30 January – 2 February 2017 | ESA-ESRIN | Frascati (Rome), Italy

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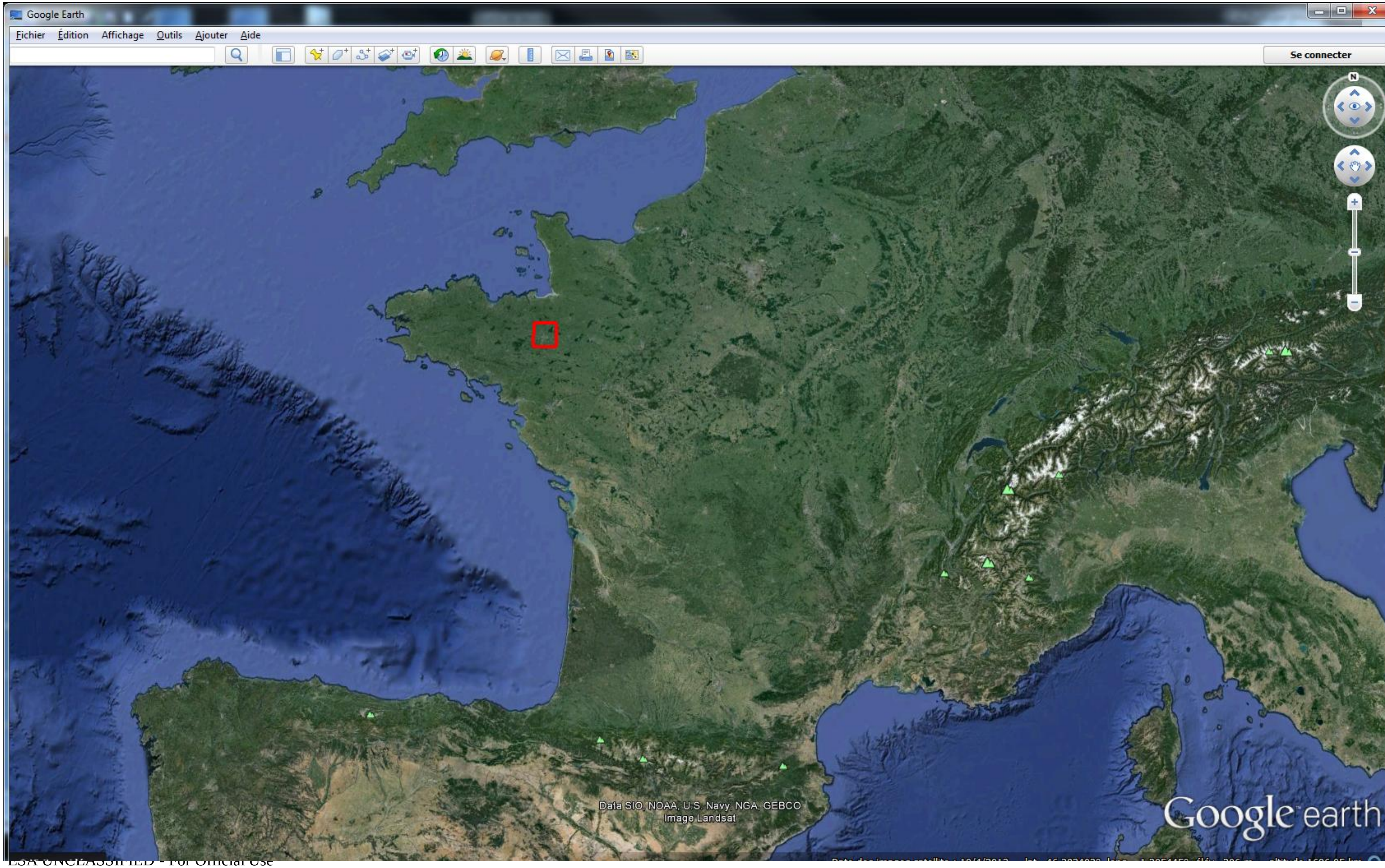
INSTITUT D'ÉLECTRONIQUE ET DE TÉLÉCOMMUNICATIONS DE RENNES

SHINE

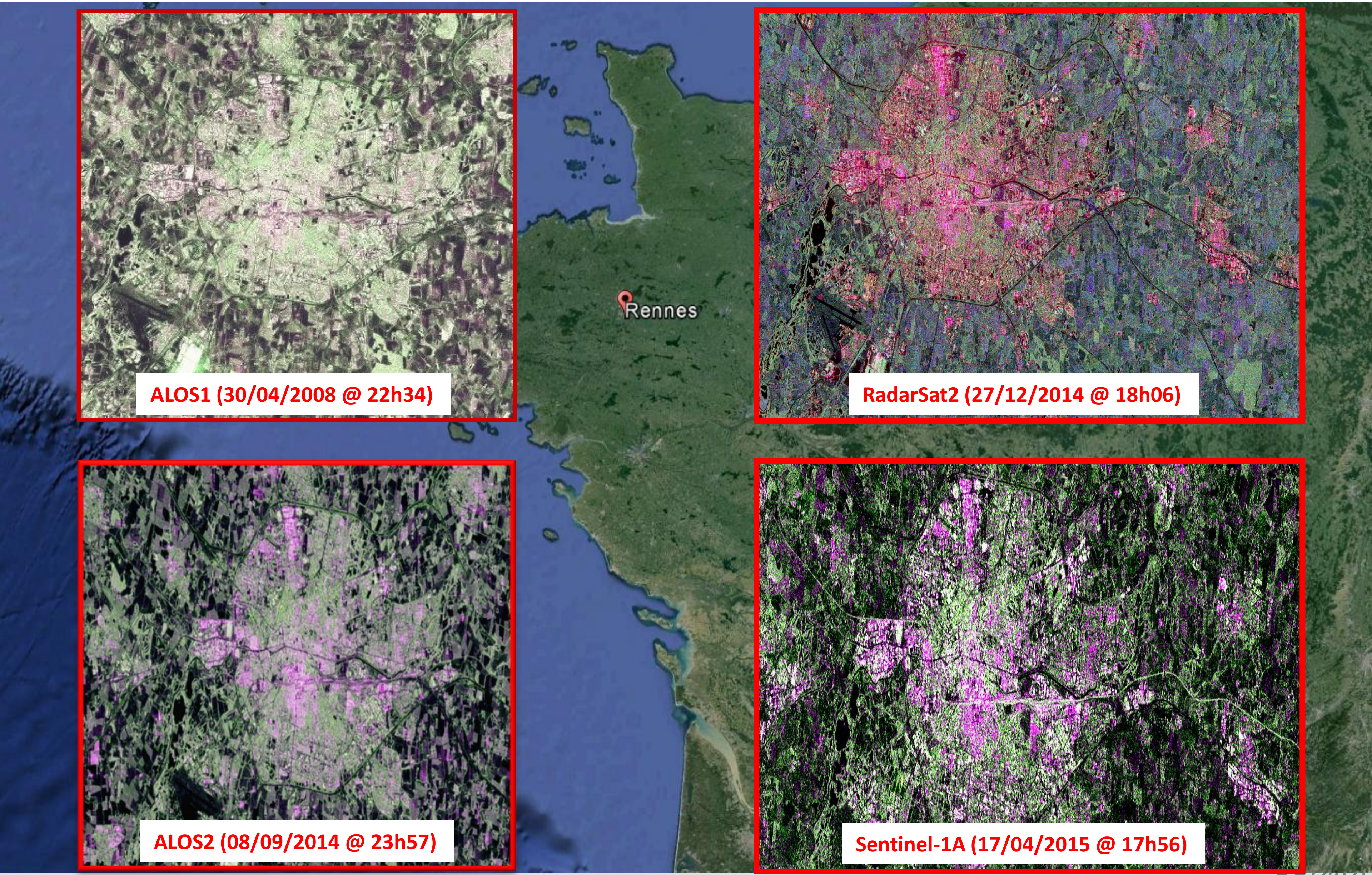
SAR & Hyperspectral multi-modal Imaging and sigNal processing, Electromagnetic modeling

I.E.T.R. - UMR CNRS 6164
Université de Rennes I - Campus de Beaulieu
Pôle Micro Ondes Radar - Bat 11D
263 Avenue Général Leclerc
CS 74205 - 35042 Rennes Cedex – France

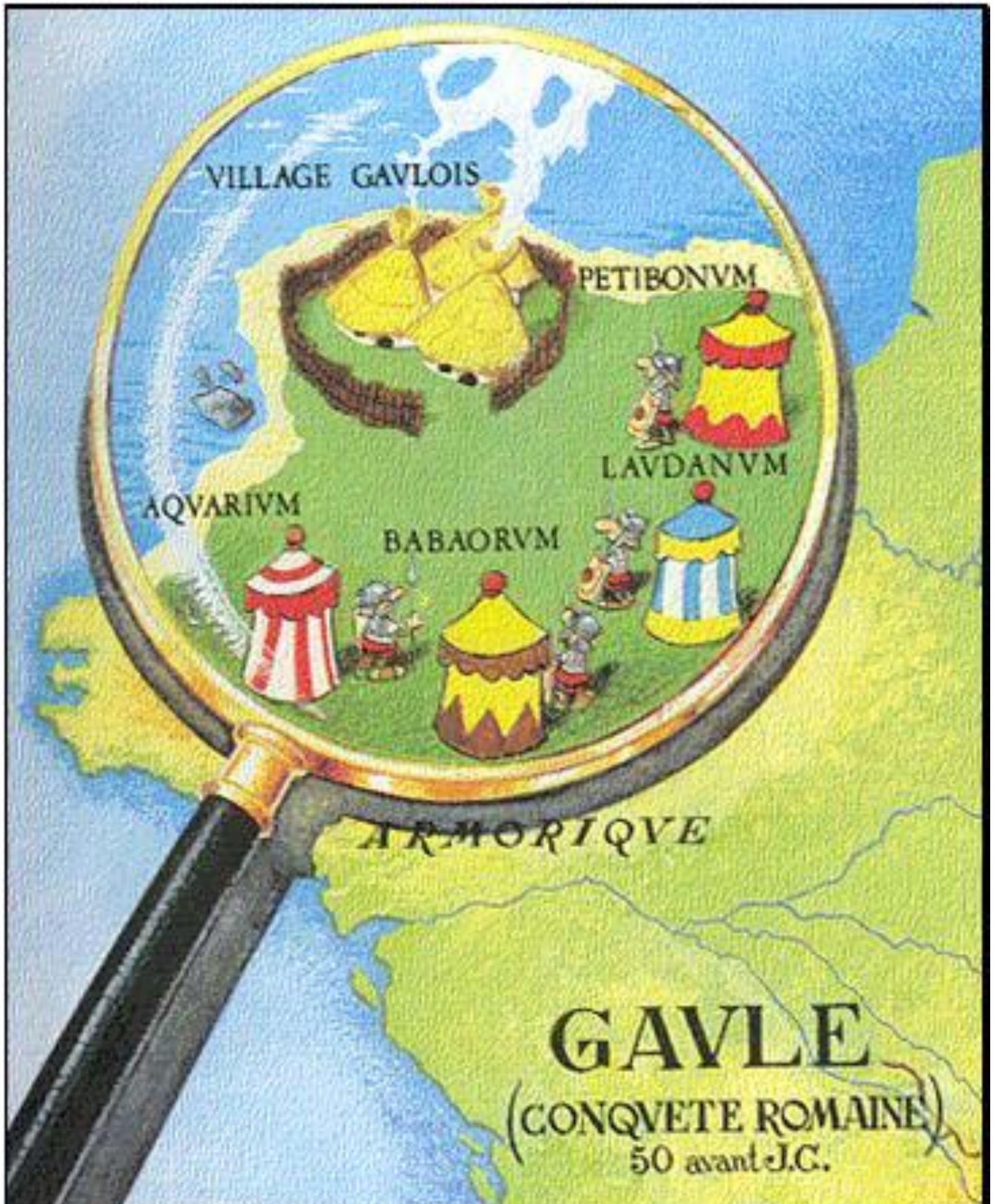
RENNES - BRITANNY - FRANCE

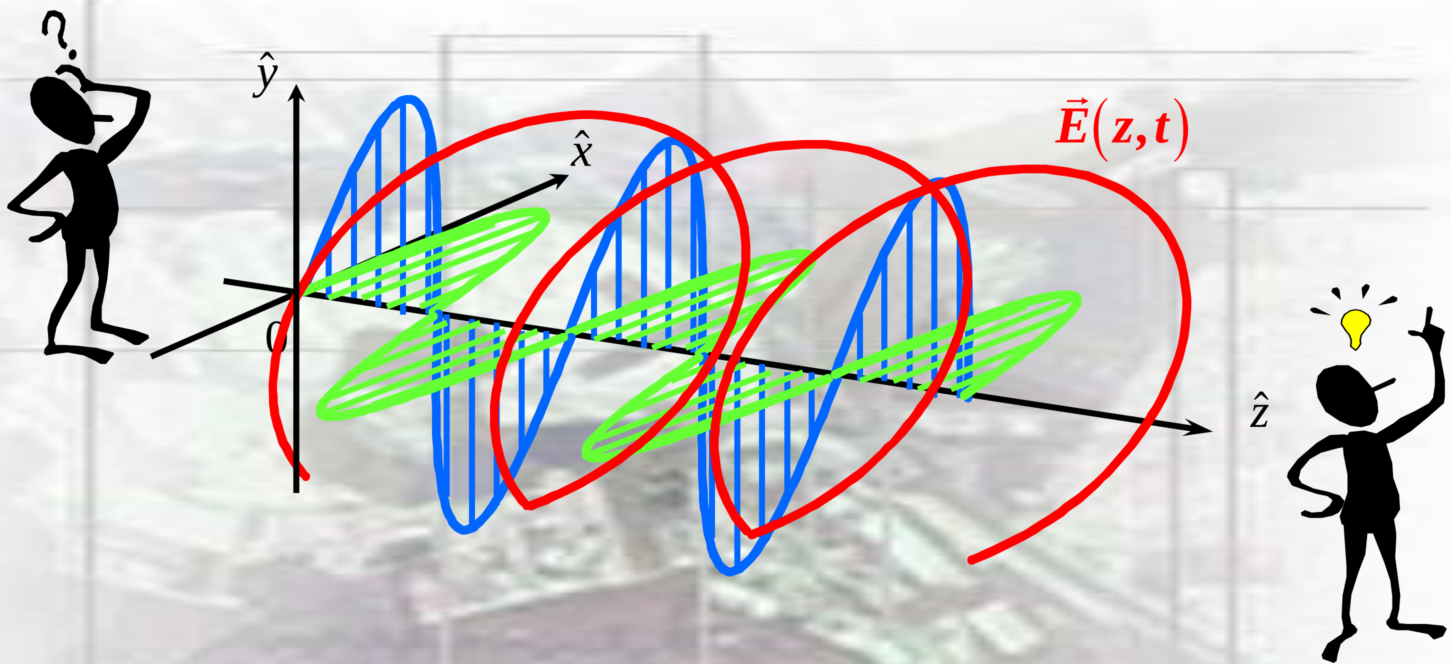


RENNES - BRITANNY - FRANCE



RENNES - BRITANNY - FRANCE





COVERED TOPICS

Objective

To provide

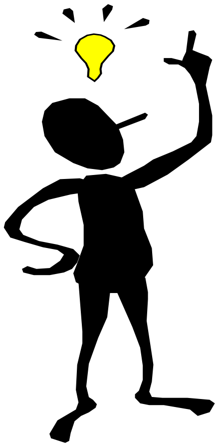
the minimum, but necessary,

amount of knowledge required

to understand

scientific works on

Radar Polarimetry



POLARIMETRIC PROCESSING CHAIN

Acquisition
Calibration
Pre-processing

→ **Image synthesis**
Co-registration
(coarse & fine)

↓

Polarimetric
Processing

↓

Post-processing

Application driven

Data inversion

→ **PolSAR**
Pol-InSAR
Pol-TomSAR
Pol-TimeSAR

Forest Vegetation

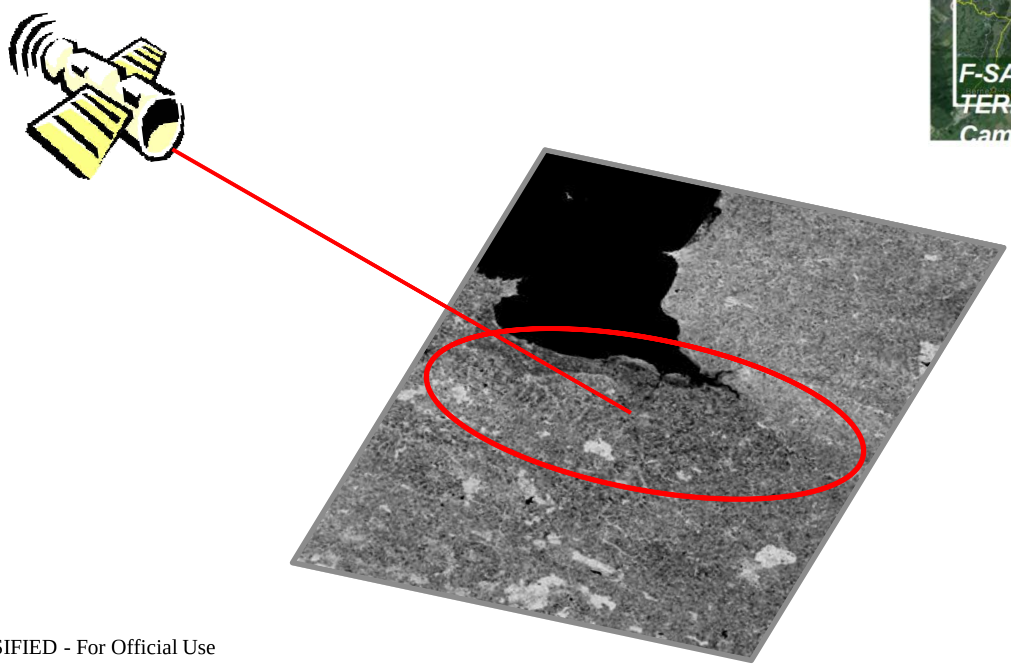
Agriculture

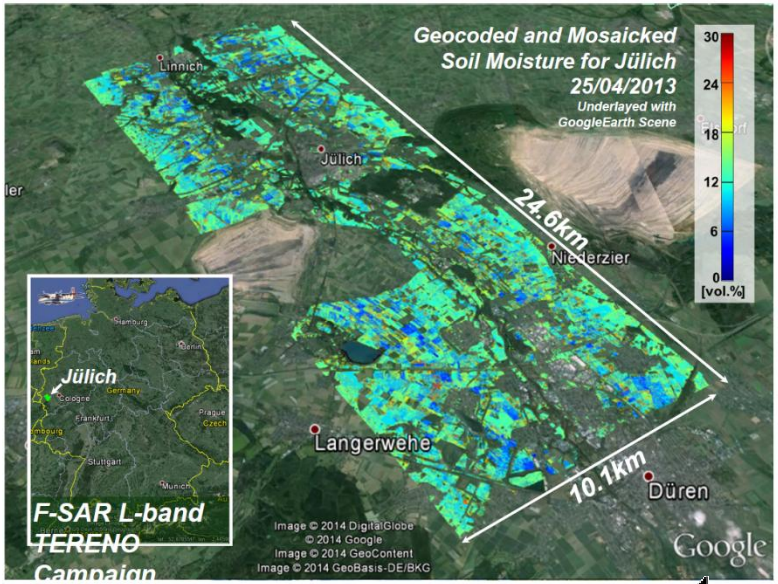
Cryosphere

Ocean

PolSAR

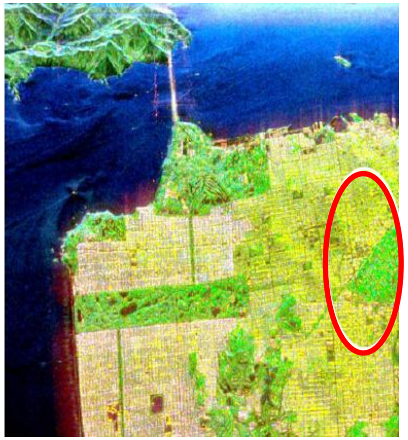
Track$_1$





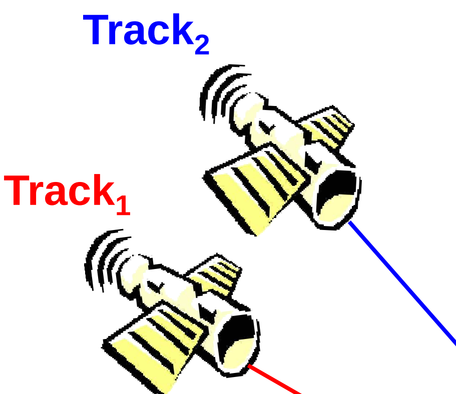
Soil moisture

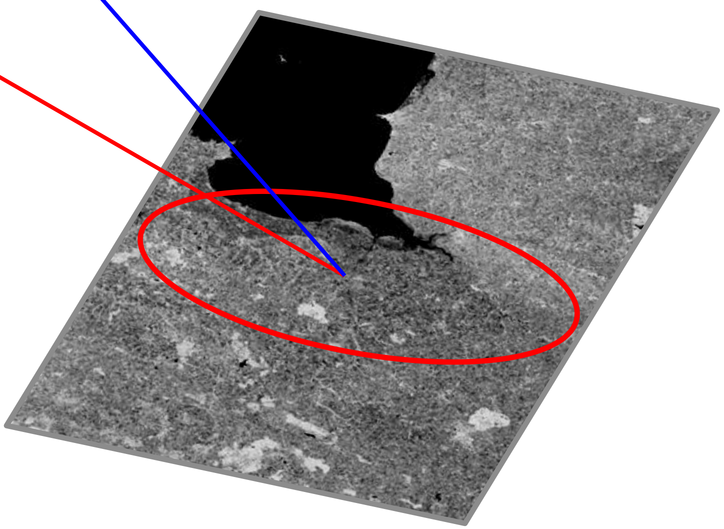
DLR

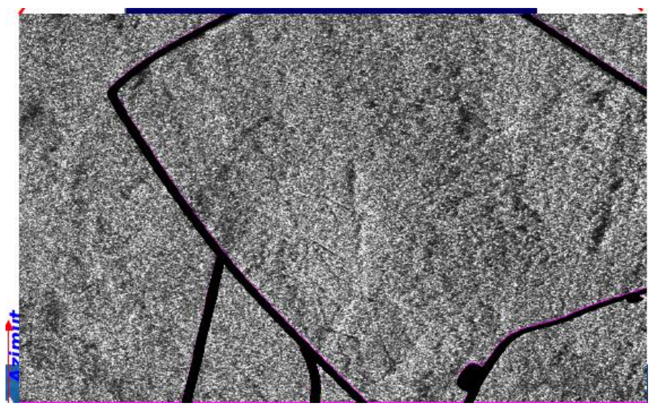


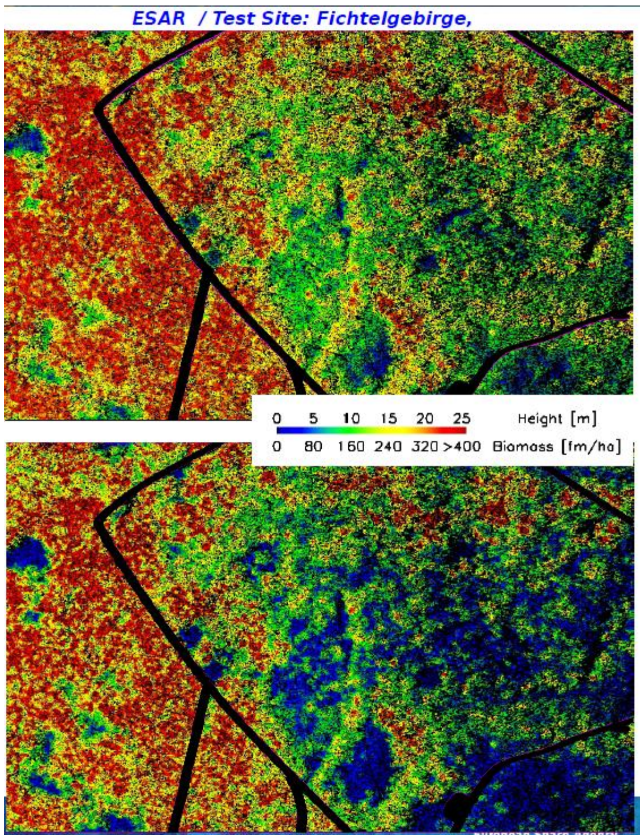
Urban monitoring

Pol-InSAR


Track$_2$
Track$_1$

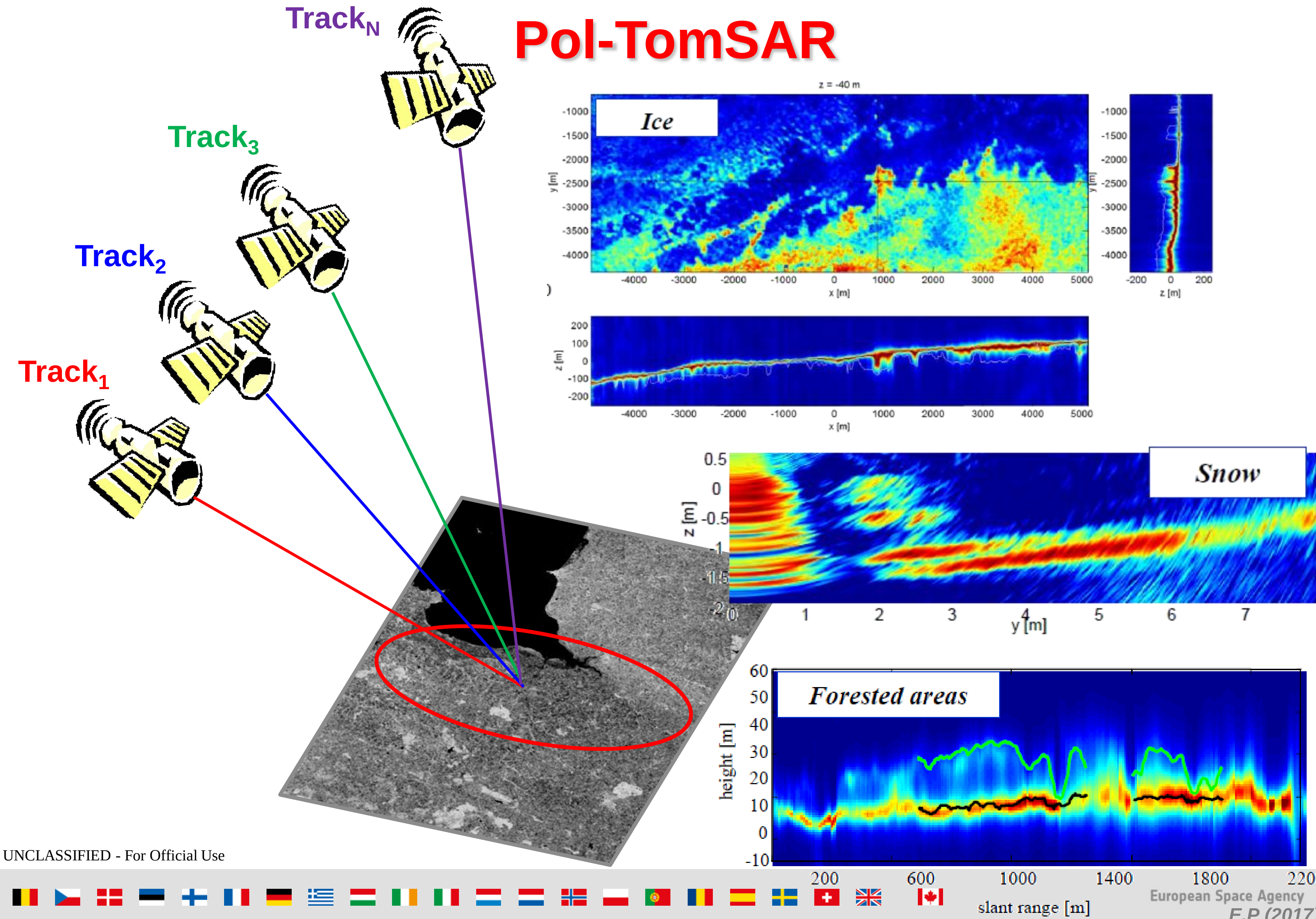





ESAR / Test Site: Fichtelgebirge,
0 5 10 15 20 25 Height [m]
0 80 160 240 320 >400 Biomass [fm/ha]

POLARIMETRY + TOMOGRAPHY

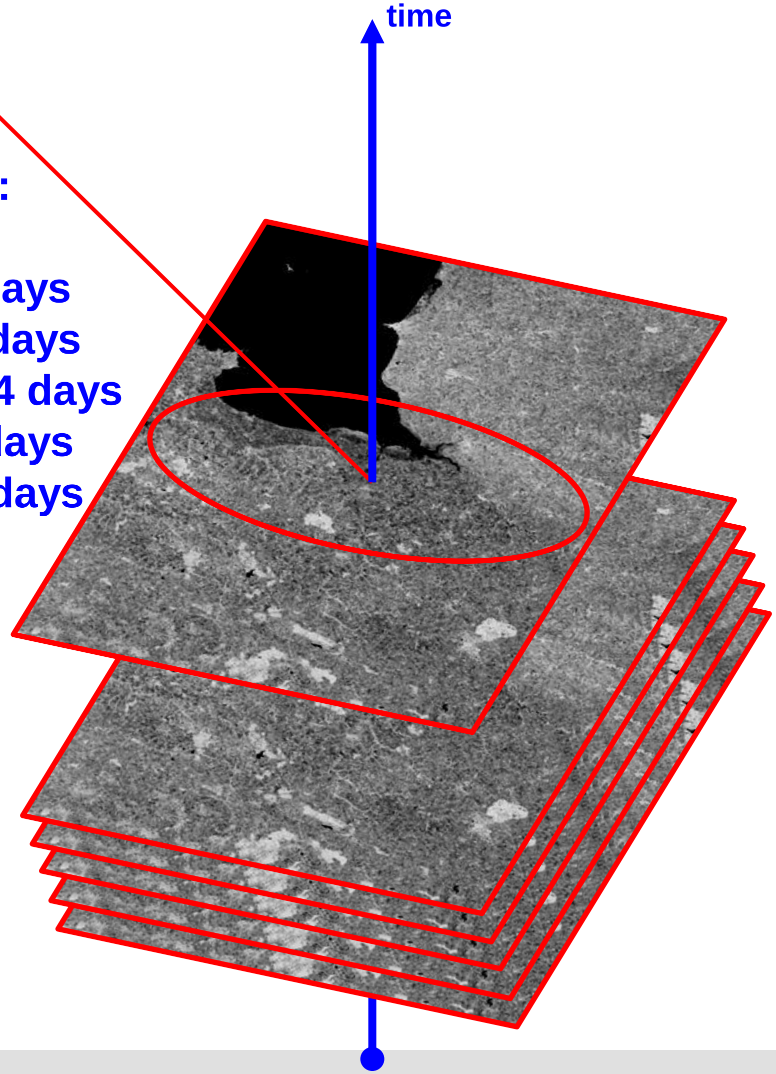
Pol-TomSAR



POLARIMETRY + TIME-SERIES

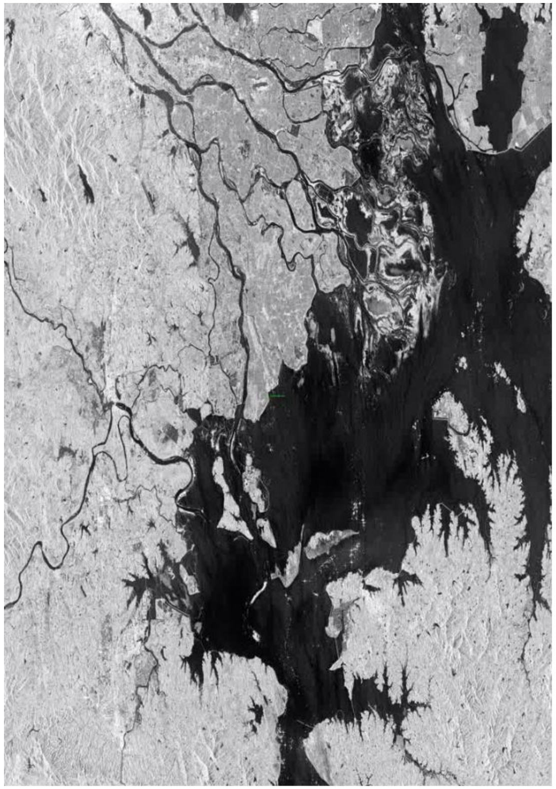
Track$_{1..N}$

Pol-TimeSAR



Revisit time :

- **ALOS-2 = 14 days**
- **BIOMASS = 4 days**
- **RADARSAT2 = 24 days**
- **RISAT-1 = 25 days**
- **Sentinel-1 = 6 days**

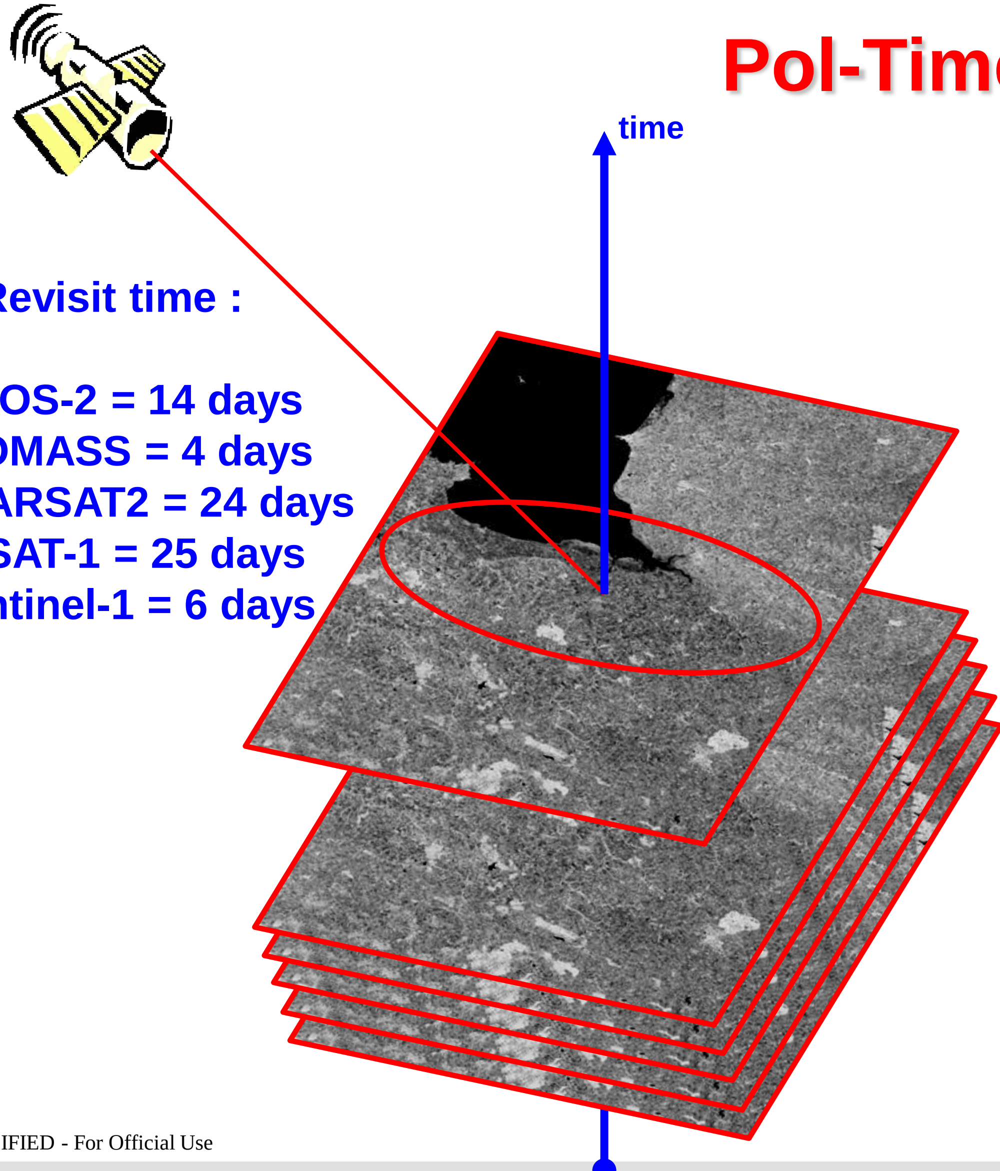


Polarimetric feature temporal evolution

POLARIMETRY + TIME-SERIES

Track$_{1..N}$

Pol-TimeSAR



Revisit time :

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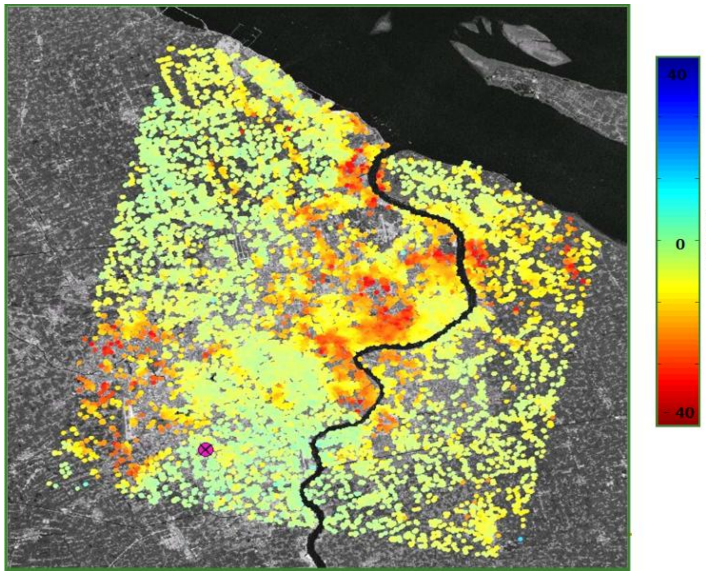


Subsidence Monitoring

Permanent scatterers
Coherent scatterers

Polarimetry = scatterer type

Basic Concepts in PolSAR Analysis

Wave Polarimetry

- Wave Propagation
- Wave Polarisation
- Jones Vector
  - Polarisation Ratio
  - Complex Polarisation Plane
  - Orthogonal Jones Vector
  - Elliptical Basis Transformation
- Stokes Vector
  - Poincaré Sphere
  - Elliptical Basis Transformation
- Partially Polarised Waves
- Wave Polarisation Dimension

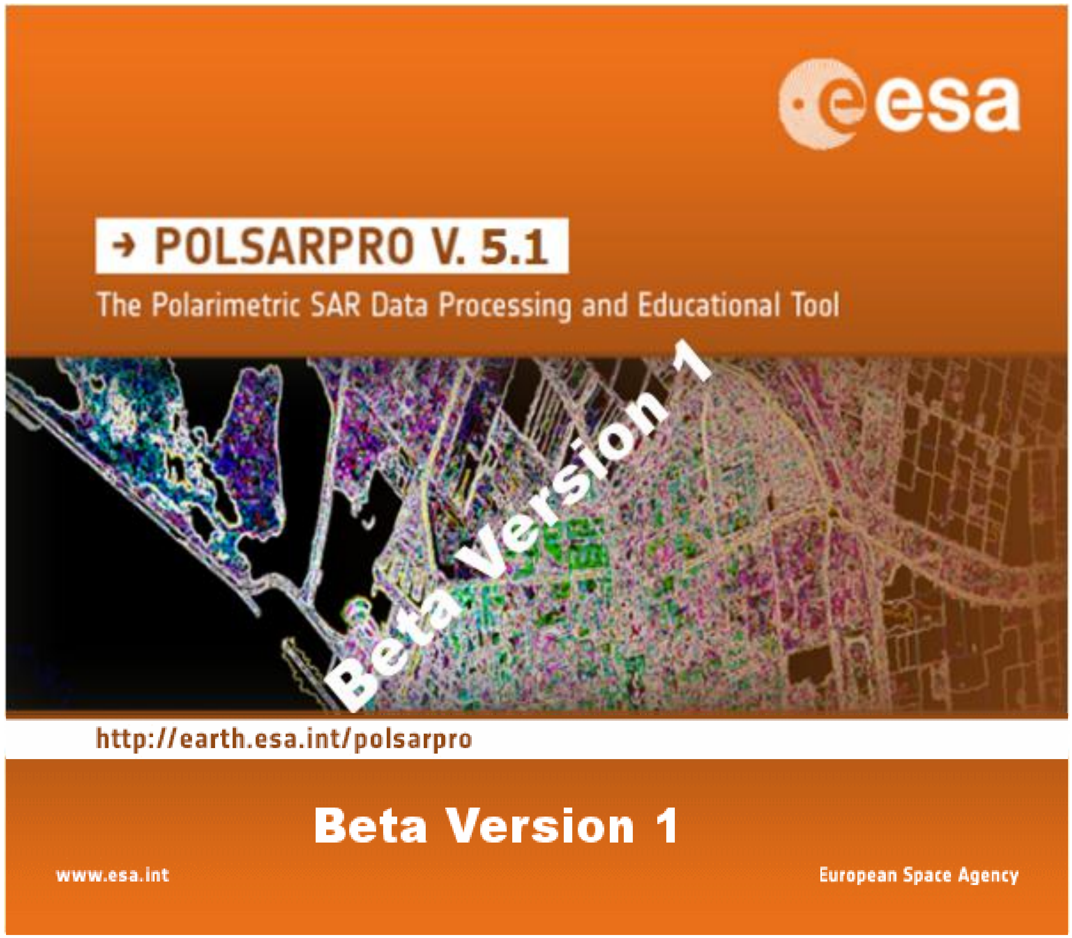
Scattering Polarimetry

- Scattering Problem
- Polarimetric Descriptors
  - Scattering / Sinclair Matrix
  - Target Vectors
  - Partially Scattering Polarimetry
  - Mueller / Kennaugh Matrix
  - Huynen Parameters
  - Coherency Matrix
  - Covariance Matrix
- Elliptical Basis Transformations
- Synthesis / Equivalence
- Polarimetric Target Dimension
  - MonostaticTarget Equations
  - Monostatic Target Diagram

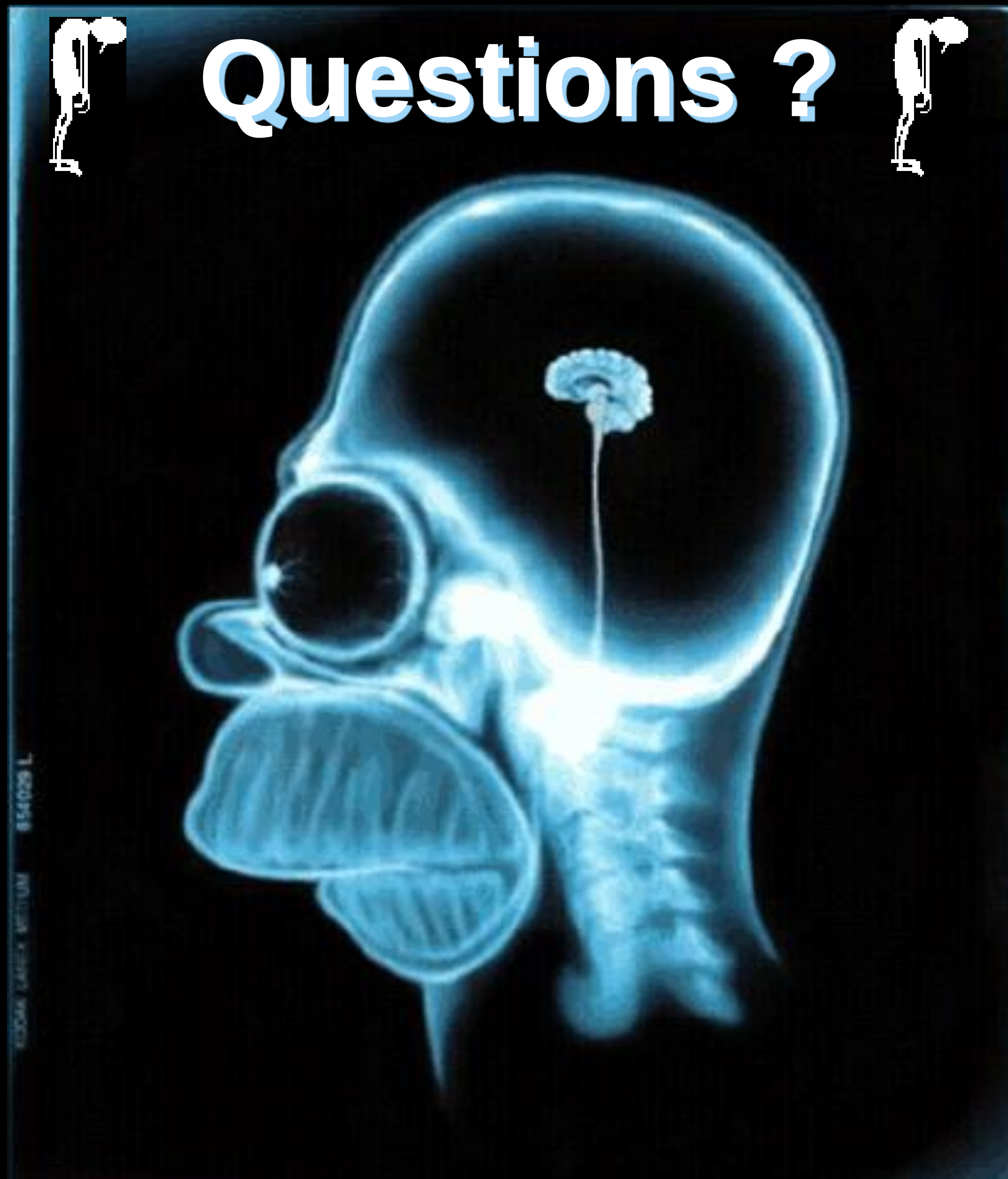
Advanced Concepts in PolSAR Analysis

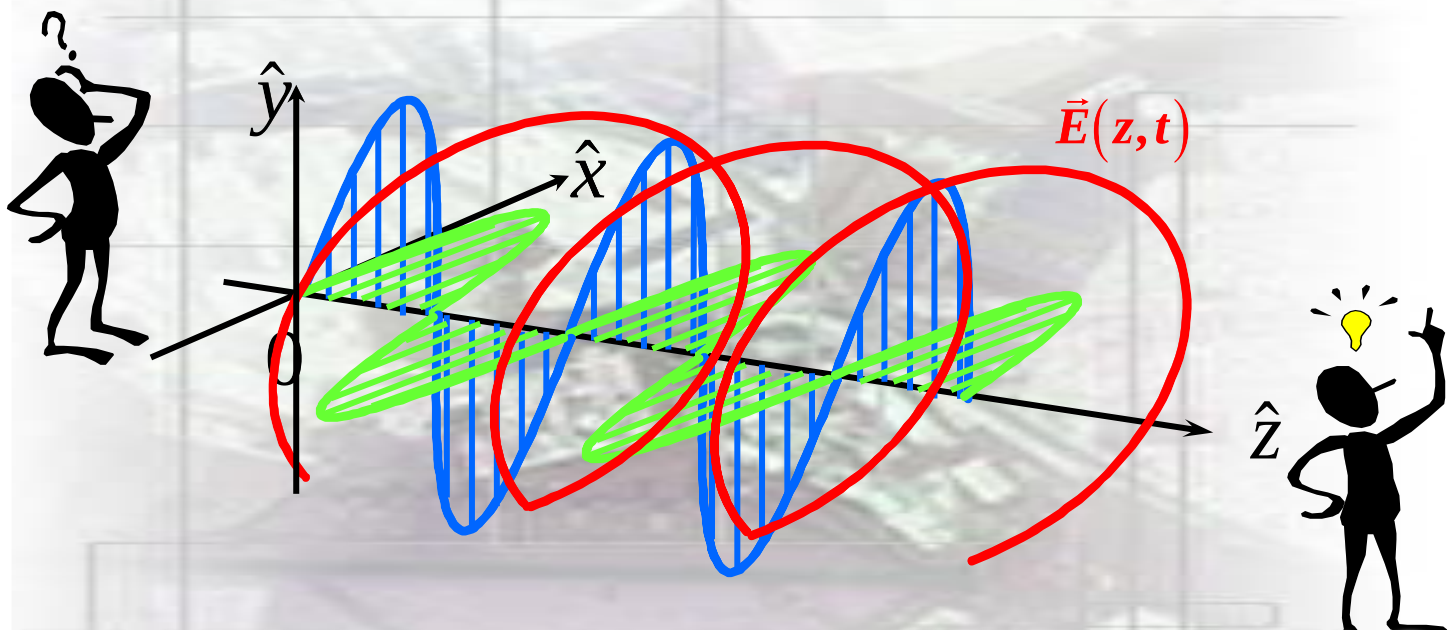
- Polarimetric Speckle Filtering
- Target Decomposition Theorems
  - Krogager Decomposition
  - Huynen / Barnes Decompositions
  - Cloude / Holm Decompositions
  - Freeman / Yamaguchi Decompositions
  - Van Zyl / Arii Decompositions
  - H / A / $\underline{\alpha}$ Decomposition
  - eigenvalues based parameters
  - TSVM Decomposition
- PolSAR Image Segmentation
  - H / $\underline{\alpha}$ Unsupervised Classification
  - Wishart Classifier
  - Wishart - H / $\underline{\alpha}$ Classification
  - Wishart - H / A / $\underline{\alpha}$ Classification
  - Wishart – Freeman Classification

Practicals


esa
POLSARPRO V. 5.1
The Polarimetric SAR Data Processing and Educational Tool
Beta Version 1
http://earth.esa.int/polsarpro
Beta Version 1
www.esa.int
European Space Agency

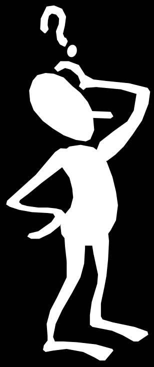
Questions ?





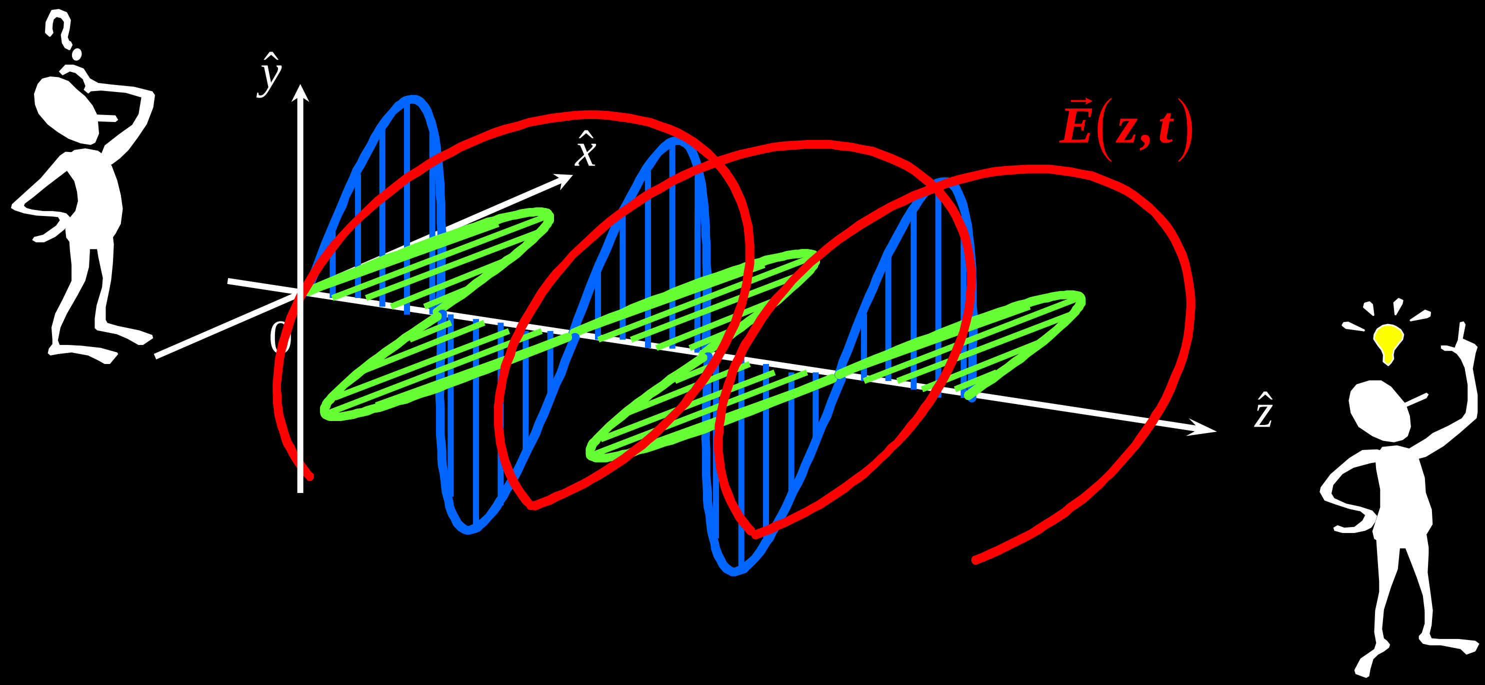
GENERAL INTRODUCTION

RADAR POLARIMETRY



- A bit of History
- Airborne and Space-borne Polarimetric SAR Sensors
- Software / Toolbox
- Learning / Training / Results

Radar Polarimetry



Radar Polarimetry (Polar : polarisation Metry: measure) is the science of acquiring, processing and analysing the polarization state of an electromagnetic field

Radar Polarimetry deals with the full vector nature of polarized electromagnetic waves

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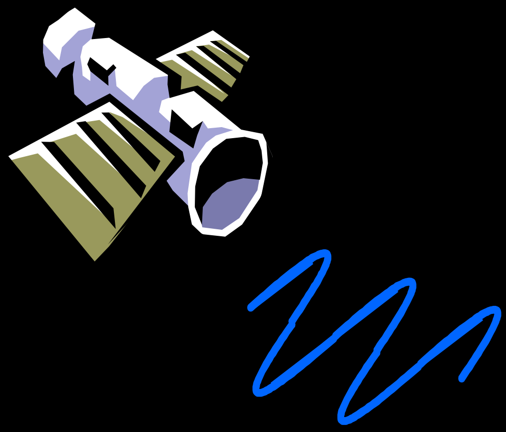
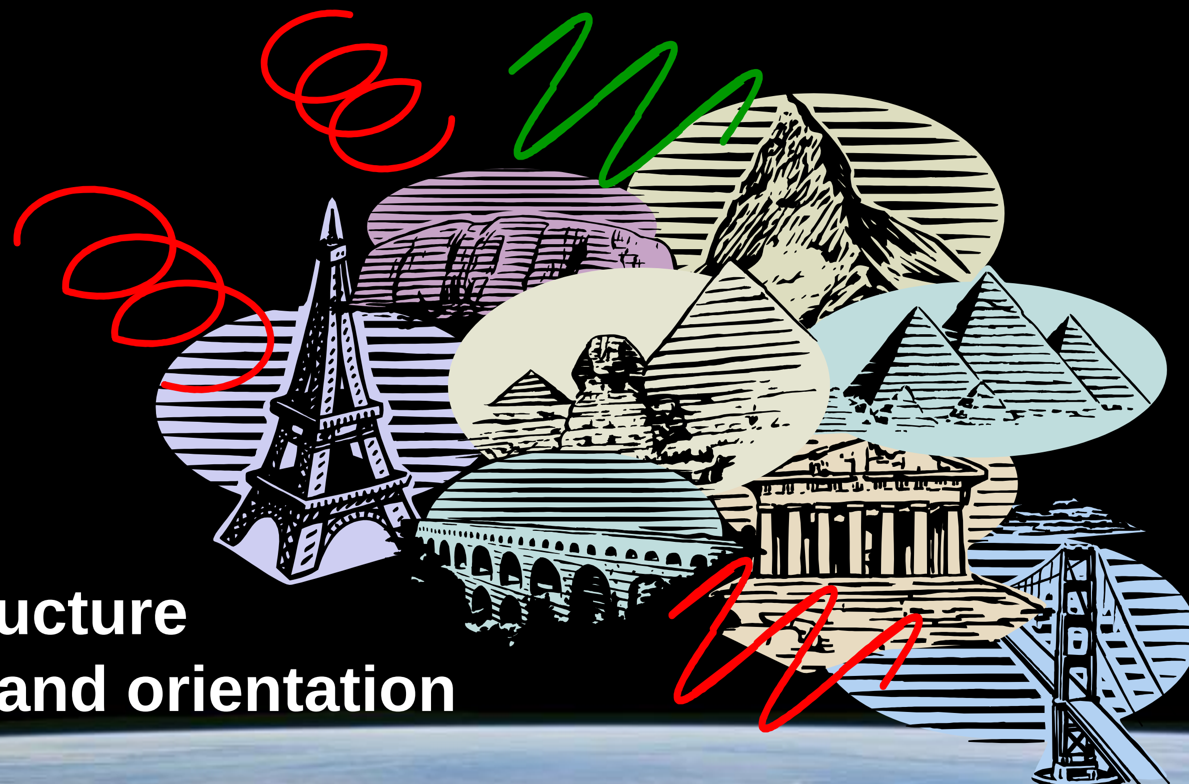
Radar Polarimetry



The POLARISATION information
Contained in the waves backscattered
from a given medium is highly related to:



its geometrical structure
reflectivity, shape and orientation

its geophysical properties such as humidity, roughness, …

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SAR Polarimetry Applications

Forest Vegetation

- Forest Height
- Forest Biomass
- Forest Structure
- Canopy Extinction
- Underlying Topography

- Forest Ecology
- Forest Management
- Ecosystem Change
- Carbon Cycle

Agriculture

- Soil Moisture Content
- Soil roughness
- Height of Vegetation Layer
- Extinction of Vegetation Layer
- Moisture of Vegetation Layer

- Farming Management
- Water Cycle
- Desretification

Snow and Ice

- Topography
- Penetration Depth / Density
- Snow Ice Layer
- Snow Ice Extinction
- Water Equivalent

- Ecosystem Change
- Water Cycle
- Water Management

Urban Areas

- Geometric Properties
- Dielectric Properties

- Urban Monitoring

DLR Courtesy of Dr. I. Hajnsek

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A Bit Of History

Radar Polarimetry

Discovery of the Phenomena of Polarized Electromagnetic Energy

AD 1000
Use of the polarized skylight to locate a hidden sun

Crystal of calcite
Iceland Spar
Sunstone

1669
First known Quantitative work on light observation

Bartholinus

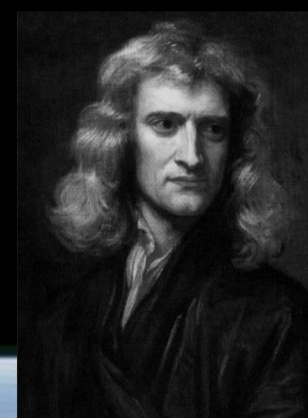
Discovery of the double refraction in calcite

1677
Wave nature of light discovery
Explanation of the double refraction

Huygens

Corpuscular model or « longitudinal » waves

1704
Corpuscular Model of light



Newton

1808
Discovery of the polarization of light (intrinsic property of light and not of crystals)

Malus
X-1795

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Non Exhaustive Chronological List of the Main Pionners who contributed to the discovery of **Polarization** leading to Radar Polarimetry

Brewster

1816

Fresnel

1820

Faraday

1832

Stokes

1852

Maxwell

1873

Helmholtz

1881

Rayleigh

1881

Kirchhoff

1883

« Transverse » nature of light waves

Electromagnetic theory of light

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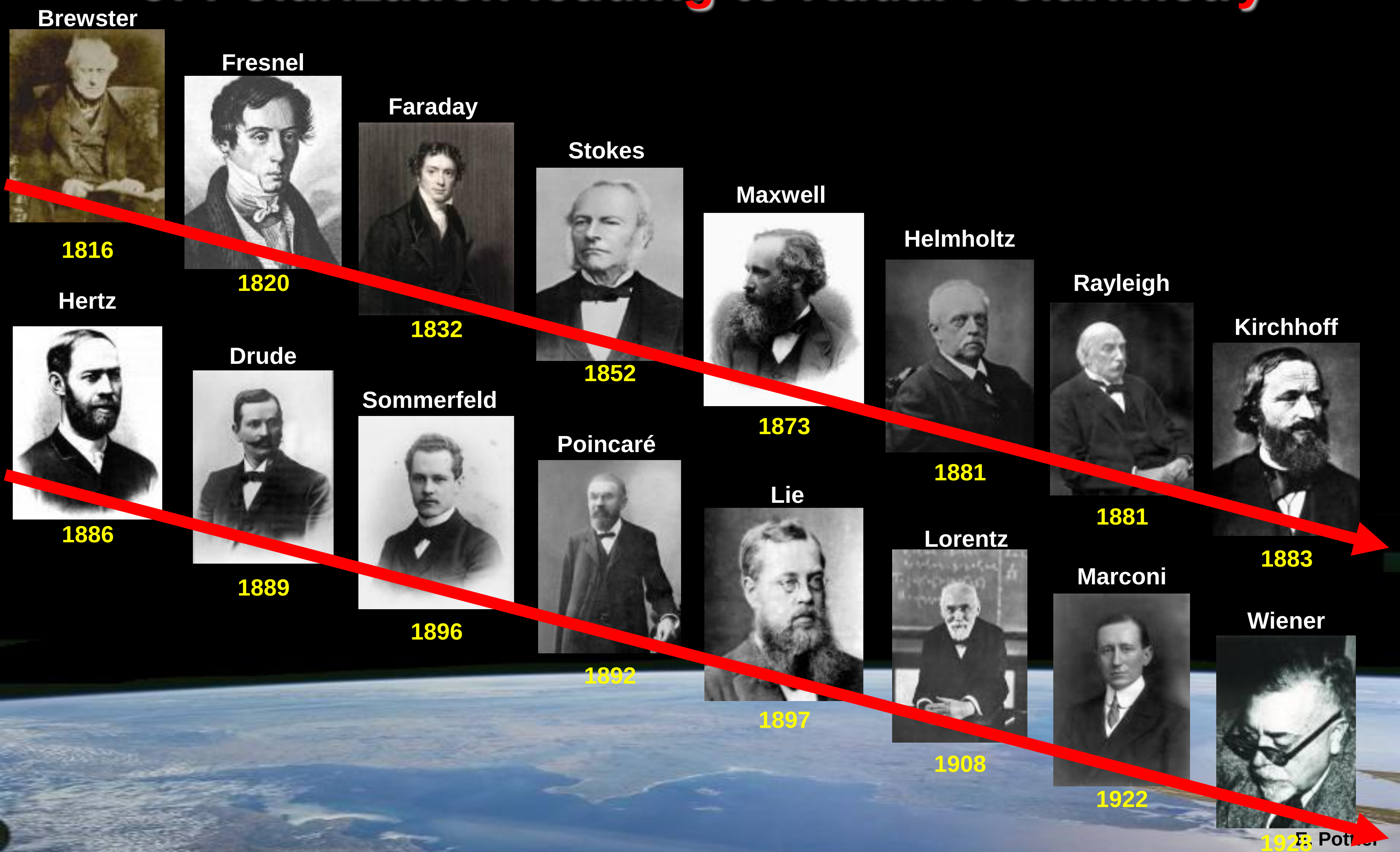
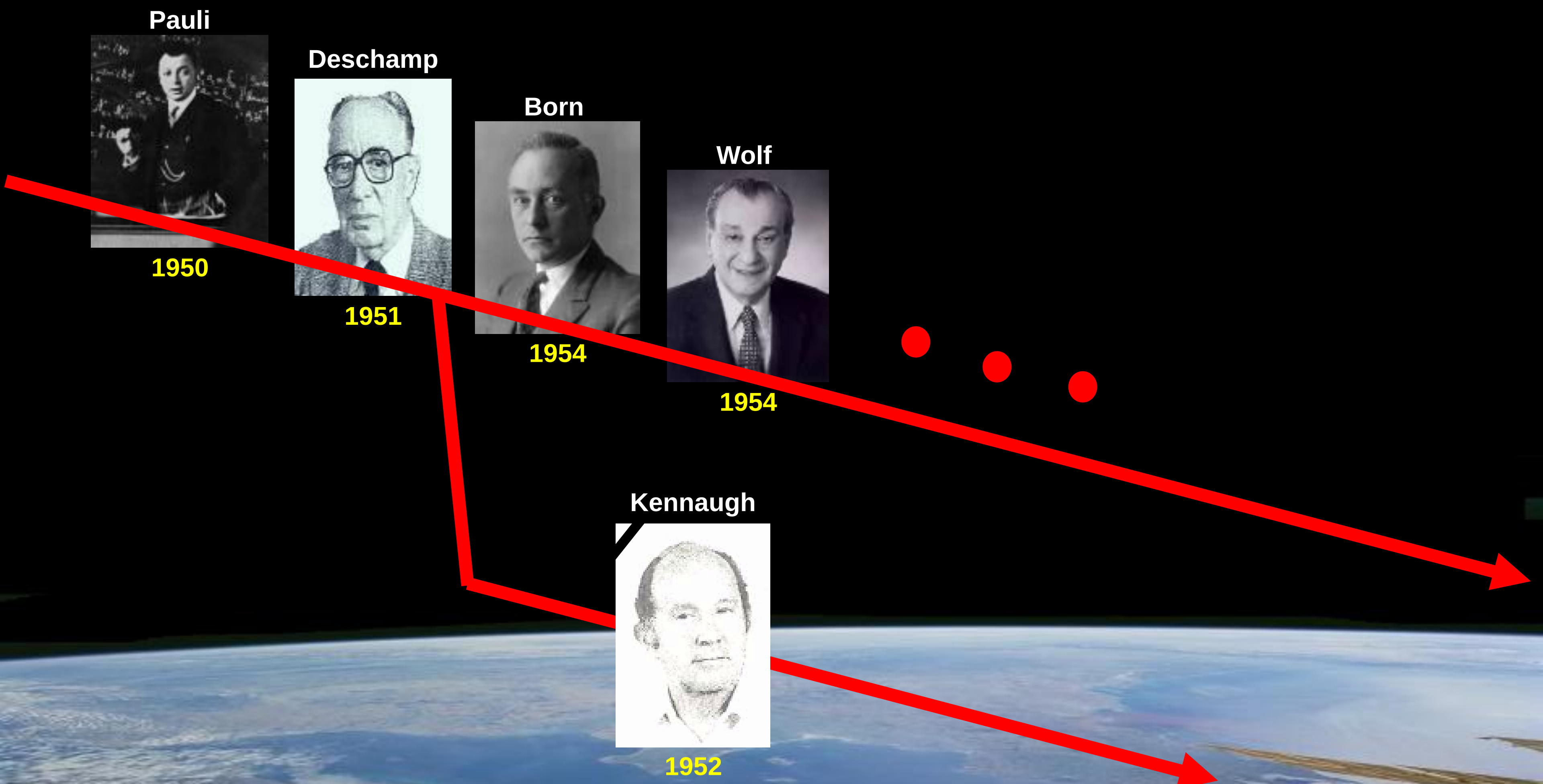
Non Exhaustive Chronological List of the Main Pionners who contributed to the discovery of **Polarization** leading to Radar Polarimetry



Non Exhaustive Chronological List of the Main Pionners who contributed to the discovery of Polarization leading to Radar Polarimetry



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Non Exhaustive Chronological List of the Main Pionners who contributed to the discovery of Polarization leading to Radar Polarimetry

Kennaugh

1952

Huynen

1970

W. M. Boerner

1980

The

Radar Polarimetric

Triptych

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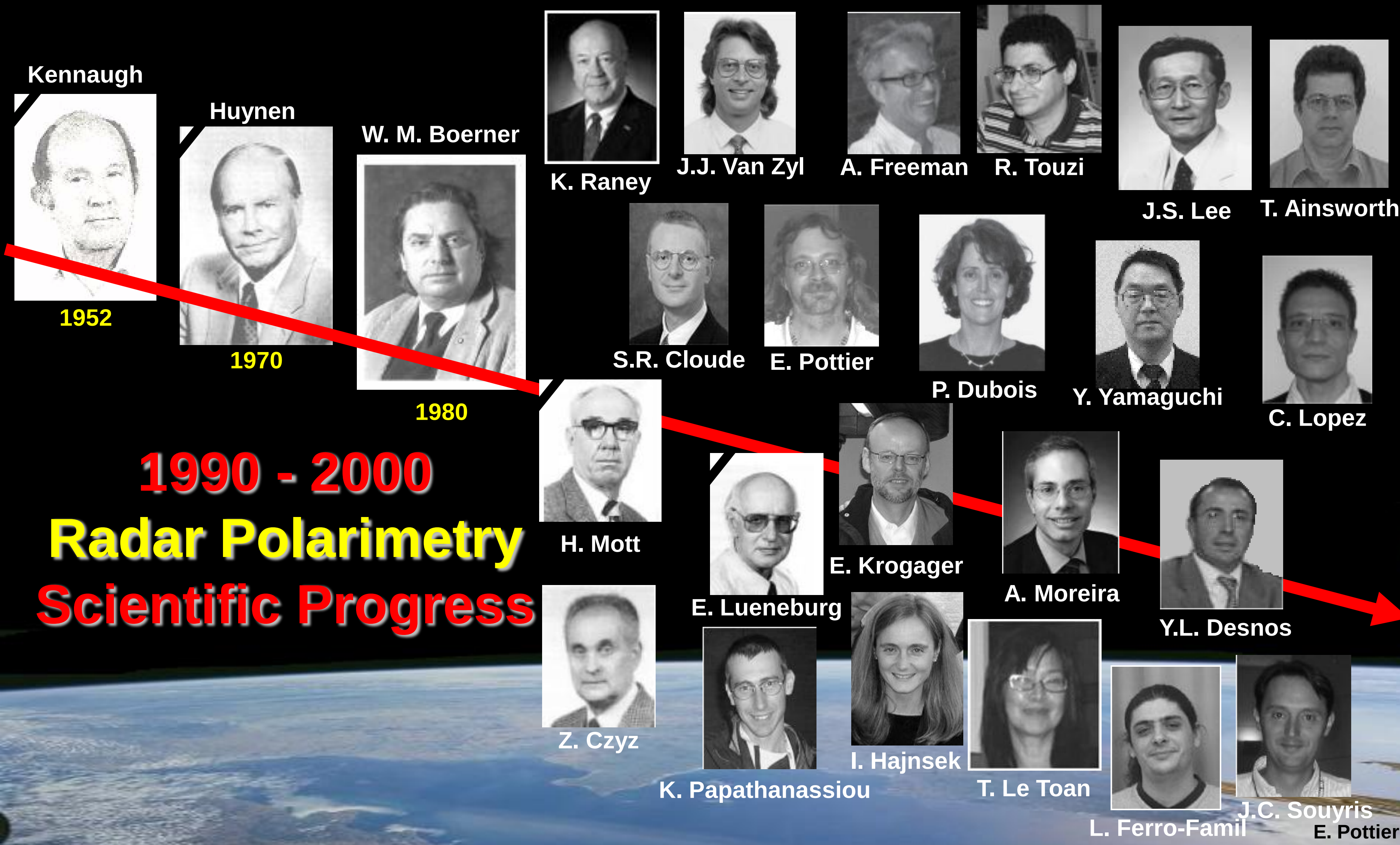
Non Exhaustive Chronological List of the Main Pionners who contributed to the discovery of Polarization leading to Radar Polarimetry



Polarimetric Radar (SAR)

Spaceborne Sensors

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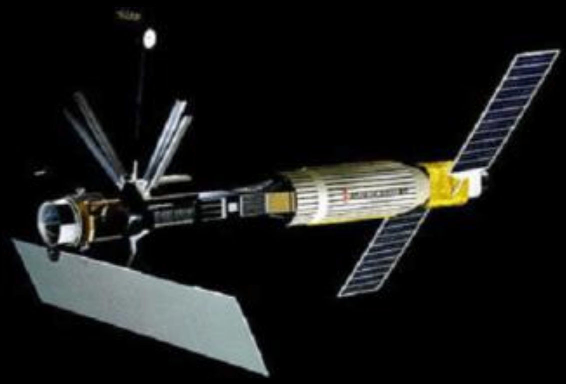
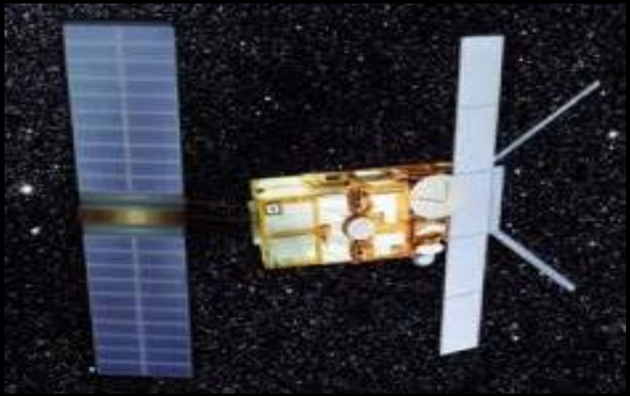
Space-borne Sensors



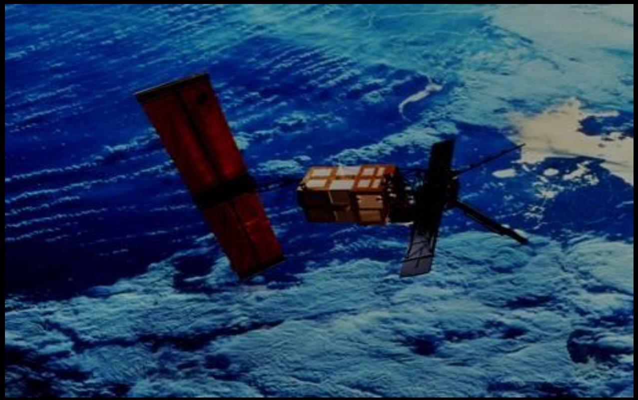
SEASAT
NASA/JPL (USA)
L-Band, 1978



ERS-1
European Space Agency (ESA)
C-Band, 1991-2000

J-ERS-1
Japanese Space Agency (NASDA)
L-Band, 1992-1998

RadarSAT-1
Canadian Space Agency (CSA)
C-Band, 1995



ERS-2
European Space Agency (ESA)
C-Band, 1995

Shuttle Radar Topography Mission
NASA/JPL (C-Band), DLR (X-Band)
February 2000

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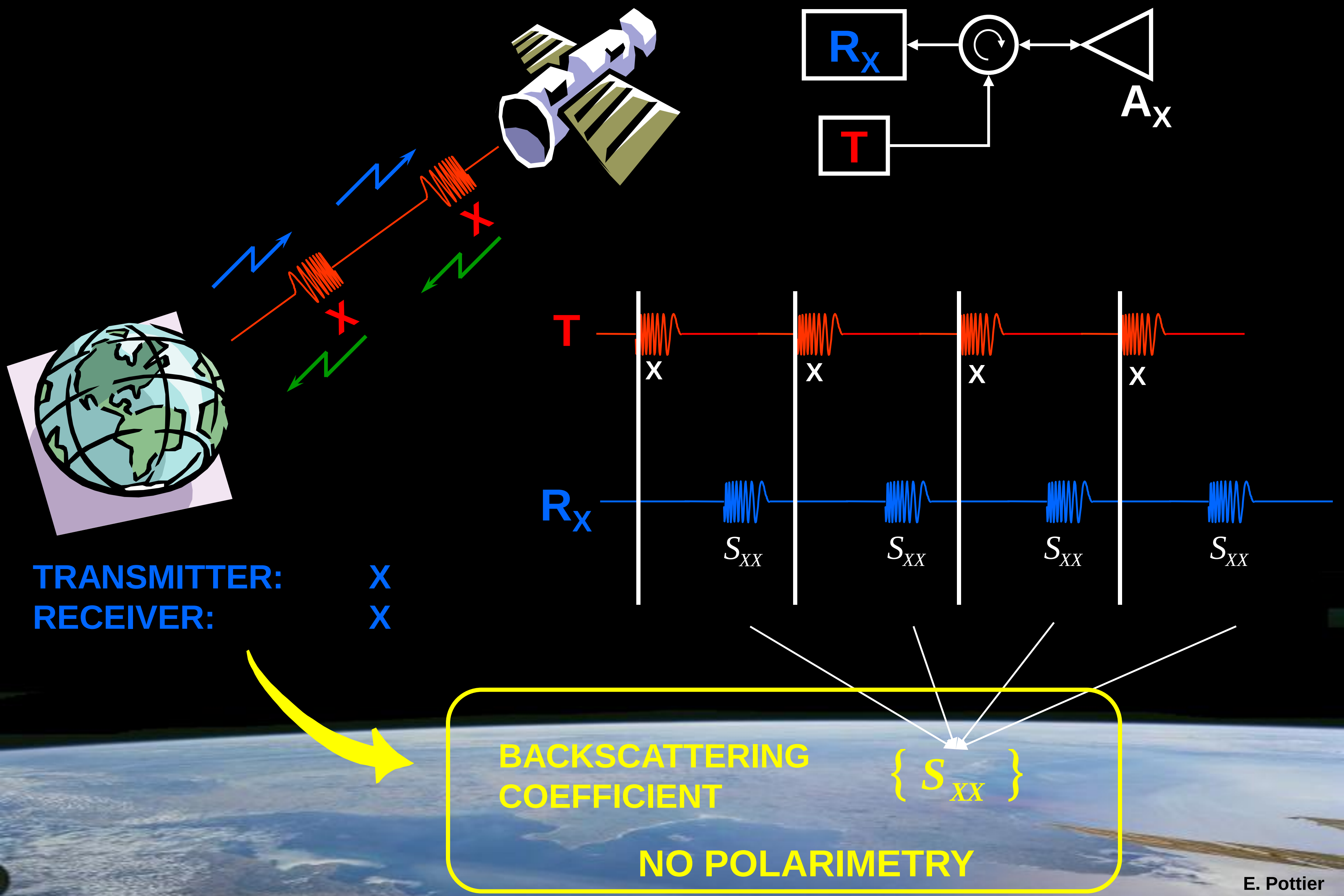
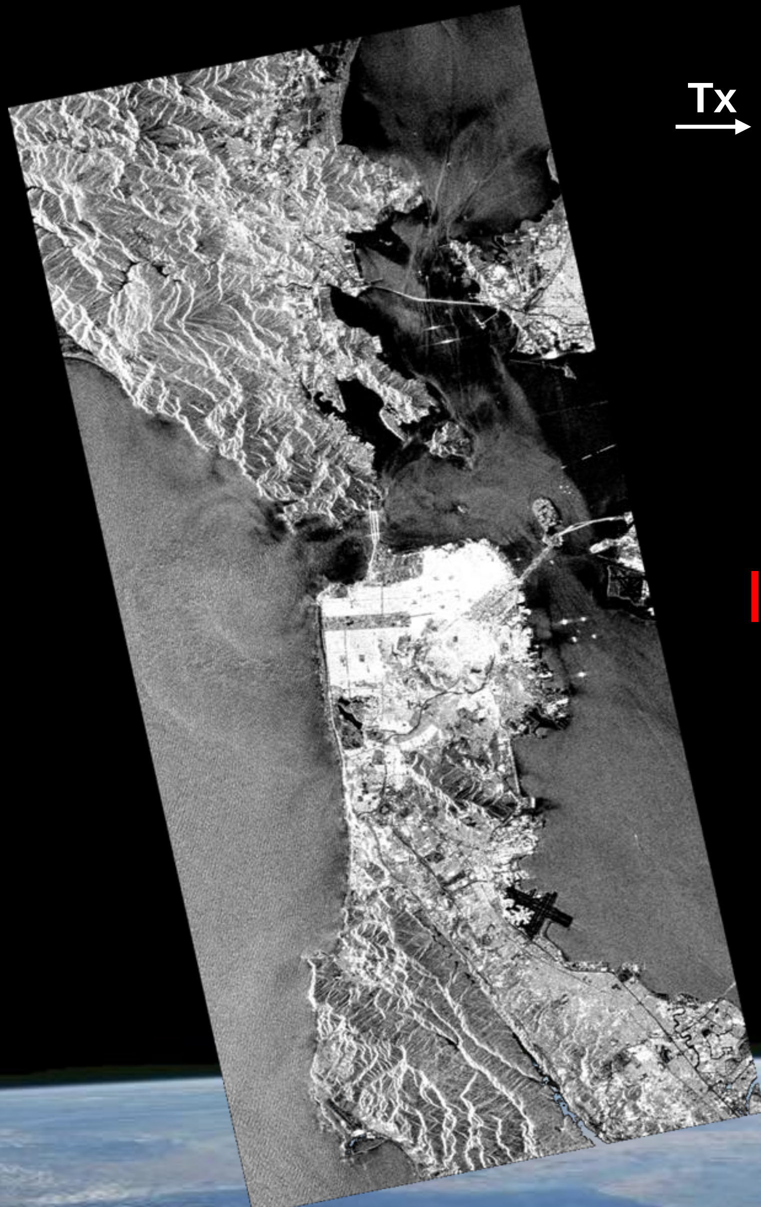
Scattering Coefficient

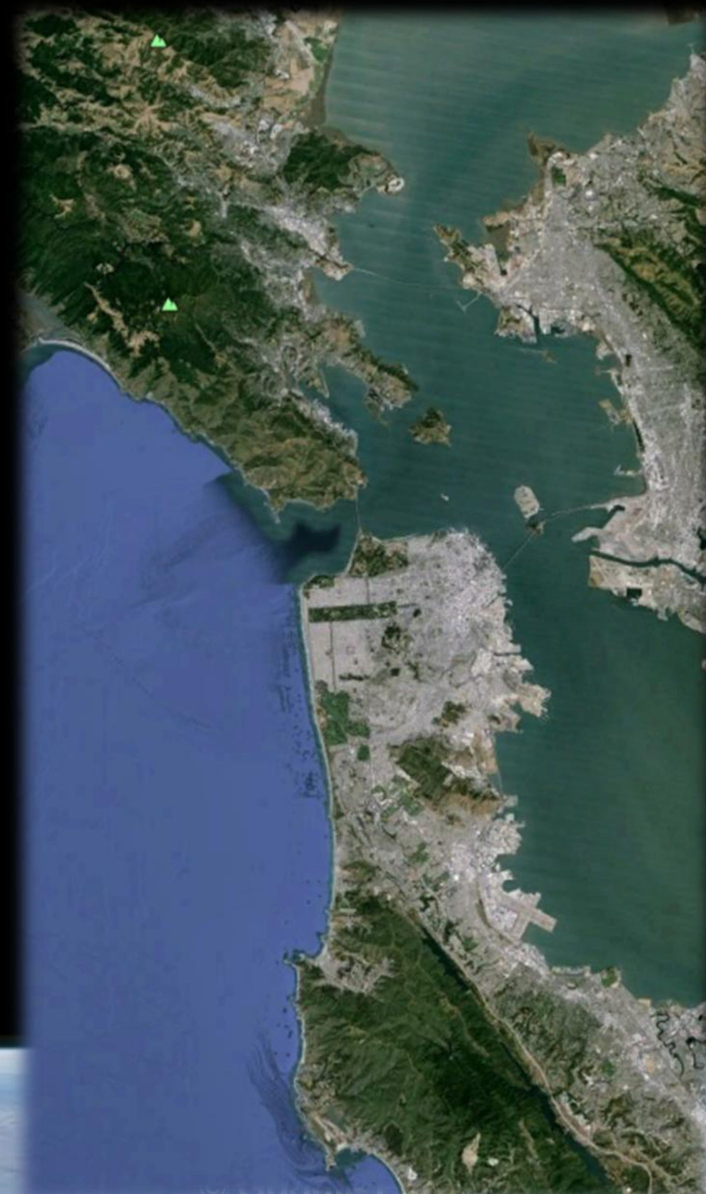


Space-borne Sensors



Tx → Rx →

$|HH|_{dB}$



San Francisco Bay – (L-Band)

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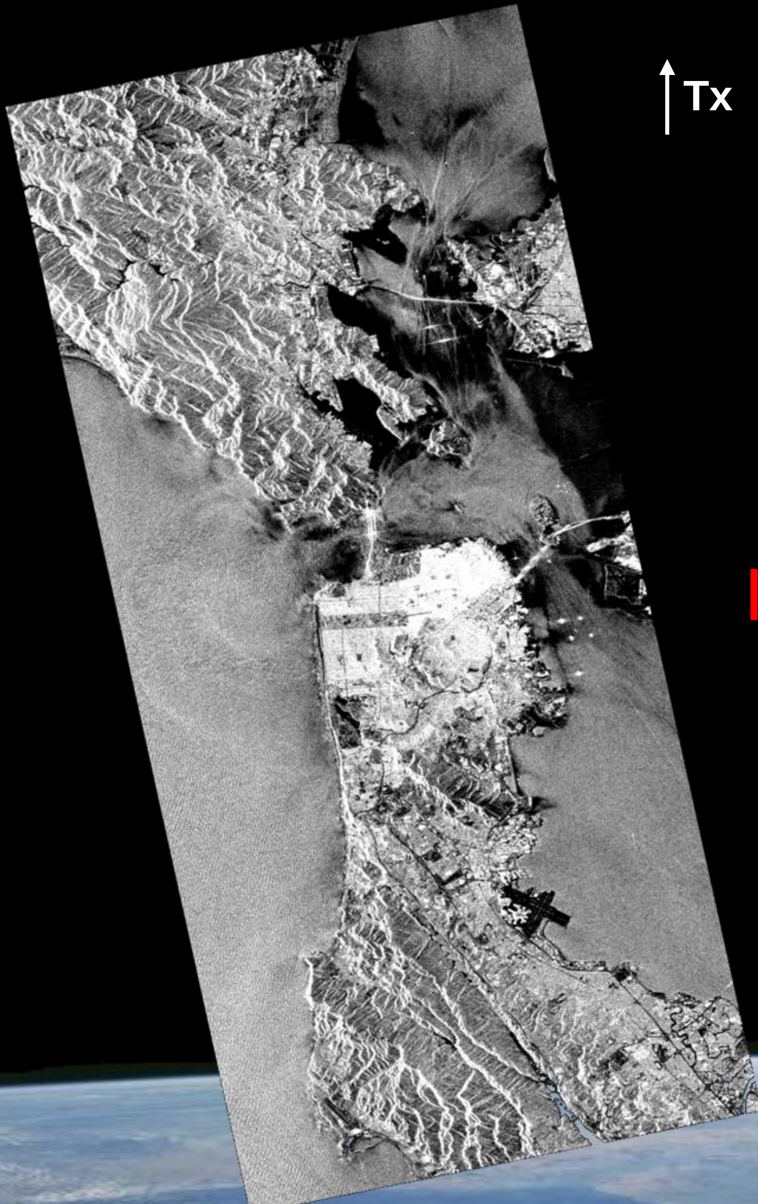
Space-borne Sensors



Tx Rx

$|VV|_{dB}$

San Francisco Bay – (L-Band)

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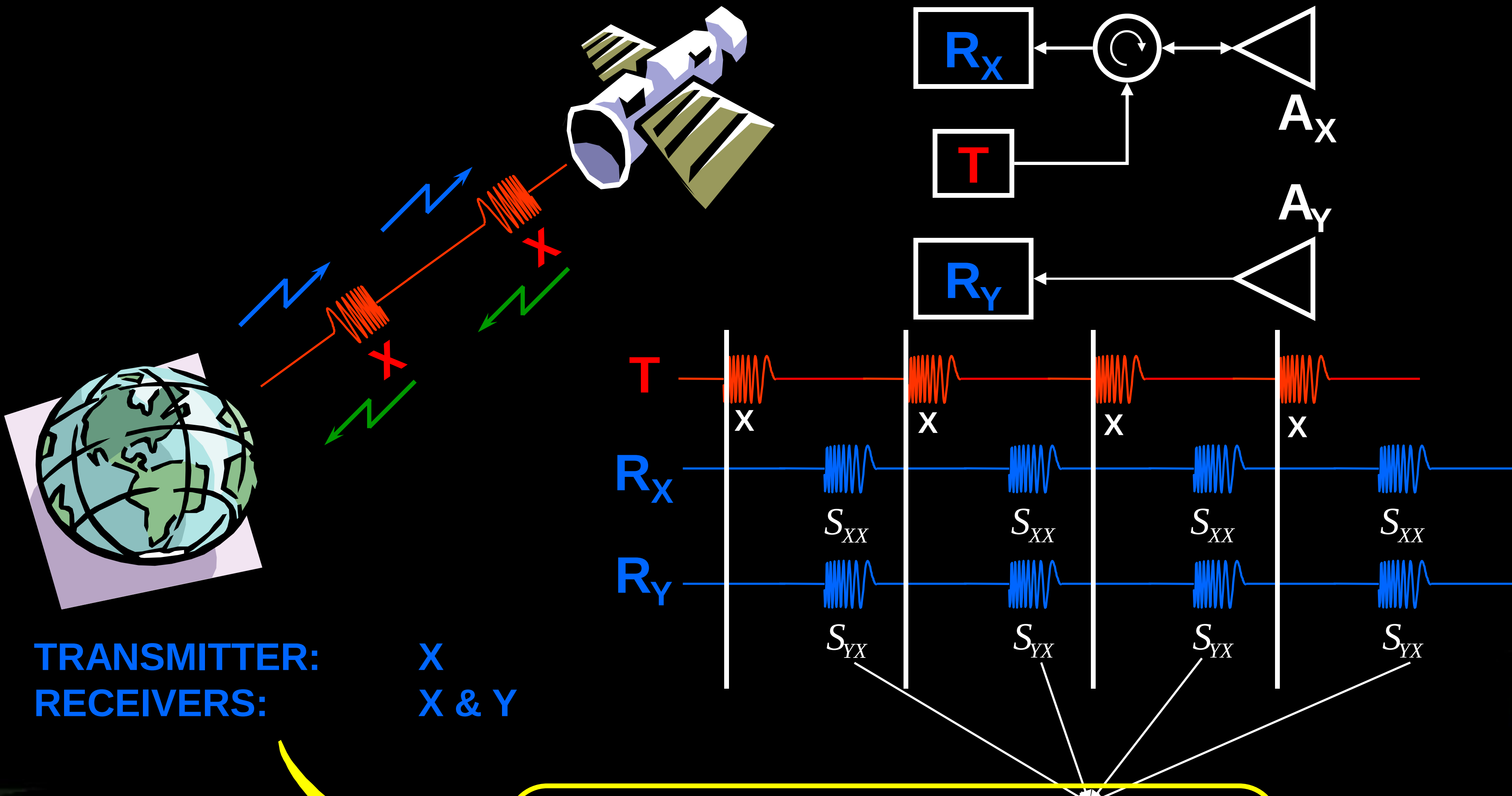
Wave Polarimetry



JONES VECTORS $\left\{ \underline{E}_S = \begin{bmatrix} S_{XX} \\ S_{YX} \end{bmatrix} \right\}$

WAVE POLARIMETRY

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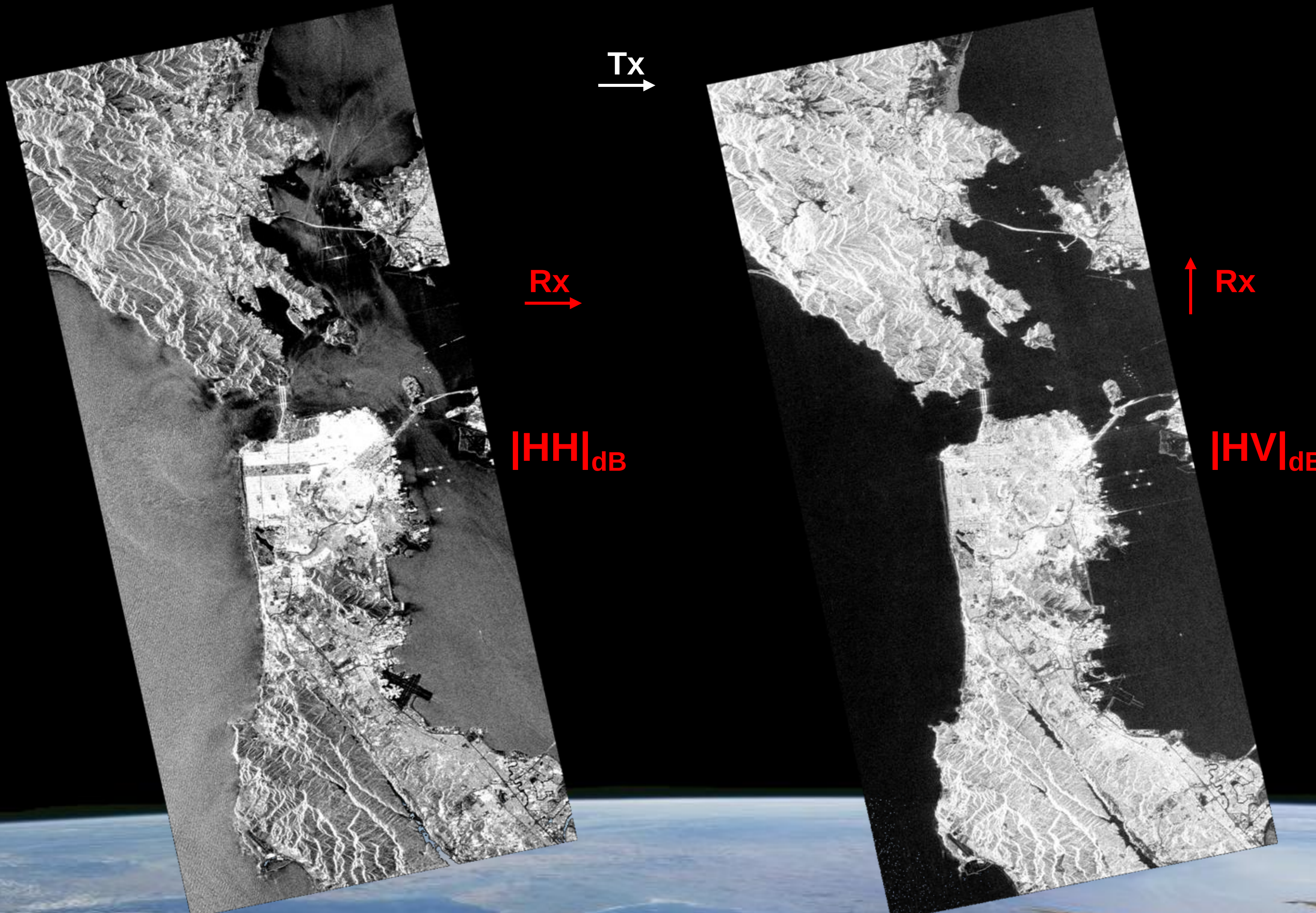
Space-borne Sensors



San Francisco Bay – (L-Band)

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Space-borne PolSAR Sensors

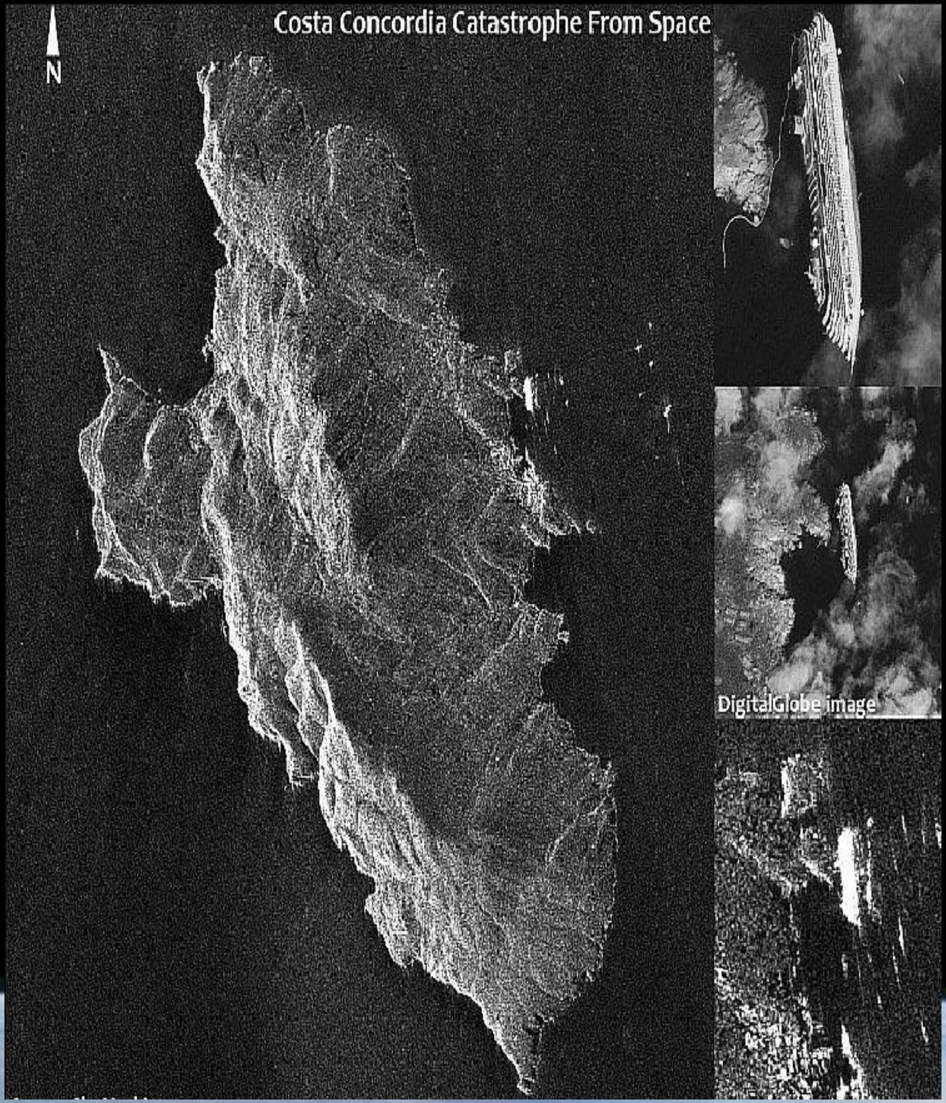
ENVISAT - ASAR

October 2001
C-Band (Sngl / Dual Inc)

Space-borne PolSAR Sensors

COSMO - SkyMed

June 2007, Dec. 2007
Oct. 2008, Nov. 2010
X-Band (Sngl / Dual)
Revisit : 1 day



Space-borne PolSAR Sensors

TerraSAR - X

June 2007

X-Band (Sngl / Twin HH-VV / Quad Exp.)

Space-borne PolSAR Sensors

RISAT-1A

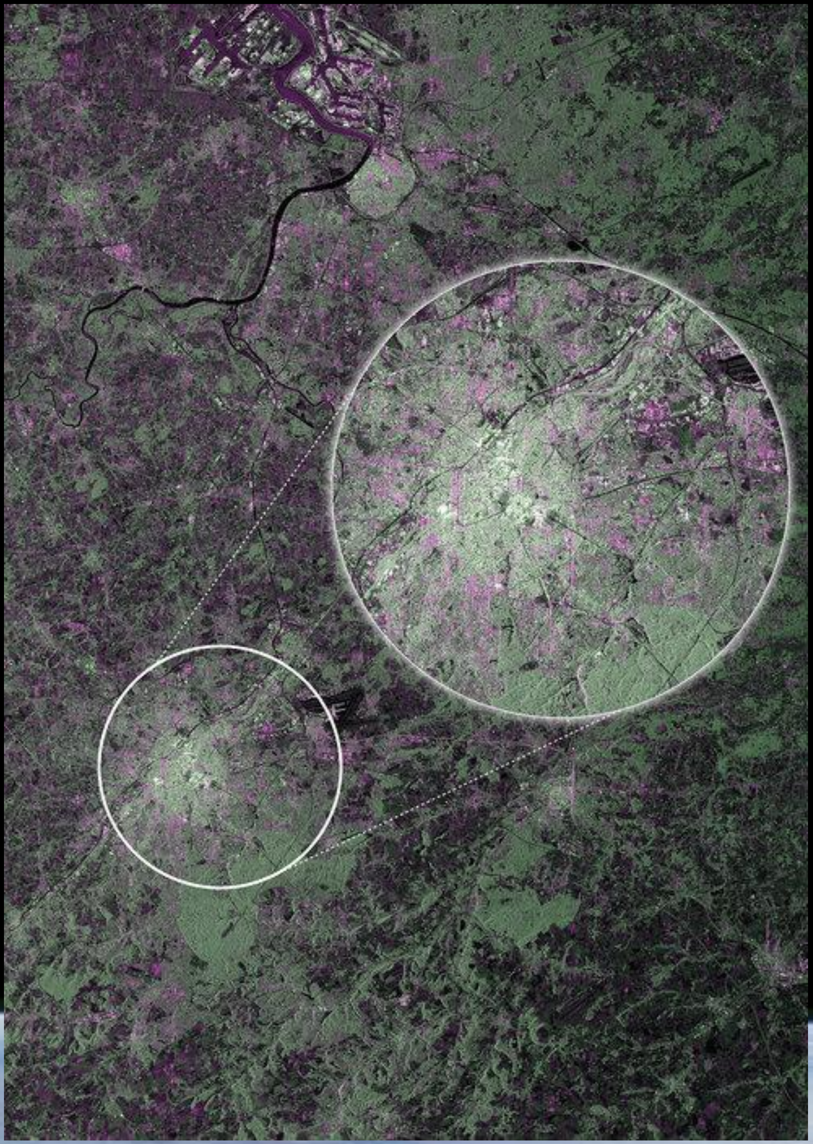
26 April 2012

C-Band (Sngl, Dual, Hybrid)

Operational since 2015

Space-borne PolSAR Sensors

SENTINEL – 1A



S1A : April 2014 **S1B : April 2016**

C-Band (Sngl, Dual)

Revisit : 6 days

Brussels – 12 April 2014

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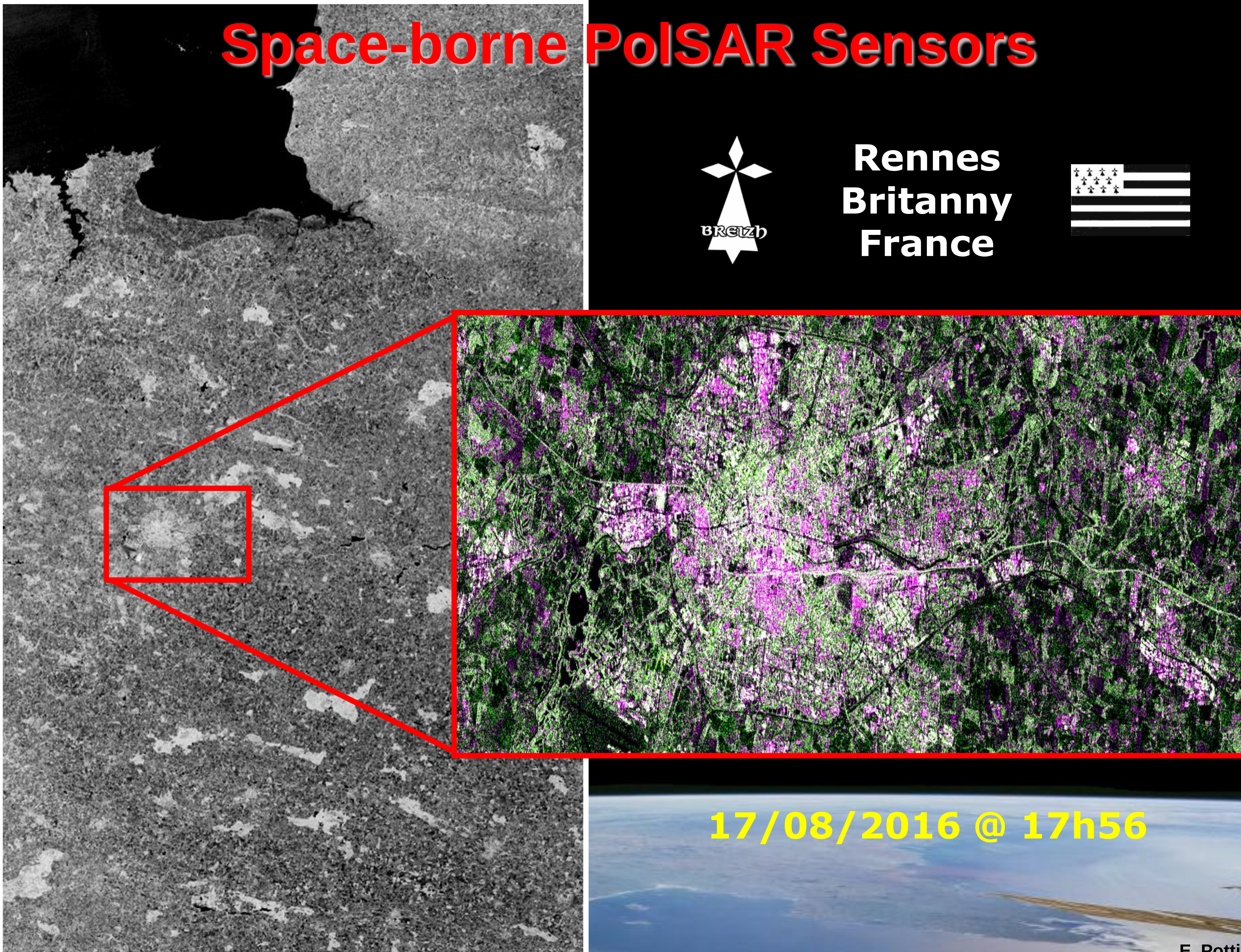
Space-borne PolSAR Sensors

Rennes
Britanny
France



17/08/2016 @ 17h56

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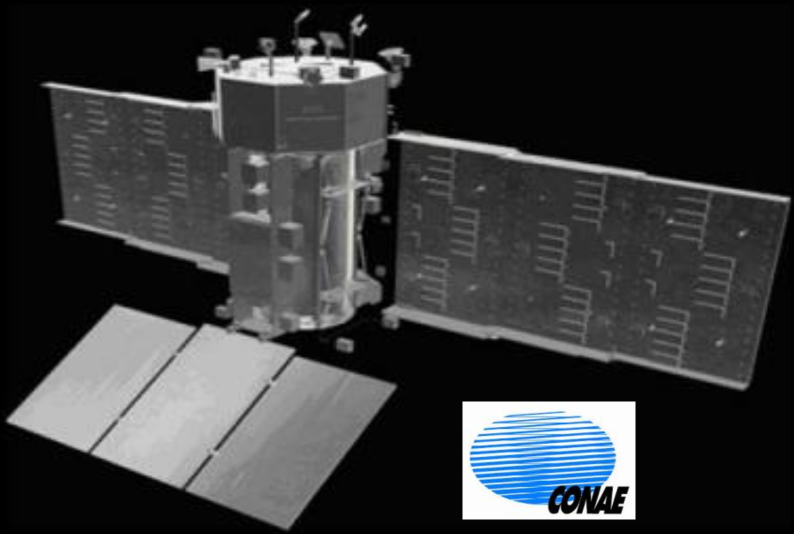
Space-borne PolSAR Sensors

SAOCOM – SAR-L

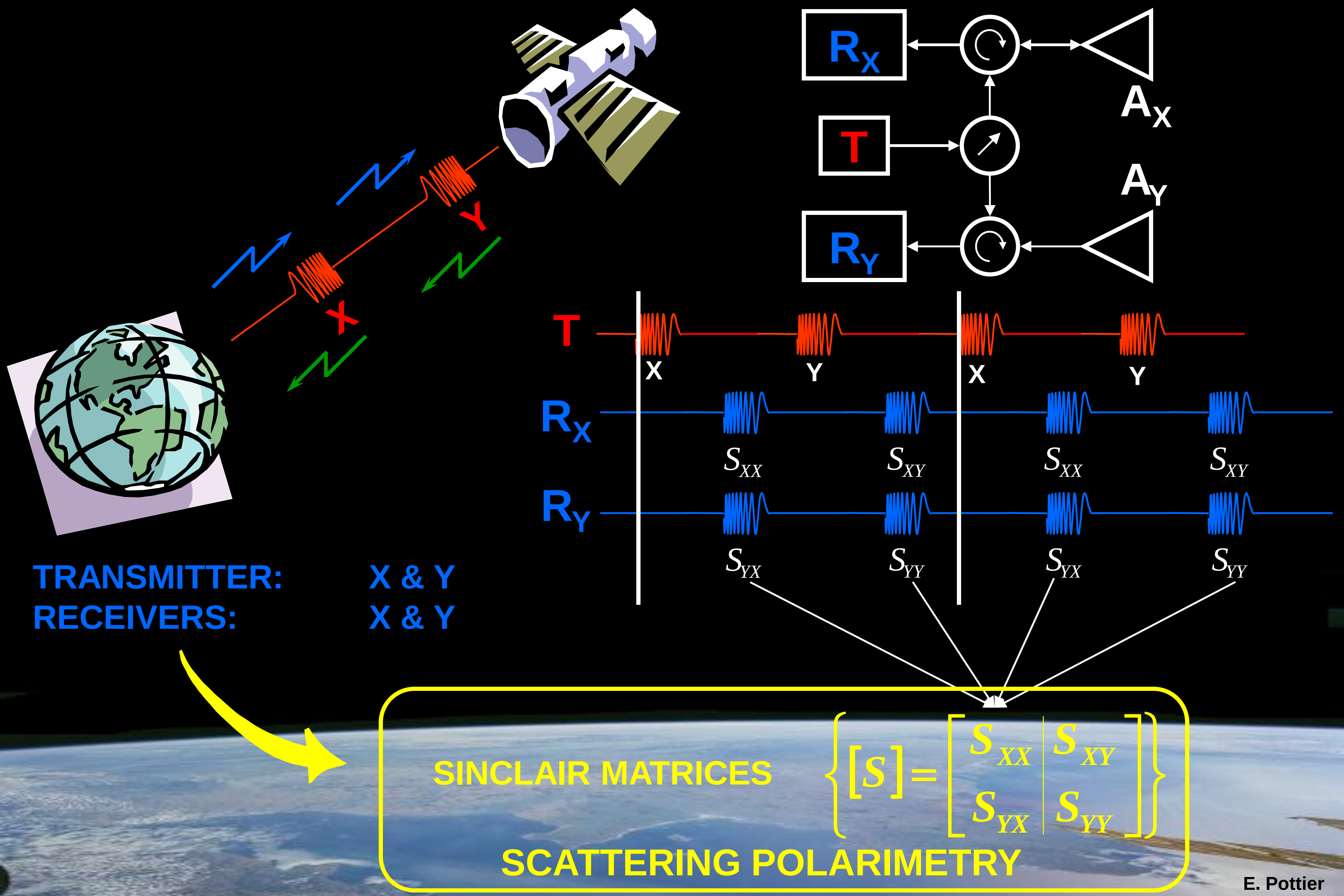


1A : 2017 1B : 2018
2A : 2019 2B : 2020
L-Band (Sngl, Dual, Twin HH-VV)
Revisit : 4 days

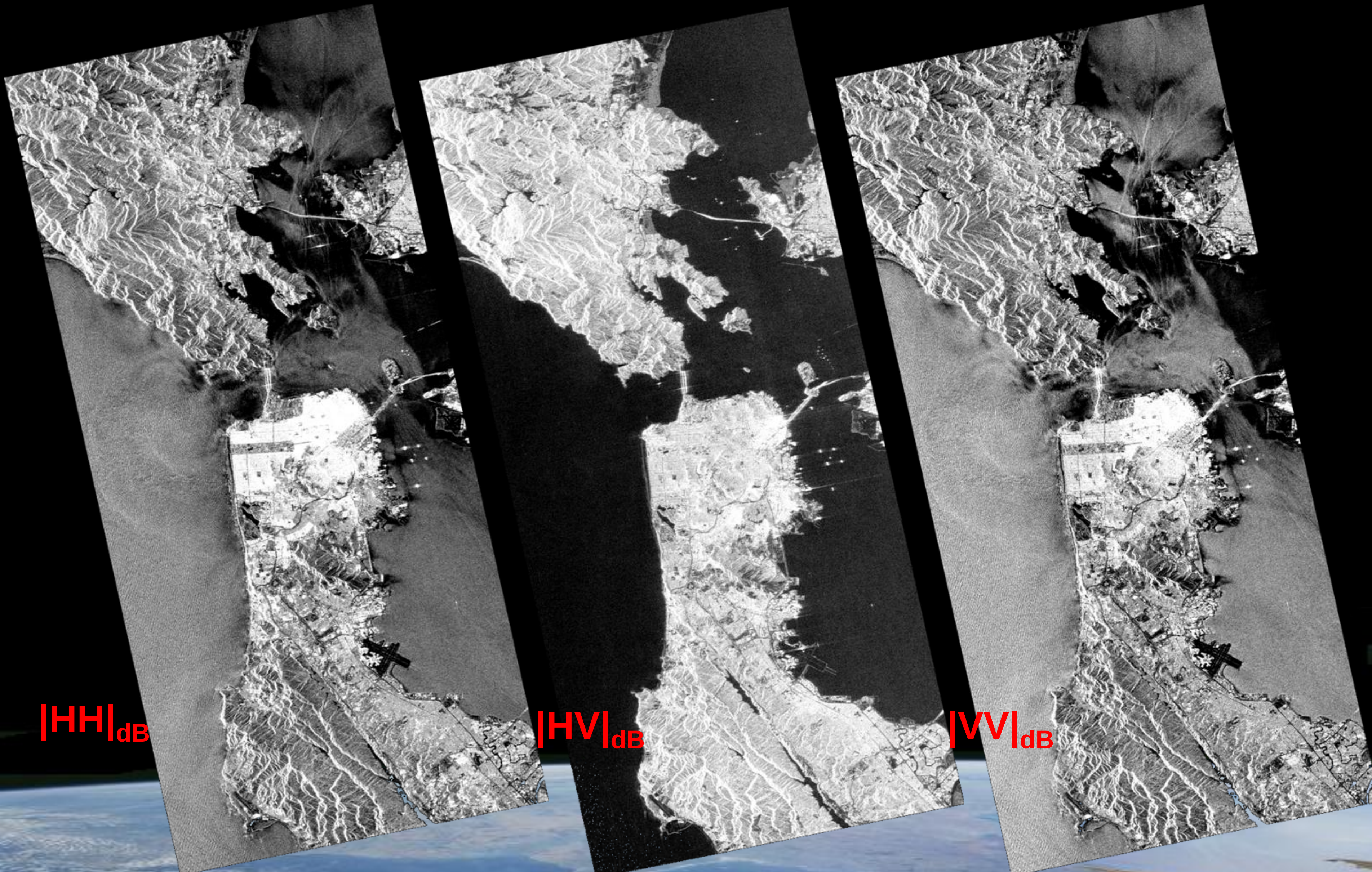
RADARSAT Constellation Mission (RCM)

1A : 2017 1B / 1C : 2018
C-Band (Sngl, Dual, Hybrid)
Revisit : 4 days

Scattering Polarimetry



Space-borne Sensors



San Francisco Bay – (L-Band)

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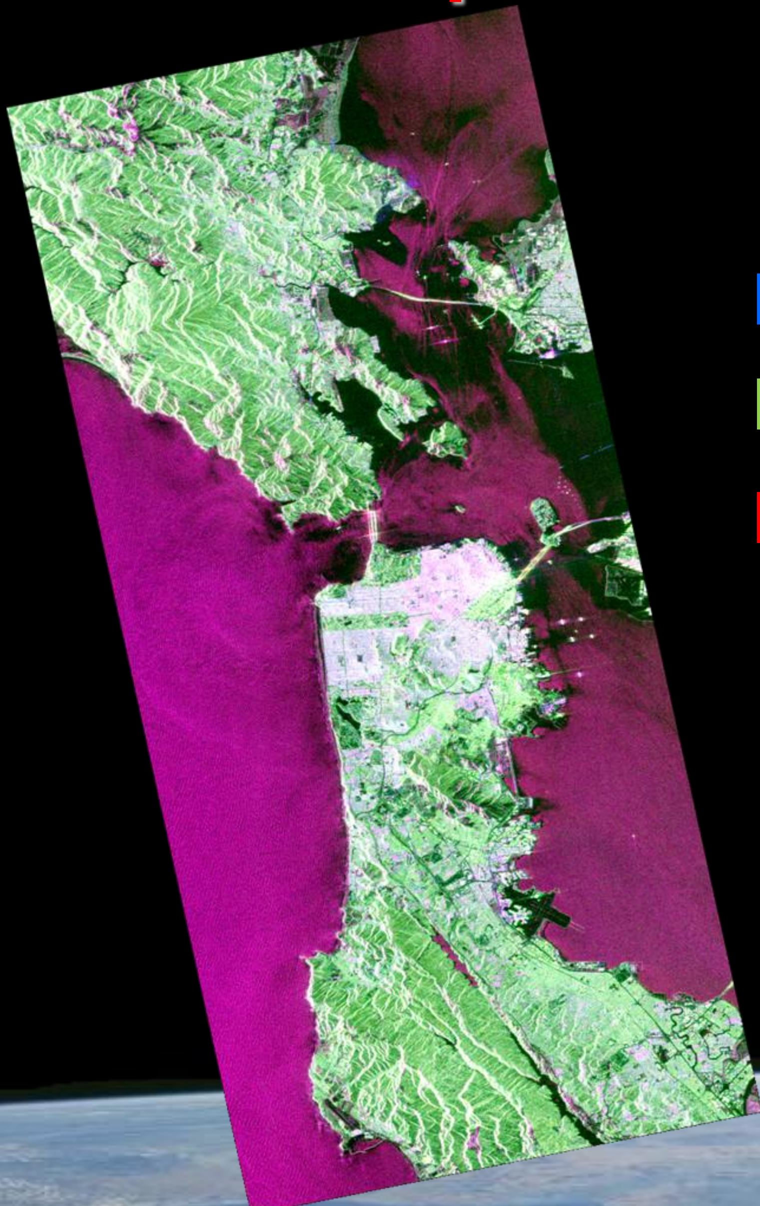
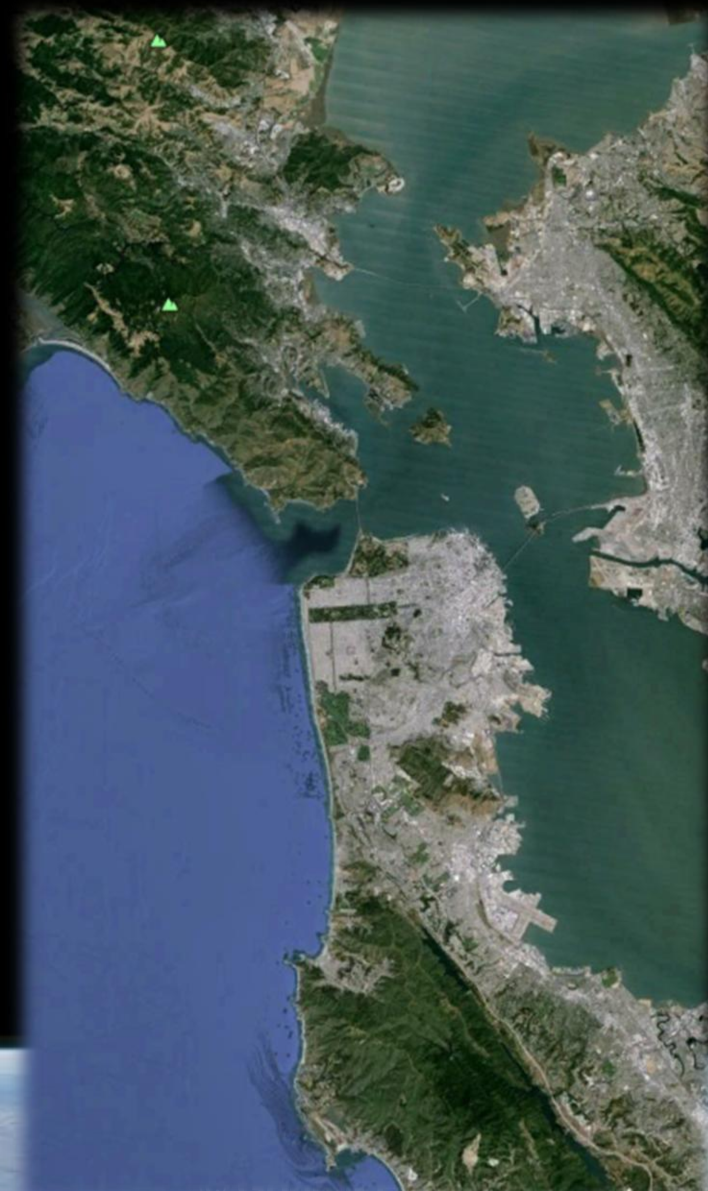
Space-borne Sensors



$|HH|_{dB}$

$|HV|_{dB}$

$|VV|_{dB}$



San Francisco Bay – (L-Band)

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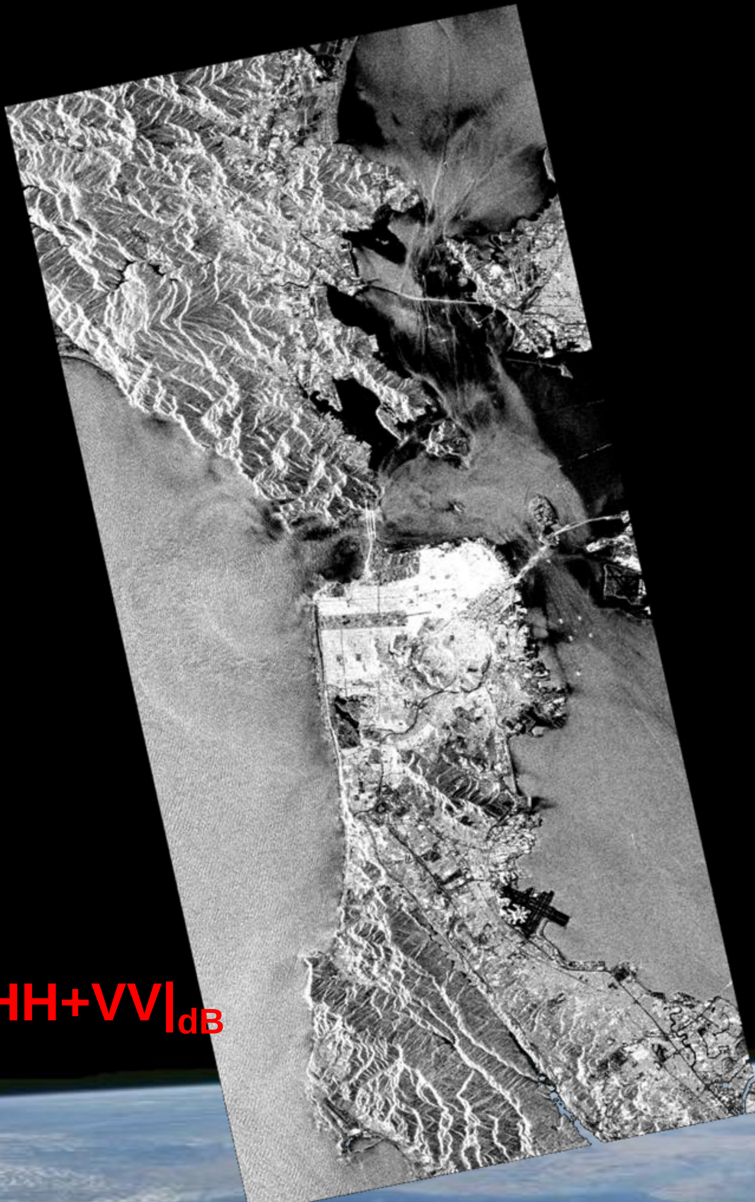
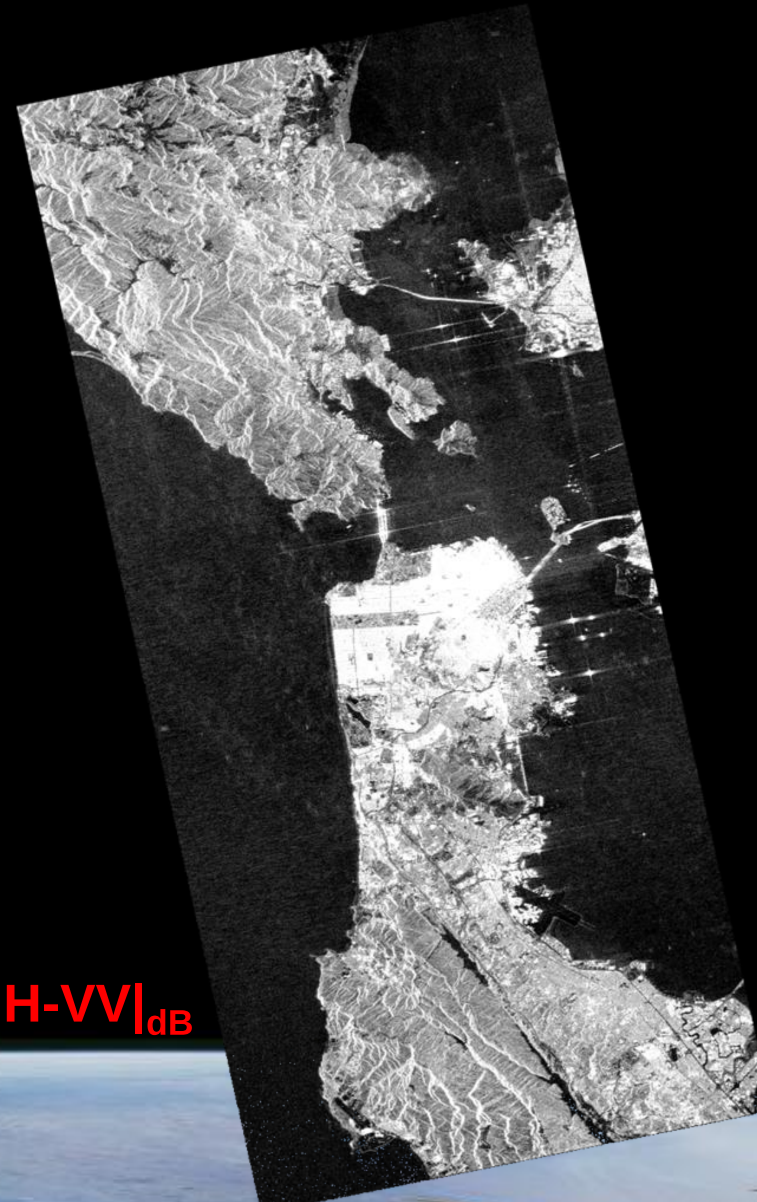
Space-borne Sensors



$|HH+VV|_{dB}$



$|HH-VV|_{dB}$

San Francisco Bay – (L-Band)

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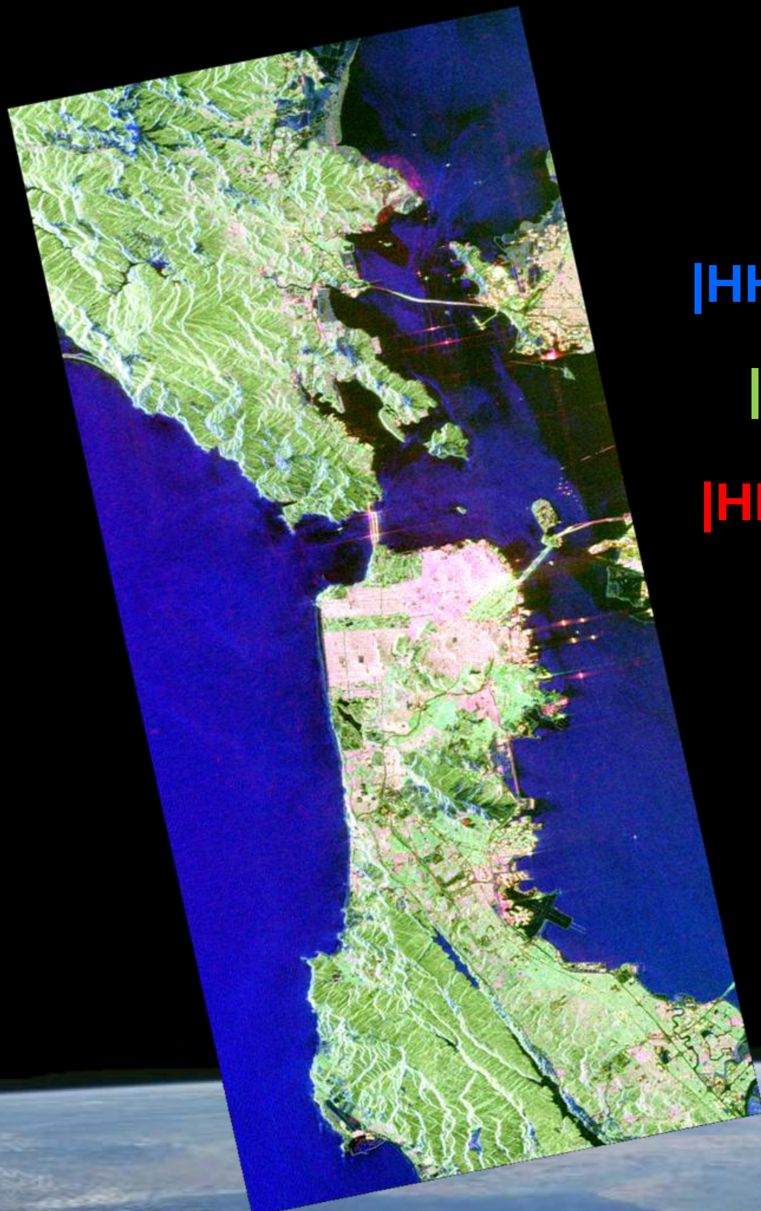
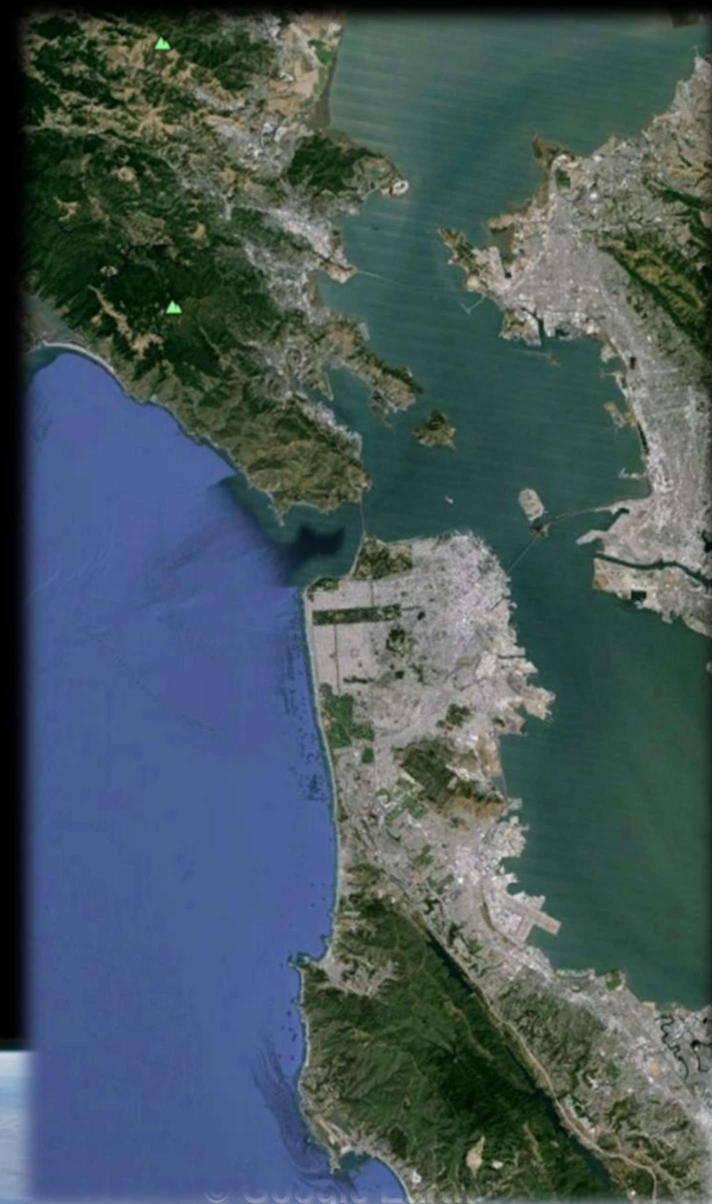
Space-borne Sensors



$|HH+VV|_{dB}$

$|HV|_{dB}$

$|HH-VV|_{dB}$



San Francisco Bay – (L-Band)

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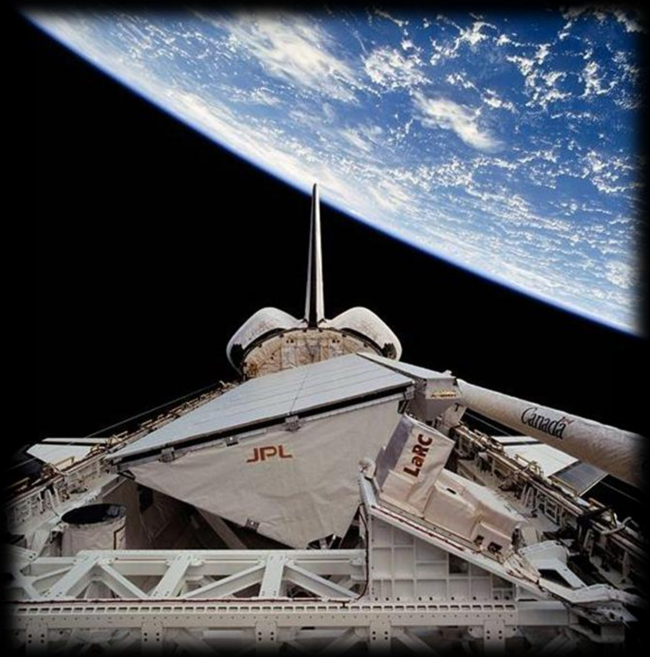
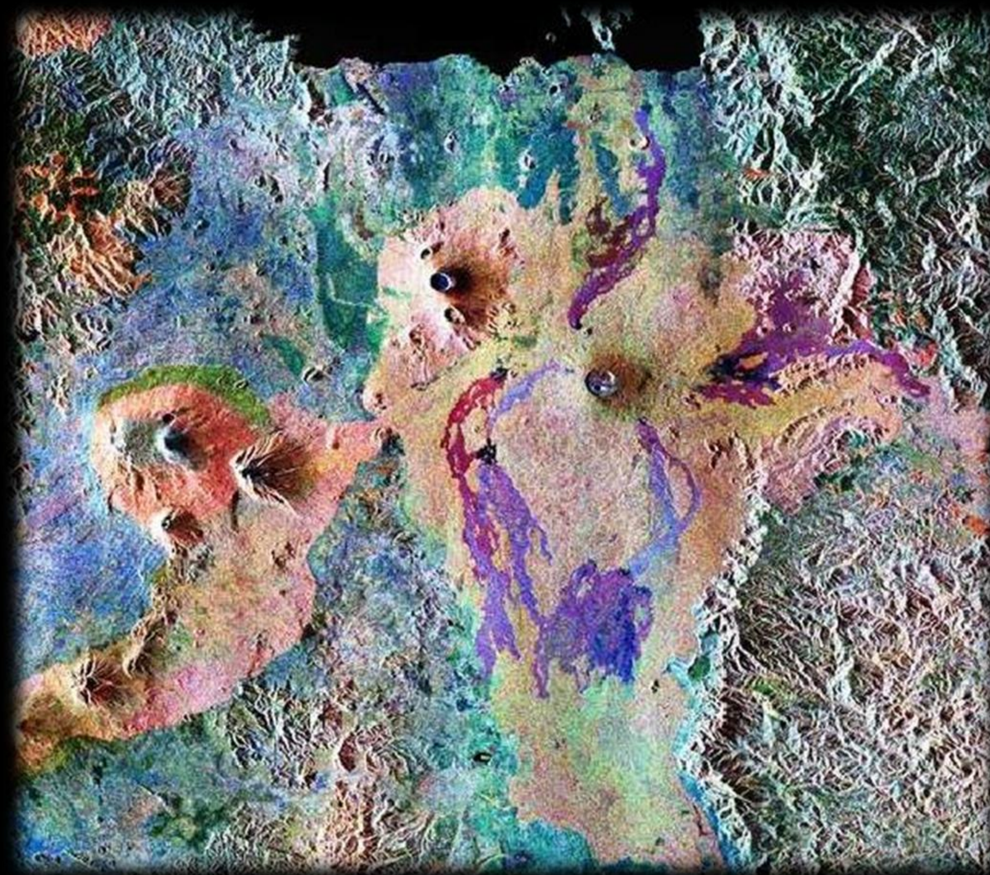
Space-borne PolSAR Sensors

SIR-C / X-SAR



April 1994
L- and C-Band (Quad)
X-Band (Sngl)



Rwanda, Zaire, Uganda

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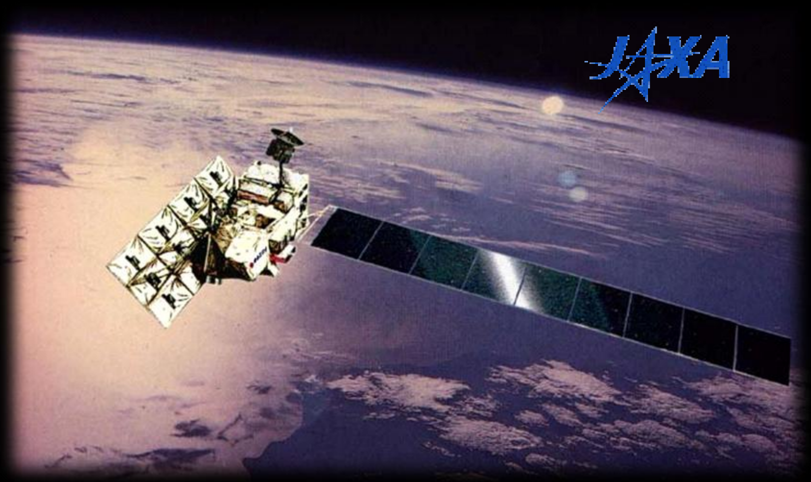
Space-borne PolSAR Sensors

ALOS - PALSAR



January 2006

L-Band (Sngl / Twin / Quad)

ALOS : Advanced Land Observing Satellite

PALSAR : Phase Array L-Band SAR

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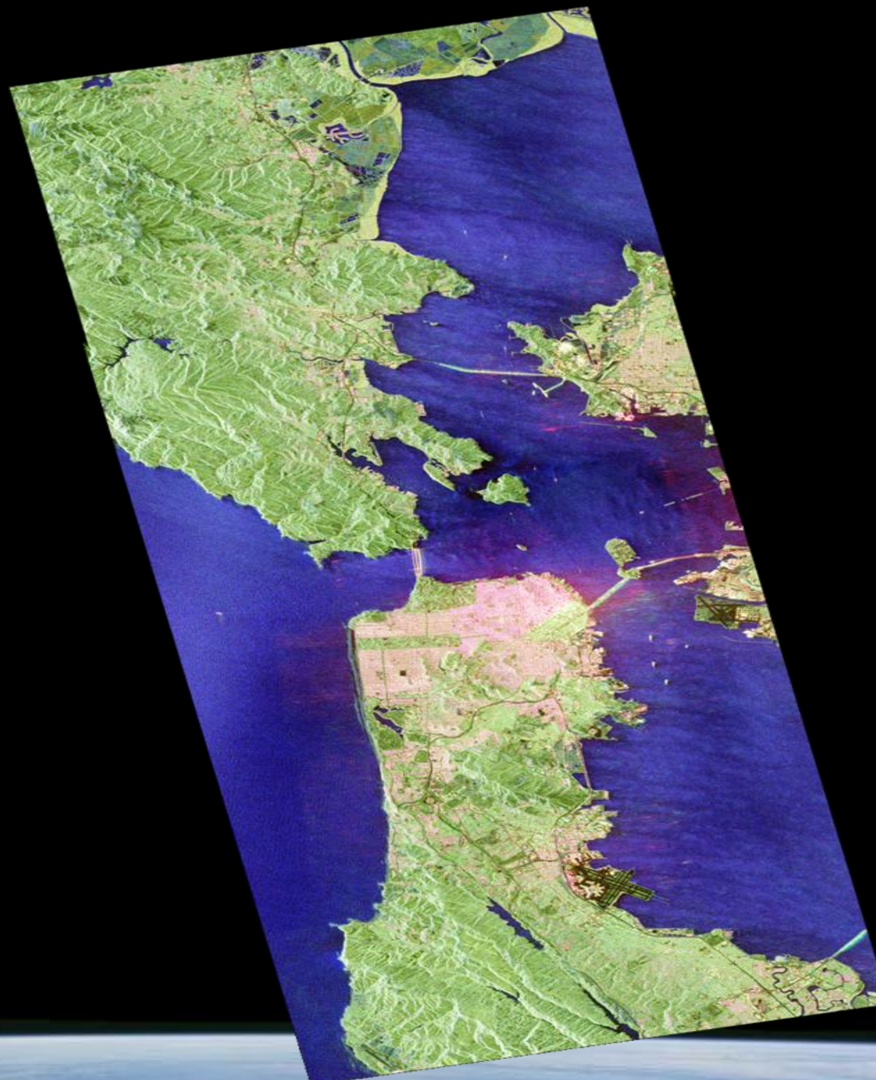
Space-borne PolSAR Sensors

RADARSAT - 2



December 2007

C-Band (Quad)

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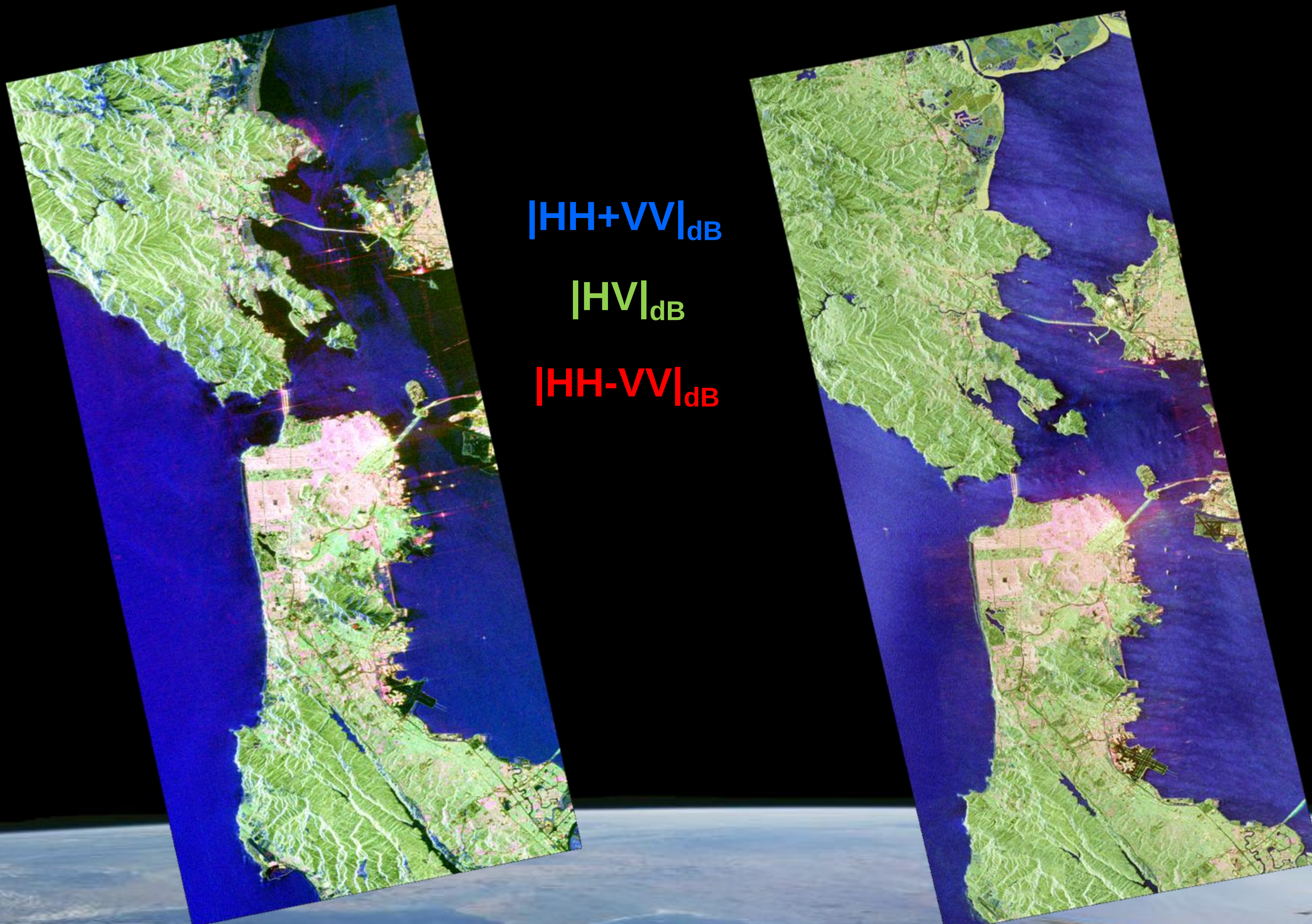
Space-borne Sensors



San Francisco Bay – (L-Band and C-Band)

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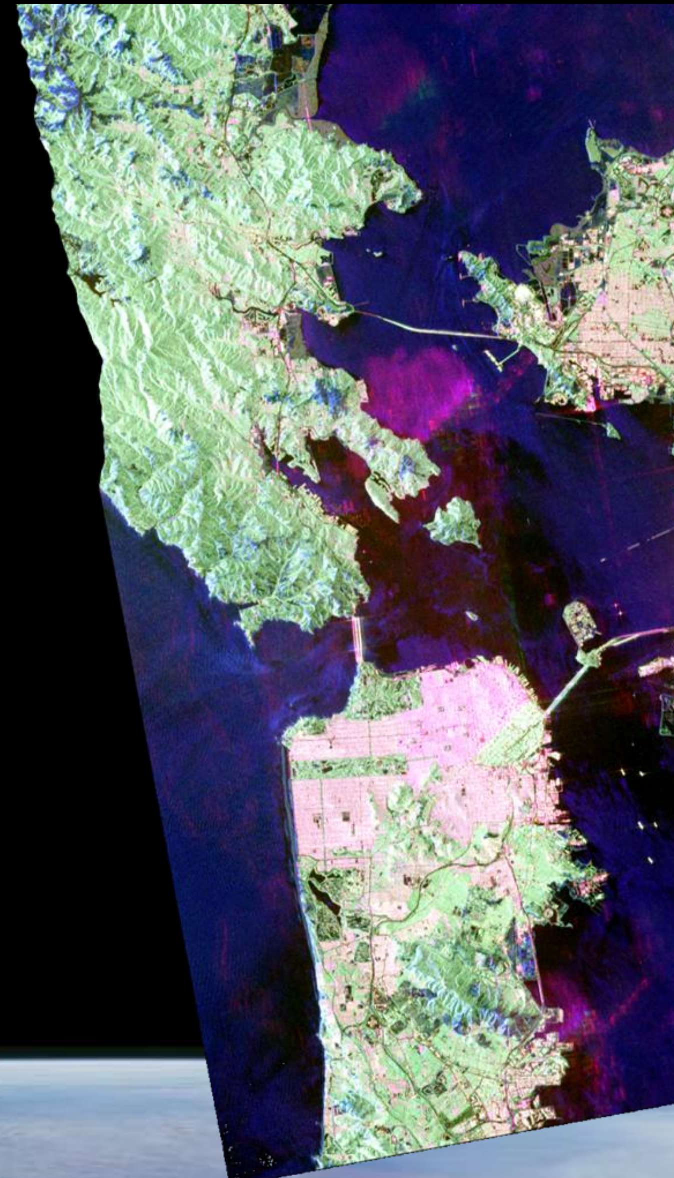
Space-borne PolSAR Sensors

ALOS - 2

May 2014
L-Band (Quad)



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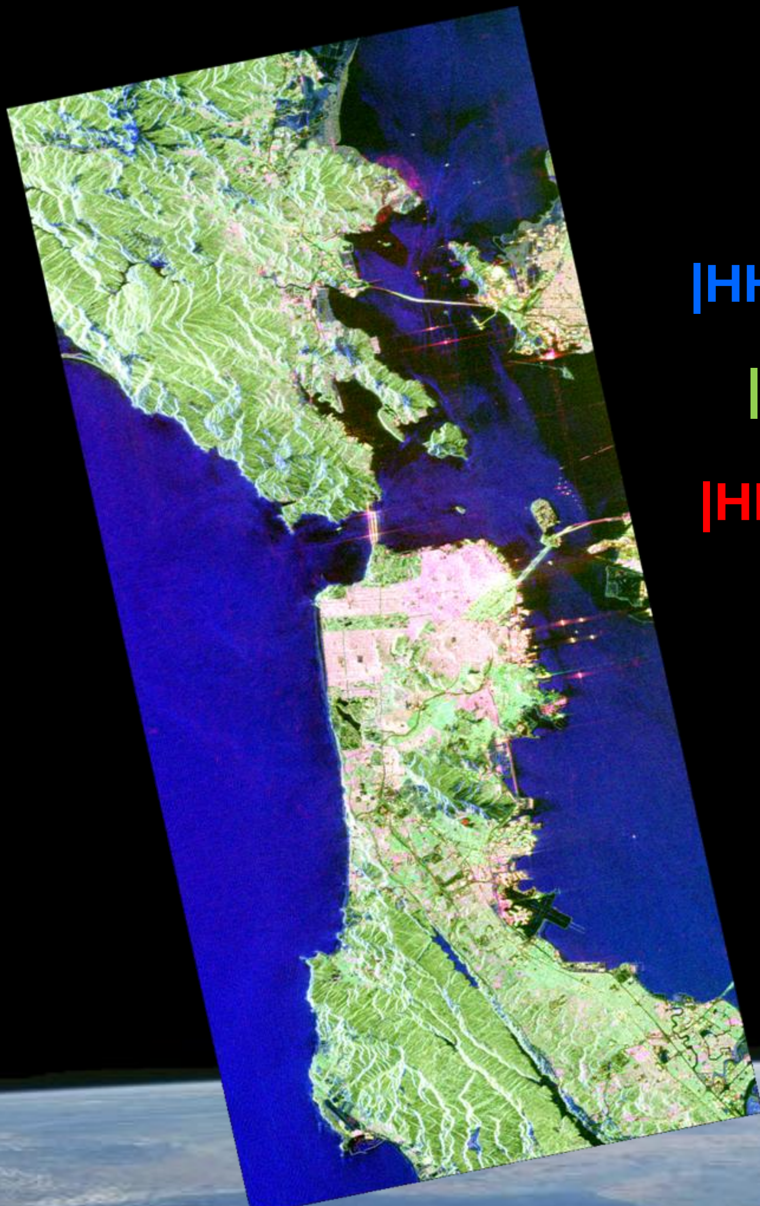
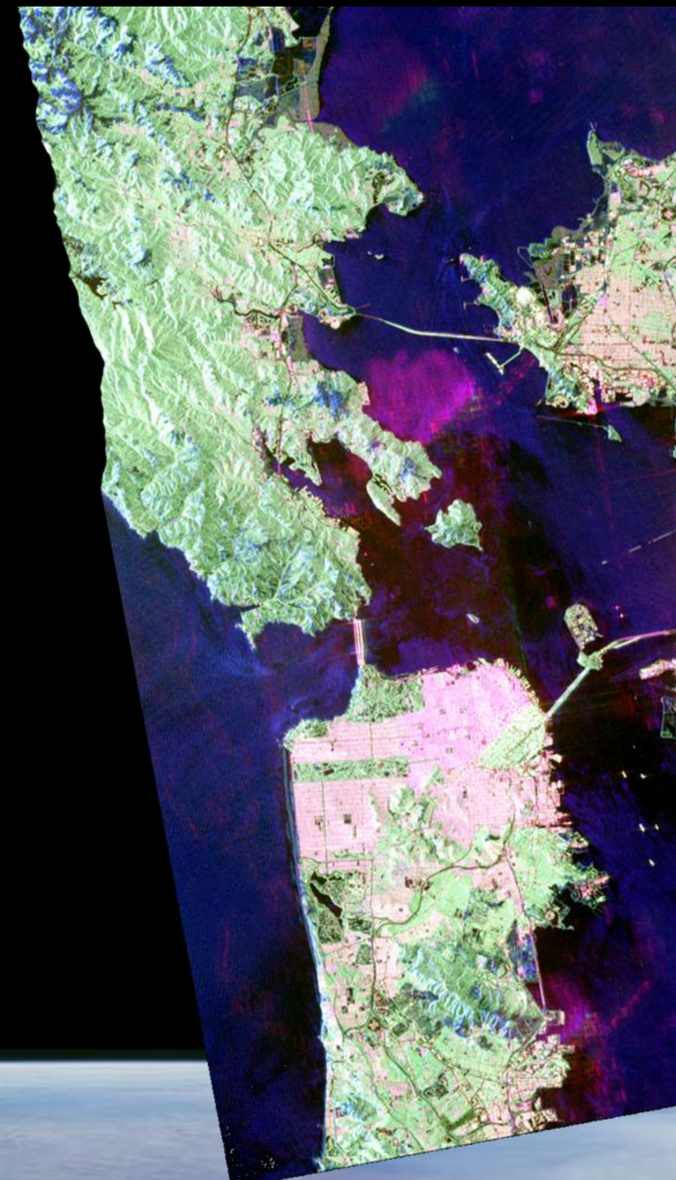
Space-borne Sensors



$|HH+VV|_{dB}$

$|HV|_{dB}$

$|HH-VV|_{dB}$



San Francisco Bay – (L-Band : ALOS-1 and ALOS-2)

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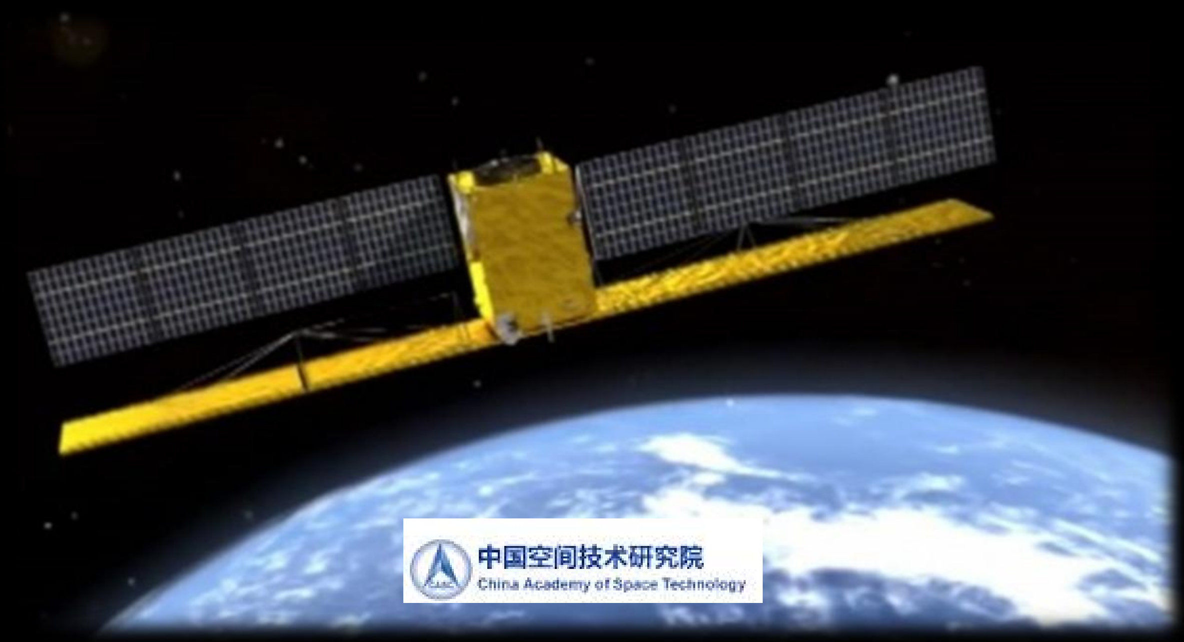
Space-borne PolSAR Sensors

Chang Zheng-4C - GaoFen-3 (GF-3)

Long March-4C - High Resolution-3



August 2016

C-Band (Quad)

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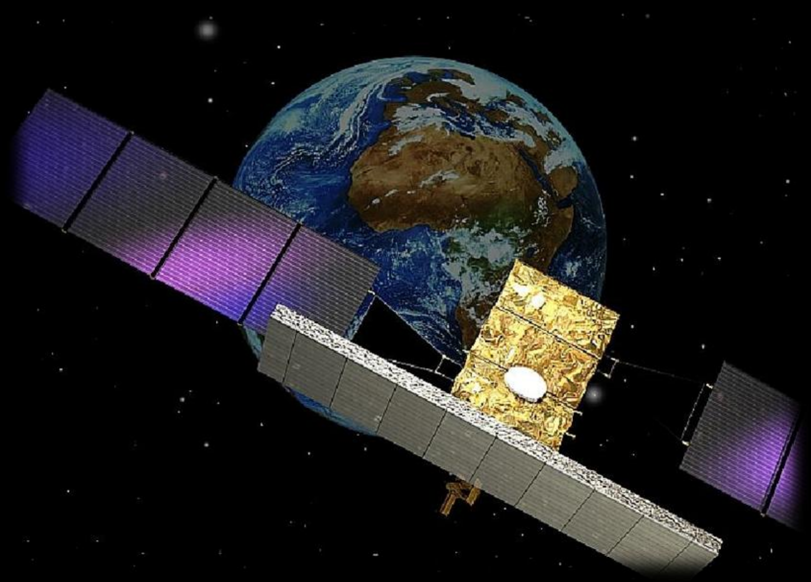
Space-borne PolSAR Sensors

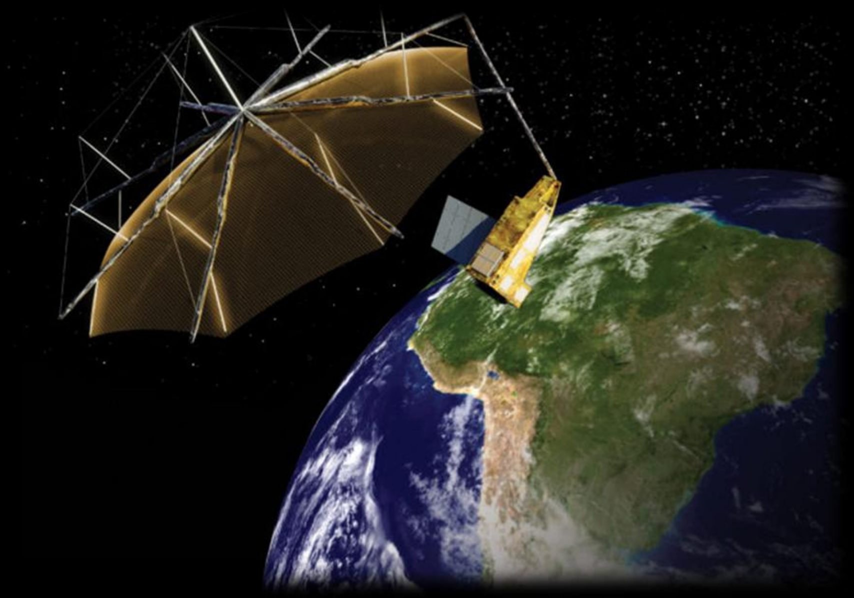
COSMO - SkyMed - CSG



2A : 2018 2B : 2019

X-Band (Sngl / Dual / Quad Exp.)

Earth Explorer - BIOMASS



2019

P-Band (Quad)

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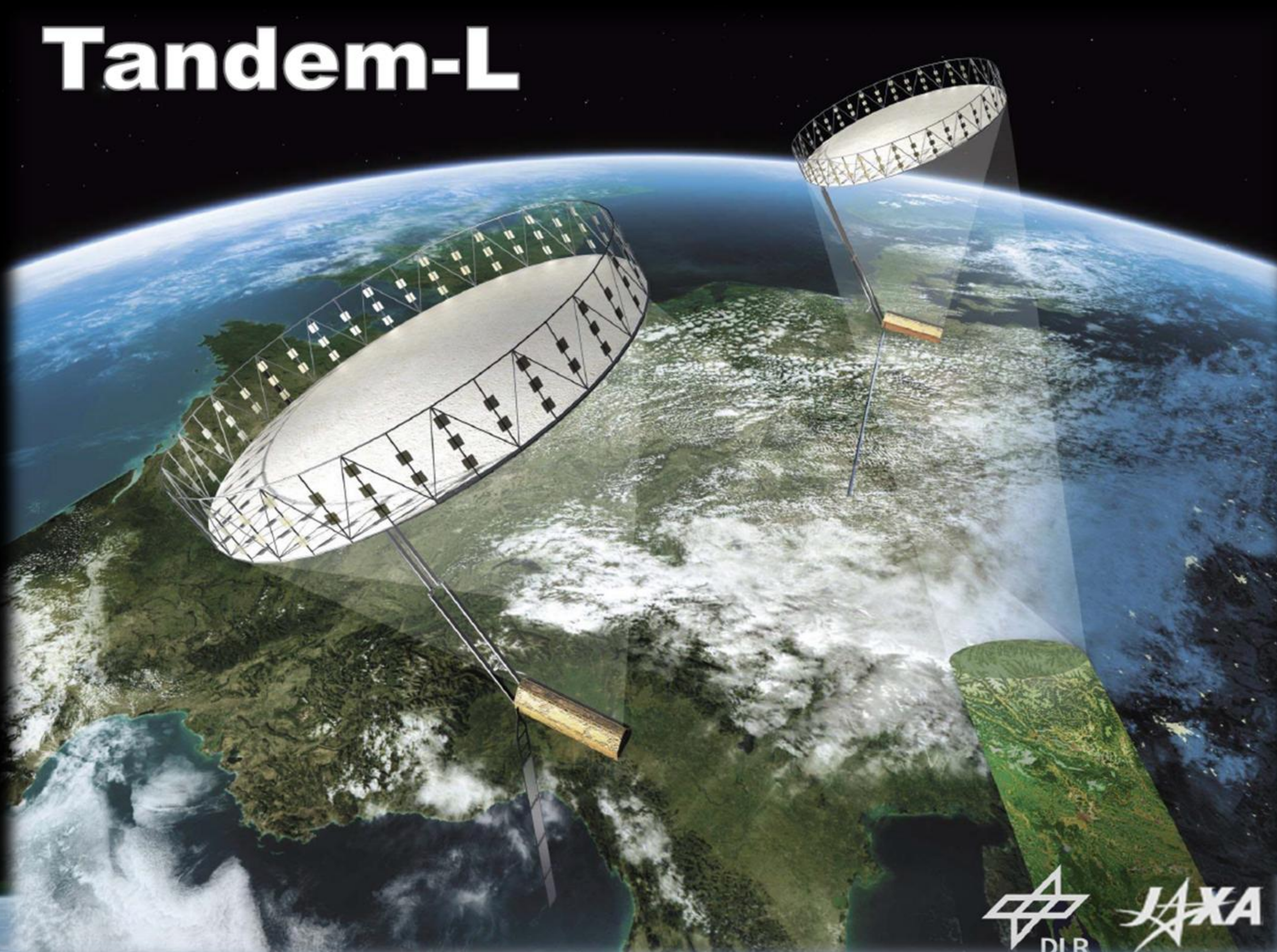
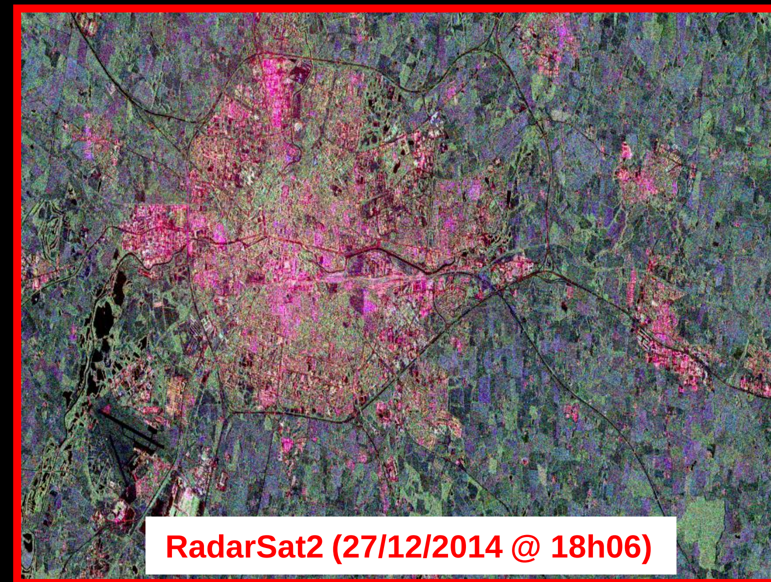
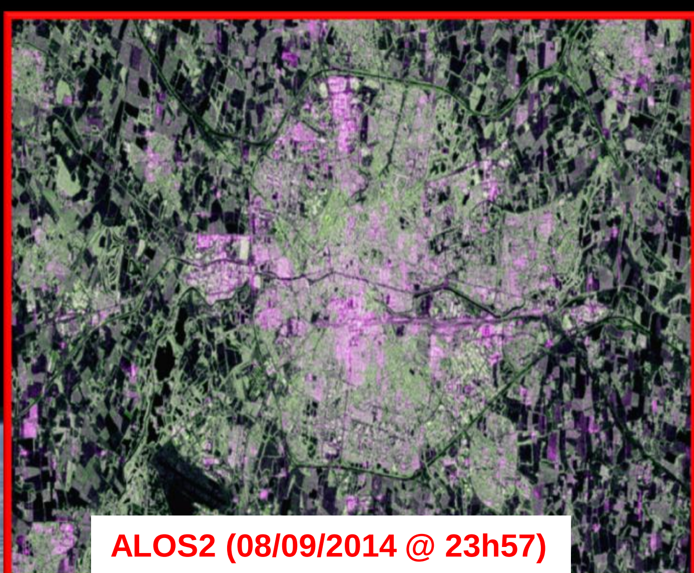
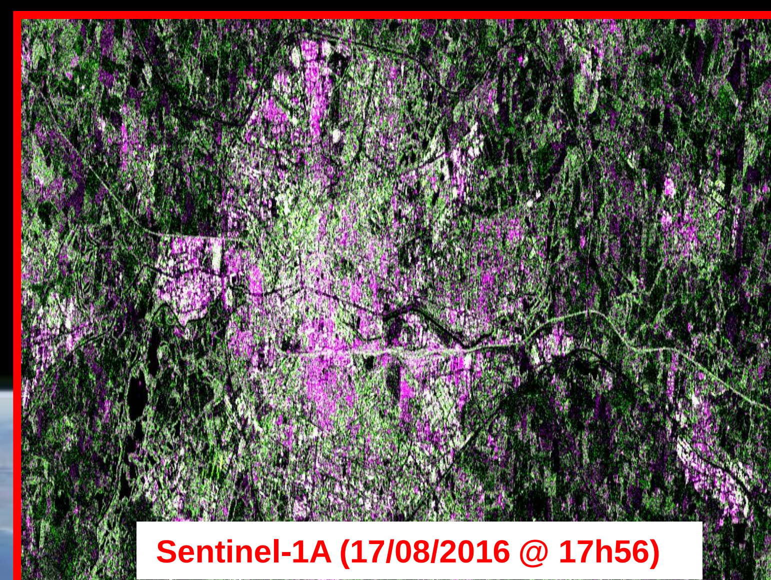
Space-borne PolSAR Sensors



Space-borne PolSAR Sensors


RadarSat2 (27/12/2014 @ 18h06)


ALOS2 (08/09/2014 @ 23h57)


Sentinel-1A (17/08/2016 @ 17h56)

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What About

Software / Toolbox ?

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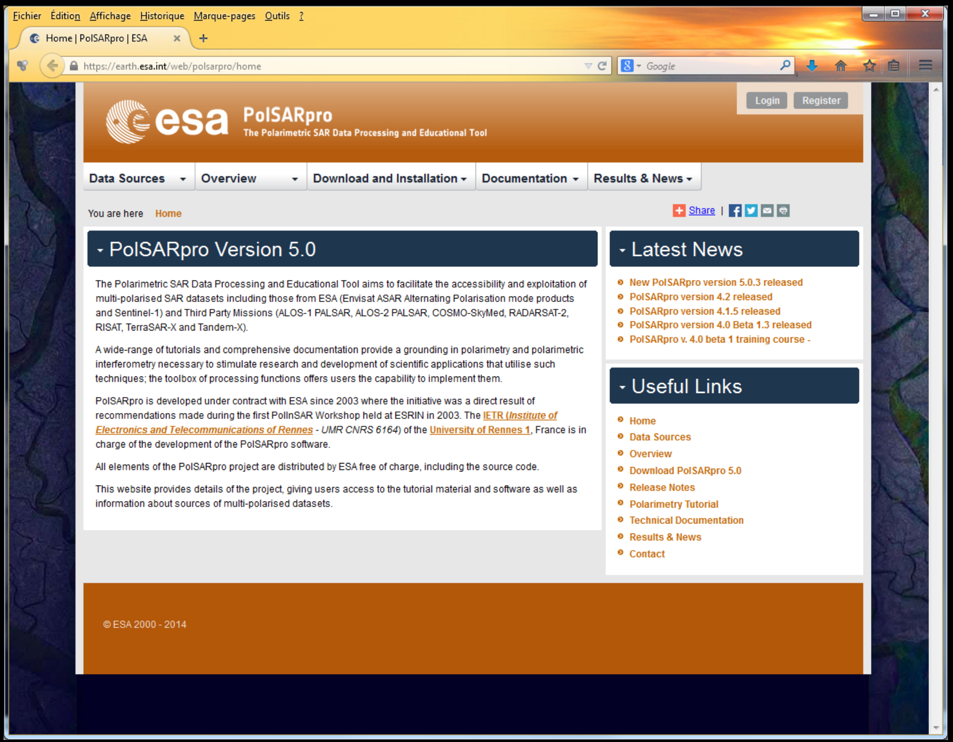
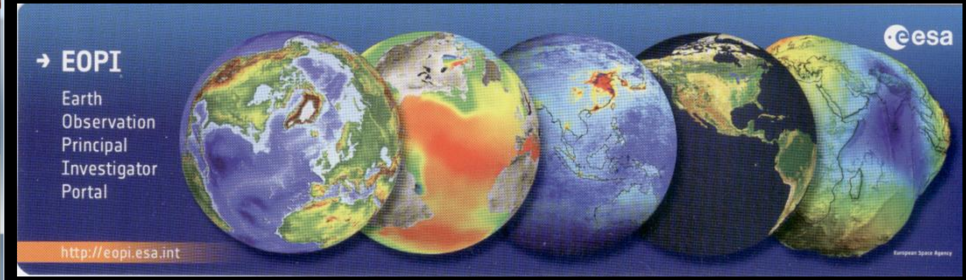
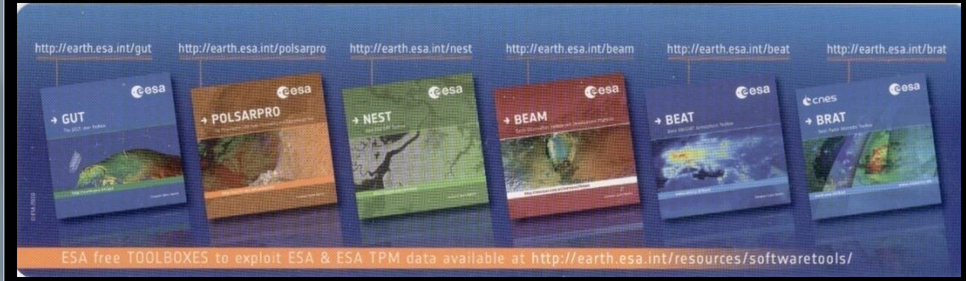
PolSARpro v5.1

The Polarimetric SAR Data Processing and Educational Tool v5.1







https://earth.esa.int/web/polsarpro

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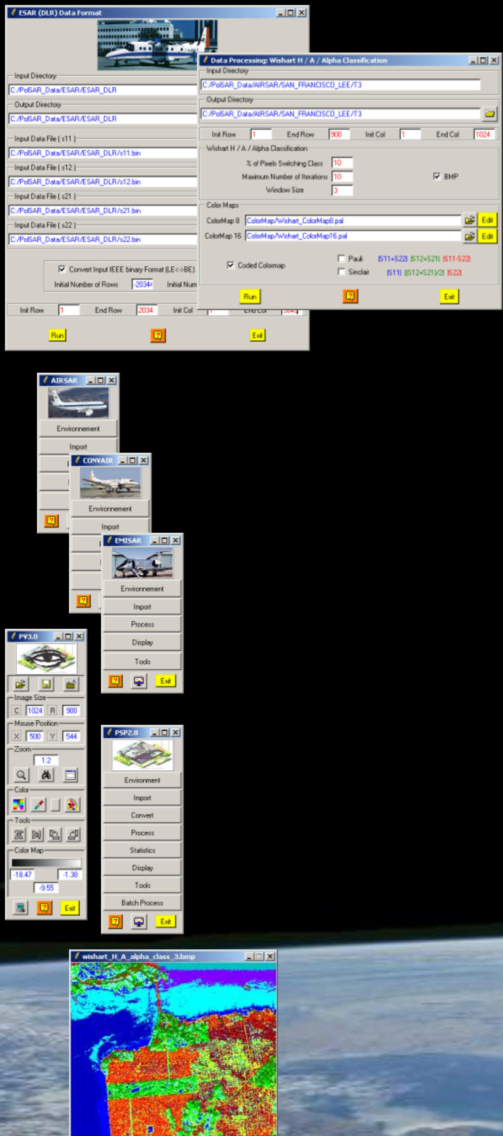
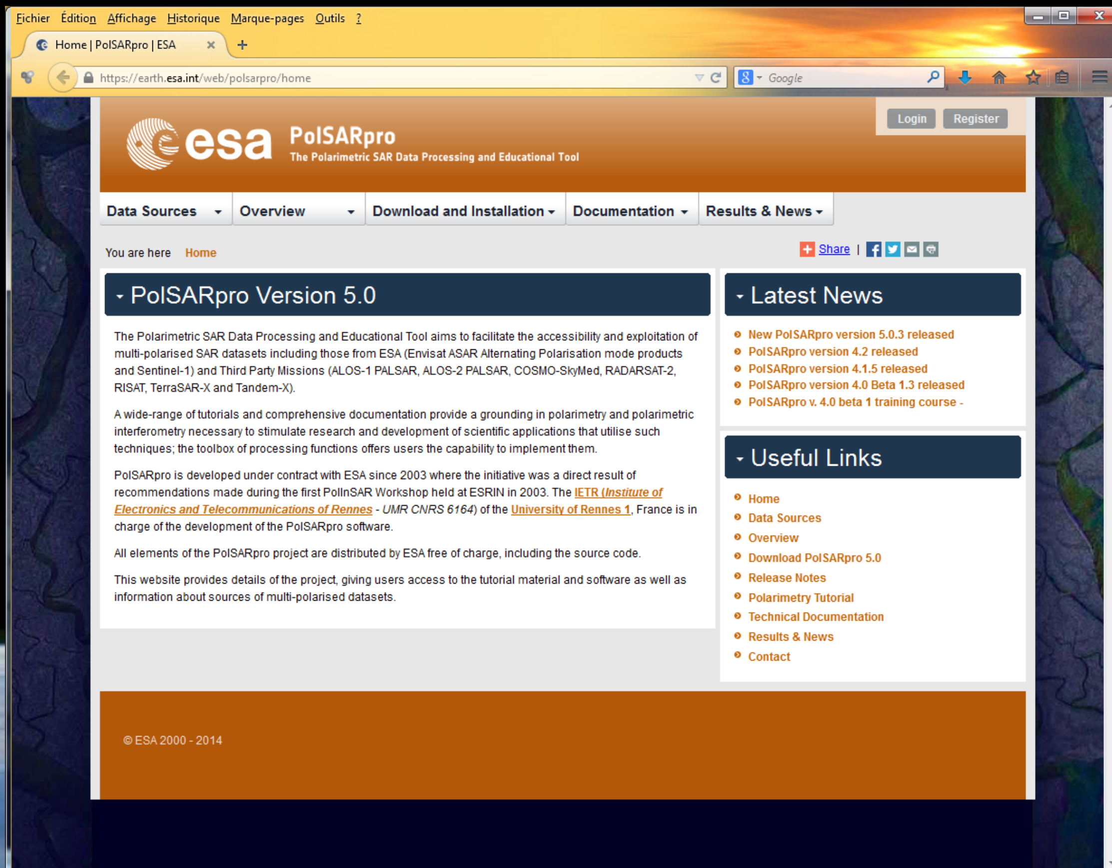
PolSARpro v5.1

OPEN SOURCE DEVELOPMENT

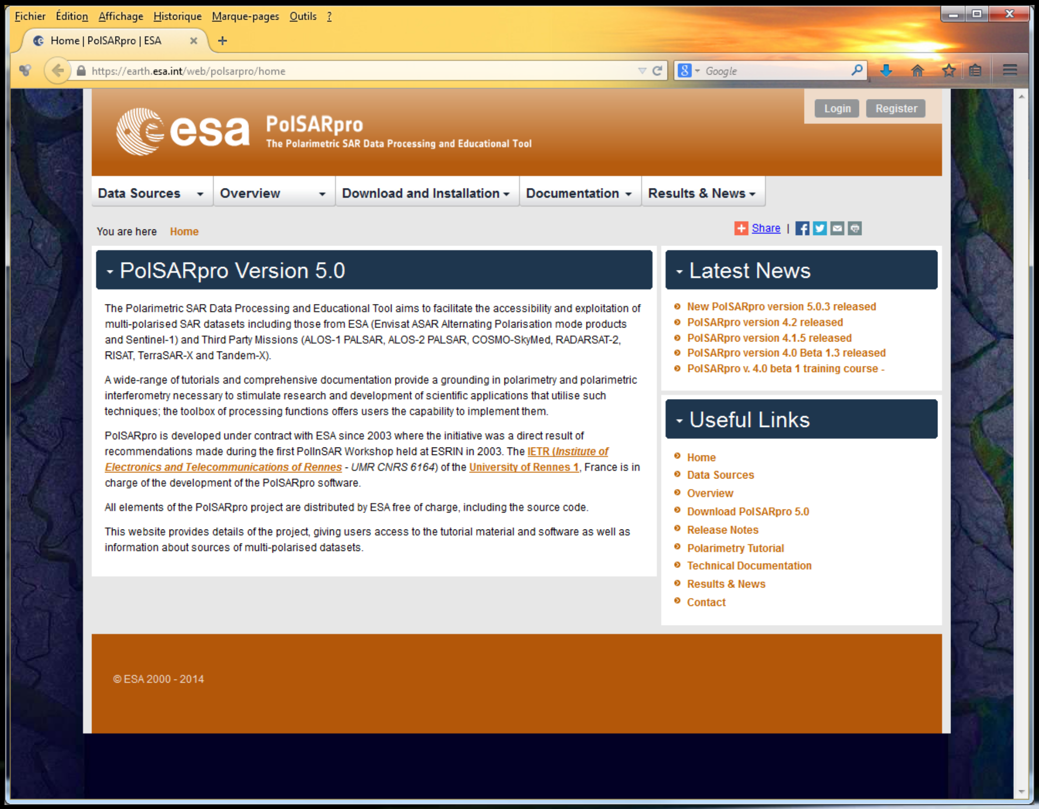


The Tool is free download on the Internet from the ESA Web Portal (Earthnet) at :
https://earth.esa.int/web/polsarpro



PolSARpro v5.1

http://earth.esa.int/web/polsarpro



The Web Site provides

- Details of the project
- Access to the tutorial and software
- Information about status of the development
- Demonstration Sample Datasets

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Learning / Training

Next P.I Generations

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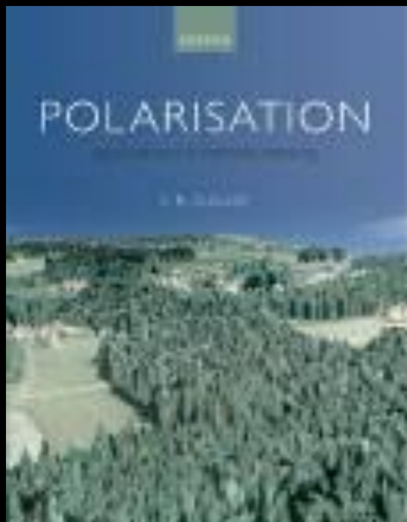
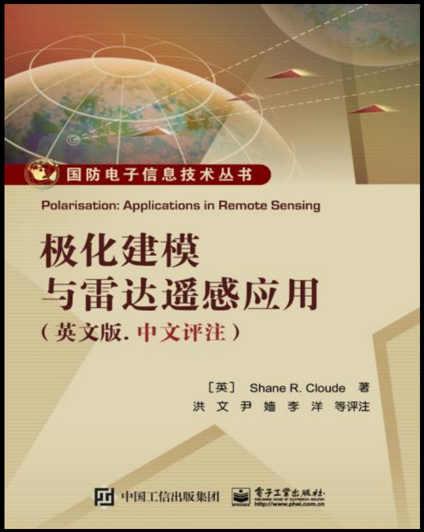
Books On Polarimetric Radar
SAR, Polarimetric Interferometry



Polarisation: Applications in Remote Sensing
Shane R. CLOUDE
Oxford University Press, October 2009, pp 352
ISBN: 978-0199569731

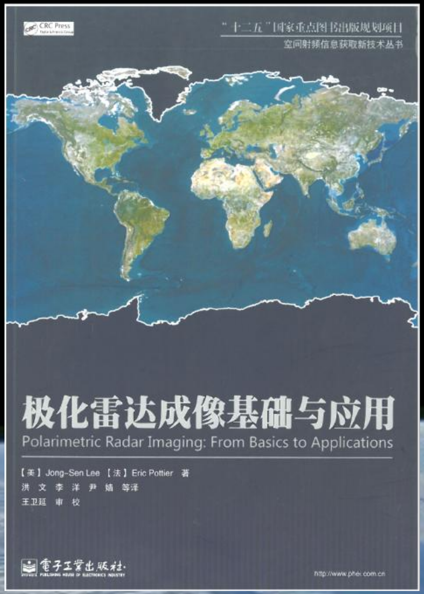
Polarimetric Radar Imaging: From basics to applications
Jong-Sen LEE – Eric POTTIER
CRC Press; 1st ed., February 2009, pp 422
ISBN: 978-1420054972

Books On Polarimetric Radar SAR, Polarimetric Interferometry



Polarisation: Applications in Remote Sensing
Shane R. CLOUDE
Oxford University Press, October 2009, pp 352
ISBN: 978-0199569731

Wen HONG et al.



Polarimetric Radar Imaging: From basics to applications
Jong-Sen LEE – Eric POTTIER
CRC Press; 1st ed., February 2009, pp 422
ISBN: 978-1420054972

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Educational Tools

Polarimetric SAR Data Processing and Educational Tool v5.0 - Menu

Environment Import Convert Process Display Calibration Quit About

PolSAR-Ap Project

WP360 : Review and update of the Basic Principles and Applications

(C. Lopez Martinez, E. Pottier)

1 Basic Principles of SAR Polarimetry

C. Lopez Martinez[1], E. Pottier[2]

[1] UPC Barcelona
[2] University of Rennes-1

1.1 Theory of radar polarimetry

1.1.1 Wave polarimetry

Polarimetry refers specifically to the vector nature of the electromagnetic waves, whereas radar polarimetry is the science of acquiring, processing and analyzing the polarization state of an electromagnetic wave in radar applications. This section summarizes the main theoretical aspects necessary for a correct processing and interpretation of the polarimetric information. As a result, the first part presents the so called wave polarimetry that deals with the representation and the understanding of the polarization state of an electromagnetic wave. The second part introduces the concept of scattering polarimetry. This concept collects the topic of inferring the properties of a given target, from a polarimetric point of view, given the incident and the scattered polarizes electromagnetic waves.

1.1.1.1 Electromagnetic waves and wave polarization descriptors

The generation, the propagation, as well as the interaction with matter of the electric and the magnetic waves are governed by the Maxwell's equations [1]. For an electromagnetic wave that is propagating in the $\hat{\mathbf{z}}$ direction, the real electric wave can be decomposed into two orthogonal components $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$, admitting the following vector formulation:

$$\vec{\mathrm{E}}(z,t)=\begin{bmatrix}E_x\\E_y\\E_z\end{bmatrix}=\begin{bmatrix}E_{0x}\cos(\omega t-kz+\delta_x)\\E_{0y}\cos(\omega t-kz+\delta_y)\\0\end{bmatrix} \quad (1.1)$$

which may be also considered in a complex form

...ering a better exploitation of the ...3], the Wishart distribution allows ...se model for all the elements of the ...del has been exploited for PolSAR ...ed that if the filtering process is ...of speckle, depending on the corre- ...an improved estimation of the dif- ...the covariance or coherency matri-

...Francisco (USA) where the colour code ...th the LLMMSE speckle filter.

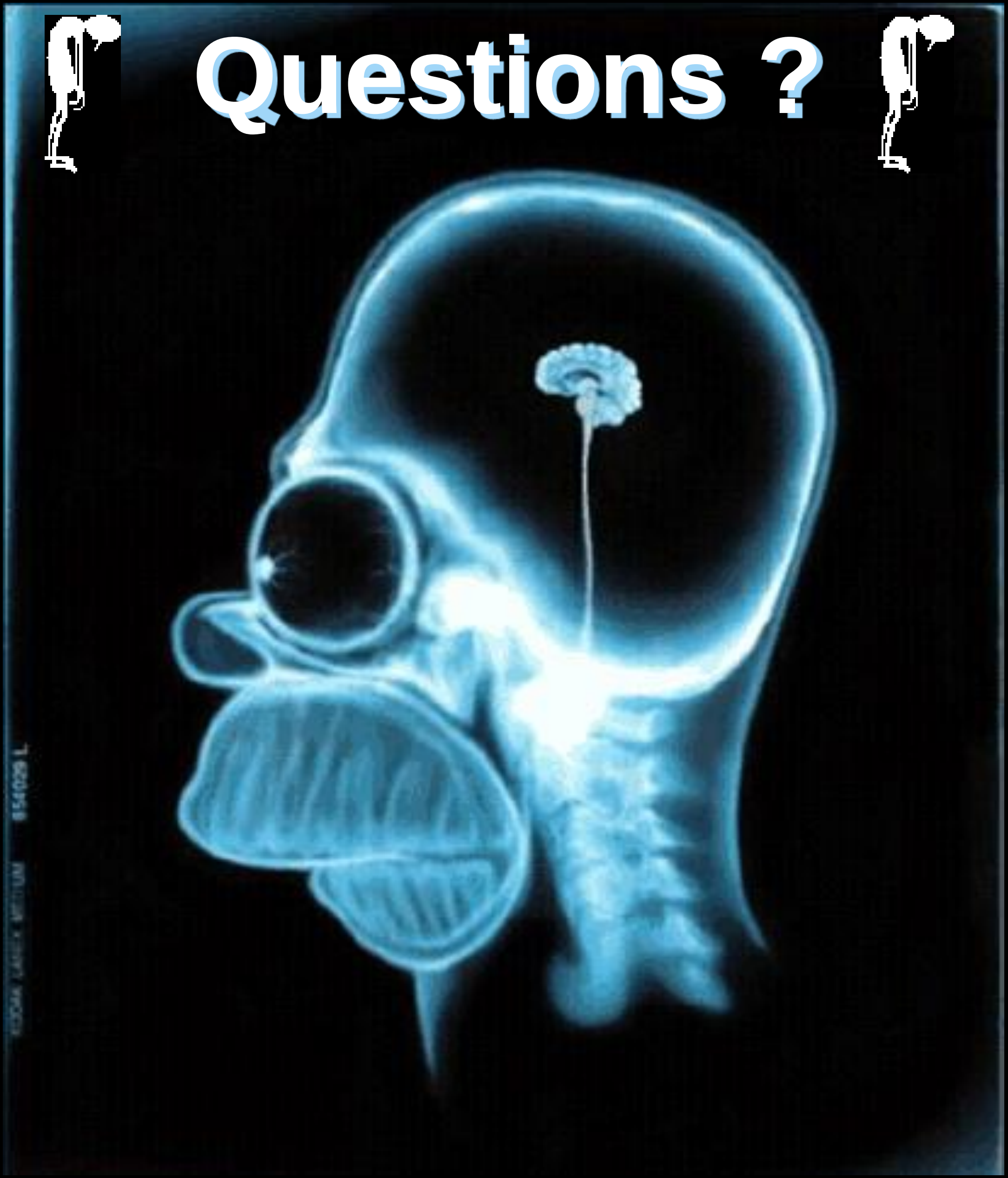
...rancisco (USA) where the colour code is: ...the BPT speckle filter.

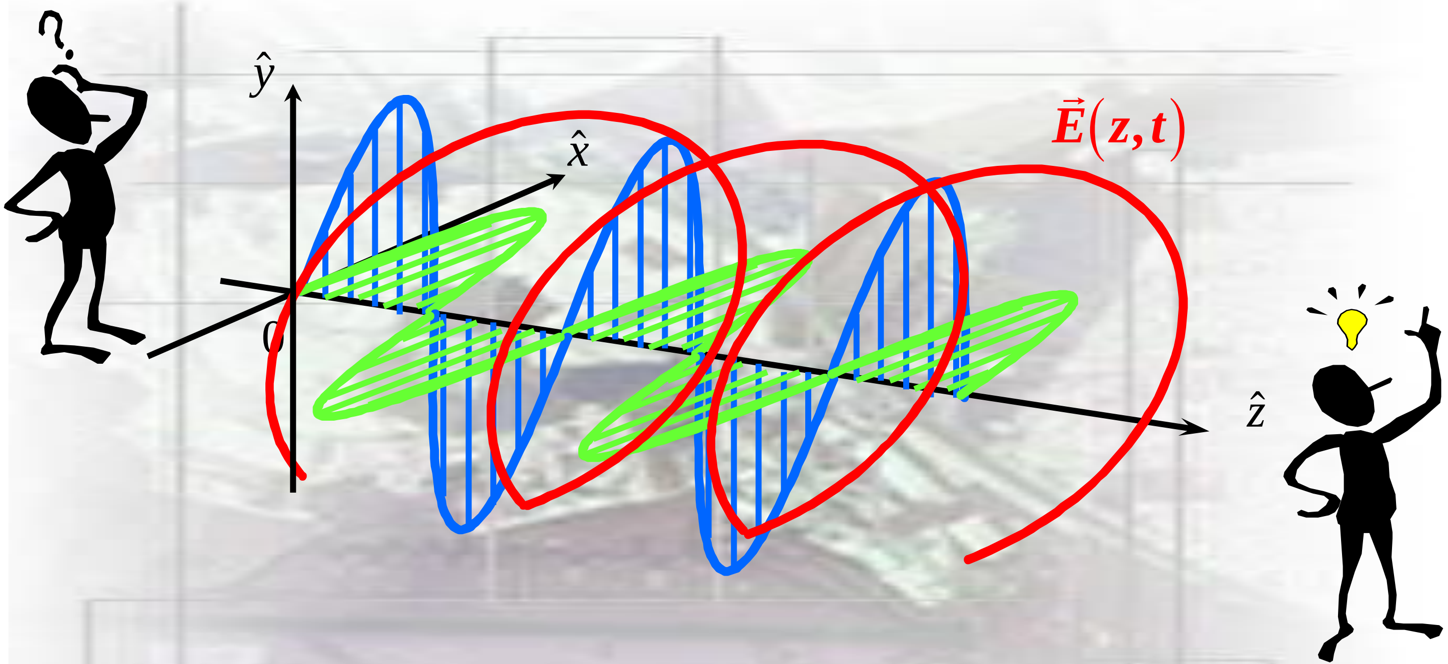
Beyond all the PolSAR data filtering techniques presented in this Section, there exist a wide variety of similar approaches in the related literature, where a com-

PolSARpro v5.0 - Run Trace

Open Window Warning
Close Window Warning

Questions ?





DATASETS

AIRSAR JPL

DC8

P, L, C-Band (Quad)

© Google Earth

SAN FRANCISCO BAY

© Google Earth

|HH+VV| $T_{11}=2A_0$

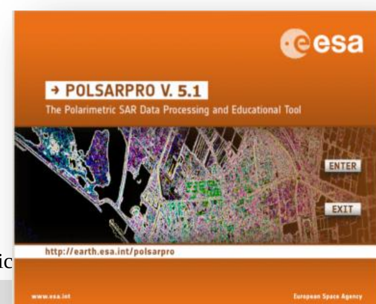
|HV| $T_{33}=B_0-B$

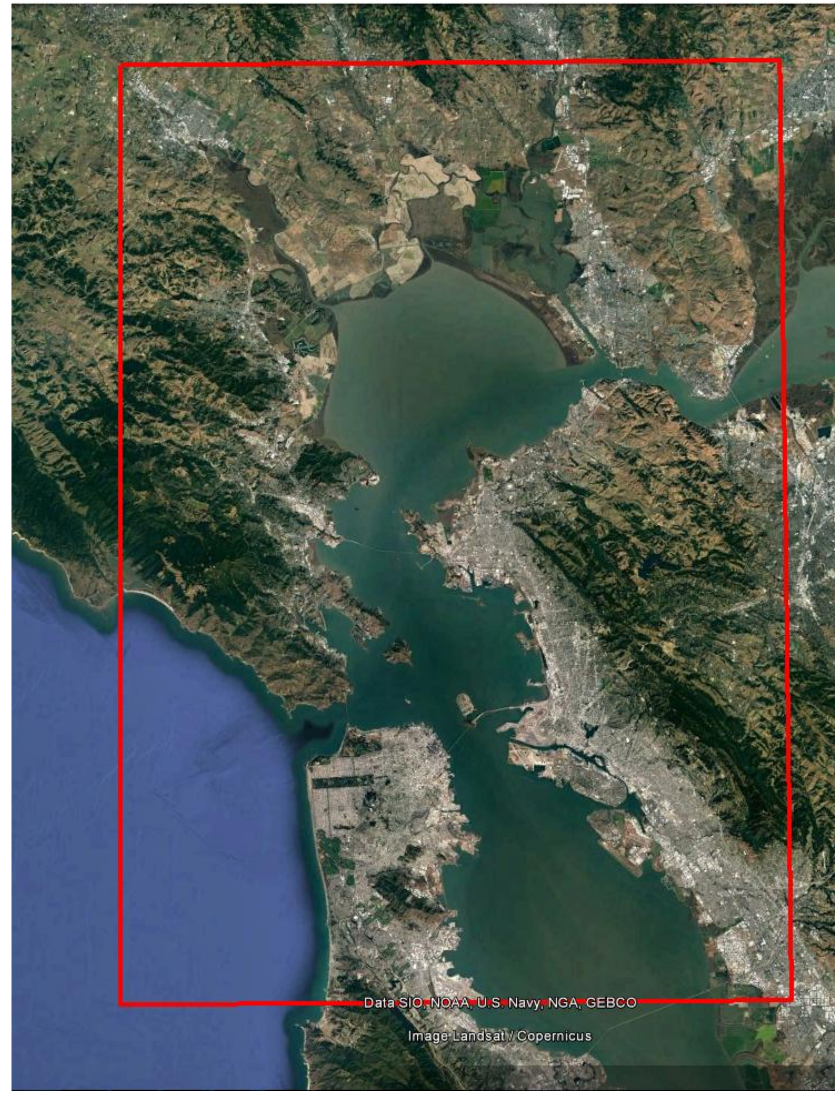
|HH-VV| $T_{22}=B_0+B$

SAN FRANCISCO BAY

ALOS2 - PALSAR

L-Band (Quad - 2015)





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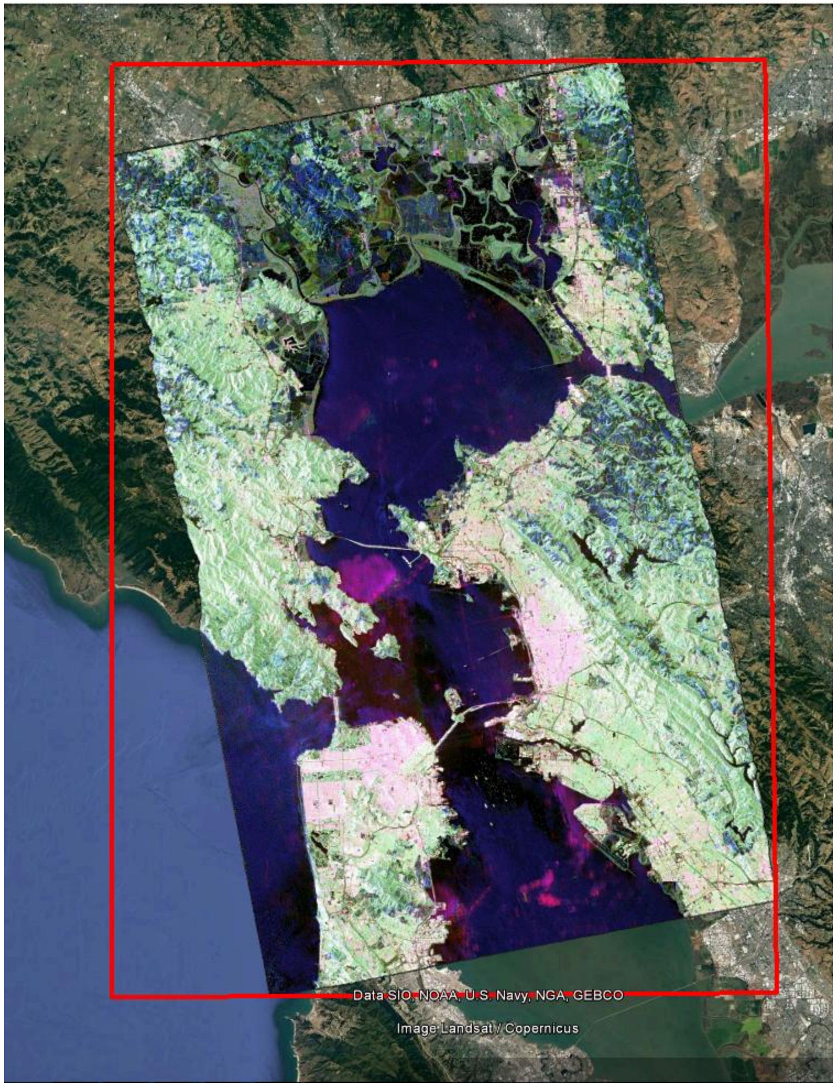
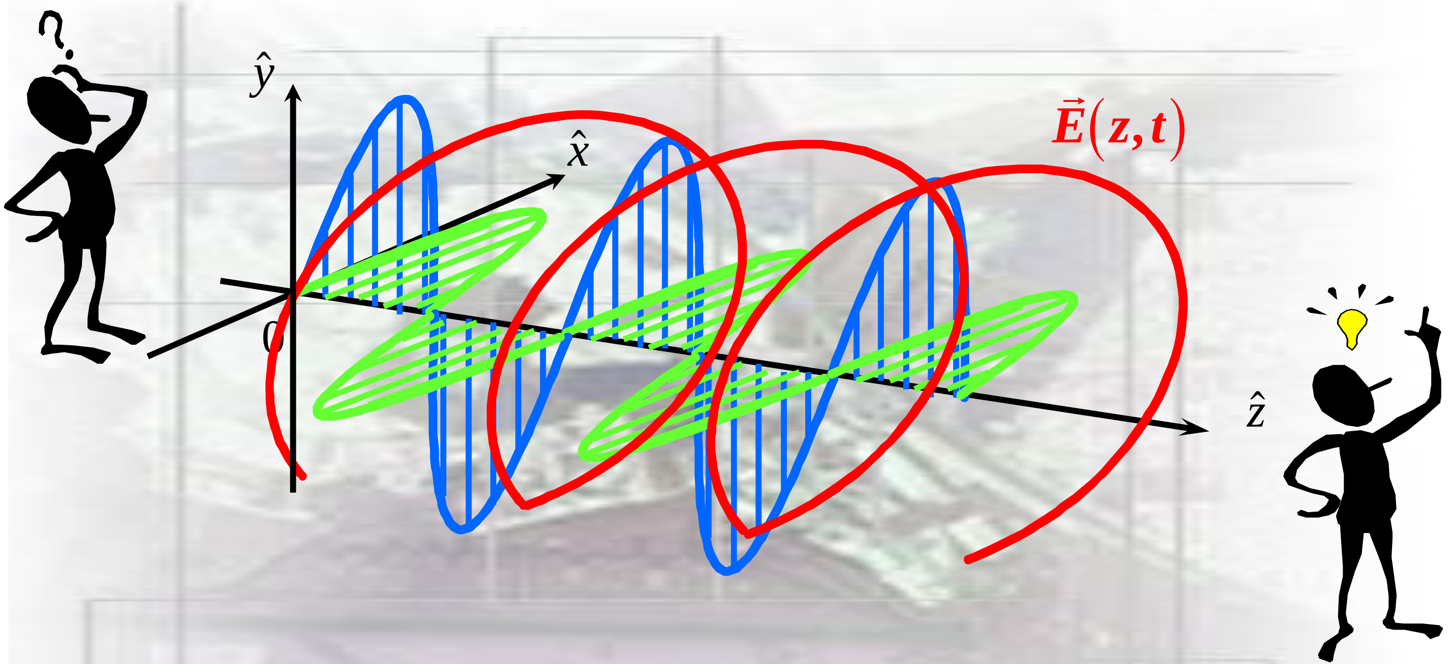
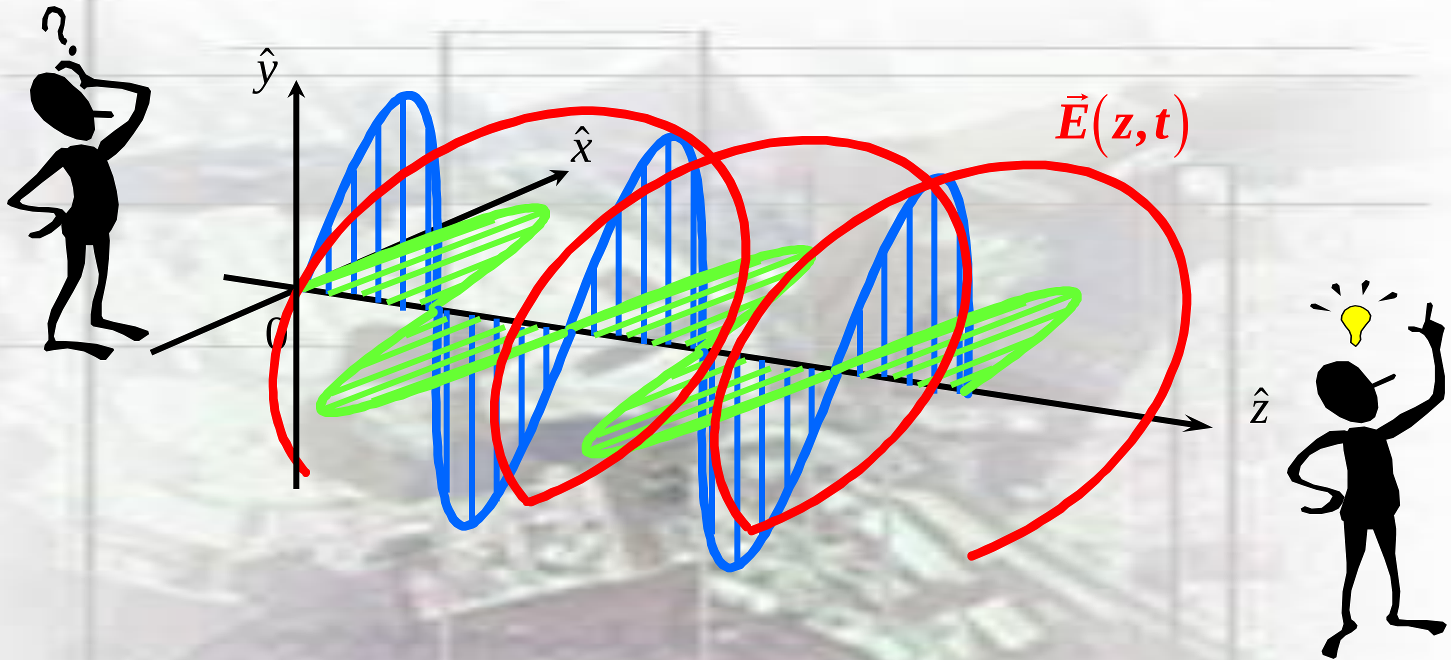
SAN FRANCISCO BAY





BASIC CONCEPTS



WAVE POLARIMETRY

PROPAGATION EQUATION

REAL ELECTRIC FIELD VECTOR $\vec{E}(z,t)$

MAXWELL EQUATIONS

MAXWELL – FARADAY EQUATION $\nabla \wedge \vec{E}(z,t) = -\frac{\partial \vec{B}(z,t)}{\partial t}$

MAXWELL – AMPERE EQUATION $\nabla \wedge \vec{H}(z,t) = \vec{J}_T(z,t)$

GAUSS THEOREM $\nabla \cdot \vec{D}(z,t) = \rho(z,t)$

$$\nabla \cdot \vec{B}(z,t) = 0$$

$$\vec{J}_T(z,t) = \vec{J}_C(z,t) + \frac{\partial \vec{D}(z,t)}{\partial t}$$

$$\vec{J}_C(z,t) = \sigma \vec{E}(z,t)$$

$$\vec{B}(z,t) = \mu \vec{H}(z,t)$$

$$\vec{D}(z,t) = \varepsilon \vec{E}(z,t)$$

σ (Conductivity)

μ (Permeability)

ε (Permittivity)

PROPAGATION EQUATION

$$\nabla \wedge \left(\nabla \wedge \vec{A}\right) = \nabla\left(\nabla \cdot \vec{A}\right) - \nabla \cdot \left(\nabla \vec{A}\right)$$

PROPAGATION EQUATION

$$\nabla^2 \vec{E}(z,t) - \mu\varepsilon \frac{\partial^2 \vec{E}(z,t)}{\partial t^2} - \mu\sigma \frac{\partial \vec{E}(z,t)}{\partial t} = -\frac{1}{\varepsilon}\frac{\partial \rho(z,t)}{\partial t}$$

HELMHOLTZ PROPAGATION EQUATION

$$\nabla^2 \vec{E}(z,t) - \mu\varepsilon \frac{\partial^2 \vec{E}(z,t)}{\partial t^2} = 0$$

Source Free, Linear, Homogeneous, Isotropic, Dielectric and lossless Medium

PROPAGATION EQUATION

COMPLEX ELECTRIC FIELD VECTOR $\underline{E}(z)$ **With:** $\vec{E}(z,t) = \Re\left(\underline{E}(z)e^{j\omega t}\right)$

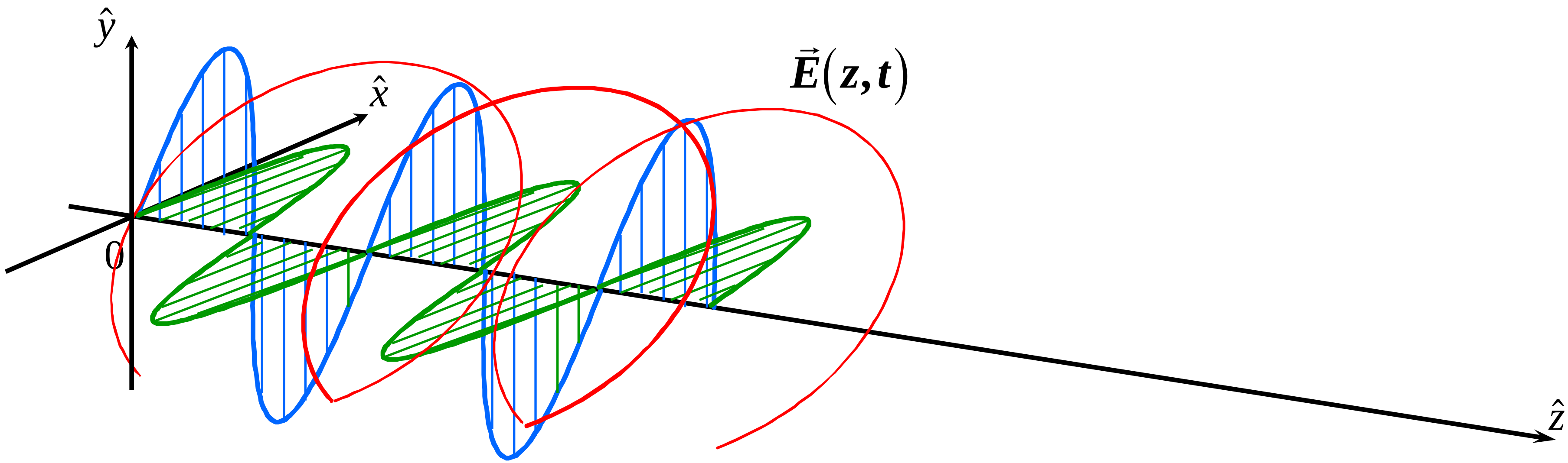
HELMHOLTZ PROPAGATION EQUATION

$$\nabla^2 \underline{E}(z) + \underline{k}^2 \underline{E}(z) = 0$$

SOLUTION: $\underline{E}(z) = \underline{E}e^{-jkz}$ **With:** $\underline{E} = \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = \begin{bmatrix} E_{ox}e^{j\delta_x} \\ E_{oy}e^{j\delta_y} \\ E_{oz}e^{j\delta_z} \end{bmatrix}$

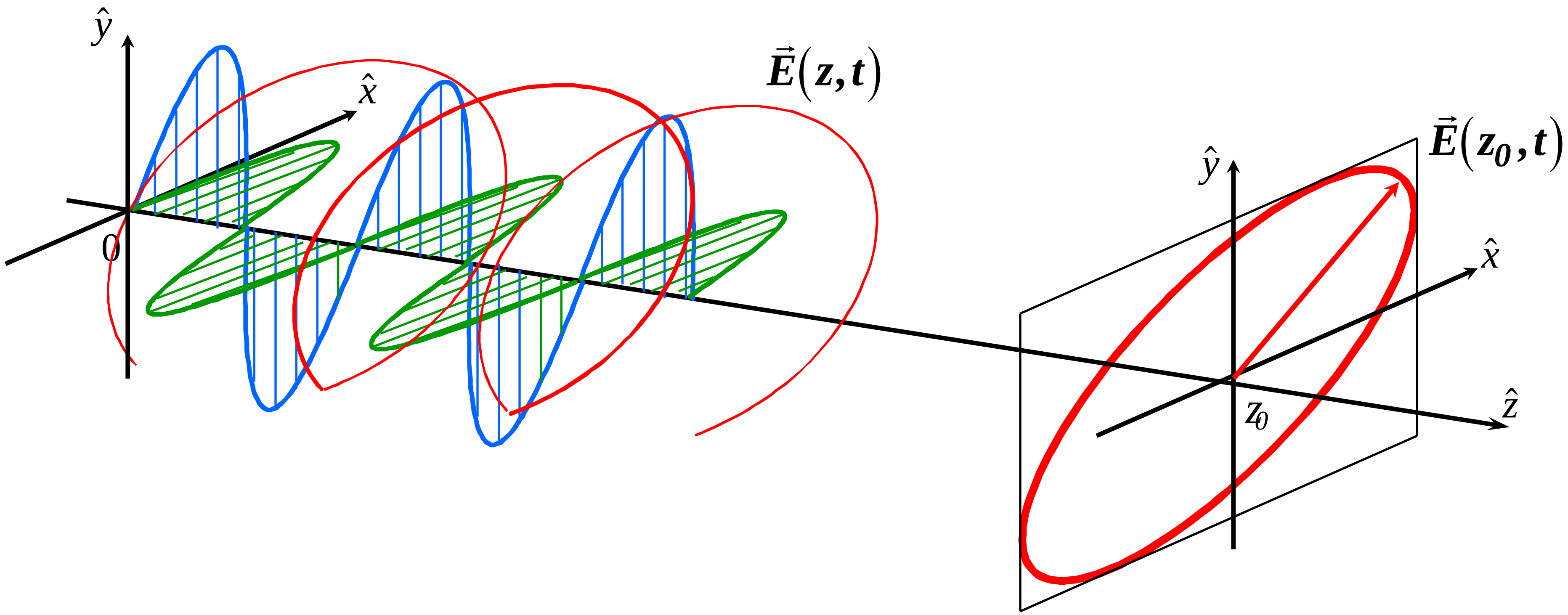
SINUSOIDAL PLANE WAVE

$$\nabla \cdot \vec{E}(z,t) = 0 \quad \Rightarrow \quad \frac{\partial E_z}{\partial z} = 0$$



REAL ELECTRIC FIELD VECTOR

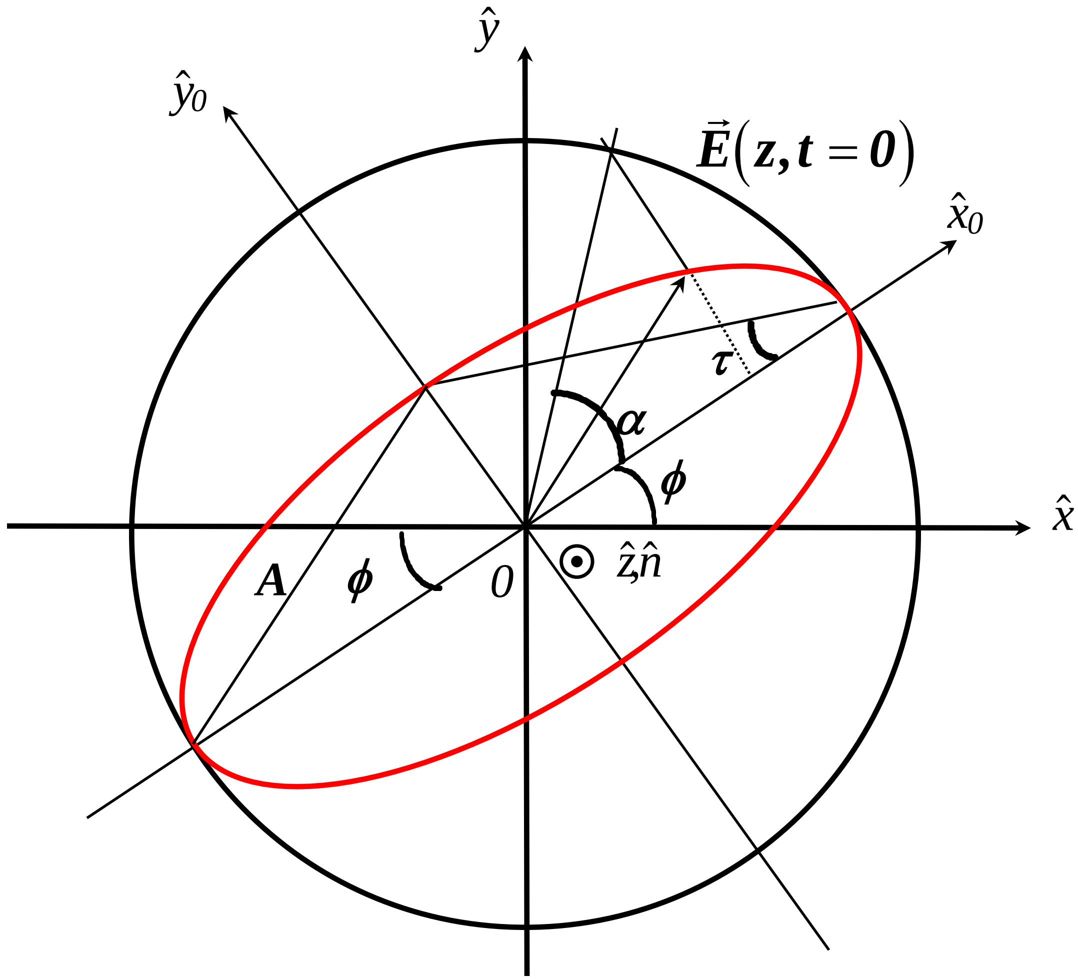
$$\vec{E}(z,t) = \begin{cases} E_x = E_{0x}\cos(\omega t - kz - \delta_x) \\ E_y = E_{0y}\cos(\omega t - kz - \delta_y) \\ E_z = 0 \end{cases}$$



THE REAL ELECTRIC FIELD VECTOR MOVES IN TIME ALONG AN ELLIPSE

$$\left(\frac{E_x}{E_{0x}}\right)^2 - 2\frac{E_x E_y}{E_{0x} E_{0y}}\cos(\delta) + \left(\frac{E_y}{E_{0y}}\right)^2 = \sin^2(\delta)$$

With: $\delta = \delta_y - \delta_x$



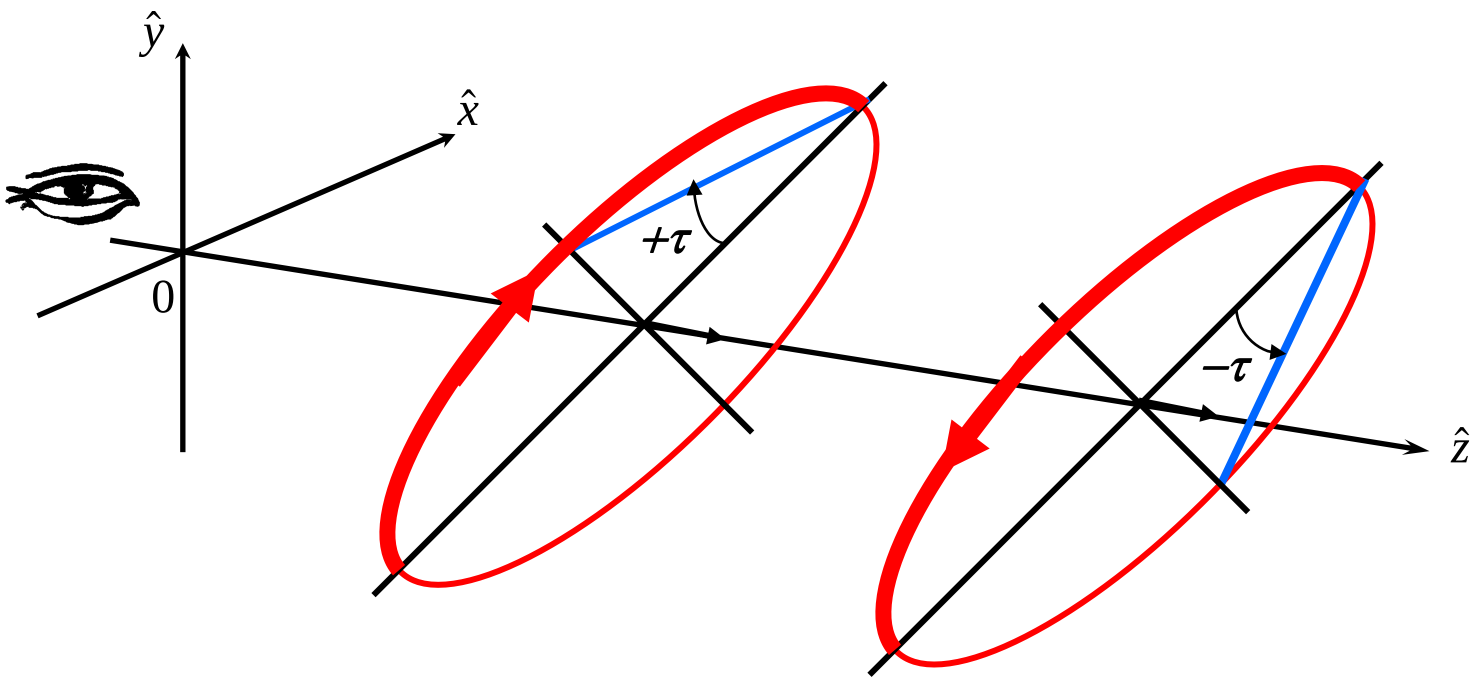
A : **WAVE AMPLITUDE**

α : **ABSOLUTE PHASE**

ϕ : **ORIENTATION ANGLE** $-\frac{\pi}{2} \leq \phi \leq \frac{\pi}{2}$

τ : **ELLIPTICITY ANGLE** $0 \leq \tau \leq \frac{\pi}{4}$

ROTATION SENSE: LOOKING INTO THE DIRECTION OF THE WAVE PROPAGATION



ANTI-CLOCKWISE ROTATION

LEFT HANDED POLARISATION

ELLIPTICITY ANGLE : $\tau > 0$

CLOCKWISE ROTATION

RIGHT HANDED POLARISATION

ELLIPTICITY ANGLE : $\tau < 0$

$$-\frac{\pi}{4} \leq \tau \leq \frac{\pi}{4}$$

JONES VECTOR

REAL ELECTRIC FIELD VECTOR

$$\vec{E}(z,t) = \begin{cases} E_x = E_{0x}\cos(\omega t - kz - \delta_x) \\ E_y = E_{0y}\cos(\omega t - kz - \delta_y) \\ E_z = 0 \end{cases}$$

PHASOR = JONES VECTOR

$$\underline{E} = \begin{bmatrix} E_x = E_{ox} e^{j\delta_x} \\ E_y = E_{oy} e^{j\delta_y} \end{bmatrix}$$

With: $\vec{E}(z,t) = \Re\left(\underline{E} e^{j(\omega t - kz)} \right)$

GEOMETRICAL PARAMETERS

ABSOLUTE PHASE

$$\alpha = \delta_x$$

AMPLITUDE

$$A = \sqrt{E_{0x}^2 + E_{0y}^2}$$

ORIENTATION ANGLE

$$\tan 2\phi = 2\frac{E_{0x}E_{0y}}{E_{0x}^2 - E_{0y}^2}\cos\delta$$

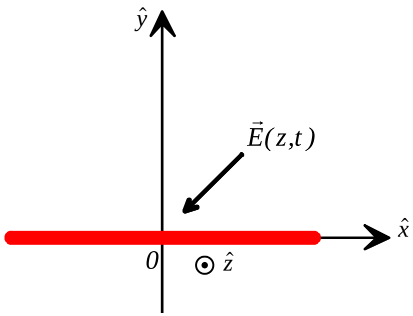
ELLIPTICITY ANGLE

$$\sin 2\tau = 2\frac{E_{0x}E_{0y}}{E_{0x}^2 + E_{0y}^2}\sin\delta$$

POLARISATION HANDENESS: $Sign(\tau)$

JONES VECTOR

HORIZONTAL POLARISATION STATE

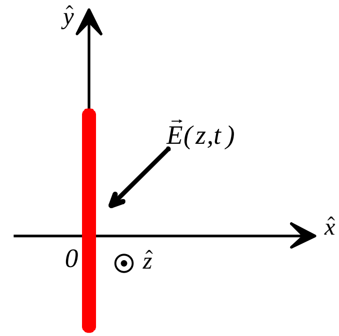


$$\underline{H} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\phi = 0$$

$$\tau = 0$$

VERTICAL POLARISATION STATE

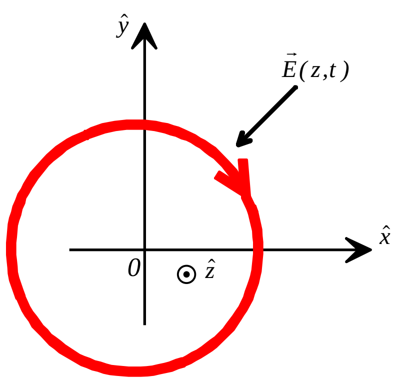


$$\underline{V} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\phi = \frac{\pi}{2}$$

$$\tau = 0$$

LEFT CIRCULAR POLARISATION STATE

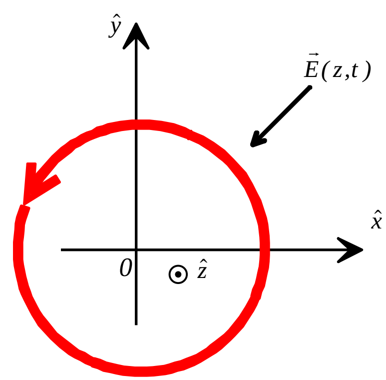


$$\underline{LC} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ j \end{bmatrix}$$

$$-\frac{\pi}{2} \leq \phi \leq +\frac{\pi}{2}$$

$$\tau = +\frac{\pi}{4}$$

RIGHT CIRCULAR POLARISATION STATE



$$\underline{RC} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -j \end{bmatrix}$$

$$-\frac{\pi}{2} \leq \phi \leq +\frac{\pi}{2}$$

$$\tau = -\frac{\pi}{4}$$

Special Unitary Matrices Group and Jones Vector

$$\underline{E}_{(\hat{x},\hat{y})} = \begin{bmatrix} E_x = E_{ox} e^{j\delta_x} \\ E_y = E_{oy} e^{j\delta_y} \end{bmatrix} = Ae^{j\alpha} \begin{bmatrix} \cos(\phi)\cos(\tau) - j\sin(\phi)\sin(\tau) \\ \sin(\phi)\cos(\tau) + j\cos(\phi)\sin(\tau) \end{bmatrix}$$

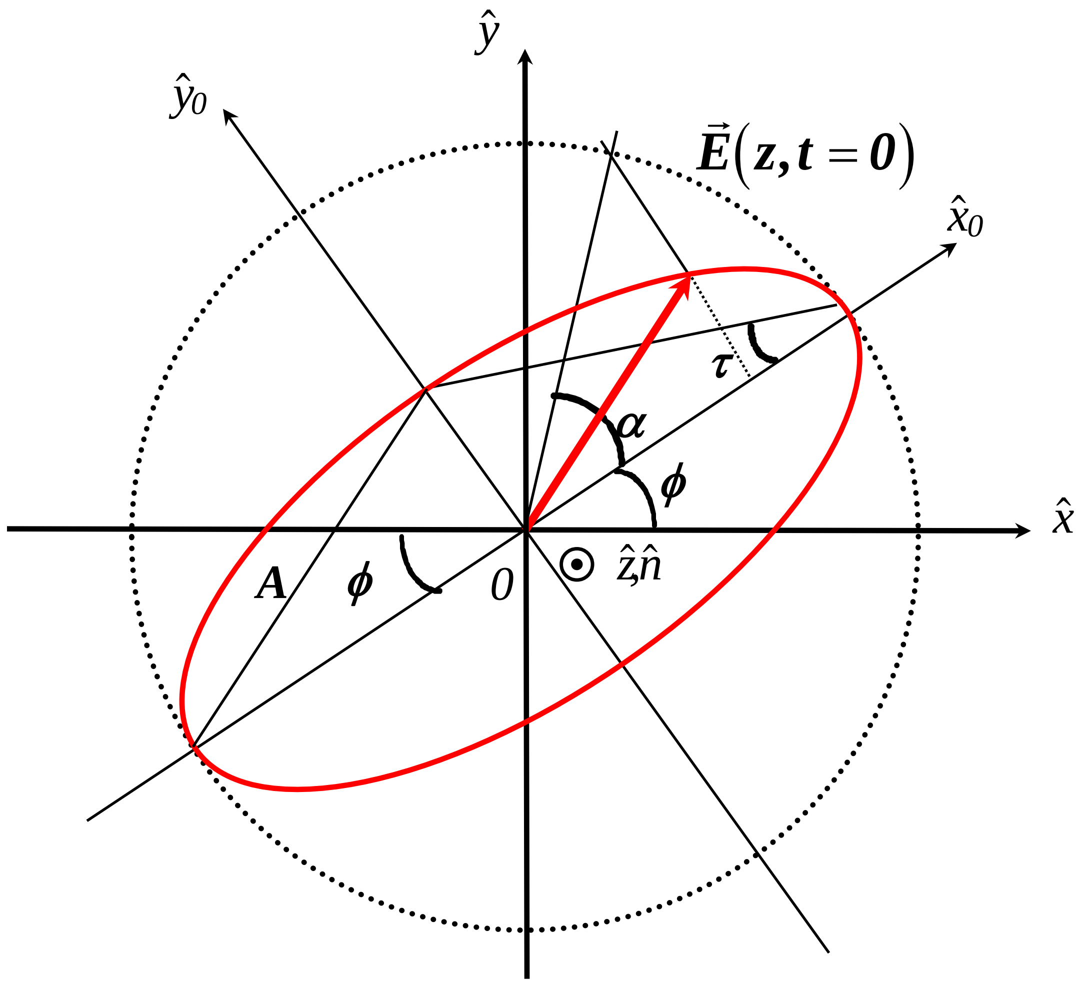
$$\Downarrow$$

$$\underline{E}_{(\hat{x},\hat{y})} = Ae^{j\alpha} \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix} \begin{bmatrix} \cos(\tau) \\ j\sin(\tau) \end{bmatrix}$$

$$\Downarrow$$

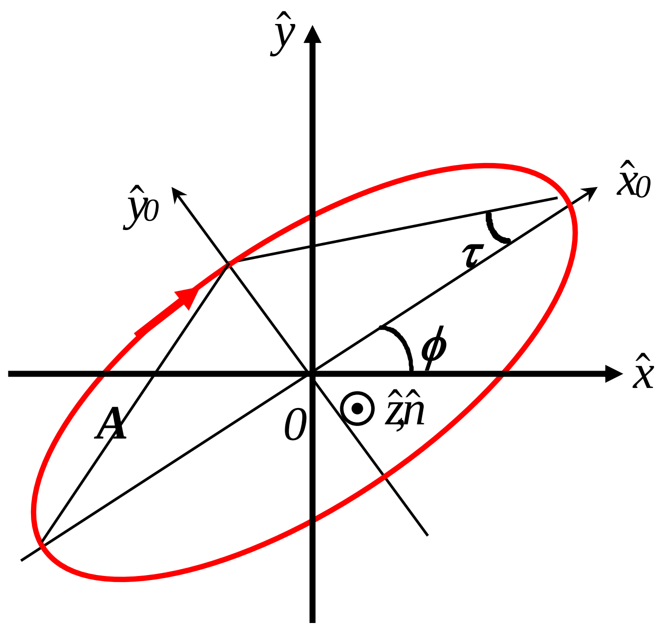
$$\underline{E}_{(\hat{x},\hat{y})} = A \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix} \begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix} \begin{bmatrix} e^{j\alpha} & 0 \\ 0 & e^{-j\alpha} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

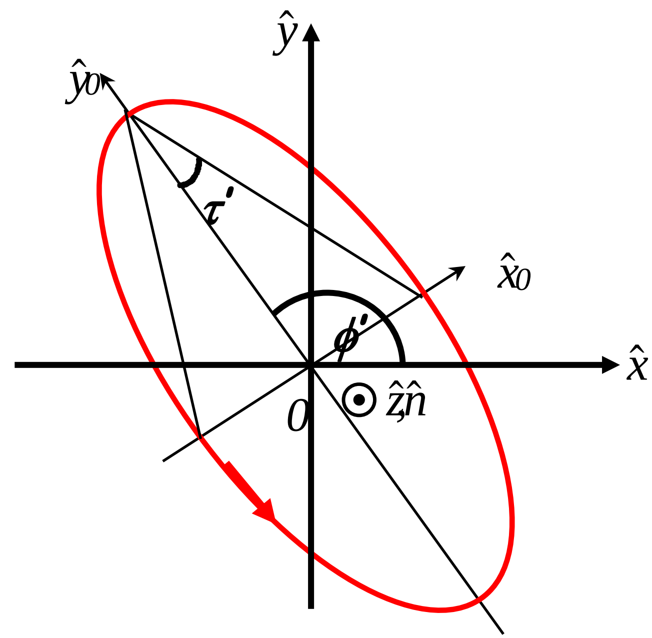
JONES VECTOR



$$\underline{E} = A\begin{bmatrix}\cos(\phi) & -\sin(\phi)\\ \sin(\phi) & \cos(\phi)\end{bmatrix}\begin{bmatrix}\cos(\tau) & j\sin(\tau)\\ j\sin(\tau) & \cos(\tau)\end{bmatrix}\begin{bmatrix}e^{-j\alpha} & 0\\ 0 & e^{j\alpha}\end{bmatrix}\hat{u}_x$$

ORTHOGONAL JONES VECTOR





ORTHOGONALITY CONDITIONS

$$(\phi, \tau) \mapsto \begin{cases} \phi' = \phi + \frac{\pi}{2} \\ \tau' = -\tau \end{cases}$$

➡ CHANGE OF POLARISATION HANDENESS

JONES VECTOR

$$\underline{E} = A\begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix}\begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix}\begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}\hat{u}_x$$

ORTHOGONAL JONES VECTOR

$$\underline{E}_{\perp} = A\begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix}\begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix}\begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}\hat{u}_y$$

↓

$$[\underline{E}, \underline{E}_{\perp}] = A\begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix}\begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix}\begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}[\hat{u}_x, \hat{u}_y]$$

↓

ELLIPTICAL BASIS TRANSFORMATION

ELLIPTICAL BASIS TRANSFORMATION

ORTHOGONAL JONES VECTORS

$$[\underline{E}, \underline{E}_{\perp}] = A \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix} \begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix} \begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix} [\hat{u}_x, \hat{u}_y]$$

↓

SU(2) : SPECIAL UNITARY TRANSFORMATION MATRIX

$$[U(\phi,\tau,\alpha)] = \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix} \begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix} \begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}$$

$[U_2(\phi)]$ $\quad$ $[U_2(\tau)]$ $\quad$ $[U_2(\alpha)]$

$[U_2][U_2]^{T*} = [I_{D2}]$ CONSERVATION OF THE WAVE ENERGY

$\det([U_2]) = +1$ ENSURES THE CORRECT PHASE DEFINITION

SU(2) : SPECIAL UNITARY TRANSFORMATION MATRIX

$$[U(\phi,\tau,\alpha)] = \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix} \begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix} \begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}$$

ELLIPTICAL BASIS TRANSFORMATION MATRIX

$$\begin{aligned} \left[U_{(\underline{A},\underline{A}_\perp)\mapsto(\underline{B},\underline{B}_\perp)}\right] &= [U(\phi,\tau,\alpha)]^{-1} \\ &= \begin{bmatrix} e^{j\alpha} & 0 \\ 0 & e^{-j\alpha} \end{bmatrix} \begin{bmatrix} \cos(\tau) & -j\sin(\tau) \\ -j\sin(\tau) & \cos(\tau) \end{bmatrix} \begin{bmatrix} \cos(\phi) & \sin(\phi) \\ -\sin(\phi) & \cos(\phi) \end{bmatrix} \end{aligned}$$

STOKES VECTOR

REAL REPRESENTATION OF THE POLARISATION STATE OF A MONOCHROMATIC WAVE

$$\underline{E} \cdot \underline{E}^{T*} = \begin{bmatrix} E_x E_x^* & E_x E_y^* \\ E_y E_x^* & E_y E_y^* \end{bmatrix}$$

PAULI MATRICES GROUP

$$\sigma_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \sigma_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad \sigma_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \sigma_3 = \begin{bmatrix} 0 & -j \\ j & 0 \end{bmatrix}$$

$$\underline{E} \cdot \underline{E}^{T*} = \frac{1}{2}\{g_0\sigma_0 + g_1\sigma_1 + g_2\sigma_2 + g_3\sigma_3\} = \frac{1}{2}\begin{bmatrix} g_0 + g_1 & g_2 - jg_3 \\ g_2 + jg_3 & g_0 - g_1 \end{bmatrix}$$

$\{g_0, g_1, g_2, g_3\}$ **STOKES PARAMETERS**

JONES VECTOR

$$\underline{E} = \begin{bmatrix} E_x = E_{ox} e^{j\delta_x} \\ E_y = E_{oy} e^{j\delta_y} \end{bmatrix}$$

STOKES VECTOR

$$\underline{g}_{\underline{E}} = \begin{bmatrix} g_0 = |E_x|^2 + |E_y|^2 \\ g_1 = |E_x|^2 - |E_y|^2 \\ g_2 = 2\Re\left(E_x E_y^*\right) \\ g_3 = -2\Im\left(E_x E_y^*\right) \end{bmatrix}$$

WAVE POLARISATION STATE ESTIMATION FROM INTENSITIES MEASUREMENTS

STOKES VECTOR

$$\underline{g}_{\underline{E}} = \begin{bmatrix} g_0 = E_{0x}^2 + E_{0y}^2 \\ g_1 = E_{0x}^2 - E_{0y}^2 \\ g_2 = 2E_{0x}E_{0y}\cos(\delta) \\ g_3 = 2E_{0x}E_{0y}\sin(\delta) \end{bmatrix} = \begin{bmatrix} g_0 = A^2 \\ g_1 = A^2 \cos 2\phi \cos 2\tau \\ g_2 = A^2 \sin 2\phi \cos 2\tau \\ g_3 = A^2 \sin 2\tau \end{bmatrix}$$

GEOMETRICAL PARAMETERS

ORIENTATION ANGLE

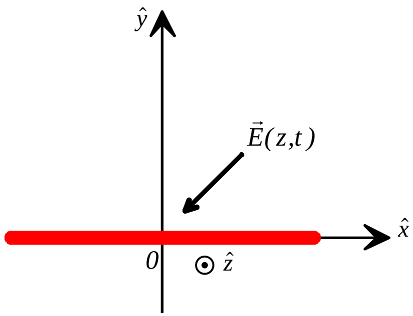
$$\tan 2\phi = 2\frac{E_{0x}E_{0y}}{E_{0x}^2 - E_{0y}^2}\cos\delta = \frac{g_2}{g_1}$$

ELLIPTICITY ANGLE

$$\sin 2\tau = 2\frac{E_{0x}E_{0y}}{E_{0x}^2 + E_{0y}^2}\sin\delta = \frac{g_3}{g_0}$$

STOKES VECTOR

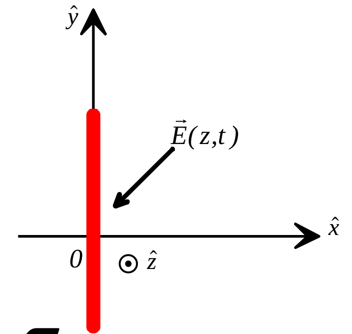
HORIZONTAL POLARISATION STATE



$$\underline{g}_{\underline{H}} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\phi = 0 \qquad \tau = 0$$

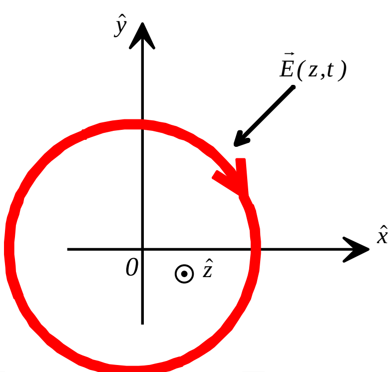
VERTICAL POLARISATION STATE



$$\underline{g}_{\underline{V}} = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

$$\phi = \frac{\pi}{2} \qquad \tau = 0$$

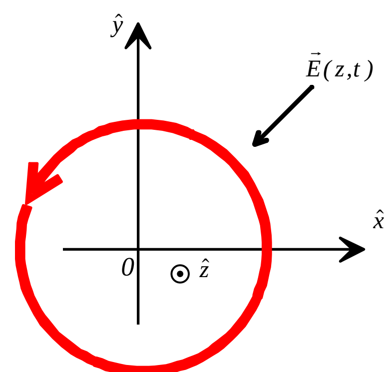
LEFT CIRCULAR POLARISATION STATE



$$\underline{g}_{\underline{LC}} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$-\frac{\pi}{2} \leq \phi \leq +\frac{\pi}{2} \qquad \tau = +\frac{\pi}{4}$$

RIGHT CIRCULAR POLARISATION STATE



$$\underline{g}_{\underline{RC}} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

$$-\frac{\pi}{2} \leq \phi \leq +\frac{\pi}{2} \qquad \tau = -\frac{\pi}{4}$$

O(4) UNITARY ROTATION ROUP

JONES VECTOR

$$\underline{E} = A\begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix}\begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix}\begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}\hat{u}_x$$

$[U_2(\phi)]$ $[U_2(\tau)]$ $[U_2(\alpha)]$

HOMOMORPHISM SU(2) - O(3)

$$[O_3(2\theta)]_{p,q} = \frac{1}{2}Tr\left([U_2(\theta)]^{T*}\sigma_p[U_2(\theta)]\sigma_q\right)$$

(σ_p, σ_q) : Pauli Matrices

STOKES VECTOR

$$\underline{g}_{\underline{E}} = A^2\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\phi) & -\sin(2\phi) & 0 \\ 0 & \sin(2\phi) & \cos(2\phi) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\tau) & 0 & -\sin(2\tau) \\ 0 & 0 & 1 & 0 \\ 0 & \sin(2\tau) & 0 & \cos(2\tau) \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos(2\alpha) & -\sin(2\alpha) \\ 0 & 0 & \sin(2\alpha) & \cos(2\alpha) \end{bmatrix}\underline{g}_{\hat{u}_x}$$

$[O_3(2\phi)]$ $[O_3(2\tau)]$ $[O_3(2\alpha)]$

ELLIPTICAL BASIS TRANSFORMATION

SPECIAL UNITARY SU(2) GROUP

$$[U_2] = \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix} \begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix} \begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}$$

$$[U_2(\phi)] \qquad [U_2(\tau)] \qquad [U_2(\alpha)]$$

HOMOMORPHISM SU(2) - O(3)

$$[O_3(2\theta)]_{p,q} = \frac{1}{2} Tr\left([U_2(\theta)]^{T*} \sigma_p [U_2(\theta)] \sigma_q\right)$$

(σ_p, σ_q) : Pauli Matrices

O(3) UNITARY GROUP

$$\begin{bmatrix} \cos 2\phi & -\sin 2\phi & 0 \\ \sin 2\phi & \cos 2\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 2\tau & 0 & -\sin 2\tau \\ 0 & 1 & 0 \\ \sin 2\tau & 0 & \cos 2\tau \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 2\alpha & -\sin 2\alpha \\ 0 & \sin 2\alpha & \cos 2\alpha \end{bmatrix}$$

$$[O_3(2\phi)] \qquad [O_3(2\tau)] \qquad [O_3(2\alpha)]$$

STOKES VECTOR

$$\underline{g}_{\underline{E}} = \begin{bmatrix} g_0 \\ g_1 \\ g_2 \\ g_3 \end{bmatrix} = \begin{bmatrix} |E_x|^2 + |E_y|^2 \\ |E_x|^2 - |E_y|^2 \\ 2\Re\left(E_x E_y^*\right) \\ -2\Im\left(E_x E_y^*\right) \end{bmatrix} = \begin{bmatrix} E_{0x}^2 + E_{0y}^2 \\ E_{0x}^2 - E_{0y}^2 \\ 2E_{0x}E_{0y}\cos(\delta) \\ 2E_{0x}E_{0y}\sin(\delta) \end{bmatrix} = \begin{bmatrix} A^2 \\ A^2 \cos 2\phi \cos 2\tau \\ A^2 \sin 2\phi \cos 2\tau \\ A^2 \sin 2\tau \end{bmatrix}$$

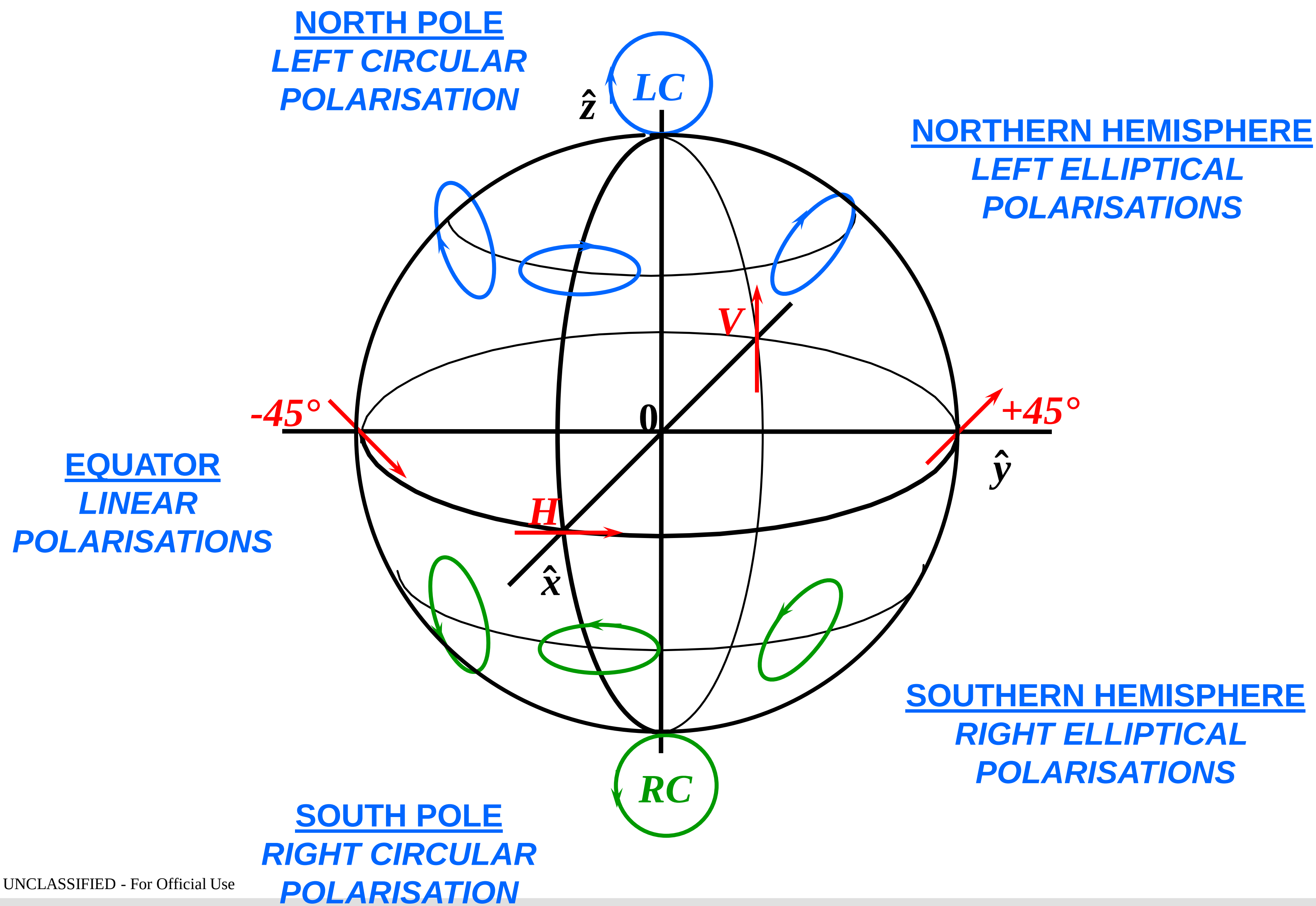
$\{g_0\}$ TOTAL WAVE INTENSITY

$\{g_1, g_2, g_3\}$ POLARISED WAVE INTENSITIES

$$g_0^2 = g_1^2 + g_2^2 + g_3^2$$

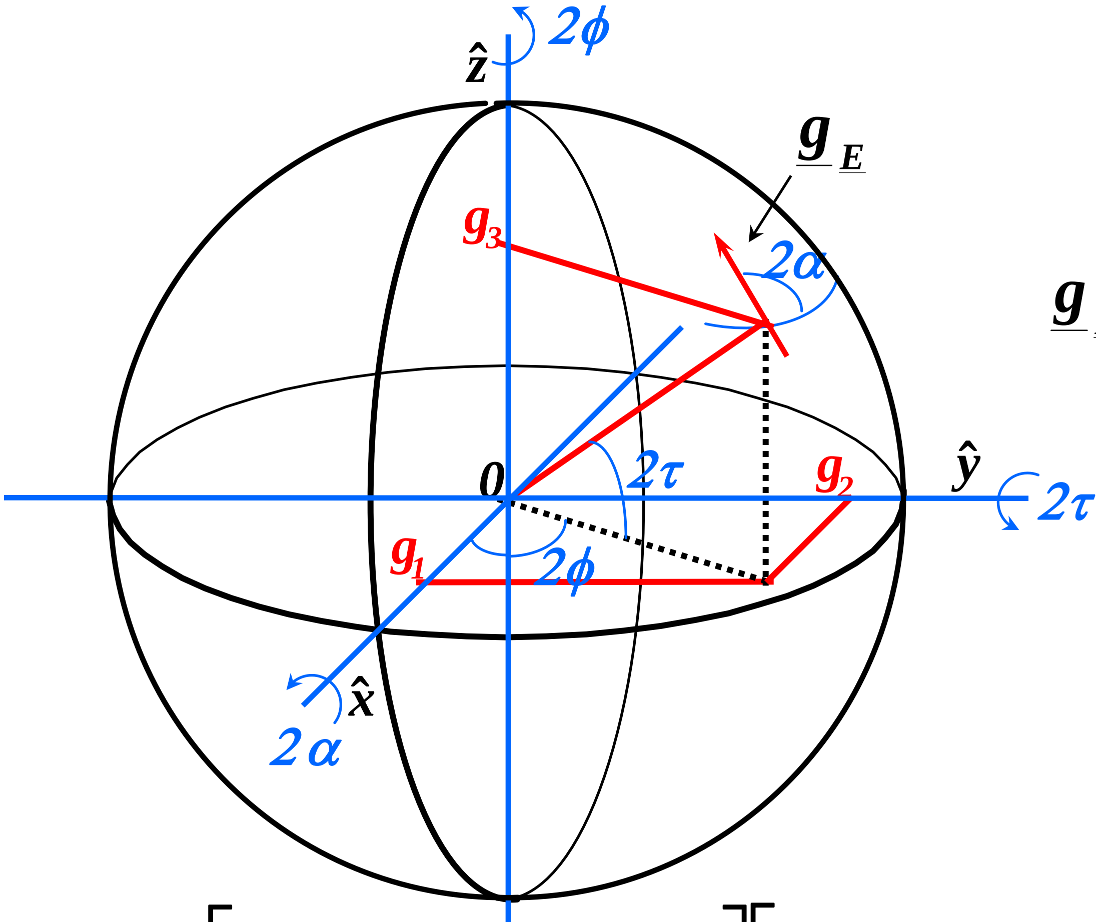
WAVE FULLY POLARISED

$\{g_1, g_2, g_3\}$ Spherical Coordinates of a point P on a sphere with radius g_0



NORTH POLE
LEFT CIRCULAR
POLARISATION
LC
ẑ
NORTHERN HEMISPHERE
LEFT ELLIPTICAL
POLARISATIONS
V
-45°
0
+45°
ŷ
EQUATOR
LINEAR
POLARISATIONS
H
x̂
SOUTHERN HEMISPHERE
RIGHT ELLIPTICAL
POLARISATIONS
RC
SOUTH POLE
RIGHT CIRCULAR
POLARISATION

POINCARE SPHERE



$$\underline{g}_E = \begin{bmatrix} g_0 \\ g_1 \\ g_2 \\ g_3 \end{bmatrix} = \begin{bmatrix} A^2 \\ A^2 \cos 2\phi \cos 2\tau \\ A^2 \sin 2\phi \cos 2\tau \\ A^2 \sin 2\tau \end{bmatrix}$$

$$\underline{g}_E = A^2 \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\phi) & -\sin(2\phi) & 0 \\ 0 & \sin(2\phi) & \cos(2\phi) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{[O_3(2\phi)]} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\tau) & 0 & -\sin(2\tau) \\ 0 & 0 & 1 & 0 \\ 0 & \sin(2\tau) & 0 & \cos(2\tau) \end{bmatrix}}_{[O_3(2\tau)]} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos(2\alpha) & -\sin(2\alpha) \\ 0 & 0 & \sin(2\alpha) & \cos(2\alpha) \end{bmatrix}}_{[O_3(2\alpha)]} \underline{g}_{\hat{u}_x}$$

POINCARE SPHERE

esa

JONES VECTOR

$$\underline{E} = \begin{bmatrix} E_x \\ E_y \end{bmatrix}$$

ORTHOGONAL JONES VECTOR

$$\underline{E}_\perp = \begin{bmatrix} E'_x \\ E'_y \end{bmatrix}$$

ORTHOGONALITY CONDITIONS

$$(\phi,\tau) \mapsto \begin{cases} \phi' = \phi + \frac{\pi}{2} \\ \tau' = -\tau \end{cases}$$

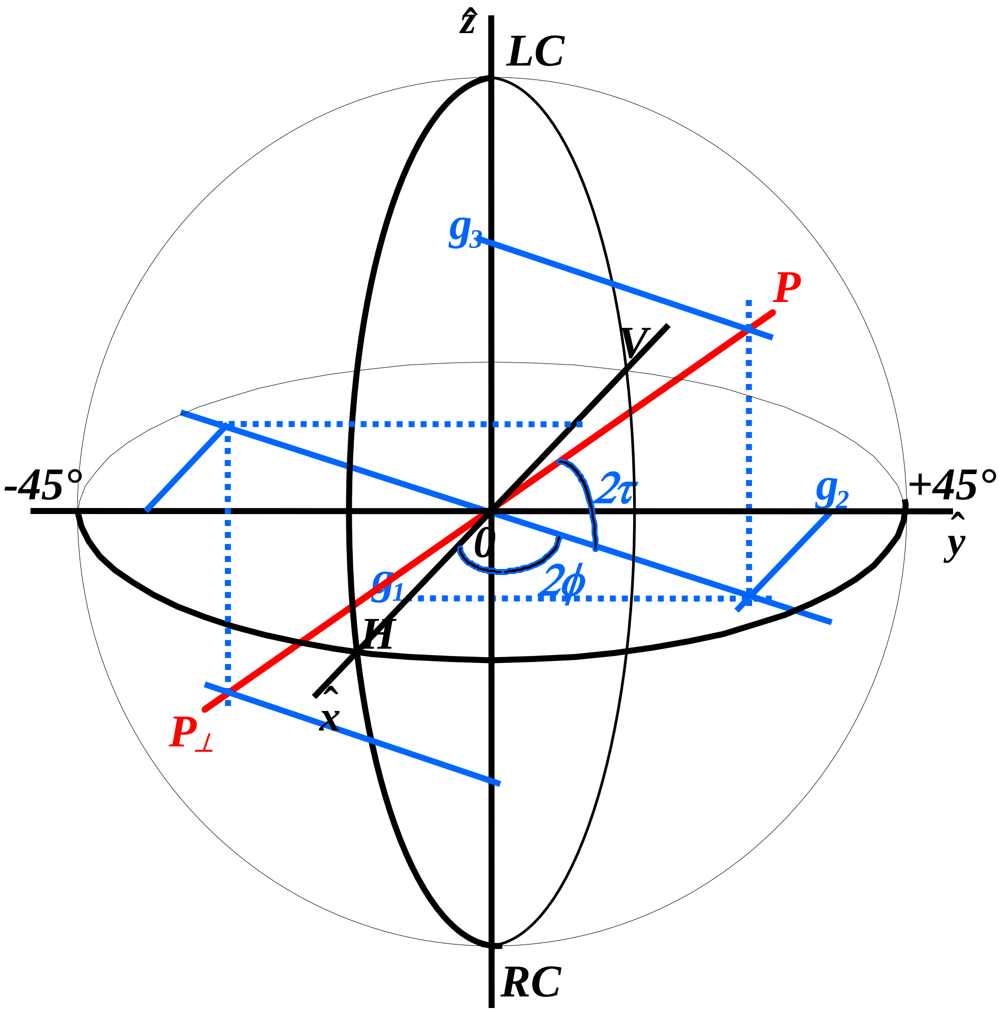
STOKES VECTOR

$$\underline{g}_{\underline{E}} = \begin{bmatrix} g_0 \\ g_1 \\ g_2 \\ g_3 \end{bmatrix} = \begin{bmatrix} A \\ A\cos 2\phi \cos 2\tau \\ A\sin 2\phi \cos 2\tau \\ A\sin 2\tau \end{bmatrix}$$

ORTHOGONAL STOKES VECTOR

$$\underline{g}_{\underline{E}_\perp} = \begin{bmatrix} g_0 \\ g_1 \\ g_2 \\ g_3 \end{bmatrix} = \begin{bmatrix} A \\ -A\cos 2\phi \cos 2\tau \\ -A\sin 2\phi \cos 2\tau \\ -A\sin 2\tau \end{bmatrix}$$

ORTHOGONALITY = ANTIPODALITY



STOKES VECTOR

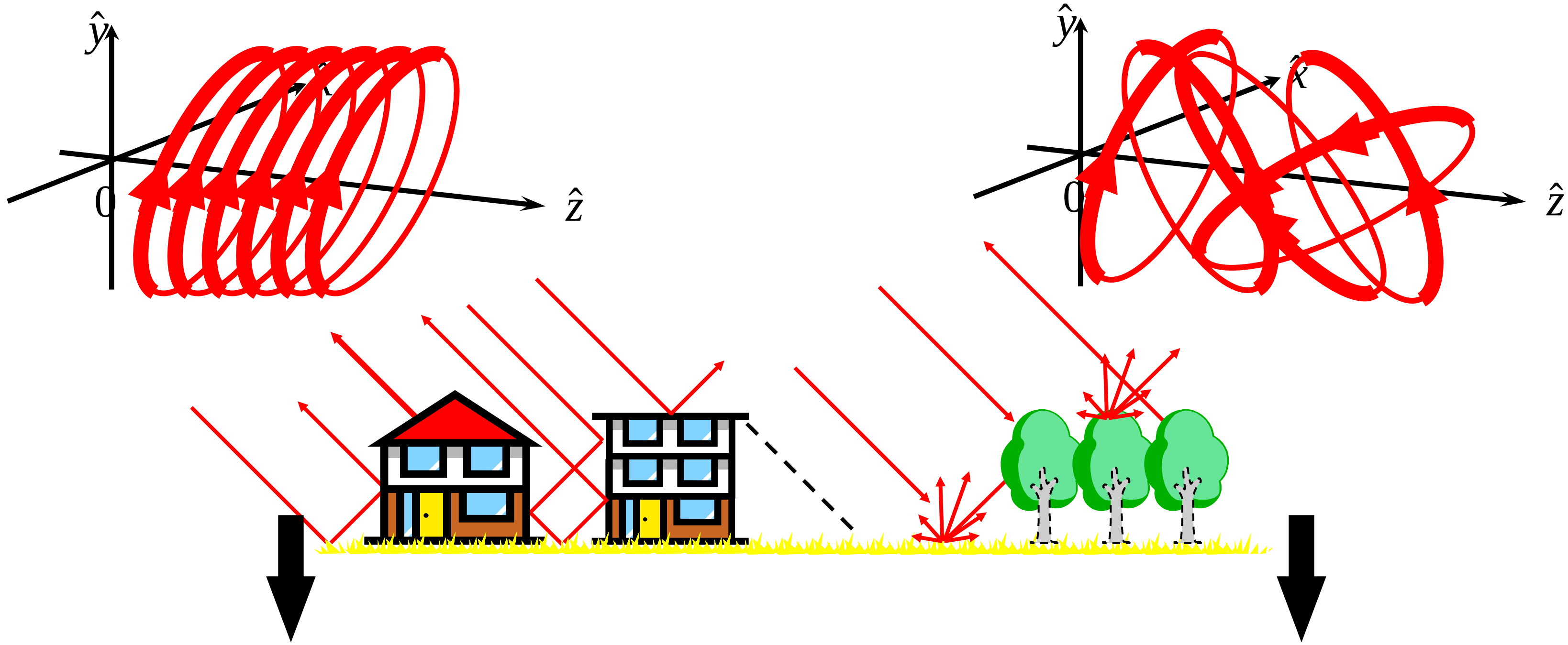
$$\underline{g}_E = \begin{bmatrix} g_0 \\ g_1 \\ g_2 \\ g_3 \end{bmatrix} = \begin{bmatrix} A \\ A\cos 2\phi \cos 2\tau \\ A\sin 2\phi \cos 2\tau \\ A\sin 2\tau \end{bmatrix}$$

ORTHOGONAL STOKES VECTOR

$$\underline{g}_{E_\perp} = \begin{bmatrix} g_0 \\ g_1 \\ g_2 \\ g_3 \end{bmatrix} = \begin{bmatrix} A \\ -A\cos 2\phi \cos 2\tau \\ -A\sin 2\phi \cos 2\tau \\ -A\sin 2\tau \end{bmatrix}$$

ORTHOGONALITY = ANTIPODALITY

PARTIALLY POLARISED WAVES



DETERMINISTIC SCATTERING

COMPLETELY POLARISED WAVE

RANDOM SCATTERING

PARTIALLY POLARISED WAVE

Polarisation Ellipse varies in time
Amplitude, Phase: Random processes

STATISTICAL DESCRIPTION

JONES VECTORS $\{\underline{E}\}$

↓

WAVE COVARIANCE MATRIX

$$\langle [J] \rangle = \langle \underline{E}\,\underline{E}^{T*} \rangle = \begin{bmatrix} \langle |E_x|^2 \rangle & \langle E_x E_y^* \rangle \\ \langle E_y E_x^* \rangle & \langle |E_y|^2 \rangle \end{bmatrix}$$

↕

$$\langle [J] \rangle = \frac{1}{2} \begin{bmatrix} \langle g_0 \rangle + \langle g_1 \rangle & \langle g_2 \rangle - j\langle g_3 \rangle \\ \langle g_2 \rangle + j\langle g_3 \rangle & \langle g_0 \rangle - \langle g_1 \rangle \end{bmatrix}$$

$$\langle g_0 \rangle^2 \geq \langle g_1 \rangle^2 + \langle g_2 \rangle^2 + \langle g_3 \rangle^2$$

PARTIALLY POLARISED WAVES

EIGENVALUES DECOMPOSITION

$$\langle [J] \rangle = [U_2] \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} [U_2]^{-1} = \lambda_1 \underline{u}_1 \underline{u}_1^{T*} + \lambda_2 \underline{u}_2 \underline{u}_2^{T*}$$

2 ORTHOGONAL EIGENVECTORS

$$[U_2] = [\underline{u}_1, \underline{u}_2]$$

2 REAL EIGENVALUES

$$\lambda_1 = \frac{1}{2}\left\{\langle g_0 \rangle + \sqrt{\langle g_1 \rangle^2 + \langle g_2 \rangle^2 + \langle g_3 \rangle^2}\right\}$$

$$\lambda_2 = \frac{1}{2}\left\{\langle g_0 \rangle - \sqrt{\langle g_1 \rangle^2 + \langle g_2 \rangle^2 + \langle g_3 \rangle^2}\right\}$$

PARTIALLY POLARISED WAVES DESCRIPTORS

Degree of Polarisation

$$DoP = \frac{\sqrt{\langle g_1\rangle^2 + \langle g_2\rangle^2 + \langle g_3\rangle^2}}{\langle g_0\rangle} = \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2} = \left(1 - \frac{4\det([J])}{Trace^2([J])}\right)$$

Polarised Wave Power / Total Wave Power

Anisotropy

$$0 \le DoP \le 1$$

Wave Entropy

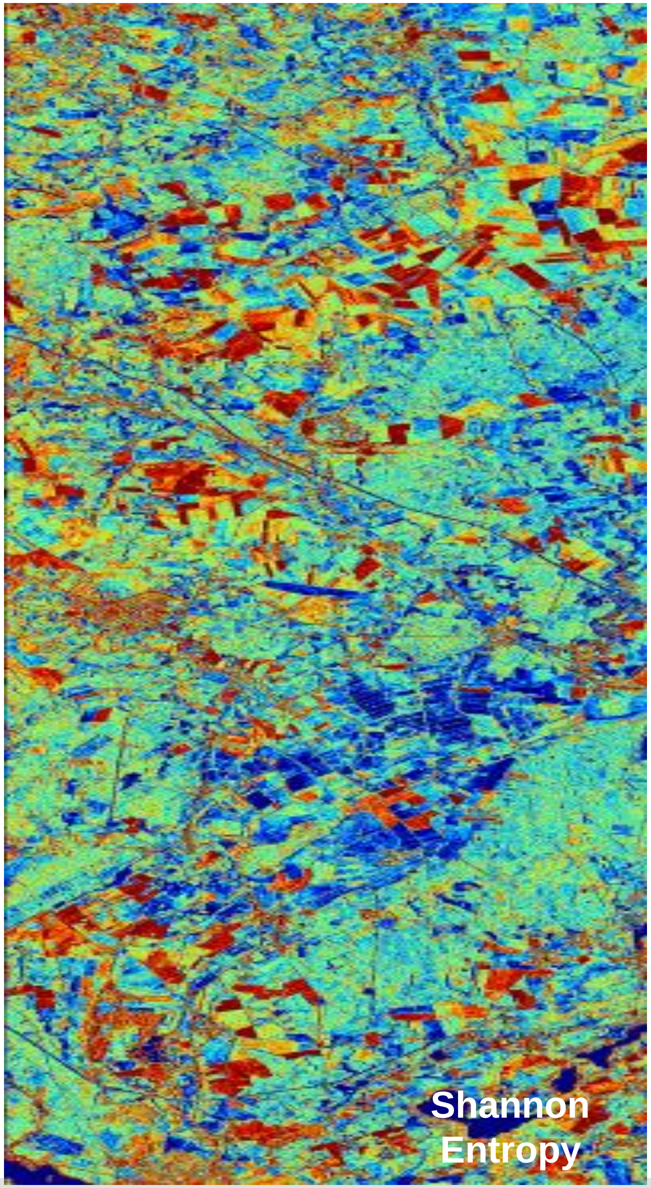
$$0 \le H \le 1$$

$$H = -\sum_{i=1}^{i=2} p_i \log_2(p_i)$$

With: $p_i = \frac{\lambda_i}{\lambda_1 + \lambda_2}$

Degree of randomness, statistical disorder

PARTIALLY POLARISED WAVES



WAVE DESCRIPTORS

MONOCHROMATIC PLANE WAVES

COMPLEX DOMAIN

JONES VECTOR $\underline{E} = \begin{bmatrix} E_x \\ E_y \end{bmatrix}$

REAL DOMAIN

STOKES VECTOR $\underline{g}_{\underline{E}} = \begin{bmatrix} g_0 \\ g_1 \\ g_2 \\ g_3 \end{bmatrix}$

PLANE WAVES FULLY DESCRIBED
BY 3 INDEPENDANT PARAMETERS

- $E_{0x}, E_{0x}, \delta = \delta_y - \delta_x$
- (A, ϕ, τ) or (A, γ, δ)
- $\{g_1, g_2, g_3\}$

WAVE POLARIMETRIC DIMENSION = 3

PARTIALLY POLARISED PLANE WAVES

COMPLEX DOMAIN

COVARIANCE MATRIX $\langle [J] \rangle = \langle \underline{E}\,\underline{E}^{T*} \rangle$

REAL DOMAIN

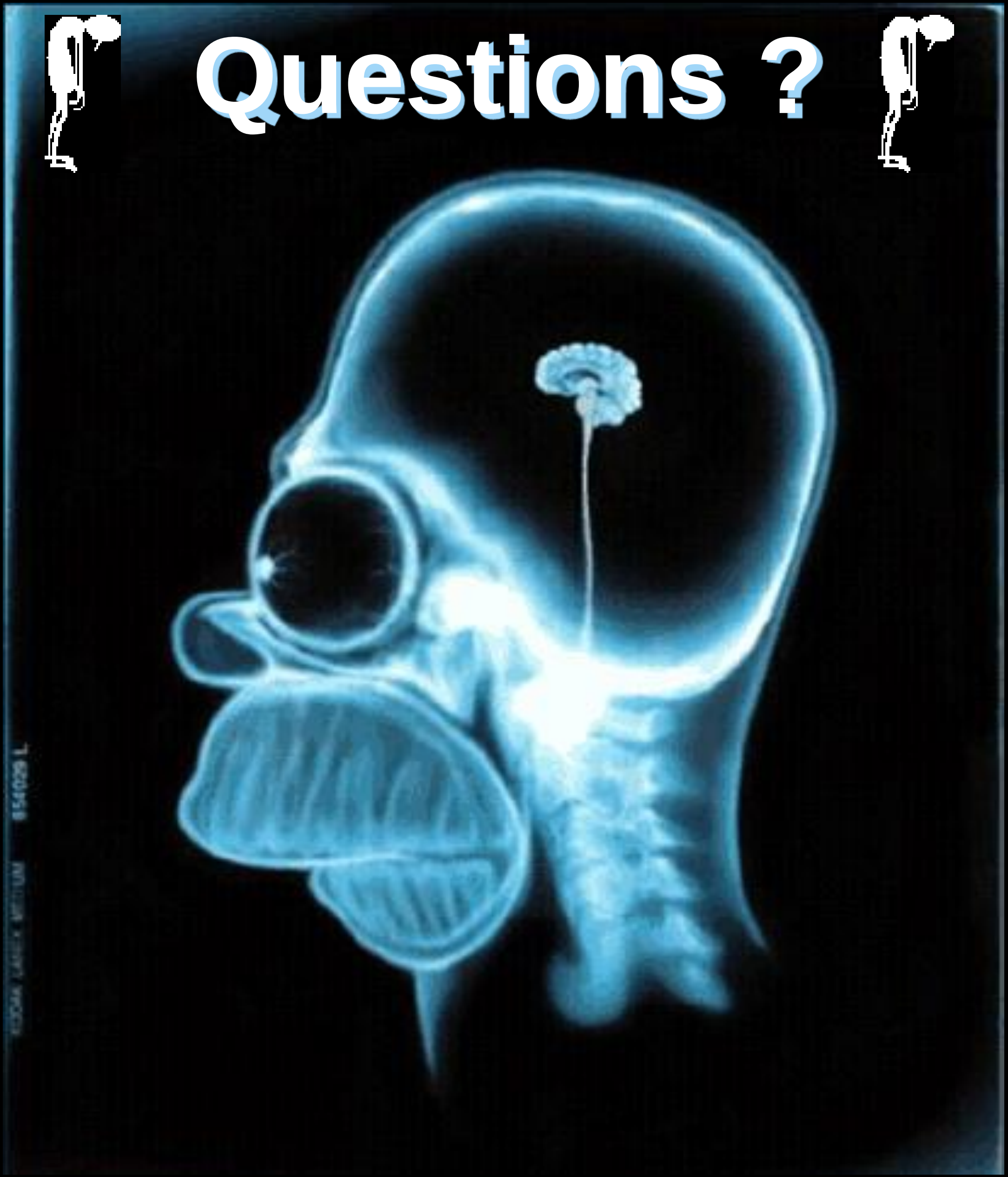
STOKES VECTOR $\langle \underline{g}_{\underline{E}} \rangle = \begin{bmatrix} \langle g_0 \rangle \\ \langle g_1 \rangle \\ \langle g_2 \rangle \\ \langle g_3 \rangle \end{bmatrix}$

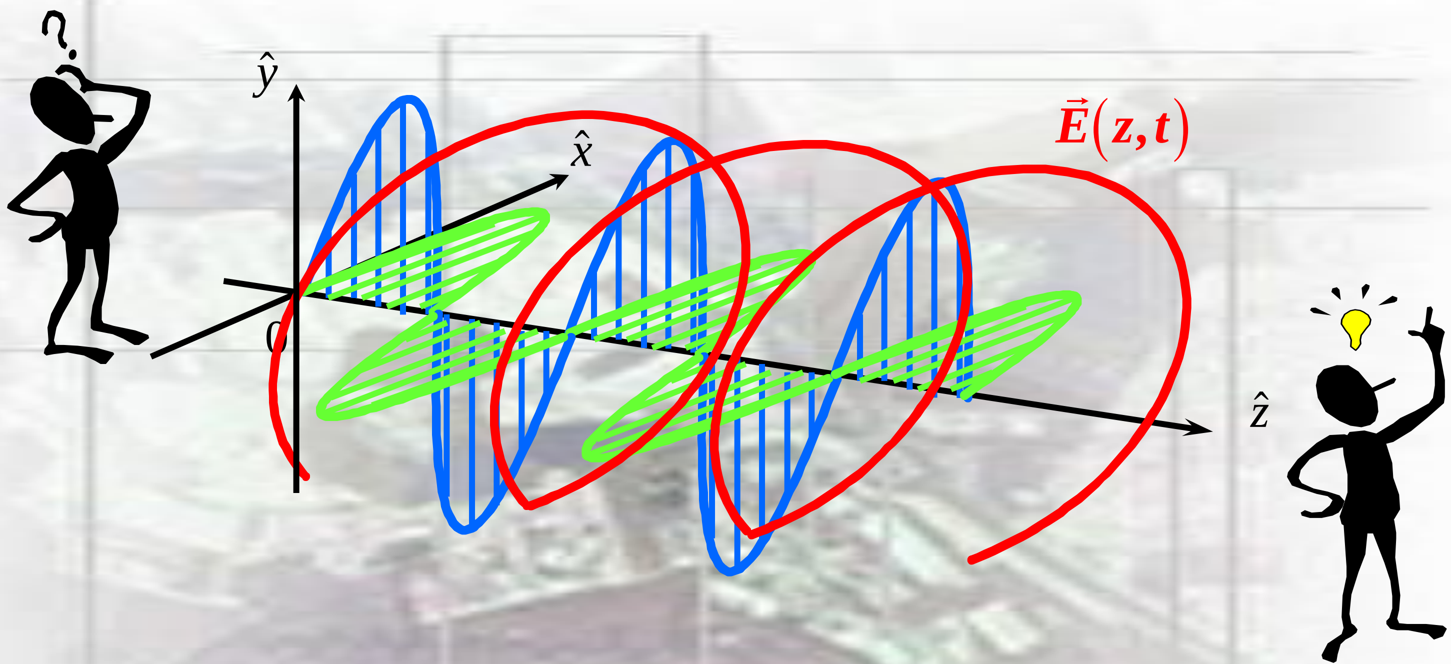
PLANE WAVES FULLY DESCRIBED BY 4 INDEPENDANT PARAMETERS

- $\langle |E_x|^2 \rangle, \langle E_x E_y^* \rangle, \langle E_y E_x^* \rangle, \langle |E_y|^2 \rangle$
- $\{ \langle g_0 \rangle, \langle g_1 \rangle, \langle g_2 \rangle, \langle g_3 \rangle \}$

WAVE POLARIMETRIC DIMENSION = 4

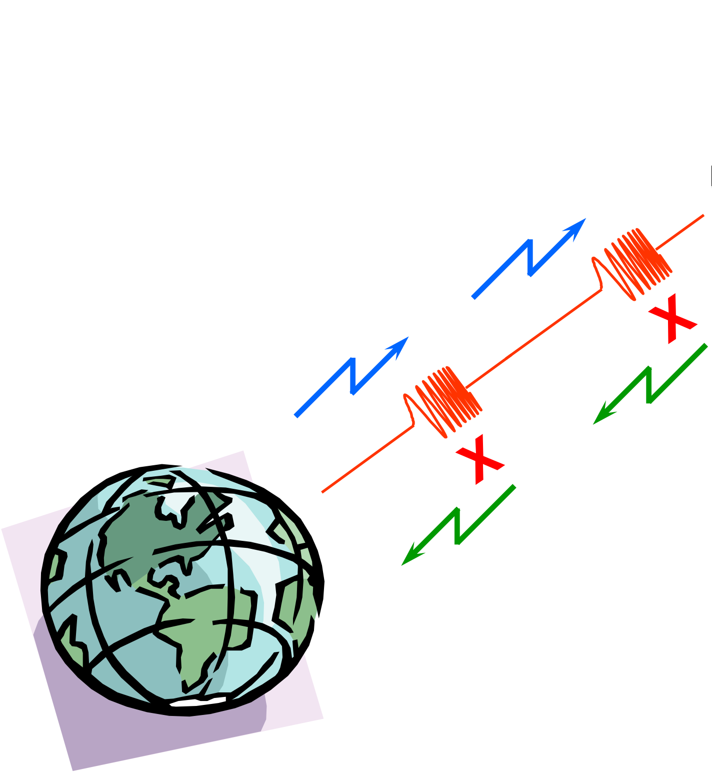
Questions ?

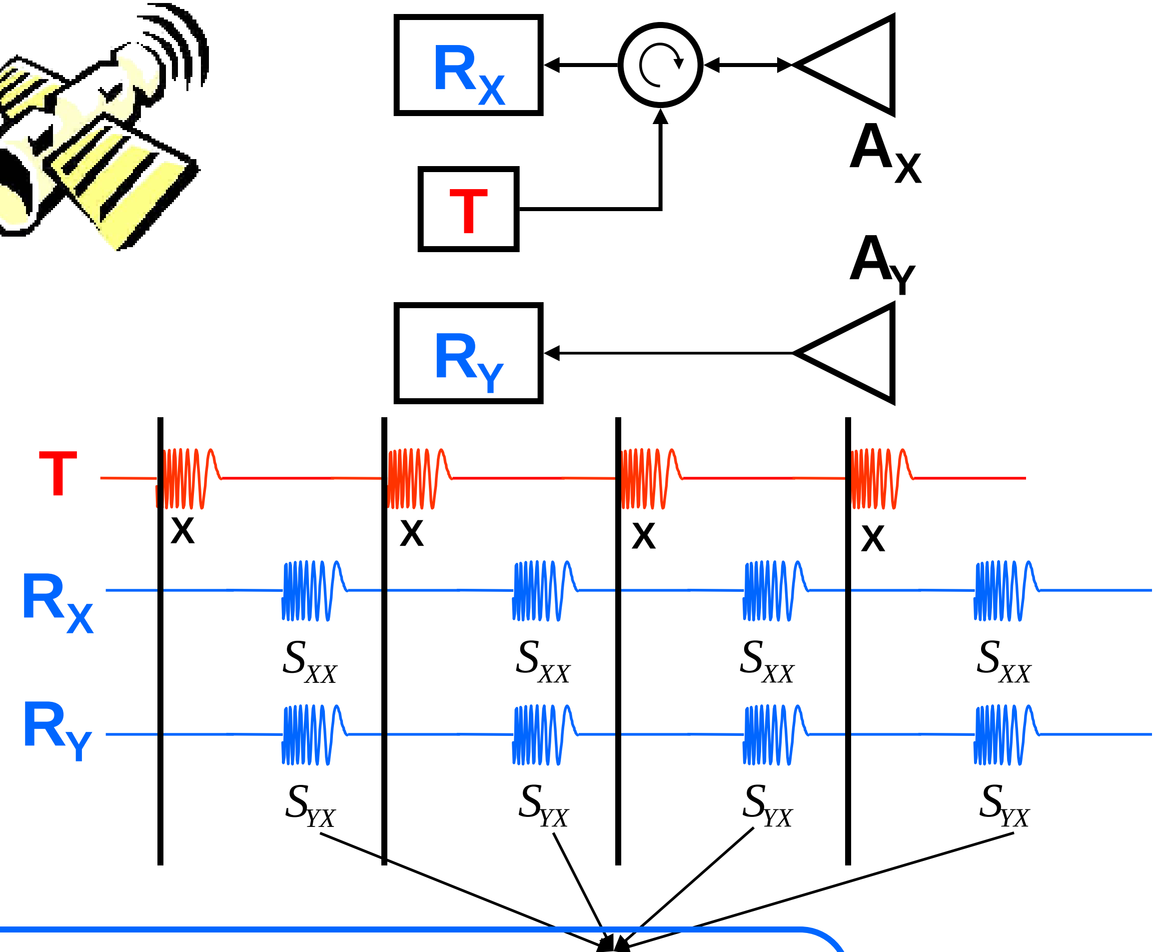




SCATTERING POLARIMETRY

WAVE POLARIMETRY





TRANSMITTER: X

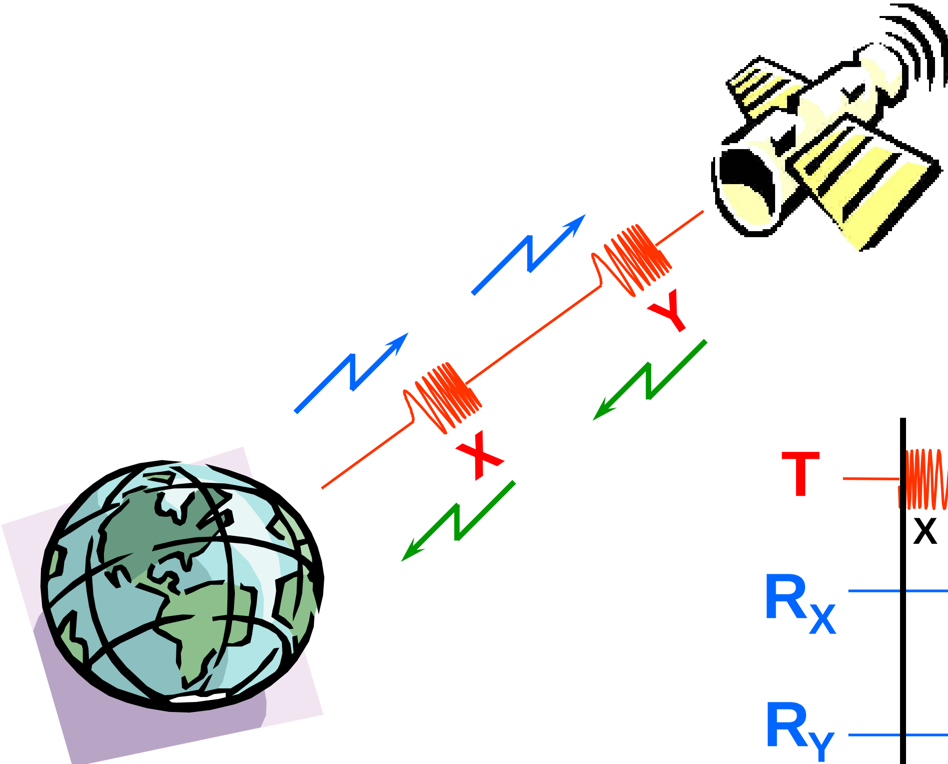
RECEIVERS: X & Y

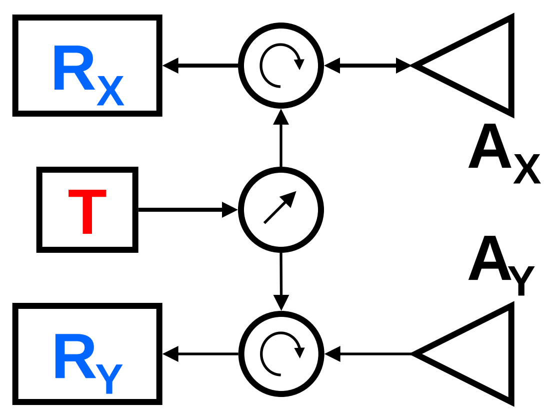
JONES VECTORS

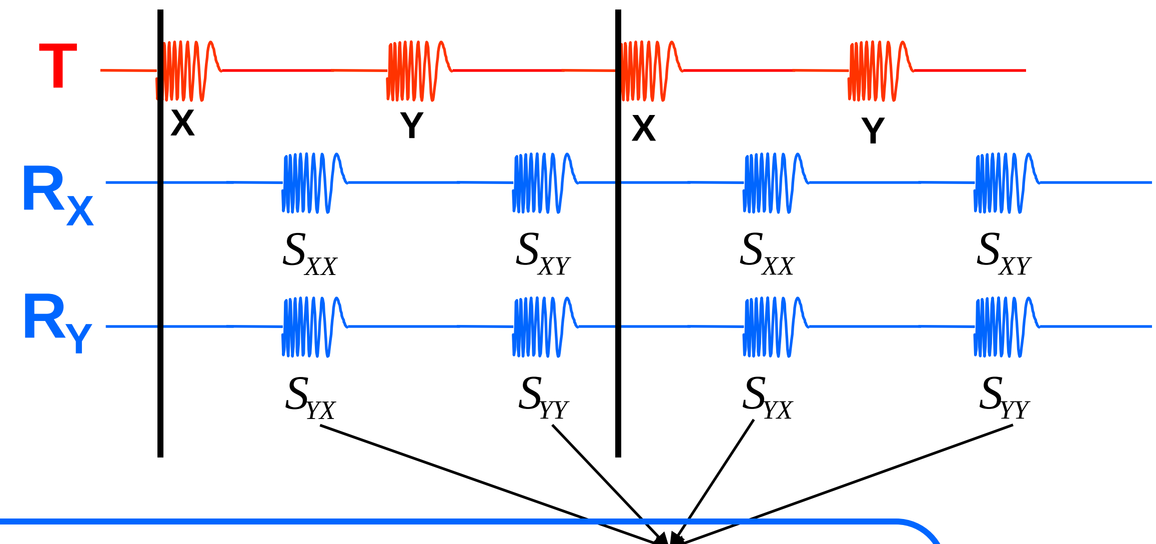
$$\left\{ \underline{E}_S = \begin{bmatrix} S_{XX} \\ S_{YX} \end{bmatrix} \right\}$$

WAVE POLARIMETRY

SCATTERING POLARIMETRY







TRANSMITTER: X & Y

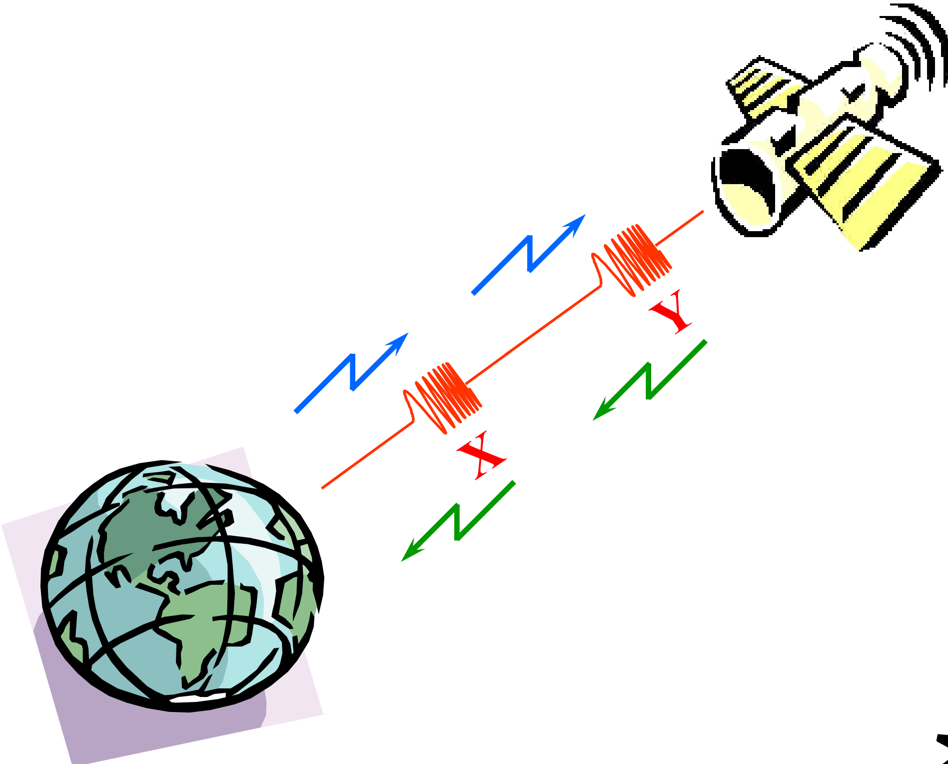
RECEIVERS: X & Y

SINCLAIR MATRICES

$$\left\{[S]=\begin{bmatrix}S_{XX} & S_{XY}\\ S_{YX} & S_{YY}\end{bmatrix}\right\}$$

SCATTERING POLARIMETRY

POLARIMETRIC DESCRIPTORS



TRANSMITTER: X & Y
RECEIVERS: X & Y

THE DIFFERENT TARGET POLARIMETRIC DESCRIPTORS

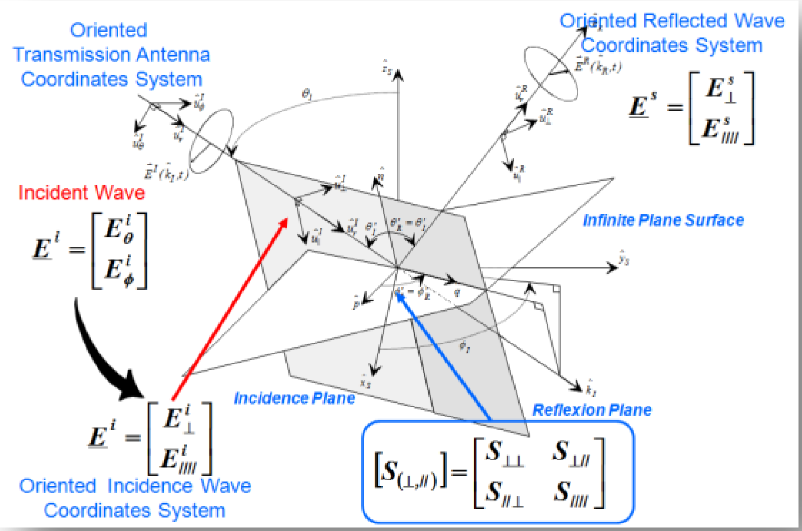
[S]	SINCLAIR Matrix
<u>k</u>, <u>Ω</u>	Target Vectors
[K]	KENNAUGH Matrix
[T]	Coherency Matrix
[C]	Covariance Matrix

JONES / SINCLAIR MATRIX

BISTATIC CASE

SCATTERING MATRIX or JONES MATRIX

$$\begin{bmatrix} E_X^s \\ E_Y^s \end{bmatrix} = \frac{e^{jkr}}{r} \begin{bmatrix} S_{XX} & S_{XY} \\ S_{YX} & S_{YY} \end{bmatrix} \begin{bmatrix} E_X^i \\ E_Y^i \end{bmatrix}$$



RECIPROCITY THEOREM

$$S_{XY} = S_{YX}$$

MONOSTATIC CASE

BACKSCATTERING MATRIX or SINCLAIR MATRIX

$$\begin{bmatrix} E_X^s \\ E_Y^s \end{bmatrix} = \frac{e^{jkr}}{r} \begin{bmatrix} S_{XX} & S_{XY} \\ S_{XY} & S_{YY} \end{bmatrix} \begin{bmatrix} E_X^i \\ E_Y^i \end{bmatrix}$$

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SCATTERER POLARIMETRIC DIMENSION = 5

SCATTERING MATRIX

$$[S]=\frac{e^{jkr}}{r}\begin{bmatrix} S_{XX} & S_{XY} \\ S_{YX} & S_{YY} \end{bmatrix}=\frac{e^{jkr}}{r}\begin{bmatrix} |S_{XX}|e^{j\varphi_{XX}} & |S_{XY}|e^{j\varphi_{XY}} \\ |S_{YX}|e^{j\varphi_{YX}} & |S_{YY}|e^{j\varphi_{YY}} \end{bmatrix}$$

ABSOLUTE SCATTERING MATRIX

$$[S]=\frac{e^{jkr}e^{j\phi_{XX}}}{r}\begin{bmatrix} |S_{XX}| & |S_{XY}|e^{j(\varphi_{XY}-\varphi_{XX})} \\ |S_{YX}|e^{j(\varphi_{YX}-\varphi_{XX})} & |S_{YY}|e^{j(\varphi_{YY}-\varphi_{XX})} \end{bmatrix}$$

Absolute Phase Factor

RELATIVE SCATTERING MATRIX

<u>Seven Parameters:</u> 4 Amplitudes and 3 Phases

SCATTERER POLARIMETRIC DIMENSION = 7

BACKSCATTERING MATRIX

$$[S]=\frac{e^{jkr}}{r}\begin{bmatrix} S_{XX} & S_{XY} \\ S_{XY} & S_{YY} \end{bmatrix}=\frac{e^{jkr}}{r}\underbrace{\begin{bmatrix} |S_{XX}|e^{j\varphi_{XX}} & |S_{XY}|e^{j\varphi_{XY}} \\ |S_{XY}|e^{j\varphi_{XY}} & |S_{YY}|e^{j\varphi_{YY}} \end{bmatrix}}$$

ABSOLUTE BACKSCATTERING MATRIX

$$[S]=\underbrace{\frac{e^{jkr}e^{j\phi_{XX}}}{r}}\underbrace{\begin{bmatrix} |S_{XX}| & |S_{XY}|e^{j(\varphi_{XY}-\varphi_{XX})} \\ |S_{XY}|e^{j(\varphi_{XY}-\varphi_{XX})} & |S_{YY}|e^{j(\varphi_{YY}-\varphi_{XX})} \end{bmatrix}}$$

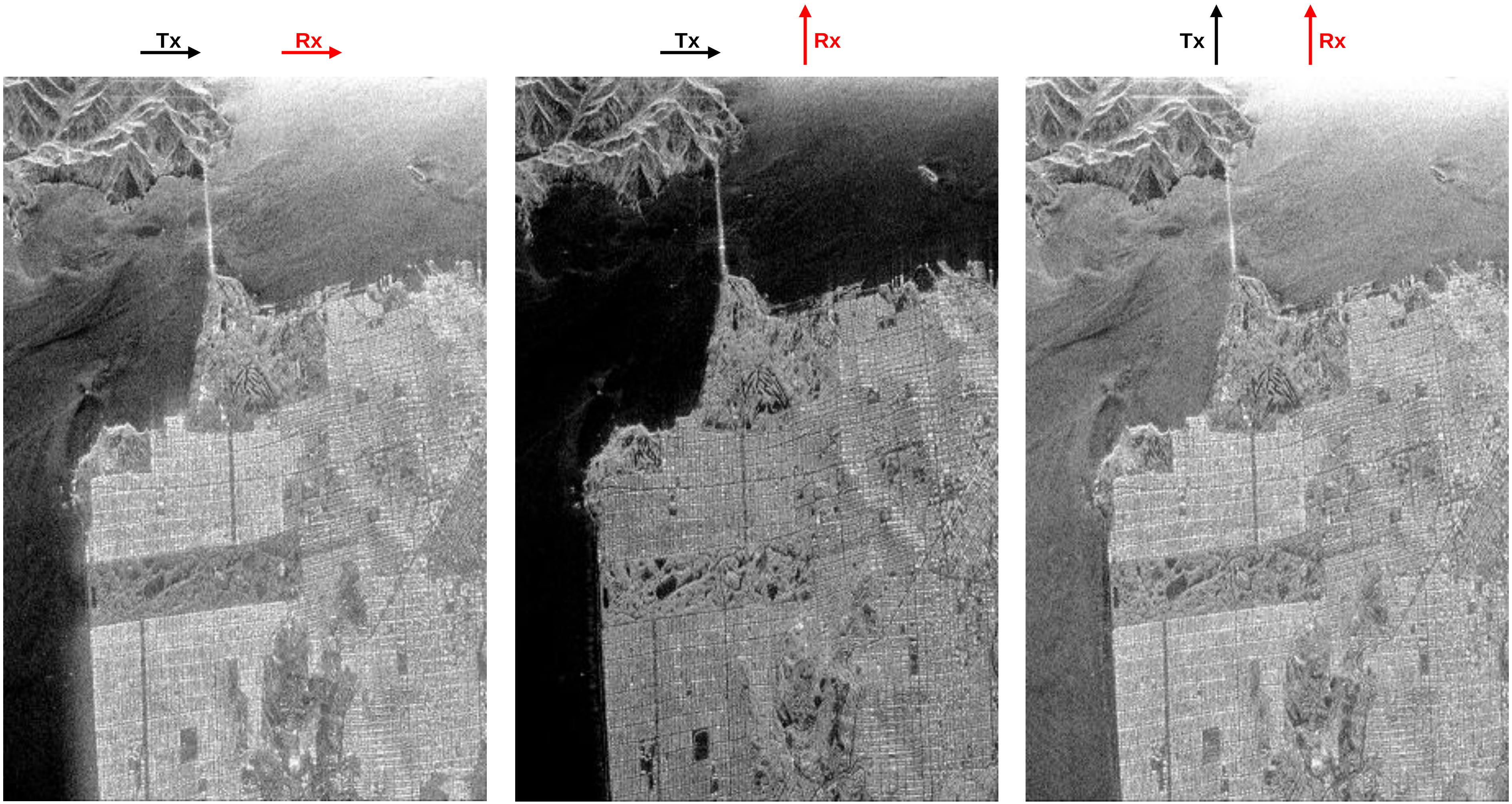
Absolute Phase Factor

RELATIVE BACKSCATTERING MATRIX

Five Parameters: 3 Amplitudes and 2 Phases

SCATTERER POLARIMETRIC DIMENSION = 5

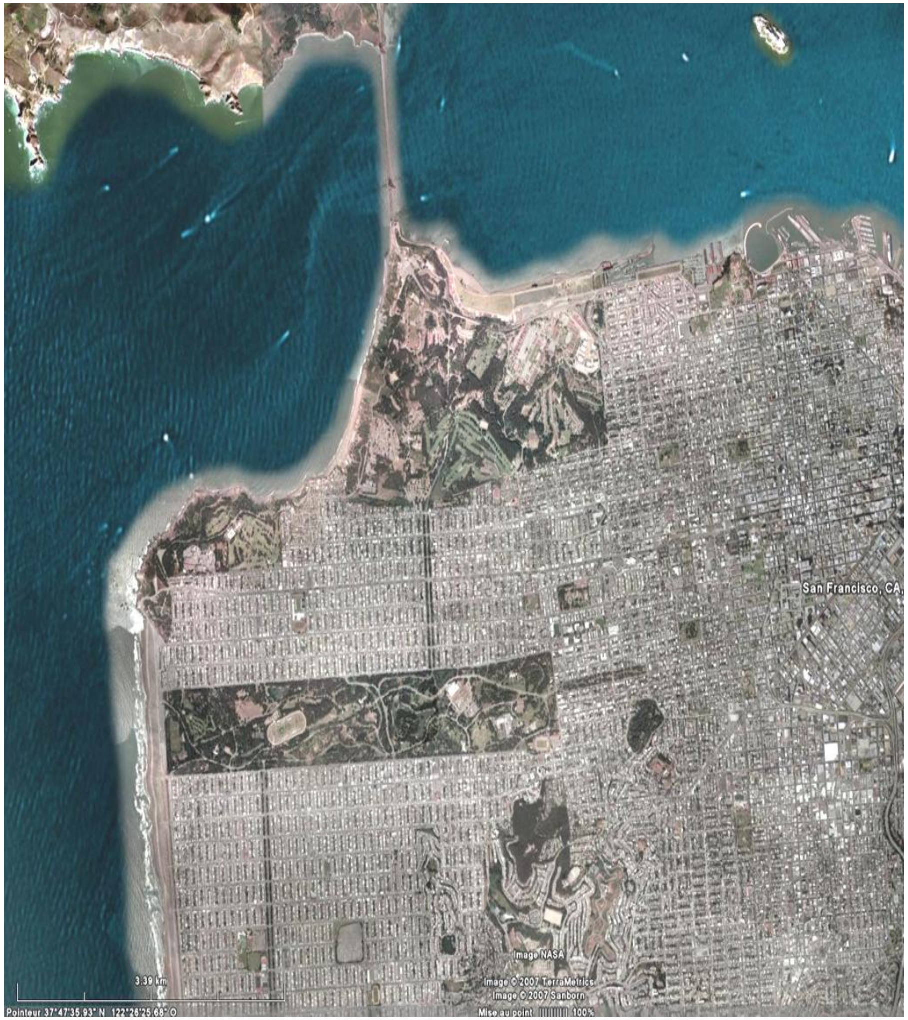
SCATTERING POLARIMETRY



$|HH|_{dB}$ $|HV|_{dB}$ $|VV|_{dB}$

-30dB -15dB 0dB

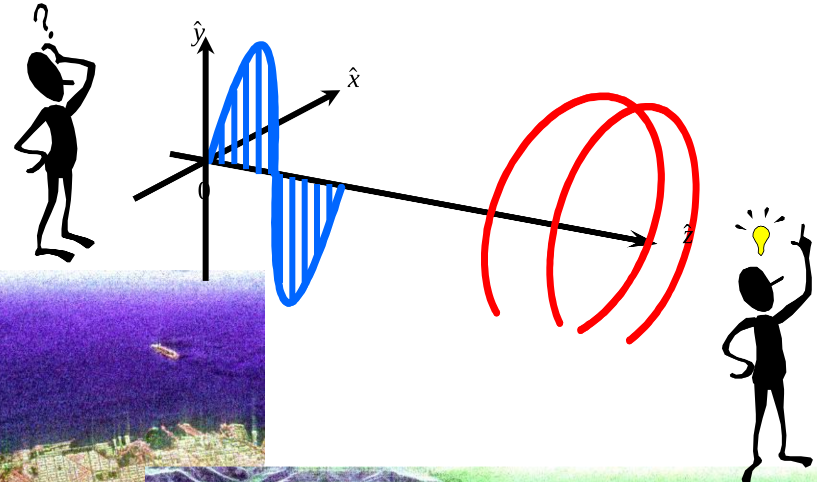
Sinclair Color Coding



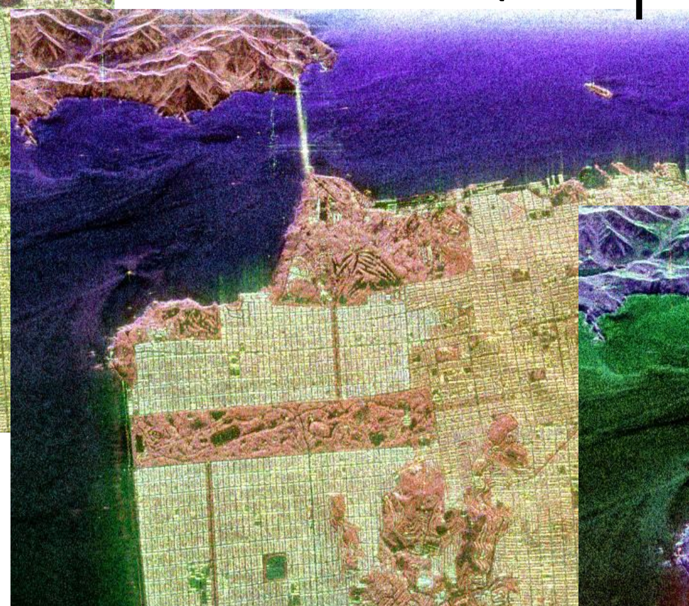
© Google Earth

|HH| |HV| |VV|

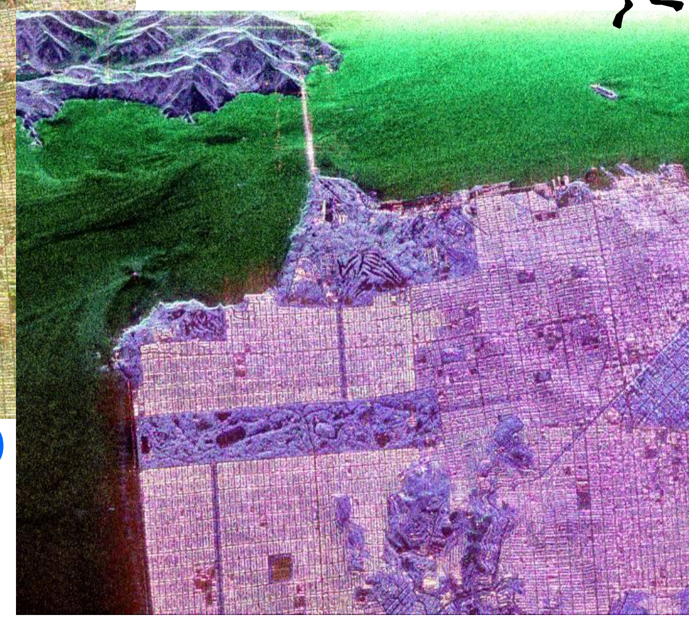
ELLIPTICAL BASIS TRANSFORMATION



Pauli Color Coding (H,V)



Pauli Color Coding (+45,-45)



Pauli Color Coding (L,R)

Ernst LÜNEBURG
(PIERS95 - Pasadena)

$$\left[S_{(B,B_\perp)}\right]=\left[U_{(A,A_\perp)\mapsto(B,B_\perp)}\right]^T\left[S_{(A,A_\perp)}\right]\left[U_{(A,A_\perp)\mapsto(B,B_\perp)}\right]$$

CON-SIMILARITY TRANSFORMATION

$$\left[U_{(A,A_\perp)\mapsto(B,B_\perp)}\right]$$

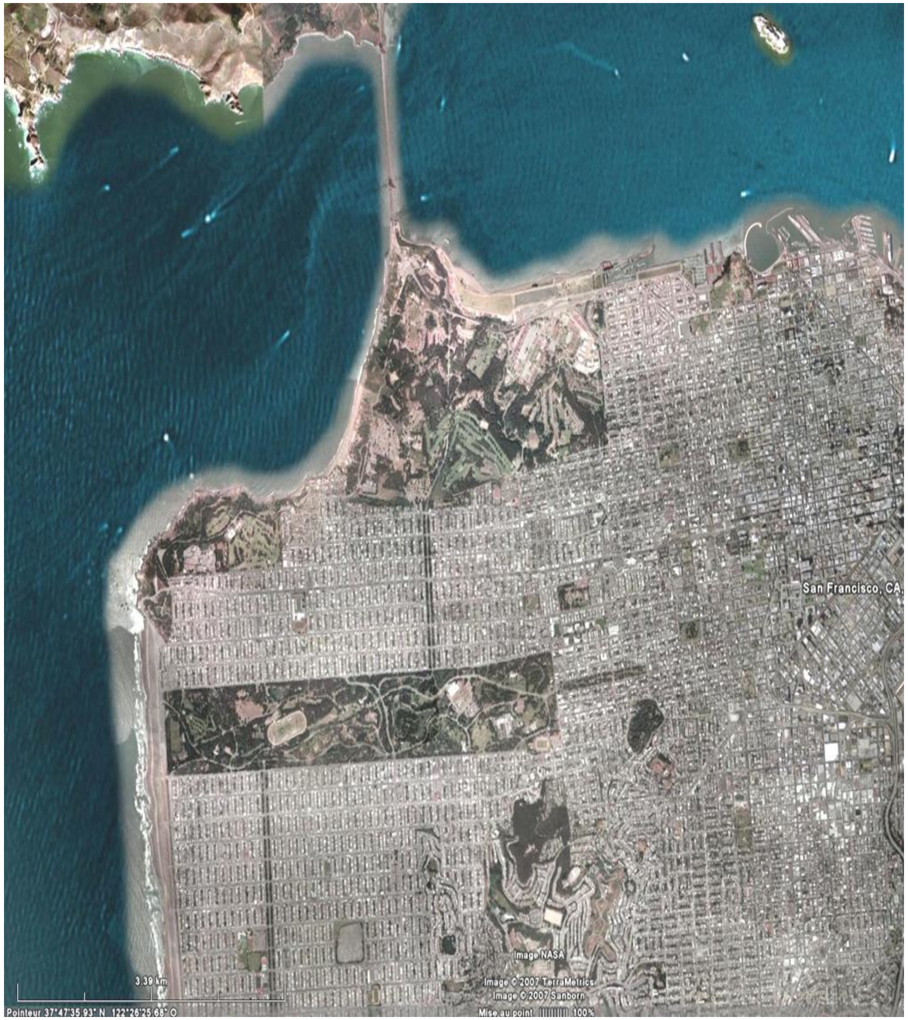
SU(2) SPECIAL UNITARY ELLIPTICAL BASIS TRANSFORMATION MATRIX

$$\left[U_{(A,A_\perp)\mapsto(B,B_\perp)}\right]=\left[U(\phi,\tau,\alpha)\right]^{-1}$$

$$=\begin{bmatrix} e^{j\alpha} & 0 \\ 0 & e^{-j\alpha} \end{bmatrix}\begin{bmatrix} \cos(\tau) & -j\sin(\tau) \\ -j\sin(\tau) & \cos(\tau) \end{bmatrix}\begin{bmatrix} \cos(\phi) & \sin(\phi) \\ -\sin(\phi) & \cos(\phi) \end{bmatrix}$$

$$\left[U_2(-\alpha)\right] \qquad \left[U_2(-\tau)\right] \qquad \left[U_2(-\phi)\right]$$

ELLIPTICAL BASIS TRANSFORMATION



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(H,V) POLARISATION BASIS

|HH+VV| |HV | |HH-VV|

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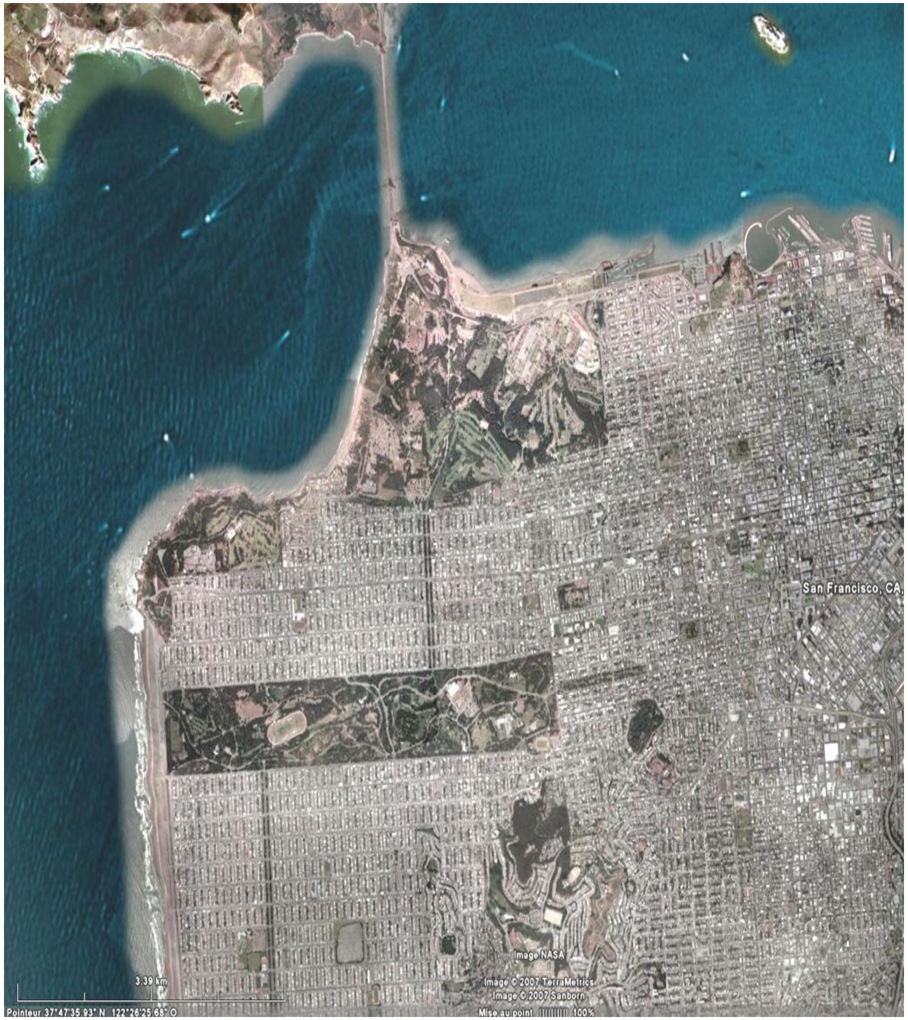
ELLIPTICAL BASIS TRANSFORMATION



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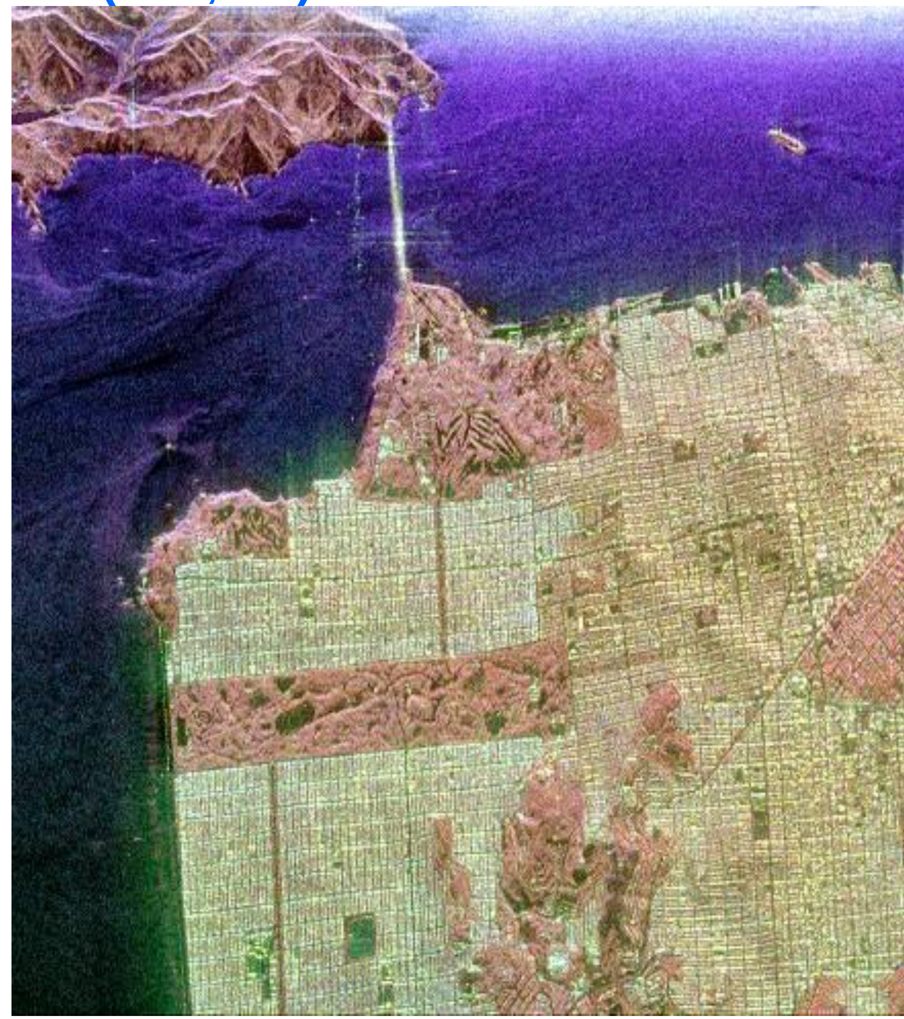
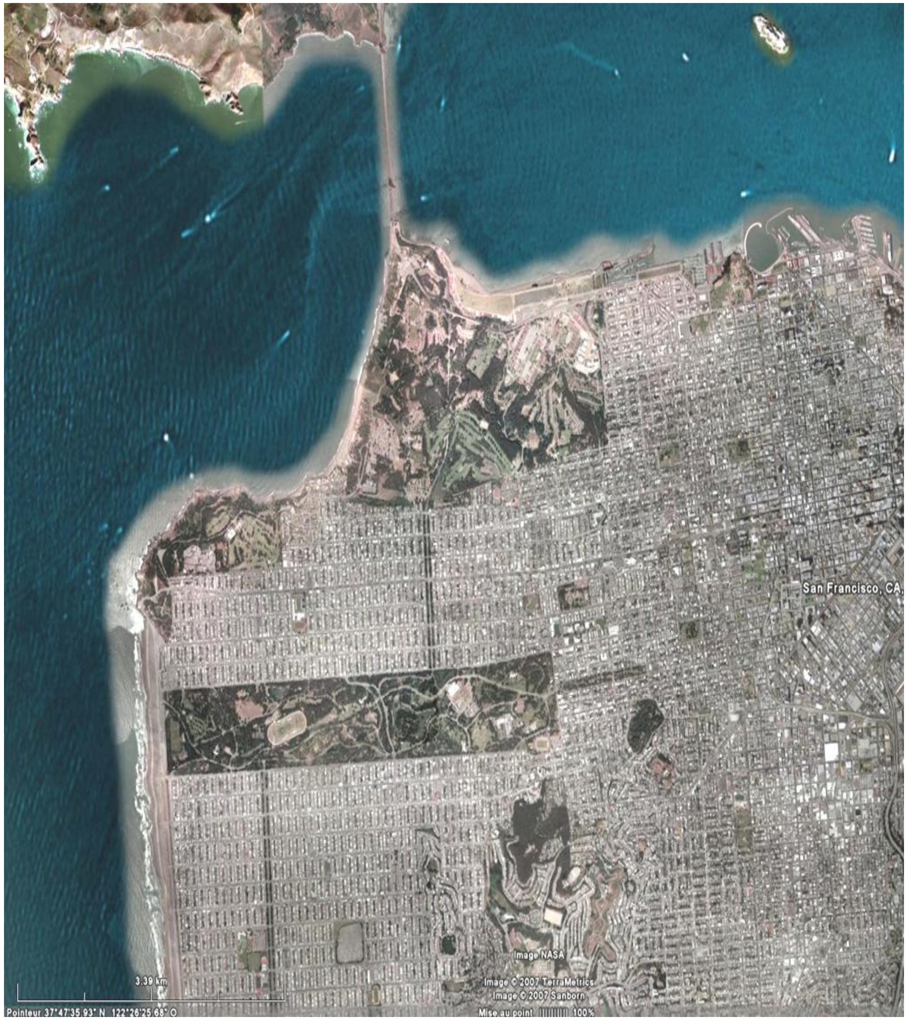
(+45°,-45°) POLARISATION BASIS



|AA+BB| |AB | |AA-BB|

With: A=Linear +45°, B=Linear –45°

ELLIPTICAL BASIS TRANSFORMATION



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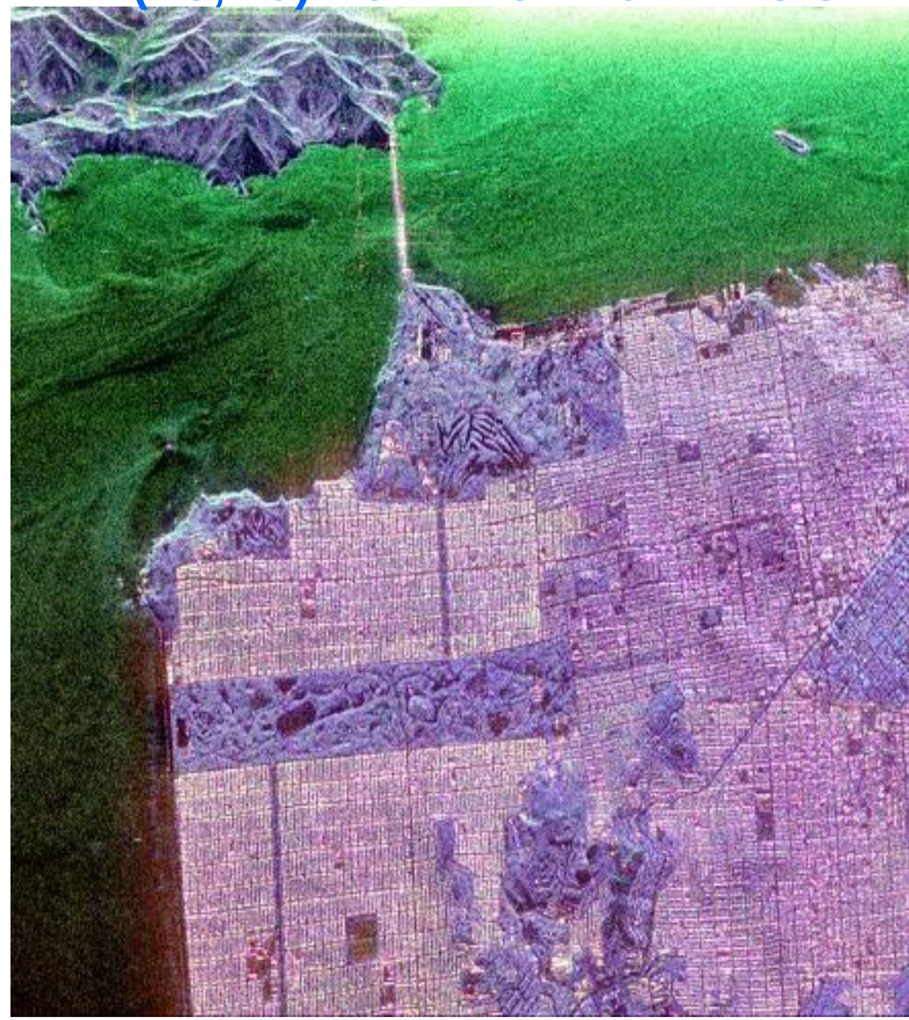
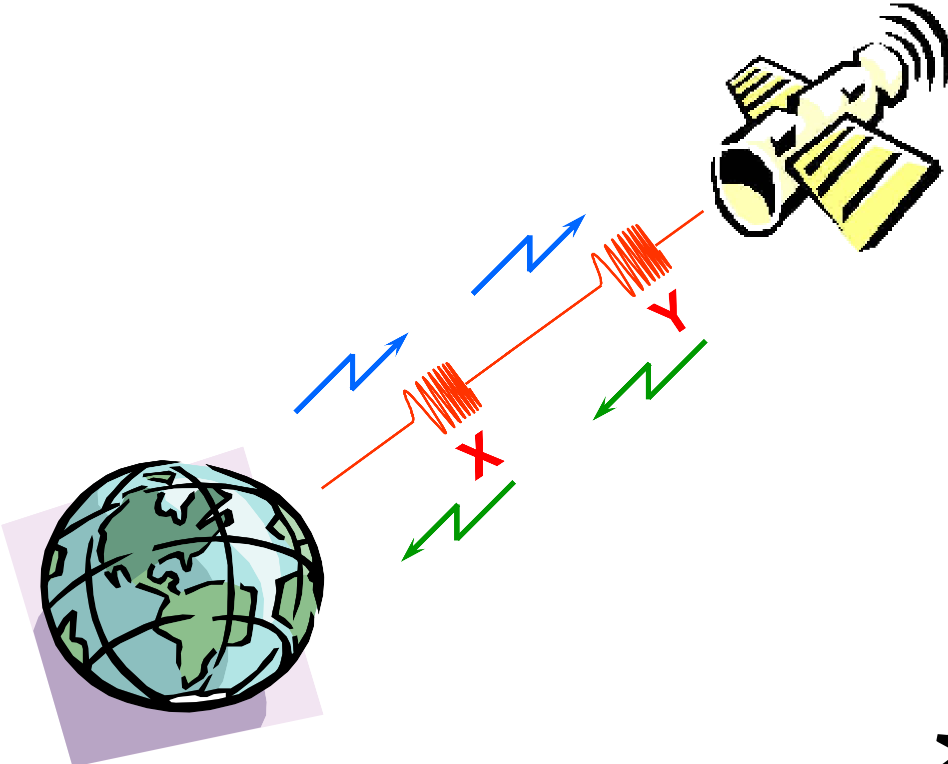
(LC,RC) POLARISATION BASIS



|LL+RR| |LR | |LL-RR|



TRANSMITTER: X & Y
RECEIVERS: X & Y

THE DIFFERENT TARGET POLARIMETRIC DESCRIPTORS

[S]	SINCLAIR Matrix
[K]	KENNAUGH Matrix
k, Ω	Target Vectors
[T]	Coherency Matrix
[C]	Covariance Matrix

STATISTICAL DESCRIPTION

PARTIAL SCATTERING POLARIMETRY

KENNAUGH MATRIX

JONES VECTOR

$$\underline{E} = \begin{bmatrix} E_x = E_{ox} e^{j\delta_x} \\ E_y = E_{oy} e^{j\delta_y} \end{bmatrix}$$

STOKES VECTOR

$$\underline{g}_{\underline{E}} = \begin{bmatrix} g_0 = |E_x|^2 + |E_y|^2 \\ g_1 = |E_x|^2 - |E_y|^2 \\ g_2 = 2\Re\left(E_x E_y^*\right) \\ g_3 = -2\Im\left(E_x E_y^*\right) \end{bmatrix}$$

$$\underline{E}_s = [S]\,\underline{E}_i$$

SINCLAIR MATRIX

$$\underline{g}_{\underline{E}_s} = [K]\,\underline{g}_{\underline{E}_i}$$

KENNAUGH MATRIX

(MUELLER MATRIX : BISTATIC)

$$[K] = \frac{1}{2}\left([V]^T\left[[S]\otimes[S]^*\right][V]\right)$$

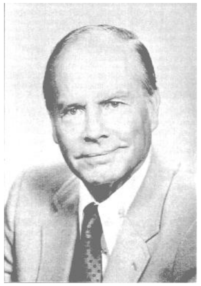
MONOSTATIC CASE

KENNAUGH MATRIX

$$[K] = \frac{1}{2}\left([V]^T\left[[S]\otimes[S]^*\right][V]\right) \qquad [V] = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -j \\ 0 & 0 & 1 & +j \\ 1 & -1 & 0 & 0 \end{bmatrix}$$

HUYNEN PARAMETERS

$$[K] = \begin{bmatrix} A_0 + B_0 & C & H & F \\ C & A_0 + B & E & G \\ H & E & A_0 - B & D \\ F & G & D & -A_0 + B_0 \end{bmatrix}$$



PHYSICAL INTERPRETATION

MAN-MADE TARGET DECOMPOSITION
IDENTIFICATION and ANALYSIS

« *PHENOMENOLOGICAL THEORY OF RADAR TARGETS* » (1970)

A0 : GENERATOR OF TARGET SYMMETRY

B0+B : GENERATOR OF TARGET NON-SYMMETRY

B0-B : GENERATOR OF TARGET IRREGULARITY

C : GENERATOR OF TARGET GLOBAL SHAPE (LINEAR)

D : GENERATOR OF TARGET LOCAL SHAPE (CURVATURE)

E : GENERATOR OF TARGET LOCAL TWIST (TORSION)

F : GENERATOR OF TARGET GLOBAL TWIST (HELICITY)

G : GENERATOR OF TARGET LOCAL COUPLING (GLUE)

H : GENERATOR OF TARGET GLOBAL COUPLING (ORIENTATION)

STOKES VECTOR

JONES VECTOR

$$\underline{E} = A \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix} \begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix} \begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix} \hat{u}_x$$

$[U_2(\phi)]$ $[U_2(\tau)]$ $[U_2(\alpha)]$

HOMOMORPHISM SU(2) - O(3)

$$[O_3(2\theta)]_{p,q} = \frac{1}{2} Tr\left([U_2(\theta)]^{T*} \sigma_p [U_2(\theta)] \sigma_q\right)$$

(σ_p, σ_q) : Pauli Matrices

STOKES VECTOR

$$\underline{g}_{\underline{E}} = A^2 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\phi) & -\sin(2\phi) & 0 \\ 0 & \sin(2\phi) & \cos(2\phi) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\tau) & 0 & -\sin(2\tau) \\ 0 & 0 & 1 & 0 \\ 0 & \sin(2\tau) & 0 & \cos(2\tau) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos(2\alpha) & -\sin(2\alpha) \\ 0 & 0 & \sin(2\alpha) & \cos(2\alpha) \end{bmatrix} \underline{g}_{\hat{u}_x}$$

$[O_3(2\phi)]$ $[O_3(2\tau)]$ $[O_3(2\alpha)]$

SINCLAIR MATRIX

$$\underline{E}^{s}_{(A,A_\perp)} = \left[S_{(A,A_\perp)}\right]\underline{E}^{i}_{(A,A_\perp)} \qquad \underline{E}^{s}_{(B,B_\perp)} = \left[S_{(B,B_\perp)}\right]\underline{E}^{i}_{(B,B_\perp)}$$

$$\left[S_{(B,B_\perp)}\right] = \left[U_{(A,A_\perp)\mapsto(B,B_\perp)}\right]^T \left[S_{(A,A_\perp)}\right]\left[U_{(A,A_\perp)\mapsto(B,B_\perp)}\right]$$

CON-SIMILARITY TRANSFORMATION

KENNAUGH MATRIX

$$\underline{g}_{\underline{E}^{s}_{(A,A_\perp)}} = \left[K_{(A,A_\perp)}\right]\underline{g}_{\underline{E}^{i}_{(A,A_\perp)}} \qquad \underline{g}_{\underline{E}^{s}_{(B,B_\perp)}} = \left[K_{(B,B_\perp)}\right]\underline{g}_{\underline{E}^{i}_{(B,B_\perp)}}$$

$$\left[K_{(B,B_\perp)}\right] = \begin{bmatrix} 1 & \underline{0}^T \\ \underline{0} & O_{3(A,A_\perp)\mapsto(B,B_\perp)} \end{bmatrix} \left[K_{(A,A_\perp)}\right] \begin{bmatrix} 1 & \underline{0}^T \\ \underline{0} & O_{3(A,A_\perp)\mapsto(B,B_\perp)} \end{bmatrix}^{-1}$$

SIMILARITY TRANSFORMATION

$$\left[O_{3(A,A_\perp)\mapsto(B,B_\perp)}\right]$$ O(3) SPECIAL UNITARY ELLIPTICAL BASIS TRANSFORMATION MATRIX

esa

SPECIAL UNITARY SU(2) GROUP

$$[U_2]=\begin{bmatrix}\cos(\phi) & -\sin(\phi)\\ \sin(\phi) & \cos(\phi)\end{bmatrix}\begin{bmatrix}\cos(\tau) & j\sin(\tau)\\ j\sin(\tau) & \cos(\tau)\end{bmatrix}\begin{bmatrix}e^{-j\alpha} & 0\\ 0 & e^{j\alpha}\end{bmatrix}$$

$$[U_2(\phi)] \qquad [U_2(\tau)] \qquad [U_2(\alpha)]$$

HOMOMORPHISM SU(2) - O(3)

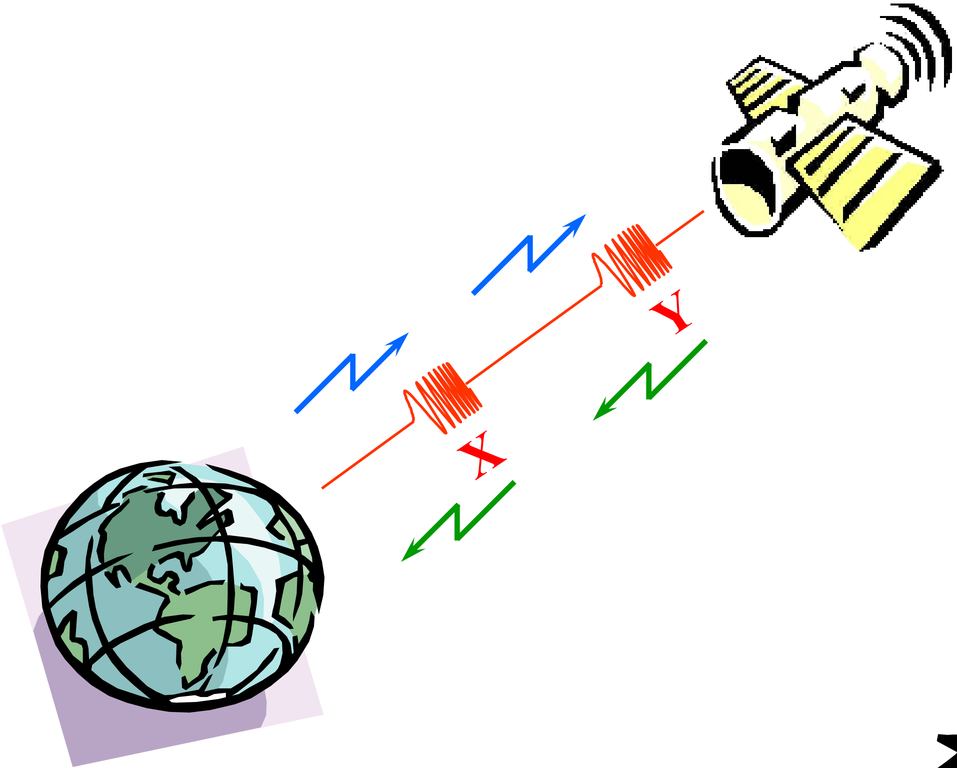
$$[O_3(2\theta)]_{p,q}=\frac{1}{2}Tr\left([U_2(\theta)]^{T*}\sigma_p[U_2(\theta)]\sigma_q\right)$$

(σ_p, σ_q) : Pauli Matrices

O(3) UNITARY GROUP

$$\begin{bmatrix}\cos 2\phi & -\sin 2\phi & 0\\ \sin 2\phi & \cos 2\phi & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos 2\tau & 0 & -\sin 2\tau\\ 0 & 1 & 0\\ \sin 2\tau & 0 & \cos 2\tau\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & \cos 2\alpha & -\sin 2\alpha\\ 0 & \sin 2\alpha & \cos 2\alpha\end{bmatrix}$$

$$[O_3(2\phi)] \qquad [O_3(2\tau)] \qquad [O_3(2\alpha)]$$



TRANSMITTER: X & Y
RECEIVERS: X & Y

THE DIFFERENT TARGET POLARIMETRIC DESCRIPTORS

- [S] SINCLAIR Matrix
- [K] KENNAUGH Matrix
- **k, Ω Target Vectors**
- [T] Coherency Matrix
- [C] Covariance Matrix

VECTORIAL FORMULATION OF THE SCATTERING PROBLEM

SCATTERING MATRIX

$$[S]=\begin{bmatrix} S_{XX} & S_{XY} \\ S_{YX} & S_{YY} \end{bmatrix}$$

SCATTERING VECTOR

$$\vec{S} := V([S]) = \frac{1}{2}Trace([S][\Psi]) = \begin{bmatrix} S1 \\ S2 \\ S3 \\ S4 \end{bmatrix} \in C_4$$

With: $V([S])$ MATRIX VECTORISATION OPERATOR

$[\Psi]$ SET OF ORTHOGONAL 2x2 MATRICES

FROBENIOUS NORM OF $\vec{S}$

$$\|\vec{S}\|^2 = \vec{S}^{T*}\cdot\vec{S} = |S_1|^2 + |S_2|^2 + |S_3|^2 + |S_4|^2$$

$$= Span([S]) = |S_{XX}|^2 + |S_{YX}|^2 + |S_{XY}|^2 + |S_{YY}|^2$$

PAULI SCATTERING VECTOR

$$\underline{k} = V([S]) = \frac{1}{2} Trace([S][\psi_P])$$

SET OF 2x2 COMPLEX MATRICES FROM THE PAULI MATRICES GROUP

$$[\psi_P] = \left\{ \sqrt{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \sqrt{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \sqrt{2}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \sqrt{2}\begin{bmatrix} 0 & -j \\ j & 0 \end{bmatrix} \right\}$$

$$\underline{k} = \frac{1}{\sqrt{2}} \left[S_{XX} + S_{YY} \quad S_{XX} - S_{YY} \quad S_{XY} + S_{YX} \quad j(S_{XY} - S_{YX}) \right]^T$$

Advantage: Closer related to physical properties of the scatterer

Note: Also known as $\underline{k}_{4P}$

TARGET VECTORS

LEXICOGRAPHIC SCATTERING VECTOR $\underline{\Omega} = V([S]) = \frac{1}{2} Trace([S][\psi_L])$

SET OF 2x2 COMPLEX MATRICES
FROM THE LEXICOGRAPHIC MATRICES GROUP

$$[\psi_L] = \left\{ 2\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, 2\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, 2\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, 2\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

$$\underline{\Omega} = [S_{XX} \quad S_{XY} \quad S_{YX} \quad S_{YY}]^T$$

Advantage: Directly related to the system measurables

Note: Also known as $\underline{k}_{4L}$

SCATTERING VECTOR TRANSFORMATIONS

Pauli Scattering Vector:

$$\underline{k} = \frac{1}{\sqrt{2}} \begin{bmatrix} S_{XX} + S_{YY} \\ S_{XX} - S_{YY} \\ S_{XY} + S_{YX} \\ j(S_{XY} - S_{YX}) \end{bmatrix}$$

Lexicographic Scattering Vector:

$$\underline{\Omega} = \begin{bmatrix} S_{XX} \\ S_{XY} \\ S_{YX} \\ S_{YY} \end{bmatrix}$$

UNITARY TRANSFORMATION

$$\underline{k} = [D_4]\underline{\Omega} \quad and \quad \underline{\Omega} = [D_4]^{-1}\underline{k} = [D_4]^{T*}\underline{k}$$

WHERE $[D_4]$ IS A SU(4) MATRIX IN ORDER TO PRESERVE THE NORM OF THE SCATTERING VECTOR

$$[D_4] = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & j & -j & 0 \end{bmatrix}$$

MONOSTATIC CASE

Pauli Scattering Vector:

$$\underline{k} = \frac{1}{\sqrt{2}}\begin{bmatrix} S_{XX} + S_{YY} \\ S_{XX} - S_{YY} \\ S_{XY} + S_{YX} \\ j(S_{XY} - S_{YX}) \end{bmatrix} \Rightarrow \underline{k} = \frac{1}{\sqrt{2}}\begin{bmatrix} S_{XX} + S_{YY} \\ S_{XX} - S_{YY} \\ 2S_{XY} \end{bmatrix}$$

Note: Also known as $\underline{k}_{3P}$

Lexicographic Scattering Vector:

$$\underline{\Omega} = \begin{bmatrix} S_{XX} \\ S_{XY} \\ S_{YX} \\ S_{YY} \end{bmatrix} \Rightarrow \underline{\Omega} = \begin{bmatrix} S_{XX} \\ \sqrt{2}S_{XY} \\ S_{YY} \end{bmatrix}$$

Note: Also known as $\underline{k}_{3L}$

SCATTERING VECTOR TRANSFORMATIONS

Pauli Scattering Vector:

$$\underline{k} = \frac{1}{\sqrt{2}} \begin{bmatrix} S_{XX} + S_{YY} \\ S_{XX} - S_{YY} \\ 2S_{XY} \end{bmatrix}$$

Lexicographic Scattering Vector:

$$\underline{\Omega} = \begin{bmatrix} S_{XX} \\ \sqrt{2}S_{XY} \\ S_{YY} \end{bmatrix}$$

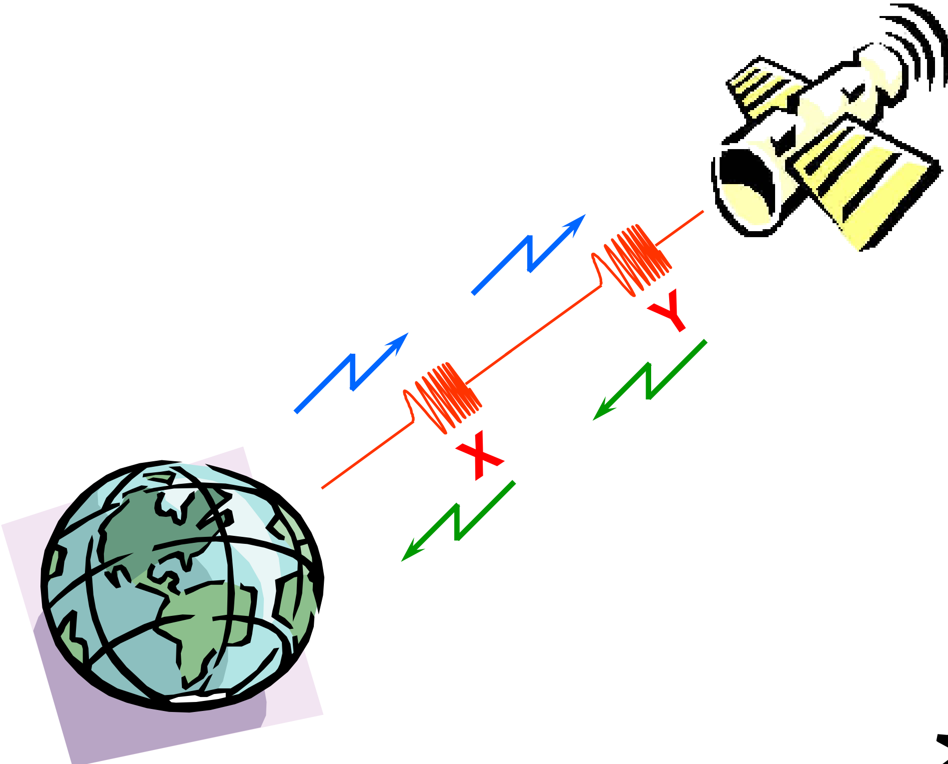
UNITARY TRANSFORMATION

$$\underline{k} = [D_3]\underline{\Omega} \quad and \quad \underline{\Omega} = [D_3]^{-1}\underline{k} = [D_3]^T \underline{k}$$

WHERE $[D_3]$ IS A SU(3) MATRIX IN ORDER TO PRESERVE THE NORM OF THE SCATTERING VECTOR

$$[D_3] = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & -1 \\ 0 & \sqrt{2} & 0 \end{bmatrix}$$

POLARIMETRIC DESCRIPTORS



TRANSMITTER: X & Y
RECEIVERS: X & Y

THE DIFFERENT TARGET POLARIMETRIC DESCRIPTORS

[S] SINCLAIR Matrix
[K] KENNAUGH Matrix
k, Ω Target Vectors
[T] Coherency Matrix
[C] Covariance Matrix

STATISTICAL DESCRIPTION

PARTIAL SCATTERING POLARIMETRY

BISTATIC CASE

PAULI SCATTERING VECTOR $\underline{k}$

$$\underline{k} = \frac{1}{\sqrt{2}}\left[S_{XX} + S_{YY} \quad S_{XX} - S_{YY} \quad S_{XY} + S_{YX} \quad j(S_{XY} - S_{YX})\right]^T$$

↓

COHERENCY MATRIX $[T]$

$$[T] = \underline{k} \cdot \underline{k}^{*T} = \begin{bmatrix} 2A_0 & C - jD & H + jG & L - jK \\ C + jD & B_0 + B & E + jF & M - jN \\ H - jG & E - jF & B_0 - B & J + jI \\ L + jK & M + jN & J - jI & 2A \end{bmatrix}$$

HERMITIAN POSITIVE SEMI DEFINITE MATRIX - RANK 1

COHERENCY MATRIX

MONOSTATIC CASE

PAULI SCATTERING VECTOR $\underline{k}$

$$\underline{k} = \frac{1}{\sqrt{2}}\left[S_{XX}+S_{YY} \quad S_{XX}-S_{YY} \quad 2S_{XY}\right]^T$$

COHERENCY MATRIX $[T]$

$$[T] = \underline{k}\cdot\underline{k}^{*T} = \begin{bmatrix} 2A_0 & C-jD & H+jG \\ C+jD & B_0+B & E+jF \\ H-jG & E-jF & B_0-B \end{bmatrix}$$

HERMITIAN POSITIVE SEMI-DEFINITE MATRIX - RANK 1

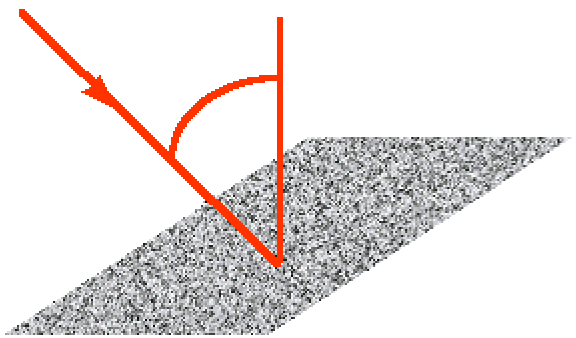
HUYNEN TARGET GENERATORS

$$T_{11} = 2A_0 = \left|S_{XX}+S_{YY}\right|^2 \qquad T_{22} = B_0 + B = \left|S_{XX}-S_{YY}\right|^2$$

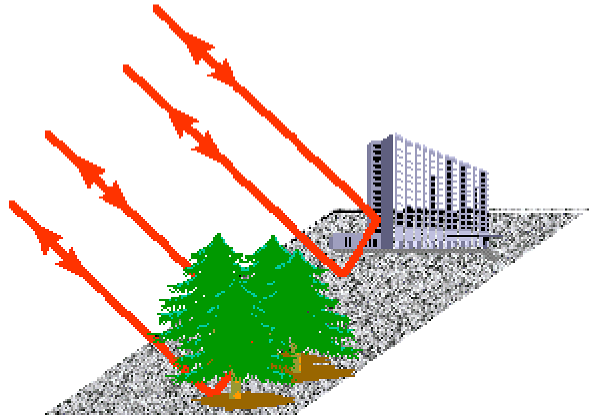
$$T_{33} = B_0 - B = 2\left|S_{XY}\right|^2$$

PHYSICAL INTERPRETATION

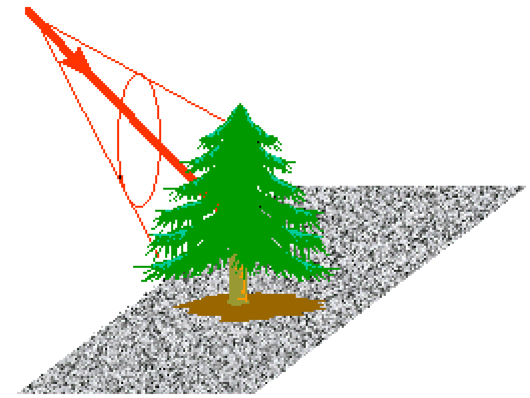
SINGLE BOUNCE SCATTERING (ROUGH SURFACE)



DOUBLE BOUNCE SCATTERING



VOLUME SCATTERING

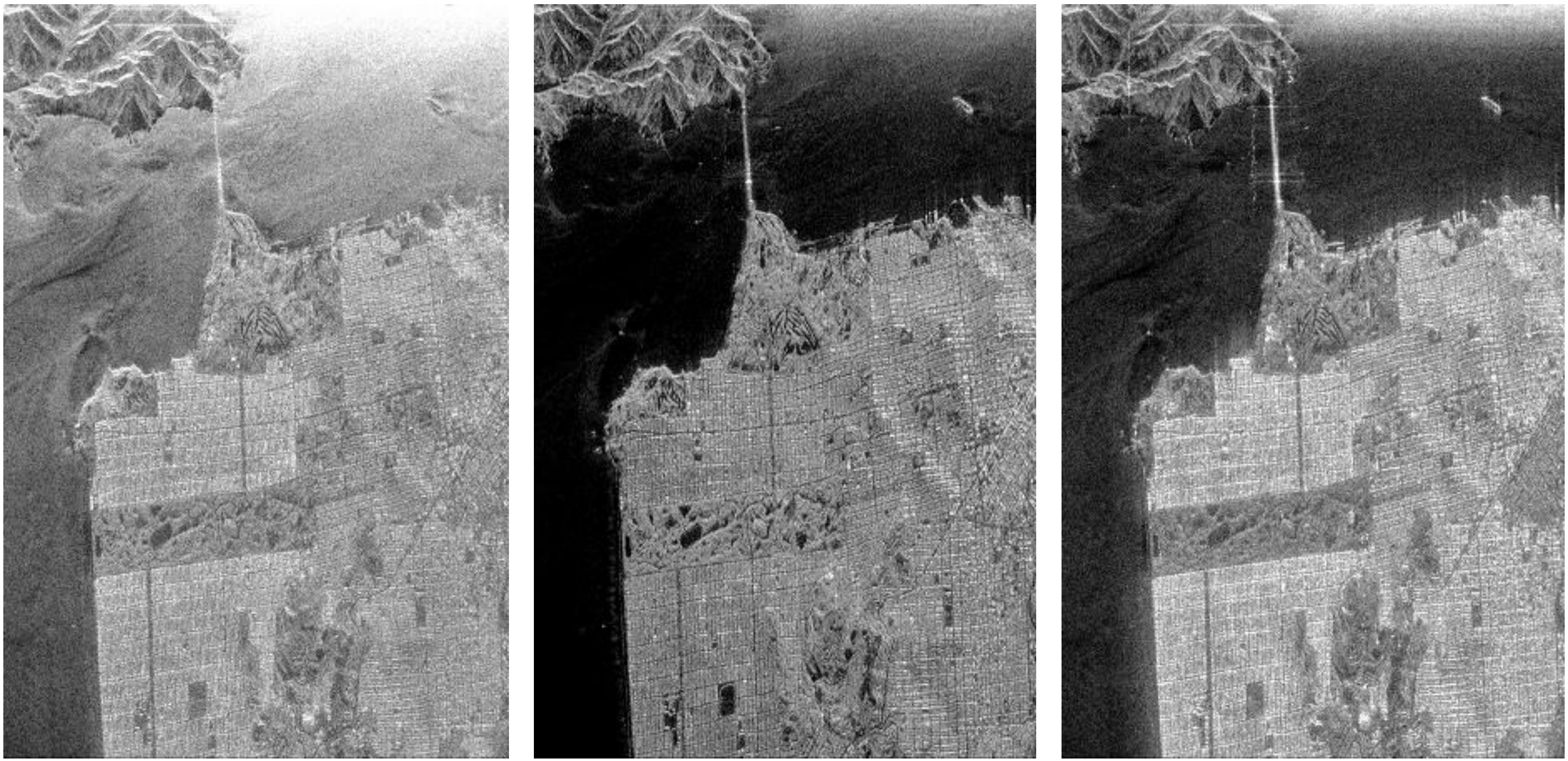


$$T_{11} = 2A_0 = \left| S_{XX} + S_{YY} \right|^2$$

$$T_{33} = B_0 - B = 2\left| S_{XY} \right|^2$$

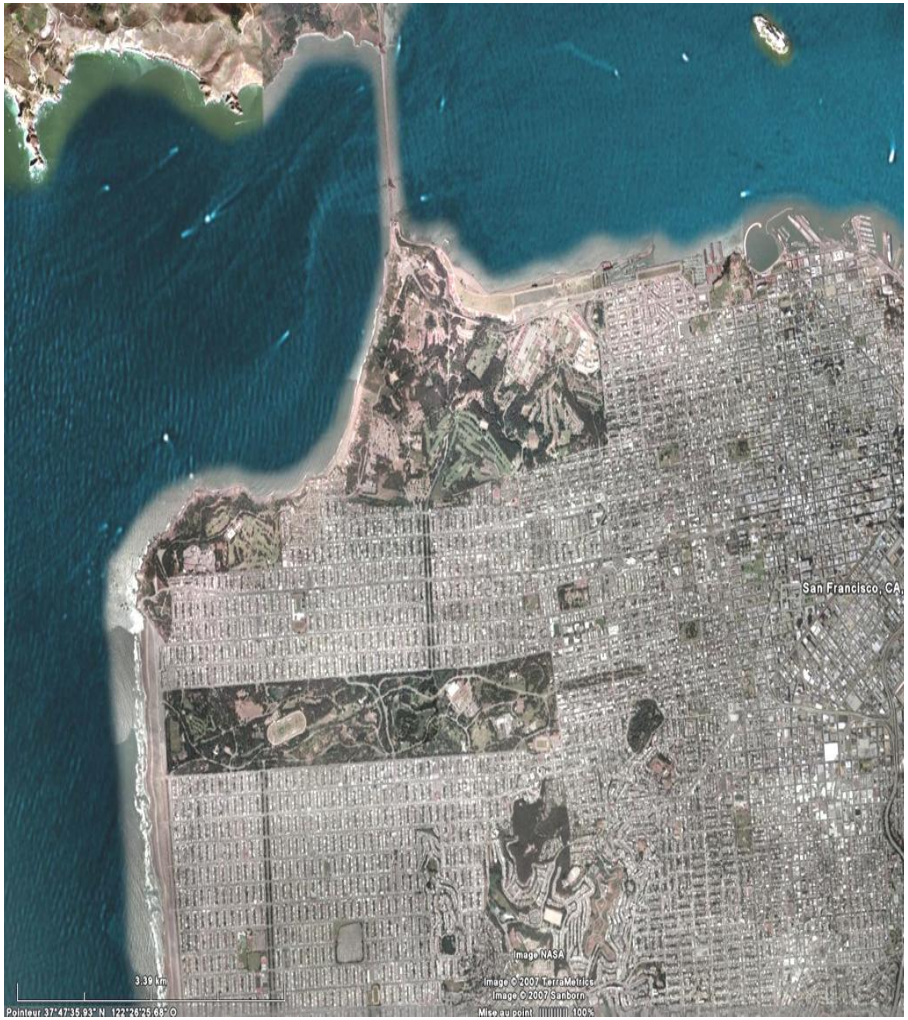
$$T_{22} = B_0 + B = \left| S_{XX} - S_{YY} \right|^2$$

TARGET GENERATORS



$|HH+VV|_{dB}$ $|HV|_{dB}$ $|HH-VV|_{dB}$

TARGET GENERATORS



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(H,V) POLARISATION BASIS

|HH+VV| |HV | |HH-VV|

ELLIPTICAL BASIS TRANSFORMATION

SPECIAL UNITARY SU(2) GROUP

$$[U_2] = \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix} \begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix} \begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}$$

$$[U_2(\phi)] \qquad [U_2(\tau)] \qquad [U_2(\alpha)]$$

SPECIAL UNITARY SU(3) GROUP

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(2\phi) & \sin(2\phi) \\ 0 & -\sin(2\phi) & \cos(2\phi) \end{bmatrix} \begin{bmatrix} \cos(2\tau) & 0 & j\sin(2\tau) \\ 0 & 1 & 0 \\ j\sin(2\tau) & 0 & \cos(2\tau) \end{bmatrix} \begin{bmatrix} \cos(2\alpha) & -j\sin(2\alpha) & 0 \\ -j\sin(2\alpha) & \cos(2\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[U_3(2\phi)] \qquad [U_3(2\tau)] \qquad [U_3(2\alpha)]$$

ELLIPTICAL BASIS TRANSFORMATION

SINCLAIR MATRIX

$$\underline{E}^{s}_{(A,A_\perp)} = \left[S_{(A,A_\perp)}\right]\underline{E}^{i}_{(A,A_\perp)} \qquad \underline{E}^{s}_{(B,B_\perp)} = \left[S_{(B,B_\perp)}\right]\underline{E}^{i}_{(B,B_\perp)}$$

$$\left[S_{(B,B_\perp)}\right] = \left[U_{(A,A_\perp)\mapsto(B,B_\perp)}\right]^T \left[S_{(A,A_\perp)}\right]\left[U_{(A,A_\perp)\mapsto(B,B_\perp)}\right]$$

CON-SIMILARITY TRANSFORMATION

COHERENCY MATRIX

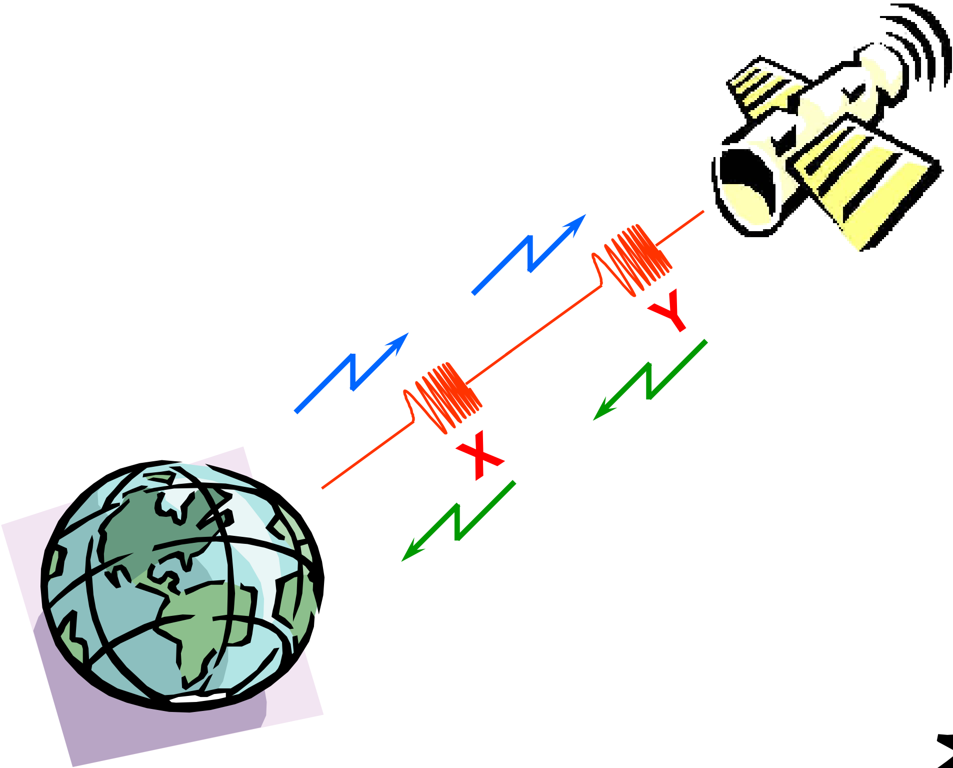
$$\left[T_{(B,B_\perp)}\right] = \left[U_{3(A,A_\perp)\mapsto(B,B_\perp)}\right]\left[T_{(A,A_\perp)}\right]\left[U_{3(A,A_\perp)\mapsto(B,B_\perp)}\right]^{-1}$$

SIMILARITY TRANSFORMATION

$$\left[U_{3(A,A_\perp)\mapsto(B,B_\perp)}\right]$$

U(3) SPECIAL UNITARY ELLIPTICAL BASIS TRANSFORMATION MATRIX

POLARIMETRIC DESCRIPTORS



TRANSMITTER: X & Y
RECEIVERS: X & Y

THE DIFFERENT TARGET POLARIMETRIC DESCRIPTORS

[S] SINCLAIR Matrix
[K] KENNAUGH Matrix
k, Ω Target Vectors
[T] Coherency Matrix
[C] Covariance Matrix

STATISTICAL DESCRIPTION

PARTIAL SCATTERING POLARIMETRY

BISTATIC CASE

LEXICOGRAPHIC SCATTERING VECTOR $\underline{\Omega}$

$$\underline{\Omega} = \begin{bmatrix} S_{XX} & S_{XY} & S_{YX} & S_{YY} \end{bmatrix}^T$$

COVARIANCE MATRIX $[C]$

$$[C] = \underline{\Omega} \cdot \underline{\Omega}^{*T} = \begin{bmatrix} S_{XX}S_{XX}^* & S_{XX}S_{XY}^* & S_{XX}S_{YX}^* & S_{XX}S_{YY}^* \\ S_{XY}S_{XX}^* & S_{XY}S_{XY}^* & S_{XY}S_{YX}^* & S_{XY}S_{YY}^* \\ S_{YX}S_{XX}^* & S_{YX}S_{XY}^* & S_{YX}S_{YX}^* & S_{YX}S_{YY}^* \\ S_{YY}S_{XX}^* & S_{YY}S_{XY}^* & S_{YY}S_{YX}^* & S_{YY}S_{YY}^* \end{bmatrix}$$

HERMITIAN POSITIVE SEMI DEFINITE MATRIX - RANK 1

MONOSTATIC CASE

LEXICOGRAPHIC SCATTERING VECTOR $\underline{\Omega}$

$$\underline{\Omega} = \begin{bmatrix} S_{XX} & \sqrt{2}S_{XY} & S_{YY} \end{bmatrix}^T$$

COVARIANCE MATRIX $[C]$

$$[C] = \underline{\Omega} \cdot \underline{\Omega}^{*T} = \begin{bmatrix} S_{XX}S_{XX}^* & \sqrt{2}S_{XX}S_{XY}^* & S_{XX}S_{YY}^* \\ \sqrt{2}S_{XY}S_{XX}^* & 2S_{XY}S_{XY}^* & \sqrt{2}S_{XY}S_{YY}^* \\ S_{YY}S_{XX}^* & \sqrt{2}S_{YY}S_{XY}^* & S_{YY}S_{YY}^* \end{bmatrix}$$

HERMITIAN POSITIVE SEMI DEFINITE MATRIX - RANK 1

ELLIPTICAL BASIS TRANSFORMATION

SINCLAIR MATRIX

$$\underline{E}^{s}_{(A,A_\perp)} = \left[S_{(A,A_\perp)}\right] \underline{E}^{i}_{(A,A_\perp)}$$

$$\underline{E}^{s}_{(B,B_\perp)} = \left[S_{(B,B_\perp)}\right] \underline{E}^{i}_{(B,B_\perp)}$$

$$\left[S_{(B,B_\perp)}\right] = \left[U_{(A,A_\perp)\mapsto(B,B_\perp)}\right]^{T} \left[S_{(A,A_\perp)}\right] \left[U_{(A,A_\perp)\mapsto(B,B_\perp)}\right]$$

CON-SIMILARITY TRANSFORMATION

COVARIANCE MATRIX

$$\left[C_{(B,B_\perp)}\right] = \left[U_{3(A,A_\perp)\mapsto(B,B_\perp)}\right] \left[C_{(A,A_\perp)}\right] \left[U_{3(A,A_\perp)\mapsto(B,B_\perp)}\right]^{-1}$$

SIMILARITY TRANSFORMATION

$$\left[U_{3(A,A_\perp)\mapsto(B,B_\perp)}\right]$$

U(3) SPECIAL UNITARY ELLIPTICAL BASIS TRANSFORMATION MATRIX

COVARIANCE-COHERENCY MATRICES

COHERENCY MATRIX **COVARIANCE MATRIX**

$$[T] = \underline{k} \cdot \underline{k}^{*T}$$

$$\underline{k} = [D_{3or4}]\underline{\Omega}$$

$$[C] = \underline{\Omega} \cdot \underline{\Omega}^{*T}$$

UNITARY TRANSFORMATION

$$[T] = [D_{3or4}][C][D_{3or4}]^{T*}$$

$[T]$ **and** $[C]$ **HAVE THE SAME EIGENVALUES**

Both contain the same information about Polarimetric Scattering Amplitudes, Phase Angles and Correlations

$[T]$ is closer related to Physical and Geometrical Properties of the Scattering Process, and thus allows a better and direct physical interpretation

$[C]$ is directly related to the system measurables

$[T]$ is directly related to the Kennaugh matrix and the Huynen parameters

POLARIMETRIC DESCRIPTORS

esa

SINCLAIR MATRIX

$$[S] = \begin{bmatrix} S_{XX} & S_{XY} \\ S_{YX} & S_{YY} \end{bmatrix}$$

KENNAUGH MATRIX

$$[K] = \frac{1}{2}\left([V]^T \left[[S] \otimes [S]^*\right][V]\right)$$

EQUIVALENCE ?

SCATTERING VECTOR $\underline{k}$

$$\underline{k} = \frac{1}{\sqrt{2}}\left[S_{XX} + S_{YY} \quad S_{XX} - S_{YY} \quad 2S_{XY}\right]^T$$

↓

COHERENCY MATRIX $[T]$

$$[T] = \underline{k} \cdot \underline{k}^{*T}$$

SCATTERING VECTOR $\underline{\Omega}$

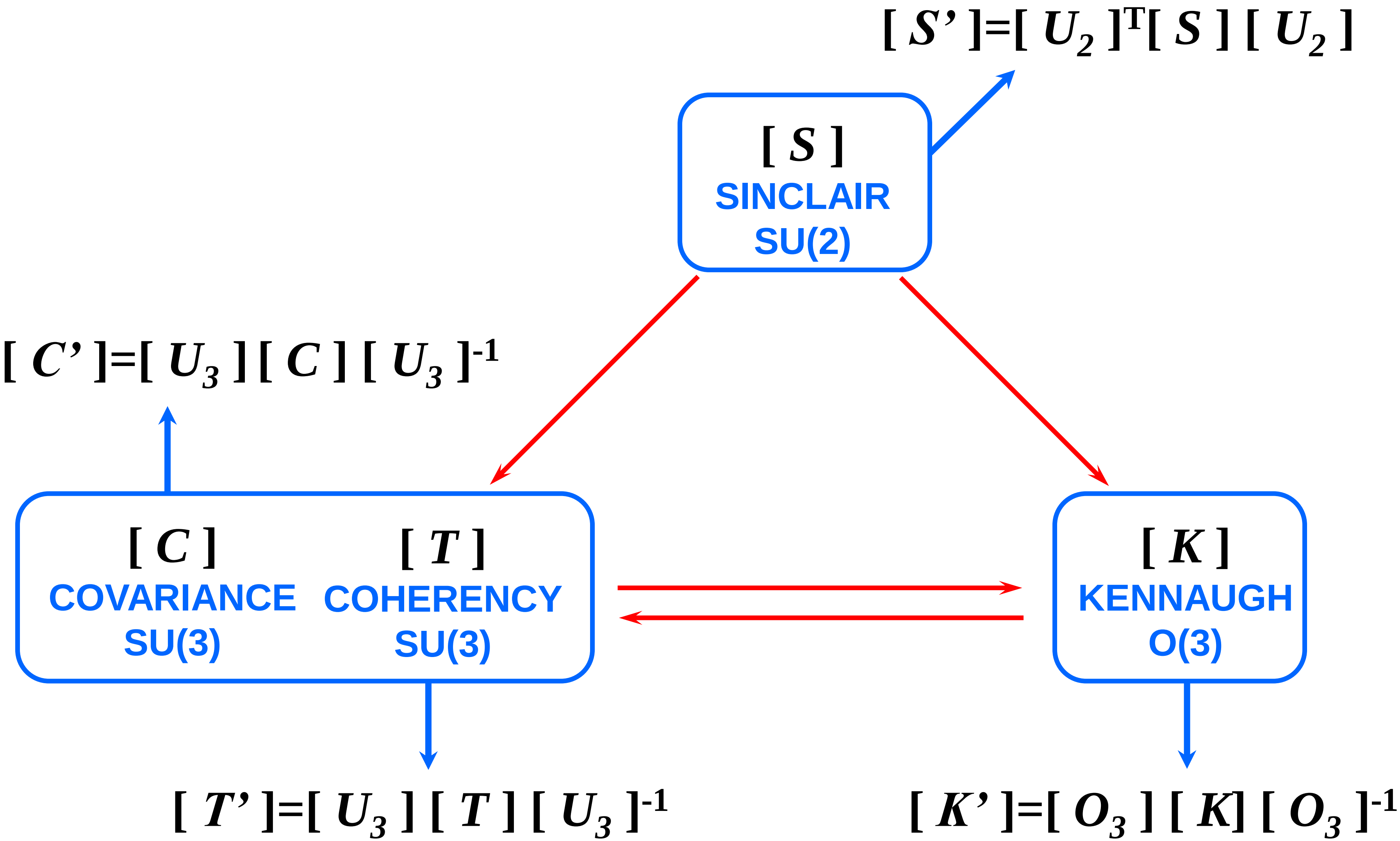
$$\underline{\Omega} = \left[S_{XX} \quad \sqrt{2}S_{XY} \quad S_{YY}\right]^T$$

↓

COVARIANCE MATRIX $[C]$

$$[C] = \underline{\Omega}\,\underline{\Omega}^{T*}$$

POLARIMETRIC DESCRIPTORS



ELLIPTICAL BASIS TRANSFORMATION

SPECIAL UNITARY SU(2) GROUP

$$\begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix} \begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix} \begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}$$

$$[U_2(\phi)] \qquad [U_2(\tau)] \qquad [U_2(\alpha)]$$

SPECIAL UNITARY SU(3) GROUP (T Matrix)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(2\phi) & \sin(2\phi) \\ 0 & -\sin(2\phi) & \cos(2\phi) \end{bmatrix} \begin{bmatrix} \cos(2\tau) & 0 & j\sin(2\tau) \\ 0 & 1 & 0 \\ j\sin(2\tau) & 0 & \cos(2\tau) \end{bmatrix} \begin{bmatrix} \cos(2\alpha) & -j\sin(2\alpha) & 0 \\ -j\sin(2\alpha) & \cos(2\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

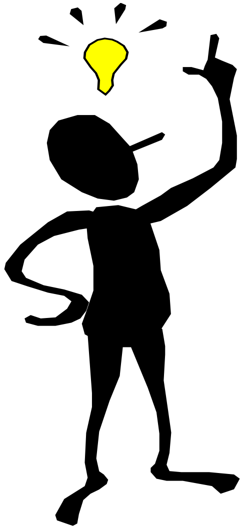
$$[U_3(2\phi)] \qquad [U_3(2\tau)] \qquad [U_3(2\alpha)]$$

O(3) UNITARY GROUP

$$\begin{bmatrix} \cos 2\phi & -\sin 2\phi & 0 \\ \sin 2\phi & \cos 2\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 2\tau & 0 & -\sin 2\tau \\ 0 & 1 & 0 \\ \sin 2\tau & 0 & \cos 2\tau \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 2\alpha & -\sin 2\alpha \\ 0 & \sin 2\alpha & \cos 2\alpha \end{bmatrix}$$

$$[O_3(2\phi)] \qquad [O_3(2\tau)] \qquad [O_3(2\alpha)]$$

TARGET EQUATIONS



POLARIMETRIC GOLDEN NUMBER

POLARIMETRIC TARGET DIMENSION

$$[S] = \begin{bmatrix} S_{XX} & S_{XY} \\ S_{YX} & S_{YY} \end{bmatrix}$$

5 DEGREES OF FREEDOM

$$|S_{XX}|, |S_{XY}|, |S_{YY}|$$

$$\phi_{XY-XX}, \phi_{YY-XX}$$

TARGET MONOSTATIC POLARIMETRIC « *DIMENSION* »

=

5

KENNAUGH MATRIX $[K]$

COHERENCY MATRIX $[T]$

9 HUYNEN REAL PARAMETERS

(A0, B0, B, C, D, E, F, G, H)

COVARIANCE MATRIX $[C]$

9 REAL PARAMETERS

$|XX|, |XY|, |YY|,$

$\mathrm{Re}(XXXY^*), \mathrm{Im}(XXXY^*)$

$\mathrm{Re}(XXYY^*), \mathrm{Im}(XXYY^*)$

$\mathrm{Re}(XYYY^*), \mathrm{Im}(XYYY^*)$

9 - 5 = 4 TARGET EQUATIONS

TARGET EQUATIONS

PURE TARGET – MONOSTATIC CASE

$$[T] = \underline{k} \cdot \underline{k}^{*T} = \begin{bmatrix} 2A_0 & C - jD & H + jG \\ C + jD & B_0 + B & E + jF \\ H - jG & E - jF & B_0 - B \end{bmatrix}$$

3x3 HERMITIAN MATRIX - RANK 1

↓

9 PRINCIPAL MINORS = 0

$$2A_0(B_0 + B) - C^2 - D^2 = 0 \qquad 2A_0(B_0 - B) - G^2 - H^2 = 0$$

$$-2A_0E + CH - DG = 0 \qquad B_0^2 - B^2 - E^2 - F^2 = 0$$

$$C(B_0 - B) - EH - GF = 0 \qquad -D(B_0 - B) + FH - GE = 0$$

$$2A_0F - CG - DH = 0 \qquad -G(B_0 + B) + FC - ED = 0$$

$$H(B_0 + B) - CE - DF = 0$$

TARGET EQUATIONS

$$[S] = \begin{bmatrix} S_{XX} & S_{XY} \\ S_{YX} & S_{YY} \end{bmatrix}$$

5 DEGREES OF FREEDOM

$$|S_{XX}|, |S_{XY}|, |S_{YY}|$$

$$\phi_{XY-XX}, \phi_{YY-XX}$$

COHERENCY MATRIX $[T]$

9 HUYNEN REAL PARAMETERS

(*A0, B0, B, C, D, E, F, G, H*)

TARGET MONOSTATIC POLARIMETRIC « *DIMENSION* »

=

5

9 - 5 = 4 TARGET EQUATIONS

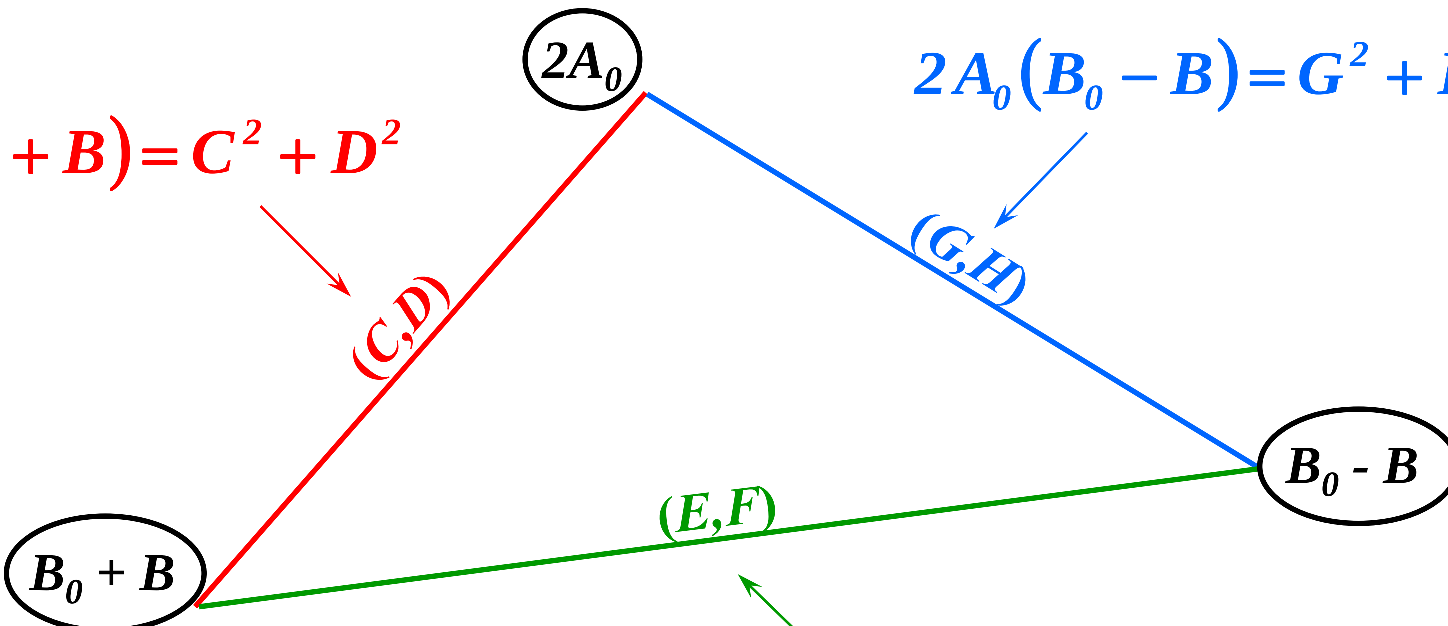
$$\begin{aligned} 2A_0(B_0+B) &= C^2+D^2 \\ 2A_0(B_0-B) &= G^2+H^2 \\ 2A_0E &= CH-DG \\ 2A_0F &= CG+DH \end{aligned}$$

MONOSTATIC TARGET DIAGRAM

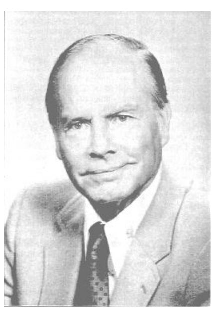
$$[T] = \underline{k} \cdot \underline{k}^{*T} = \begin{bmatrix} 2A_0 & C - jD & H + jG \\ C + jD & B_0 + B & E + jF \\ H - jG & E - jF & B_0 - B \end{bmatrix}$$

$$2A_0(B_0 + B) = C^2 + D^2$$

$$2A_0(B_0 - B) = G^2 + H^2$$

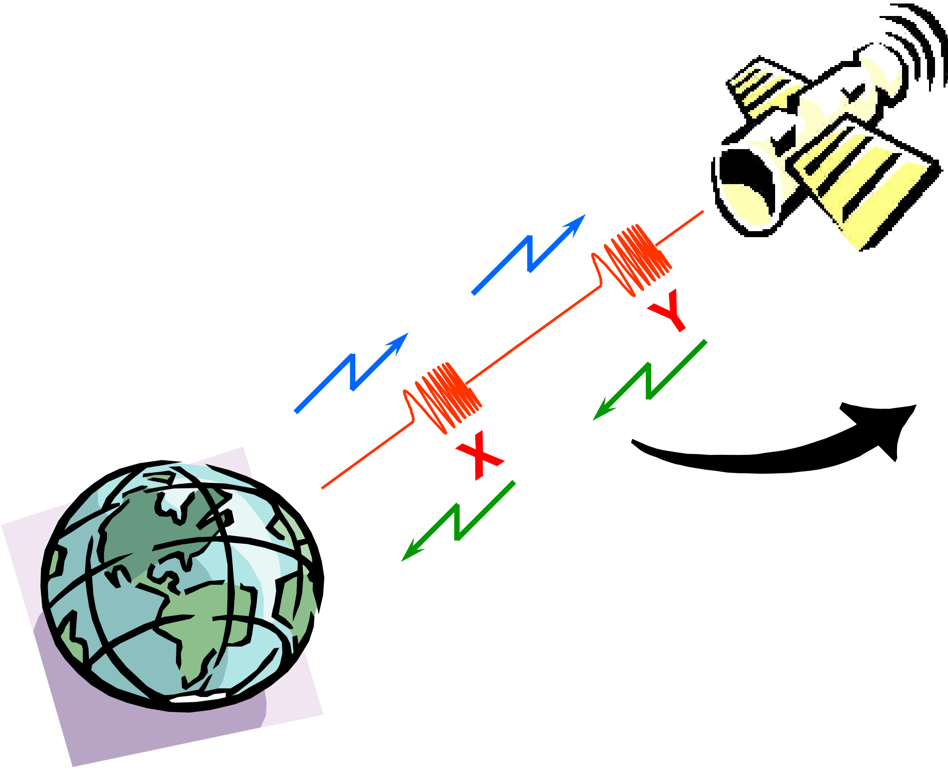


$$(B_0 + B)(B_0 - B) = E^2 + F^2$$

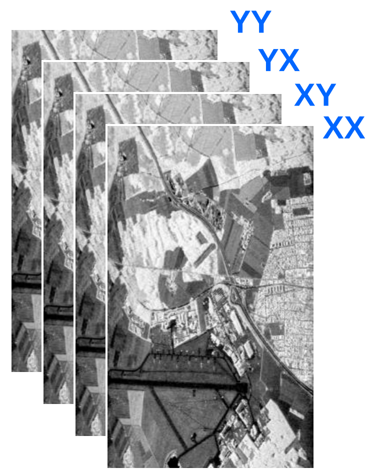


J.R. HUYNEN
(1920 – 2007)

SCATTERING POLARIMETRY



SINCLAIR MATRIX $[S]$



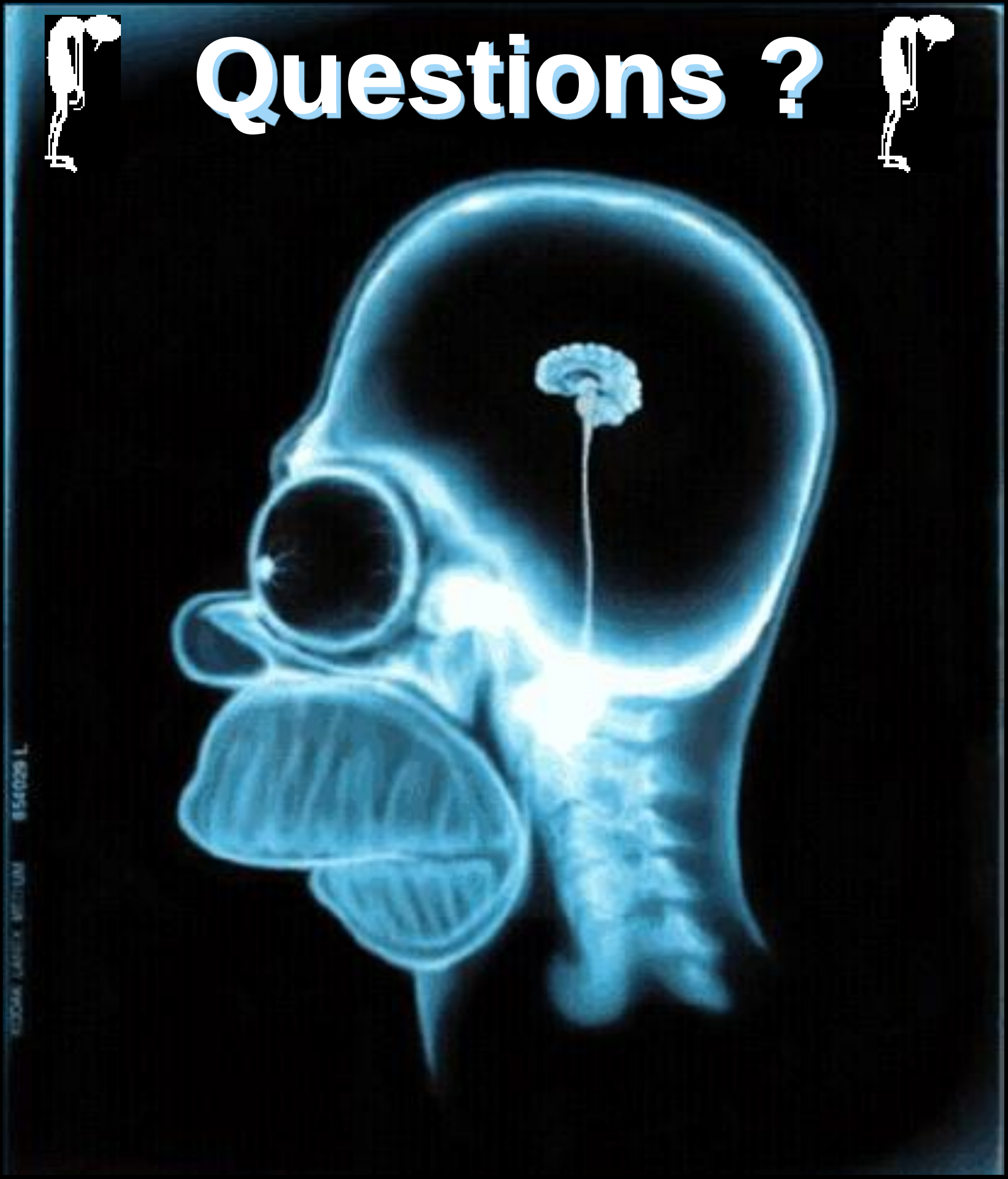
THE DIFFERENT TARGET POLARIMETRIC DESCRIPTORS

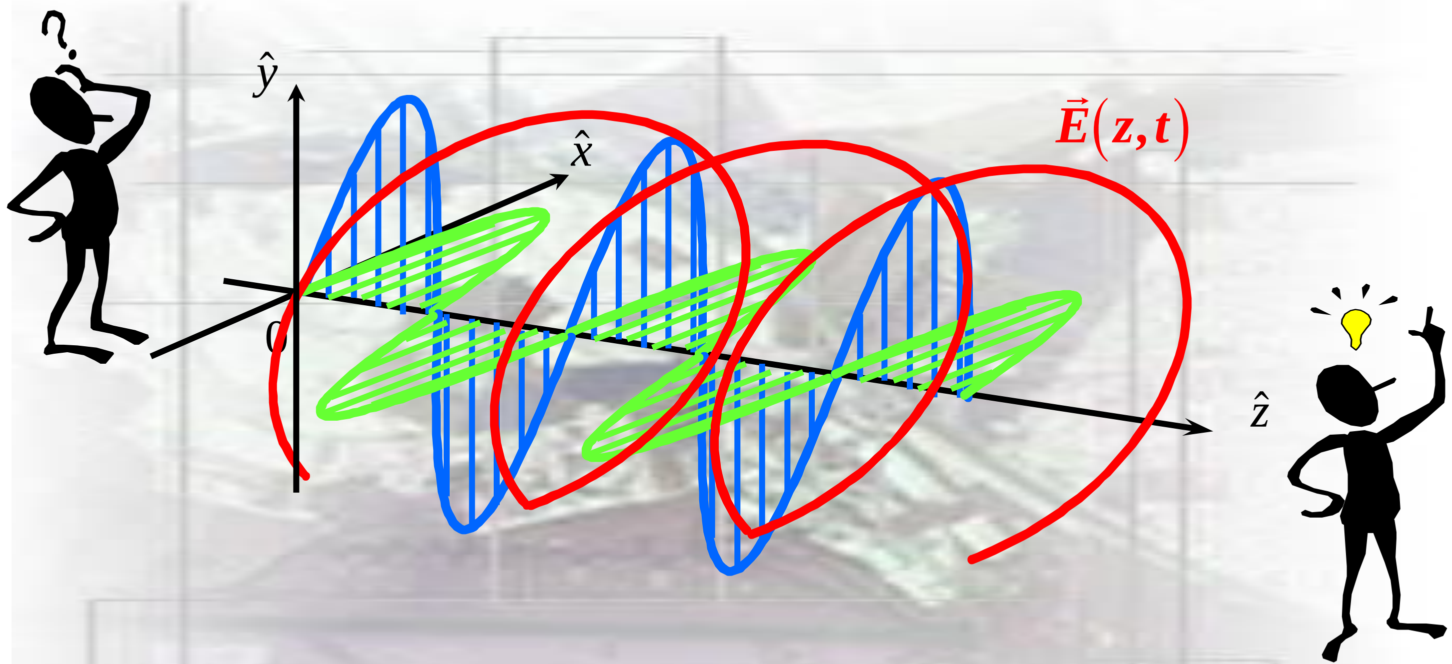
[S]	SINCLAIR Matrix
$\underline{k}$, $\underline{\Omega}$	Target Vectors
[K]	KENNAUGH Matrix
[T]	Coherency Matrix
[C]	Covariance Matrix

TRANSMITTER: X & Y
RECEIVERS: X & Y

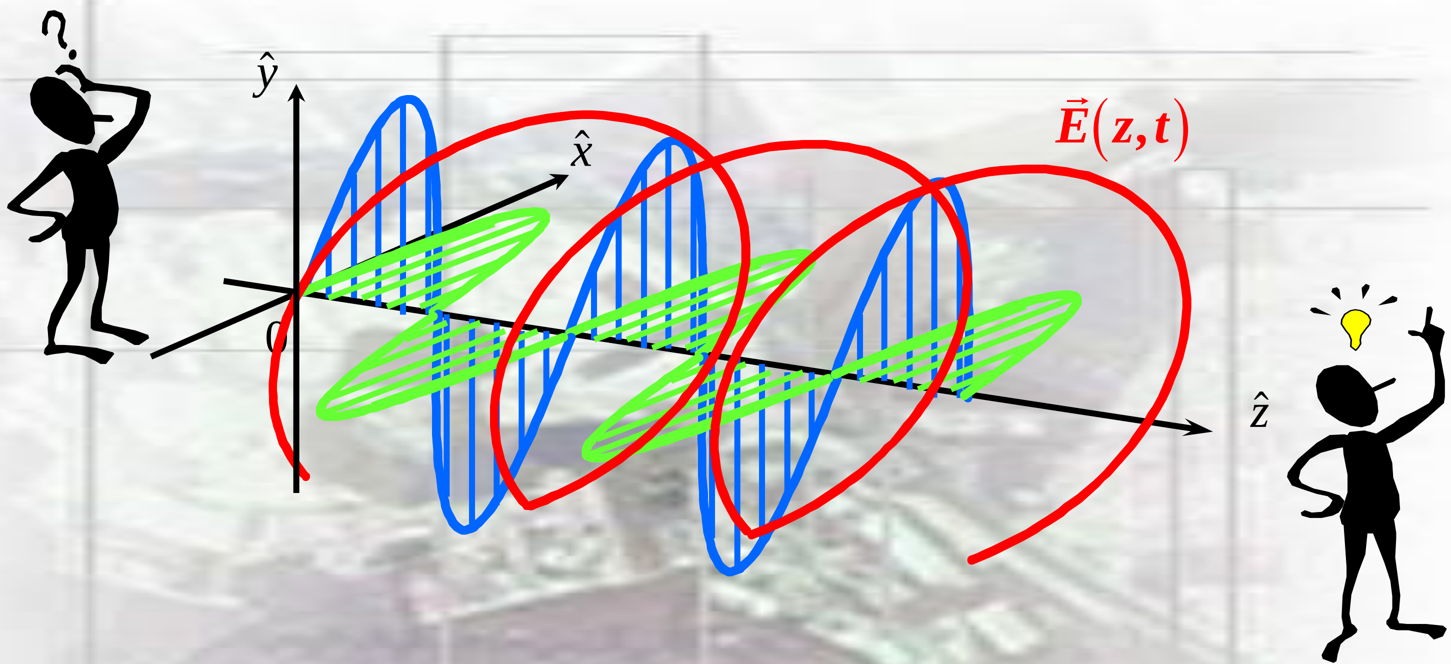
POLARIMETRIC REMOTE SENSING

Questions ?



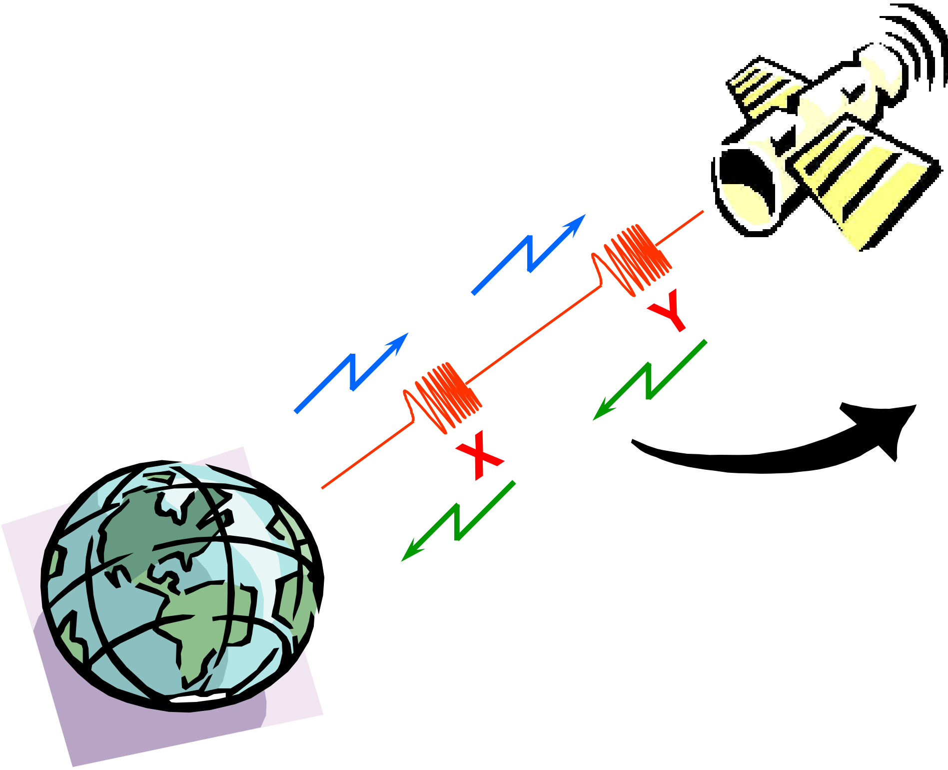


ADVANCED CONCEPTS

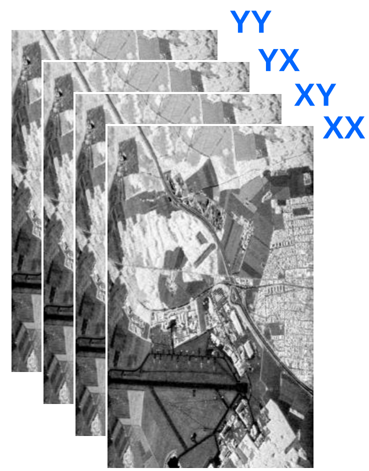


POLARIMETRIC REMOTE SENSING

SCATTERING POLARIMETRY



SINCLAIR MATRIX $[S]$



TRANSMITTER: X & Y
RECEIVERS: X & Y

THE DIFFERENT TARGET POLARIMETRIC DESCRIPTORS

[S]	SINCLAIR Matrix
k, Ω	Target Vectors
[K]	KENNAUGH Matrix
[T]	Coherency Matrix
[C]	Covariance Matrix

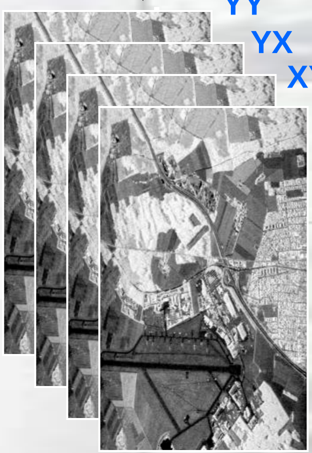
POLARIMETRIC REMOTE SENSING

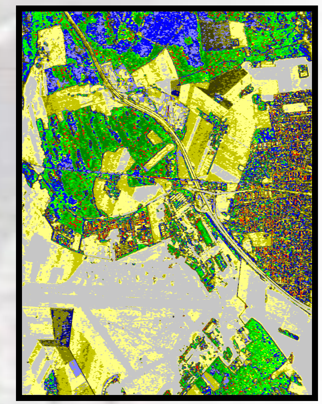
POLARIMETRIC REMOTE SENSING

POL-SAR PROCESSING

PHENOMENOLOGIC

QUALITATIVE ANALYSIS





POLARIMETRIC SPECKLE FILTERING

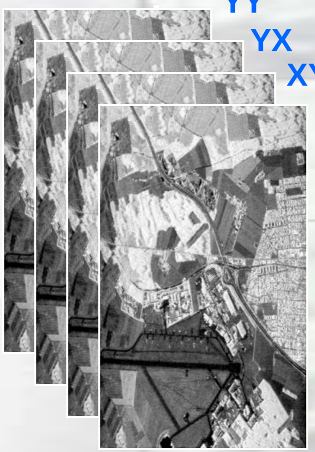
POLARIMETRIC TARGET DECOMPOSITION

POLARIMETRIC CLASSIFICATION

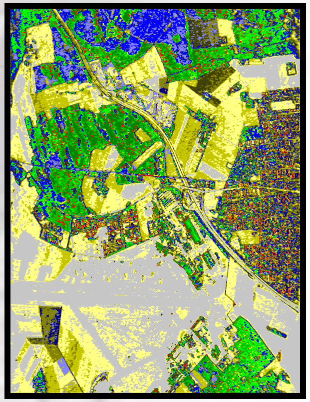
MONO/DUAL CHANNELS

POLARIMETRIC REMOTE SENSING

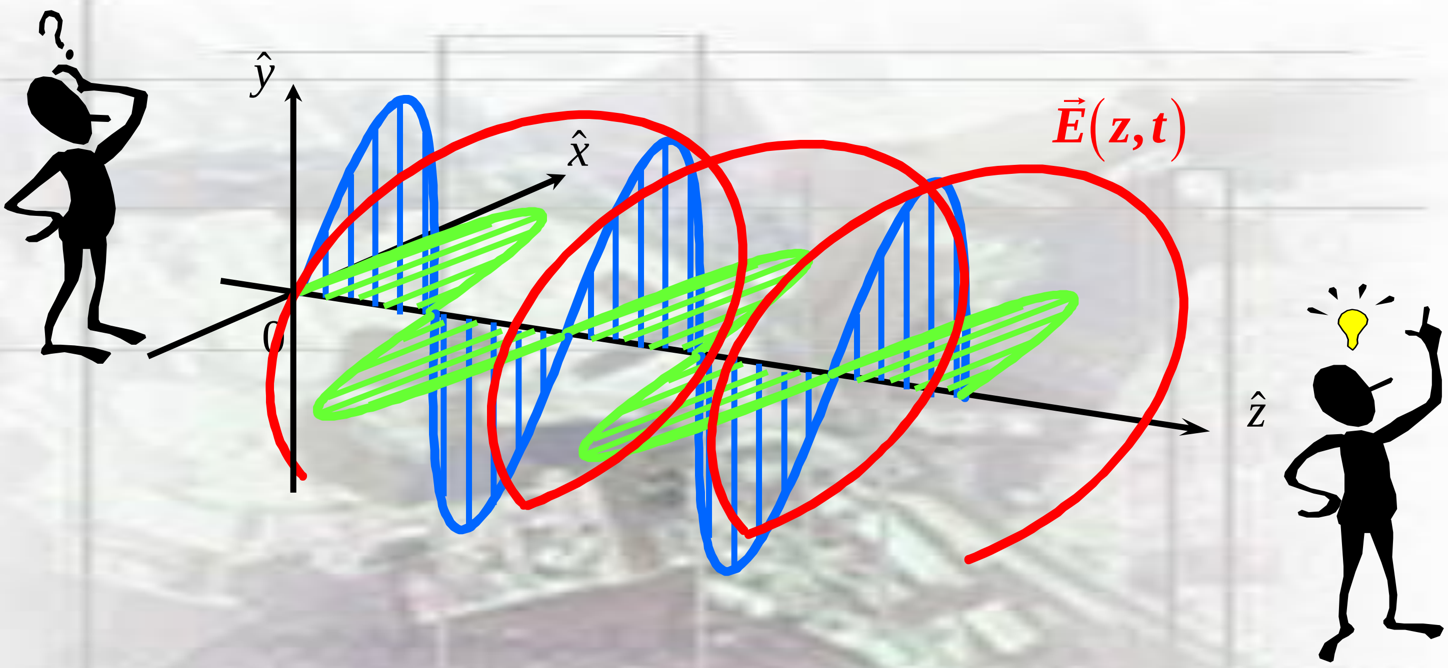
esa



POL-SAR PROCESSING
PHENOMENOLOGIC
QUALITATIVE ANALYSIS

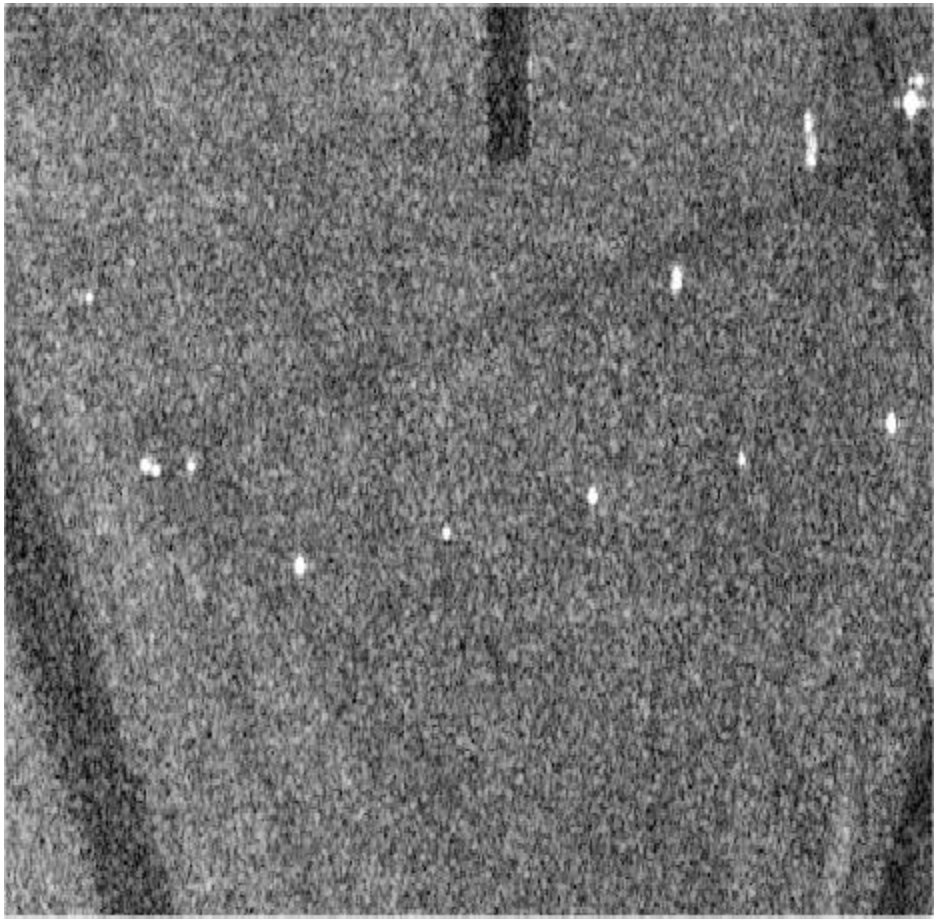


POLARIMETRIC SPECKLE FILTERING → **POLARIMETRIC TARGET DECOMPOSITION** → **POLARIMETRIC CLASSIFICATION** MONO/DUAL CHANNELS

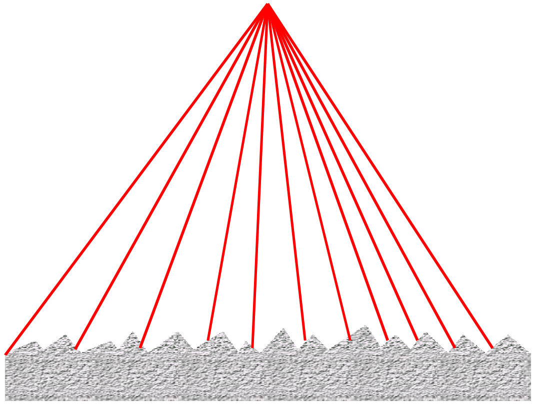


POLARIMETRIC SPECKLE FILTERING

An Introduction



OBSERVATION POINT



SURFACE ROUGHNESS
WAVELENGTH

SCATTERING FROM DISTRIBUTED SCATTERERS

↓

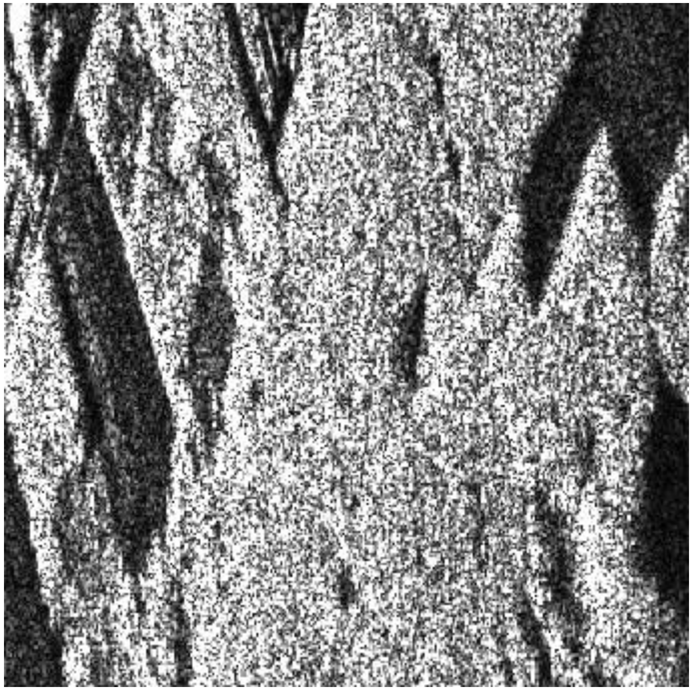
COHERENT INTERFERENCES OF WAVES SCATTERED FROM MANY RANDOMLY DISTRIBUTED ELEMENTARY SCATTERERS INSIDE THE RESOLUTION CELL

↓

GRANULAR NOISE

↓

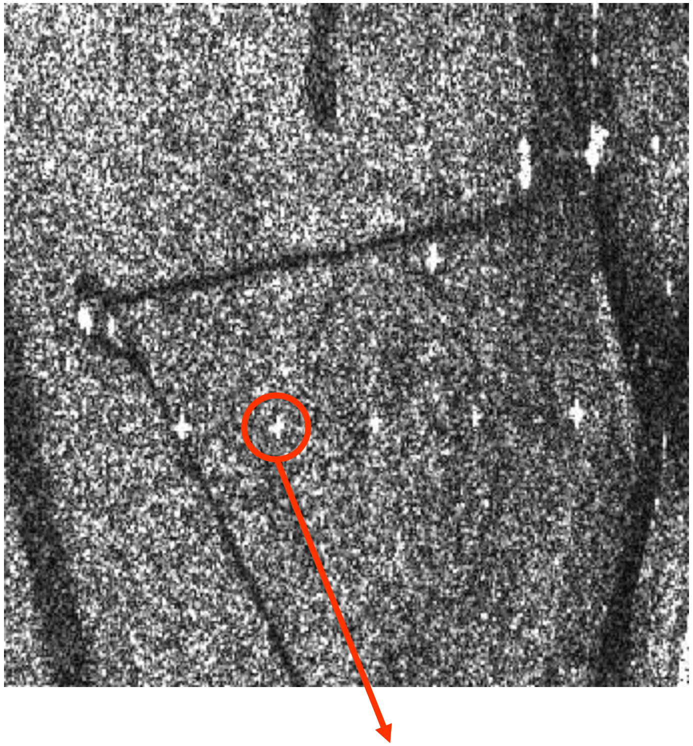
SPECKLE PHENOMENON



Fully Developed speckle

Bright points: Points where the interference is constructive

Dark points: Points where the interference is destructive



Corner reflector

Dominant scatter

No speckle

S_{hh} amplitude

E-SAR L-band system

Courtesy of Dr C. Lopez Martinez

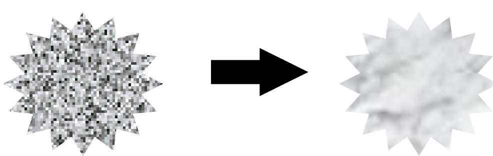
SPECKLE FILTERING

SPECKLE PHENOMENON

DISTORTION OF THE INTERPRETATION

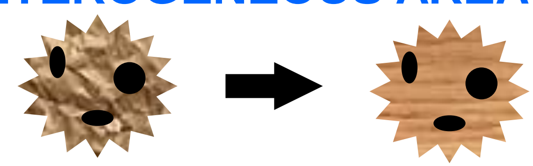
SPECKLE FILTERING

HOMOGENEOUS AREA



SPECKLE REDUCTION (RADIOMETRIC RESOLUTION)

HETEROGENEOUS AREA



DETAILS PRESERVATION (SPATIAL RESOLUTION)

SPECKLE : MULTIPLICATIVE NOISE MODEL

« SPECKLE is a scattering phenomenon and not a noise. However, from the image SAR processing point of vue, the speckle can be modeled as multiplicative noise for extended target » (Lee, IGARSS-98)

$$\underline{y} = \begin{bmatrix} y_{HH} \\ y_{HV} \\ y_{VV} \end{bmatrix} = \begin{bmatrix} n_{HH} & 0 & 0 \\ 0 & n_{HV} & 0 \\ 0 & 0 & n_{VV} \end{bmatrix} \begin{bmatrix} x_{HH} \\ x_{HV} \\ x_{VV} \end{bmatrix} = \begin{bmatrix} x_{HH} n_{HH} \\ x_{HV} n_{HV} \\ x_{VV} n_{VV} \end{bmatrix}$$

SCATTERING FIELD — NOISE — REFLECTIVITY DENSITY

$$I_{pqpq} = y_{pq} y^*_{pq} = X_{pqpq} v_{pqpq}$$

INTENSITY

$$A_{pqpq} = \sqrt{I_{pqpq}} = \sqrt{y_{pq} y^*_{pq}}$$

AMPLITUDE

esa

LINEAR SPECKLE FILTERS
Intensity / Amplitude – Single / Multi Look – Single Pol Channel

Median Filter
MAP Filter (Kuan)
Gradient Filter
Nagao Filter (Nagao)
Sigma Filter (Lee)
Frost Filter (Frost)
Geometrical Filter (Crimmins)
Morphological Filter (Safa, Flouzat)
.
Local Statistics Filter (Lee 80)
Refined Lee Filter (Lee 81)

J.S. Lee, et al. "Speckle Filtering of SAR images: A Review," Remote Sensing Reviews, Vol. 8, pp. 313-340, 1994.

J.S. Lee,"Speckle analysis and smoothing of SAR images," Computer Graphics and Image Processing, Vol. 17, 1981.

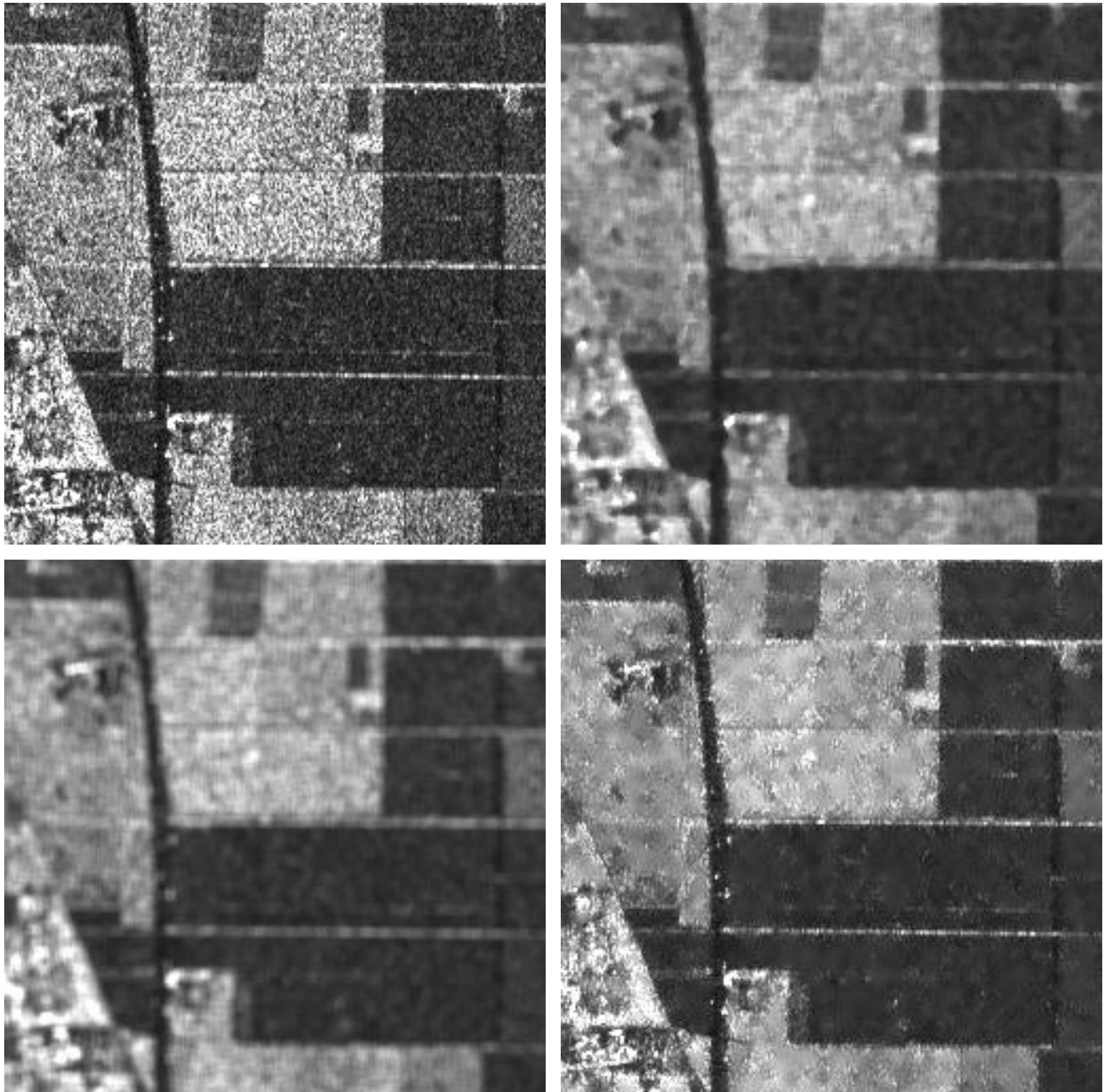
J.S. Lee,"Digital image enhancement and noise filtering by use of local statistics," IEEE PAMI, Vol. 2 No. 2, 1980.

J.S. Lee,"Refined filtering of image noise using local statistics," CVGIP, vol.15, 380-389, 1981.

SPECKLE FILTERING

Original
4-look
amplitude

5x5 Median



5x5 Boxcar

Lee Refined
(7x7)

POLSAR SPECKLE FILTERING

- Preserving polarimetric properties
  - Filter all elements equally like multi-look Processing
  - Select pixels with the same scattering property
- Introduce no cross-talk
  - Filter each element separately but equally
- Reduce speckle while preserving image quality

J.S. Lee, M.R. Grunes and G. De Grandi, "Polarimetric SAR Speckle Filtering and Its Impact on Terrain Classification" *IEEE TGRS*, September 1999

POLARIMETRIC VECTORIAL SPECKLE FILTER

$$\langle [T] \rangle = \frac{1}{N}\sum_{i=1}^{N} \underline{k}_i \underline{k}_i^{*T}$$

$$[\hat{T}] = E([T]) - k[E([T]) - [T]] \rightarrow [\hat{T}]$$

SPAN IMAGE

$$S_S = \langle T_{11} \rangle + \langle T_{22} \rangle + \langle T_{33} \rangle$$

LINEAR SCALAR LEE FILTER

$$\hat{S} = E(S_S) - k[E(S_S) - S_S]$$

$$k = \frac{var(S)}{var(S_S)} = \frac{CV_{S_S}^2 - \sigma_v^2}{CV_{S_S}^2 [1 + \sigma_v^2]}$$

J.S. LEE

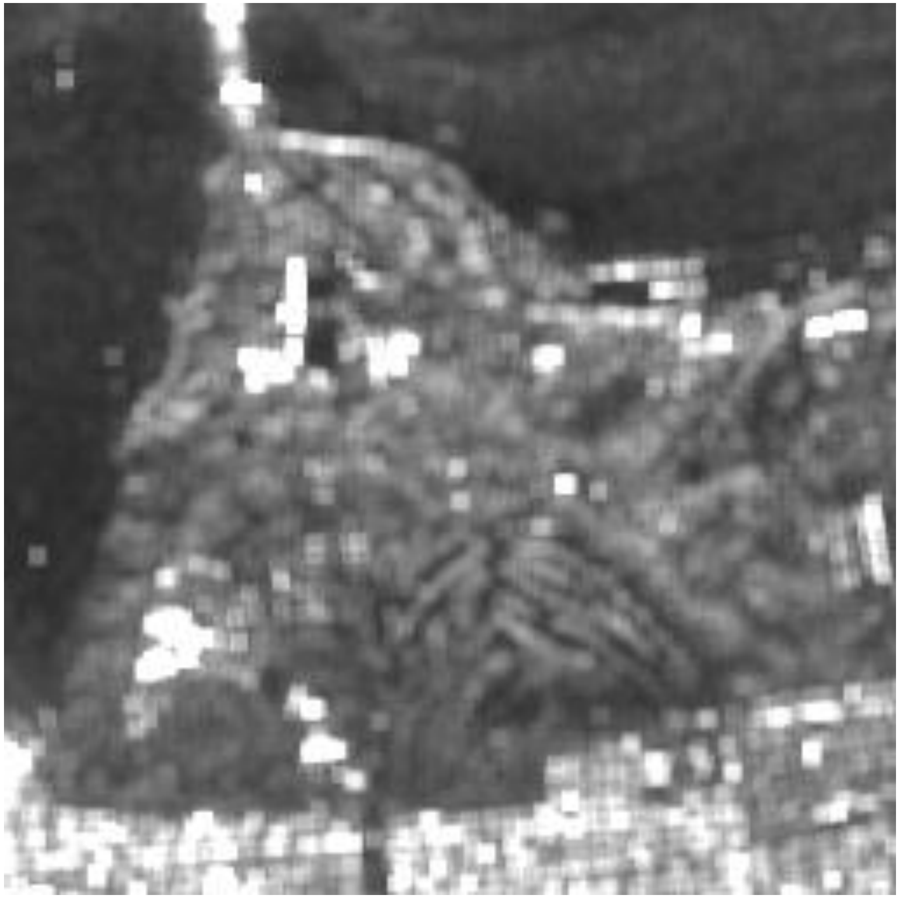
Homogeneous Areas

$$var(S) \approx 0 \Rightarrow k = 0 \Rightarrow \hat{S} = E(S_S)$$

Highly Inhomogeneous Areas

$$var(S) \mapsto var(S_s) \Rightarrow k = 1 \Rightarrow \hat{S} = S_S$$

REFINED FILTER



SAN FRANCISCO BAY JPL - AIRSAR L-band 1988

BoxCar Filter

SAN FRANCISCO BAY JPL - AIRSAR L-band 1988

J.S. Lee, M.R. Grunes and G. De Grandi, "Polarimetric SAR Speckle Filtering and Its Impact on Terrain Classification" *IEEE TGRS*, September 1999

POLSAR SPECKLE FILTERING

SAN FRANCISCO BAY JPL - AIRSAR L-band 1988

J.S. Lee, D.L. Schuler, T.L. Ainsworth, M.R. Grunes, E Pottier, L. Ferro-Famil, "*Scattering Model Based Speckle Filetring of Polarimetric SAR Data*" IEEE – TGRS, vol 1, January 2006

SAN FRANCISCO BAY JPL - AIRSAR L-band 1988

J.S. Lee, J.H Wen, T.L. Ainsworth, K.S. Chen, A.J. Chen, "*Improved Sigma Filter for Speckle Filtering of SAR Imagery*" IEEE – TGRS, vol 1, January 2009

POLARIMETRIC SPECKLE FILTERING IS NOT AN EXACT SCIENCE
SUBJECTIVE, IMAGE DEPENDENT

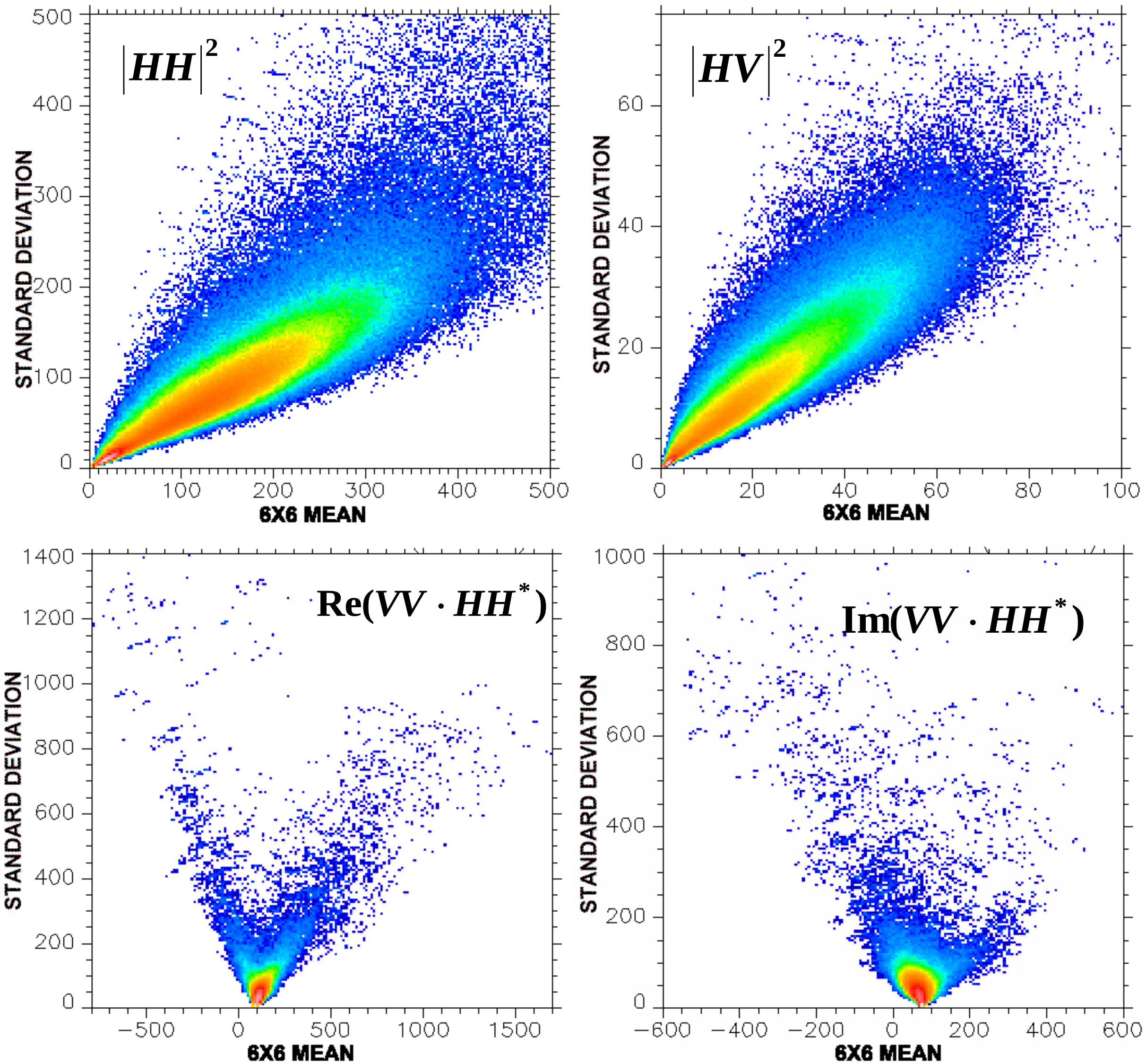
Quantitative Criteria (J.S. Lee - IGARSS 98)

- Speckle Reduction (E.N.L)
- Edge Sharpness Preservation
- Line and Point Target Contrast Preservation
- Retention of Mean Values in Homogeneous Regions
- Retention of Texture Information
- Retention of Polarimetric Information (co, cross-correlations)
- Computational Efficiency
- Implementation Complexity

$$[\hat{T}] = E([T]) - k[E([T]) - [T]]$$

THE POLARIMETRIC SPECKLE LEE FILTER IS TODAY A GOOD COMPROMISE

POLSAR SPECKLE NOISE MODEL



Diagonal Terms

Multiplicative

Off-Diagonal Terms

Additive/Multiplicative

J.S. Lee, M.R. Grunes and G. De Grandi, "Polarimetric SAR Speckle Filtering and Its Impact on Terrain Classification" *IEEE TGRS*, vol. 37, N°5, September 1999

MULTIPLICATIVE-ADDITIVE NOISE MODEL

$$S_i S_j^* = \underbrace{\psi \bar{z}_n n_m N_c \exp(j\phi_x)}_{\textit{Multiplicative term}} + \underbrace{\psi(|\rho| - N_c \bar{z}_n)\exp(j\phi_x) + \psi(n_{ar} + jn_{ai})}_{\textit{Additive term}}$$

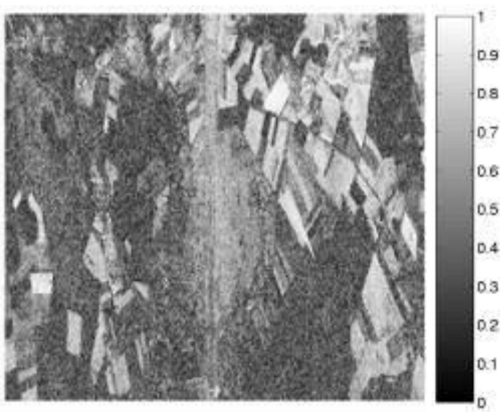
C. LOPEZ MARTINEZ

Multiplicative speckle noise component: n_m → Important for high coherence areas

Additive speckle noise component: $n_{ar}+jn_{ai}$ → Important for low coherence areas

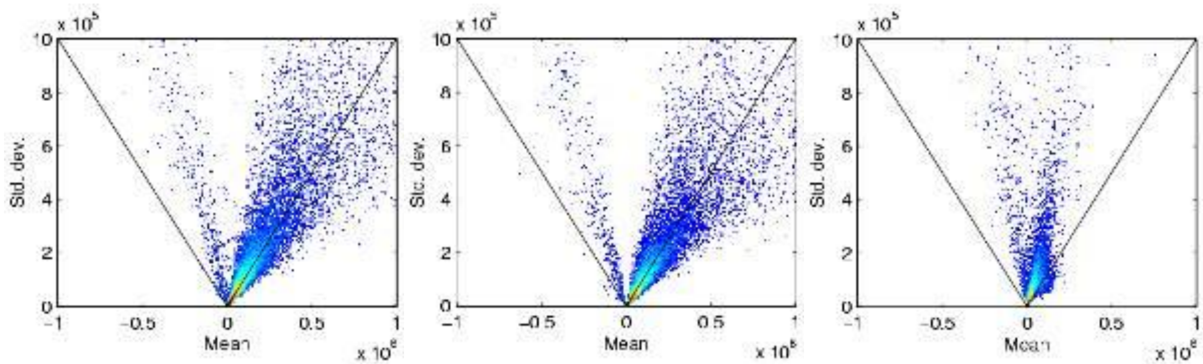
Combination controlled by complex coherence

$S_{hh}S_{vv}^*$



$0.8 \leq |\rho| \leq 1$

Total $\Re\{S_{hh}S_{vv}^*\}$ Mult. term $\Re\{S_{hh}S_{vv}^*\}$ Add. term $\Re\{S_{hh}S_{vv}^*\}$

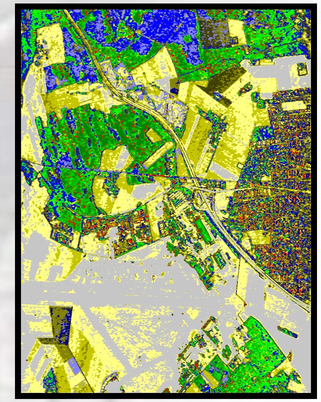


C. López-Martínez and X. Fàbregas, "Polarimetric SAR Speckle Noise Model," IEEE TGRS, vol. 41, no. 10, pp. 2232 – 2242, Oct. 2003

Courtesy of Dr C. Lopez Martinez

POLARIMETRIC REMOTE SENSING

POL-SAR PROCESSING
PHENOMENOLOGIC
QUALITATIVE ANALYSIS



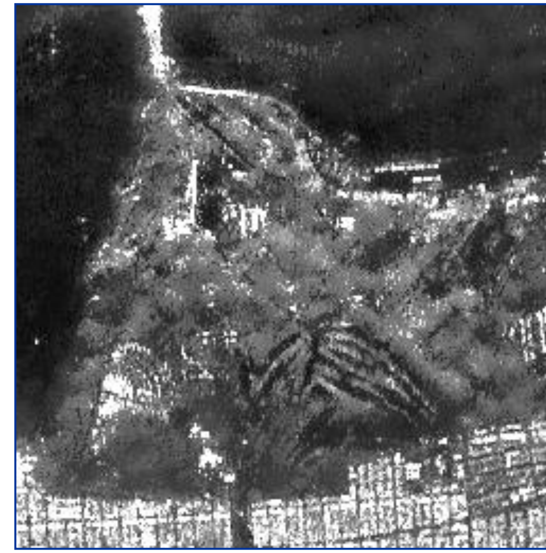
POLARIMETRIC SPECKLE FILTERING

POLARIMETRIC TARGET DECOMPOSITION

POLARIMETRIC CLASSIFICATION
MONO/DUAL CHANNELS

SPECKLE FILTERING

SPECKLE FILTER



$$[T] = \underline{k}\,\underline{k}^{*T}$$

AVERAGING DATA

SECOND ORDER STATISTICS

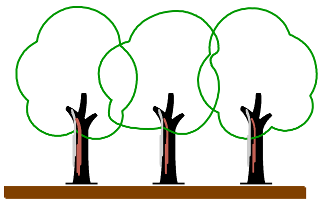
COHERENCY MATRICES

$$\langle [T] \rangle = \frac{1}{N}\sum_{i=1}^{N} \underline{k}_i\,\underline{k}_i^{*T}$$

SMOOTHING AVERAGING

CONCEPT OF THE DISTRIBUTED TARGET

TARGET DECOMPOSITIONS



PURE TARGET

COHERENCY MATRIX [T]

9 REAL DEPENDANT HUYNEN PARAMETERS (A0,B0,B,C,D,E,F,G,H)

POLARIMETRIC DISTRIBUTED TARGET « *DIMENSION* » = 5

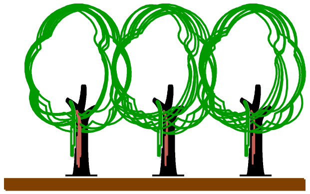
9 - 5 = 4 TARGET EQUATIONS

$$2A_0(B_0+B) = C^2+D^2$$

$$2A_0(B_0-B) = G^2+H^2$$

$$2A_0E = CH-DG$$

$$2A_0F = CG+DH$$



DISTRIBUTED TARGET

POLARIMETRIC DISTRIBUTED TARGET « *DIMENSION* » = 9

COHERENCY MATRIX <[*T*]>

9 REAL INDEPENDANT HUYNEN PARAMETERS

(<A0>,<B0>,<B>,<C>,<D>,<E>,<F>,<G>,<H>)

9 TARGET INEQUATIONS

$$2\langle A_0\rangle\left(\langle B_0\rangle+\langle B\rangle\right)\geq\langle C\rangle^2+\langle D\rangle^2$$

$$2\langle A_0\rangle\left(\langle B_0\rangle-\langle B\rangle\right)\geq\langle G\rangle^2+\langle H\rangle^2$$

$$2\langle A_0\rangle\langle E\rangle\geq\langle C\rangle\langle H\rangle-\langle D\rangle\langle G\rangle$$

$$2\langle A_0\rangle\langle F\rangle\geq\langle C\rangle\langle G\rangle+\langle D\rangle\langle H\rangle$$

$$\langle B_0\rangle^2\geq\langle B\rangle^2+\langle E\rangle^2+\langle F\rangle^2$$

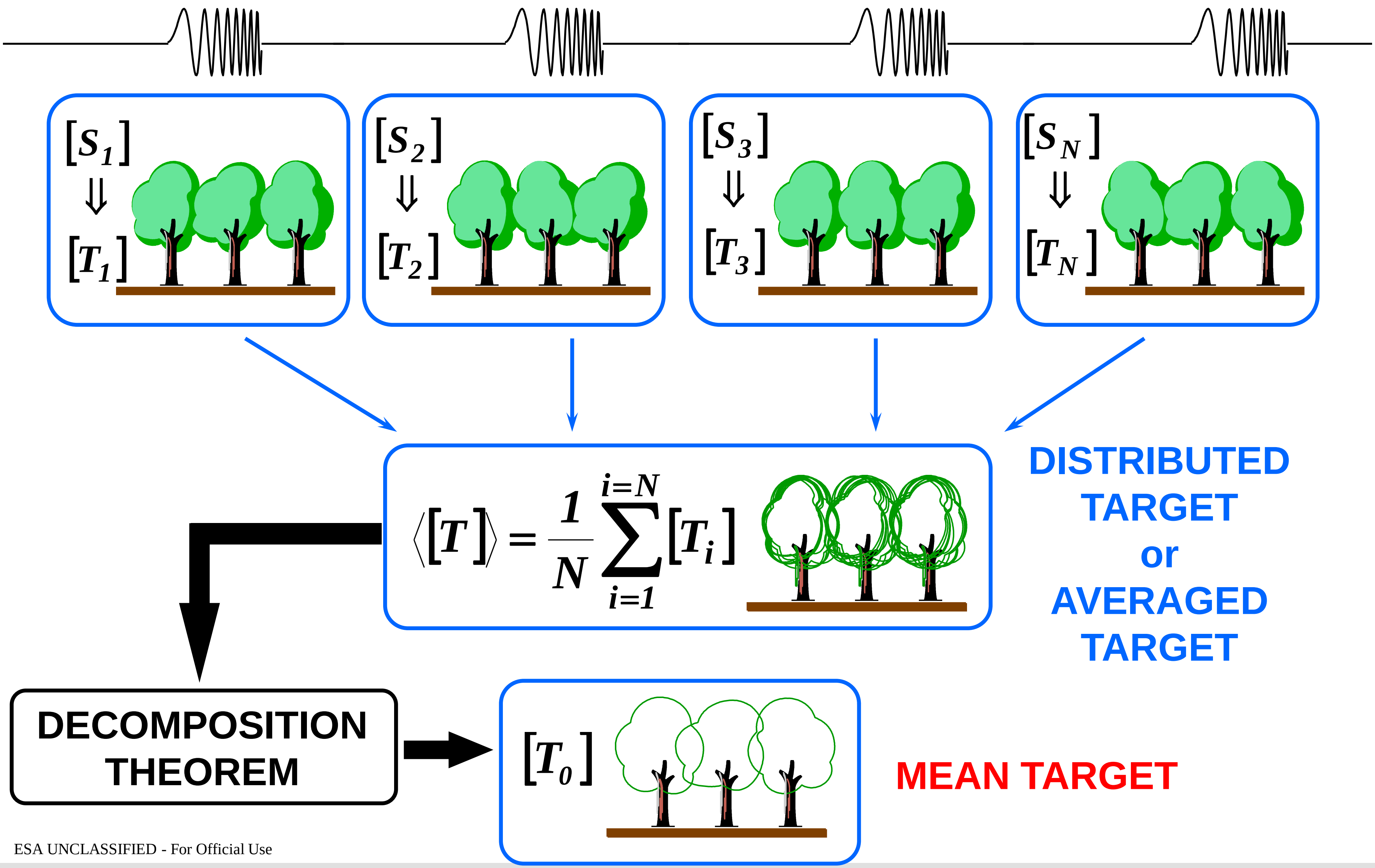
$$\langle H\rangle\left(\langle B_0\rangle+\langle B\rangle\right)\geq\langle C\rangle\langle E\rangle+\langle D\rangle\langle F\rangle$$

$$\langle G\rangle\left(\langle B_0\rangle+\langle B\rangle\right)\geq\langle C\rangle\langle F\rangle-\langle D\rangle\langle E\rangle$$

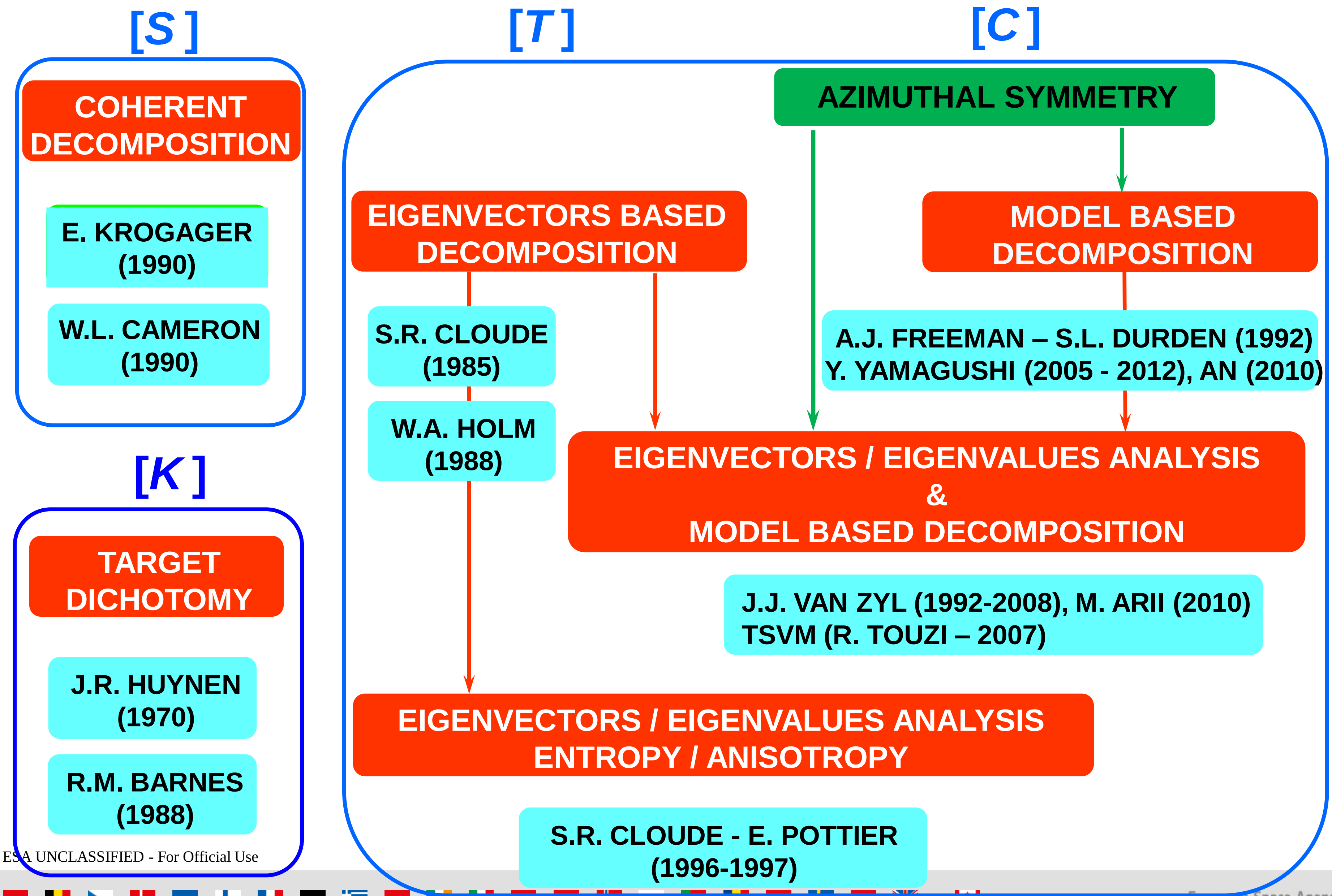
$$\langle C\rangle\left(\langle B_0\rangle-\langle B\rangle\right)\geq\langle H\rangle\langle E\rangle+\langle F\rangle\langle G\rangle$$

$$\langle D\rangle\left(\langle B_0\rangle-\langle B\rangle\right)\geq\langle F\rangle\langle H\rangle-\langle G\rangle\langle E\rangle$$

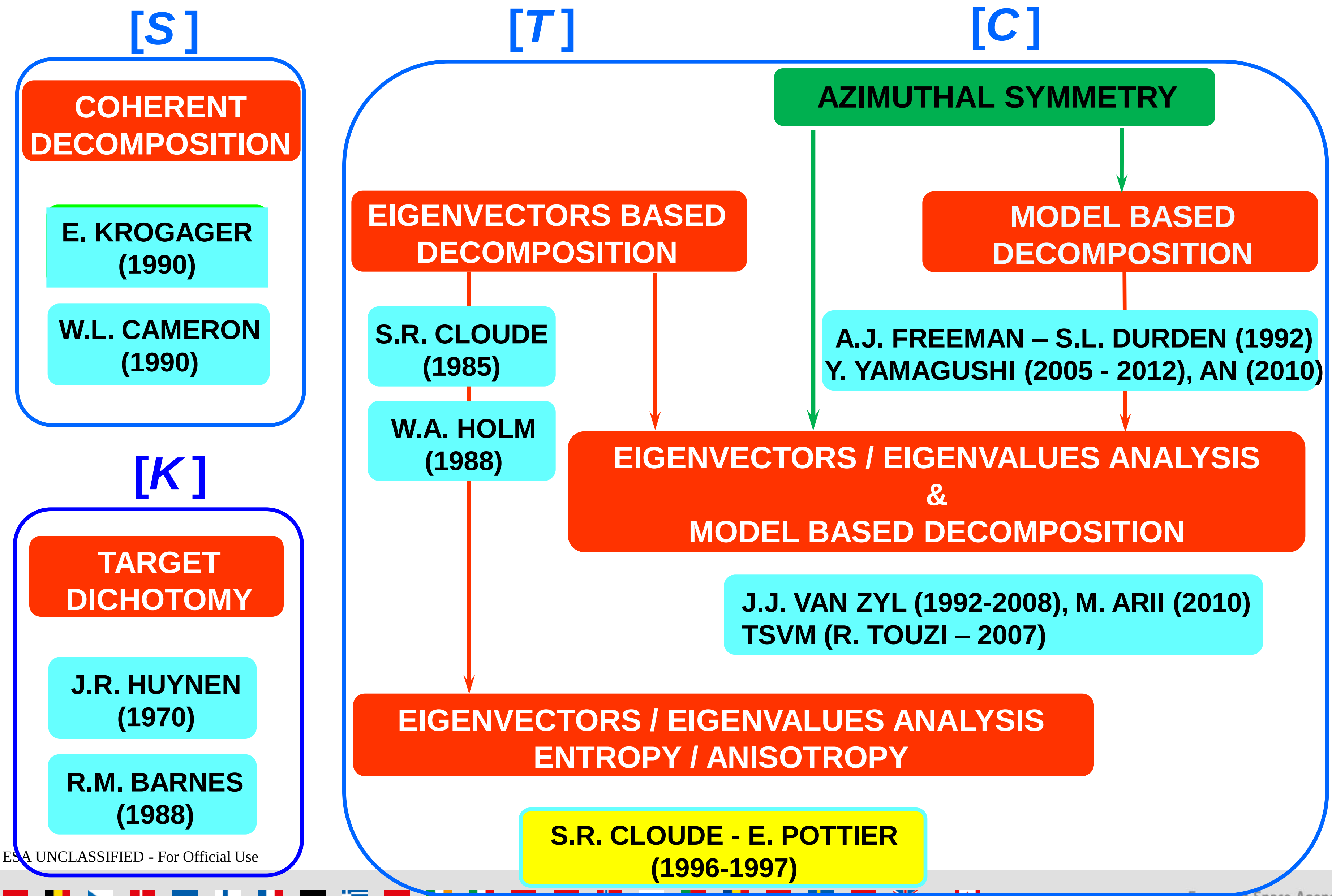
TARGET DECOMPOSITIONS



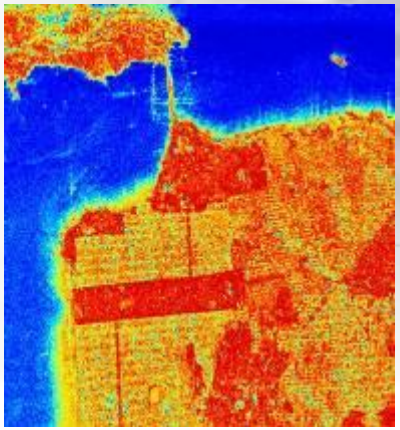
TARGET DECOMPOSITIONS

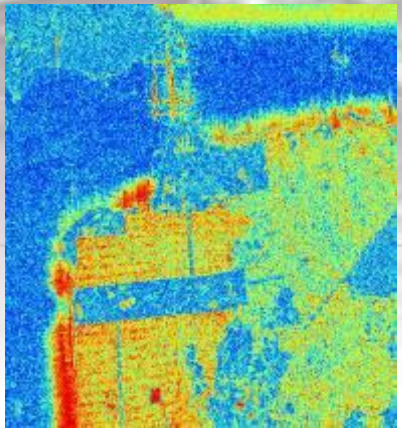


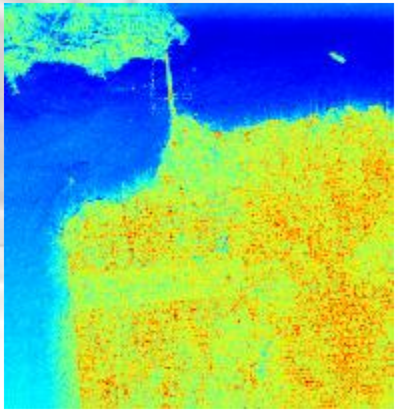
TARGET DECOMPOSITIONS



THE H / A / $\underline{\alpha}$ POLARIMETRIC TARGET DECOMPOSITION THEOREM







S.R. CLOUDE - E. POTTIER (1995 - 1996)

H / A / α DECOMPOSITION

TARGET VECTOR

$$\underline{k} = \frac{1}{\sqrt{2}}\left[S_{XX} + S_{YY} \quad S_{XX} - S_{YY} \quad 2S_{XY}\right]^T$$

LOCAL ESTIMATE OF THE COHERENCY MATRIX

$$\langle [T] \rangle = \frac{1}{N}\sum_{i=1}^{N} \underline{k}_i \cdot \underline{k}_i^{*T} = \frac{1}{N}\sum_{i=1}^{N} [T_i]$$

EIGENVECTORS / EIGENVALUES ANALYSIS

$$\langle [T] \rangle = [U_3][\Sigma][U_3]^{-1} = \begin{bmatrix} \underline{u}_1 & \underline{u}_2 & \underline{u}_3 \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \begin{bmatrix} \underline{u}_1 & \underline{u}_2 & \underline{u}_3 \end{bmatrix}^{*T}$$

ORTHOGONAL EIGENVECTORS

REAL EIGENVALUES
$\lambda_1 > \lambda_2 > \lambda_3$

$$P_i = \frac{\lambda_i}{\sum_{k=1}^{3} \lambda_k}$$

H / A / α DECOMPOSITION

$$\langle [T] \rangle = [U_3][\Sigma][U_3]^{-1} = \begin{bmatrix} \underline{u}_1 & \underline{u}_2 & \underline{u}_3 \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \begin{bmatrix} \underline{u}_1 & \underline{u}_2 & \underline{u}_3 \end{bmatrix}^{*T}$$

ORTHOGONAL EIGENVECTORS

REAL EIGENVALUES

$\lambda_1 > \lambda_2 > \lambda_3$

PARAMETERISATION OF THE SU(3) UNITARY MATRIX

$$[U_3] = \begin{bmatrix} \cos\alpha_1 e^{j\phi_1} & \cos\alpha_2 e^{j\phi_2} & \cos\alpha_3 e^{j\phi_3} \\ \sin\alpha_1 \cos\beta_1 e^{j\phi_1} e^{j\delta_1} & \sin\alpha_2 \cos\beta_2 e^{j\phi_2} e^{j\delta_2} & \sin\alpha_3 \cos\beta_3 e^{j\phi_3} e^{j\delta_3} \\ \sin\alpha_1 \sin\beta_1 e^{j\phi_1} e^{j\gamma_1} & \sin\alpha_2 \sin\beta_2 e^{j\phi_2} e^{j\gamma_2} & \sin\alpha_3 \sin\beta_3 e^{j\phi_3} e^{j\gamma_3} \end{bmatrix}$$

TARGET 1 | TARGET 2 | TARGET 3

PROBABILITIES

$$P_i = \frac{\lambda_i}{\sum_{k=1}^{3} \lambda_k}$$

AVERAGED PARAMETERS

$$\underline{\alpha} = P_1\alpha_1 + P_2\alpha_2 + P_3\alpha_3 \qquad \underline{\beta} = P_1\beta_1 + P_2\beta_2 + P_3\beta_3$$

$$\underline{\gamma} = P_1\gamma_1 + P_2\gamma_2 + P_3\gamma_3 \qquad \underline{\delta} = P_1\delta_1 + P_2\delta_2 + P_3\delta_3$$

UNITARY TARGET VECTOR ($\underline{u}_0$) OF THE MEAN DOMINANT MECHANISM

$$\underline{u}_0 = \begin{bmatrix} \cos(\underline{\alpha}) & \sin(\underline{\alpha})\cos(\underline{\beta})\, e^{j\underline{\delta}} & \sin(\underline{\alpha})\sin(\underline{\beta})\, e^{j\underline{\gamma}} \end{bmatrix}^T$$

MEAN SCATTERING MECHANISM

UNITARY VECTOR $\underline{u}_0$

$$\underline{u}_0 = \begin{bmatrix} \cos(\underline{\alpha}) \\ \sin(\underline{\alpha})\cos(\underline{\beta})e^{j\underline{\delta}} \\ \sin(\underline{\alpha})\sin(\underline{\beta})e^{j\underline{\gamma}} \end{bmatrix}$$

TARGET MAGNITUDE

$$\underline{\lambda} = P_1\lambda_1 + P_2\lambda_2 + P_3\lambda_3 = \frac{\sum_{i=1}^{3}\lambda_i^2}{\sum_{k=1}^{3}\lambda_k}$$

TARGET VECTOR $\underline{k}_0$

$$\underline{k}_0 = \sqrt{\underline{\lambda}} \begin{bmatrix} \cos(\underline{\alpha}) \\ \sin(\underline{\alpha})\cos(\underline{\beta})e^{j\underline{\delta}} \\ \sin(\underline{\alpha})\sin(\underline{\beta})e^{j\underline{\gamma}} \end{bmatrix}$$

H / A / $\underline{\alpha}$ DECOMPOSITION

$2A_0$ $\quad B_0 + B$ $\quad B_0 - B$

$\sqrt{\underline{\lambda}}\,cos(\underline{\alpha})$ $\quad \sqrt{\underline{\lambda}}\,sin(\underline{\alpha})\,cos(\underline{\beta})$ $\quad \sqrt{\underline{\lambda}}\,sin(\underline{\alpha})\,sin(\underline{\beta})$

ROLL INVARIANCE PROPERTY

SAME PHYSICAL PHENOMENOUS WHATEVER THE ANTENNA *ORIENTATION ANGLE* AROUND THE *RADAR LINE OF SIGHT*

ORIENTED (θ) COHERENCY MATRIX

$$\langle [T(\theta)] \rangle = [U_R(\theta)] \langle [T] \rangle [U_R(\theta)]^{-1}$$

SU(3) UNITARY ROTATION MATRIX (θ)

$$[U_R(\theta)] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 2\theta & \sin 2\theta \\ 0 & -\sin 2\theta & \cos 2\theta \end{bmatrix}$$

EIGENVECTORS / EIGENVALUES ANALYSIS

$$\langle [T(\theta)] \rangle = [U_3(\theta)][\Sigma][U_3(\theta)]^{-1}$$

EIGENVALUES $\lambda_1 \;\; \lambda_2 \;\; \lambda_3$: ROLL INVARIANT

PROBABILITIES $P_1 \;\; P_2 \;\; P_3$: ROLL INVARIANT

EIGENVECTORS UNITARY MATRIX

$$[U_3(\theta)] = [U_R(\theta)][U_3]$$

PARAMETERIZATION OF THE UNITARY MATRIX

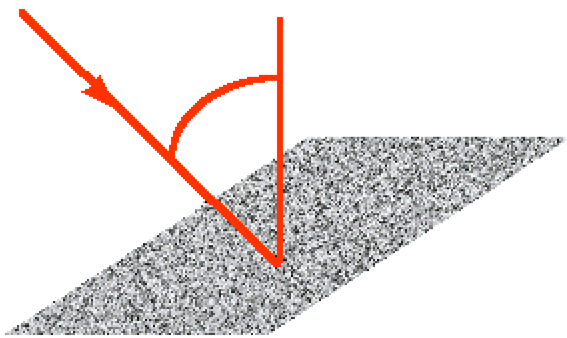
$$[U_3] = \begin{bmatrix} \cos\alpha_1 e^{j\phi'_1} & \cos\alpha_2 e^{j\phi'_2} & \cos\alpha_3 e^{j\phi'_3} \\ \sin\alpha_1 \cos\beta'_1 e^{j\phi'_1} e^{j\delta'_1} & \sin\alpha_2 \cos\beta'_2 e^{j\phi'_2} e^{j\delta'_2} & \sin\alpha_3 \cos\beta'_3 e^{j\phi'_3} e^{j\delta'_3} \\ \sin\alpha_1 \sin\beta'_1 e^{j\phi'_1} e^{j\gamma'_1} & \sin\alpha_2 \sin\beta'_2 e^{j\phi'_2} e^{j\gamma'_2} & \sin\alpha_3 \sin\beta'_3 e^{j\phi'_3} e^{j\gamma'_3} \end{bmatrix}$$

$\underline{\alpha} = P_1\alpha_1 + P_2\alpha_2 + P_3\alpha_3$: ROLL INVARIANT

PHYSICAL INTERPRETATION

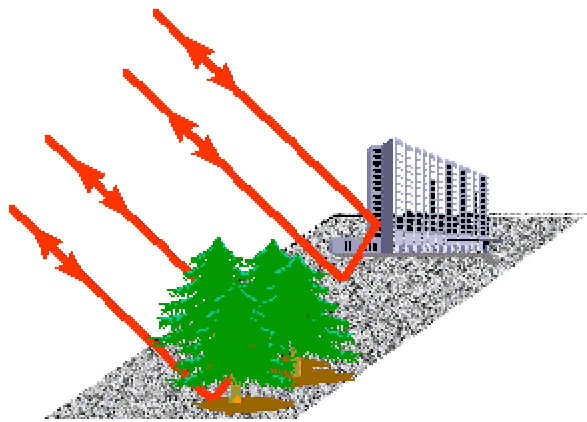
$\underline{\alpha}$ PHYSICAL INTERPRETATION

SINGLE BOUNCE SCATTERING (ROUGH SURFACE)



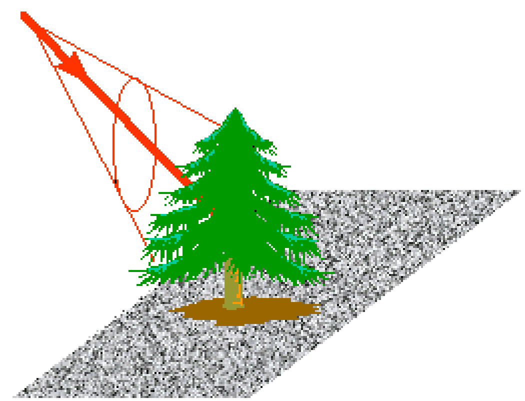
$$a \mapsto b \Rightarrow \nu \mapsto 0$$
$$\Downarrow$$
$$\underline{\alpha} \mapsto 0$$

DOUBLE BOUNCE SCATTERING



$$a \mapsto -b \Rightarrow \varepsilon \mapsto 0$$
$$\Downarrow$$
$$\underline{\alpha} \mapsto \frac{\pi}{2}$$

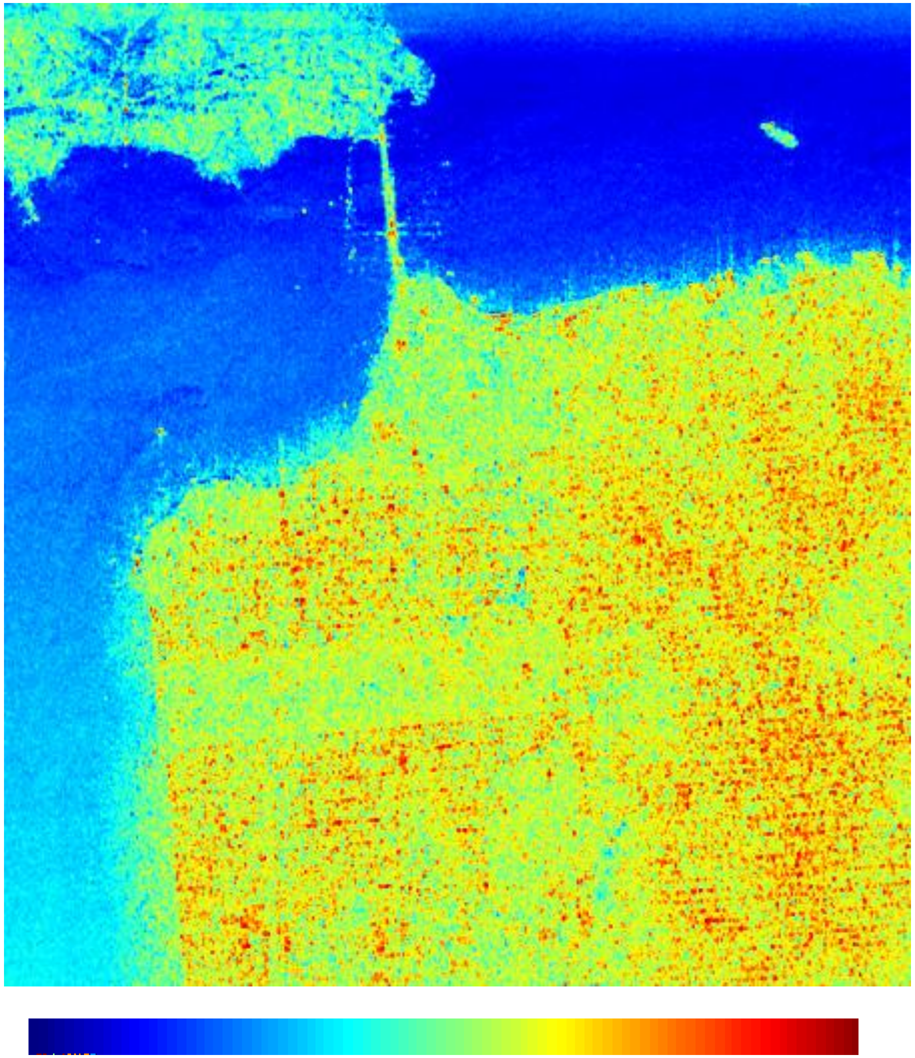
VOLUME SCATTERING



$$a >> b \Rightarrow \varepsilon \approx \nu$$
$$\Downarrow$$
$$\underline{\alpha} \mapsto \frac{\pi}{4}$$

H / A / α DECOMPOSITION

$2A_0$ $B_0 + B$ $B_0 - B$



α PARAMETER

H / A / α DECOMPOSITION

EIGENVALUES $\lambda_1\ \lambda_2\ \lambda_3$: ROLL INVARIANT

PROBABILITIES $P_1\ P_2\ P_3$: ROLL INVARIANT

↓

ENTROPY

(DEGREE OF RANDOMNESS
STATISTICAL DISORDER)

$$H = -\sum_{i=1}^{3} P_i \log_3(P_i)$$

PURE TARGET

$\lambda_1 = SPAN \quad \lambda_2 = 0 \quad \lambda_3 = 0$

$H = 0$

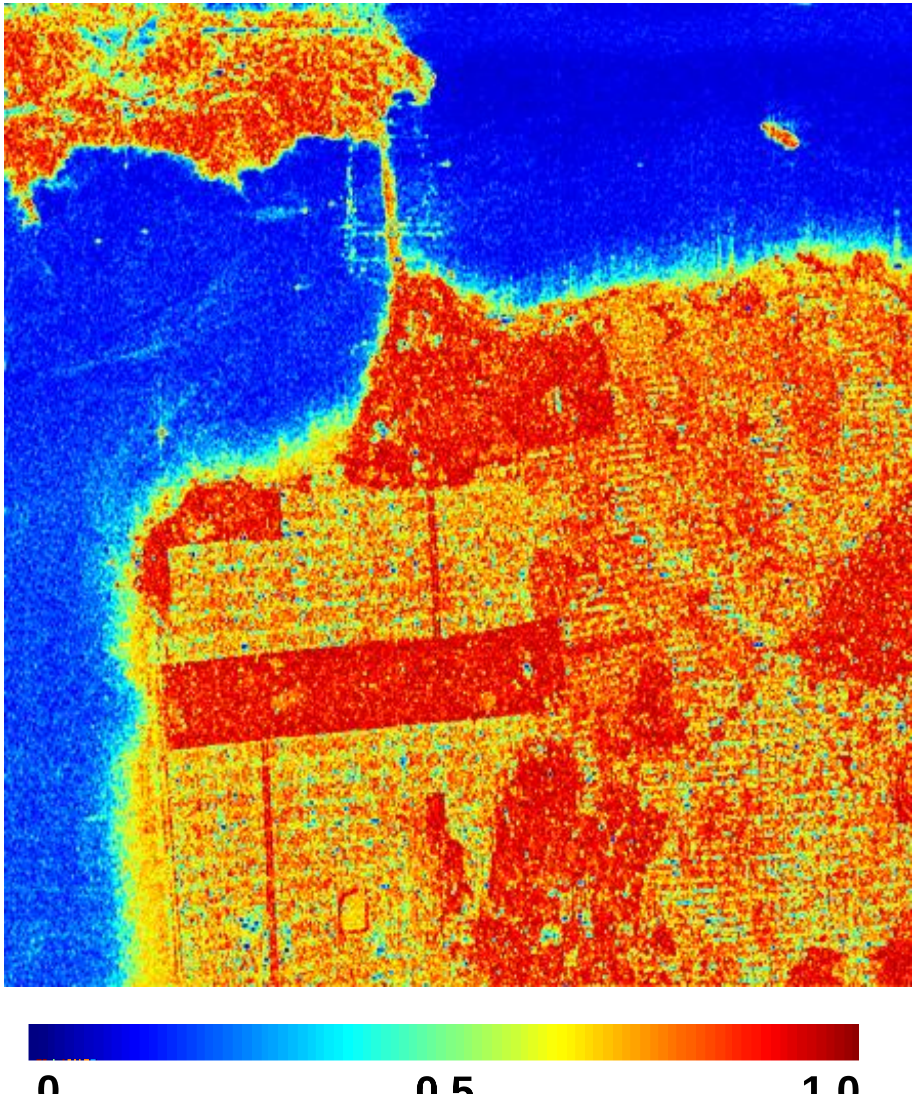
DISTRIBUTED TARGET

$\lambda_1 = \lambda_2 = \lambda_3 = SPAN / 3$

$H = 1$

H / A / α DECOMPOSITION

$2A_0$ $B_0 + B$ $B_0 - B$



H / A / α DECOMPOSITION

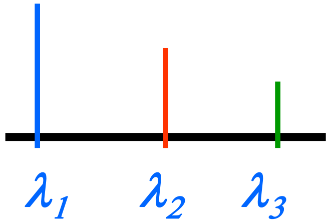
esa

DIFFICULT MECHANISM DISCRIMINATION WHEN : H > 0.7

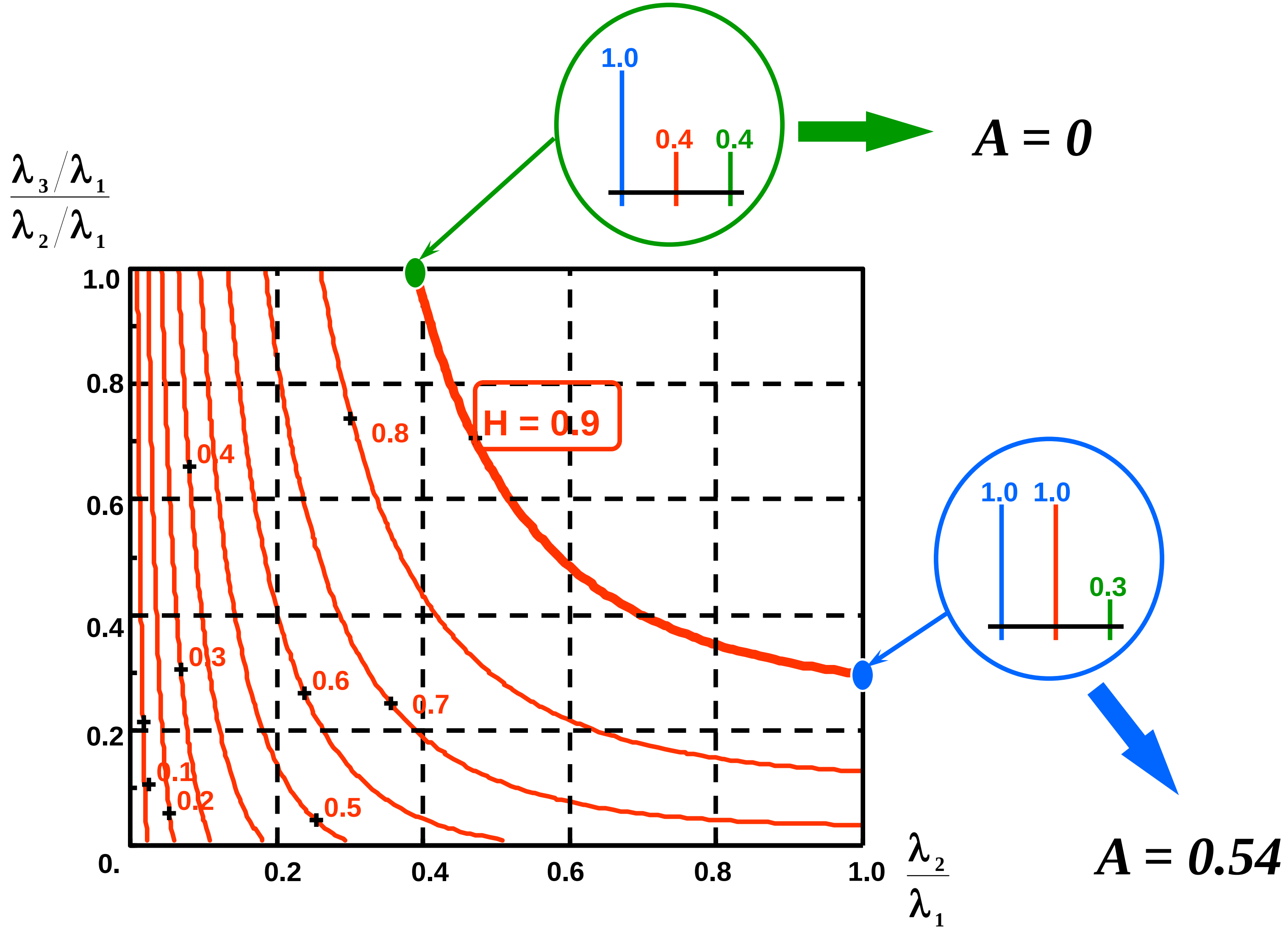
ANISOTROPY

(EIGENVALUES SPECTRUM)

$$A = \frac{\lambda_2 - \lambda_3}{\lambda_2 + \lambda_3}$$



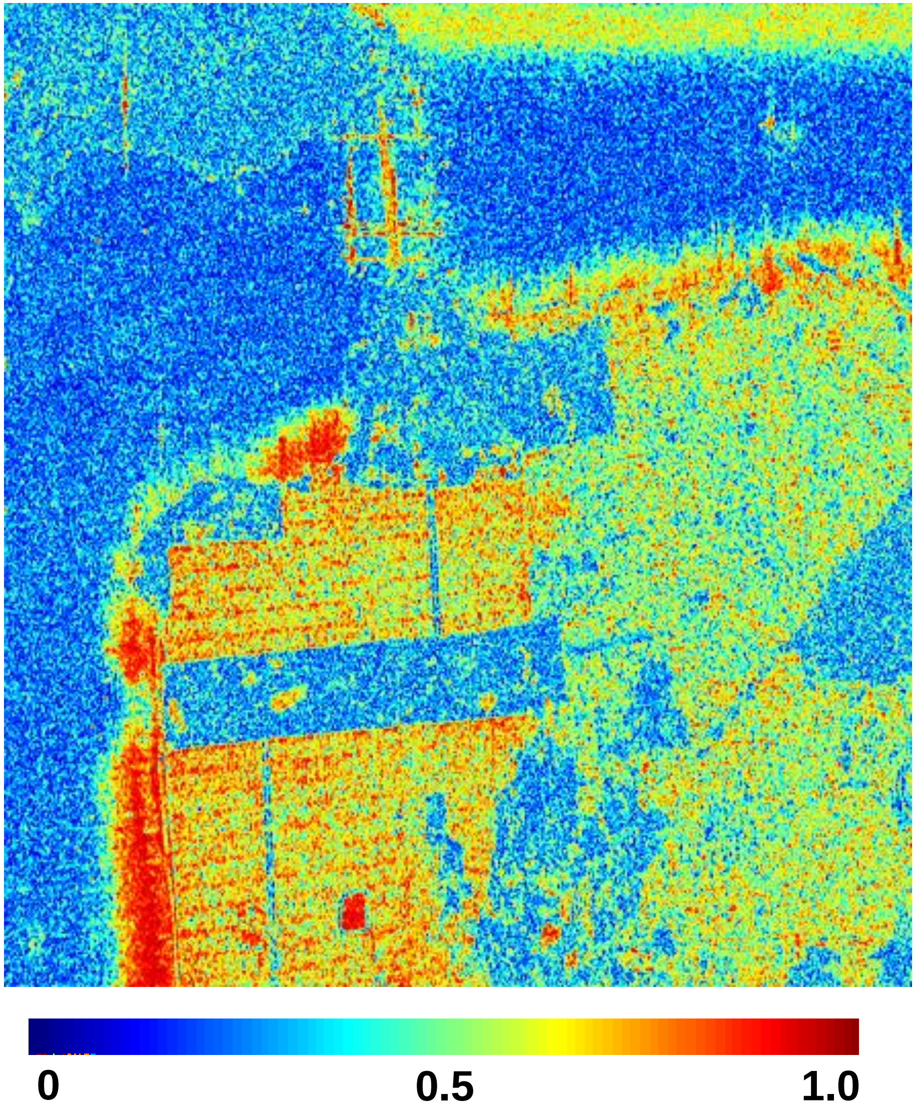
- COMPLEMENTARY TO ENTROPY
- DISCRIMINATION WHEN H > 0.7
- ROLL INVARIANT



1.0
0.4
0.4
A = 0
λ3/λ1
λ2/λ1
1.0
0.8
0.6
0.4
0.2
0.
0.2
0.4
0.6
0.8
1.0
λ2
λ1
H = 0.9
0.8
0.4
0.3
0.6
0.7
0.1
0.2
0.5
1.0 1.0
0.3
A = 0.54

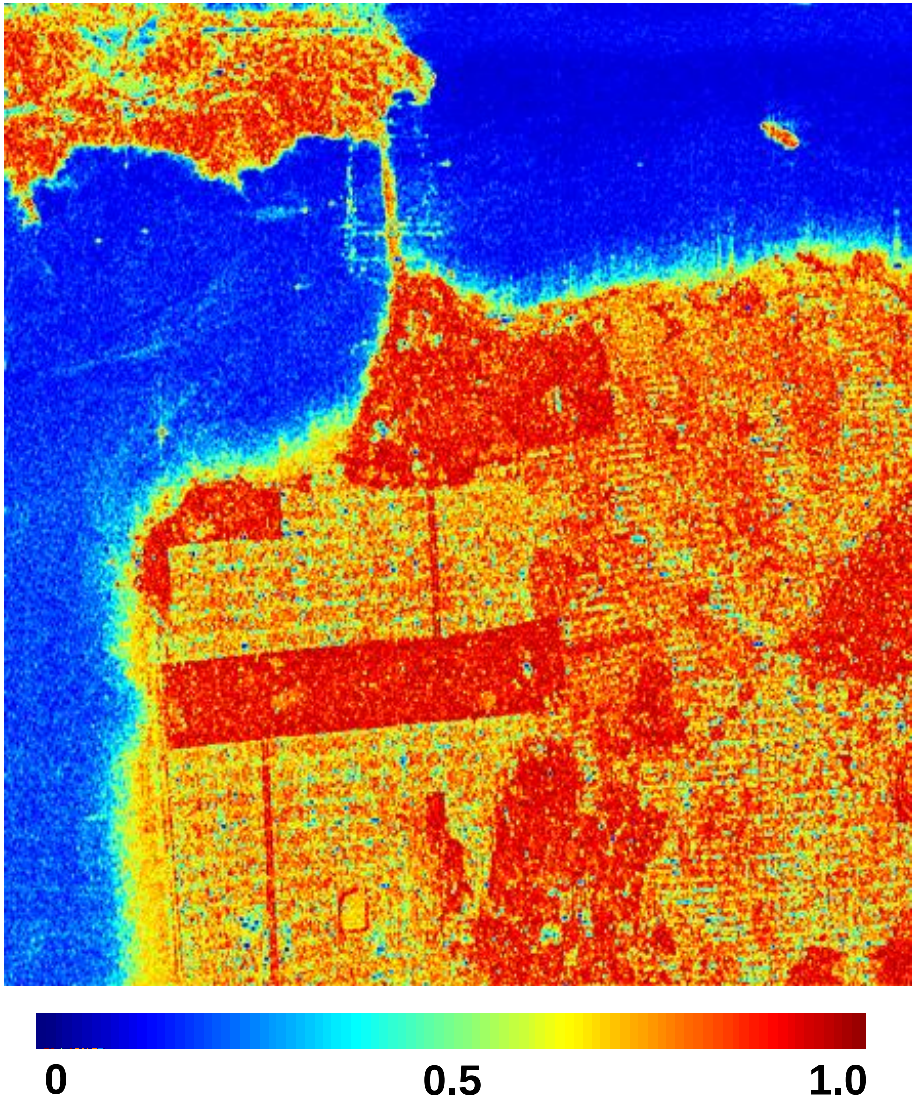
H / A / $\underline{\alpha}$ DECOMPOSITION

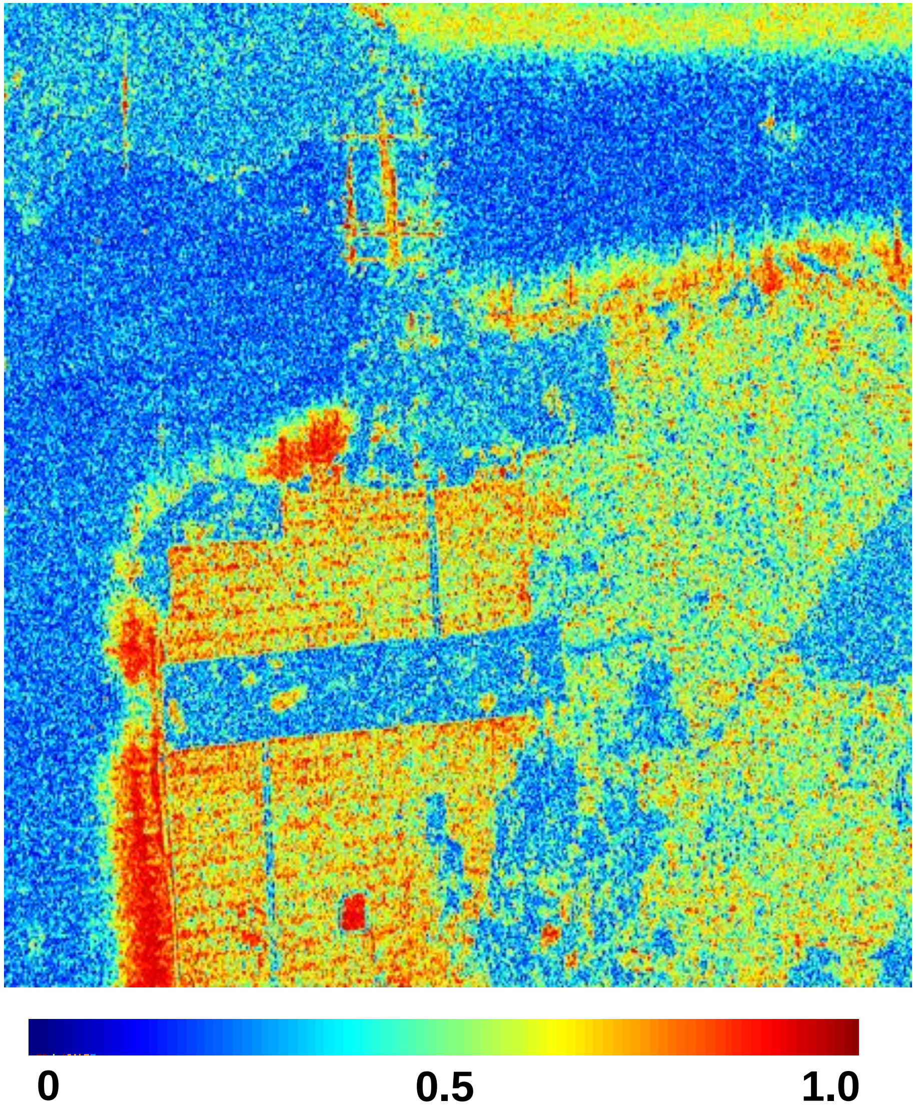
$2A_0$ $B_0 + B$ $B_0 - B$



ANISOTROPY (A)

H / A / α DECOMPOSITION



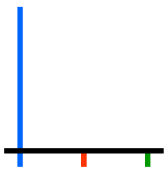


ENTROPY (H)

ANISOTROPY (A)

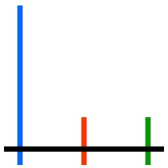
H / A / α DECOMPOSITION

(1-H)(1-A)

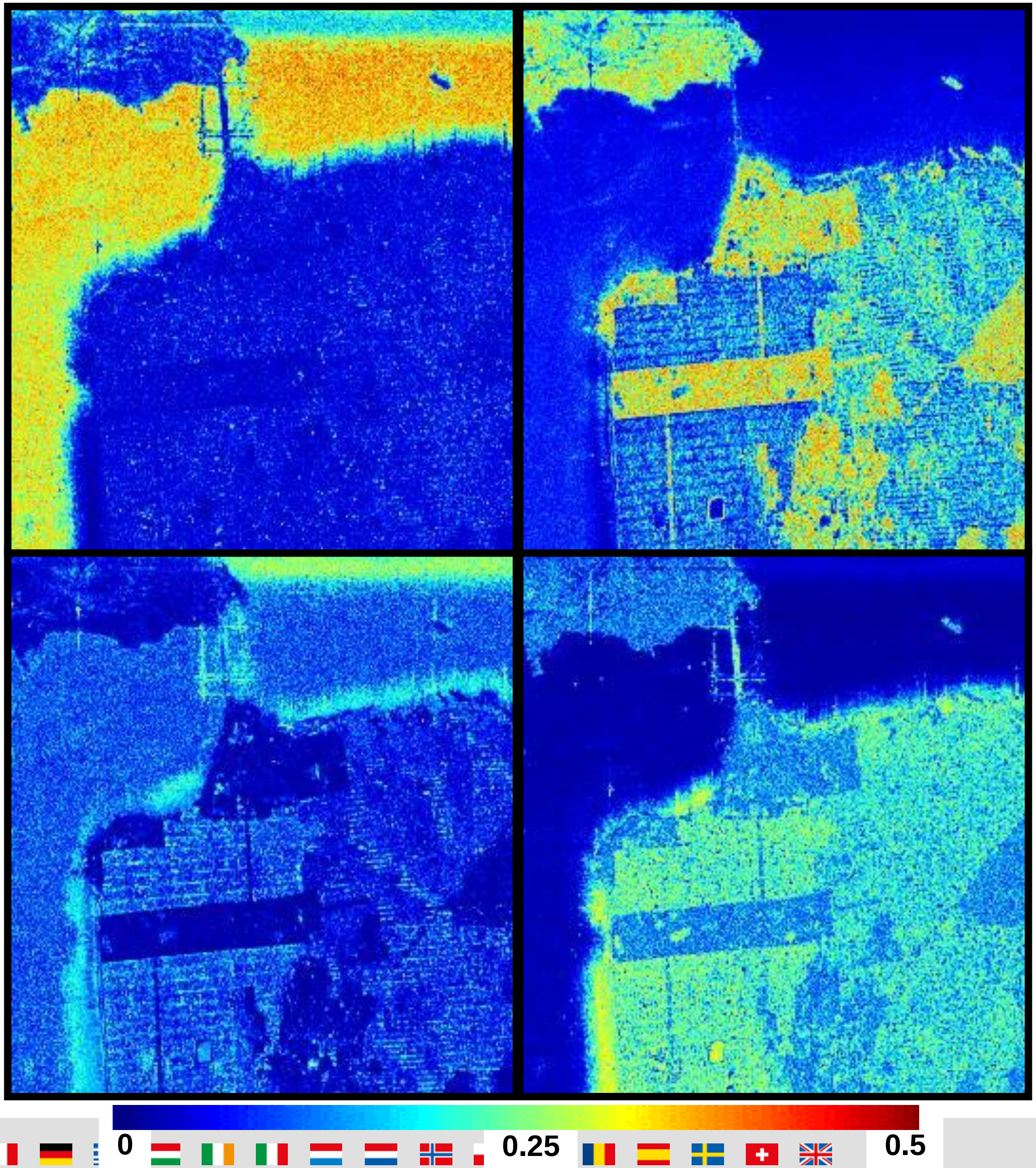


1 MECHANISM

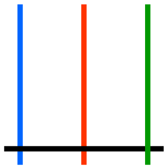
A(1-H)



2 MECHANISMS

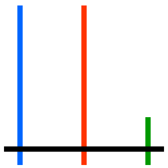


H(1-A)



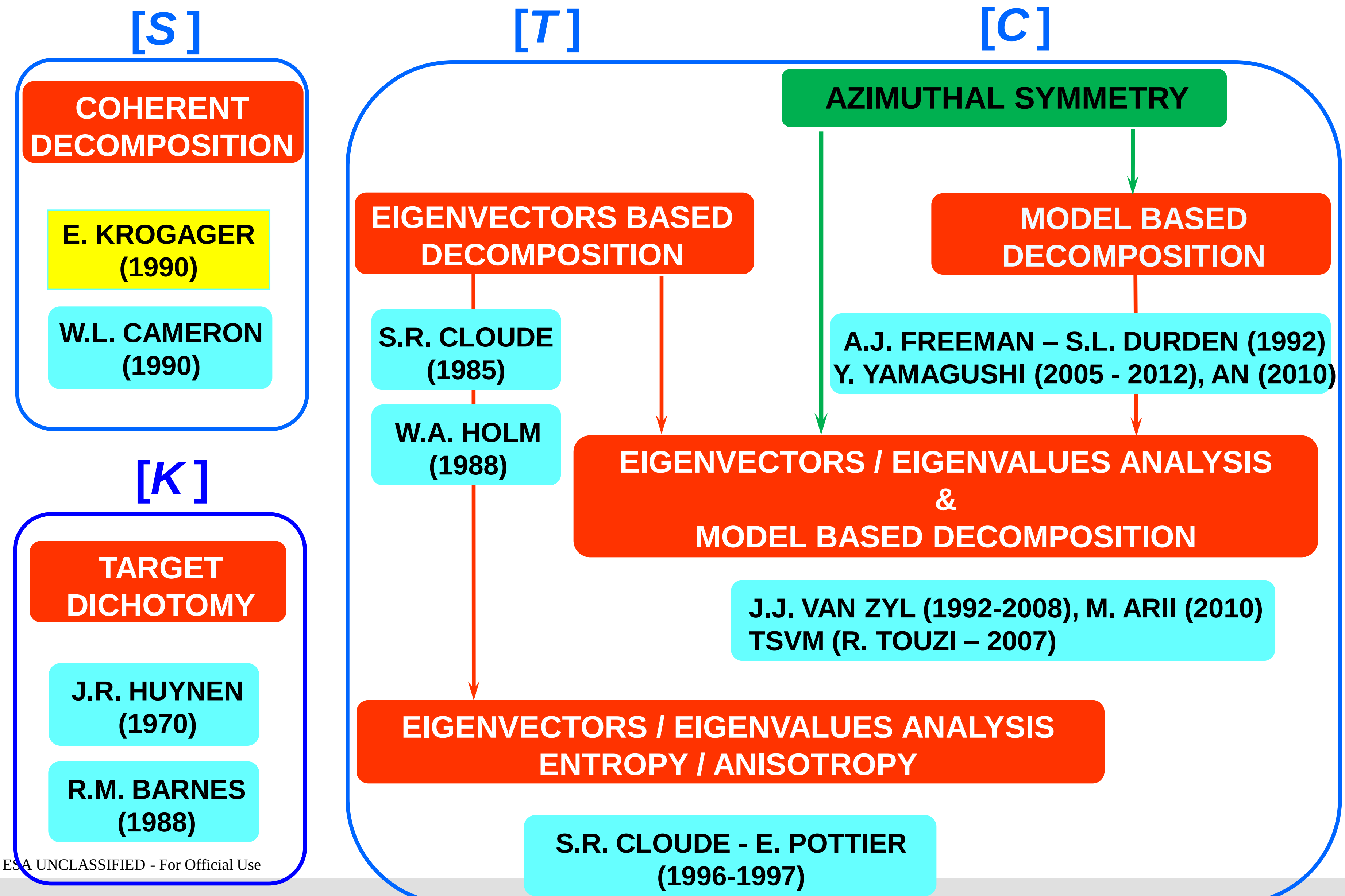
3 MECHANISMS

HA



2 MECHANISMS

TARGET DECOMPOSITIONS



COHERENT

TARGET DECOMPOSITION

(1990)

ERNST KROGAGER

(1990)

DECOMPOSITION

$[S]$ → THREE COHERENT COMPONENTS

$$[S]=\begin{bmatrix} a+b & c \\ c & a-b \end{bmatrix}=e^{j\phi}\left\{k_S[S_S]+e^{j\phi_R}\left(k_D[S_D]+k_H[S_H]\right)\right\}$$

SINGLE BOUNCE SCATTERING

DOUBLE BOUNCE SCATTERING

HELICAL SCATTERING

COHERENT DECOMPOSITION

$$[S] = e^{j\phi}\left\{k_S\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} + e^{j\phi_R}\left(k_D\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix} + \frac{k_H}{2}\begin{bmatrix}1 & \pm j\\ \pm j & -1\end{bmatrix}\right)\right\}$$

ROTATION AROUND THE RADAR LINE OF SIGHT

$$[U] = \begin{bmatrix}\cos(\theta) & \sin(\theta)\\ -\sin(\theta) & \cos(\theta)\end{bmatrix}$$

$$\begin{aligned}[S(\theta)] &= [U]^T[S][U]\\ &= e^{j\phi}\left\{k_S\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} + e^{j\phi_R}\left(k_D\begin{bmatrix}\cos(2\theta) & \sin(2\theta)\\ \sin(2\theta) & -\cos(2\theta)\end{bmatrix}\cdots\right.\right.\\ &\left.\left.\cdots + \frac{k_H e^{\mp j2\theta}}{2}\begin{bmatrix}1 & \pm j\\ \pm j & -1\end{bmatrix}\right)\right\}\end{aligned}$$

COHERENT DECOMPOSITION

$$[S] = e^{j\phi}\begin{bmatrix} k_S + e^{j\phi_R}\left\{k_D\cos(2\theta) + \frac{\hat{k}_H}{2}\right\} & e^{j\phi_R}\left\{k_D\sin(2\theta) \pm j\frac{\hat{k}_H}{2}\right\} \\ e^{j\phi_R}\left\{k_D\sin(2\theta) \pm j\frac{\hat{k}_H}{2}\right\} & k_S - e^{j\phi_R}\left\{k_D\cos(2\theta) + \frac{\hat{k}_H}{2}\right\} \end{bmatrix}$$

AVEC : $\hat{k}_H = k_H e^{\mp j2\theta}$

$$\underline{k} = \sqrt{2}e^{j\phi}\begin{bmatrix} k_S & e^{j\phi_R}\left\{k_D\cos(2\theta) + \frac{\hat{k}_H}{2}\right\} & e^{j\phi_R}\left\{k_D\sin(2\theta) \pm j\frac{\hat{k}_H}{2}\right\} \end{bmatrix}^T$$

$$\underline{k} = \sqrt{2}k_S e^{j\phi}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \hat{k}_H e^{j(\phi+\phi_R)}\frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \\ \pm j \end{bmatrix} + \sqrt{2}k_D e^{j(\phi+\phi_R)}\begin{bmatrix} 0 \\ \cos(2\theta) \\ \sin(2\theta) \end{bmatrix}$$

esa

SINGLE SCATTERING CONTRIBUTION

$$k_S = \sqrt{A_0} \qquad [S_S] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

DOUBLE SCATTERING CONTRIBUTION

$$k_D = \sqrt{B_0 - |F|} \qquad [S_D] = \begin{bmatrix} cos(2\theta) & sin(2\theta) \\ sin(2\theta) & -cos(2\theta) \end{bmatrix}$$

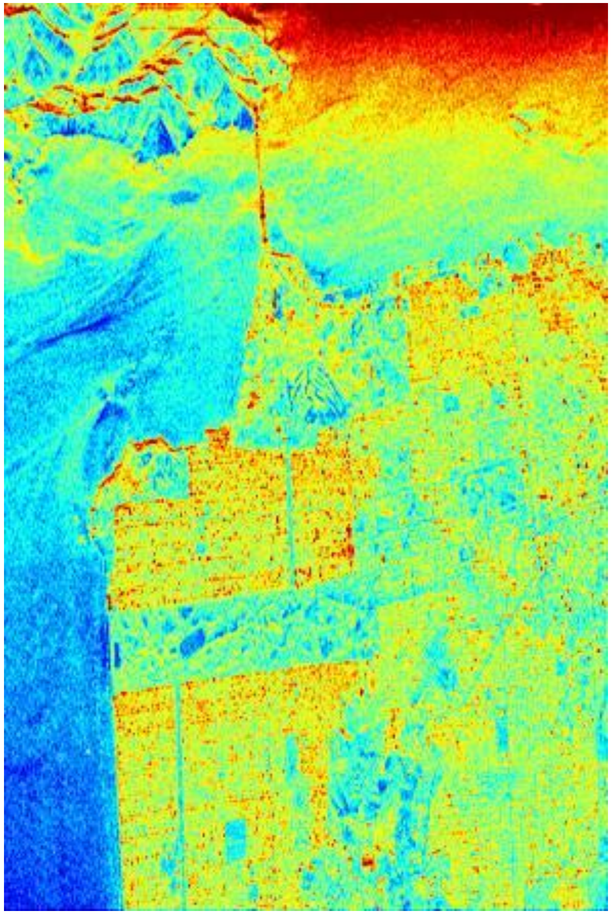
$$tan(4\theta) = \frac{E}{B}$$

DIPLANE ORIENTATION ANGLE
INSIDE THE TARGET

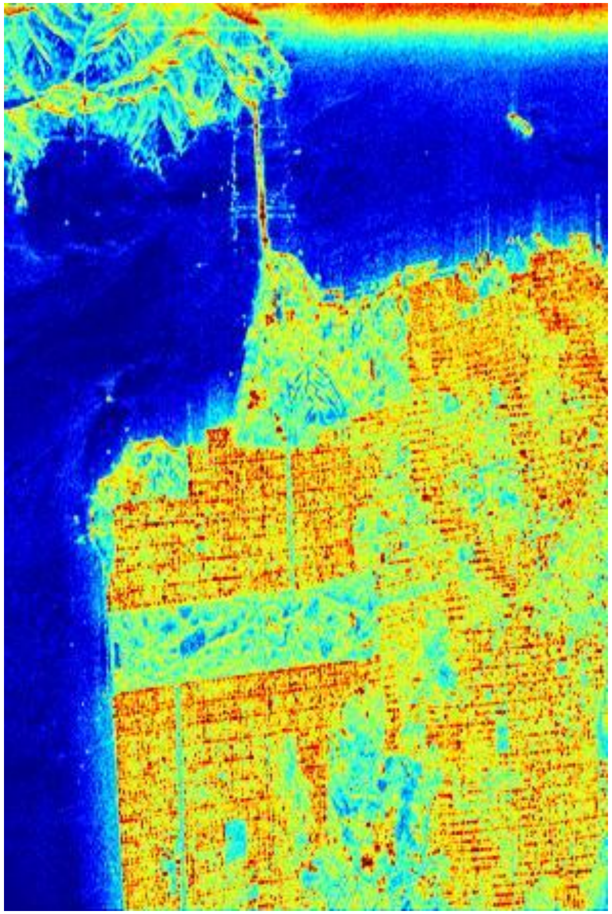
HELICAL SCATTERING CONTRIBUTION

$$k_H = \sqrt{B_0 + |F|} - \sqrt{B_0 - |F|} \qquad [S_H] = \frac{1}{2}\begin{bmatrix} 1 & \pm j \\ \pm j & -1 \end{bmatrix}$$

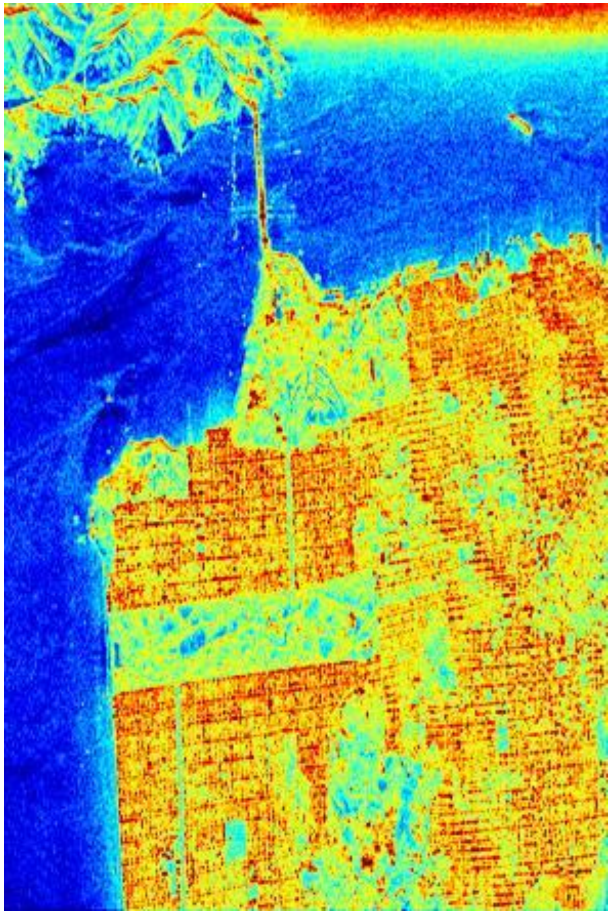
COHERENT DECOMPOSITION



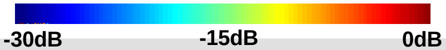
$(k_S)_{dB}$



$(k_D)_{dB}$

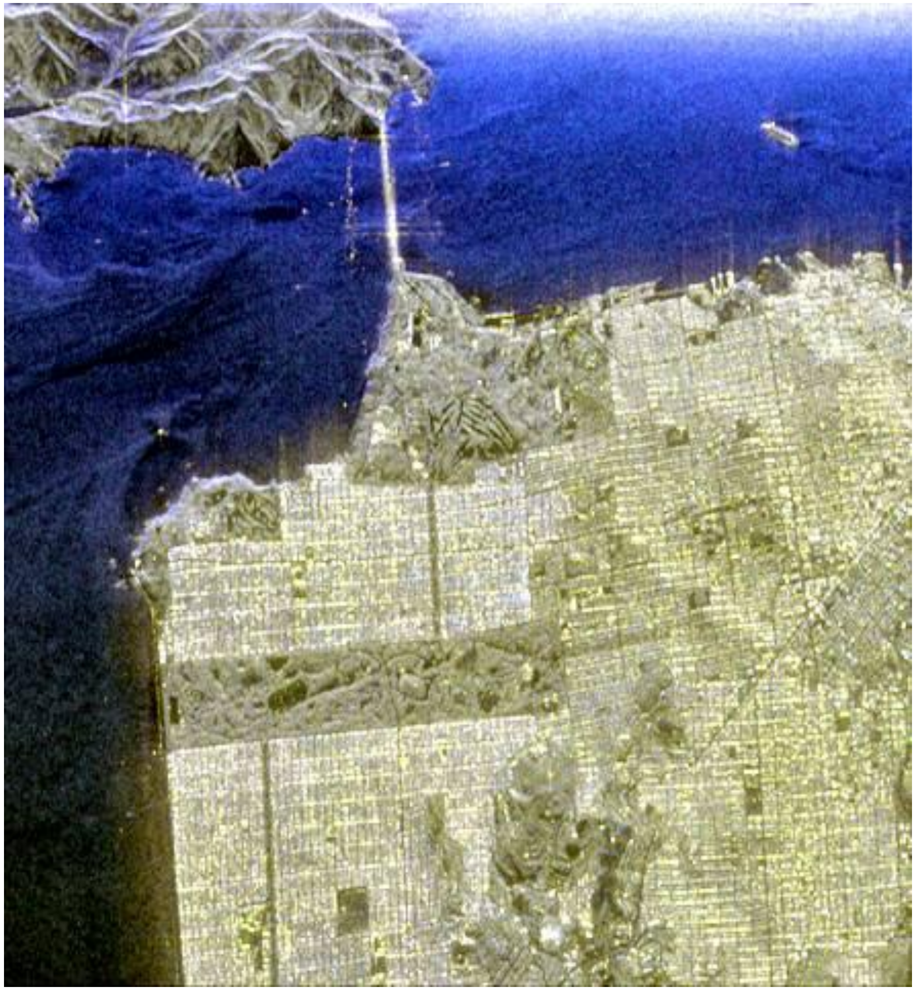


$(k_H)_{dB}$



COHERENT DECOMPOSITION

$2A_0$ $B_0 + B$ $B_0 - B$



k_S k_D k_H

COHERENT DECOMPOSITION

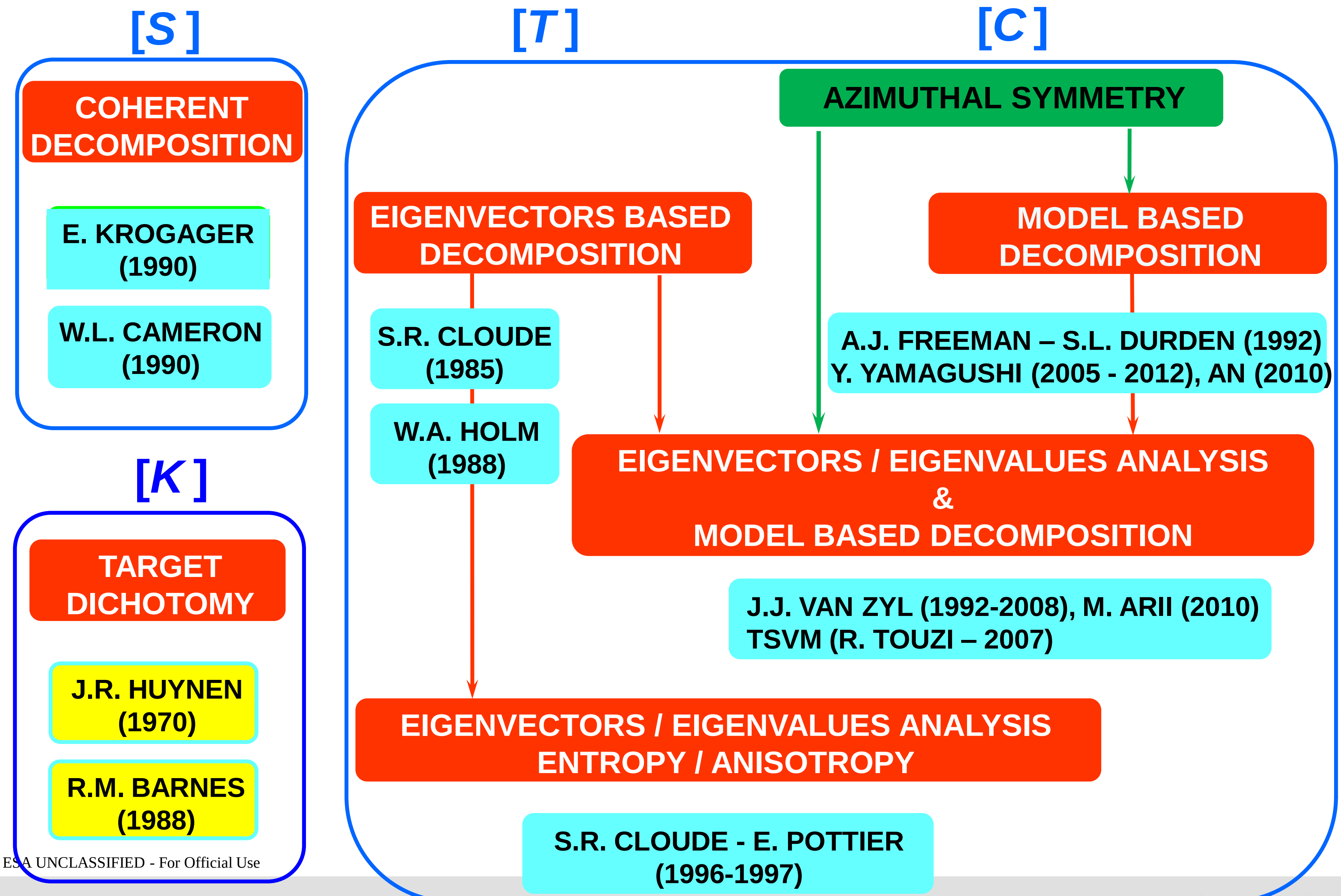
$$\underline{k} = \sqrt{2}k_S e^{j\phi}\begin{bmatrix}1\\0\\0\end{bmatrix} + \hat{k}_H e^{j(\phi+\phi_R)}\frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\\\varepsilon j\end{bmatrix} + \sqrt{2}k_D e^{j(\phi+\phi_R)}\begin{bmatrix}0\\\cos(2\theta)\\\sin(2\theta)\end{bmatrix}$$

EIGENVECTORS OF $[U_{3R}(\phi)]$ (ROLL INVARIANCE)

→ NO ORTHOGONALITY OF THE TARGETS COMPONENTS

→ COHERENT DECOMPOSITION and SPECKLE FILTERING ?

TARGET DECOMPOSITIONS



J.R. HUYNEN

DECOMPOSITION

(1970)

esa

DISTRIBUTED TARGET

$$\langle [T] \rangle = \frac{1}{N} \sum_{i=1}^{i=N} [T_i]$$

DISTRIBUTED TARGET EQUATIONS

$$2\langle A_0 \rangle \left(\langle B_0 \rangle + \langle B \rangle \right) \geq \langle C \rangle^2 + \langle D \rangle^2$$

$$2\langle A_0 \rangle \left(\langle B_0 \rangle - \langle B \rangle \right) \geq \langle G \rangle^2 + \langle H \rangle^2$$

$$\langle B_0 \rangle^2 \geq \langle B \rangle^2 + \langle E \rangle^2 + \langle F \rangle^2$$

$$2\langle A_0 \rangle \langle E \rangle \geq \langle C \rangle \langle H \rangle - \langle D \rangle \langle G \rangle$$

$$2\langle A_0 \rangle \langle F \rangle \geq \langle C \rangle \langle G \rangle + \langle D \rangle \langle H \rangle$$

DECOMPOSITION - TARGET DICHOTOMY

$$\langle B_0 \rangle^2 \geq \langle B \rangle^2 + \langle E \rangle^2 + \langle F \rangle^2$$

$$\begin{bmatrix} \langle B_0 \rangle \\ \langle B \rangle \\ \langle E \rangle \\ \langle F \rangle \end{bmatrix} = \begin{bmatrix} B_{0T} \\ B_T \\ E_T \\ F_T \end{bmatrix} + \begin{bmatrix} B_{0N} \\ B_N \\ E_N \\ F_N \end{bmatrix}$$

J.R. HUYNEN (1970)

PURE TARGET

NON SYMMETRIC TARGET (N-TARGET)

$$B_{0T}^{\ 2} = B_T^{\ 2} + E_T^{\ 2} + F_T^{\ 2}$$

J.R. HUYNEN DECOMPOSITION

$$\langle [T] \rangle = \begin{bmatrix} \langle 2A_0 \rangle & \langle C \rangle - j\langle D \rangle & \langle H \rangle + j\langle G \rangle \\ \langle C \rangle + j\langle D \rangle & \langle B_0 \rangle + \langle B \rangle & \langle E \rangle + j\langle F \rangle \\ \langle H \rangle - j\langle G \rangle & \langle E \rangle - j\langle F \rangle & \langle B_0 \rangle - \langle B \rangle \end{bmatrix} = [T_0] + [T_N]$$

↓

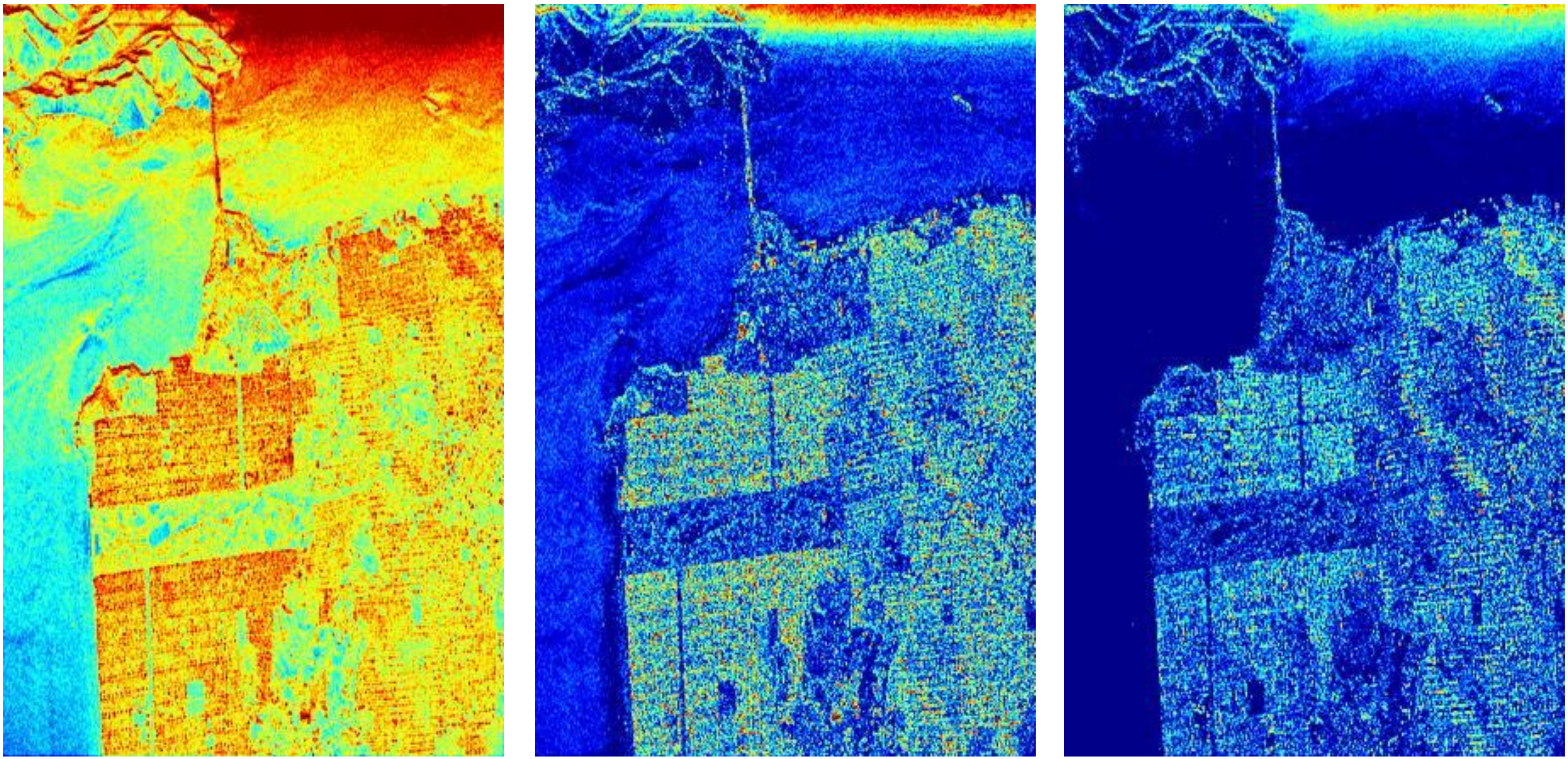
→ PURE TARGET

$$[T_0] = \begin{bmatrix} \langle 2A_0 \rangle & \langle C \rangle - j\langle D \rangle & \langle H \rangle + j\langle G \rangle \\ \langle C \rangle - j\langle D \rangle & B_{0T} + B_T & E_T + jF_T \\ \langle H \rangle - j\langle G \rangle & E_T - jF_T & B_{0T} - B_T \end{bmatrix}$$

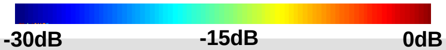
→ N-TARGET

$$[T_N] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & B_{0N} + B_N & E_N + jF_N \\ 0 & E_N - jF_N & B_{0N} - B_N \end{bmatrix}$$

J.R. HUYNEN DECOMPOSITION



$\langle 2A_0 \rangle$ | $\left(\langle C\rangle^2 + \langle D\rangle^2\right) / 2A_0$ | $\left(\langle H\rangle^2 + \langle G\rangle^2\right) / 2A_0$



J.R. HUYNEN DECOMPOSITION

$2A_0$ $B_0 + B$ $B_0 - B$

$\langle 2A_0 \rangle$ $\left(\langle C \rangle^2 + \langle D \rangle^2\right) / 2A_0$

$\left(\langle H \rangle^2 + \langle G \rangle^2\right) / 2A_0$

BARNES DECOMPOSITION

3 SINGLE TARGET VECTORS

$$\underline{k}_{01} = \frac{\langle[T]\rangle \underline{q}_1}{\sqrt{\underline{q}_1^{T*}\langle[T]\rangle \underline{q}_1}} = \frac{1}{\sqrt{\langle 2A_0\rangle}} \begin{bmatrix} \langle 2A_0\rangle \\ \langle C\rangle + j\langle D\rangle \\ \langle H\rangle - j\langle G\rangle \end{bmatrix}$$

→ HUYNEN DECOMPOSITION

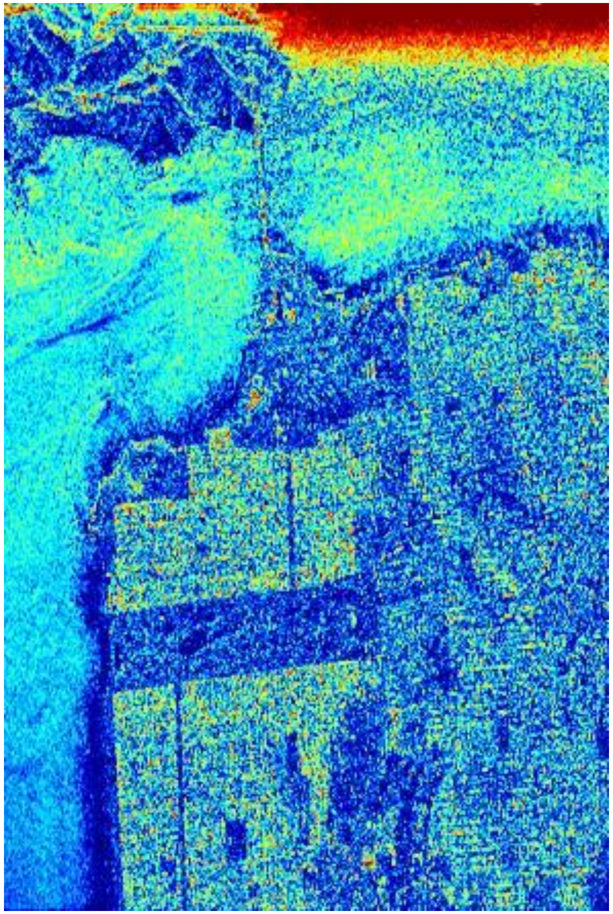
(SYMMETRIC WORLD)

$$\underline{k}_{02} = \frac{\langle[T]\rangle \underline{q}_2}{\sqrt{\underline{q}_2^{T*}\langle[T]\rangle \underline{q}_2}} = \frac{1}{\sqrt{2(\langle B_0\rangle - \langle F\rangle)}} \begin{bmatrix} \langle C\rangle - \langle G\rangle + j\langle H\rangle - j\langle D\rangle \\ \langle B_0\rangle + \langle B\rangle - \langle F\rangle + j\langle E\rangle \\ \langle E\rangle + j\langle B_0\rangle - j\langle B\rangle - j\langle F\rangle \end{bmatrix}$$

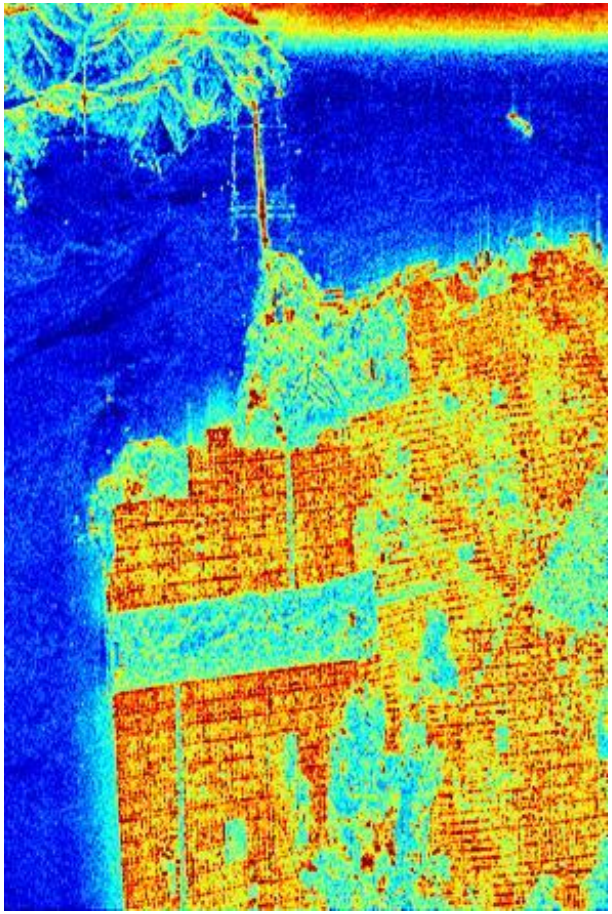
$$\underline{k}_{03} = \frac{\langle[T]\rangle \underline{q}_3}{\sqrt{\underline{q}_3^{T*}\langle[T]\rangle \underline{q}_3}} = \frac{1}{\sqrt{2(\langle B_0\rangle + \langle F\rangle)}} \begin{bmatrix} \langle H\rangle + \langle D\rangle + j\langle C\rangle + j\langle G\rangle \\ \langle E\rangle + j\langle B_0\rangle + j\langle B\rangle + j\langle F\rangle \\ \langle B_0\rangle - \langle B\rangle + \langle F\rangle + j\langle E\rangle \end{bmatrix}$$

(HELICITY - NON SYMMETRIC COMPONENTS = EXOTIC WORLD)

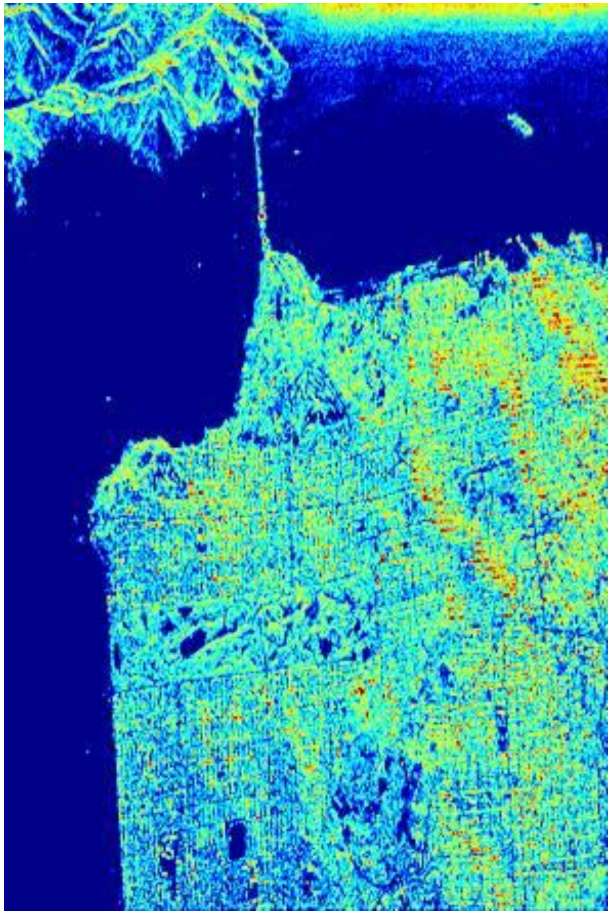
BARNES DECOMPOSITION



$$\frac{\sqrt{(\langle C\rangle-\langle G\rangle)^2+(\langle H\rangle-\langle D\rangle)^2}}{\sqrt{2(\langle B_0\rangle-\langle F\rangle)}}$$



$$\frac{\sqrt{(\langle B_0\rangle+\langle B\rangle-\langle F\rangle)^2+\langle E\rangle^2}}{\sqrt{2(\langle B_0\rangle-\langle F\rangle)}}$$



$$\frac{\sqrt{(\langle B_0\rangle-\langle B\rangle-\langle F\rangle)^2+\langle E\rangle^2}}{\sqrt{2(\langle B_0\rangle-\langle F\rangle)}}$$

-30dB -15dB 0dB

BARNES DECOMPOSITION

$2A_0$ $B_0 + B$ $B_0 - B$

$$\frac{\sqrt{\left(\langle C\rangle-\langle G\rangle\right)^2+\left(\langle H\rangle-\langle D\rangle\right)^2}}{\sqrt{2\left(\langle B_0\rangle-\langle F\rangle\right)}}$$

$$\frac{\sqrt{\left(\langle B_0\rangle+\langle B\rangle-\langle F\rangle\right)^2+\langle E\rangle^2}}{\sqrt{2\left(\langle B_0\rangle-\langle F\rangle\right)}}$$

$$\frac{\sqrt{\left(\langle B_0\rangle-\langle B\rangle-\langle F\rangle\right)^2+\langle E\rangle^2}}{\sqrt{2\left(\langle B_0\rangle-\langle F\rangle\right)}}$$

HUYNEN DECOMPOSITION

TARGET DICHOTOMY : PURE TARGET + N TARGET

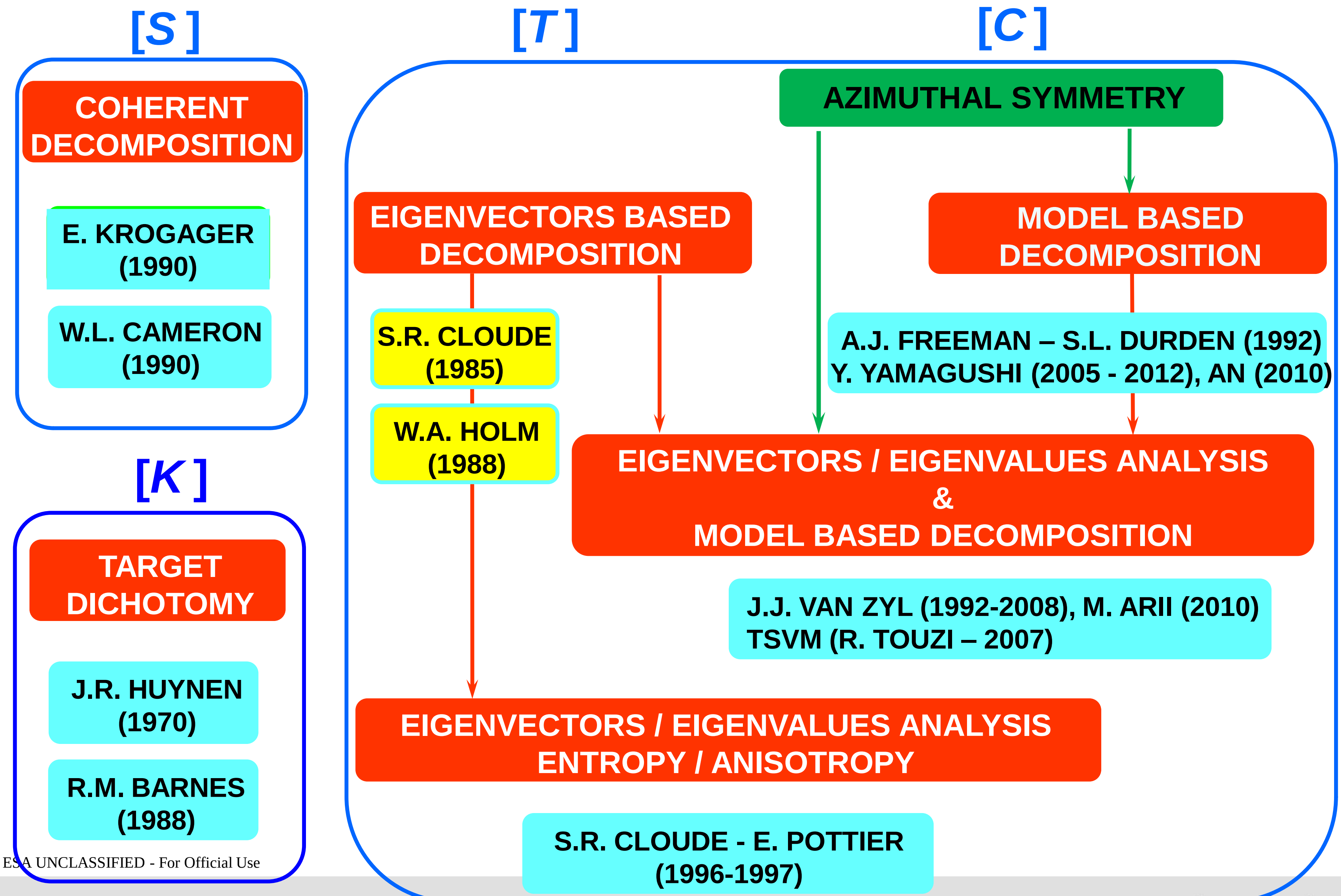
ROLL INVARIANCE OF THE FORM OF THE N-TARGET

NO UNICITY : 3 DIFFERENT DECOMPOSITIONS

↓

MAN-MADE TARGET DECOMPOSITION
IDENTIFICATION - ANALYSIS

TARGET DECOMPOSITIONS



EIGENVECTOR BASED

DECOMPOSITION

→ **SHANE R. CLOUDE**

(1985-1992)

→ **WILLIAM A. HOLM**

(1988)

EIGENVECTOR-BASED DECOMPOSITION

PROPRIETY

EIGENVALUE PROBLEM IS AUTOMATICALLY

BASIS INVARIANT

→ GENERATE A DIAGONAL FORM OF THE COHERENCY MATRIX

→ PHYSICALLY INTERPRETATION AS STATISTICAL INDEPENDENCE BETWEEN A SET OF VECTORS

→ GENERAL DECOMPOSITION INTO INDEPENDENT SCATTERING PROCESSES

EIGENVECTOR-BASED DECOMPOSITION

COHERENCY MATRIX

$$\langle[T]\rangle = \begin{bmatrix} \langle 2A_0\rangle & \langle C\rangle - j\langle D\rangle & \langle H\rangle + j\langle G\rangle \\ \langle C\rangle + j\langle D\rangle & \langle B_0\rangle + \langle B\rangle & \langle E\rangle + j\langle F\rangle \\ \langle H\rangle - j\langle G\rangle & \langle E\rangle - j\langle F\rangle & \langle B_0\rangle - \langle B\rangle \end{bmatrix}$$

$$\langle[T]\rangle = [U_3][\Sigma][U_3]^{-1}$$

$$[\Sigma] = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}_{\lambda_1 \geq \lambda_2 \geq \lambda_3}$$

3x3 DIAGONAL MATRIX OF EIGENVALUES

$$[U_3] = [\underline{u}_1, \underline{u}_2, \underline{u}_3]$$

SU(3) UNITARY MATRIX (EIGENVECTORS)

SHANE R. CLOUDE

(1985-1992)

DECOMPOSITION

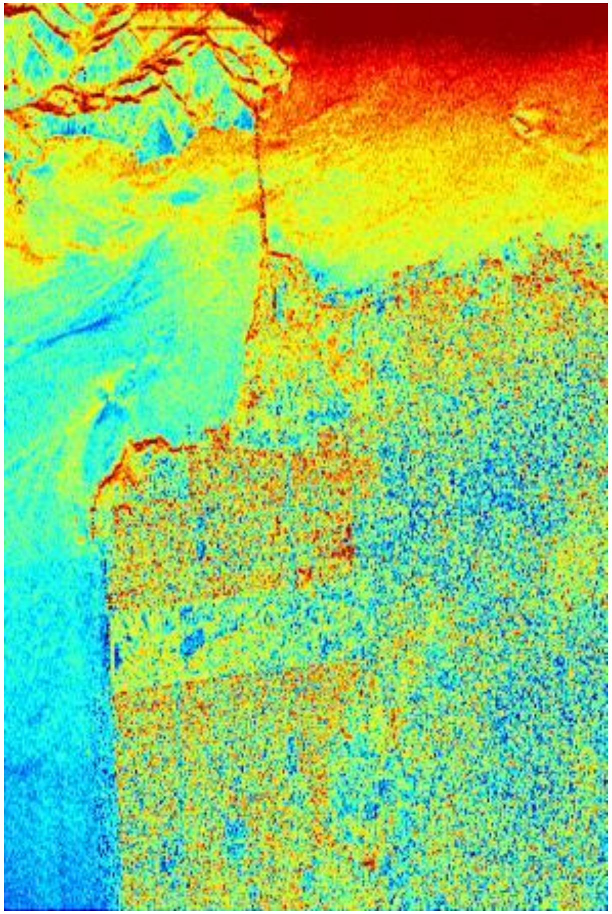
IDENTIFICATION OF THE DOMINANT SCATTERING MECHANISM

VIA THE

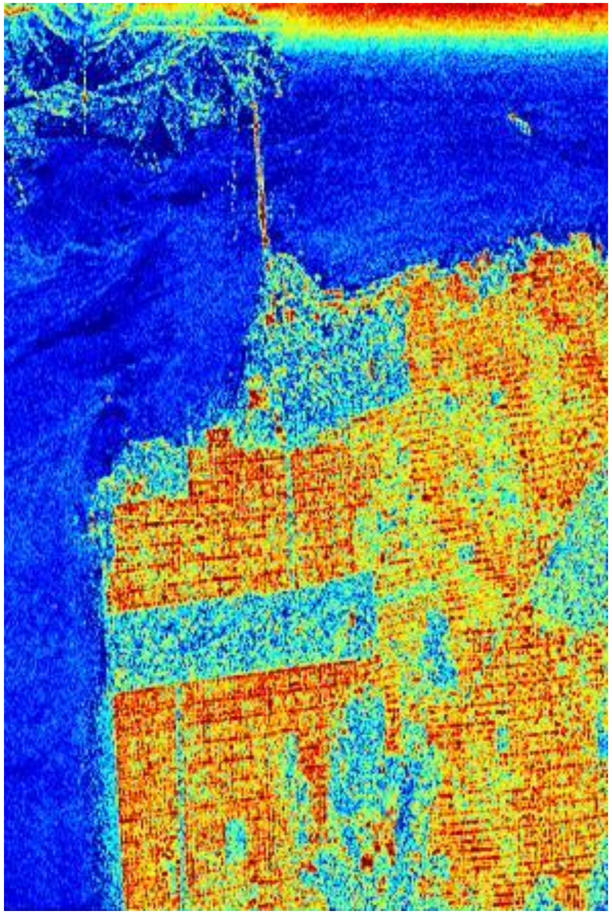
EXTRACTION OF THE LARGEST EIGENVALUE

$$\langle [T] \rangle = [U_3][\Sigma][U_3]^{-1} \quad \Rightarrow \quad [T_1] = \lambda_1 \underline{u}_1 \underline{u}_1^{T*} = \underline{k}_1 \underline{k}_1^{T*}$$

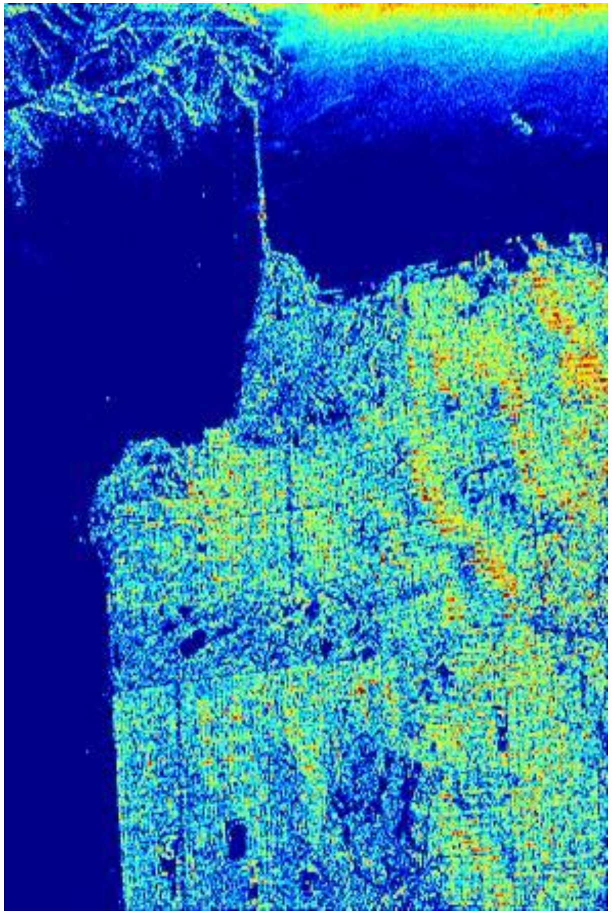
S.R. CLOUDE DECOMPOSITION



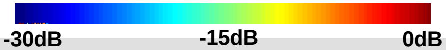
$$\sqrt{\lambda_1}\,|u_{11}|$$



$$\sqrt{\lambda_1}\,|u_{12}|$$



$$\sqrt{\lambda_1}\,|u_{13}|$$



S.R. CLOUDE DECOMPOSITION

$2A_0$ $\quad B_0 + B$ $\quad B_0 - B$

$\sqrt{\lambda_1}\,|u_{11}|$ $\quad \sqrt{\lambda_1}\,|u_{12}|$ $\quad \sqrt{\lambda_1}\,|u_{13}|$

WILLIAM A. HOLM

(1988)

DECOMPOSITION

ALTERNATIVE PHYSICAL INTERPRETATION

OF THE EIGENVALUES SPECTRUM

$$[\Sigma]=\begin{bmatrix}\lambda_1 & 0 & 0\\ 0 & \lambda_2 & 0\\ 0 & 0 & \lambda_3\end{bmatrix}$$

$$\Downarrow$$

$$[\Sigma]=\begin{bmatrix}\lambda_1-\lambda_2 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}+\begin{bmatrix}\lambda_2-\lambda_3 & 0 & 0\\ 0 & \lambda_2-\lambda_3 & 0\\ 0 & 0 & 0\end{bmatrix}+\begin{bmatrix}\lambda_3 & 0 & 0\\ 0 & \lambda_3 & 0\\ 0 & 0 & \lambda_3\end{bmatrix}$$

W.A HOLM DECOMPOSITION

$$\langle[T]\rangle = \lambda_1 \underline{u}_1 \underline{u}_1^{T*} + \lambda_2 \underline{u}_2 \underline{u}_2^{T*} + \lambda_3 \underline{u}_3 \underline{u}_3^{T*}$$

$$\langle[T]\rangle = (\lambda_1 - \lambda_2)\underline{u}_1 \underline{u}_1^{T*} + (\lambda_2 - \lambda_3)\left(\underline{u}_1 \underline{u}_1^{T*} + \underline{u}_2 \underline{u}_2^{T*}\right) + \lambda_3 [I_{3D}]$$

PURE TARGET
(AVERAGE)

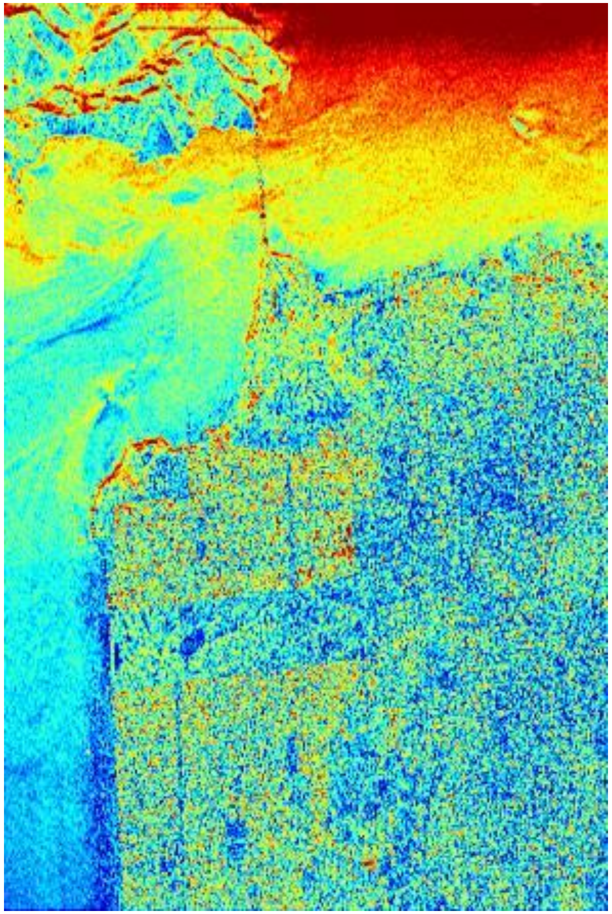
MIXED TARGET
(VARIANCE)

NOISE
(UNPOLARIZED)

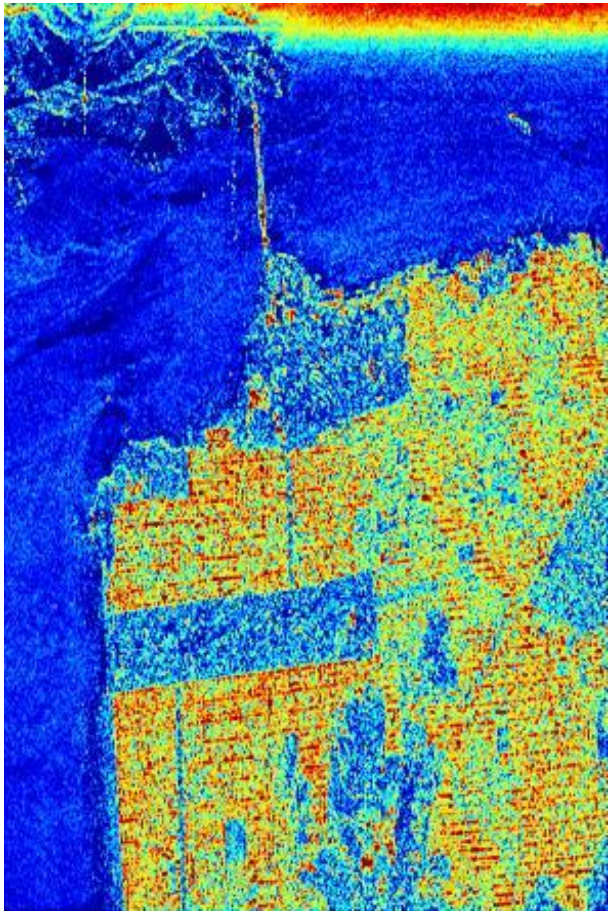
CONCEPT OF : **TARGET** PLUS **NOISE**

HYBRID APPROACH OF THE HUYNEN MODEL

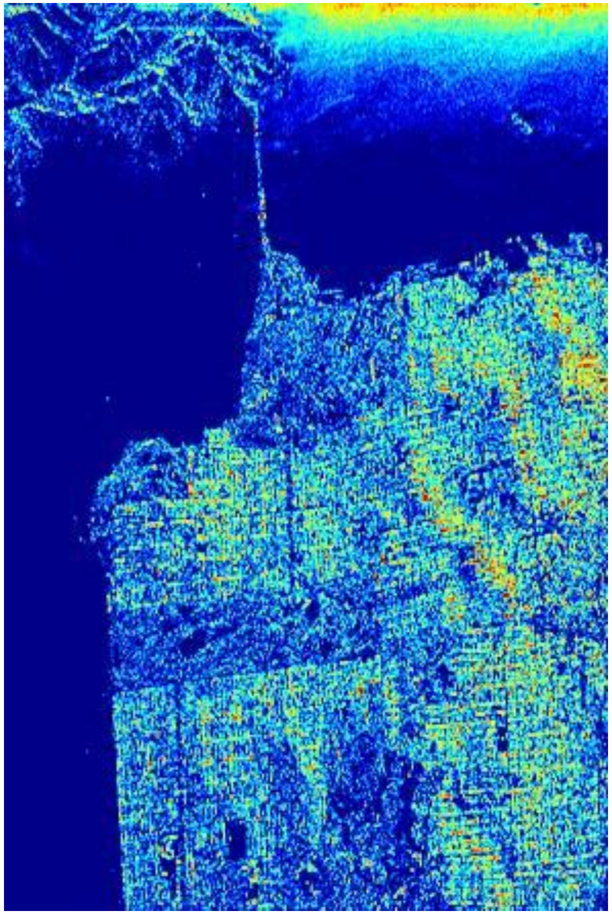
W.A HOLM DECOMPOSITION



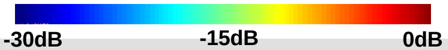
$$\sqrt{\lambda_1 - \lambda_2}\,|u_{11}|$$



$$\sqrt{\lambda_1 - \lambda_2}\,|u_{12}|$$



$$\sqrt{\lambda_1 - \lambda_2}\,|u_{13}|$$

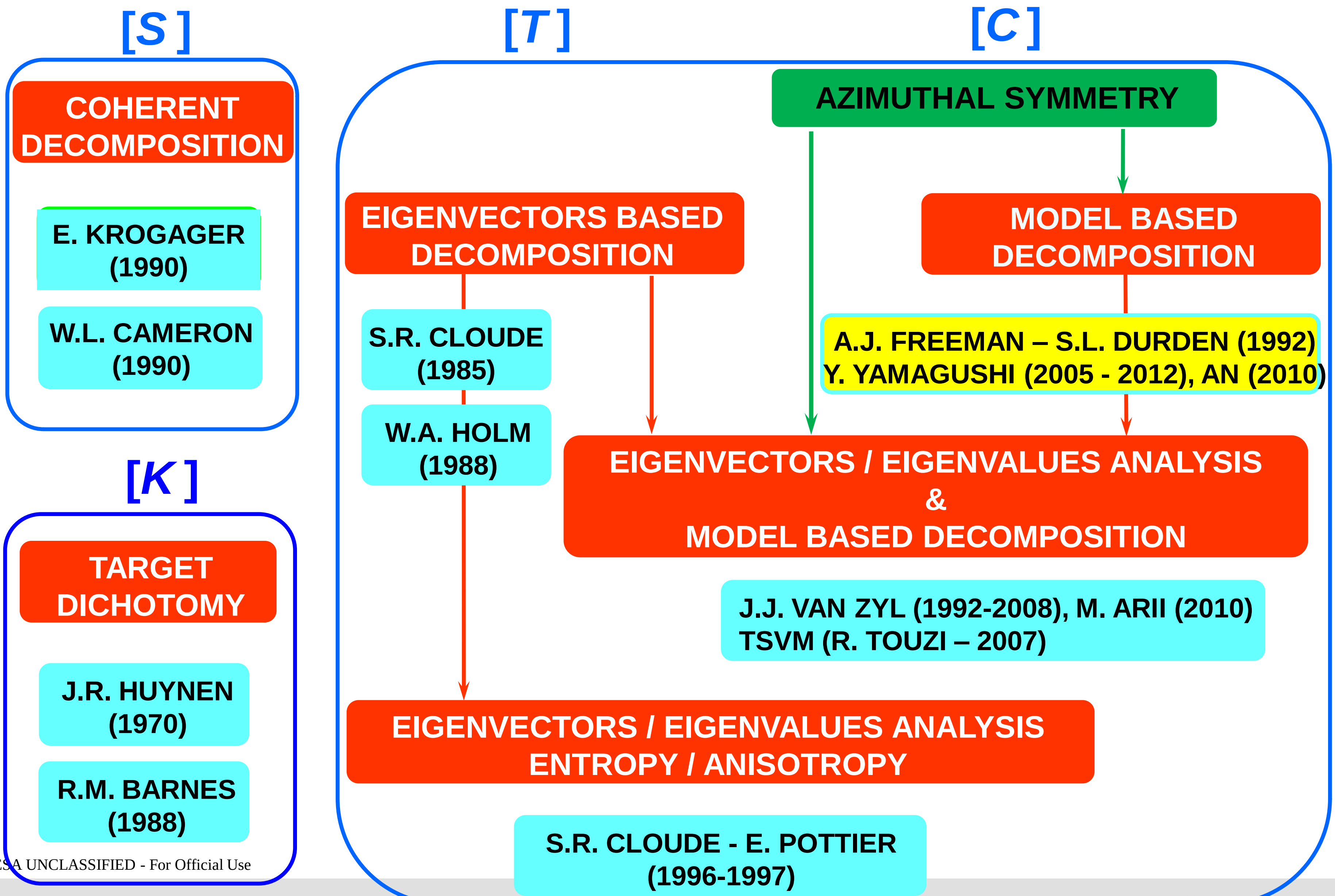


W.A HOLM DECOMPOSITION

$2A_0$ $B_0 + B$ $B_0 - B$

$\sqrt{\lambda_1 - \lambda_2}\,|u_{11}|$ $\sqrt{\lambda_1 - \lambda_2}\,|u_{12}|$ $\sqrt{\lambda_1 - \lambda_2}\,|u_{13}|$

TARGET DECOMPOSITIONS



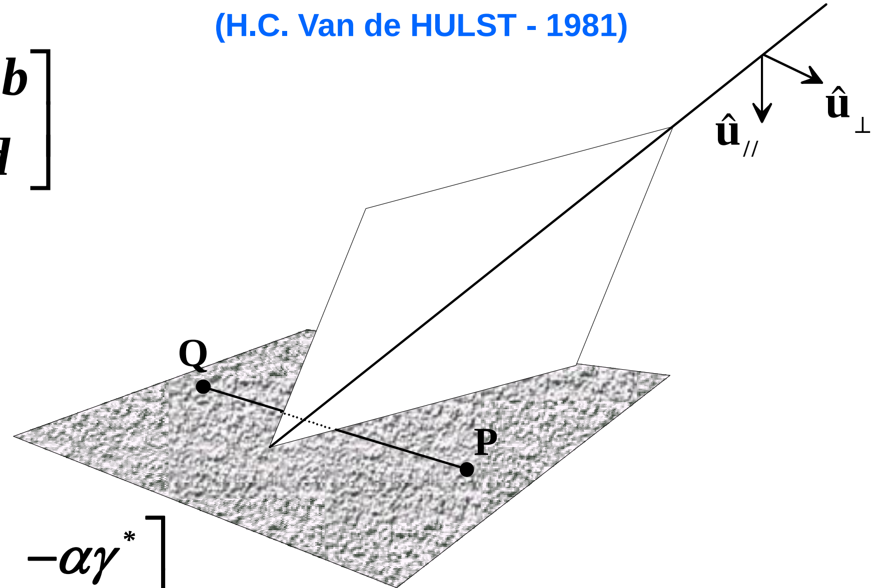
TARGET DECOMPOSITION FOR TARGETS WITH REFLECTION SYMMETRY

MEDIUM WITH REFLECTION SYMMETRY

(H.C. Van de HULST - 1981)

$$[S_Q]=\begin{bmatrix} a & -b \\ -b & d \end{bmatrix}$$

$$[S_P]=\begin{bmatrix} a & b \\ b & d \end{bmatrix}$$



$$\underline{k}_Q=\begin{bmatrix} \alpha \\ \beta \\ -\gamma \end{bmatrix}$$

$$\underline{k}_P=\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}$$

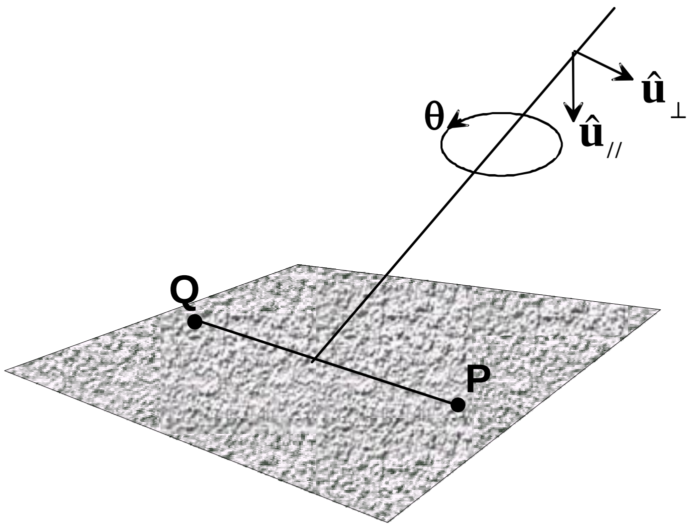
$$[T_Q]=\begin{bmatrix} |\alpha|^2 & \alpha\beta^* & -\alpha\gamma^* \\ \beta\alpha^* & |\beta|^2 & -\beta\gamma^* \\ -\gamma\alpha^* & -\gamma\beta^* & |\gamma|^2 \end{bmatrix}$$

$$[T_P]=\begin{bmatrix} |\alpha|^2 & \alpha\beta^* & \alpha\gamma^* \\ \beta\alpha^* & |\beta|^2 & \beta\gamma^* \\ \gamma\alpha^* & \gamma\beta^* & |\gamma|^2 \end{bmatrix}$$

$$\langle[T]\rangle=[T_P]+[T_Q]=\begin{bmatrix} |\alpha|^2 & \alpha\beta^* & 0 \\ \beta\alpha^* & |\beta|^2 & 0 \\ 0 & 0 & |\gamma|^2 \end{bmatrix}$$

MEDIUM WITH ROTATION SYMMETRY

(H.C. Van de HULST - 1981)



$$\langle [T(\theta)] \rangle = [R_3(\theta)] \langle [T] \rangle [R_3(\theta)]^{-1}$$

With:

$$[R_3(\theta)] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(2\theta) & \sin(2\theta) \\ 0 & -\sin(2\theta) & \cos(2\theta) \end{bmatrix}$$

EIGENVECTORS OF $[U_{3P}^{R}]$

$$[U_{3P}^{R}]\underline{q} = \lambda \underline{q}$$

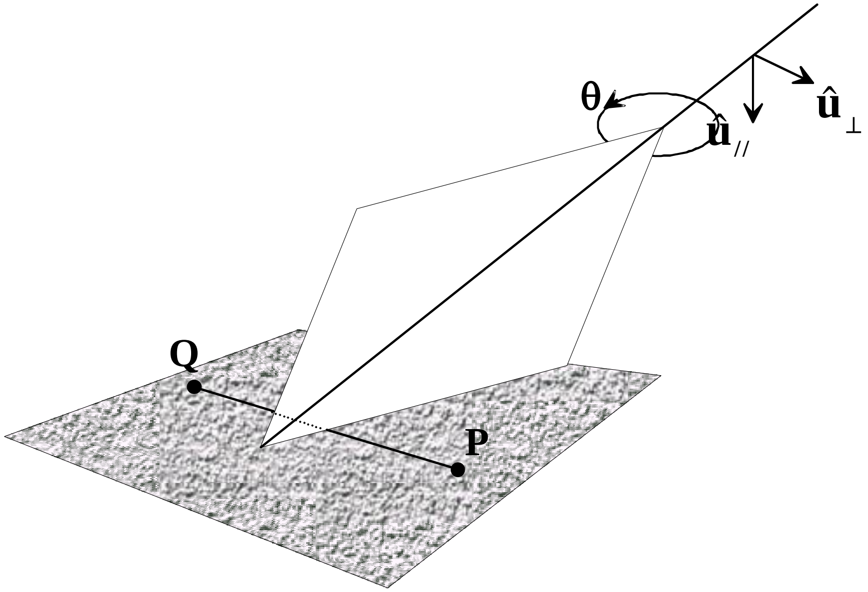
$$\underline{q}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \underline{q}_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ j \end{bmatrix} \quad \underline{q}_3 = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ j \\ 1 \end{bmatrix}$$

$$\langle [T_R] \rangle = \alpha \underline{q}_1 \underline{q}_1^{T*} + \beta \underline{q}_2 \underline{q}_2^{T*} + \gamma \underline{q}_3 \underline{q}_3^{T*}$$

$$= \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \beta+\gamma & -j(\beta-\gamma) \\ 0 & j(\beta-\gamma) & \beta+\gamma \end{bmatrix}$$

MEDIUM WITH AZIMUTHAL SYMMETRY

(S.H. NGHIEM et al. - 1992)



$$[T_{PR}] = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \beta+\gamma & -j(\beta-\gamma) \\ 0 & j(\beta-\gamma) & \beta+\gamma \end{bmatrix}$$

$$[T_{QR}] = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \beta+\gamma & j(\beta-\gamma) \\ 0 & -j(\beta-\gamma) & \beta+\gamma \end{bmatrix}$$

AZIMUTHAL SYMMETRY = REFLECTION + ROTATION SYMMETRIES

$$\langle [T] \rangle = [T_{PR}] + [T_{QR}] = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \beta+\gamma & 0 \\ 0 & 0 & \beta+\gamma \end{bmatrix}$$

COHERENCY MATRIX

General Case

$$\langle[T]\rangle=\begin{bmatrix} T_1 & T_2 & T_3 \\ T_2^* & T_4 & T_5 \\ T_3^* & T_5^* & T_6 \end{bmatrix}$$

Reflection Symmetry

$$\langle[T]\rangle=\begin{bmatrix} T_1 & T_2 & 0 \\ T_2^* & T_4 & 0 \\ 0 & 0 & T_6 \end{bmatrix}$$

Rotation Symmetry

$$\langle[T]\rangle=\begin{bmatrix} T_1 & 0 & 0 \\ 0 & T_4 & T_5 \\ 0 & T_5^* & T_4 \end{bmatrix}$$

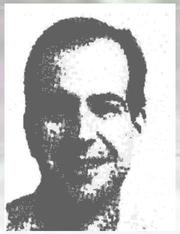
Azimuthal Symmetry

$$\langle[T]\rangle=\begin{bmatrix} T_1 & 0 & 0 \\ 0 & T_4 & 0 \\ 0 & 0 & T_4 \end{bmatrix}$$

TARGET DECOMPOSITION FOR TARGETS WITH REFLECTION SYMMETRY

MODEL BASED DECOMPOSITION

A. FREEMAN – S. DURDEN (1992)



A. Freeman and S.L. Durden, "A Three-Component Scattering Model for Polarimetric SAR Data"
IEEE TGRS, vol. 36, no. 3, May 1998

MODEL BASED DECOMPOSITION

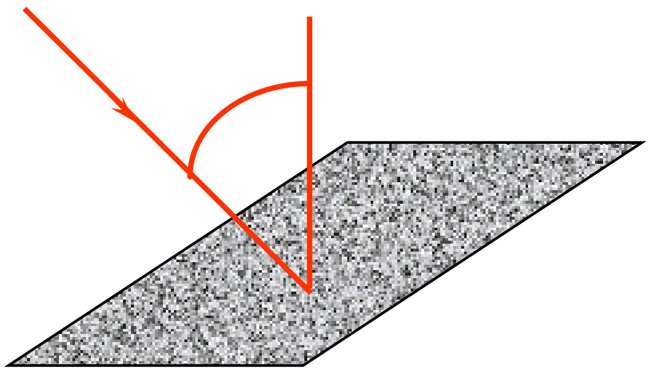
esa

3 COMPONENTS SCATTERING MECHANISM MODEL

$$\langle [T] \rangle = [T_S] + [T_D] + [T_V]$$

SINGLE SCATTERING | DOUBLE SCATTERING | VOLUME SCATTERING

SINGLE SCATTERING
(ROUGH SURFACE)



MECHANISM

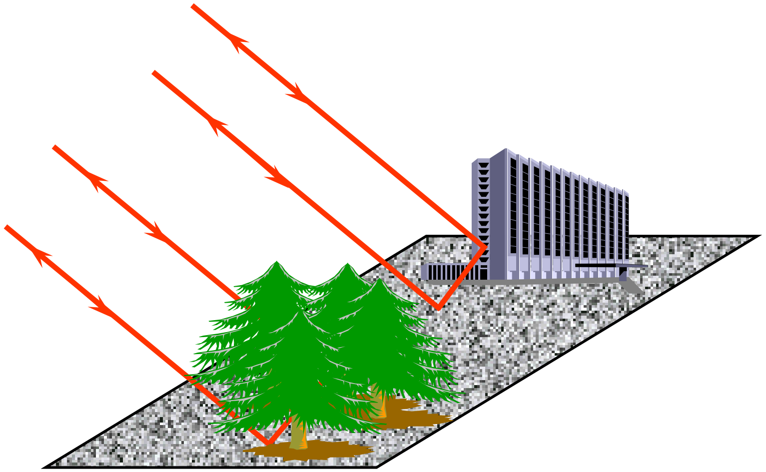
$$[S_s] = \begin{bmatrix} R_H & 0 \\ 0 & R_V \end{bmatrix} \Rightarrow \underline{k}_S = \begin{bmatrix} R_H + R_V \\ R_H - R_V \\ 0 \end{bmatrix}$$

COHERENCY MATRIX

$$[T_s] = f_s \begin{bmatrix} |\beta+1|^2 & (\beta+1)(\beta-1)^* & 0 \\ (\beta+1)^*(\beta-1) & |\beta-1|^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$f_s = |R_V|^2 \qquad \beta = \frac{R_H}{R_V}$$

DOUBLE SCATTERING MECHANISM



$$[S_D] = \begin{bmatrix} R_{GH} R_{TH} & 0 \\ 0 & -R_{GV} R_{TV} \end{bmatrix}$$

$$\Rightarrow \underline{k}_D = \begin{bmatrix} R_{GH} R_{TH} - R_{GV} R_{TV} \\ R_{GH} R_{TH} + R_{GV} R_{TV} \\ 0 \end{bmatrix}$$

COHERENCY MATRIX

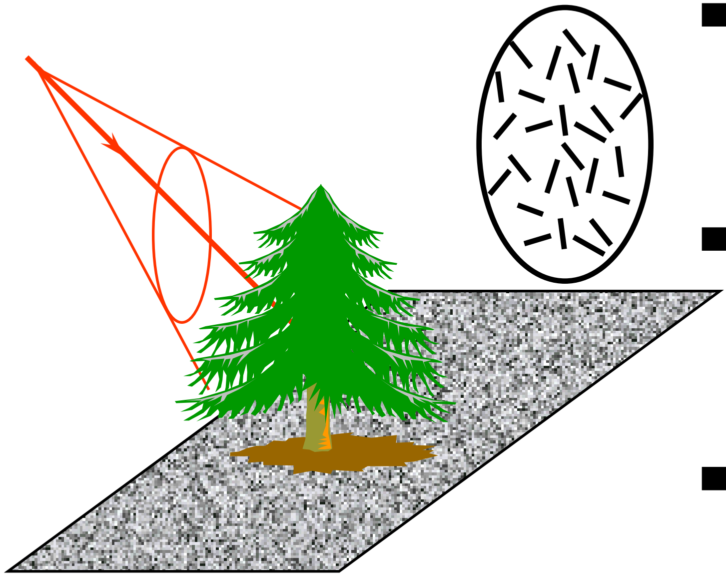
$$[T_D] = f_D \begin{bmatrix} |\alpha-1|^2 & (\alpha-1)(\alpha+1)^* & 0 \\ (\alpha-1)^*(\alpha+1) & |\alpha+1|^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$f_D = |R_{GV} R_{TV}|^2$$

$$\alpha = \frac{R_{GH} R_{TH}}{R_{GV} R_{TV}}$$

MODEL BASED DECOMPOSITION

VOLUME SCATTERING
(RANDOMLY ORIENTED VERY THIN CYLINDER-LIKE SCATTERERS)



MECHANISM (CYLINDER)

$$[S] = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

ORIENTED MECHANISM

$$[S_\theta] = [U_2(\theta)]^T [S][U_2(\theta)]$$

UNIFORM ORIENTATION

$$P(\theta) = \frac{1}{2\pi}$$

SECOND-ORDER STATISTICS

$$[T_V] = \langle [T_\theta] \rangle = \int_0^{2\pi} [T_V] P(\theta) d\theta$$

COVARIANCE MATRIX

(THIN CYLINDERS)

$$\mathbf{a} \mapsto \mathbf{1} \quad \mathbf{b} \mapsto \mathbf{0}$$

$$[T_V] = \frac{f_V}{3} \begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

MODEL BASED DECOMPOSITION

3 COMPONENTS SCATTERING MECHANISM MODEL

$$\langle [T] \rangle = [T_S] + [T_D] + [T_V]$$

SINGLE SCATTERING — DOUBLE SCATTERING — VOLUME SCATTERING

$$T_{11} = f_S |\beta + 1|^2 + f_D |\alpha - 1|^2 + \frac{4 f_V}{3}$$

$$T_{12} = f_S (\beta + 1)(\beta - 1)^* + f_D (\alpha - 1)(\alpha + 1)^*$$

$$T_{22} = f_S |\beta - 1|^2 + f_D |\alpha + 1|^2 + \frac{2 f_V}{3}$$

$$T_{33} = \frac{2 f_V}{3}$$

→ 5 UNKNOWN REAL COEFFICIENTS

→ 4 OBSERVED EQUATIONS

MODEL BASED DECOMPOSITION

$$if\ \Re\left(\left\langle S_{XX} S_{YY}^{*} \right\rangle - \frac{f_V}{3}\right) \geq 0 \quad \Rightarrow \quad \alpha = +1$$

$$if\ \Re\left(\left\langle S_{XX} S_{YY}^{*} \right\rangle - \frac{f_V}{3}\right) \leq 0 \quad \Rightarrow \quad \beta = +1$$

$$\left\{ f_S, |\beta|, f_D, |\alpha|, f_V \right\}$$

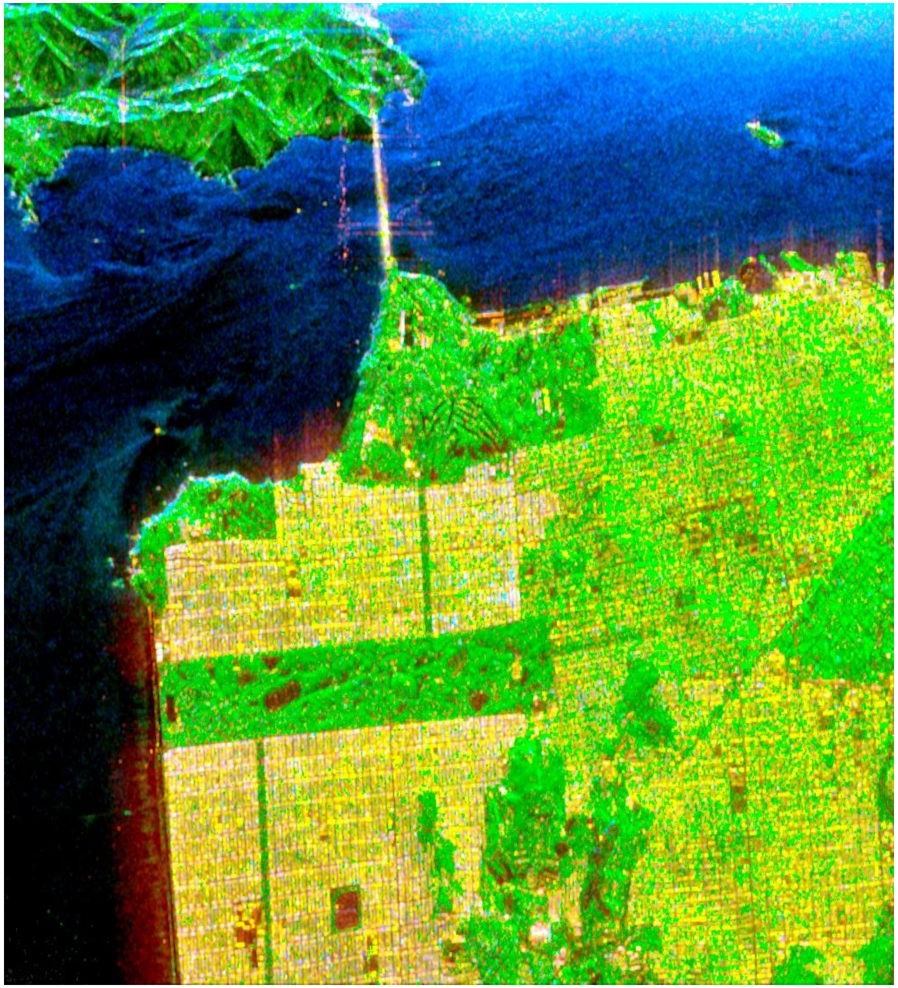
$$span = \langle T_{11} \rangle + \langle T_{22} \rangle + \langle T_{33} \rangle = f_S\left(1 + \beta^2\right) + f_D\left(1 + |\alpha|^2\right) + \frac{2}{3} f_V$$

SINGLE BOUNCE SCATTERING (ODD)

DOUBLE DOUBLE SCATTERING (DBL)

VOLUME SCATTERING (VOL)

MODEL BASED DECOMPOSITION



$2A_0$ $B_0 + B$ $B_0 - B$

$ODD = f_S\left(1+\beta^2\right)$

$DBL = f_D\left(1+\alpha^2\right)$

$VOL = \frac{2f_V}{3}$

2 COMPONENTS SCATTERING MECHANISM MODEL

$$\langle [T] \rangle = [T_G] + [T_V]$$

GROUND SCATTERING | **VOLUME SCATTERING**

Bragg scatter from a moderately rough surface

Double-bounce scatter from a pair of orthogonal surfaces

Freeman A., "Fitting a Two-Component Scattering Model to Polarimetric SAR Data from Forests", IEEE Trans. Geosci. Remote Sensing, vol. 45, no. 8, pp. 2583–2592, Aug. 2007.

2007

TARGET DECOMPOSITION FOR TARGETS WITHOUT REFLECTION SYMMETRY

MODEL BASED - 4 COMPONENTS DECOMPOSITION

Y. YAMAGUCHI et al. (2005 - 2013)

MEDIUM WITHOUT ANY REFLECTION SYMMETRY

4 COMPONENTS SCATTERING MECHANISM MODEL

$$\langle [T] \rangle = [T_S] + [T_D] + [T_V] + [T_H]$$

SINGLE SCATTERING | DOUBLE SCATTERING | VOLUME SCATTERING | HELIX SCATTERING

$$[S]_{\pm Helix} = \frac{1}{2}\begin{bmatrix} 1 & \pm j \\ \pm j & -1 \end{bmatrix} \qquad \langle [T] \rangle_{Helix} = \frac{1}{2}\left\langle \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & \pm j \\ 0 & \mp j & 1 \end{bmatrix} \right\rangle$$

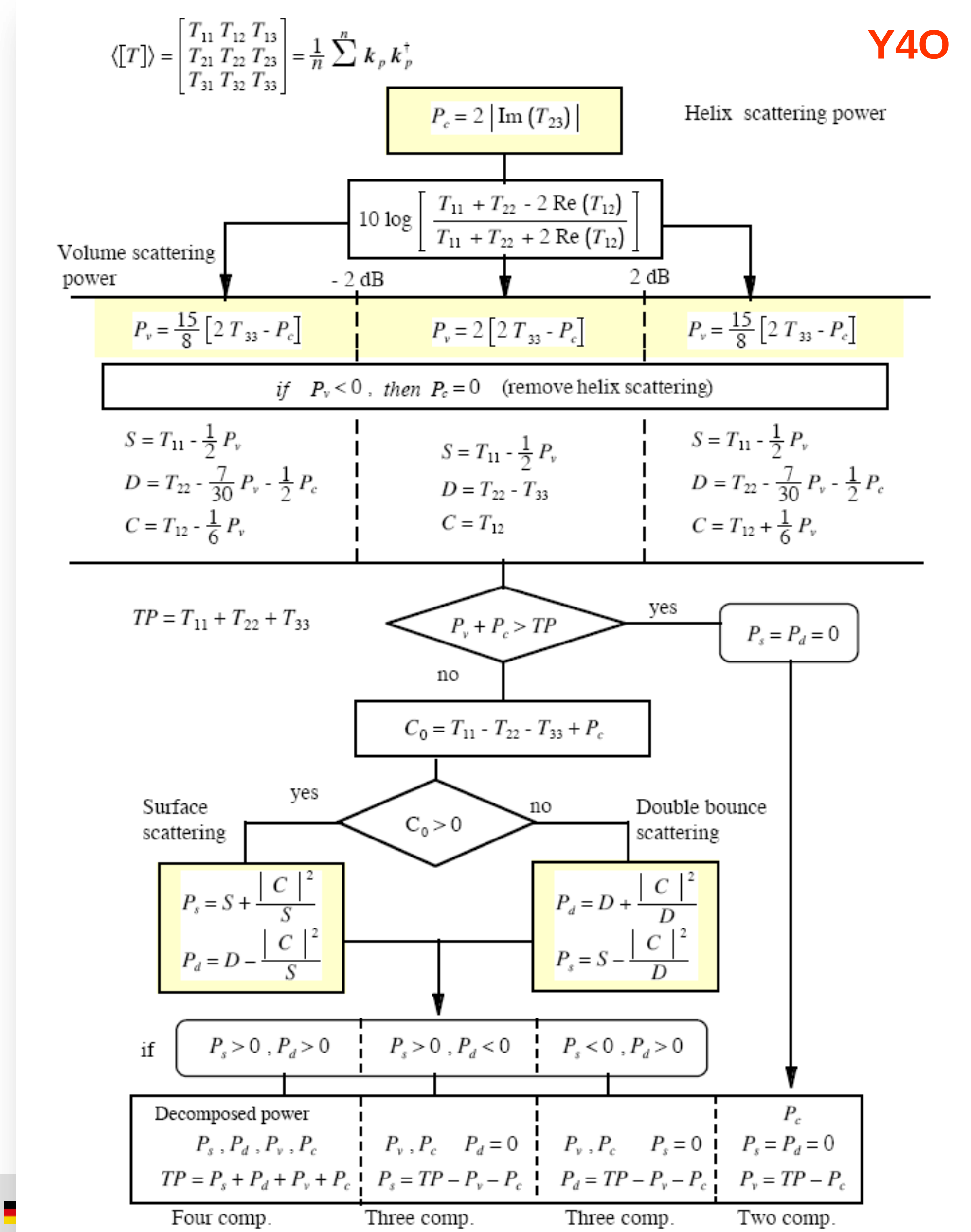
Non reflection Symmetric cases

Yamaguchi Y., Moriyama T., Ishido M. and Yamada H., "*Four-Component Scattering Model for Polarimetric SAR Image Decomposition*", IEEE Trans. Geos. Remote Sens., vol. 43, no. 8, August 2005.

Yamaguchi Y., Yajima Y. and Yamada H., "A Four-Component Decomposition of POLSAR Images Based on the Coherency Matrix", IEEE Geos. Rem. Sens. Letters, vol. 3, no. 3, July 2006.

2005 - 2006

MODEL BASED DECOMPOSITION



MODEL BASED DECOMPOSITION

$2A_0$ $\quad B_0 + B$ $\quad B_0 - B$

$$ODD = f_S\left(1+\beta^2\right)$$

$$DBL = f_D\left(1+\alpha^2\right)$$

$$VOL = \frac{2f_V}{3}$$

MODEL BASED DECOMPOSITION

ODD *DBL* *VOL*

Freeman decomposition **Yamaguchi decomposition**

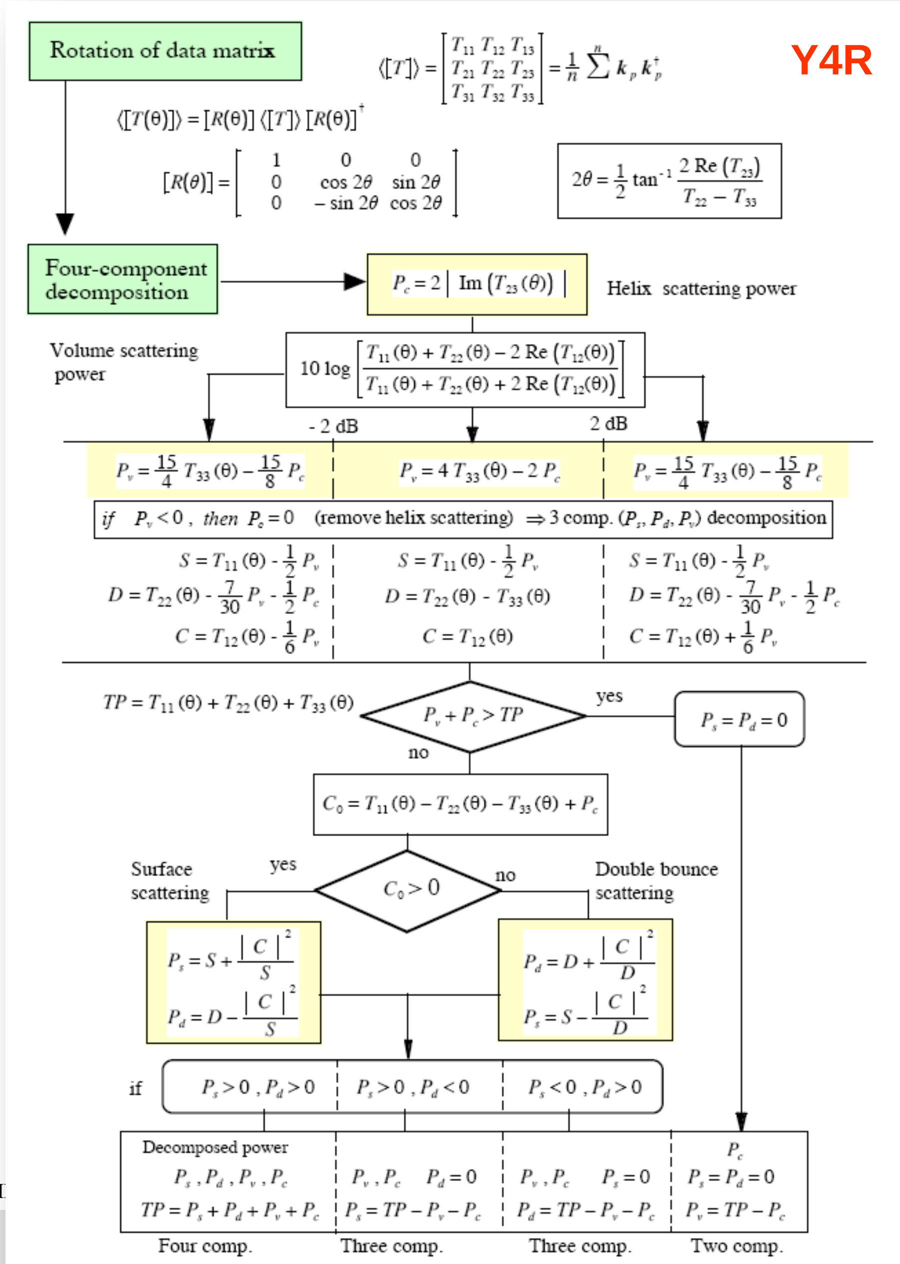
Y. Yamaguchi, A. Sato, W.M. Boerner, R. Sato, H. Yamada, “*4-component scattering power decomposition with rotation of coherency matrix*”, IEEE TGRS vol. 49, no. 6, June 2011.

A. Sato, Y. Yamaguchi, G. Singh, and S.-E. Park, “*4-component scattering power decomposition with extended volume scattering model*”, IEEE GRS Letters, vol. 9, no. 2, pp. 166–170, Mar. 2012.

G. Singh, Y. Yamaguchi, S.E. Park*, « General Four-Component Scattering Power Decomposition With Unitary Transformation of Coherency Matrix »* IEEE TGRS in press

G. Singh, Y. Yamaguchi, S.E. Park*, Y. Cui, H. Kobayashi, « Hybrid Freeman/Eigenvalue Decomposition Method With Extended Volume Scattering Model »* IEEE GRS Letters, vol. 10, no. 1, Jan. 2013

MODEL BASED DECOMPOSITION



$$2\theta = \frac{1}{2}\tan^{-1}\left(\frac{2\,\mathrm{Re}\{T_{23}\}}{T_{22} - T_{33}}\right)$$

$$[R(\theta)] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 2\theta & \sin 2\theta \\ 0 & -\sin 2\theta & \cos 2\theta \end{bmatrix}$$

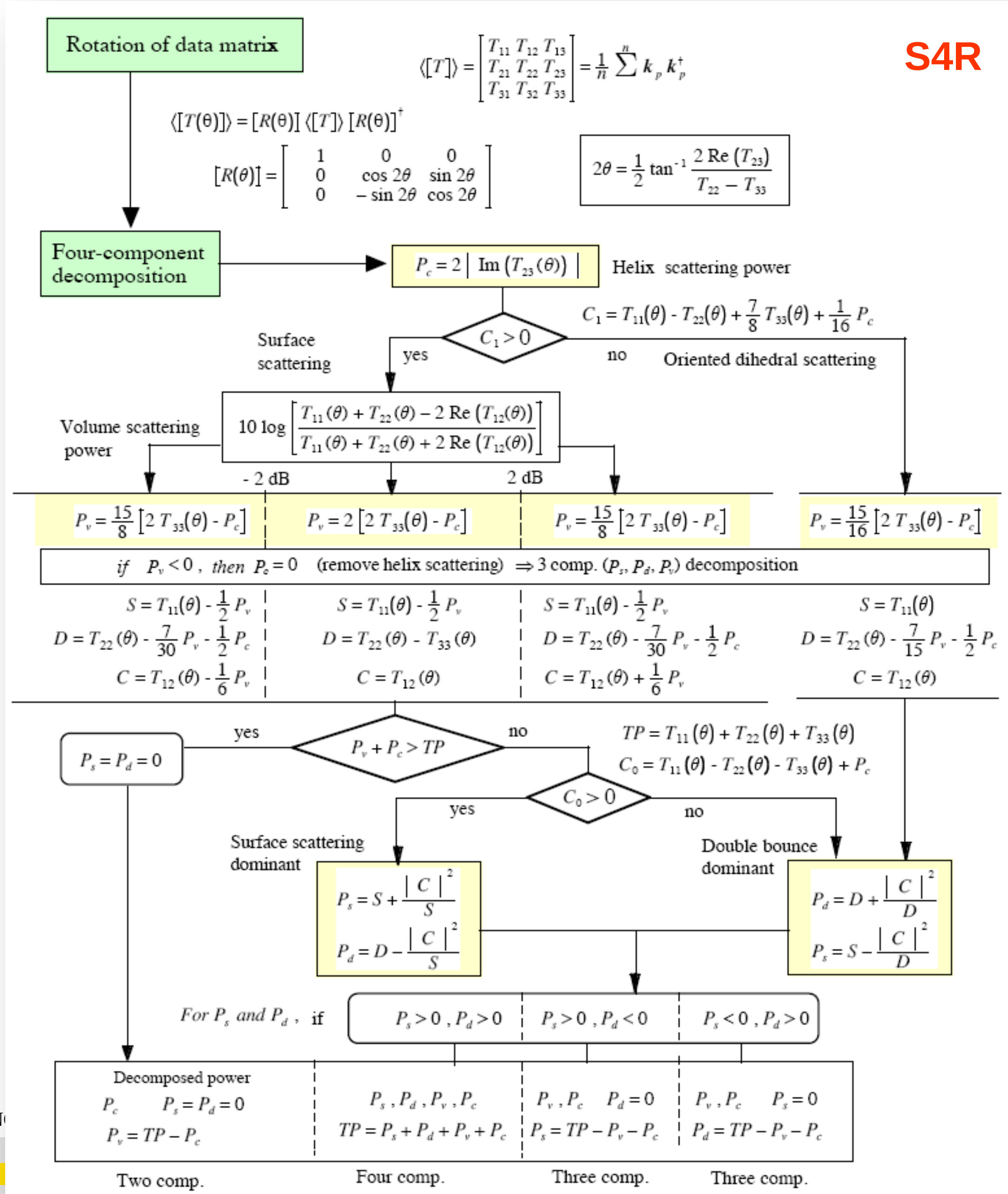
$$\langle[T(\theta)]\rangle = [R(\theta)]\,\langle[T]\rangle\,[R(\theta)]^{\dagger}$$

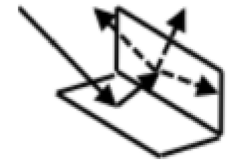
MODEL BASED DECOMPOSITION

$2A_0$ $B_0 + B$ $B_0 - B$

ODD *DBL* *VOL*

MODEL BASED DECOMPOSITION





$$\frac{1}{15}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

HV from oriented dihedral components

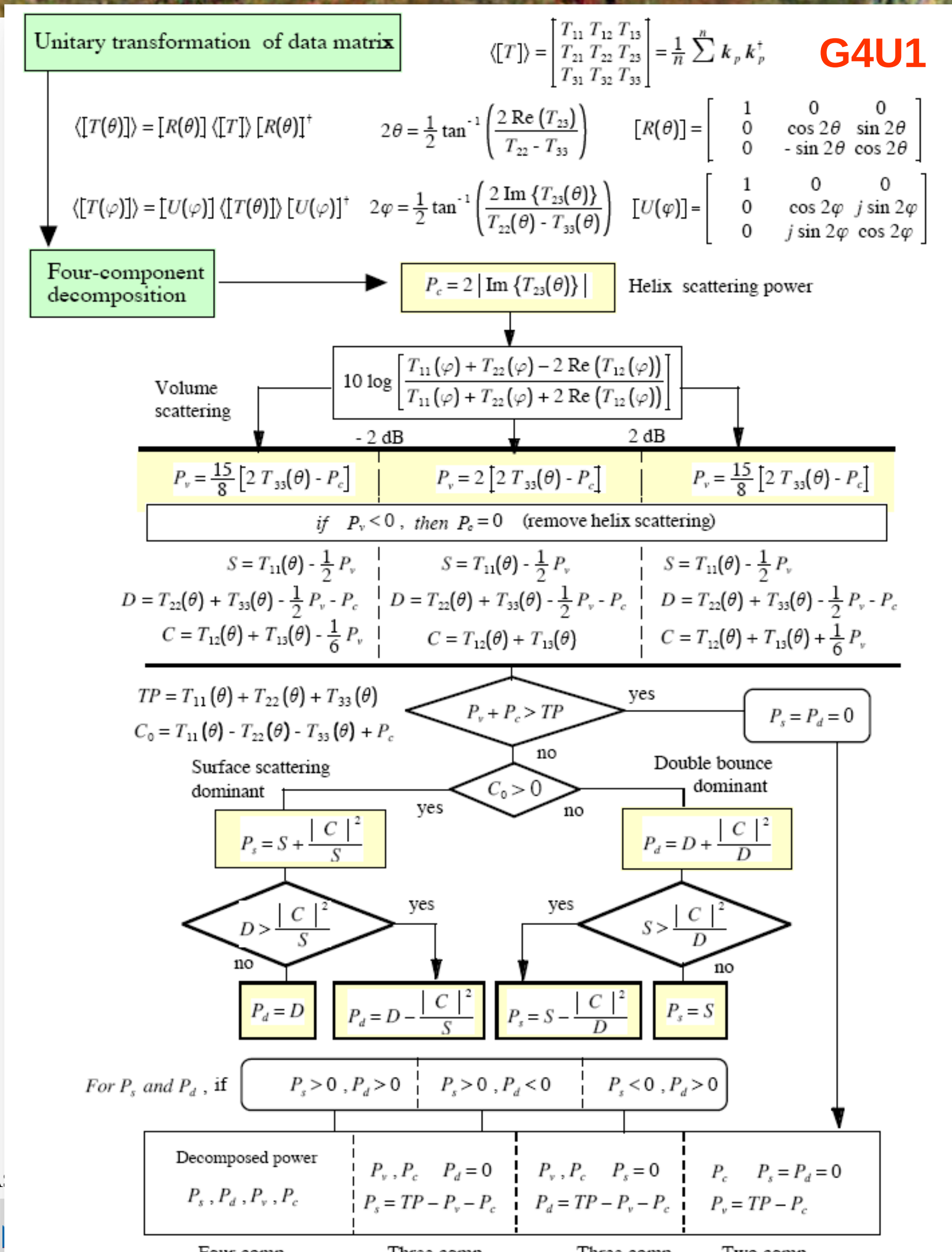
Extended volume scattering model

MODEL BASED DECOMPOSITION

$2A_0$ $B_0 + B$ $B_0 - B$

ODD *DBL* *VOL*

MODEL BASED DECOMPOSITION



$$2\varphi = \frac{1}{2} \tan^{-1} \left(\frac{2 \operatorname{Im}\{T_{23}(\theta)\}}{T_{22}(\theta) - T_{33}(\theta)} \right)$$

$$[U(\varphi)] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 2\varphi & j \sin 2\varphi \\ 0 & j \sin 2\varphi & \cos 2\varphi \end{bmatrix}$$

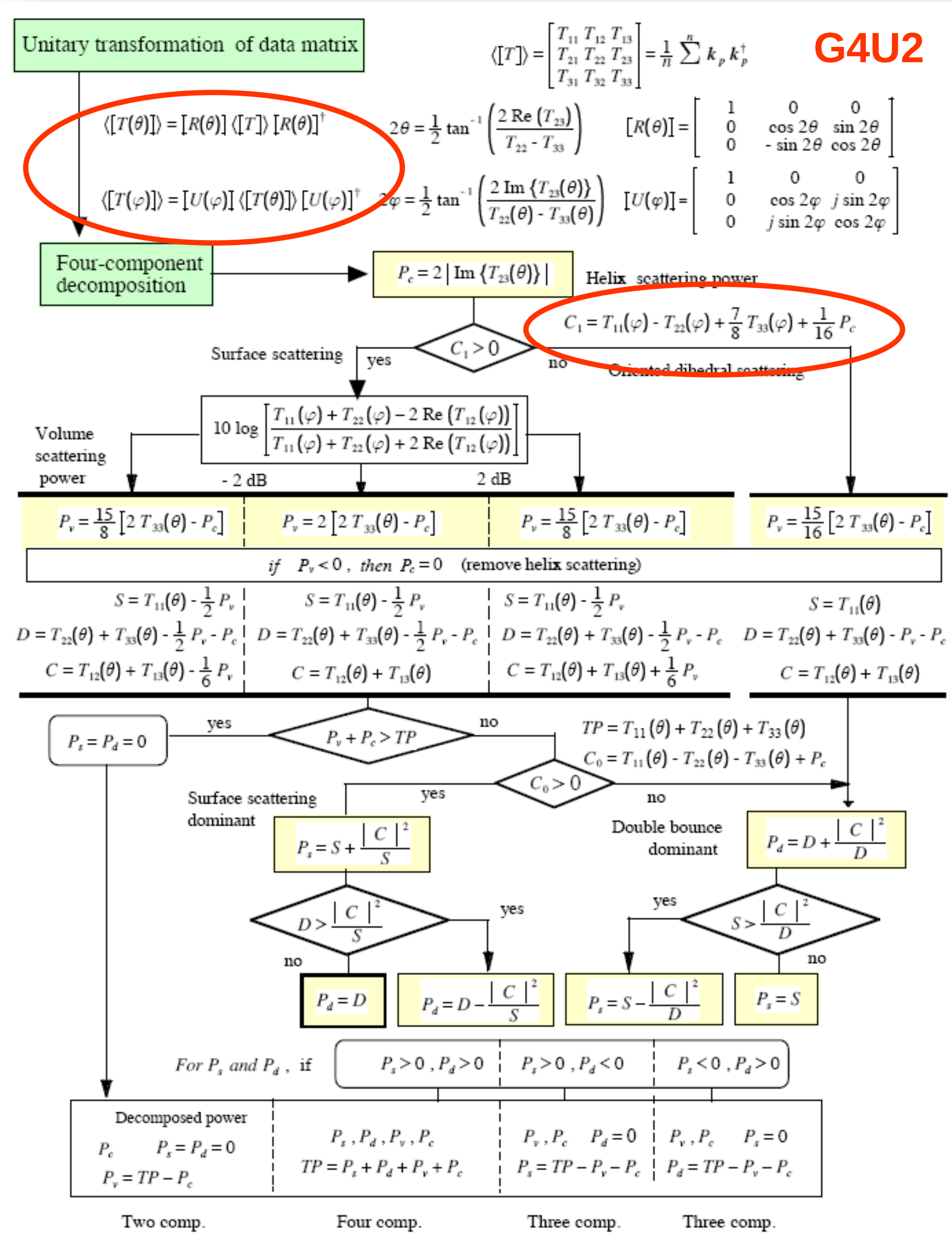
$$\langle [T(\varphi)] \rangle = [U(\varphi)] \langle [T(\theta)] \rangle [U(\varphi)]^{\dagger}$$

MODEL BASED DECOMPOSITION

$2A_0$ $B_0 + B$ $B_0 - B$

ODD *DBL* *VOL*

MODEL BASED DECOMPOSITION

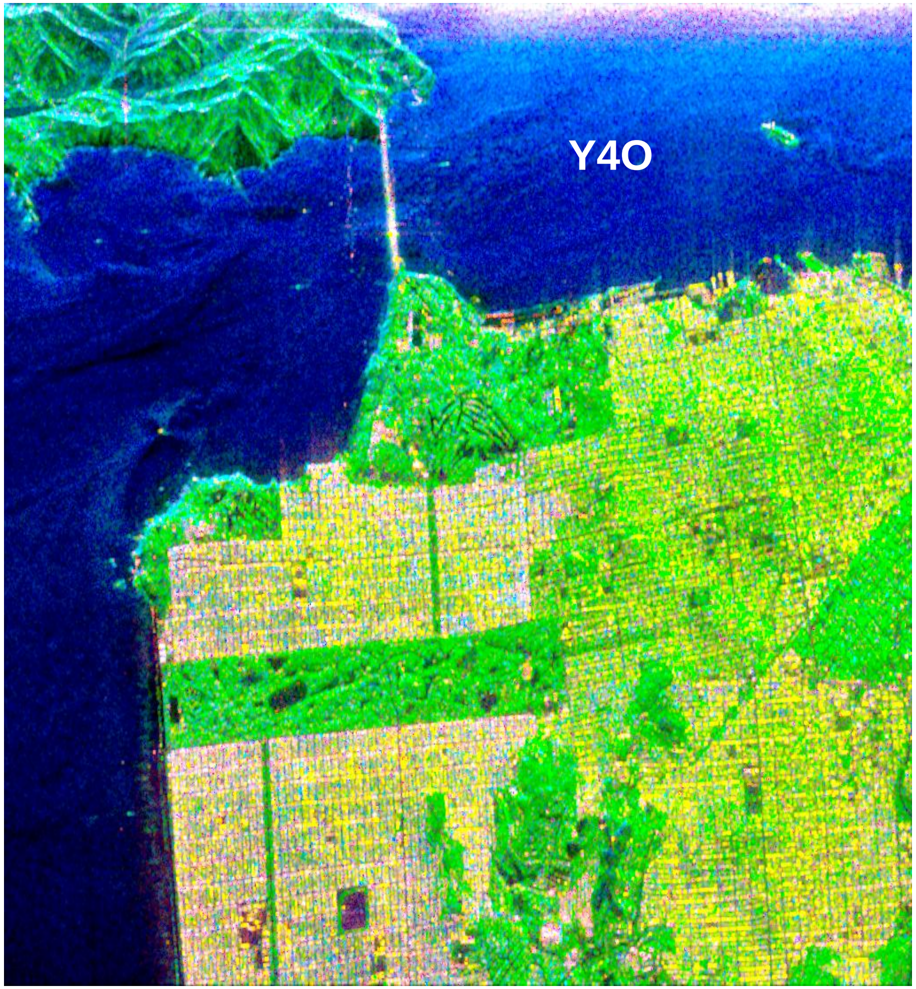


MODEL BASED DECOMPOSITION

$2A_0$ $B_0 + B$ $B_0 - B$

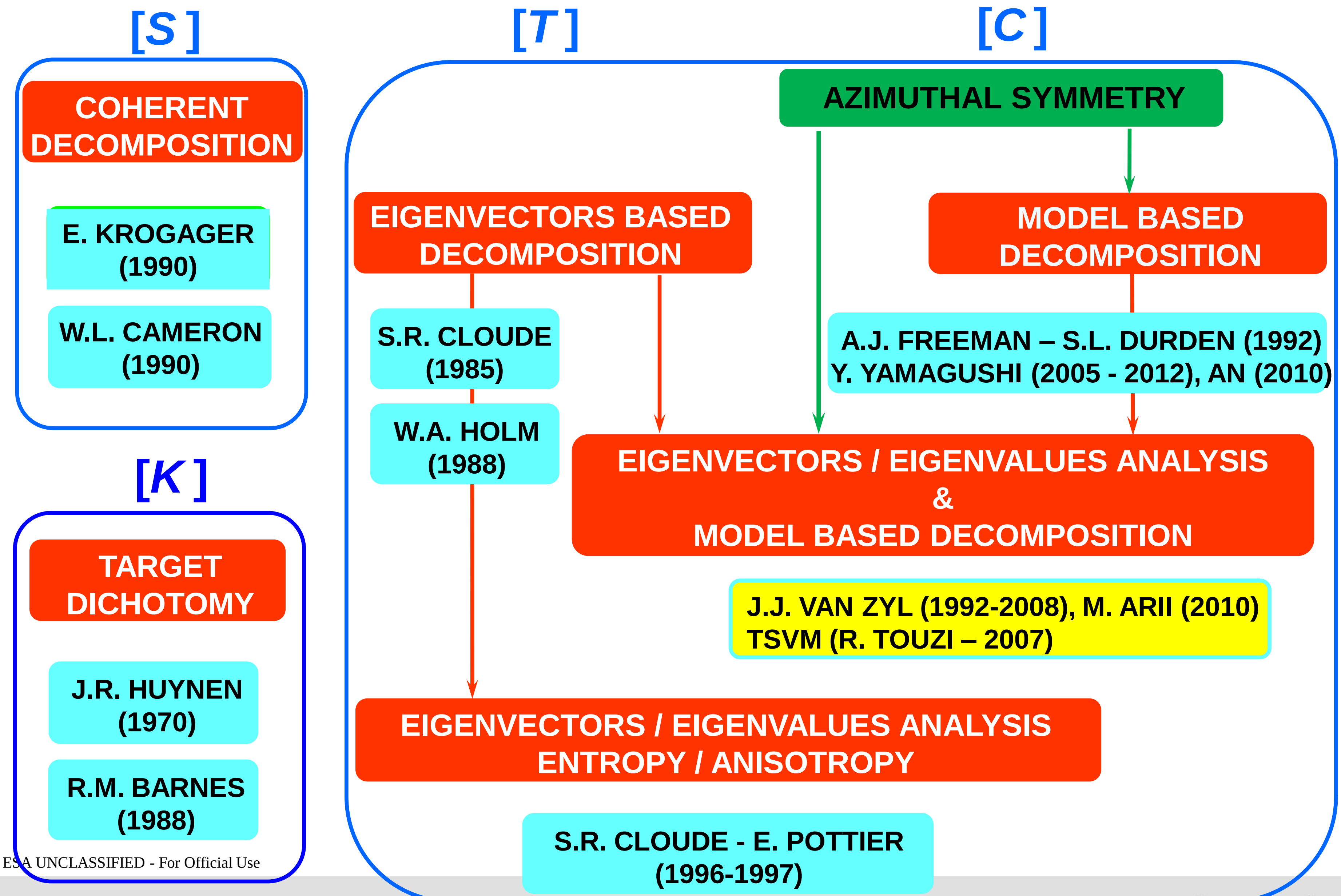
ODD DBL VOL

MODEL BASED DECOMPOSITION



ODD DBL VOL

TARGET DECOMPOSITIONS



TARGET DECOMPOSITION FOR TARGETS WITH / WITHOUT REFLECTION SYMMETRY

REQUIEREMENTS FOR MODEL BASED POLARIMETRIC DECOMPOSITIONS

J.J. VAN ZYL – M. ARII – Y. KIM (2010)



J. J. Van Zyl, M. Arii, Y. Kim, "*Model-Based Decomposition of Polarimetric SAR Covariance Matrices Constrained for Nonnegative Eigenvalues*" IEEE TGRS, vol. 49, n°9, Sept. 2011.

3 COMPONENTS SCATTERING MECHANISM MODEL

$$\langle [C] \rangle = [C_S] + [C_D] + [C_V]$$

The algorithm uses the cross-polarized term to calculate the volume scattering contribution, and subtract that from the observed matrix.

$$[C_{remainder}] = \langle [C] \rangle - [C_V] = [C_S] + [C_D]$$

$[C_{remainder}]$ **COULD BE NOT POSITIVE SEMI-DEFINITE HERMITIAN MATRIX** → $\lambda_i \leq 0$

$$[C_{remainder}] = \langle [C] \rangle - a[C_V]$$

Subtract the volume contribution from a covariance matrix for terrain with reflection symmetry

$$[C_{remainder}] = \langle [C] \rangle - a[C_V] = \begin{bmatrix} \xi & 0 & \rho \\ 0 & \eta & 0 \\ \rho^* & 0 & \zeta \end{bmatrix} - a\begin{bmatrix} 3 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 3 \end{bmatrix}$$

The eigenvalues are:

$$\lambda_1 = \frac{1}{2}\left\{\xi + \zeta - 6a + \sqrt{(\xi + \zeta - 6a)^2 - 4(\xi - 3a)(\zeta - 3a) + 4|\rho - a|^2}\right\}$$

$$\lambda_2 = \frac{1}{2}\left\{\xi + \zeta - 6a - \sqrt{(\xi + \zeta - 6a)^2 - 4(\xi - 3a)(\zeta - 3a) + 4|\rho - a|^2}\right\}$$

$$\lambda_3 = \eta - 2a$$

- Find those values of ***a*** that will ensure that all three eigenvalues are positive or zero. The solution is

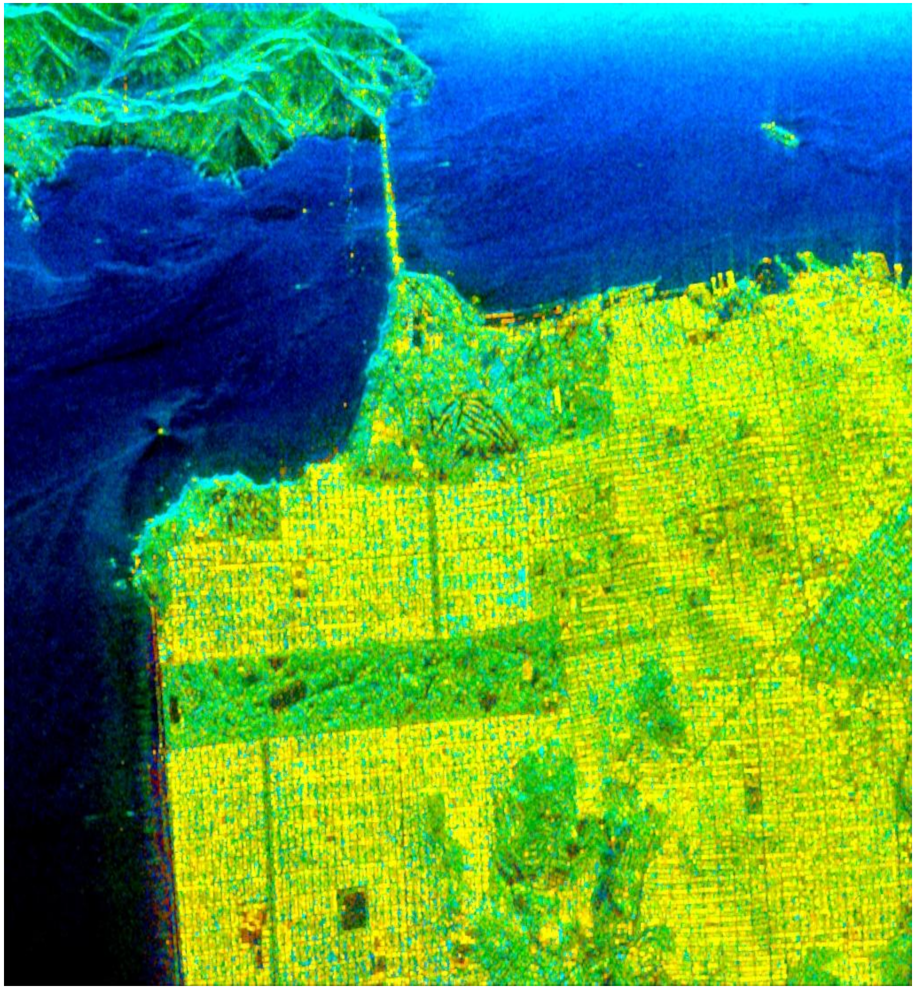
$$a_{\max} = \min \begin{cases} \eta/2 \\ \dfrac{1}{16}\left\{3(\xi+\zeta)-\rho-\rho^{*}-\sqrt{\left[3(\xi+\zeta)-\rho-\rho^{*}\right]^{2}-32\left(\xi\zeta-|\rho|^{2}\right)}\right\} \end{cases}$$

- Values of ***a*** larger than this, will leave either the second or third eigenvalue negative, resulting in a non-physical solution.

$$\downarrow$$

$$\left[C_{remainder}\right] = \langle [C] \rangle - a_{\max}\left[C_V\right] = \left[C_S\right] + \left[C_D\right]$$

MODEL BASED DECOMPOSITION



$2A_0$ $B_0 + B$ $B_0 - B$

ODD *DBL* *VOL*

ADAPTATIVE MODEL-BASED DECOMPOSITION

$$[C'_{\text{remainder}}] = \langle [C] \rangle - f_v \langle [C_{\text{vol}}(\theta_0, \sigma)] \rangle$$

$$\langle [C_{\text{vol}}(\theta_0, \sigma)] \rangle = [C_\alpha] + p(\sigma)[C_\beta] + q(\sigma)[C_\gamma].$$

$$[C_\alpha] = \frac{1}{8}\begin{bmatrix} 3 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 3 \end{bmatrix}$$

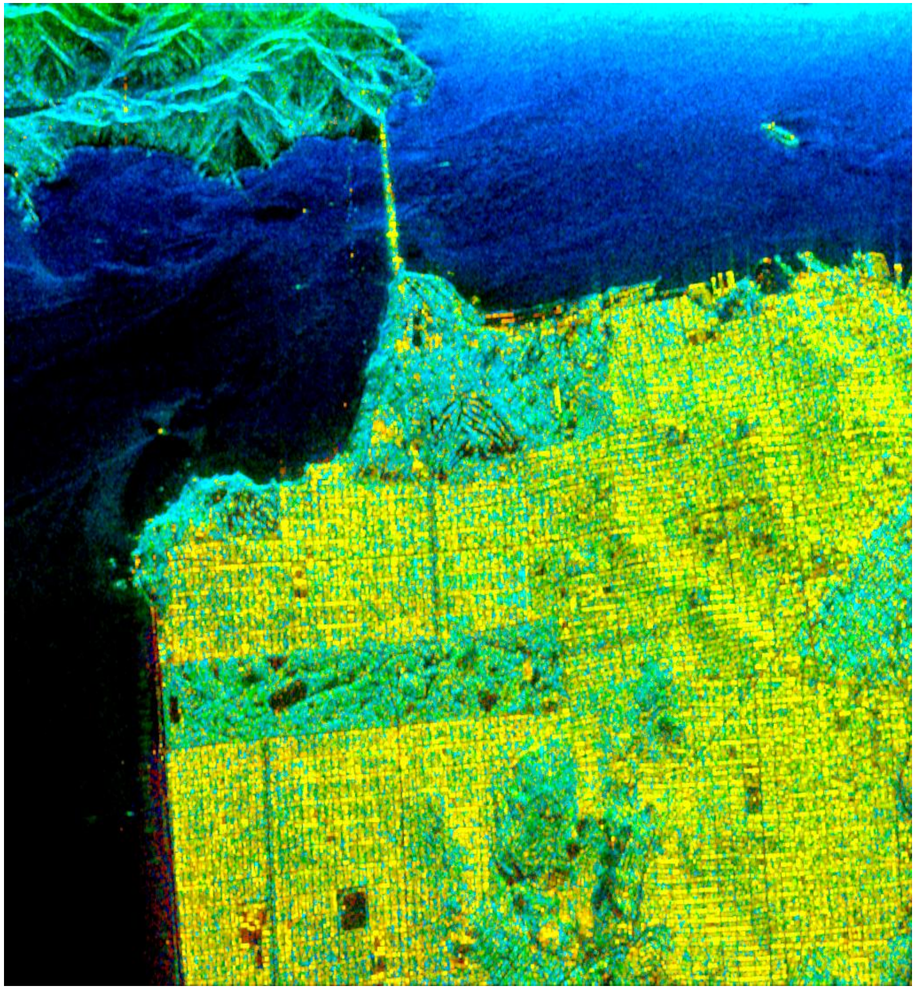
$$[C_\beta(2\theta_0)] = \frac{1}{8}\begin{bmatrix} -2\cos 2\theta_0 & \sqrt{2}\sin 2\theta_0 & 0 \\ \sqrt{2}\sin 2\theta_0 & 0 & \sqrt{2}\sin 2\theta_0 \\ 0 & \sqrt{2}\sin 2\theta_0 & 2\cos 2\theta_0 \end{bmatrix}$$

$$[C_\gamma(4\theta_0)] = \frac{1}{8}\begin{bmatrix} \cos 4\theta_0 & -\sqrt{2}\sin 4\theta_0 & -\cos 4\theta_0 \\ -\sqrt{2}\sin 4\theta_0 & -2\cos 4\theta_0 & \sqrt{2}\sin 4\theta_0 \\ -\cos 4\theta_0 & \sqrt{2}\sin 4\theta_0 & \cos 4\theta_0 \end{bmatrix}.$$

$$\begin{aligned} p(\sigma) =& 2.0806\sigma^6 - 6.3350\sigma^5 + 6.3864\sigma^4 \\ & - 0.4431\sigma^3 - 3.9638\sigma^2 - 0.0008\sigma + 2.000 \\ q(\sigma) =& 9.0166\sigma^6 - 18.7790\sigma^5 + 4.9590\sigma^4 \\ & + 14.5629\sigma^3 - 10.8034\sigma^2 + 0.1902\sigma + 1.000. \end{aligned}$$

M. Arii, J. J. Van Zyl, Y. Kim, "Adaptative *Model-Based Decomposition of Polarimetric SAR Covariance Matrices*" IEEE TGRS, vol. 49, n°9, Sept. 2011.

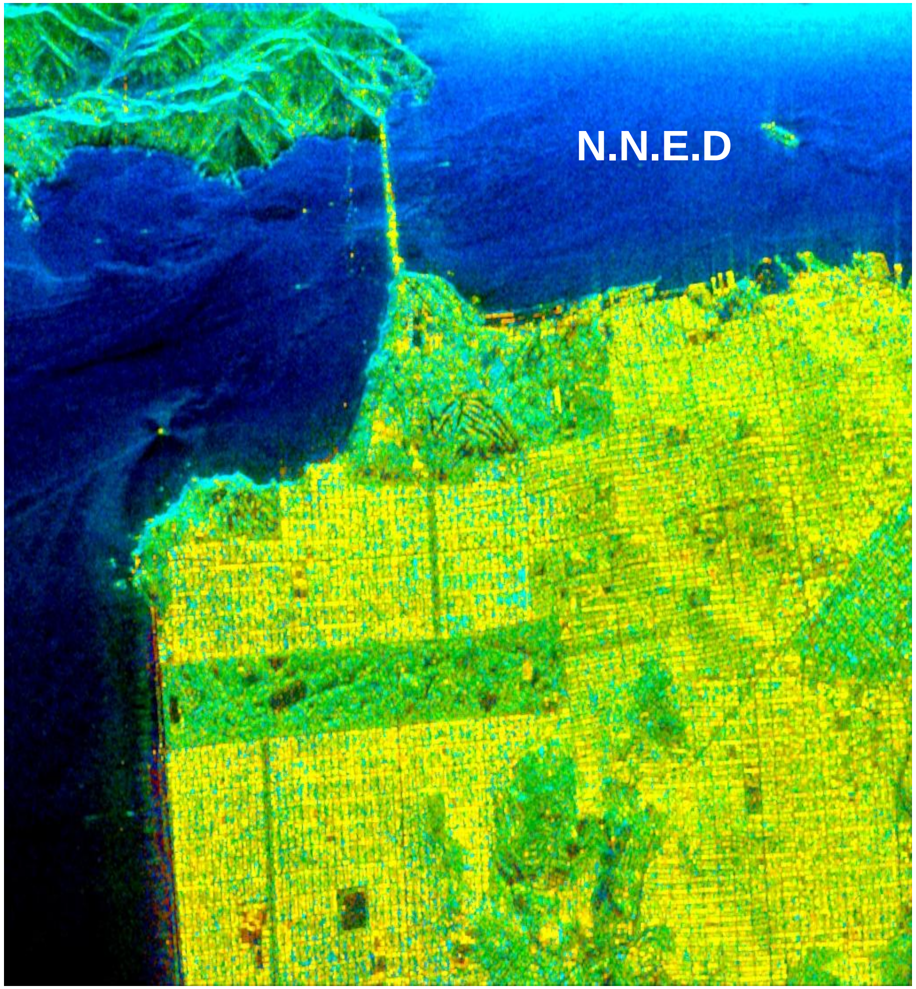
MODEL BASED DECOMPOSITION

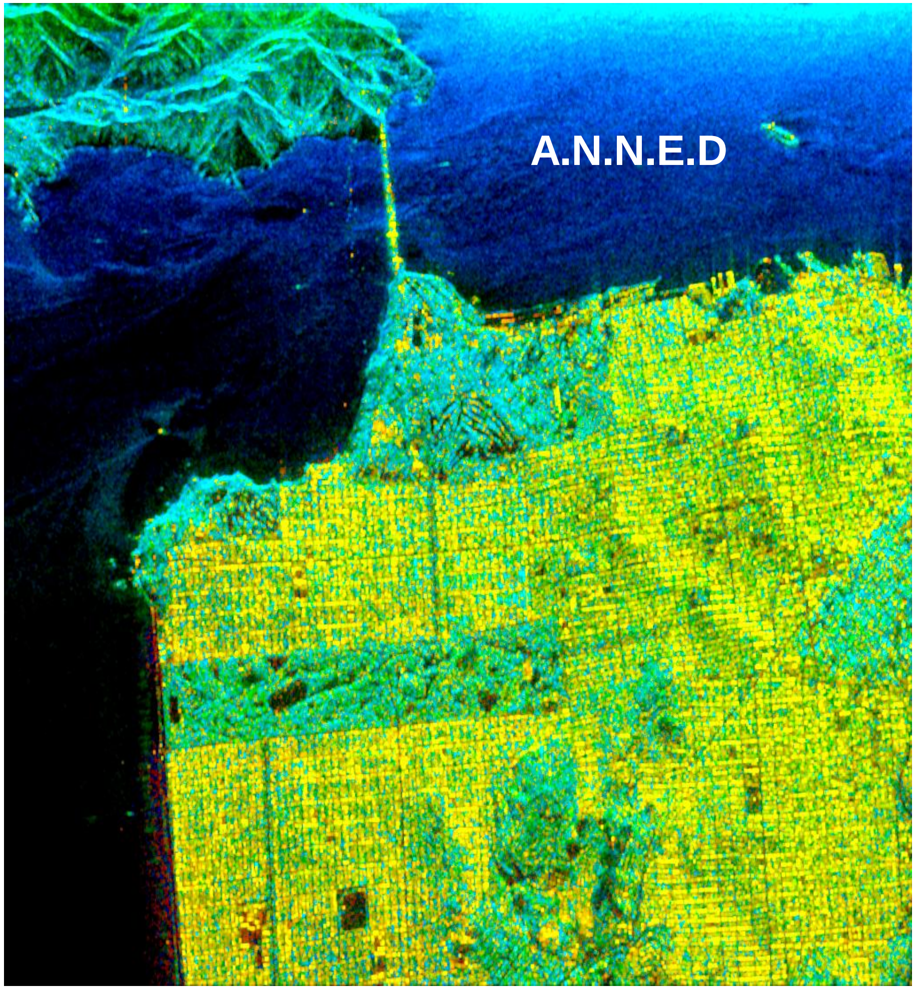


$2A_0$ $B_0 + B$ $B_0 - B$

ODD *DBL* *VOL*

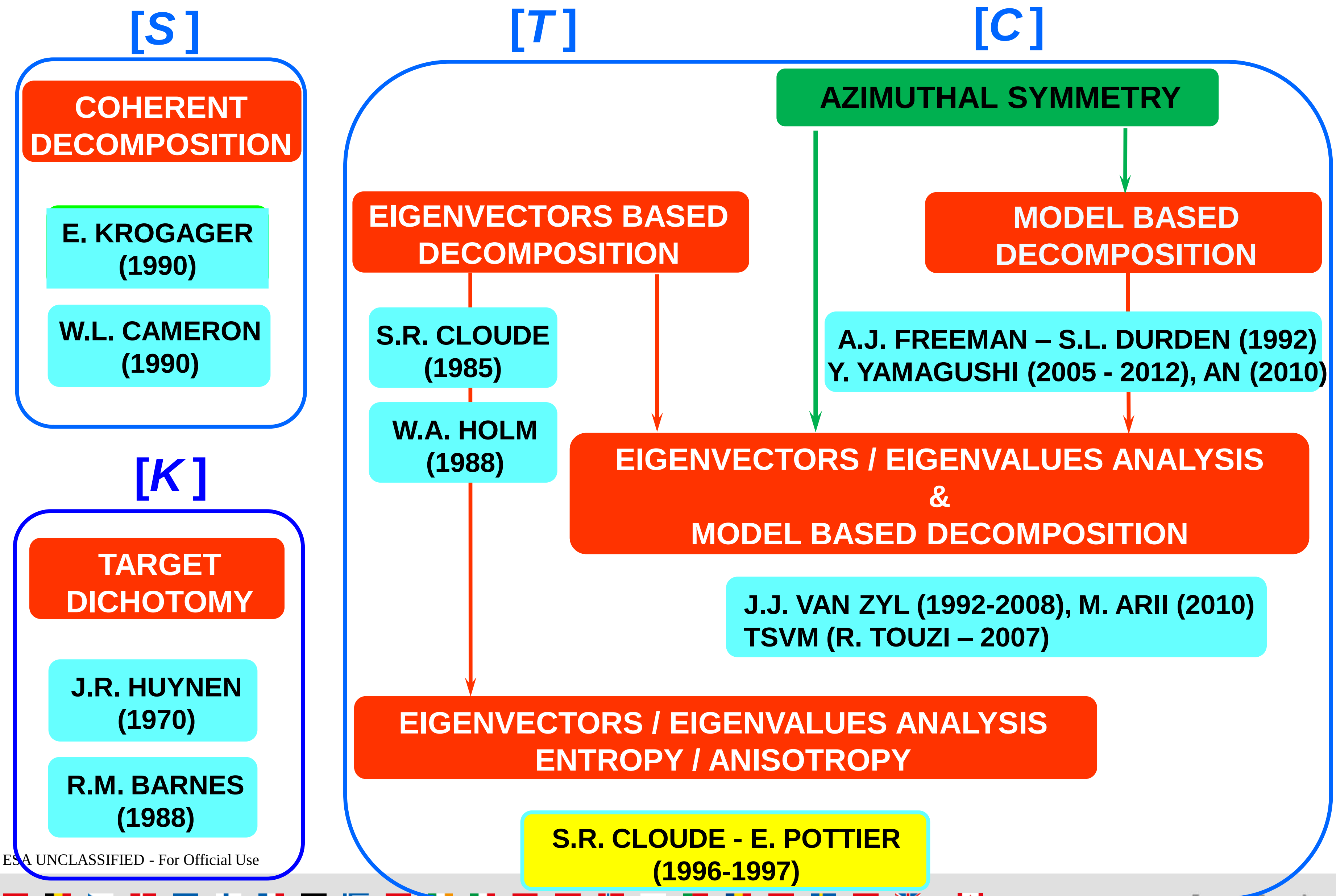
MODEL BASED DECOMPOSITION





ODD DBL VOL

TARGET DECOMPOSITIONS



H / A / α DECOMPOSITION

TARGET VECTOR

$$\underline{k} = \frac{1}{\sqrt{2}}\left[S_{XX} + S_{YY} \quad S_{XX} - S_{YY} \quad 2S_{XY}\right]^T$$

LOCAL ESTIMATE OF THE COHERENCY MATRIX

$$\langle[T]\rangle = \frac{1}{N}\sum_{i=1}^{N}\underline{k}_i \cdot \underline{k}_i^{*T} = \frac{1}{N}\sum_{i=1}^{N}[T_i]$$

EIGENVECTORS / EIGENVALUES ANALYSIS

$$\langle[T]\rangle = [U_3][\Sigma][U_3]^{-1} = \begin{bmatrix} \underline{u}_1 & \underline{u}_2 & \underline{u}_3 \end{bmatrix}\begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}\begin{bmatrix} \underline{u}_1 & \underline{u}_2 & \underline{u}_3 \end{bmatrix}^{*T}$$

ORTHOGONAL EIGENVECTORS

REAL EIGENVALUES
$\lambda_1 > \lambda_2 > \lambda_3$

$$P_i = \frac{\lambda_i}{\sum_{k=1}^{3}\lambda_k}$$

S. Allain

S.E.R.D and D.E.R.D PARAMETERS
(*Single- and Double-bounce Eigenvalue Relative Difference*)

$$SERD = \frac{\lambda_S - \lambda_{3_{NOS}}}{\lambda_S + \lambda_{3_{NOS}}} \qquad DERD = \frac{\lambda_D - \lambda_{3_{NOS}}}{\lambda_D + \lambda_{3_{NOS}}}$$

T. Ainsworth

POLARIZATION FRACTION

$$PF = 1 - \frac{3\lambda_3}{Span} = 1 - \frac{3\lambda_3}{\lambda_1 + \lambda_2 + \lambda_3} \qquad 0 \leq PF \leq 1$$

POLARIZATION ASYMMETRY

$$PA = \frac{(\lambda_1 - \lambda_3) - (\lambda_2 - \lambda_3)}{(\lambda_1 - \lambda_3) + (\lambda_2 - \lambda_3)} = \frac{\lambda_1 - \lambda_2}{Span - 3\lambda_3} \qquad 0 \leq PA \leq 1$$

EIGENVALUE-BASED PARAMETERS

J. Van Zyl

RADAR VEGETATION INDEX

$$RVI = \frac{4\lambda_3}{\lambda_1 + \lambda_2 + \lambda_3} \qquad 0 \le RVI \le \frac{4}{3}$$

S.L. Durden

PEDESTAL HEIGHT

$$PH = \frac{min(\lambda_1, \lambda_2, \lambda_3)}{max(\lambda_1, \lambda_2, \lambda_3)} = \frac{\lambda_3}{\lambda_1} \qquad 0 \le PH \le 1$$

E. Luneuburg

TARGET RANDOMNESS

$$p_R = \sqrt{\frac{3}{2}}\sqrt{\frac{\lambda_2^2 + \lambda_3^2}{\lambda_1^2 + \lambda_2^2 + \lambda_3^2}} \qquad 0 \le p_R \le 1$$

ALTERNATIVE ENTROPY PARAMETERS DERIVATION

J. Praks

E. Colin

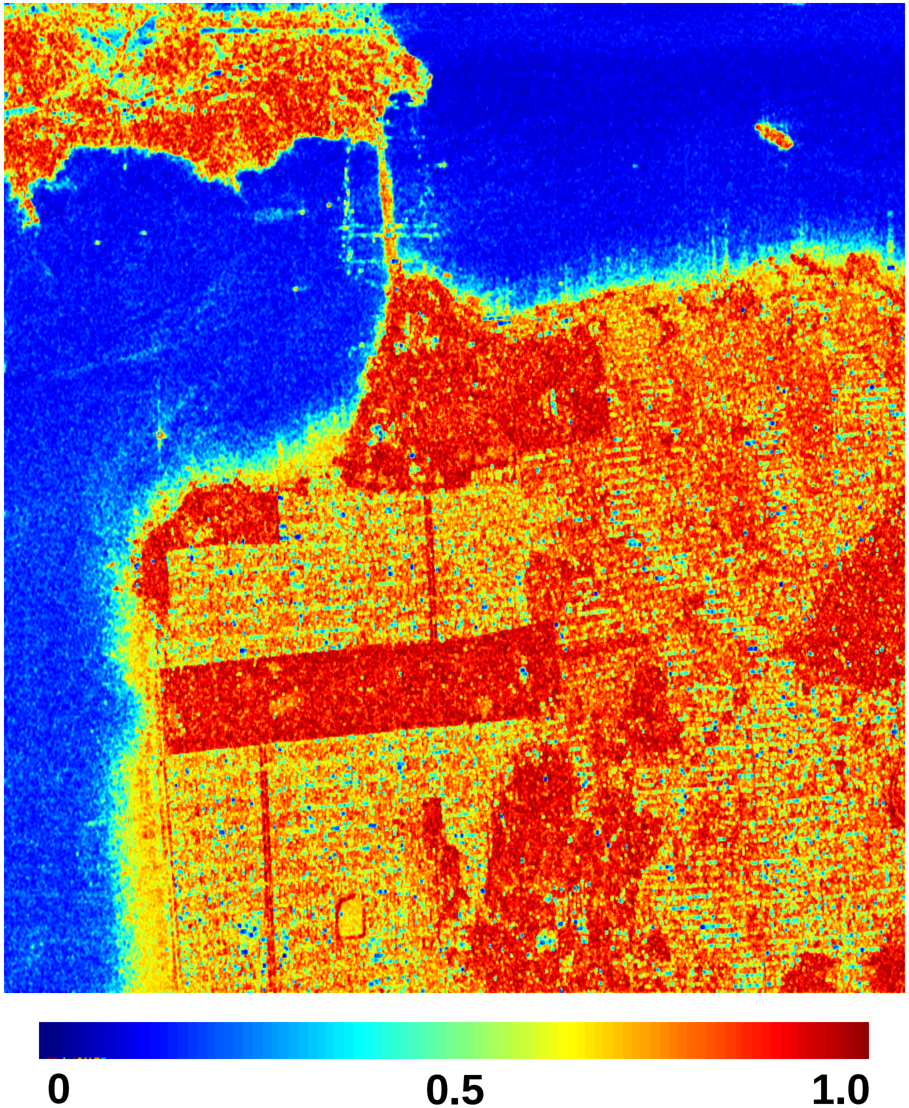
Normalized Coherency Matrix

$$\mathbf{N}_3 = \left\langle \underline{\mathbf{k}}^{T*} \cdot \underline{\mathbf{k}} \right\rangle^{-1} \left\langle \underline{\mathbf{k}} \cdot \underline{\mathbf{k}}^{T*} \right\rangle = \frac{\mathbf{T}_3}{\mathrm{Tr}\left(\mathbf{T}_3\right)}$$

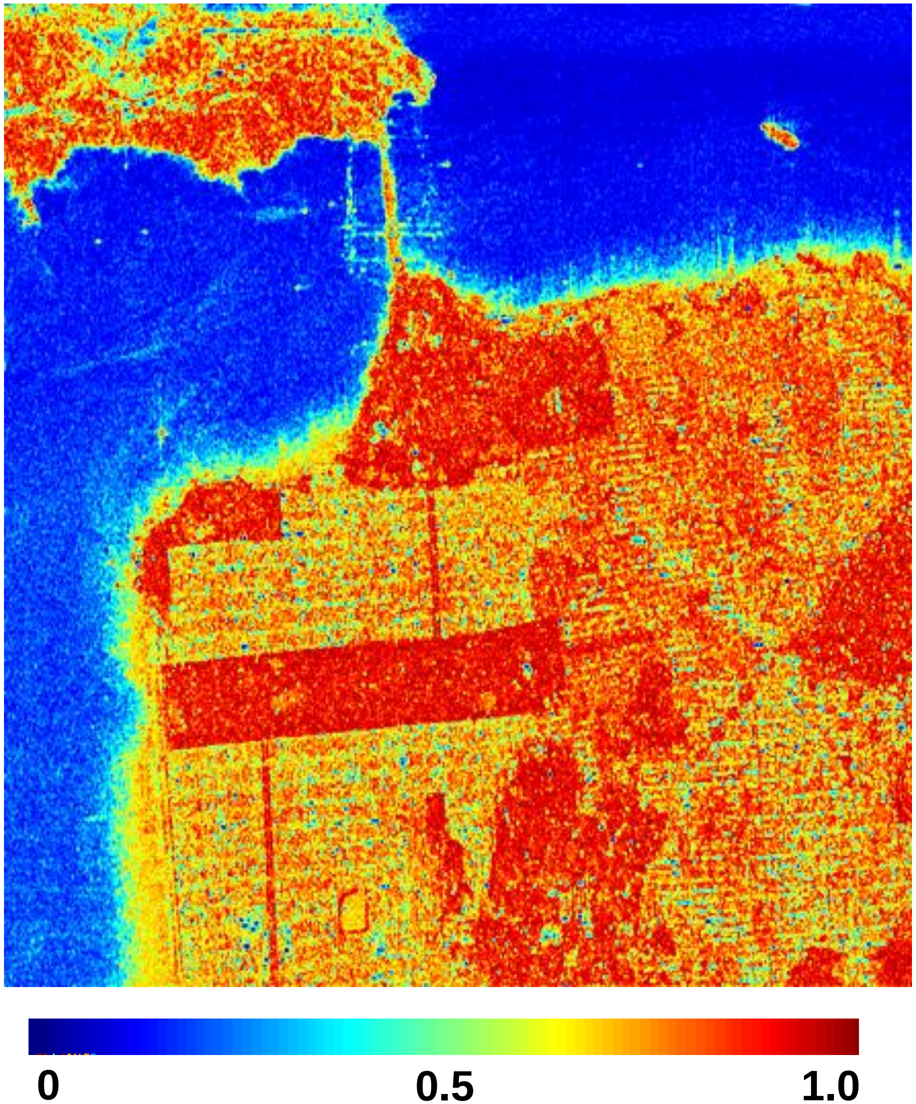
$$H \approx 2.52 + 0.78\, log_3\left(\left| \mathbf{N}_3 + 0.16\,\mathbf{I}_{D3} \right| \right)$$

ENTROPY

EIGENVALUE-BASED PARAMETERS



ENTROPY – Praks Colin



ENTROPY (H)

EIGENVALUE-BASED PARAMETERS

ALTERNATIVE ENTROPY PARAMETERS DERIVATION


J. Praks


E. Colin


J. Morio


F. Réfrégier

Normalized Coherency Matrix

$$\mathbf{N}_3 = \left\langle \underline{\mathbf{k}}^{T*} \cdot \underline{\mathbf{k}} \right\rangle^{-1} \left\langle \underline{\mathbf{k}} \cdot \underline{\mathbf{k}}^{T*} \right\rangle = \frac{\mathbf{T}_3}{\mathrm{Tr}(\mathbf{T}_3)}$$

$$H \approx 2.52 + 0.78\, log_3\left(\left| \mathbf{N}_3 + 0.16\,\mathbf{I}_{D3} \right| \right)$$

ENTROPY

SHANNON POLARIMETRIC ENTROPY (2006)

$$SE = log\left(\pi^3 e^3 \left|\mathbf{T}_3\right|\right) = SE_I + SE_P$$

$$SE_I = 3\, log\left(\frac{\pi\, e\, I_T}{3}\right) = 3\, log\left(\frac{\pi\, e\, \mathrm{Tr}(\mathbf{T}_3)}{3}\right)$$

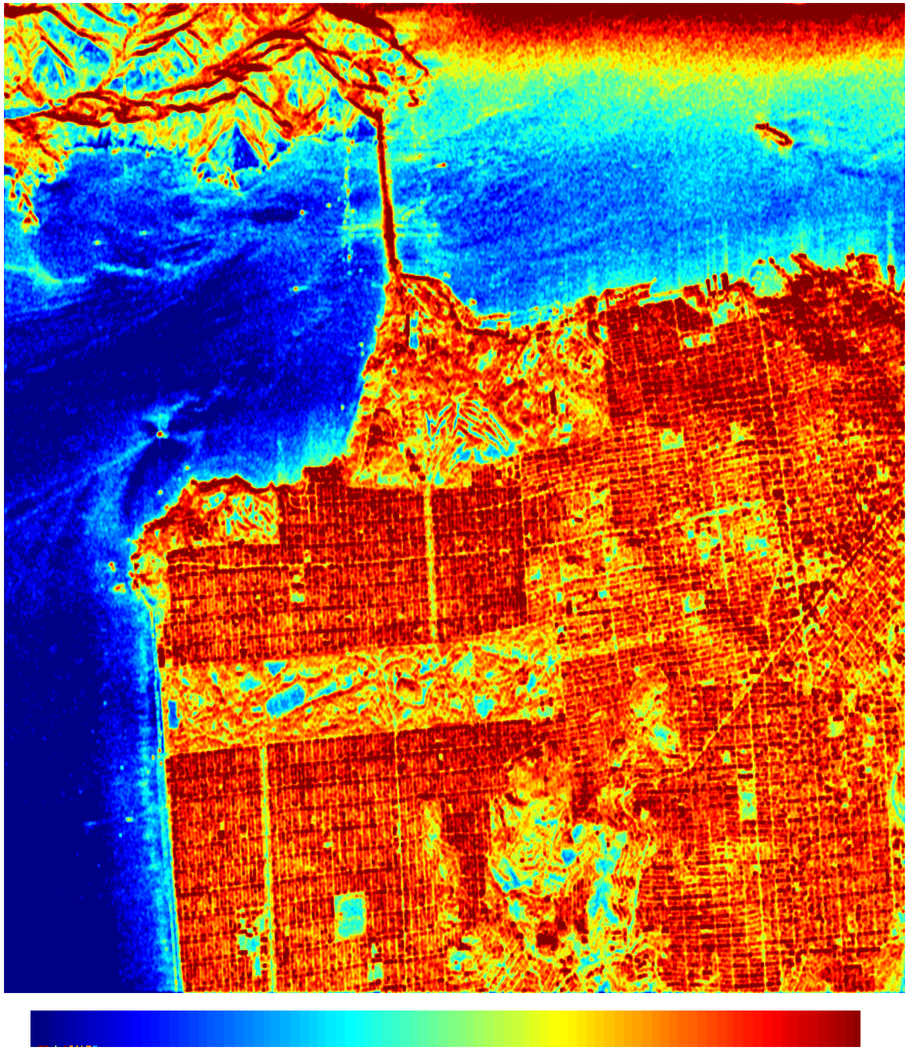
INTENSITY

$$SE_P = log\left(1 - p_T^2\right) = log\left(27 \frac{\left|\mathbf{T}_3\right|}{\mathrm{Tr}(\mathbf{T}_3)^3}\right)$$

DEGREE OF POLARIZATION

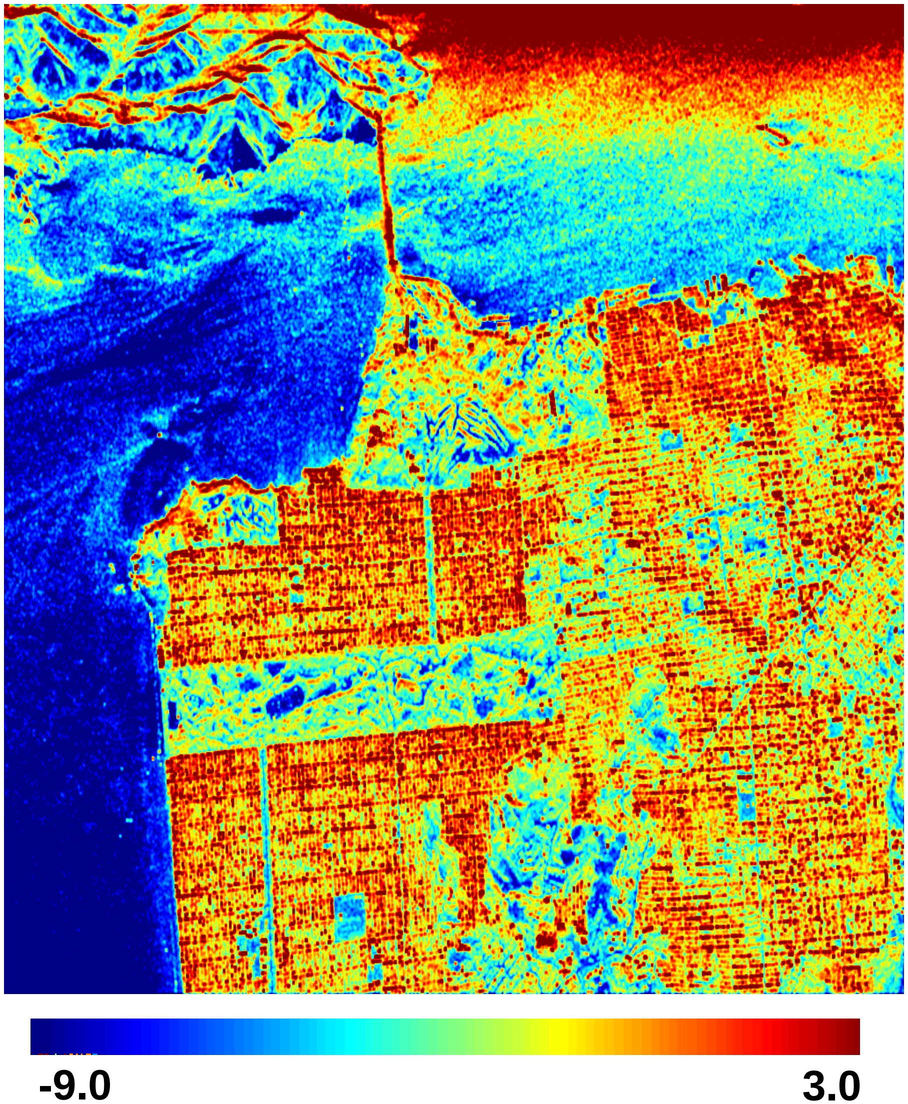
EIGENVALUE-BASED PARAMETERS

$2A_0$ $B_0 + B$ $B_0 - B$

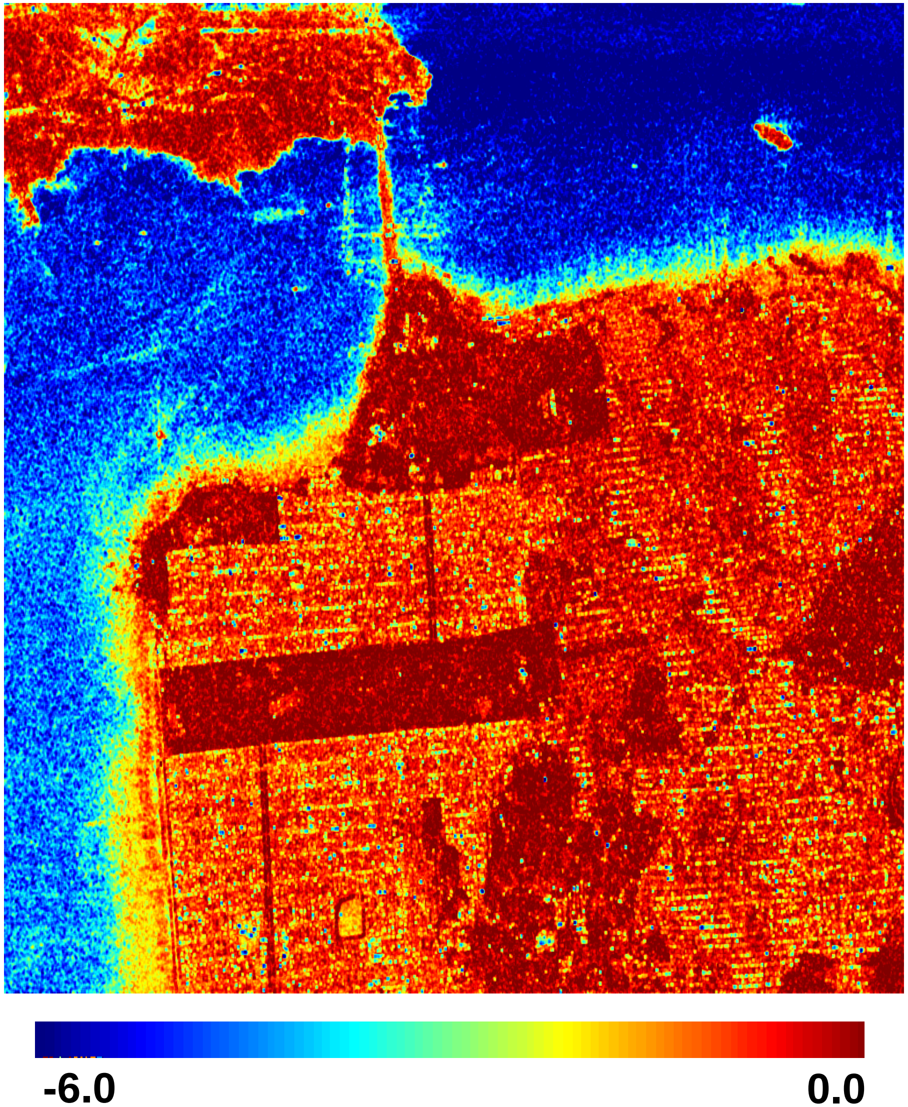


SHANNON ENTROPY (SE-norm)

EIGENVALUE-BASED PARAMETERS



SHANNON ENTROPY (SE-I)



SHANNON ENTROPY (SE-P)

TARGET SCATTERING VECTOR

MODEL DECOMPOSITION

(2007)

T.S.V.M DECOMPOSITION

$$\langle [T] \rangle = [U_3][\Sigma][U_3]^{-1} = \begin{bmatrix} \underline{u}_1 & \underline{u}_2 & \underline{u}_3 \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \begin{bmatrix} \underline{u}_1 & \underline{u}_2 & \underline{u}_3 \end{bmatrix}^{*T}$$

ORTHOGONAL EIGENVECTORS

REAL EIGENVALUES $\lambda_1 > \lambda_2 > \lambda_3$

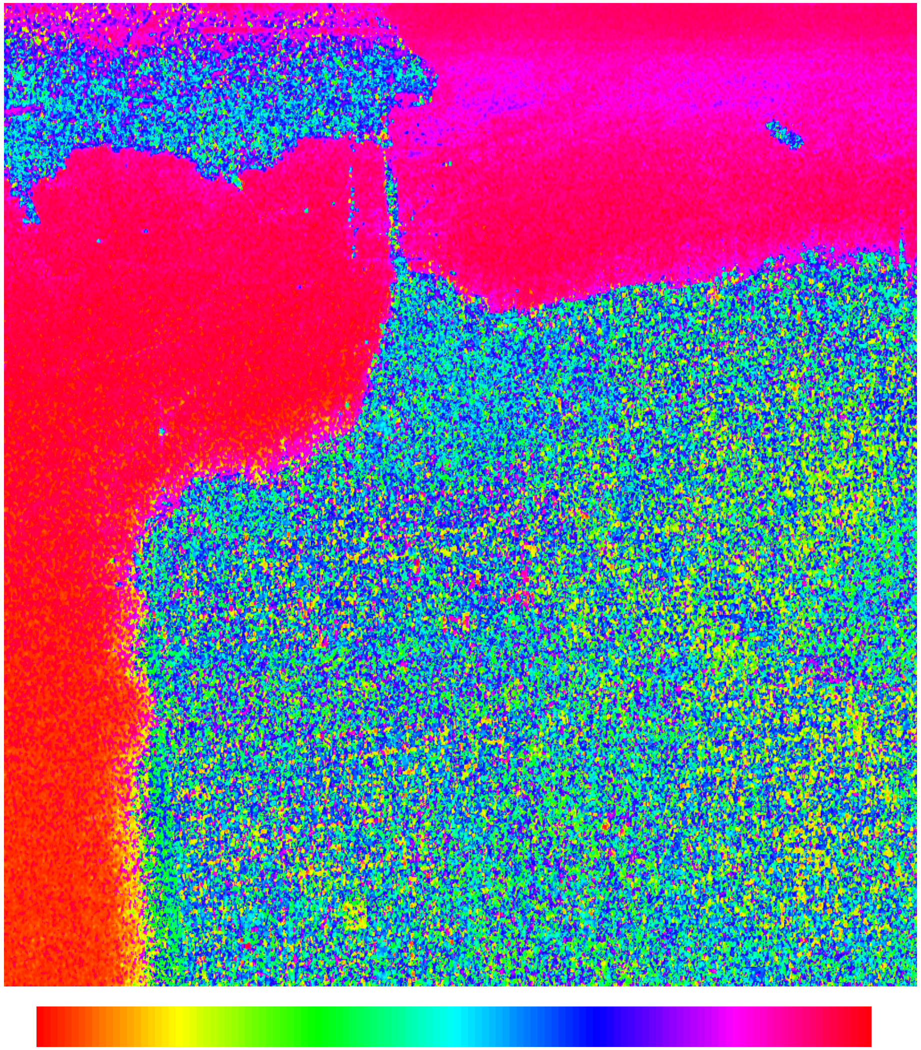
PARAMETERISATION OF THE EIGENVECTOR

$$\begin{bmatrix} \cos\alpha\, e^{j\phi} \\ \sin\alpha \cos\beta\, e^{j\phi} e^{j\delta} \\ \sin\alpha \sin\beta\, e^{j\phi} e^{j\gamma} \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 2\psi & -\sin 2\psi \\ 0 & \sin 2\psi & \cos 2\psi \end{bmatrix} \begin{bmatrix} \cos\alpha_s \cos 2\tau_m \\ \sin\alpha_s\, e^{j\phi_{\alpha_s}} \\ -j\cos\alpha_s \sin 2\tau_m \end{bmatrix}$$

ψ : Target Orientation τ_m : Target Helicity

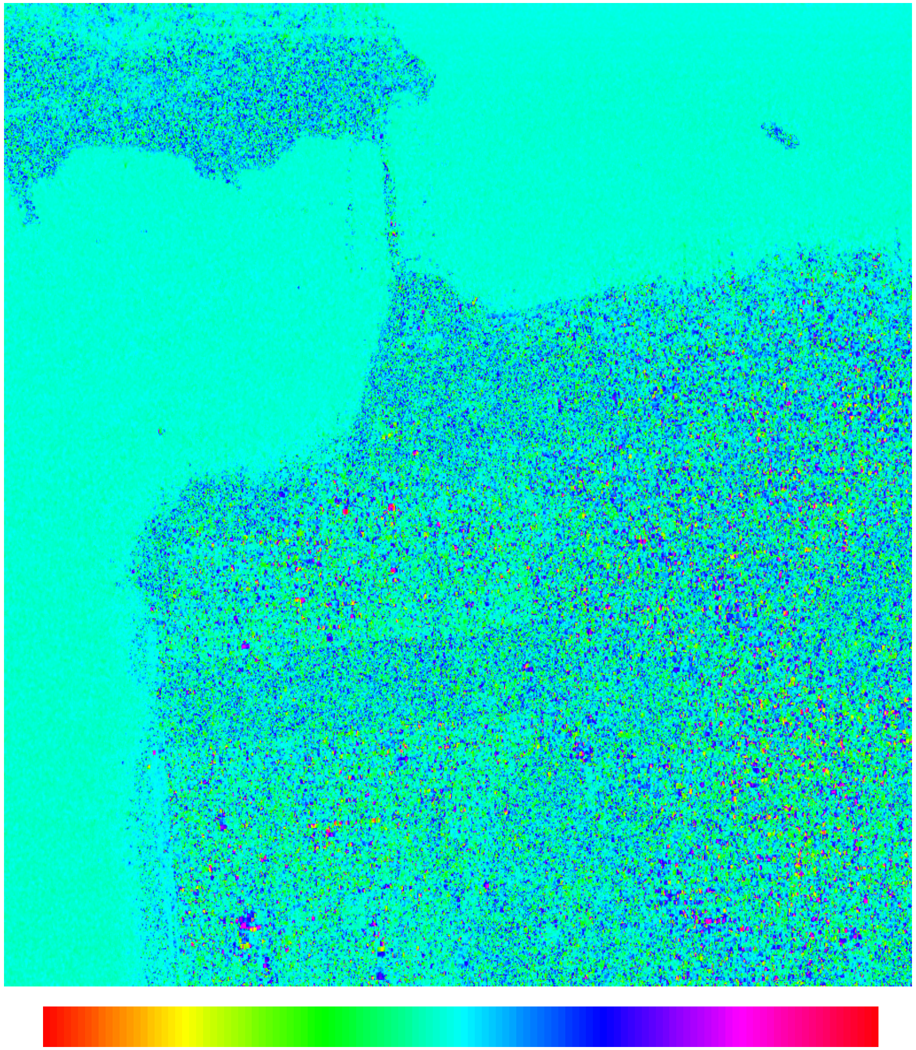
α_s , ϕ_{α_s} : Symmetric scattering type vector parameters

T.S.V.M DECOMPOSITION



-90 0 +90

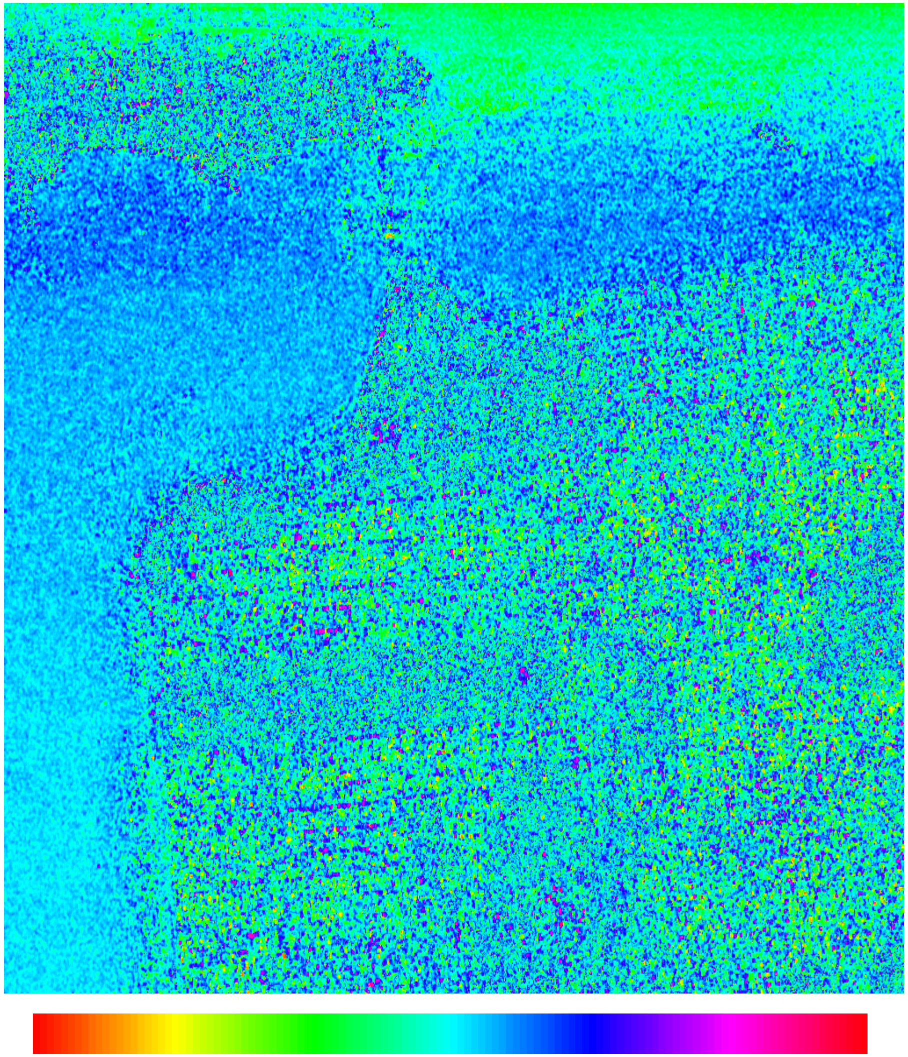
Target Orientation (ψ)



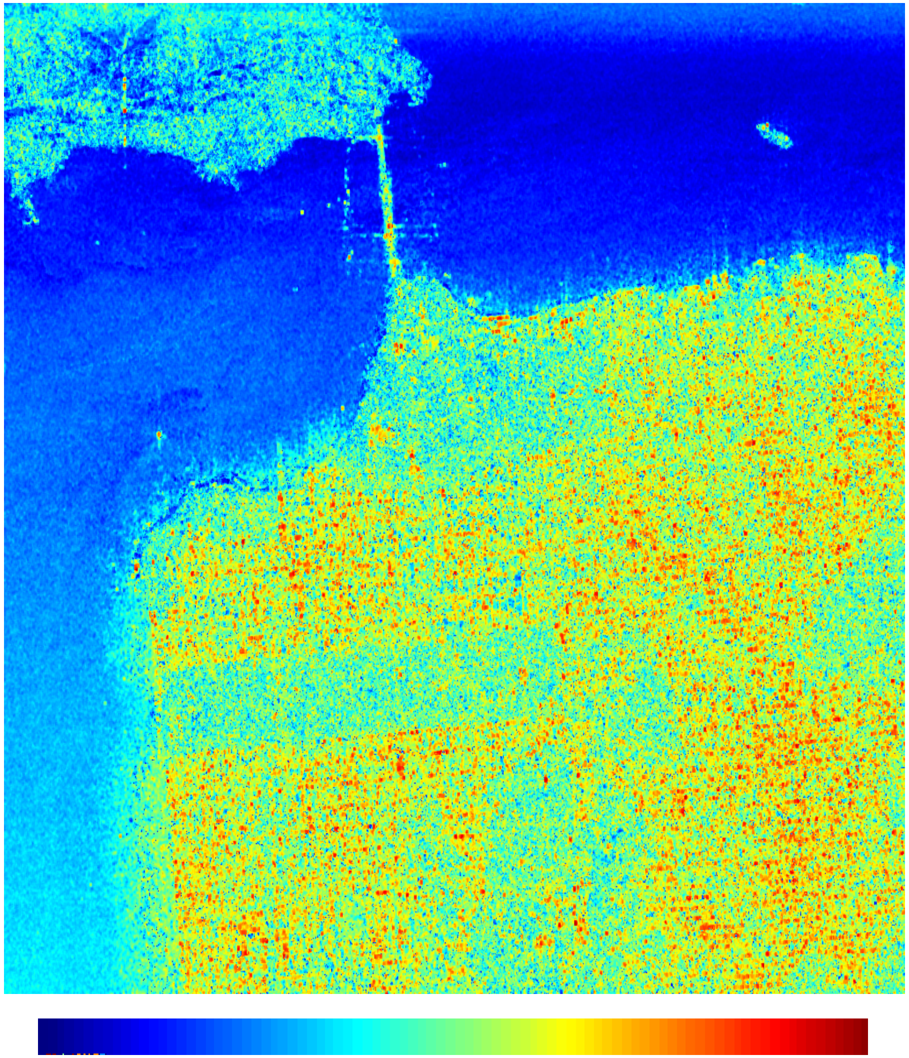
-45 0 +45

Target Helicity (τ_m)

T.S.V.M DECOMPOSITION



(ϕ_{α_s})



(α_s)

H / A / $\underline{\alpha}$ DECOMPOSITION

ENTROPY

$$H = -\sum_{i=1}^{3} P_i \log_3(P_i)$$

$\underline{\alpha}$ PARAMETER

$$\underline{\alpha} = P_1\alpha_1 + P_2\alpha_2 + P_3\alpha_3$$

ANISOTROPY

$$A = \frac{\lambda_2 - \lambda_3}{\lambda_2 + \lambda_3}$$

3 ROLL INVARIANT PARAMETERS

$$\underline{I} = \begin{bmatrix} \underline{\alpha} \\ HA \\ H(1-A) \\ (1-H)A \\ (1-H)(1-A) \end{bmatrix}$$

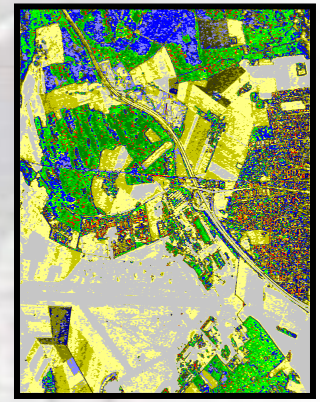
→ **PHYSICAL SCATTERING MECHANISM**

→ **TYPE OF SCATTERING PROCESS**

SEGMENTATION / CLASSIFICATION

POLARIMETRIC REMOTE SENSING

POL-SAR PROCESSING
PHENOMENOLOGIC
QUALITATIVE ANALYSIS



POLARIMETRIC SPECKLE FILTERING → **POLARIMETRIC TARGET DECOMPOSITION** → **POLARIMETRIC CLASSIFICATION**
MONO/DUAL CHANNELS

PolSAR TERRAIN and LAND-USE CLASSIFICATION

J.S. Lee, M.R. Grunes, E. Pottier, L. Ferro-Famil, "Unsupervised terrain classification preserving scattering characteristics," IEEE Transactions on Geoscience and Remote Sensing,vol. 42, no.4, pp. 722-731, April, 2004.

J.S. Lee, M. R. Grunes and E. Pottier, "Quantitative Comparison of Classification Capability: Fully polarimetric versus Dual- and Single polarization SAR," IEEE TGRS, November 2002

E. Pottier and J.S. Lee, "Application of the « H / A / $\underline{\alpha}$ » polarimetric decomposition theorem for unsupervised classification of fully polarimetric SAR data based on the Wishart distribution" Proceedings of EUSAR2000

J.S. Lee, M.R. Grunes, T.L. Ainsworth, L. Du, D.L. Schuler, and S.R. Cloude, " Unsupervised Classification of Polarimetric SAR Imagery Based on Target Decomposition and Wishart Distribution," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 37, no. 5, 2249-2258, September 1999.

J.S. Lee, M. R. Grunes and R. Kwok," Classification of Polarimetric SAR Images Based on the Complex Wishart Distribution," ***Int. J. Remote Sensing, vol.32, No. 5, Sept. 1994.***

J.S. Lee, E. Pottier, *Polarimetric Radar Imaging: From Basics to Applications,* Taylor & Francis/CRC, 2009

Target Vector

$$\underline{X} = \begin{bmatrix} S_{HH} & \sqrt{2}S_{HV} & S_{VV} \end{bmatrix}^T \qquad P(\underline{X}) = \frac{1}{\pi^3 \left|[C]\right|} e^{-\underline{X}^{*T}[C]^{-1}\underline{X}}$$

$$\underline{k} = \frac{1}{\sqrt{2}} \begin{bmatrix} S_{HH}+S_{VV} & S_{HH}-S_{VV} & 2S_{HV} \end{bmatrix}^T \qquad P(\underline{k}) = \frac{1}{\pi^3 \left|[T]\right|} e^{-\underline{k}^{*T}[T]^{-1}\underline{k}}$$

Coherency Matrix

$$\langle [T] \rangle = \frac{1}{N} \sum_{i=1}^{N} \underline{k}_i \cdot \underline{k}_i^{*T} = \frac{1}{N} \sum_{i=1}^{N} [T_i]$$

$$P(\langle [T] \rangle / [T_m]) = \frac{L^{Lp} \left| \langle [T] \rangle \right|^{L-p} e^{-LTr\left([T_m]^{-1} \langle [T] \rangle\right)}}{\pi^{\frac{p(p-1)}{2}} \Gamma(L) \ldots \Gamma(L-p+1) \left|[T_m]\right|^{L}}$$

COMPLEX WISHART DISTRIBUTION

L: Number of Look   p: Polarimetric Dimension

WISHART CLASSIFIER

$$P(\langle[T]\rangle/[T_m]) = \frac{L^{Lp}\left|\langle[T]\rangle\right|^{L-p} e^{-LTr\left([T_m]^{-1}\langle[T]\rangle\right)}}{\pi^{\frac{p(p-1)}{2}}\Gamma(L)\ldots\Gamma(L-p+1)\left|[T_m]\right|^{L}}$$

↓

BAYES MAXIMUM LIKELIHOOD CLASSIFICATION PROCEDURE

$$\langle[T]\rangle \in [T_m] \quad if \quad P([T_m]/\langle[T]\rangle) \geq P([T_j]/\langle[T]\rangle) \quad \forall\, j \neq m$$

Applying Bayes rule $$P([T_m]/\langle[T]\rangle) = \frac{P(\langle[T]\rangle/[T_m])}{P(\langle[T]\rangle)} P([T_m])$$

It follows

$$\langle[T]\rangle \in [T_m] \quad if \quad P(\langle[T]\rangle/[T_m])P([T_m]) \geq P(\langle[T]\rangle/[T_j])P([T_j]) \quad \forall\, j \neq m$$

$[T_m]$: Cluster Center of the class *m*

WISHART CLASSIFIER

$$P\left(\langle[T]\rangle / [T_m]\right) = \frac{L^{Lp} \left|\langle[T]\rangle\right|^{L-p} e^{-L Tr\left([T_m]^{-1}\langle[T]\rangle\right)}}{\pi^{\frac{p(p-1)}{2}} \Gamma(L) \ldots \Gamma(L-p+1) \left|[T_m]\right|^{L}}$$

$$\Downarrow$$

BAYES MAXIMUM LIKELIHOOD CLASSIFICATION PROCEDURE

$$\langle[T]\rangle \in [T_m] \quad \text{if} \quad d_m\left(\langle[T]\rangle\right) < d_j\left(\langle[T]\rangle\right) \quad \forall\, j \neq m$$

with

$$d_m\left(\langle[T]\rangle\right) = L Tr\left([T_m]^{-1}\langle[T]\rangle\right) + L \ln\left(\left|[T_m]\right|\right) - \ln\left(P\left([T_m]\right)\right) + K$$

$[T_m]$: Cluster Center of the class *m*

ROBUSTENESS OF WISHART CLASSIFIER

$$d_m(\langle[T]\rangle) = L\,Tr\left([T_m]^{-1}\langle[T]\rangle\right) + L\ln\left(|[T_m]|\right) - \ln\left(P([T_m])\right) + K$$

INDEPENDENT OF # OF LOOKS

INDEPENDENT OF POLARIZATION BASIS

[T] or [C] IDENTICAL CLASSIFICATION RESULTS

INDEPENDENT OF WEIGHTING

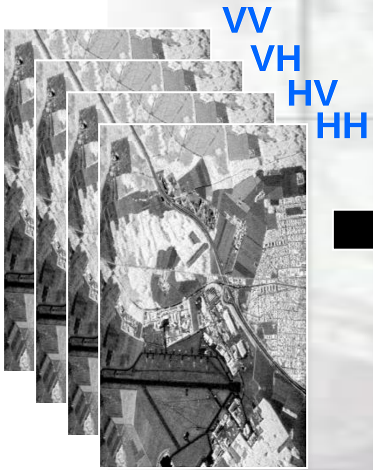
$$u = \begin{bmatrix} S_{hh} \\ \sqrt{2}S_{hv} \\ S_{vv} \end{bmatrix} \qquad u_1 = \begin{bmatrix} S_{hh} \\ S_{hv} \\ S_{vv} \end{bmatrix}$$

For Dual-Pol ($p=2$), PolSAR ($p=3$), Pol-InSAR ($p=6$)

J.S. Lee, E. Pottier, *Polarimetric Radar Imaging: From Basics to Applications,* Taylor & Francis/CRC, 2009

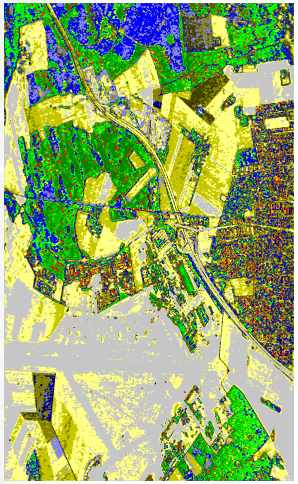
WISHART PDF

$$P\left(\langle[T]\rangle/[T_m]\right)=\frac{L^{Lp}\left|\langle[T]\rangle\right|^{L-p}e^{-LTr\left([T_m]^{-1}\langle[T]\rangle\right)}}{\pi^{\frac{p(p-1)}{2}}\Gamma(L)\ldots\Gamma(L-p+1)\left|[T_m]\right|^{L}}$$



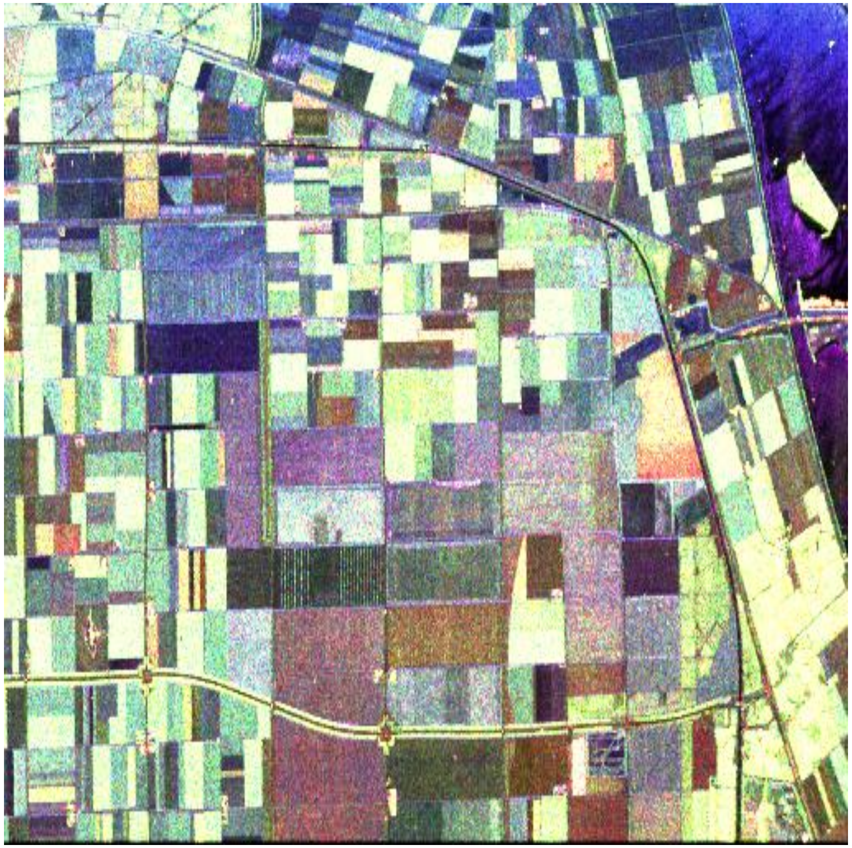
SUPERVISED POLSAR CLASSIFICATION

J.S LEE, M.R GRUNES, E.POTTIER (2002)



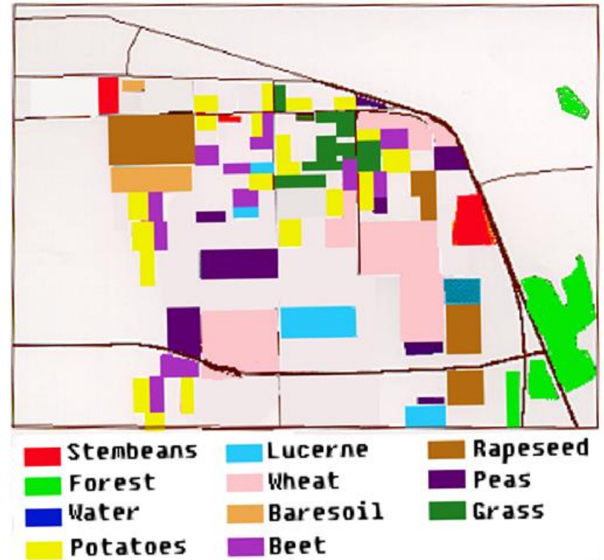
WISHART CLASSIFIER

Courtesy of Dr J.S Lee



$2A_0$ $\quad B_0 + B$ $\quad B_0 - B$

JPL AIRSAR
P-L-C Band Flevoland Data

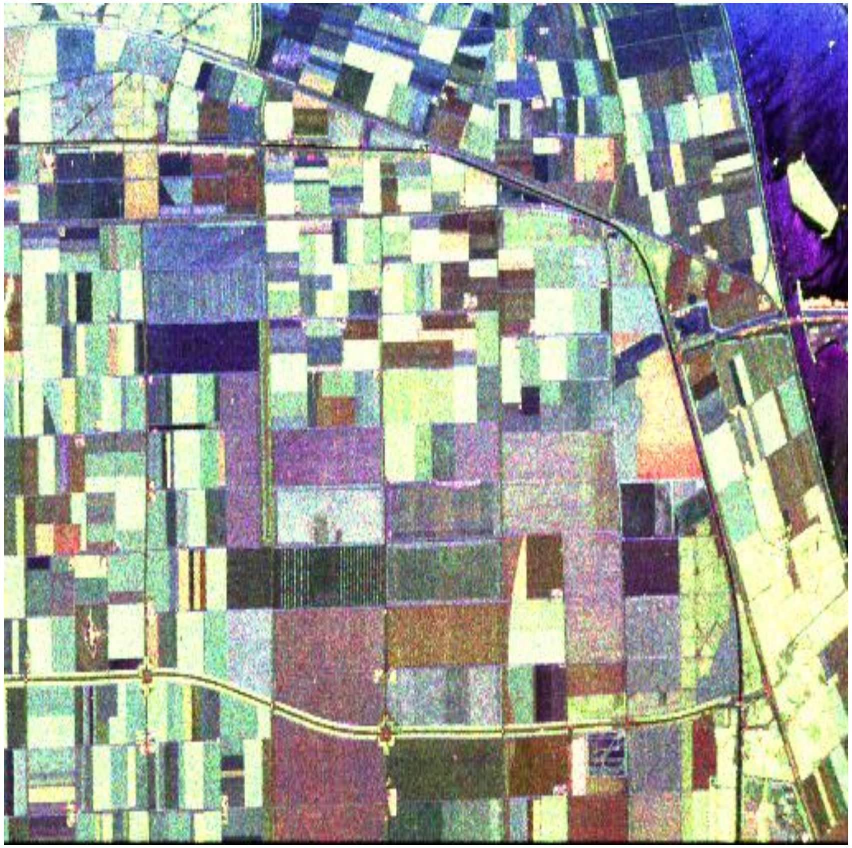


Original Ground- Truth

Training Sets / Reference map

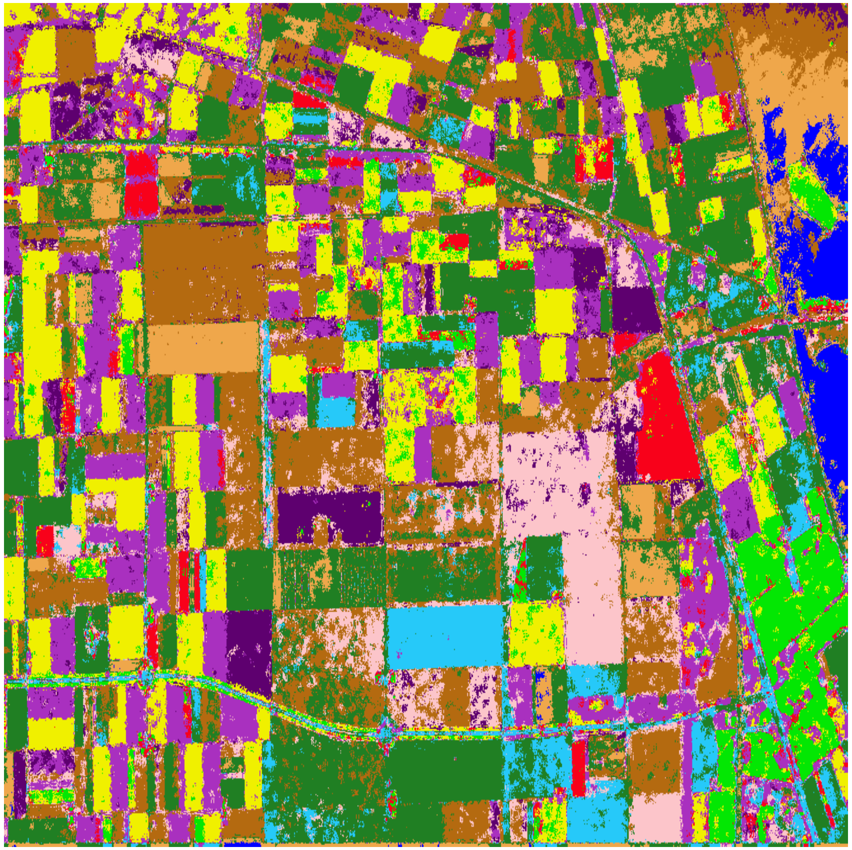
WISHART CLASSIFIER

Courtesy of Dr J.S Lee



$2A_0$ $B_0 + B$ $B_0 - B$

JPL AIRSAR
L-Band Flevoland Data

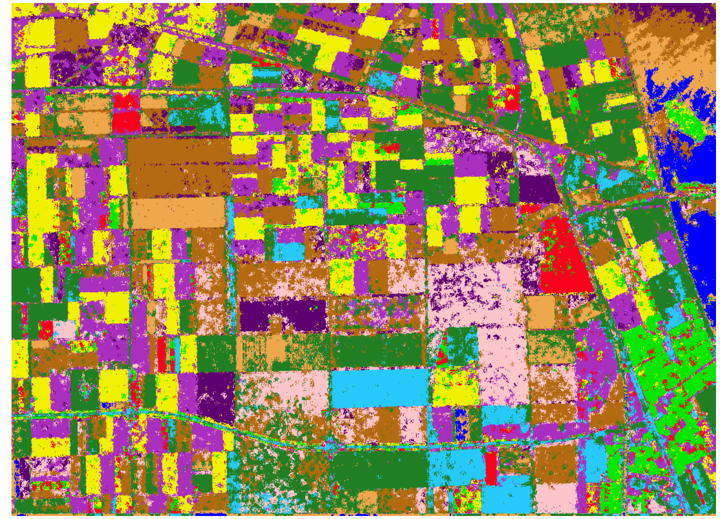


L-band (81.63%)

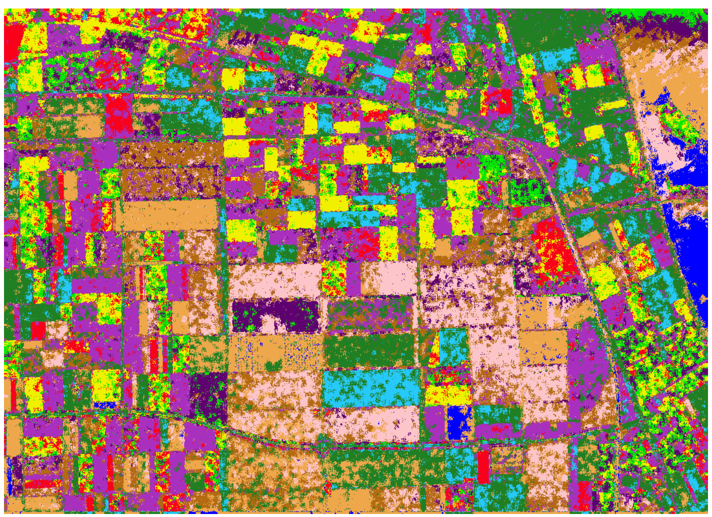
SUPERVISED CLASSIFIER

Courtesy of Dr J.S Lee

L-band Fully Pol. (81.63%)



L-band complex HH and VV (80.91%)



L-band HH and VV Intensities (56.35%)

Reference map for comparison

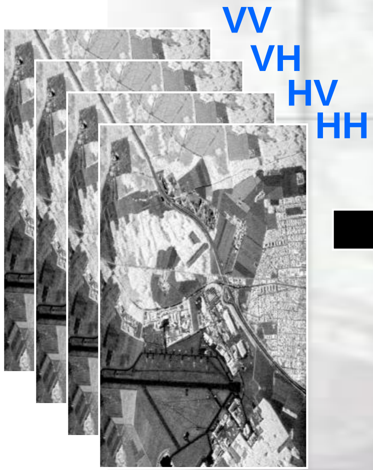
Courtesy of Dr J.S Lee

	Fully Polarimetric	Complex HH, HV	Intensity HH, HV	Complex HH, VV	Intensity HH, VV	Complex VV, HV	Intensity VV, HV
Stem Bean	95.32	51.16	63.27	90.64	61.73	35.97	31.29
Forest	81.07	66.73	68.39	75.75	33.83	60.05	60.91
Potatoes	82.89	67.53	66.36	81.52	49.35	54.40	59.15
Lucerne	97.91	39.29	38.23	99.26	65.15	67.49	65.30
Wheat	64.80	49.77	44.27	68.02	53.72	49.43	41.65
Bare Soil	99.36	90.04	82.86	98.42	93.15	90.93	63.74
Beet	89.26	68.80	66.36	86.22	81.98	75.94	74.77
Rape Seed	89.05	55.01	53.23	87.18	49.85	82.31	77.12
Peas	86.47	50.77	39.25	84.59	65.21	81.82	79.59
Grass	91.05	66.44	65.06	90.13	71.08	75.36	75.19
Water	100	90.39	87.33	100	99.86	96.30	70.53
TOTAL	81.63	59.16	55.38	80.91	56.35	64.72	60.12

L-Band Crop Classification Results

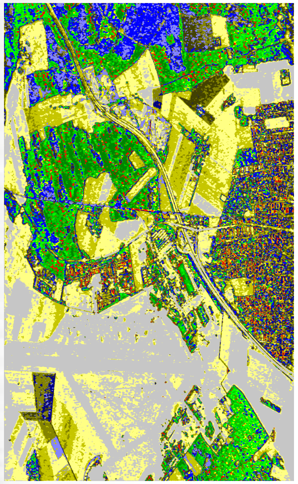
WISHART PDF

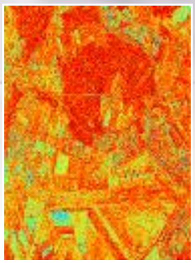
$$P\left(\langle[T]\rangle/[T_m]\right)=\frac{L^{Lp}\left|\langle[T]\rangle\right|^{L-p}e^{-LTr\left([T_m]^{-1}\langle[T]\rangle\right)}}{\pi^{\frac{p(p-1)}{2}}\Gamma(L)\ldots\Gamma(L-p+1)\left|[T_m]\right|^{L}}$$

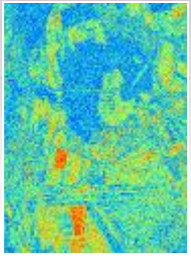


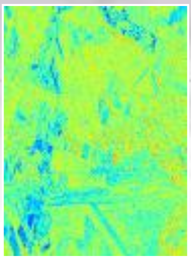
UNSUPERVISED POLSAR CLASSIFICATION

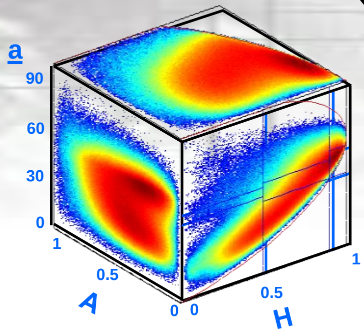
E.POTTIER, J.S LEE (2000)




H


A

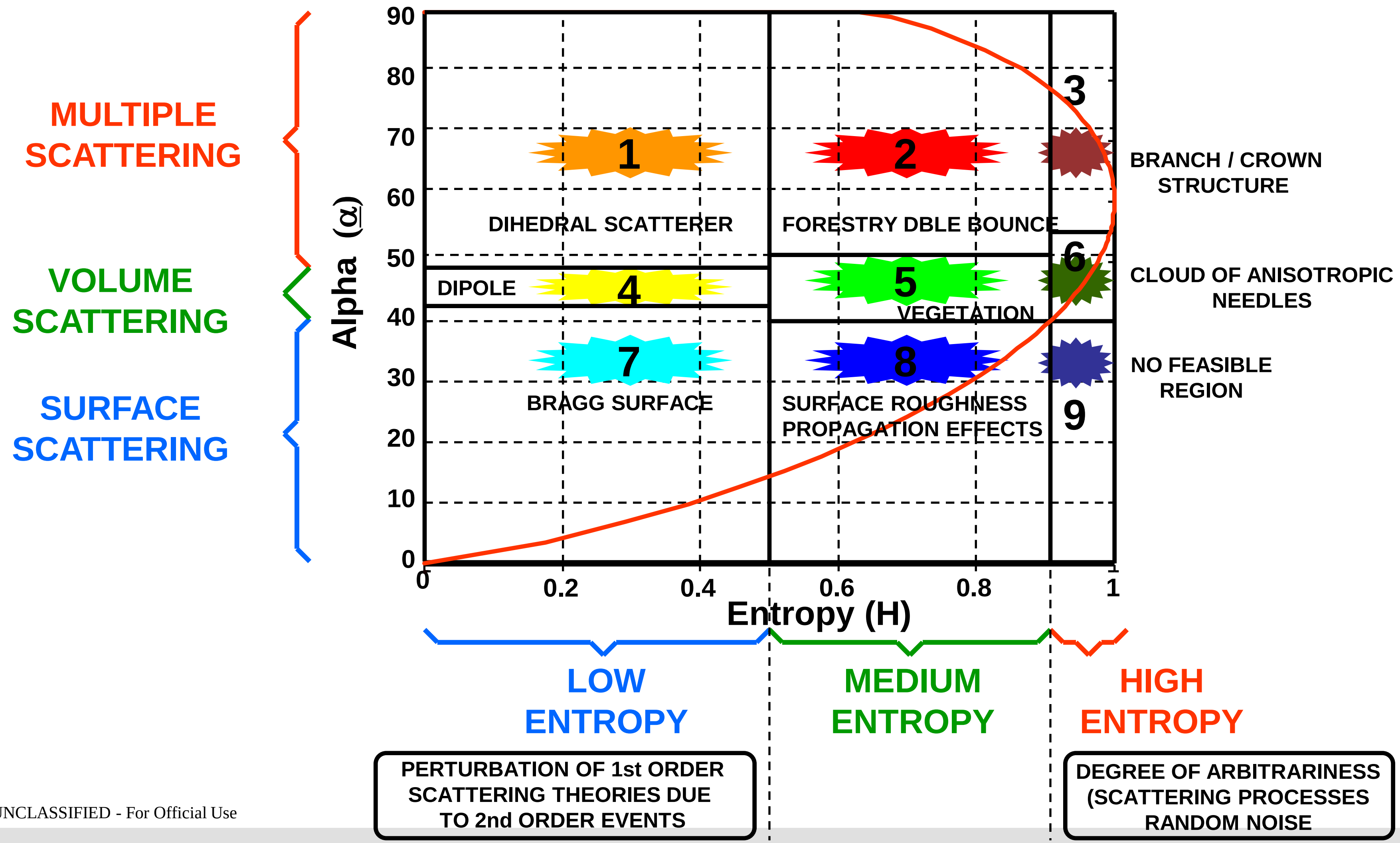

α



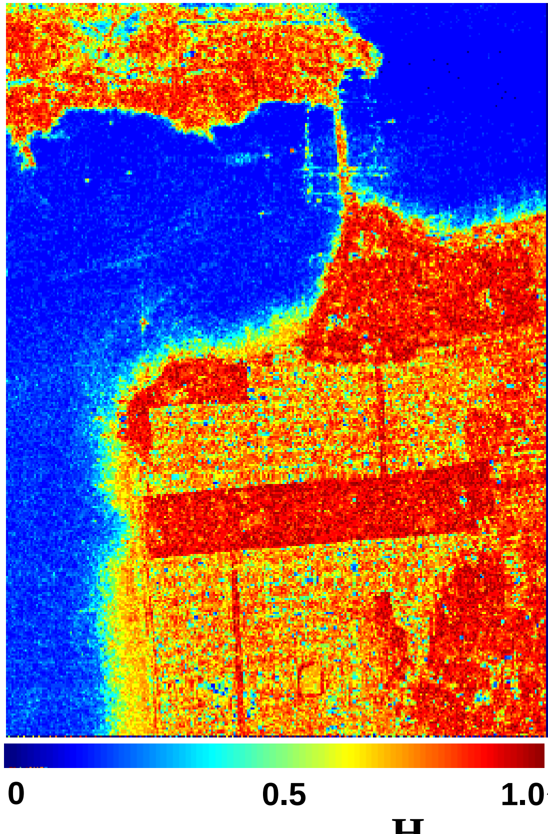
H / A / a DECOMPOSITION THEOREM

H / α CLASSIFICATION

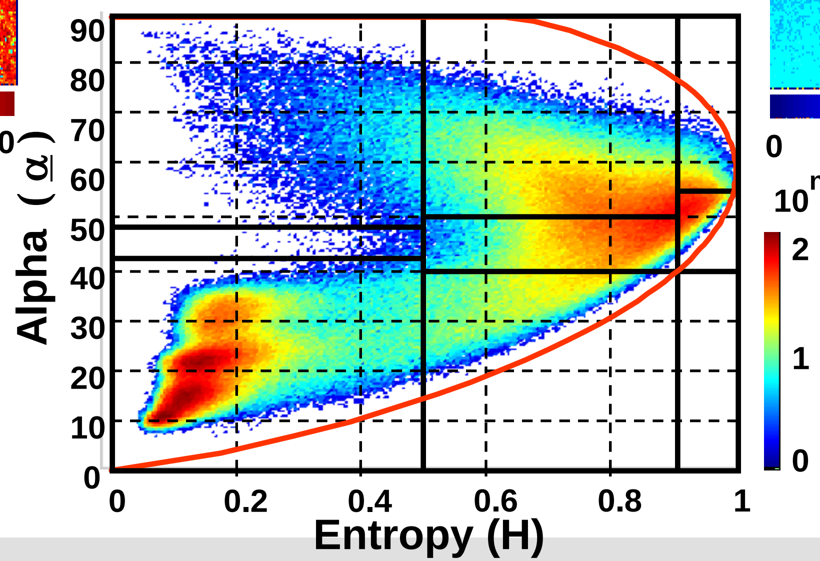
SEGMENTATION OF THE H / α SPACE

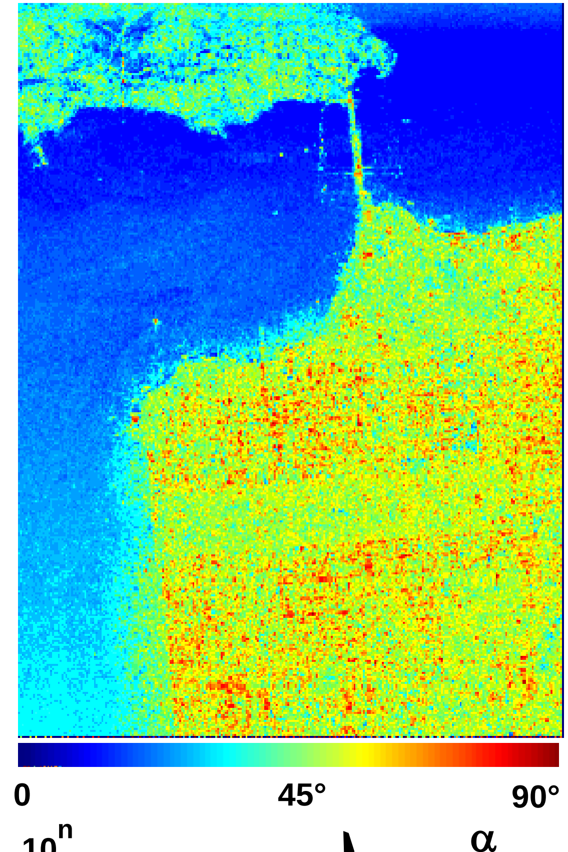


H / α CLASSIFICATION



POLSAR DATA DISTRIBUTION IN THE H / α PLANE

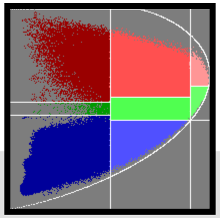




H / $\underline{\alpha}$ CLASSIFICATION

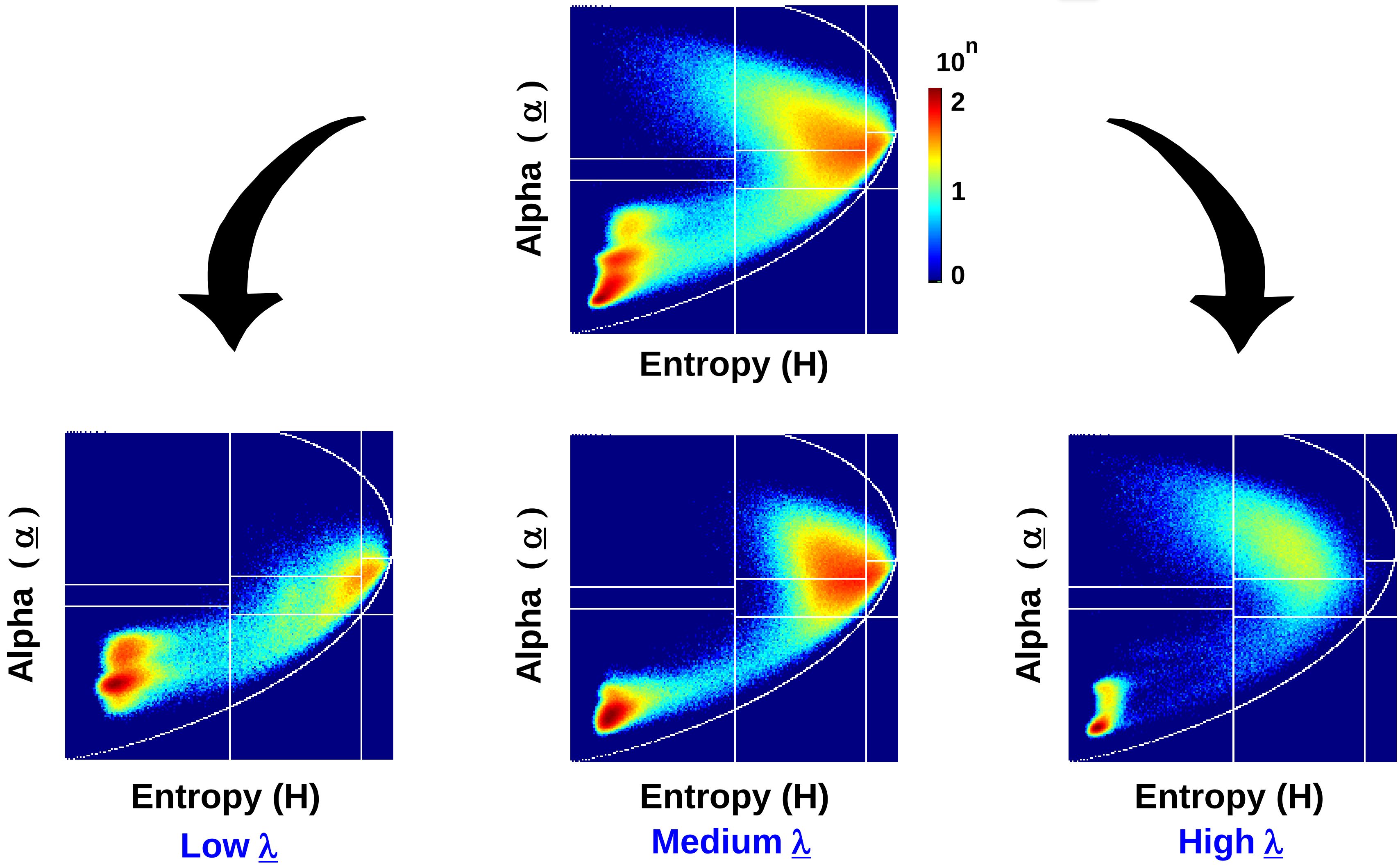
$2A_0$ $\quad B_0 + B$ $\quad B_0 - B$

H - $\underline{\alpha}$ classification



H / α / span CLASSIFICATION

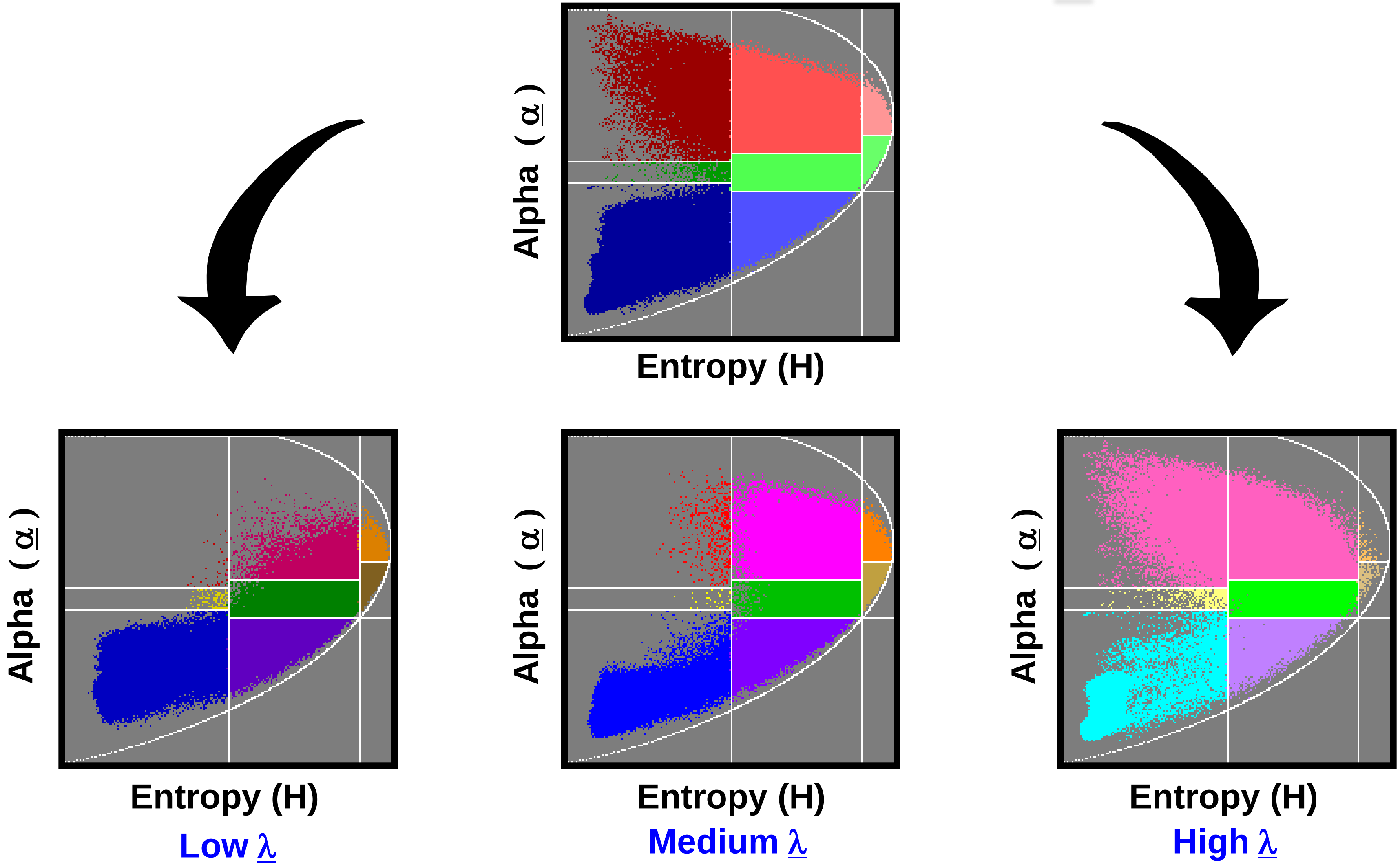
POLSAR DATA DISTRIBUTION IN THE H / α PLANE



Cao Fang, Hong Wen *A New Classification Method Based on Cloude-Pottier Eigenvalue / Eigenvector Decomposition,* IGARSS 05, Seoul, Korea

H / α / span CLASSIFICATION

POLSAR DATA DISTRIBUTION IN THE H / α PLANE

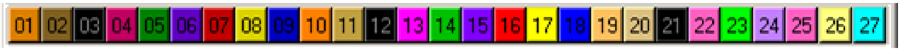


Cao Fang, Hong Wen *A New Classification Method Based on Cloude-Pottier Eigenvalue / Eigenvector Decomposition,* IGARSS 05, Seoul, Korea

H / α / span CLASSIFICATION

esa

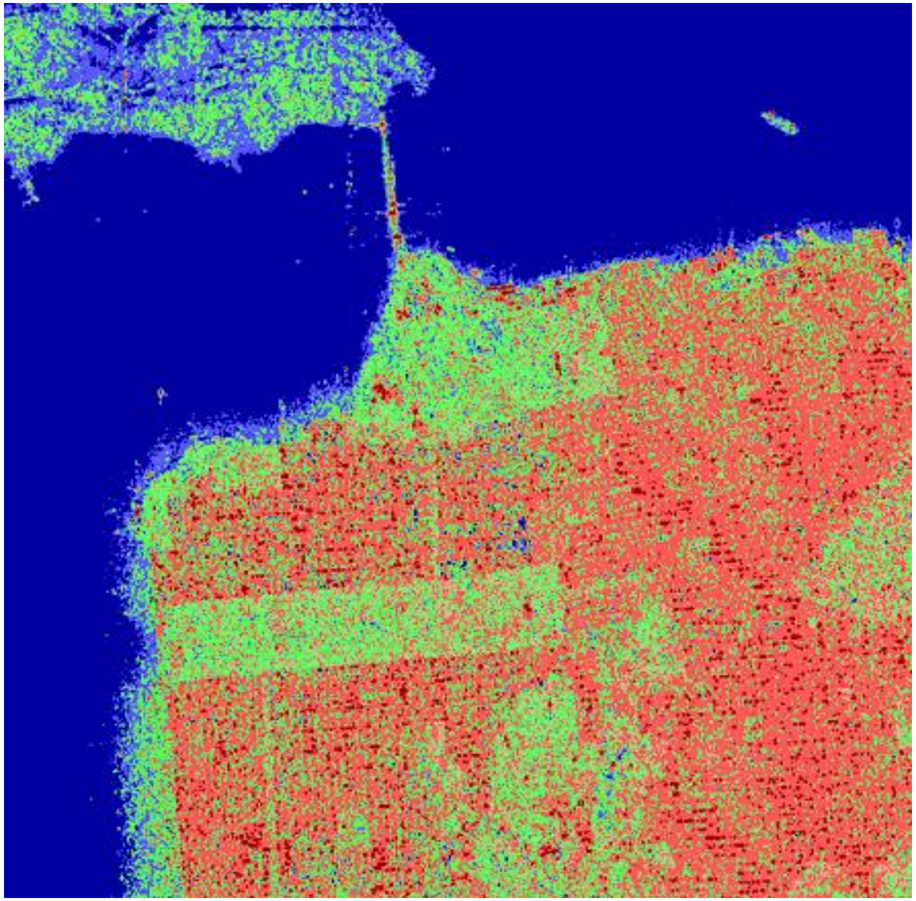
H – α (λ) classification

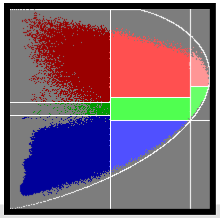


$2A_0$ $B_0 + B$ $B_0 - B$

H / $\underline{\alpha}$ CLASSIFICATION

H-$\underline{\alpha}$ classification





H / $\underline{\alpha}$ Classification Space
Sub-divised into 9 basic zones

↓

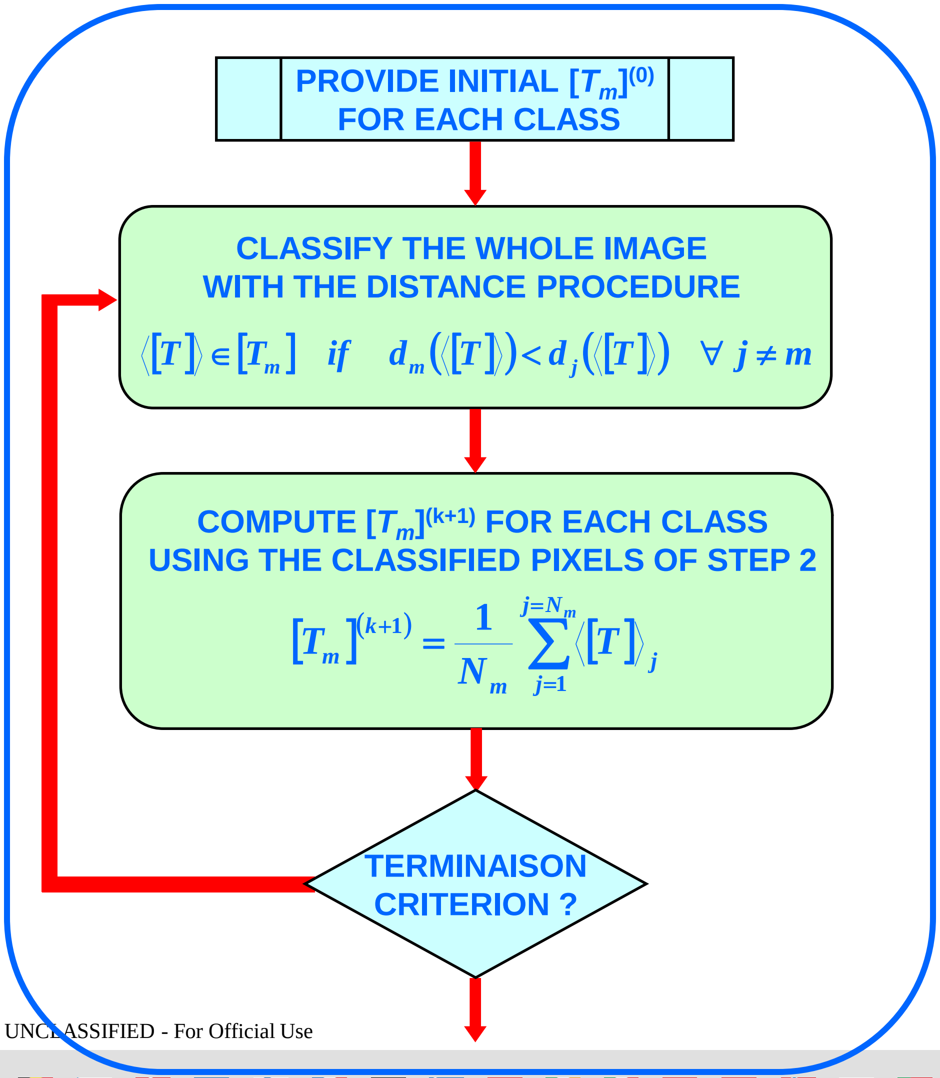
Location of the boundaries is arbitrary and generically

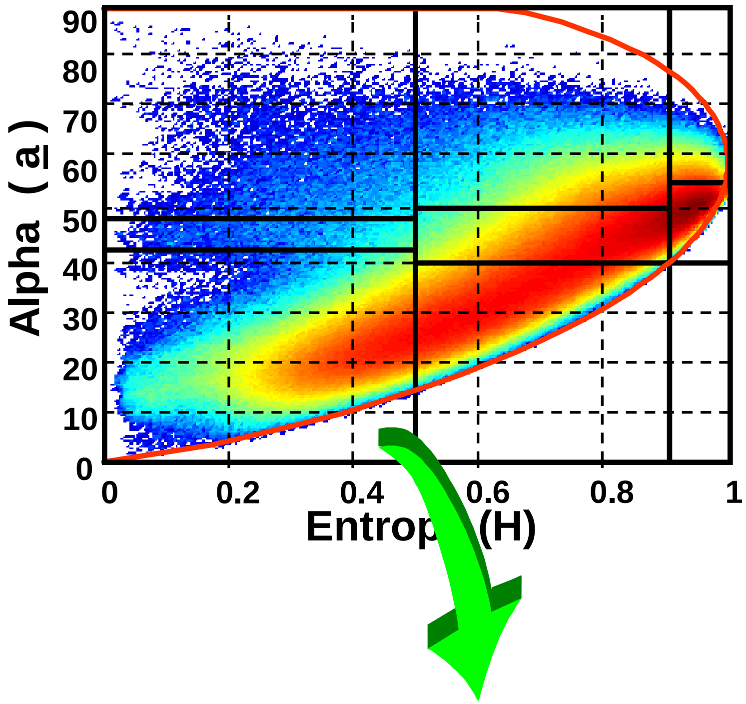
Degree of arbitrariness on the setting of these boundaries

↓

Segmentation is offered merely to illustrate the unsupervised classification strategy and to emphasize the geometrical segmentation of physical scattering processes

k - mean CLASSIFICATION PROCEDURE





$$[T_m]^{(0)} = \frac{1}{N_m} \sum_{k=1}^{k=N_m} \langle[T]\rangle_k$$

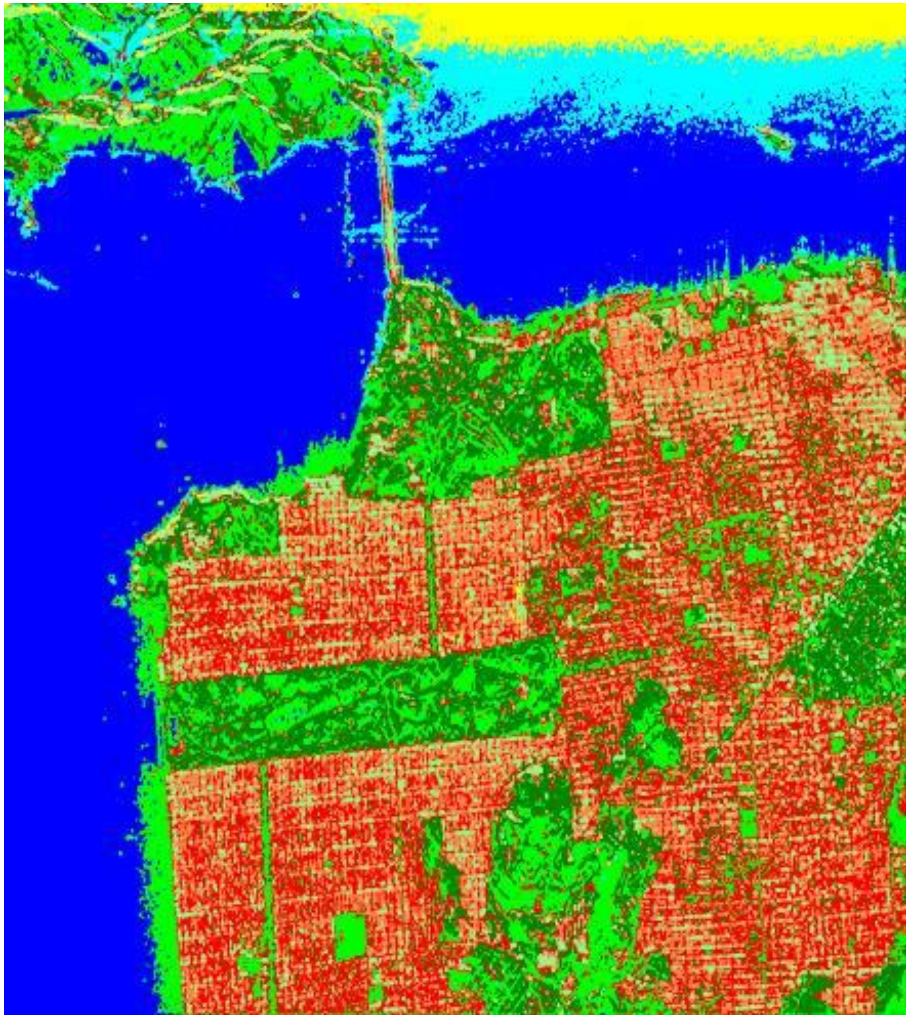
Cluster Center of the class *m*

(Lee 1998)

H / $\underline{\alpha}$ - WISHART CLASSIFIER

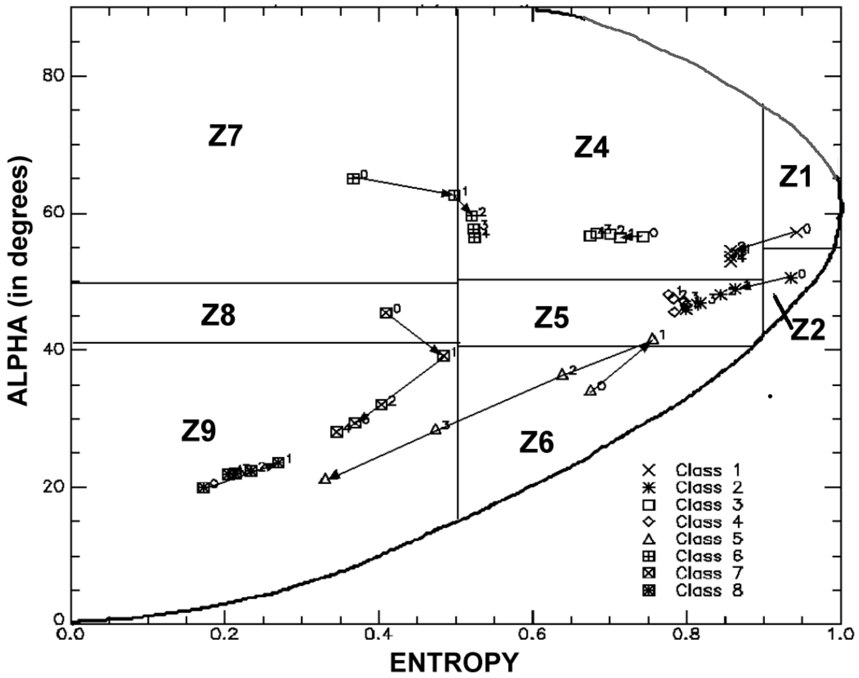
SAN FRANCISCO BAY JPL - AIRSAR L-band 1988

4th ITERATION



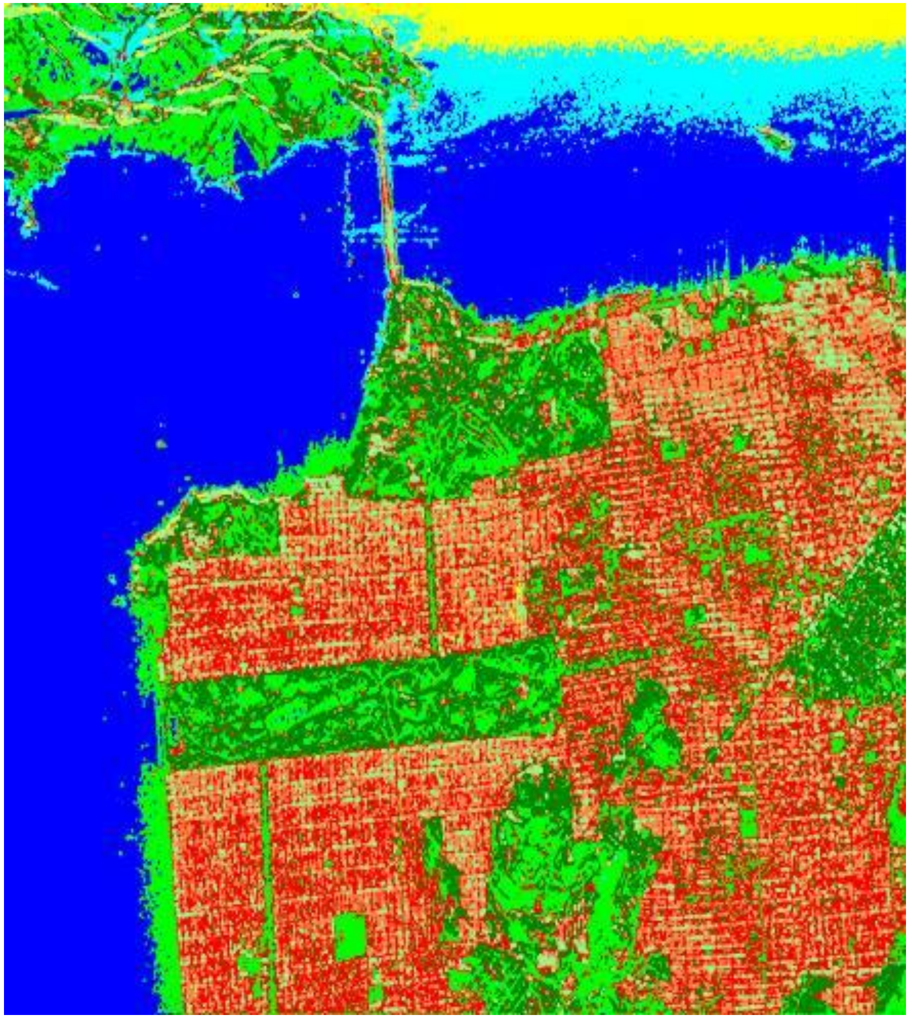
$2A_0$ $B_0 + B$ $B_0 - B$

C1 C2 C3 C4 C5 C6 C7 C8



Cluster centers shifting after each iteration

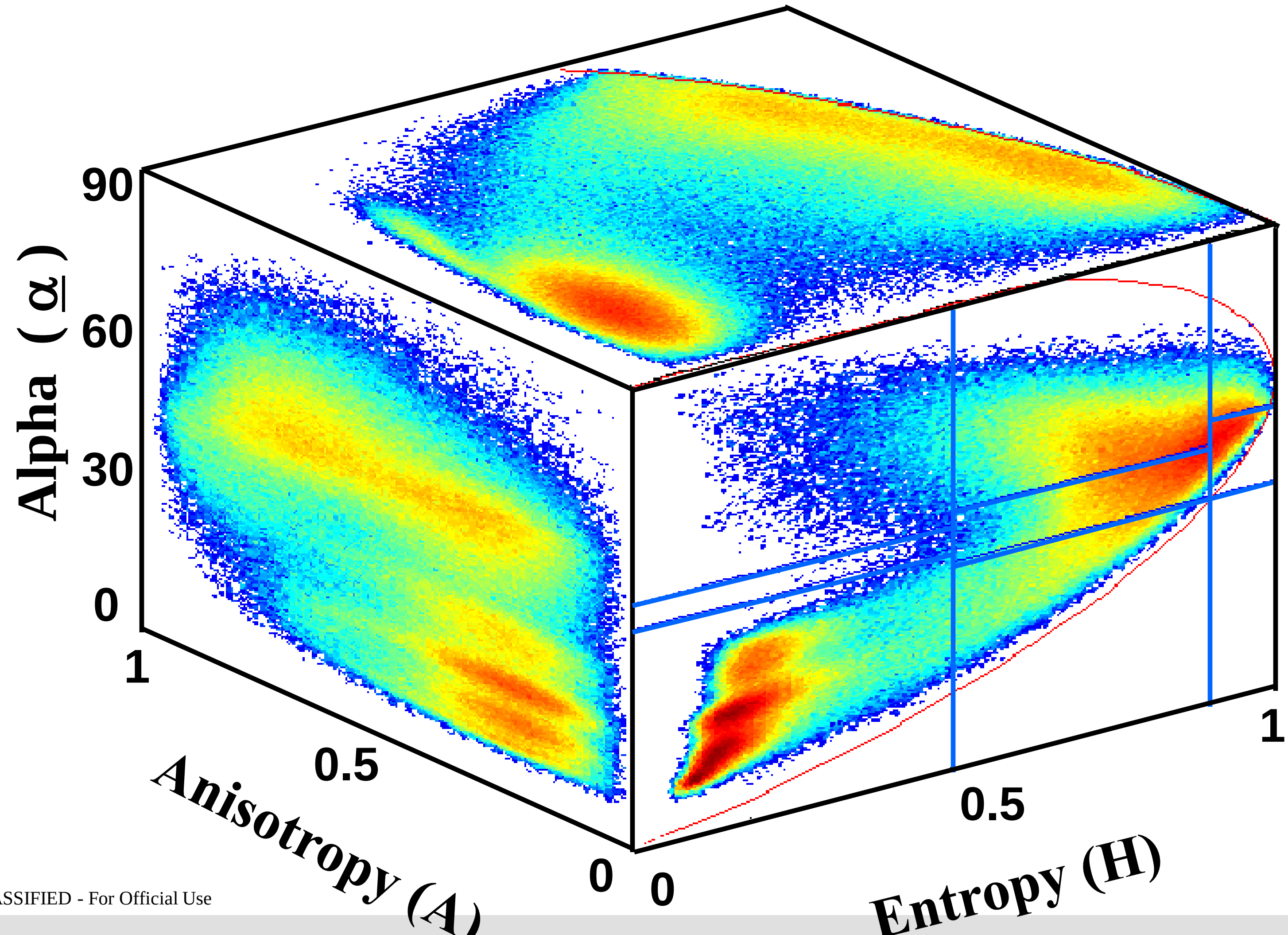
4th ITERATION



C1 C2 C3 C4 C5 C6 C7 C8

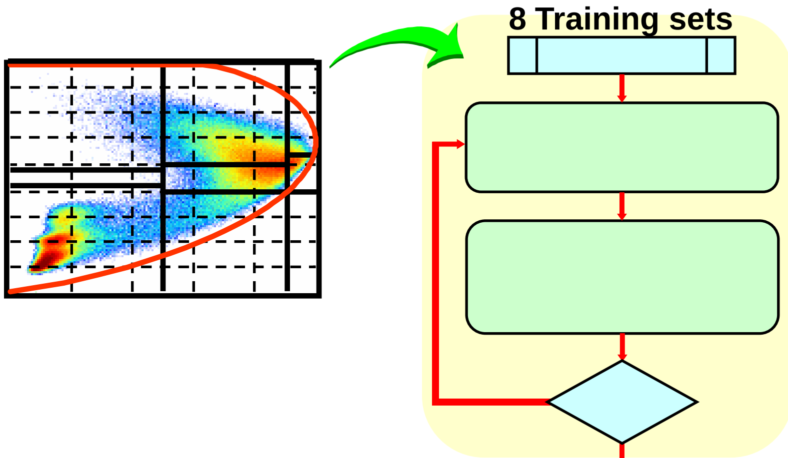
H / A / $\underline{\alpha}$ - WISHART CLASSIFIER

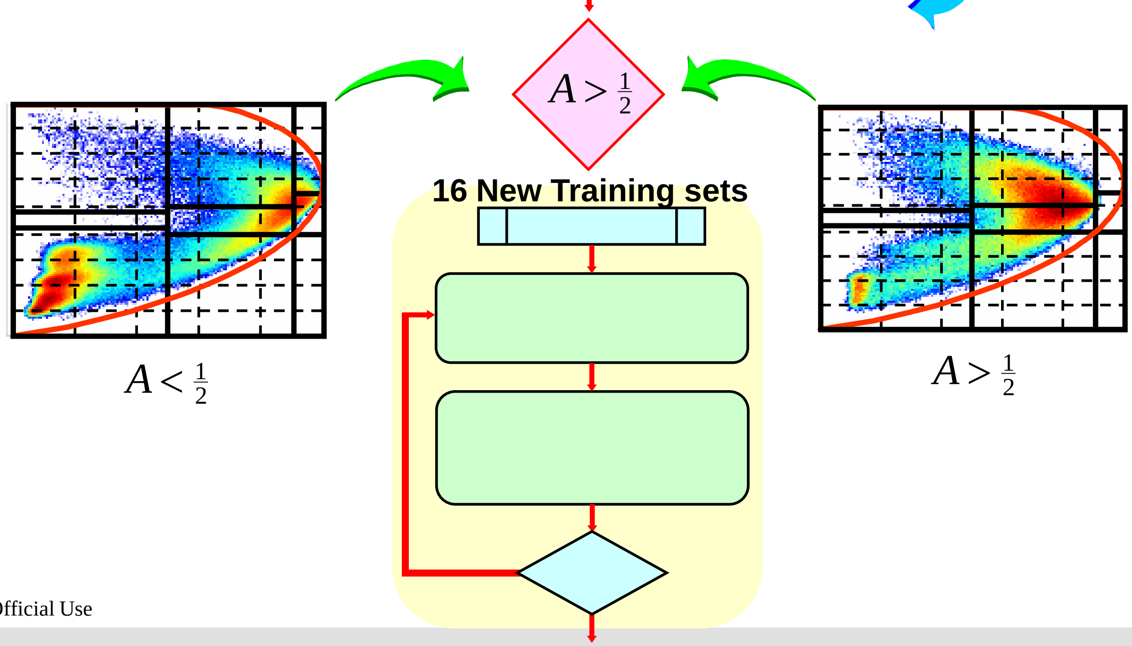
POLSAR DATA DISTRIBUTION IN THE H / A / $\underline{\alpha}$ SPACE



2 Successive k - mean Classification procedures

Introduction of the Anisotropy (A) information once the first Classification procedure has met its termination criterion



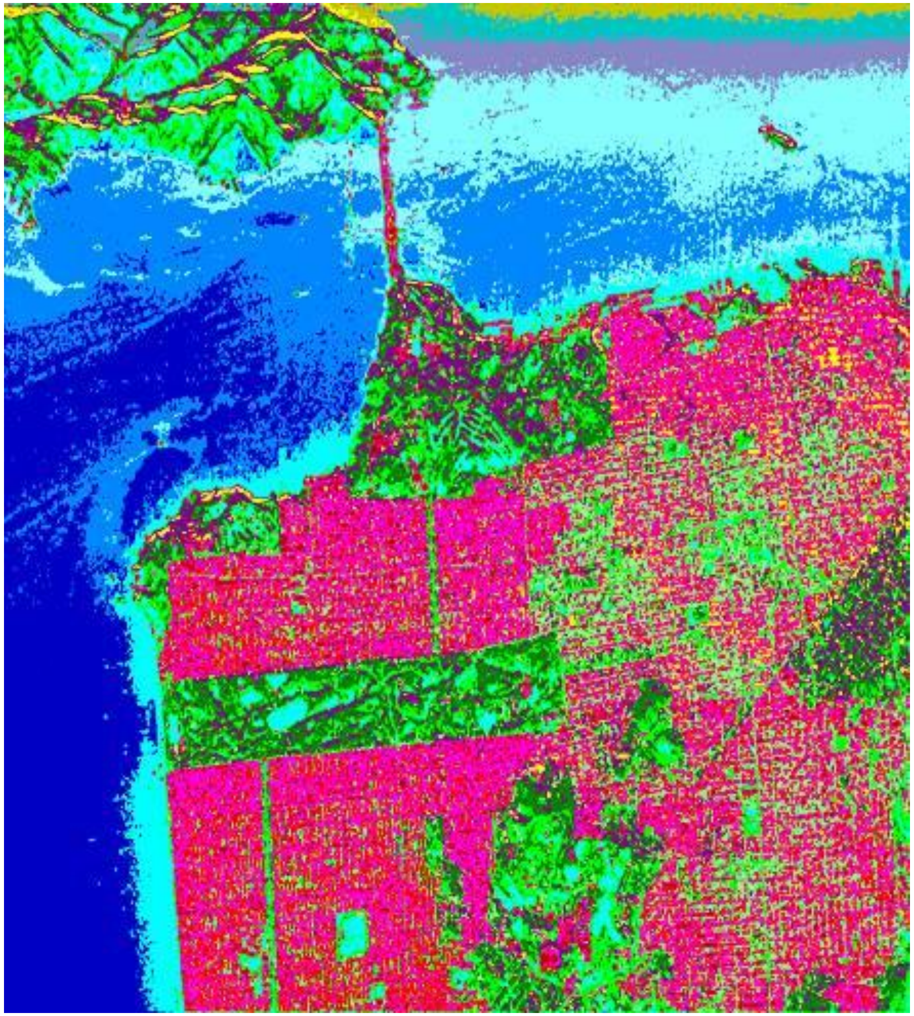


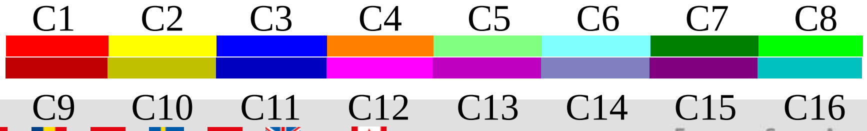
H / A / $\underline{\alpha}$ - WISHART CLASSIFIER

SAN FRANCISCO BAY JPL - AIRSAR L-band 1988

$2A_0$ B_0+B B_0-B

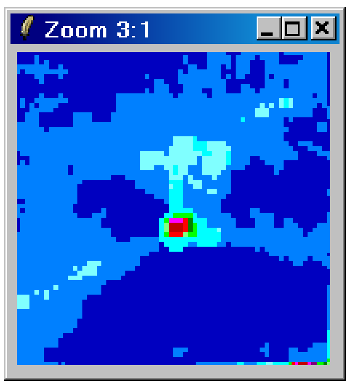
4th ITERATION

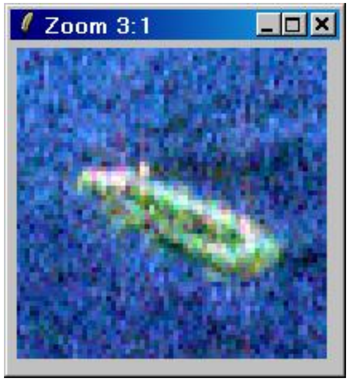


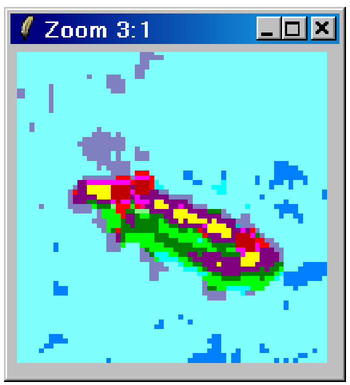


H / A / $\underline{\alpha}$ - WISHART CLASSIFIER

SAN FRANCISCO BAY JPL - AIRSAR L-band 1988



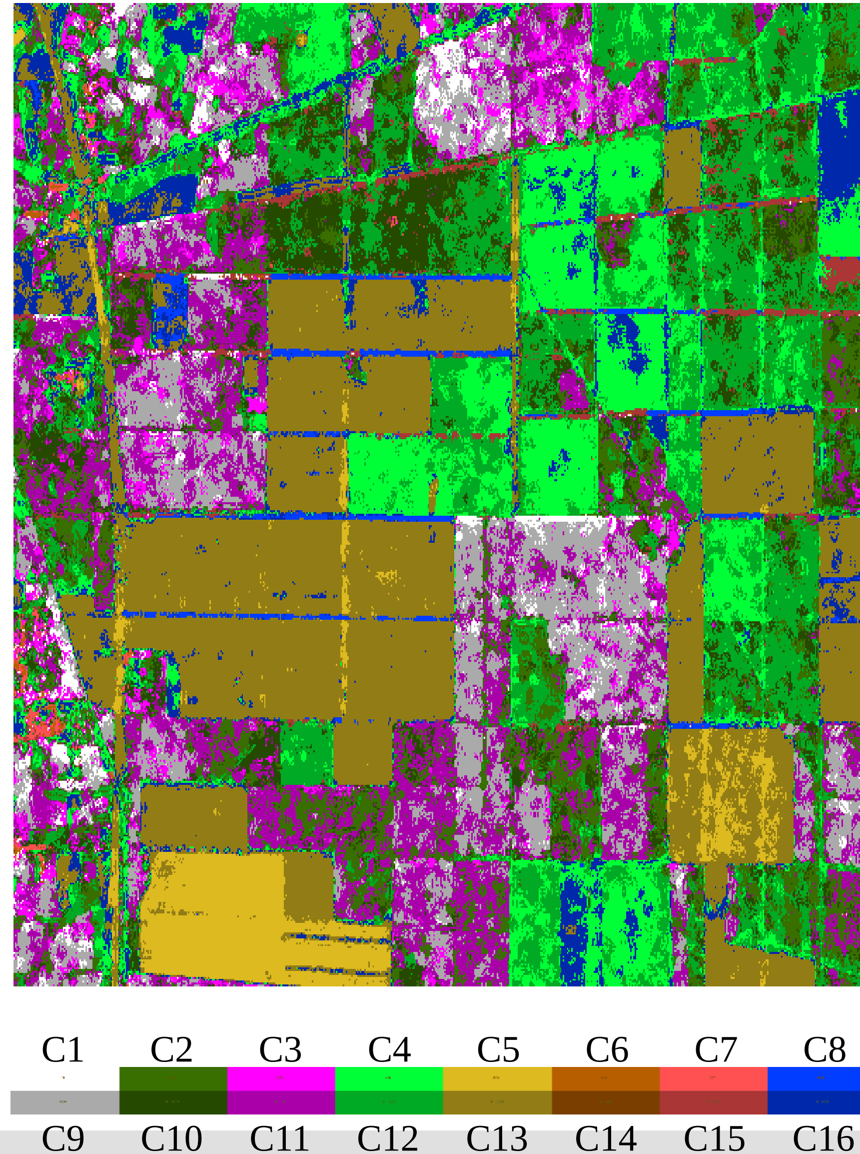




$2A_0$ $\quad B_0 + B$ $\quad B_0 - B$

H / A / $\underline{\alpha}$ - WISHART CLASSIFIER

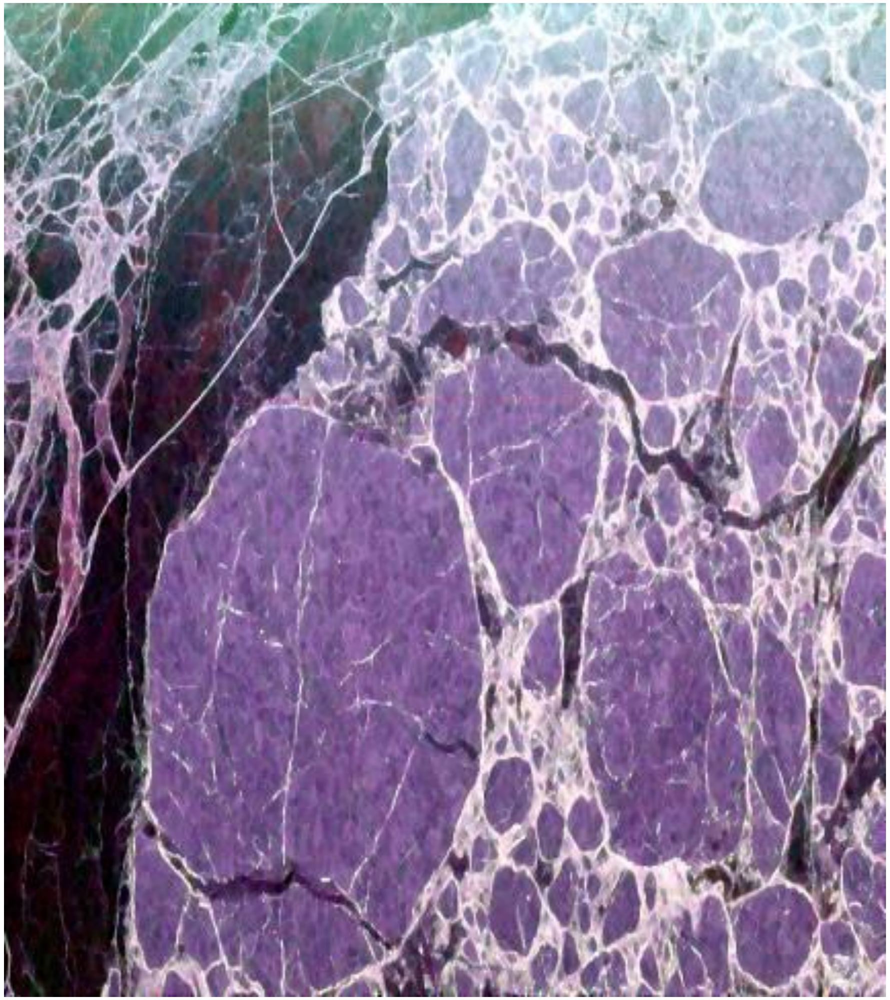
NEZER FOREST JPL - AIRSAR L-band

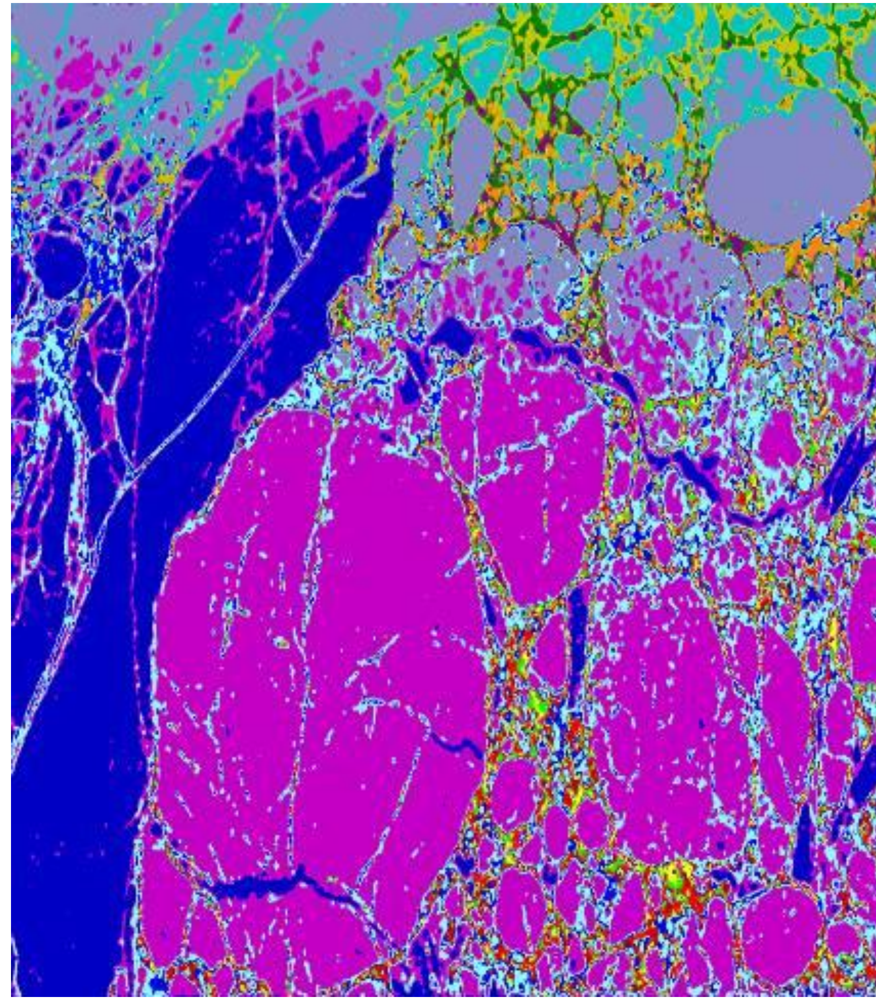


$2A_0$ $\quad B_0 + B$ $\quad B_0 - B$

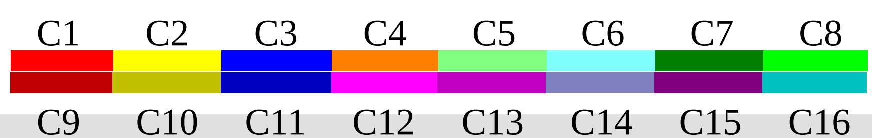
H / A / $\underline{\alpha}$ - WISHART CLASSIFIER

ICE AREA JPL - AIRSAR L-band





$2A_0$ $B_0 + B$ $B_0 - B$

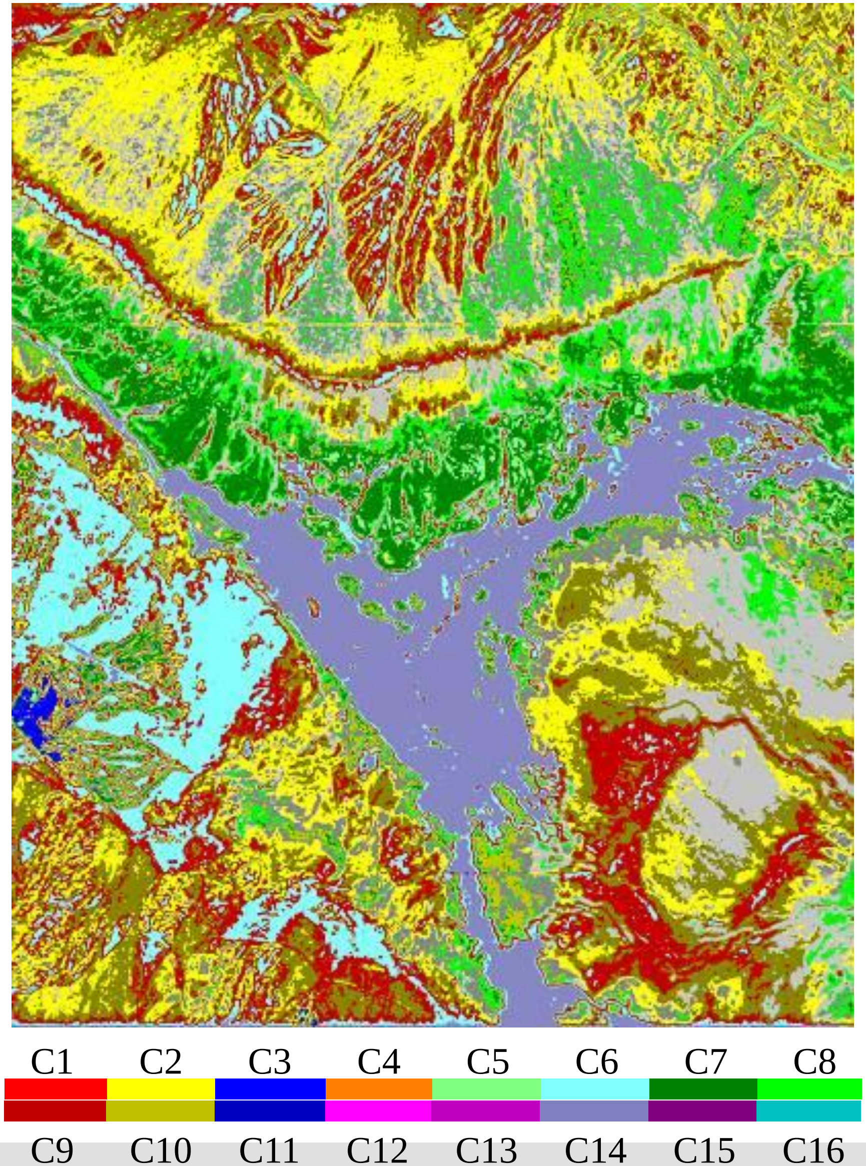


H / A / $\underline{\alpha}$ - WISHART CLASSIFIER

esa

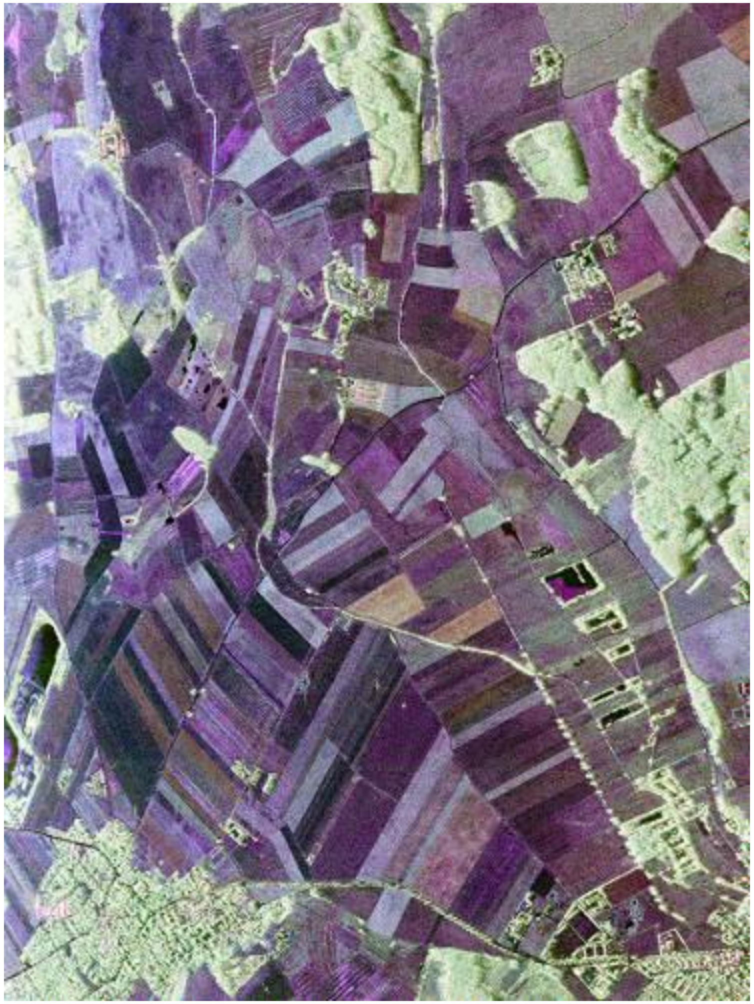
DEATH VALLEY JPL - AIRSAR L-band

$2A_0$ $B_0 + B$ $B_0 - B$



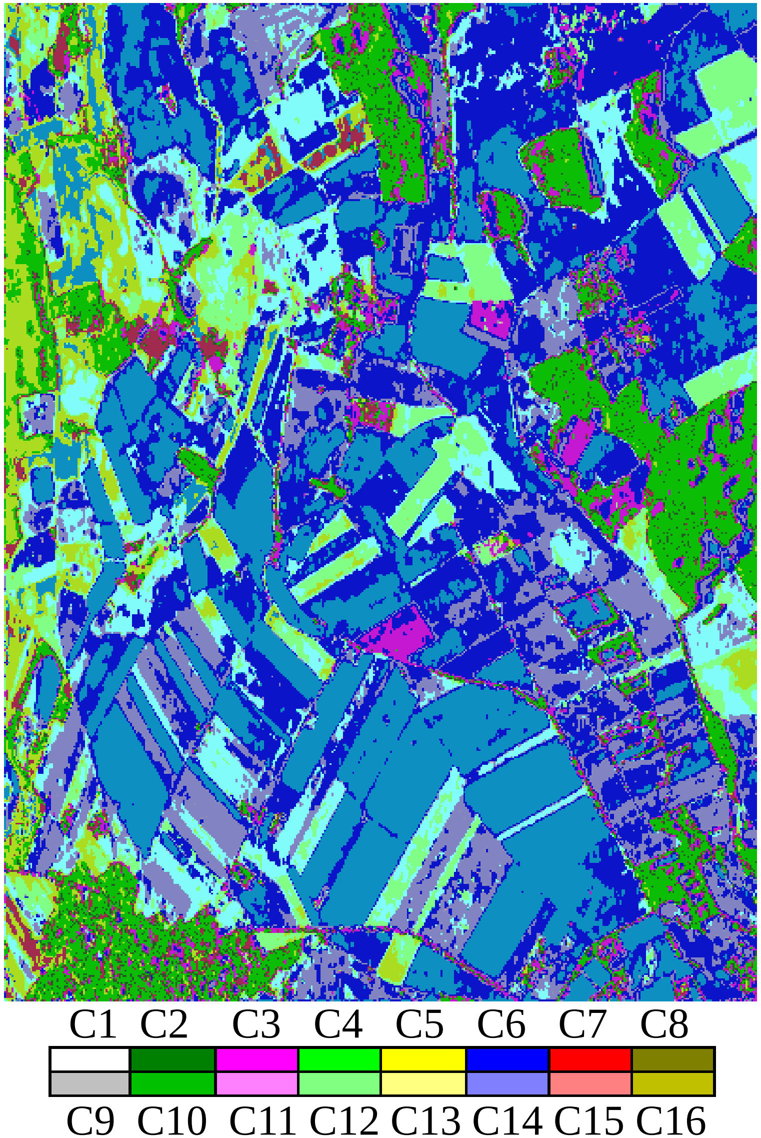
H / A / $\underline{\alpha}$ - WISHART CLASSIFIER

ALLING - ESAR L-band



$2A_0$ $B_0 + B$ $B_0 - B$

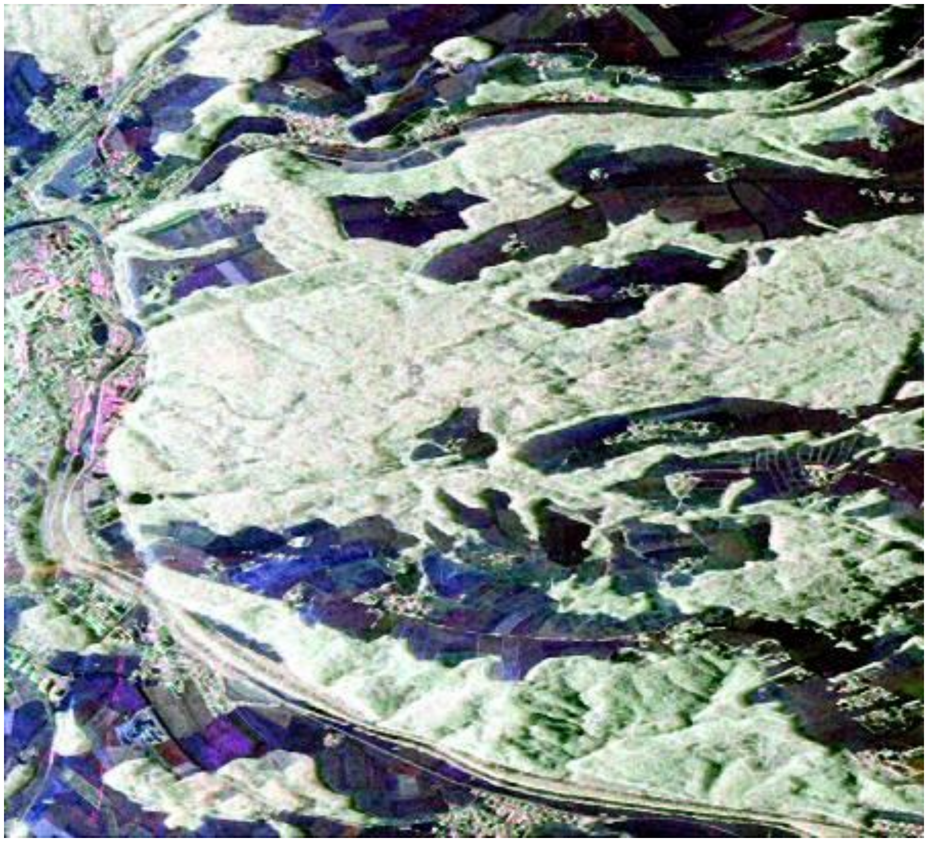
H / A / $\underline{\alpha}$ and WISHART CLASSIFIER



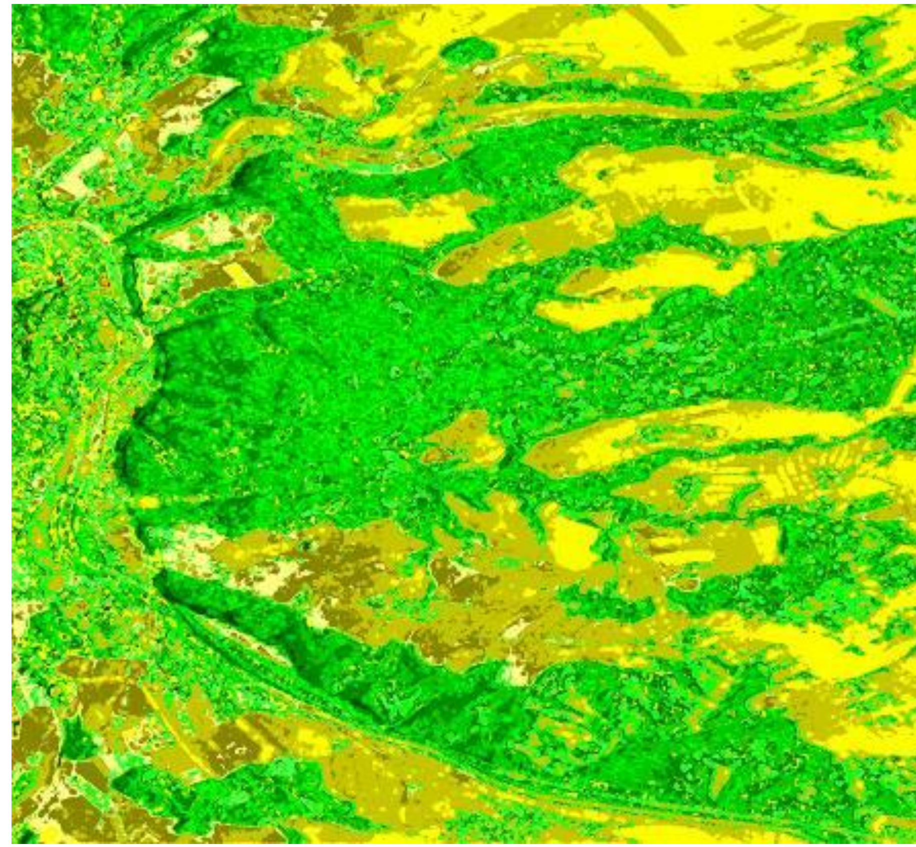
H / A / $\underline{\alpha}$ - WISHART CLASSIFIER

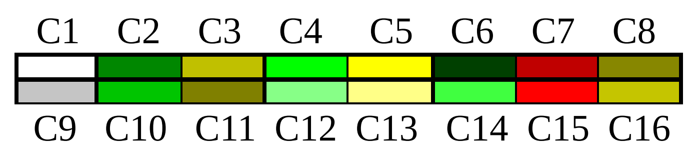
esa POLinSAR Project

TRAUNSTEIN - ESAR L-band



H / A / $\underline{\alpha}$ and WISHART CLASSIFIER

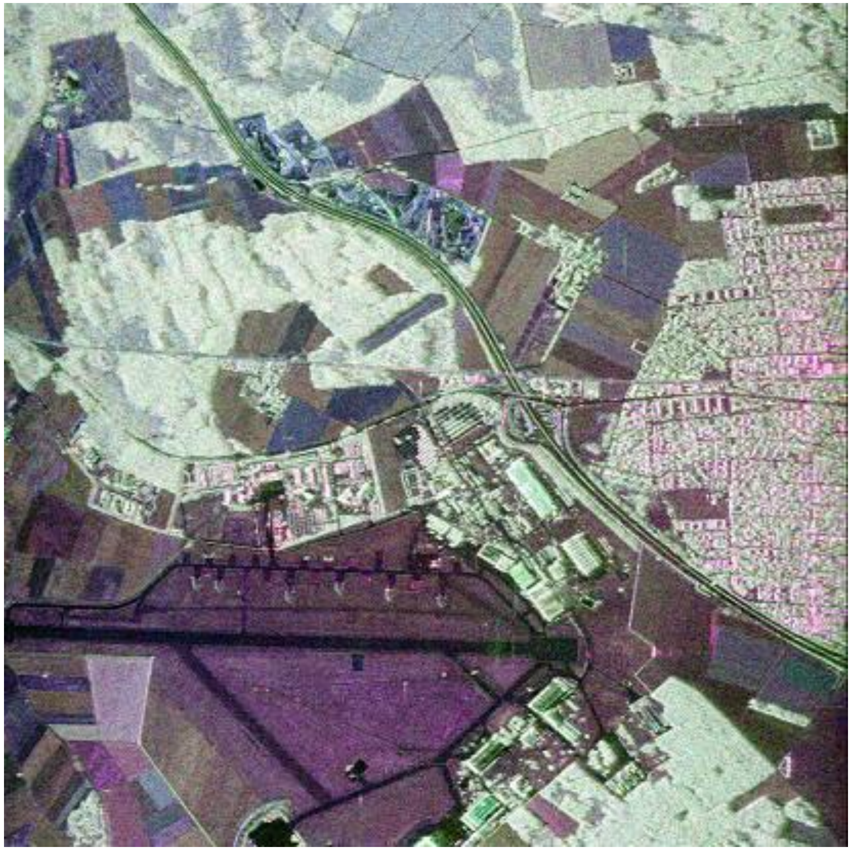




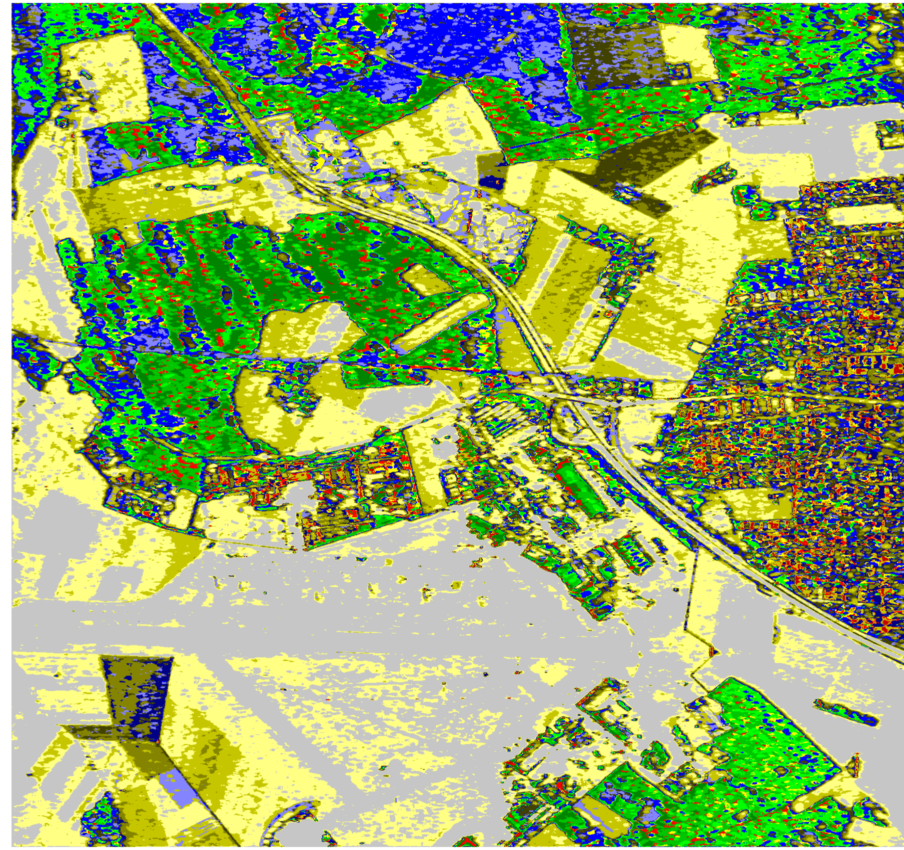
$2A_0$ $B_0 + B$ $B_0 - B$

H / A / $\underline{\alpha}$ - WISHART CLASSIFIER

OBERPFAFFENHOFEN - ESAR L-band



H / A / $\underline{\alpha}$ and WISHART CLASSIFIER



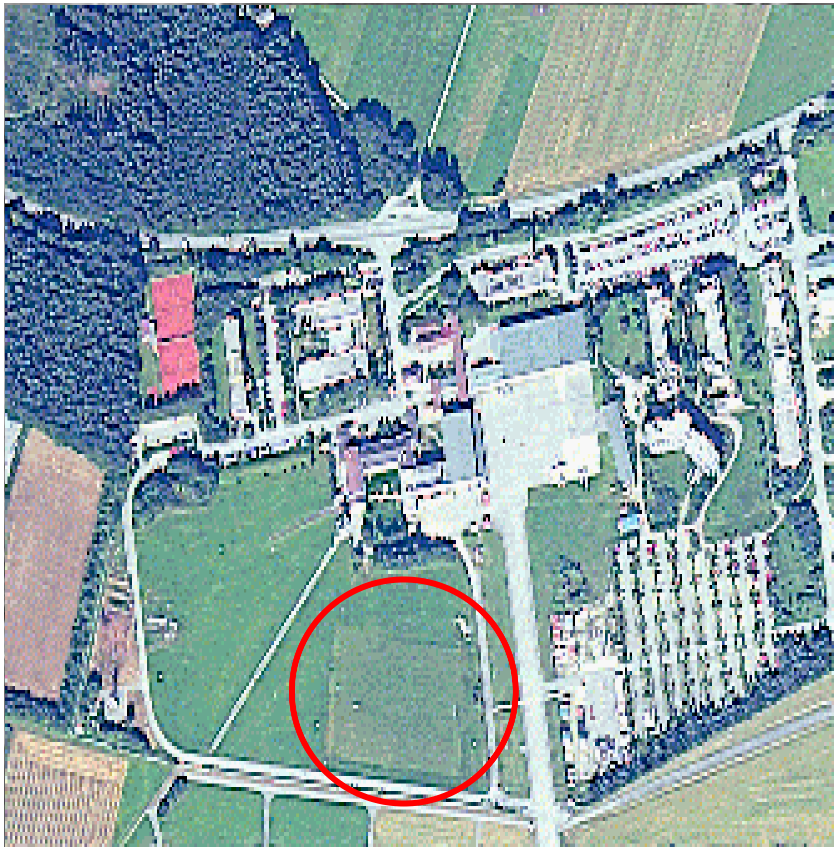
$2A_0$ $\quad B_0 + B$ $\quad B_0 - B$

C1 C2 C3 C4 C5 C6 C7 C8

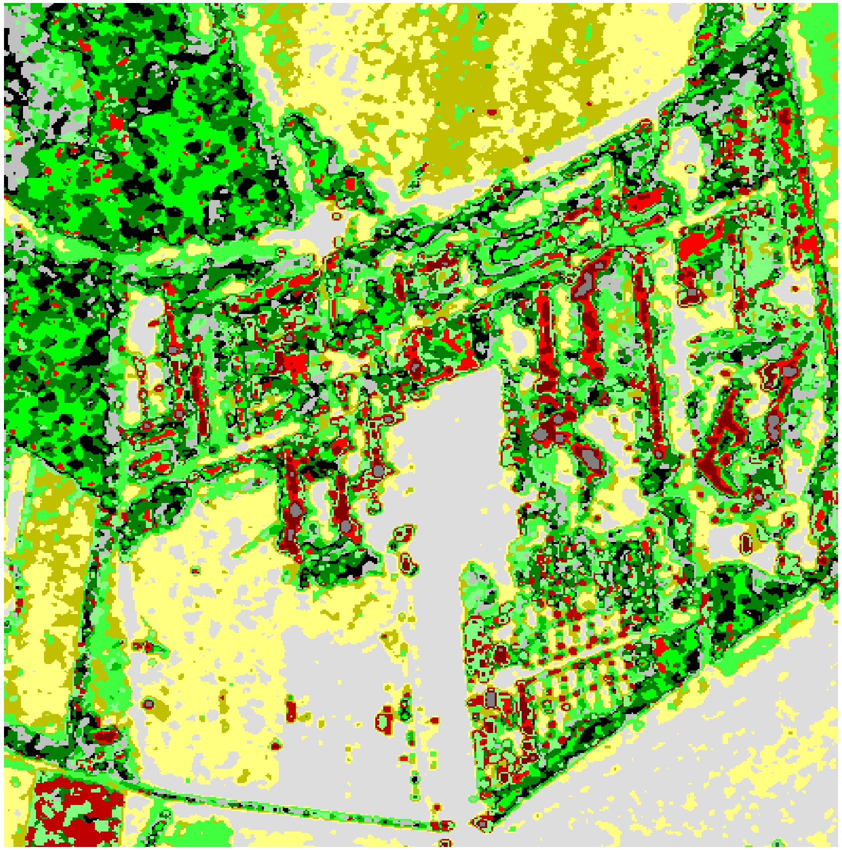
C9 C10 C11 C12 C13 C14 C15 C16

H / A / $\underline{\alpha}$ - WISHART CLASSIFIER

OBERPFAFFENHOFEN - ESAR L-band



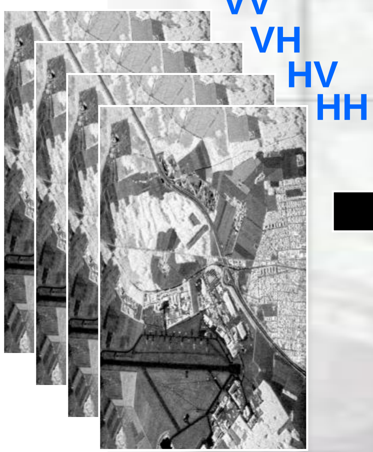
H / A / $\underline{\alpha}$ and WISHART CLASSIFIER



C1	C2	C3	C4	C5	C6	C7	C8
C9	C10	C11	C12	C13	C14	C15	C16

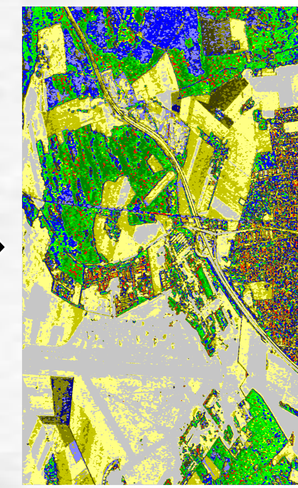
WISHART PDF

$$P\left(\langle[T]\rangle/[T_m]\right)=\frac{L^{Lp}\left|\langle[T]\rangle\right|^{L-p}e^{-L\,Tr\left([T_m]^{-1}\langle[T]\rangle\right)}}{\pi^{\frac{p(p-1)}{2}}\Gamma(L)\ldots\Gamma(L-p+1)\left|[T_m]\right|^{L}}$$



UNSUPERVISED POLSAR CLASSIFICATION

E.POTTIER, J.S LEE (2000)



Unsupervised Classification Preserving Scattering Mechanisms

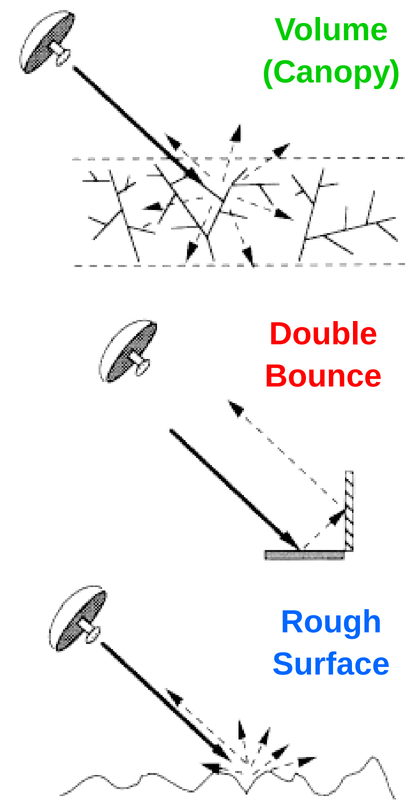
J.S. Lee, M.R. Grunes, E. Pottier and L. Ferro-Famil, "Segmentation of polarimetric SAR images that preserves scattering mechanisms" Proceedings of EUSAR2002

FREEMAN DECOMPOSITION

Courtesy of Dr J.S Lee

|HH-VV|, |HV|, |HH+VV|

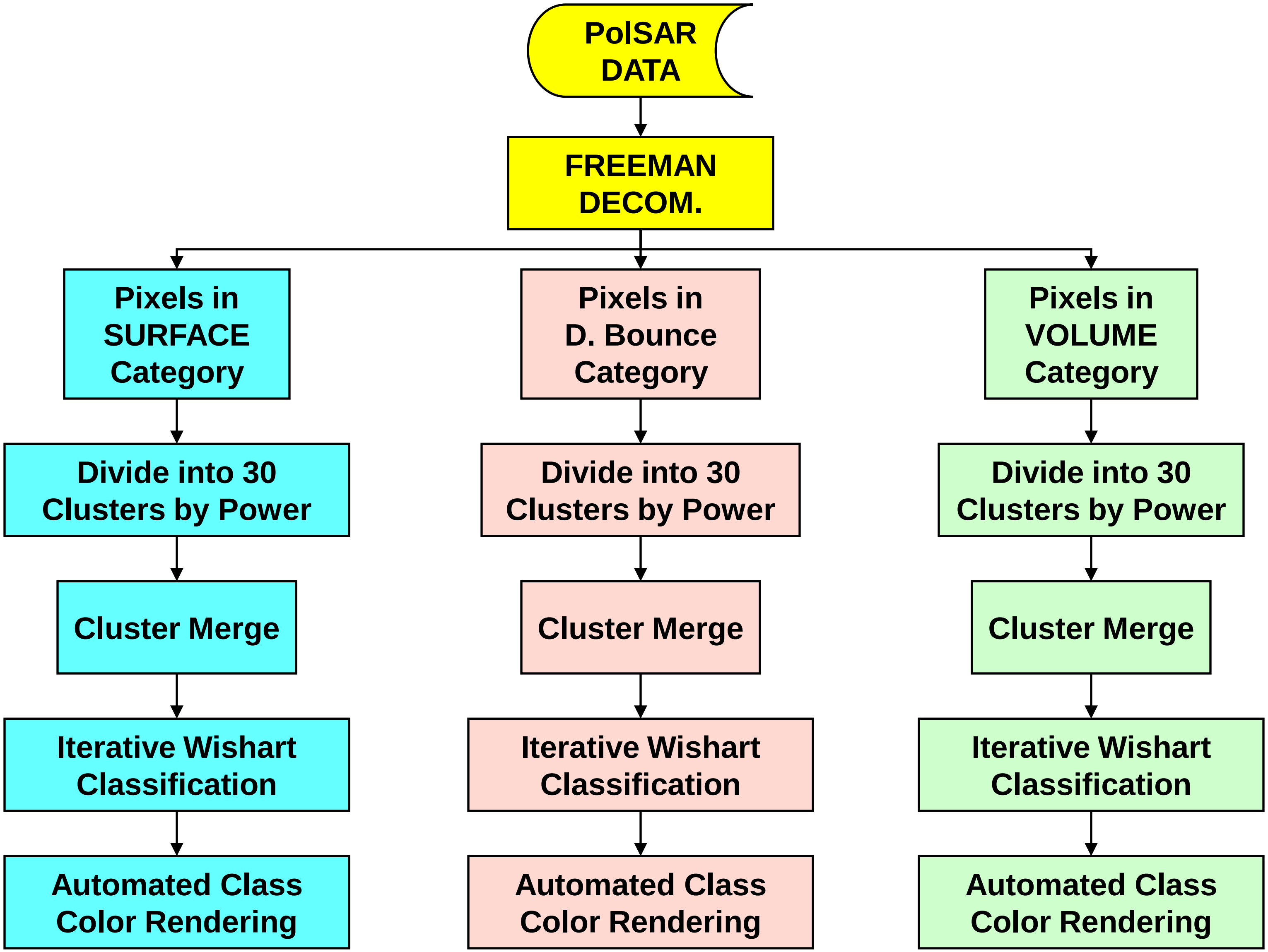
Freeman and Durden



A. Freeman and S.L. Durden, “A Three-Component Scattering Model for Polarimetric SAR Data” IEEE TGRS, vol. 36, no. 3, May 1998

PROCEDURE – FLOW CHART

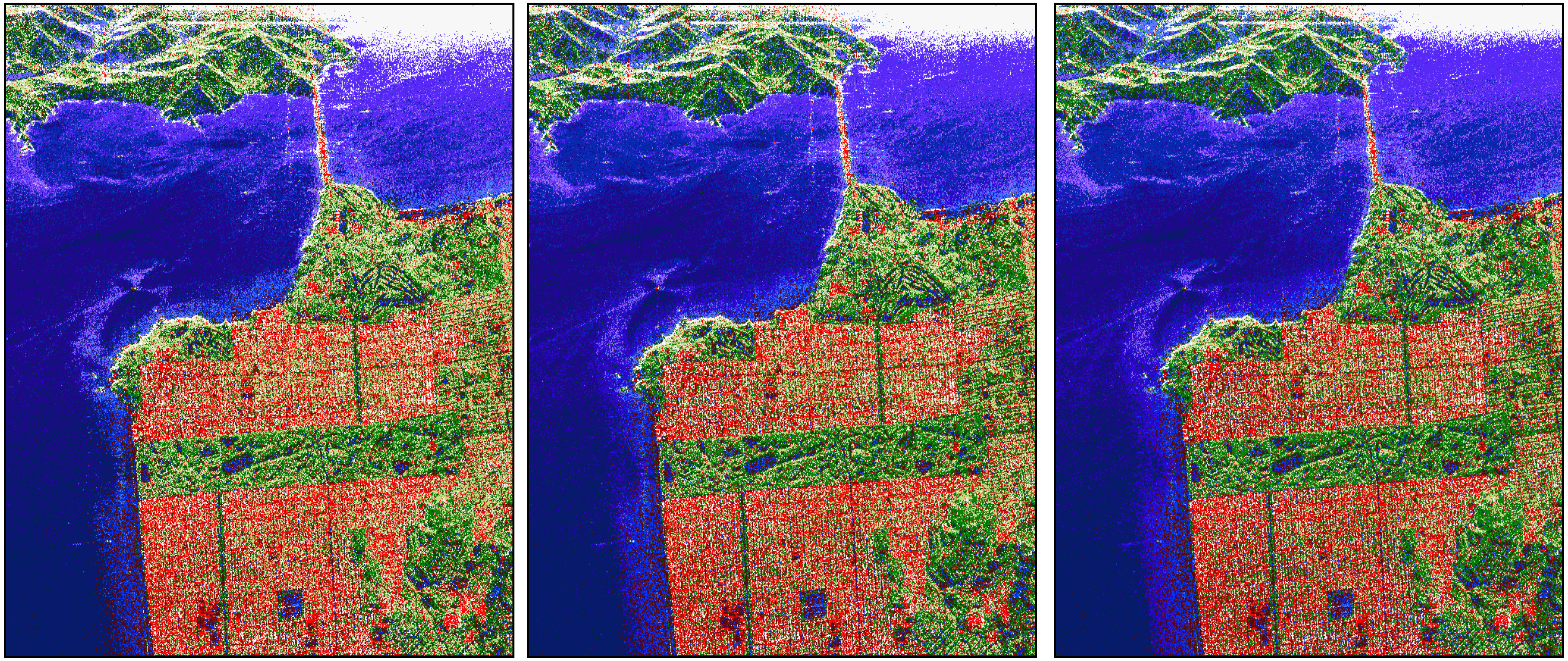
esa



Cluster Merging $D_{ij} = \frac{1}{2}\{ln(|V_i|) + ln(|V_j|) + Tr(V_i^{-1}V_j + V_j^{-1}V_i)\}$

Wishart Iteration – After Class Merge

Classification Maps



First Iteration | Second Iteration | Third Iteration

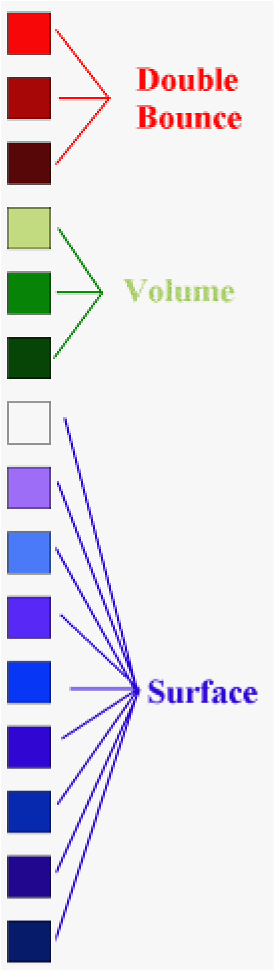
Note: Stability insures good convergence

FREEMAN - WISHART CLASSIFIER

Courtesy of Dr J.S Lee

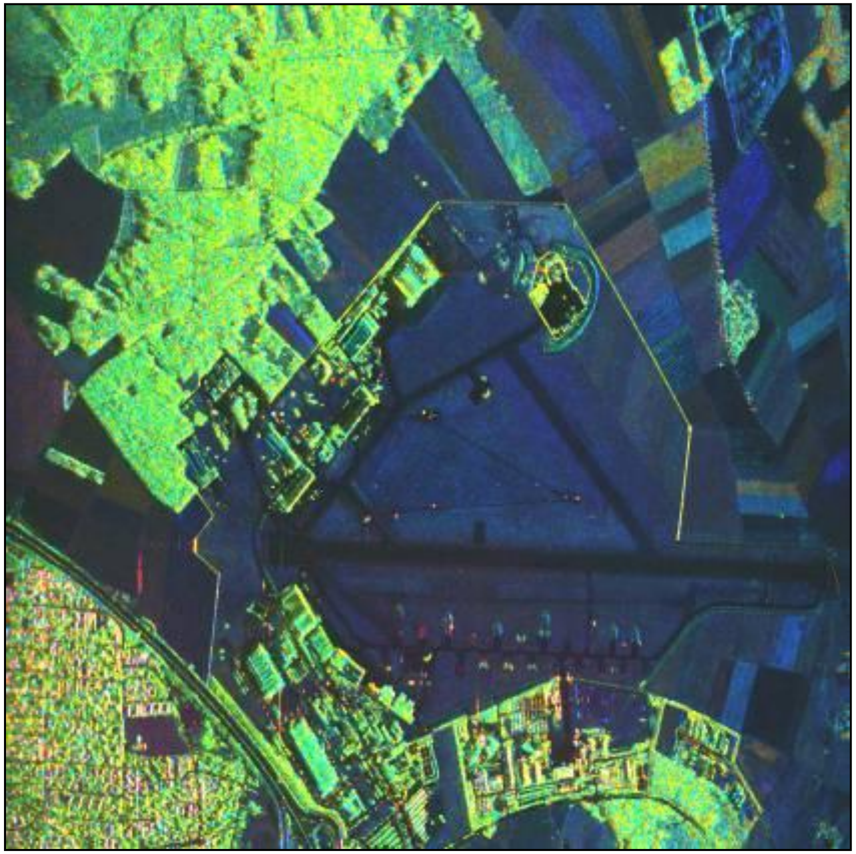
|HH-VV|, |HV|, |HH+VV|

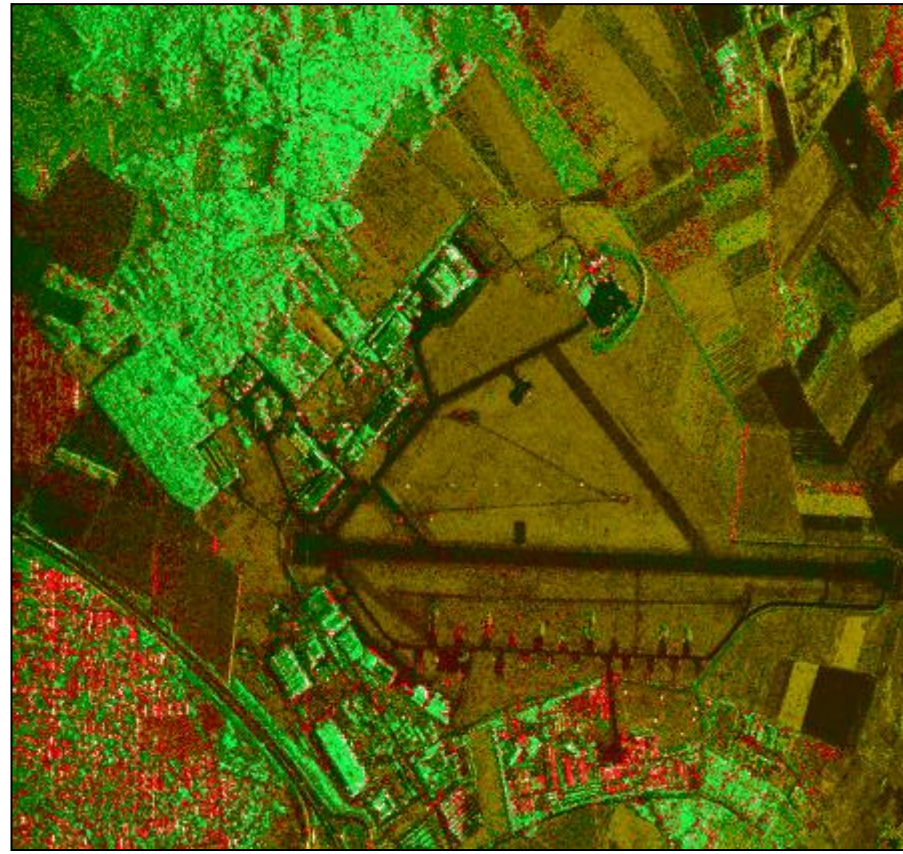
4th Iteration (15 classes)



FREEMAN - WISHART CLASSIFIER

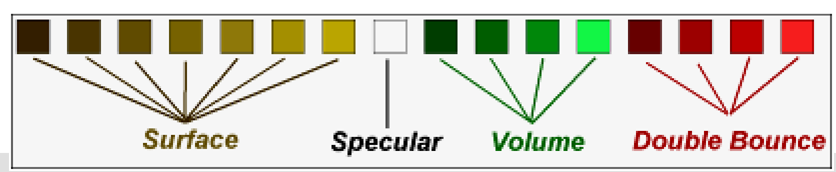
Courtesy of Dr J.S Lee





$2A_0$ $B_0 + B$ $B_0 - B$

4th Iteration (15 classes)

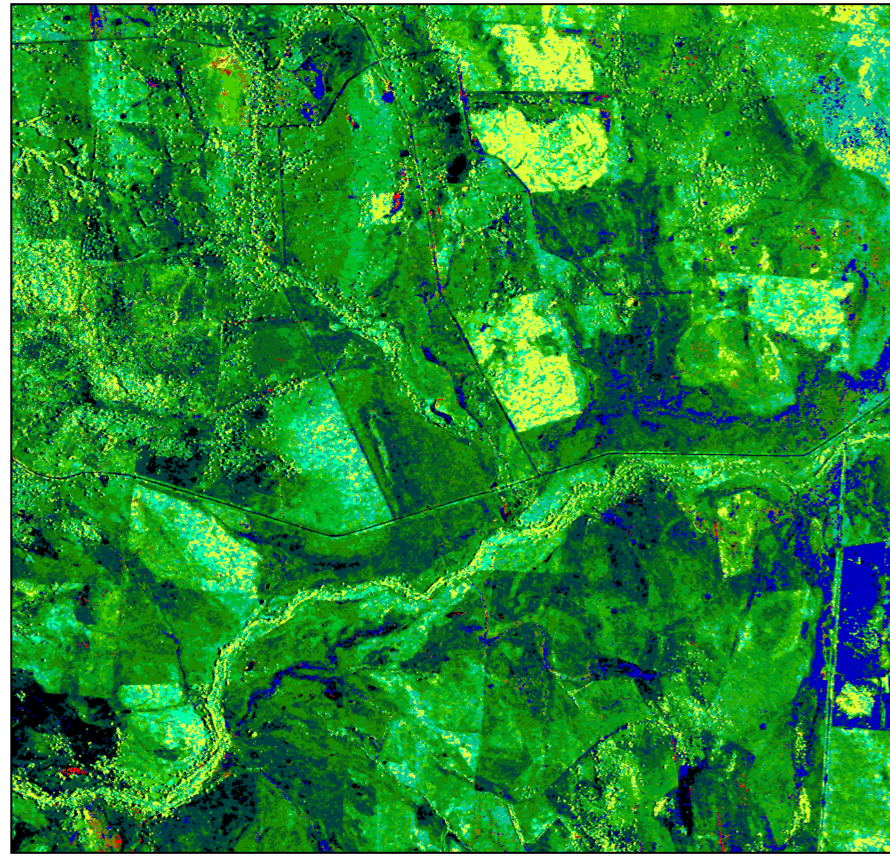


FREEMAN - WISHART CLASSIFIER

Courtesy of Dr J.S Lee

$2A_0$ $B_0 + B$ $B_0 - B$

Australian Pasture



4th Iteration (15 classes)

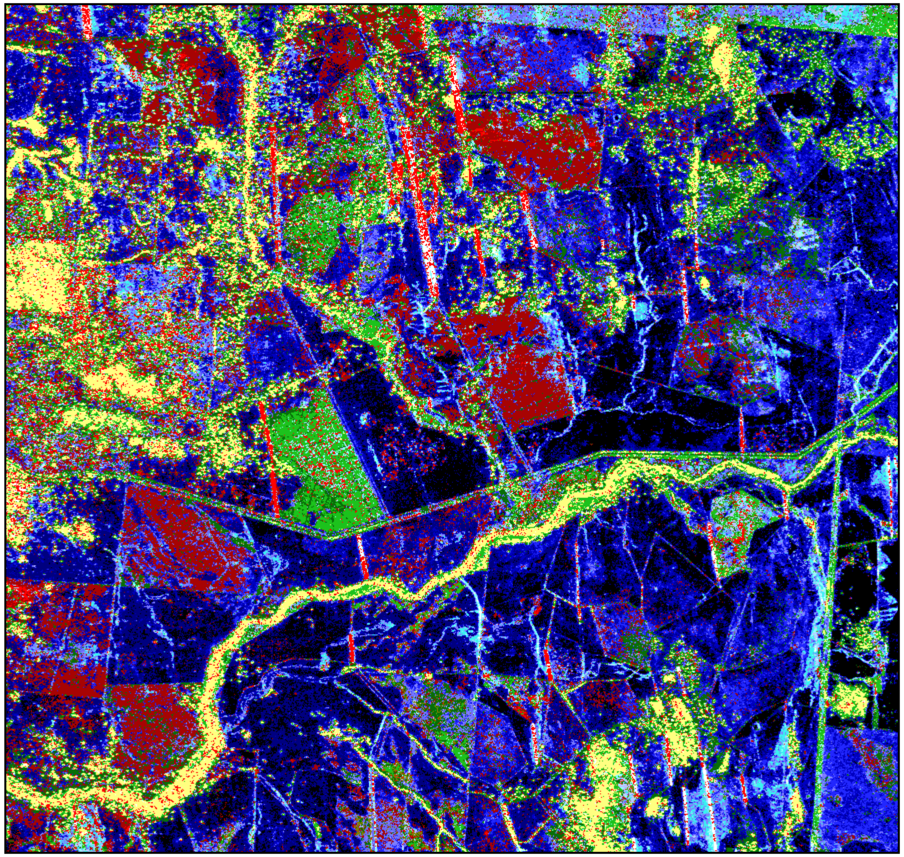
C-Band Volume Dominated

FREEMAN - WISHART CLASSIFIER

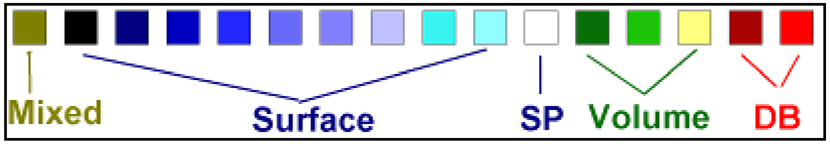
Courtesy of Dr J.S Lee

$2A_0$ $B_0 + B$ $B_0 - B$

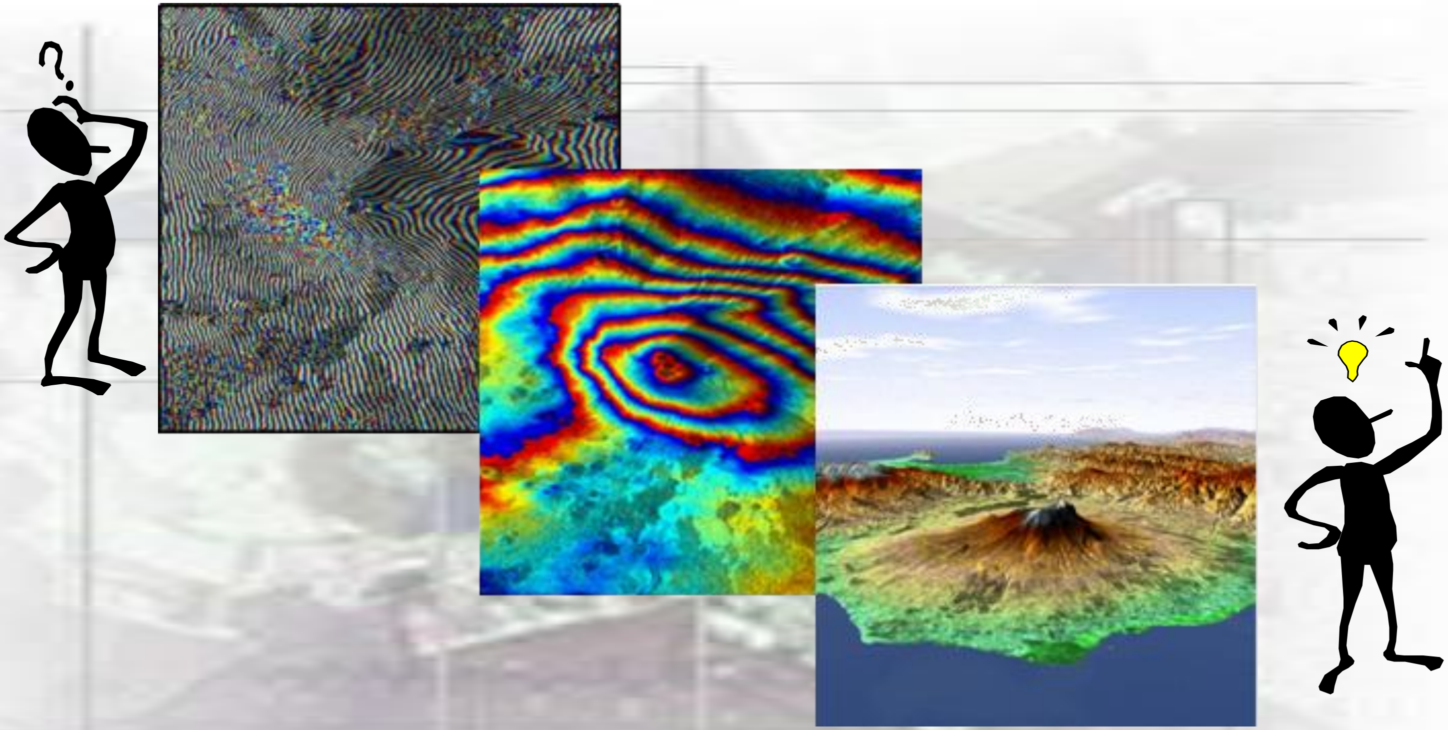
Australian Pasture



4th Iteration (15 classes)

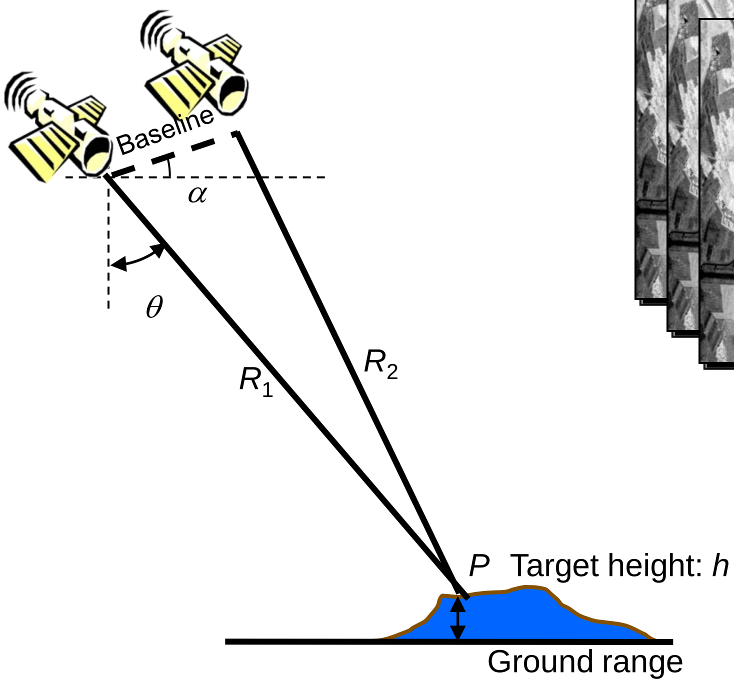


L-Band Volume Dominated



POLARIMETRIC INTERFEROMETRIC SAR POL-InSAR

POL-InSAR



$$\underline{k}_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} S_{XX_1} + S_{YY_1} \\ S_{XX_1} - S_{YY_1} \\ 2S_{XY_1} \end{bmatrix}$$

$$\underline{\underline{k}} = \begin{bmatrix} \underline{k}_1 \\ \underline{k}_2 \end{bmatrix}$$

POLARIMETRIC INTERFEROMETRIC TARGET VECTOR

$$\underline{k}_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} S_{XX_2} + S_{YY_2} \\ S_{XX_2} - S_{YY_2} \\ 2S_{XY_2} \end{bmatrix}$$

$$\underline{\underline{k}} = \begin{bmatrix} \underline{k}_1 \\ \underline{k}_2 \end{bmatrix}$$

POLARIMETRIC INTERFEROMETRIC TARGET VECTOR

$$\langle [T_6] \rangle = \left\langle \underline{\underline{k}} \cdot \underline{\underline{k}}^{T*} \right\rangle = \begin{bmatrix} \left\langle \underline{k}_1 \cdot \underline{k}_1^{T*} \right\rangle & \left\langle \underline{k}_1 \cdot \underline{k}_2^{T*} \right\rangle \\ \left\langle \underline{k}_2 \cdot \underline{k}_1^{T*} \right\rangle & \left\langle \underline{k}_2 \cdot \underline{k}_2^{T*} \right\rangle \end{bmatrix} = \begin{bmatrix} \langle [T_1] \rangle & \langle [\Omega_{12}] \rangle \\ \left\langle [\Omega_{12}]^{T*} \right\rangle & \langle [T_2] \rangle \end{bmatrix}$$

POLARIMETRIC INTERFEROMETRIC COHERENCY MATRIX (6x6)

$\langle [T_1] \rangle$ **HERMITIAN POLARIMETRIC COHERENCY MATRIX (3x3)**

$\langle [T_2] \rangle$ **HERMITIAN POLARIMETRIC COHERENCY MATRIX (3x3)**

$\langle [\Omega_{12}] \rangle$ **NON HERMITIAN POLARIMETRIC INTER-COHERENCY MATRIX (3x3)**

DUAL CHANNELS POLINSAR UNSUPERVISED SEGMENTATION

$$\langle [T_6] \rangle = \langle \underline{\underline{k}} \cdot \underline{\underline{k}}^{T*} \rangle = \begin{bmatrix} \langle \underline{k}_1 \cdot \underline{k}_1^{T*} \rangle & \langle \underline{k}_1 \cdot \underline{k}_2^{T*} \rangle \\ \langle \underline{k}_2 \cdot \underline{k}_1^{T*} \rangle & \langle \underline{k}_2 \cdot \underline{k}_2^{T*} \rangle \end{bmatrix} = \begin{bmatrix} \langle [T_1] \rangle & \langle [\Omega_{12}] \rangle \\ \langle [\Omega_{12}]^{T*} \rangle & \langle [T_2] \rangle \end{bmatrix}$$

POLARIMETRIC INTERFEROMETRIC COHERENCY MATRIX (6x6)

$\langle [T_6] \rangle$ FOLLOWS A WISHART DISTRIBUTION

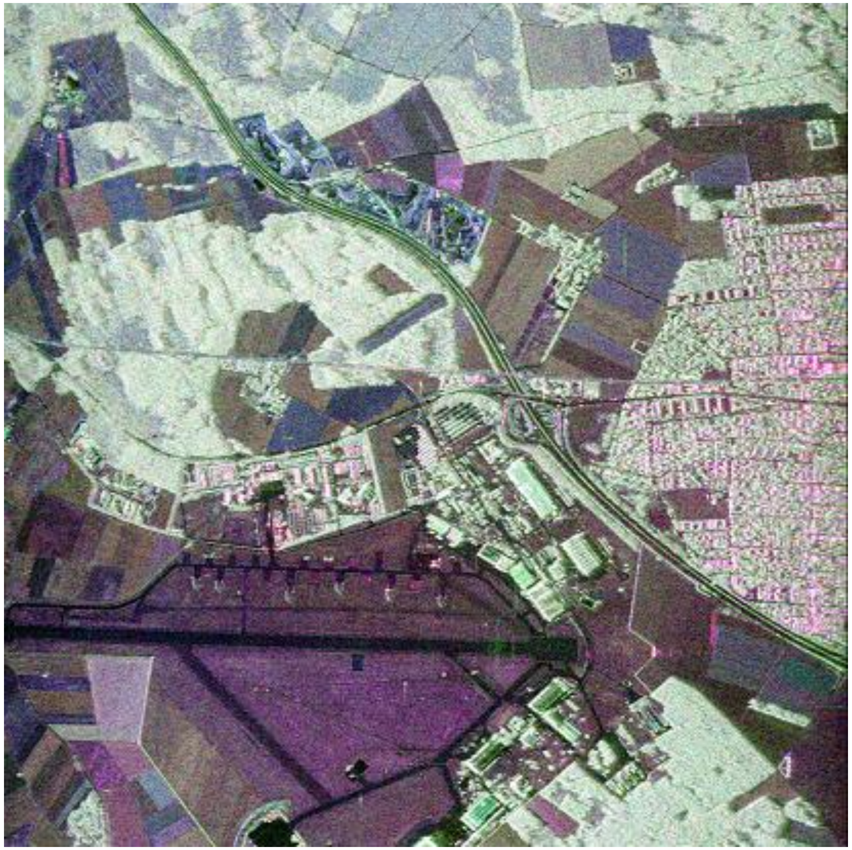
$$P(\langle [T_6] \rangle / [\Sigma_m]) = \frac{|\langle [T_6] \rangle|^{L-p} \exp\left(-tr\left([\Sigma_m]^{-1} \langle [T_6] \rangle\right)\right)}{K(L,p) |[\Sigma_m]|^{L}} = W_C(L, [\Sigma_m])$$

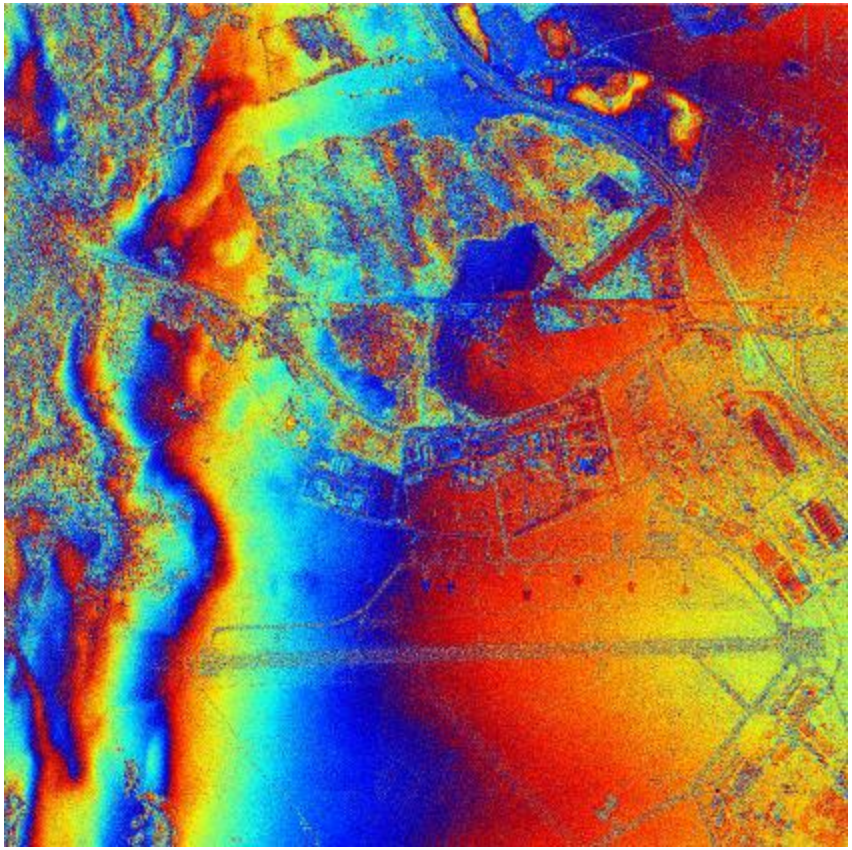
L: Number of Look
p: Polarimetric Dimension

With: $K(L,p) = \frac{\pi^{\frac{p(p-1)}{2}}}{L^{Lp}} \Gamma(L) \ldots \Gamma(L-p+1)$

$[\Sigma m]$: Cluster Center of the class *m*

POL-InSAR



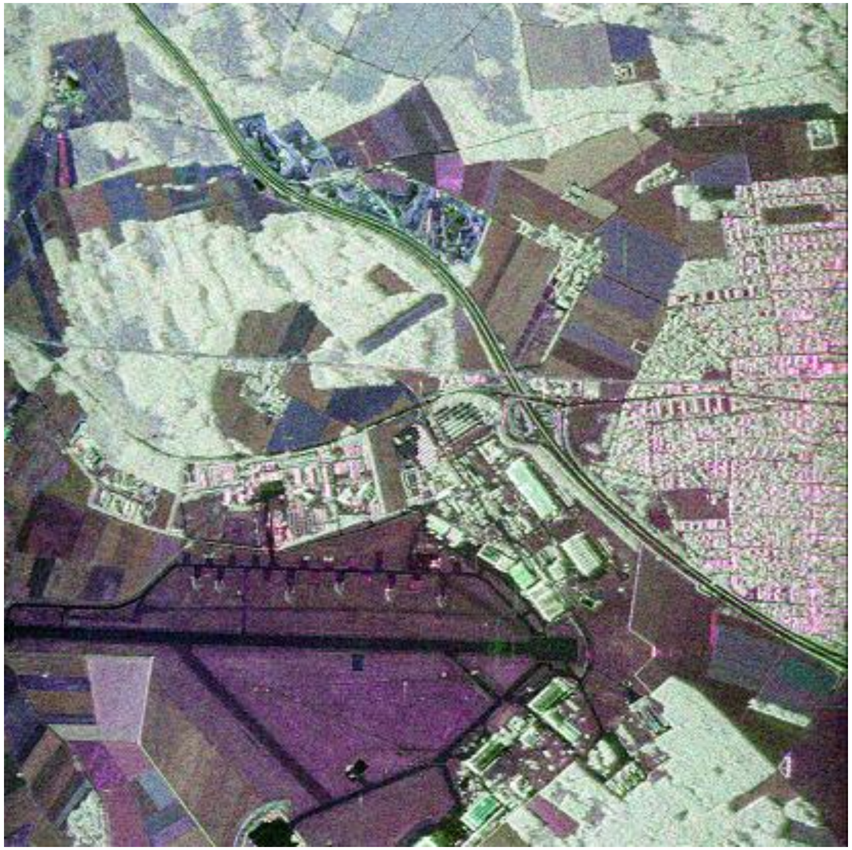


DLR E-SAR L Band
Pol-In SAR (1.5m x 3m) – Baseline 15m

POL-SAR INFORMATION

IN-SAR INFORMATION **Arg(γ)**

POL-InSAR

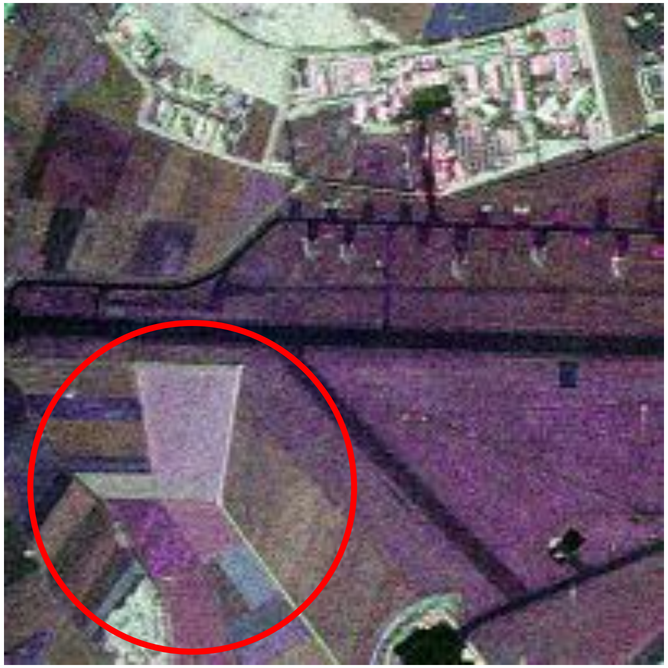


DLR E-SAR L Band
Pol-In SAR (1.5m x 3m) – Baseline 5m

POL-SAR INFORMATION

IN-SAR INFORMATION $|\gamma|$

COMPLEMENTARY INFORMATION



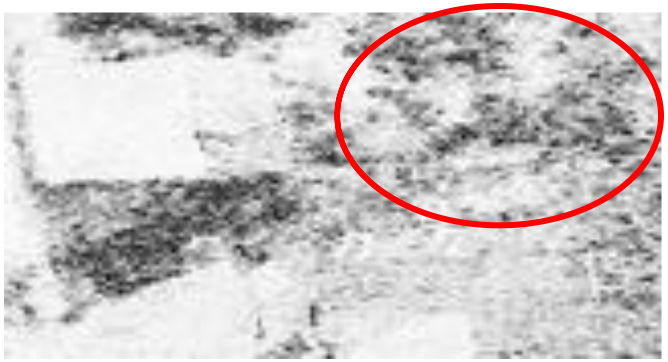
HETEROGENEOUS AREA

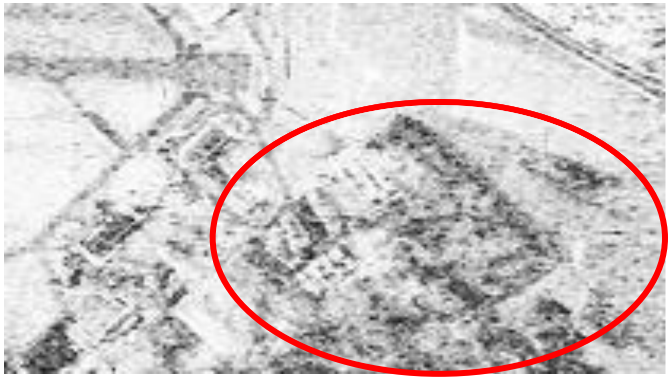
HOMOGENEOUS AREA

DIFFERENT POLARIMETRIC SCATTERING MECHANISMS

CONSTANT INTERFEROMETRIC COHERENCE

POL-InSAR





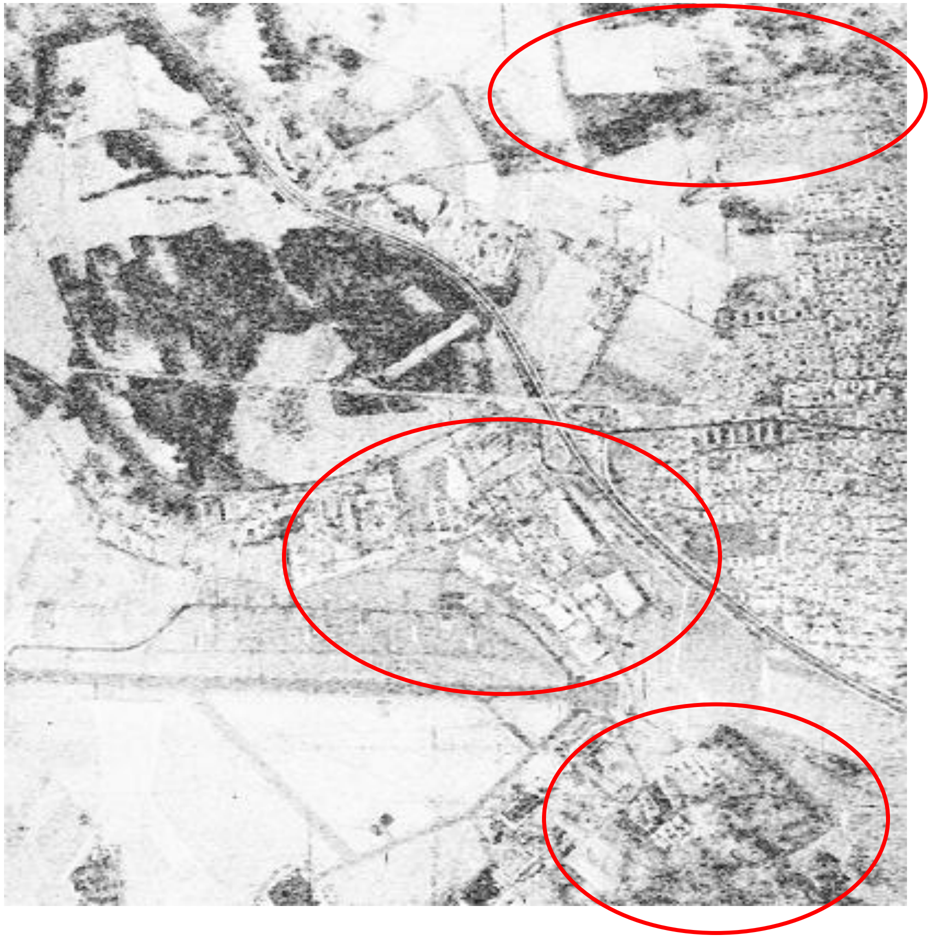
HOMOGENEOUS AREA

HETEROGENEOUS AREA

SAME POLARIMETRIC SCATTERING MECHANISMS

DIFFERENT INTERFEROMETRIC COHERENCE

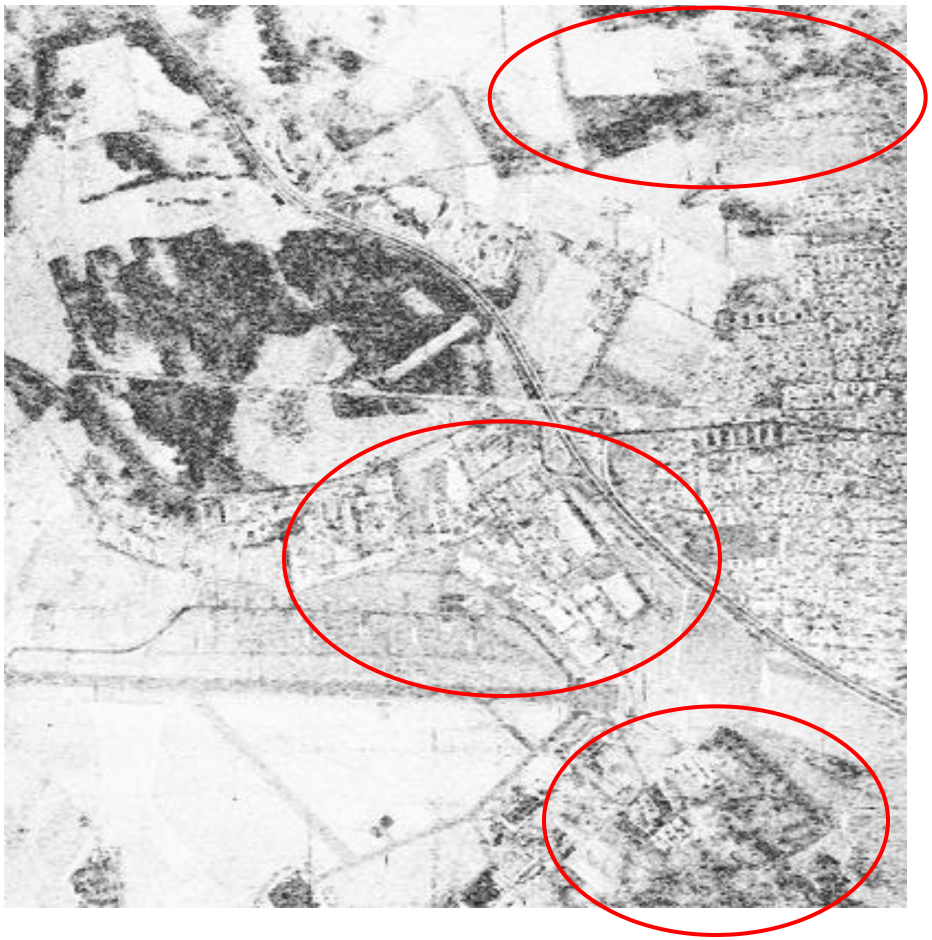
POL-InSAR



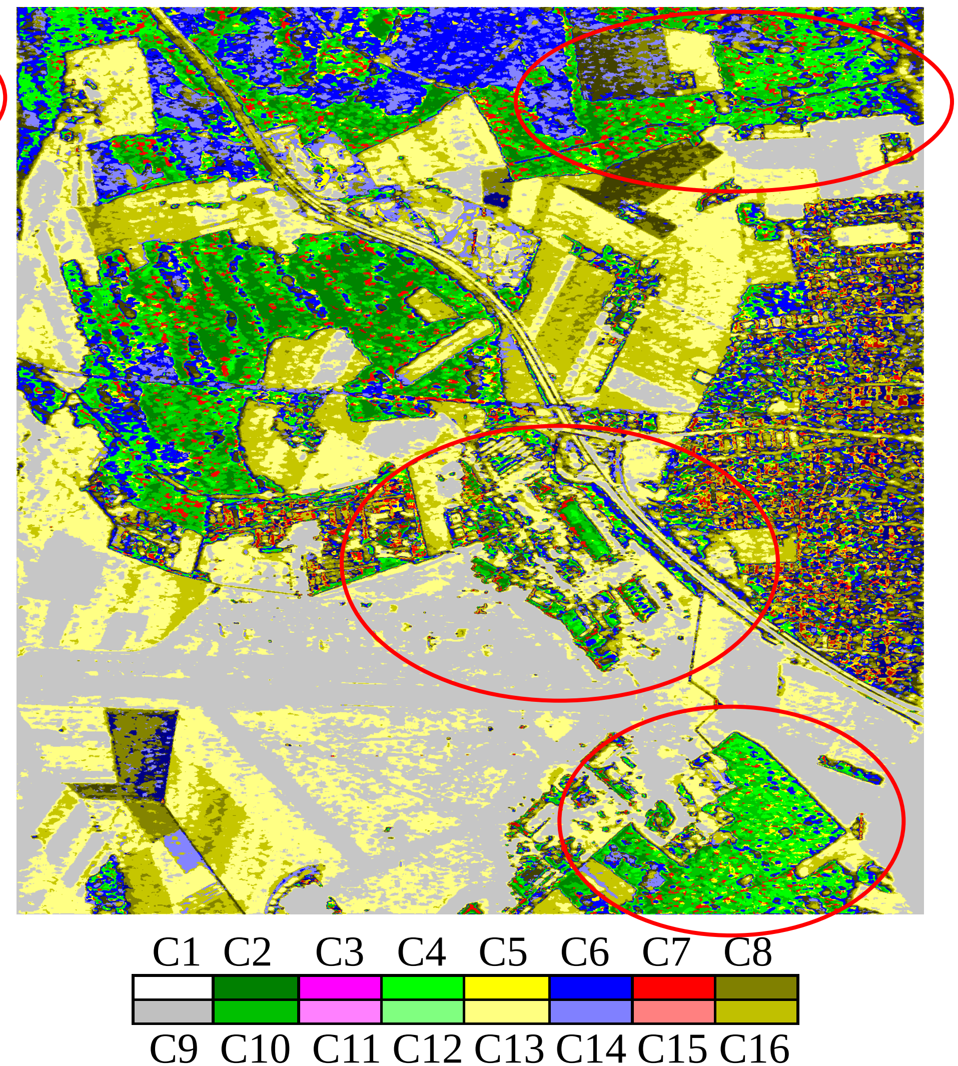
INTERFEROMETRIC COHERENCE γ

$2A_0$ $B_0 + B$ $B_0 - B$

Wishart H-A-α segmentation



INTERFEROMETRIC COHERENCE γ

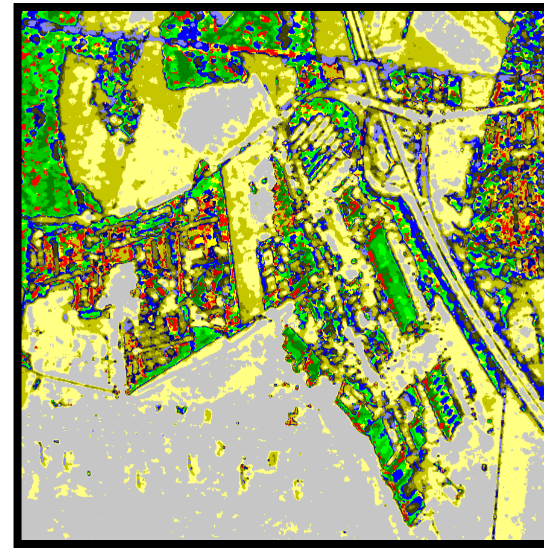


POL-InSAR

Optical Image

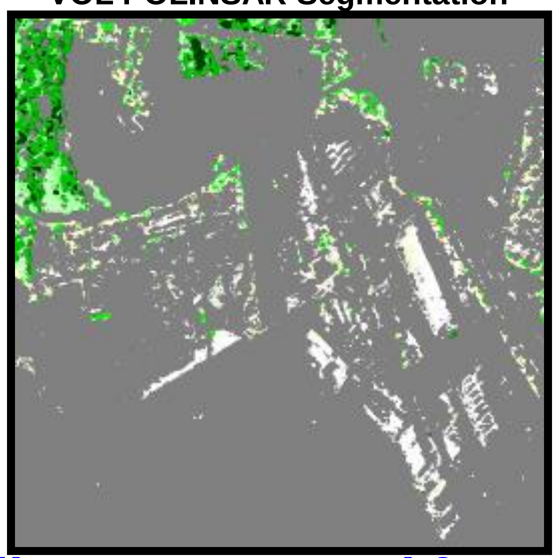
POLSAR Image

POLSAR Segmentation

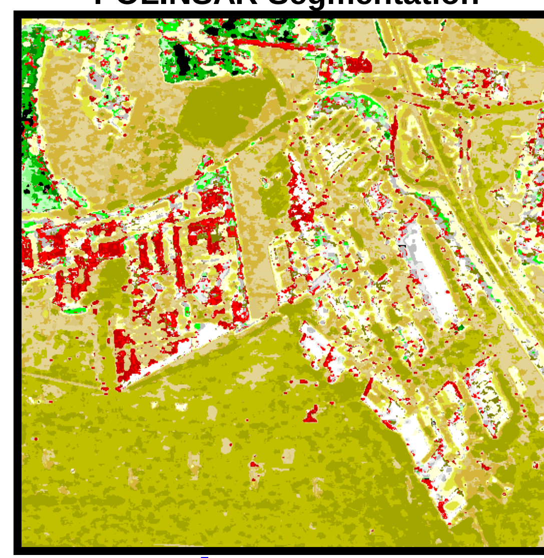


INSAR Image

VOL POLINSAR Segmentation



POLINSAR Segmentation



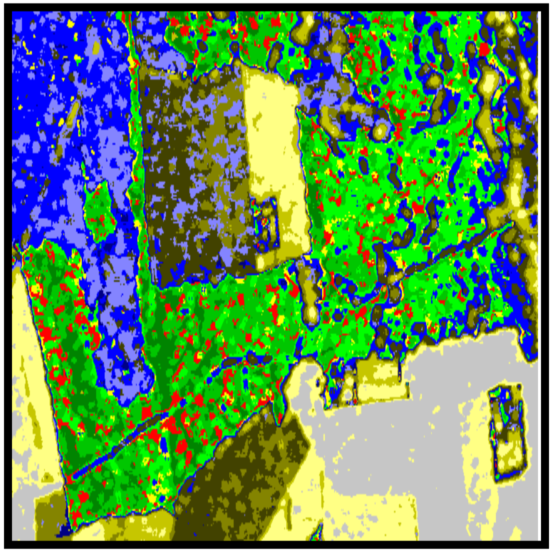
Oriented buildings segmented from vegetated areas

POL-InSAR

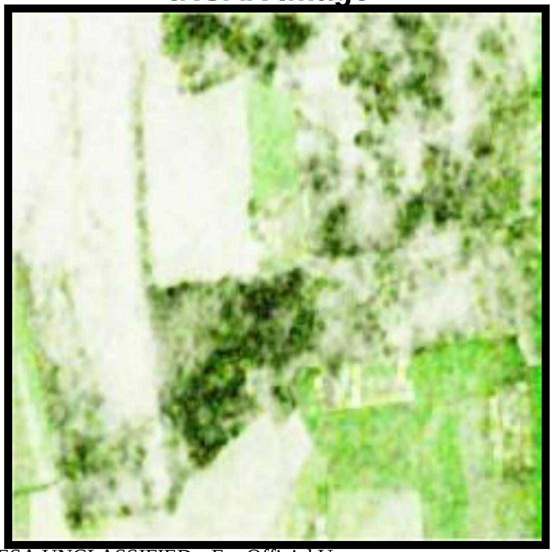
Optical Image

POLSAR Image

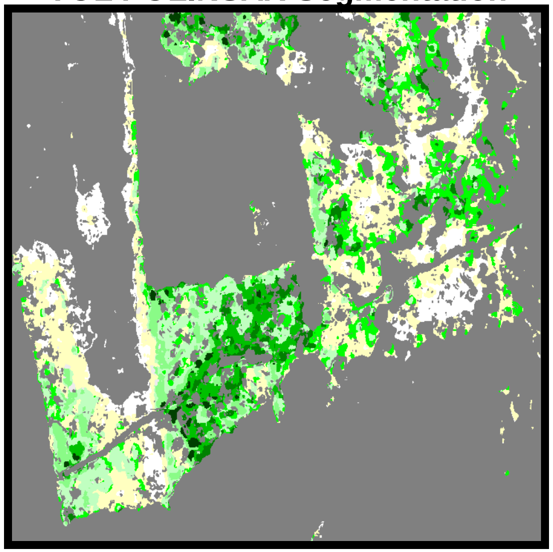
POLSAR Segmentation



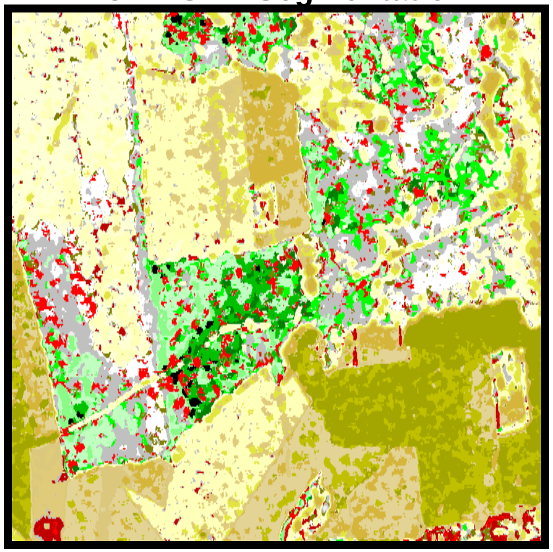
INSAR Image



VOL POLINSAR Segmentation

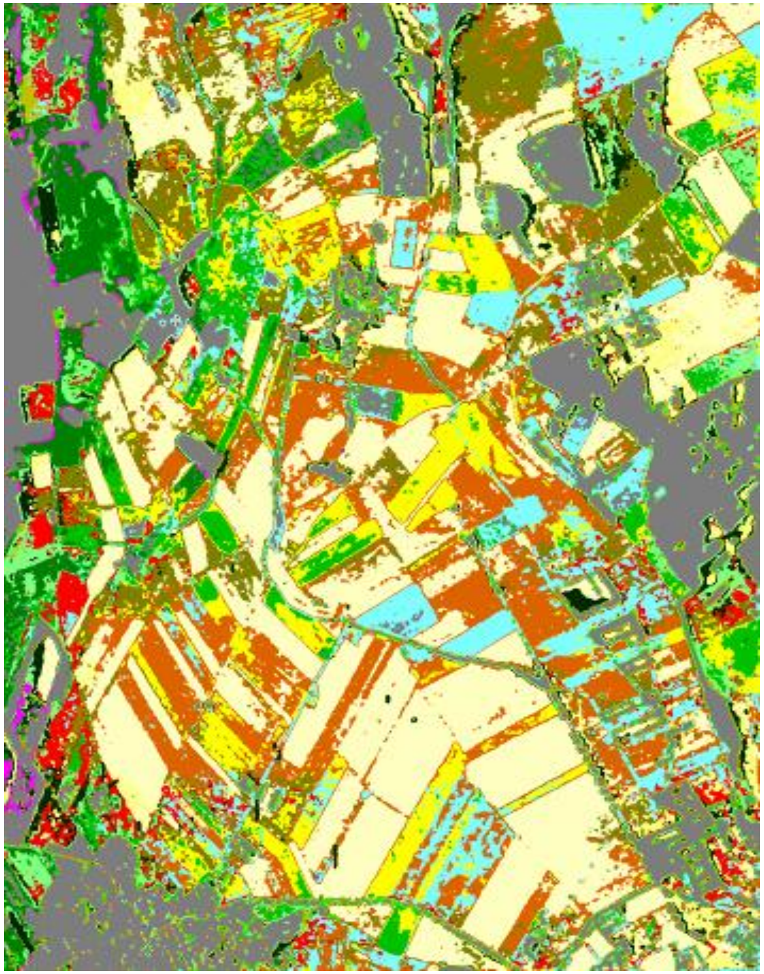


POLINSAR Segmentation



Low density forested areas segmented from dense forest

ALLING - ESAR L-band



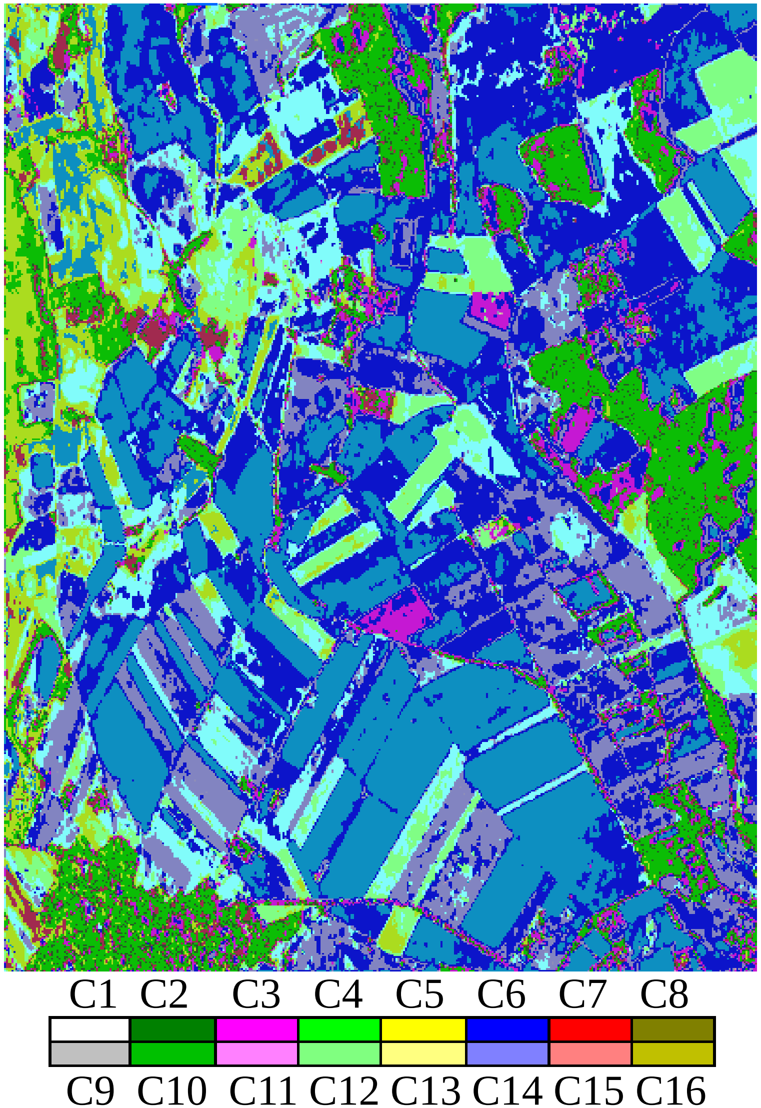
$2A_0$ $B_0 + B$ $B_0 - B$

POL-InSAR

ALLING - ESAR L-band

$2A_0$ $B_0 + B$ $B_0 - B$

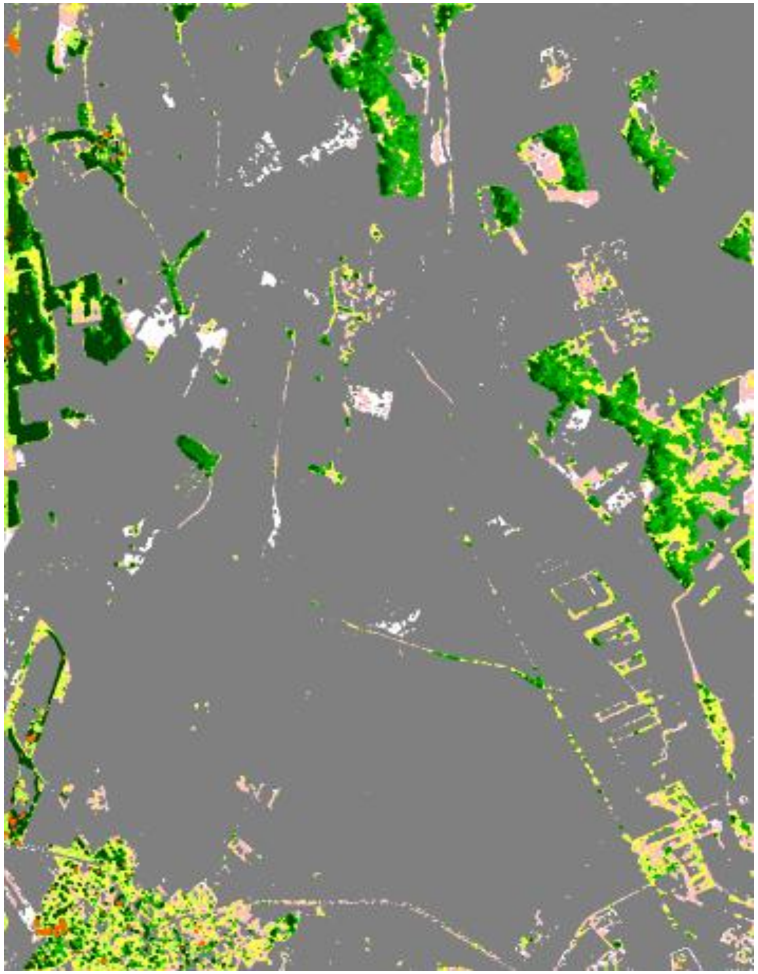
H / A / α and WISHART CLASSIFIER



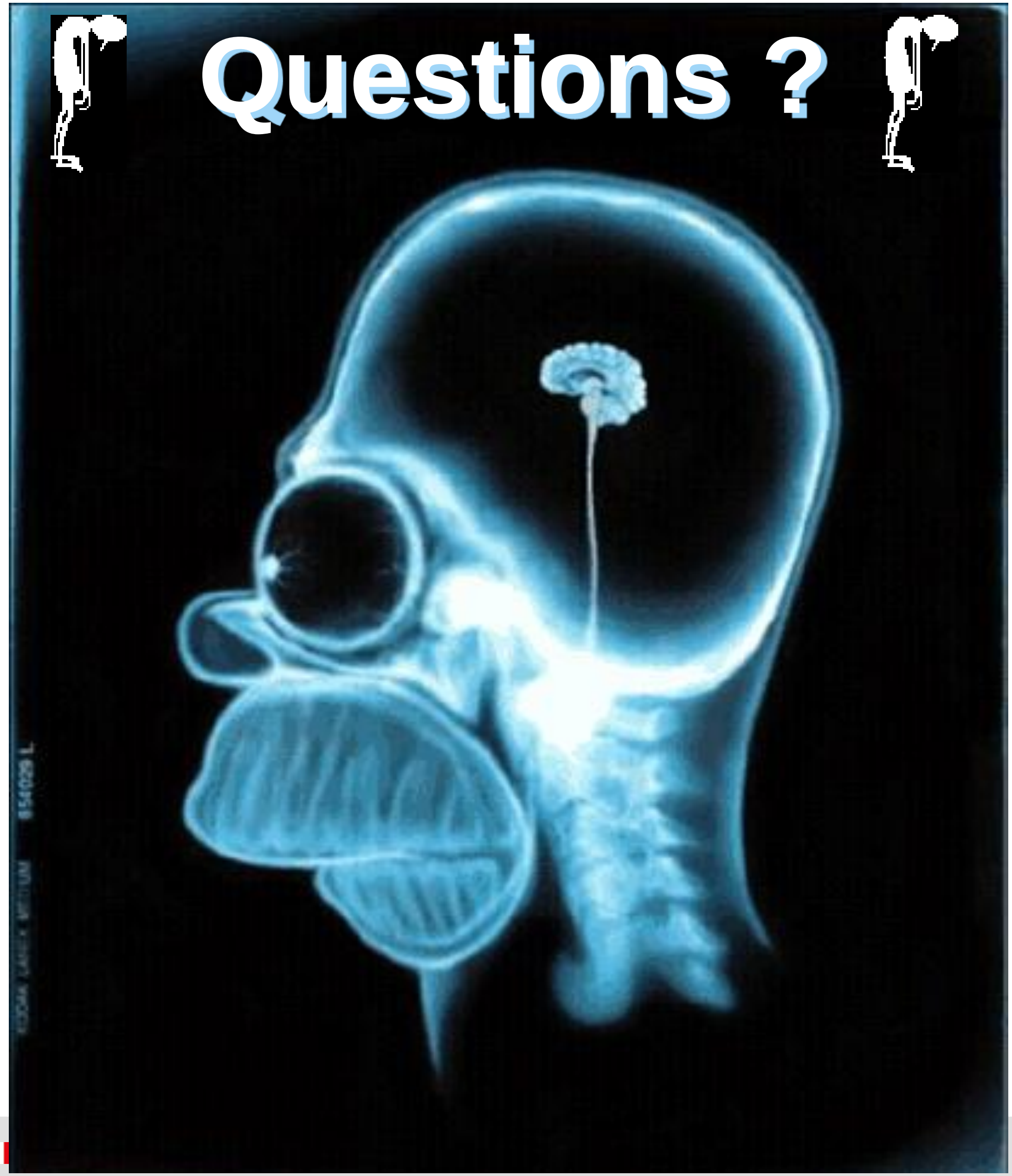
DLR E-SAR L Band – Pol-InSAR (1.5m x 3m) – Baseline 5m

POL-SAR INFORMATION

IN-SAR INFORMATION $|\gamma|$



Oriented Targets segmented from Vegetated Areas


Questions ?