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High-quality fringe pattern generation using binary pattern optimization through symmetry and periodicity

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ABSTRACT

This paper presents a novel method to construct binary patterns for high-quality 3D shape measurement. The algorithm generates small patches using symmetry and periodicity, randomly initializes each pixels, optimizes the small patches through mutations, and finally tiles the optimized patches into full size patterns using again symmetry and periodicity. We will demonstrate that the proposed method can achieve substantial phase quality improvements over the dithering techniques for different amounts of defocusing.

1. Introduction

In the past decade, digital fringe projection (DFP) techniques have been increasingly used for high-quality 3D shape measurement due to its flexibility [1], but have the major limitations of speed bottleneck (typically 120 Hz) and projection nonlinearity. Our recently proposed binary defocusing technique [2,3] has demonstrated the great promise of overcoming the limitations of conventional DFP technique. Yet, the high-frequency harmonics substantially influence the measurement quality if the measurement depth range is large and the required measurement speed is high [4]. Xu et al. [4] proposed a passive error compensation method to alleviate the high-frequency harmonic influences. This technique has demonstrated its success if high-quality calibration is performed pixel by pixel. However, the improvement is rather limited if the fringe stripes are wide.

Actively modulating the squared binary patterns has been extensively studied and shows even greater improvements. Among these methods, the pulse width modulation (PWM) techniques [5–8] change the squared binary patterns in one dimension. These techniques either shift the high-order harmonics further away from fundamentally frequency such that they are easier to be suppressed by defocusing [5], or theoretically eliminate those most influential harmonics [6–8]. These techniques indeed could improve measurement quality. However, due to the discrete nature of fringe generation, the PWM techniques can only generate high-quality sinusoidal fringe patterns when the fringe stripes are narrow [9]. PWM techniques only modulate the patterns in 1D, and thus their ultimate enhancements are rather limited. 2D area modulation techniques [10] could further improve the quality. However, it is difficult for all these techniques to generate high quality fringe pattern when fringe stripes are wide.

It turns out that the dithering techniques [11–16], developed to represent high-bit number images with binary images, could improve fringe quality for wider fringe stripes [17]. These techniques endeavor to maintain low-frequency information such that the overall image appears to be the original once a low-pass filter is applied. However, we found that if the fringe stripes are narrow, the improvement was rather small. Unfortunately, for 3D shape measurement, high-frequency sinusoidal fringe patterns are usually desirable since they provide better measurement quality. Optimizing dithered patterns could improve the fringe quality. We have recently developed a genetic algorithm to drastically improve the phase quality when fringe stripes are narrow [18]. However, this technique is very time consuming (taking hours), and sometimes the algorithm does not converge if the initial genes are not good.

The objective of all these optimization techniques is to obtain the best fit of the binary patterns to the ideal sinusoidal pattern. In other words, the optimized binary patterns should be as close as possible to the ideal sinusoidal patterns after applying Gaussian smoothing. Mathematically, we are minimizing the functional

$$\min_{B,G} \| I(x,y) - G(x,y) \otimes B(x,y) \|_F \tag{1}$$

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where $\|\cdot\|_F$ is the Frobenius norm, $I(x,y)$ is the ideal sinusoidal intensity pattern, $G(x,y)$ is a 2D Gaussian kernel, $B(x,y)$ is the desired 2D binary pattern, and $\otimes$ represents convolution. Unfortunately, the problem is Non-deterministic Polynomial-time (NP) hard, making it impractical to solve the problem mathematically. Furthermore, the desired pattern should perform well for different amounts of defocusing (i.e., varying $G(x,y)$), making the problem even more complex.

This paper presents a method to conquer these challenges. This technique comes from our two observations: (1) the binary patterns should be symmetric for one fringe stripe since the resultant sinusoidal patterns are symmetric, and (2) the binary pattern should be periodical in both x and y directions since the desired sinusoidal patterns are periodical in both directions (with a period of 1 pixel for one direction). The optimization is to minimize the error between the defocused (or blurred) binary pattern and the desired ideal sinusoidal pattern. Since the ultimate goal is to generate high-quality phase for a large depth range, the proposed technique selects the fringe patterns that consistently perform well with different amounts of defocusing.

Section 2 explains the principle of the phase-shifting algorithm and the dithering technique. Section 3 presents the proposed framework for constructing binary patterns. Section 4 shows simulation results. Section 5 presents the experimental results. Section 6 discusses the merits and shortcomings of the proposed technique, and finally Section 7 summarizes this paper.

2. Principle

2.1. Three-step phase-shifting algorithm

Phase-shifting algorithms have been extensively used in optical metrology [19]. Typically, the more fringe patterns used, the better measurement quality can be achieved. For high-speed 3D shape measurement, a three-step phase-shifting algorithm is usually adopted since it requires the minimum number of patterns to solve for the phase uniquely point by point. Since our research focuses on high-speed 3D shape measurement, a simple three-step phase-shifting algorithm with a phase shift of $2\pi/3$ was used to test the generated patterns. Three fringe images can be described as

$$I_1(x,y) = I'(x,y) + I''(x,y)\cos[\phi - 2\pi/3], \tag{2}$$

$$I_2(x,y) = I'(x,y) + I''(x,y)\cos[\phi], \tag{3}$$

$$I_3(x,y) = I'(x,y) + I''(x,y)\cos[\phi + 2\pi/3]. \tag{4}$$

where $I'(x,y)$ is the average intensity, $I''(x,y)$ the intensity modulation, and $\phi(x,y)$ the phase to be solved for

$$\phi(x,y) = \tan^{-1}\frac{\sqrt{3}(I_1 - I_3)}{2I_2 - I_1 - I_3}. \tag{5}$$

This equation provides the wrapped phase ranging $[-\pi, +\pi)$ with 2π discontinuities. A continuous phase map can be obtained by adopting a spatial [20] or temporal phase unwrapping algorithm. In this research, we used the temporal phase unwrapping framework introduced in [21].

2.2. Error-diffusion dithering technique

Compared with the Bayer-ordered dithering technique, the error-diffusion dithering technique is more complicated but achieves higher quality. For an error-diffusion algorithm, the pixels are quantized in a specific order, and the quantization error for the current pixel is propagated forward to local unprocessed pixels through the following equation:

$$\tilde{f}(i,j) = f(i,j) + \sum_{k,l \in S} h(k,l)e(i-k, j-l). \tag{6}$$

Here, $f(i,j)$ is the original image, $\tilde{f}(i,j)$ the quantized image, and $e(i,j)$ the quantization error: the difference between the quantized image pixel and the diffused image pixel. $e(i,j)$ is further diffused to its neighboring pixels through a two-dimensional weighting function $h(i,j)$, known as the diffusion kernel. There are numerous error-diffusion dithering algorithms differing on the diffusion kernel selection. In this paper, we use one of the most accurate methods proposed by Floyd–Steinberg, with the following diffusion kernel:

$$h(i,j) = \frac{1}{16}\begin{bmatrix} - & * & 7 \\ 3 & 5 & 1 \end{bmatrix} \tag{7}$$

Here − represents the processed pixel, $*$ represents the pixel in process. It should be noted that the kernel coefficients sum to one, and thus the local average value of the quantized image will be equal to the local average of the original one.

3. Binary pattern construction framework

As discussed earlier, the objective of all these optimization techniques is to obtain the best fit of the binary patterns to the ideal sinusoidal pattern. In other words, the optimized binary patterns should be as close as possible to the ideal sinusoidal patterns after applying Gaussian smoothing. Mathematically, we are minimizing the functional

$$\min_{B,G} \|I(x,y) - G(x,y) \otimes B(x,y)\|_F. \tag{8}$$

This problem is NP hard, making it impractical to solve the problem mathematically. Furthermore, the desired pattern should perform well for different amounts of defocusing (i.e., varying $G(x,y)$), making the problem even more complex.

Instead of optimizing the desired fringe pattern as a whole (e.g., 800×600) as our previously proposed [18,22], we propose to optimize a subset called *binary patch*, and then tile the patch to generate the full-size patterns using symmetry and periodicity. Unlike those PWM techniques, the proposed technique belongs to area modulation technique where the modulations occur in both x and y directions. Compared with the dithering techniques, the proposed technique strives to generate higher quality fringe patterns with narrow fringe stripes, and similar quality for broad fringe stripes. In addition, unlike the previously proposed method [18] where the optimization is performed under one defocusing level, the proposed method improves fringe quality for different amounts of defocusing.

Assume that the desired sinusoidal fringe patterns vary along x direction: the best-fit binary pattern should be symmetric along x direction for one fringe period (T); and it should be periodic along the y direction. *Row period*, S_y, is defined as the period along y direction. We believe that different breadths of fringe patterns require different optimization strategies, and thus we could utilize different row periods for different breadths of fringe patterns. Instead of directly solving the best-fit *NP*-hard problem, we propose to modulate a small binary patch for each fringe pattern, and then tile the patch together using symmetry and periodicity of the fringe pattern. The process of modulating a binary patch to generate the whole binary pattern can be divided into the following major steps:

*Step*1: *Patch formation*. This step initializes the S_y (2 to 10), and defines the number of pixels along x direction. The patch is formed as a dimension of $S_x \times S_y$, here $S_x = T/2$ is one half fringe period.

*Step*2: *Patch initialization*. Randomly assign each pixel of the $S_x \times S_y$ patch with 0 or 1.

*Step*3: *Patch optimization*. For each pixel in the binary patch, its binary status is mutated (i.e., 1 to 0 or 0 to 1). If this mutation improves fringe quality, i.e., the intensity root-mean-square (rms) difference between the Gaussian smoothed pattern and the ideal sinusoidal pattern is smaller, the mutation is regarded as a good mutation. It should be noted that mutating one pixel will influence its neighborhood after applying a Gaussian filter. Therefore, the optimization is iteratively performed until the algorithm converges when the rms error difference for a new round of iterations is less than 0.01%.

*Step*4: *Patch variation*. Repeat Steps 2–3 for a number of times (ranging from 50–500 times) to generate a number of good candidates for each S_y, a number of good patches could be generated because of the random initialization.

*Step*5: *Patch dimension mutation*. Change S_y to another value (i.e., 2 to 10), and go to Step 2.

*Step*6: *Patch selection*. After a number of patch mutations, a set of optimized patches are generated. From these patches, the *best patch* is selected based on the following two rules: (1) phase error does not change drastically if a different size of Gaussian filter is applied; and (2) the resultant phase error is consistently small. These two rules imply that the best patch under one amount of defocusing may not be chosen. This is one of the fundamental difference between our algorithm and the previously developed genetic algorithm [18].

*Step*7: *Fringe pattern generation*. Utilizing the symmetry and periodicity properties of the fringe patterns, the desired size fringe pattern was generated by tiling the best patch together.

Fig. 1 illustrates how to select the final pattern to use. Fig. 1a–c shows three examples of optimized patterns when fringe period $T=18$. Pattern 1 performs the best when the amounts of defocusing is larger (Gaussian filter size 11 or larger). If one considers the well defocused projector, this might be a good candidate. However, our major focus is to improve the fringe quality when the projector is nearly focused (i.e., Gaussian filter size is small). However, this candidate does not perform well with small amount of defocusing. Therefore, we did not choose this candidate. Pattern 2 depicts the smallest phase error when the filter size is 7 or 11, but does not perform consistently well. This one was not chosen since we require the pattern consistently perform well over different amounts of defocusing. Pattern 3 is the one we chose because this pattern performs consistently across different amounts of defocusing.

From this figure, one may also notice that the phase rms error fluctuates with the increased size of Gaussian filter. This is because our proposed optimization was performed under a special amount of defocusing, i.e., a fixed size matrix G of Eq. (1) (5×5 in our case). However, because the optimized pattern is the best pattern among those candidates, is not the result of exhaustive search, we cannot guarantee that the phase rms error is still minimized for different amounts of defocusing. Nevertheless, even with such fluctuations, the phase rms error is always smaller than the

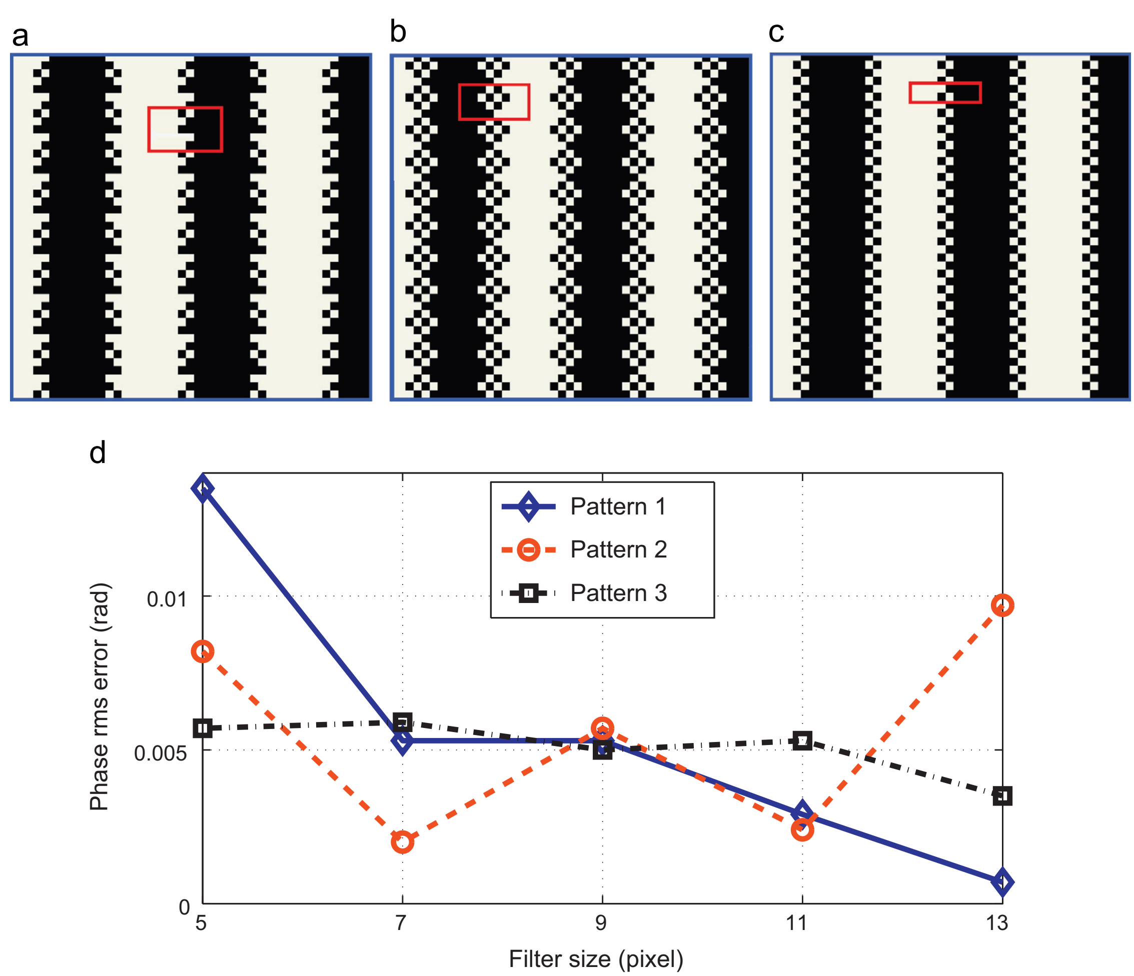

Fig. 1. Example of selecting the pattern from the optimized binary patches. (a) Pattern 1: $T=18$, $S_y=5$; (b) pattern 2: $T=18$, $S_y=4$; (c) pattern 3: $T=18$, $S_y=2$; (d) phase rms error with different amounts of defocusing.

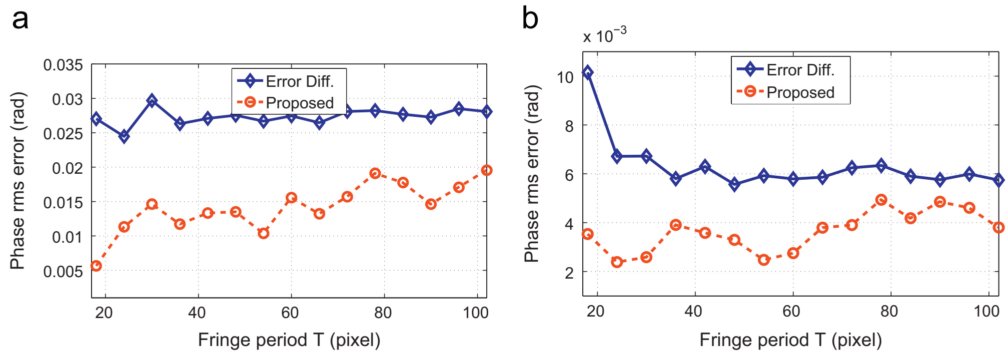

Fig. 2. Comparing the phase quality between the proposed method and the Floyd–Steinberg error-diffusion technique. (a) Gaussian filter size of 5 × 5 pixels and standard deviation of 5/3 pixels; (b) Gaussian filter size of 13 × 13 pixels and standard deviation of 13/3 pixels.

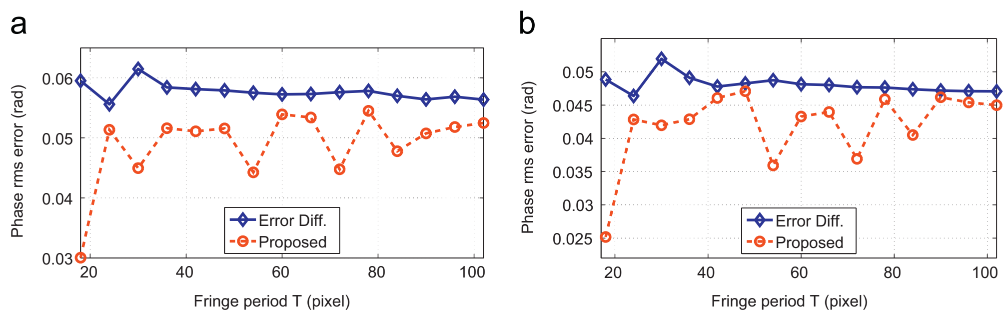

Fig. 3. Experimentally comparing the phase quality between the proposed method and the Floyd–Steinberg error-diffusion technique. (a) Nearly focused; (b) slightly defocused.

error-diffusion dithering technique, as will be shown in Figs. 2 and 3, indicating the success of the proposed method.

4. Simulations

We simulated different amounts of defocusing by applying different sizes of Gaussian filters. The smallest Gaussian filter was 5 × 5 with a standard deviation of 5/3 pixels, and the largest was 13 × 13 with a standard deviation of 13/3 pixels. Gaussian filter size of 5 × 5 represents the case that the projector is nearly focused, whilst 13 × 13 represents the case when the projector is defocused to a certain degree. We did not use larger filter sizes as they will jeopardize the fringe contrast, which is not usually used in real measurements. The phase error $\Delta\Phi$ was calculated by taking the difference between the phase obtained from the smoothed binary patterns, Φ^b, and the phase obtained from the ideal sinusoidal fringe patterns, Φ^i. $\Delta\Phi_o = \Phi_o^b - \Phi^i$ is the phase error obtained from the optimized binary patterns, and $\Delta\Phi_e = \Phi_e^b - \Phi^i$ is the phase error using the error-diffusion dithered patterns.

Fig. 2 shows the simulation results. The simulation results clearly show that the proposed method can substantially improve the fringe quality for different amounts of defocusing. For instance, the improvement is over 40% when fringe period $T = 18$ pixels. It also indicates that when the fringe period increases, the improvement decreases. This is because the error-diffusion technique has already generated good quality sinusoidals for low-frequency patterns.

5. Experiments

We also conducted experiments to verify the performance of the proposed technique. The 3D shape measurement system includes a digital-light-processing (DLP) projector (Samsung SP-P310MEMX) and a charge-coupled-device (CCD) camera (Jai Pulnix TM-6740CL). The camera is attached with a 16 mm focal length Mega-pixel lens (Computar M1614-MP) with F/1.4 to 16C. The camera has a resolution of 640 × 480, and the projector has a native resolution of 800 × 600 with a projection distance of 0.49–2.80 m.

We experimentally verified the simulation results by measuring a flat white board using all these fringe patterns. Fig. 3 shows the results. The phase errors were determined by taking the difference between the phase obtained from the binary patterns (the dithered patterns and the proposed patterns) after Gaussian smoothing and the phase obtained from the ideal sinusoidal patterns. Again, the proposed algorithm generated better results than the error-diffusion algorithm at different amounts of defocusing.

A more complex 3D statue was measured to visually compare these methods. Fig. 4 shows the results. Fig. 4(a) shows one of the binary patterns, indicating that the projector was nearly focused. In this experiment, we used the fringe period of $T = 18$ pixels, and converted the phase to depth using the simple reference-plane based method discussed in Ref. [4]. This figure shows that at different amounts of defocusing, the results obtained by using the proposed method are much better than the error-diffusion method, or the squared binary technique.

Fig. 5 shows the zoom-in views around the nose areas for the measurement results. These results clearly show that when the projector is nearly focused, neither the squared binary method nor the dithering technique could provide reasonable quality measurement; and the proposed technique could perform much better than both methods. When projector is slightly defocused, all these techniques can perform well with the proposed method yielding the best result, again. It is interesting to notice that when the projector is nearly focused, the standard error-diffusion dithering

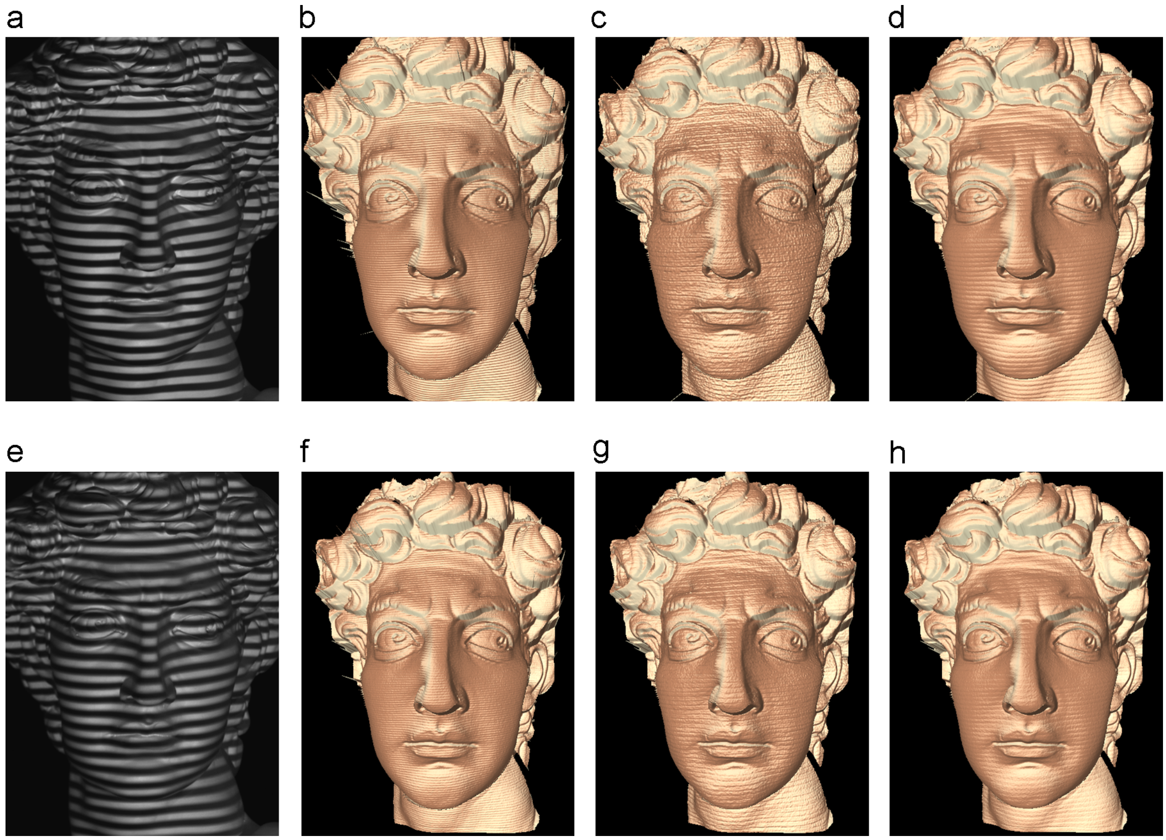

Fig. 4. Measurement results of a complex 3D statue. (a) One nearly focused fringe pattern; (b), (c), and (d) respectively shows the 3D result with squared binary pattern, the dithered patterns, and the proposed patterns when the projector is nearly focused; (e) one slightly defocused fringe pattern; (f)–(h) respective shows the 3D result with the squared binary pattern, the dithered patterns, and the proposed patterns when the projector is slightly defocused.

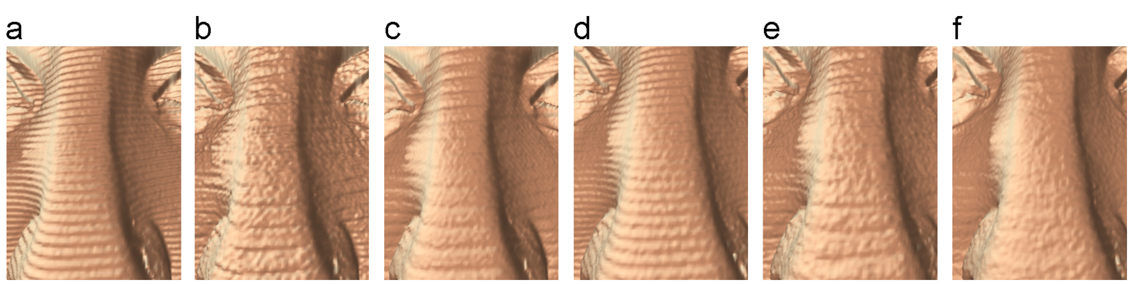

Fig. 5. Zoom-in views around the nose areas for measurement results with different methods. (a)–(c) respectively shows the zoom-in view of the results shown in Fig. 4(b)–(d) when the fringe is nearly focused; (d)–(f) respectively shows the zoom-in view of the results shown in Fig. 4(b)–(d) when the projector is slightly defocused.

technique actually cannot outperform the squared binary method. This is because the error-diffusion technique tries to keep low frequency information while sacrifices high frequency information, and the fringe frequency here is quite high for fringe period of 18 pixels.

6. Discussions

Compared with the genetic optimization algorithm proposed previously [18], the proposed method has the merit of speed (seconds instead of hours). It also has the advantage of generating periodical patterns, making it easier to be realized on hardware since the element used is very small. In addition, the proposed algorithm can ensure high-quality phase for different amounts of defocusing while the genetic algorithm cannot.

Compared with all our previous research along the same direction [18,17,22], this proposed method is fundamentally different from any of them, where they either directly generated binary patterns using a dithering technique [17], or modified the dithered patterns through some optimization strategies [18,22]. In other words, they all started from the dithered patterns. The proposed technique, in contrast, starts with randomly assigned patterns, and performs optimization.

However, the proposed algorithm does involve some manual process when two candidates generate similar results. For example, for fringe period $T=24$, we have two candidate patches shown in Fig. 6. Pattern 1 was chosen because the phase error performs more consistently across different amounts of defocusing, as shown in Fig. 6(c). One may notice that Pattern 1 actually has larger phase error when filter size is 9×9 than Pattern 2.

7. Conclusion

This paper has presented a method to generate high-quality sinusoidal fringe patterns with binary patterns. We found that this

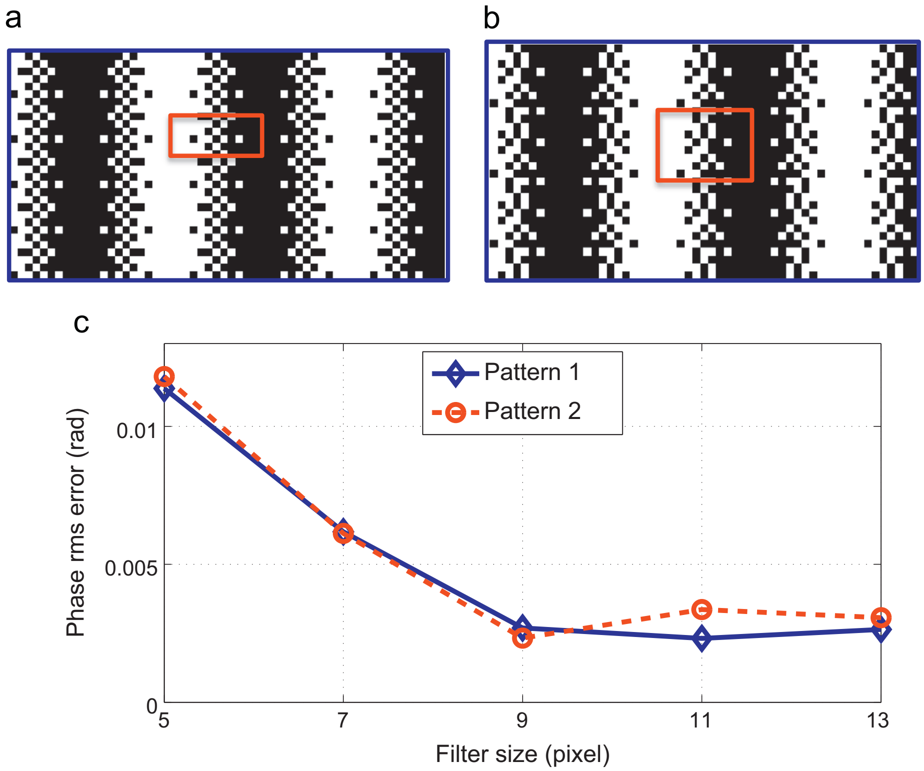

Fig. 6. Example of selecting the pattern from the optimized binary patches. (a) pattern 1: $T=24$, $S_y=6$; (b) pattern 2: $T=24$, $S_y=9$; (c) phase rms error with different amounts of defocusing.

method could drastically improve the high-accurate error-diffusion method for different amounts of defocusing. Both simulation and experimental results demonstrated the success of the proposed technique.

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