Spatial and Spatiotemporal Matching Framework for Causal Inference

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Abstract

Matching is a procedure aimed at reducing the impact of observational data bias in causal analysis. Designing matching methods for spatial data reflecting static spatial or dynamic spatio-temporal processes is complex because of the effects of spatial dependence and spatial heterogeneity. Both may be compounded with temporal lag in the dependency effects on the study units. Current matching techniques based on similarity indexes and pairing strategies need to be extended with optimal spatial matching procedures. Here, we propose a decision framework to support analysts through the choice of existing matching methods and anticipate the development of specialized matching methods for spatial data. This framework thus enables to identify knowledge gaps.

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1 Introduction

When collecting data to analyze causal effects of an intervention, Randomised Controlled Trials (RCTs) are the theoretical best practice. Yet RCTs are costsly and complex [15, 18]. Quasi-experimental methods on observational data have been proposed, i.e. on data collected for different purposes and only re-analyzed to identify causal effects of interventions. Causal studies on observational data lack control over the design and data collection process, making it impossible to manage the selection and confounding bias. Matching is the analytical step that aims to reduce such bias by controlling for the imbalance between the characteristics of the units in the treated and control groups based on the distribution of covariates [16].

Matching on spatial data is an emerging topic in spatial causal inference [1], where theoretical assumptions of independent random processes do not hold due to first-order effects (spatial heterogeneity) and second-order effects (spatial lags) [10]. Matching methods find pairs of units from the treated and control groups based on the similarity measured from baseline covariates. For non-spatial data, established methods are available, e.g., Propensity Score (PS) [12] and Mahalanobis Distance (MD) matching [13]. These methods do not consider the effects of spatial heterogeneity and spatial autocorrelation and may lead to biased estimates of causal effects in spatial data.

Here we investigate the requirements for considering spatial data characteristics in *spatial matching*, addressing the question: *How to measure the similarity and match spatial units of control and treated groups in spatial causal inference?* We describe the challenges along the analytical process of (1) spatiotemporal dependence estimation, (2) covariate selection and

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prioritization, (3) similarity measurement, and finally, (4) the optimal matching and pairing of spatial units. We present an outline of a decision-making workflow, enabling to reduce bias in spatial causal analysis.

2 Spatial/Spatiotemporal Matching Framework

2.1 Data Generation Processes

Causal inference on spatial data should lead to better insights into the spatial data generation process, whether static (i.e., single snapshot) or dynamic (spatio-temporal change). Static spatial data are cross-sectional, i.e., reflecting spatial dependence in the system but not capturing change over time. A general static spatial causal model can be expressed by Equation 1, where β and ρ_j are the coefficients that quantify the direct and indirect causal effects, respectively. Y, D, W, X, and ε refer to the outcome variable, treatment, weight matrix, observed covariates, and the error term sequentially. Correspondingly, $\beta, \theta, \rho, \alpha, \gamma, \delta$ are coefficients.

$$\begin{gathered}
Y_i = \overbrace{\theta X_i}^{\text{covariates}} + \underbrace{\beta D_i + \textstyle\sum_j^n \rho_j W D_j}_{\text{causal effects}} + \overbrace{\textstyle\sum_j^n \alpha_j W Y_j}^{\text{spatial lag of outcomes}} + \overbrace{\textstyle\sum_j^n \gamma_j W X_j}^{\text{spatial lag of covariates}} + \upsilon_i \\
\upsilon_i = \underbrace{\textstyle\sum_j^n \delta W \upsilon_j}_{\text{spatial lag of residuals}} + \varepsilon_i
\end{gathered} \tag{1}$$

In dynamic spatial systems, temporal dependence should be considered in addition to spatial dependence. Data capturing dynamical systems are *spatial panel data*, and these require spatiotemporal causal models. Equation 2 shows the structure of the general spatiotemporal data generation process resulting in spatial panel data. There, β_2 and ρ_j are the coefficients that need to be determined to quantify the direct and indirect causal effects sequentially. t is the time variable, where Y_{jt} and X_{jt} are the outcome variable and covariates of adjacent neighbors for spatial unit i at time t. $Y_{i,t-l}$ and $X_{i,t-l}$ are the outcome and covariates of time lag l of spatial unit i where L is the number of effective temporal lags chosen by the researcher. Similarly, $Y_{j,t-l}$ and $X_{j,t-l}$ are the outcome and covariates of different time lags of neighbours of the spatial unit i. Spatial and spatiotemporal observational data generation processes thus hold different characteristics which should be reflected in matching strategies.

$$\begin{gathered}
Y_{it} = \overbrace{\theta X_{it}}^{\text{covariates}} + \overbrace{\beta_1 t}^{\text{time effect}} + \overbrace{\beta_2 D_{it} + \textstyle\sum_j^n \rho_j W D_{jt}}^{\text{causal effects}} + \overbrace{\textstyle\sum_j^n \alpha_j W Y_{jt}}^{\text{spatial lag of outcomes}} + \overbrace{\textstyle\sum_j^n \gamma_j W X_{jt}}^{\text{spatial lag of covariates}} + \\
\underbrace{\textstyle\sum_l^L (\phi_l Y_{i,t-l} + \varphi_l X_{i,t-l})}_{\text{temporal lags}} + \underbrace{\textstyle\sum_l^L (\kappa_l W Y_{j,t-l} + \lambda_l W X_{j,t-l})}_{\text{spatio-temporal lags}} + \upsilon_{it} \\
\upsilon_{it} = \underbrace{\textstyle\sum_j^n \delta W \upsilon_{jt}}_{\text{spatio-temporal lag of residuals}} + \varepsilon_{it}
\end{gathered} \tag{2}$$

2.2 The Spatial and Spatiotemporal Matching Process

Imagine the need to measure the causal effect of an intervention, e.g., the effect of opening new train stations on the average suburb property prices. There are two groups of suburbs: suburbs with new stations (treated group) and those without (control group). Because we lack control over the assignment of suburbs to treated and control groups, a matching process is needed to manage the selection and confounding biases on the measured causal effects, i.e., price increases may be due to other effects. Figure 1 shows a simple matching process for five suburbs. A similarity index is computed based on the values of different covariates, and similar suburbs from the treated and control groups are then matched (suburb 1 $\longrightarrow$ suburb 4, and suburb 2 $\longrightarrow$ suburb 5). During the matching process, some units may be pruned (e.g., suburb 3).

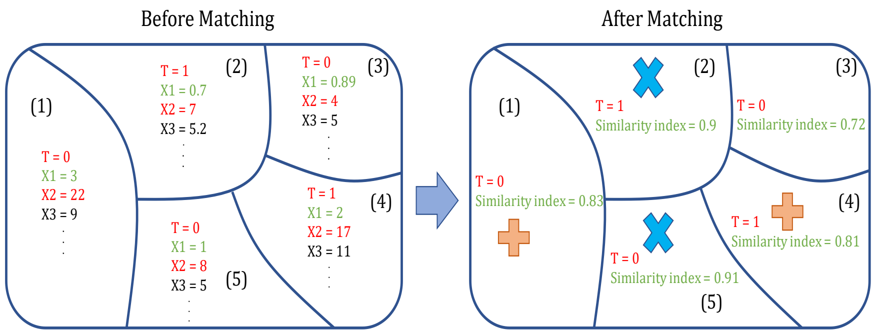

Figure 1 Matching process for suburbs (spatial units); T - treatment, X_i - covariates.

In spatial and spatiotemporal matching (S/STM), we are analyzing data that are sets of variables $\{X_i\}_{i \in S}$ over a study area (S) partitioned into a set of spatial units (locations) in the d-dimensional Euclidean space R_d (where is typically 2). X_i refers to a measured variable at unit i [9]. For each study area S, we may observe a distinct spatial process. The outcome variable $\{Y_i\}_{i \in S}$ then captures a multivariate spatial process. Moreover, the similarity index measured based on the spatial processes $\{X_i\}_{i \in S}$ will be, in turn, a type of a multivariate spatial process, too. We define the similarity index of spatial units $\{Sim_i\}_{i \in S}$ as a multivariate spatial process in S, where $\{Sim_i\}_{i \in S} = g(\{X_i\}_{i \in S})$. g is a similarity measurement function based on the baseline spatial processes for each unit i. For spatial matching we define M with a function of pairing and matching based on the measured Sim_i for the treated and control units, $M = f(k, j \in S | \{Sim_k\}_{k \in S_1}, \{Sim_j\}_{j \in S_2}, W)$. f is a function that matches, i.e., pairs of units – a unit k from the treated group (S_1) and a matching unit j from the control group (S_2), conditional on the similarity based on the values of covariates and the values of a spatial function W, representative of the variation in spatial dependence and heterogeneity.

Optimal matching seeks to find pairs close to a hypothetical exact matching process. The difference between the output of a defined matching procedure and exact matching is known as *imbalance* (I). If I_S is a degree of imbalance for the whole set of spatial units before matching and I_z the imbalance after matching for a subset of the entire dataset z, we aim to reduce imbalance, $I_z < I_S$. Imbalance can thus be defined as a mean of absolute differences of similarity index values between spatial treated and control units, $I(z_m) = \frac{\sum_{k,j \in z_m} (|Sim_k - Sim_j|)}{n}$. Where z_m refers to the set of spatial observations after matching by method m. Sim_k and Sim_j are similarity index values of units in the treated and control groups, respectively, and n is the number of matched pairs identified using method m.

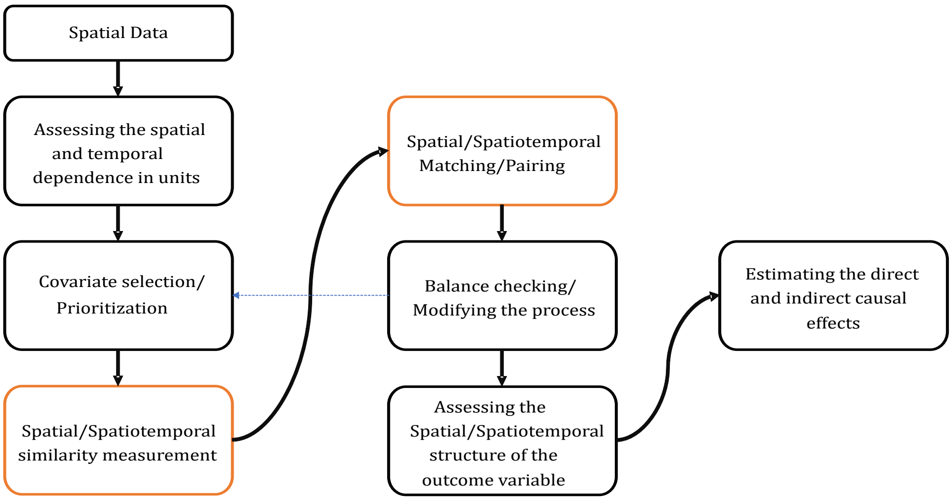

Figure 2 Framework of Spatial Matching Process.

2.3 Components of the Framework

Figure 2 shows our proposed framework of an S/STM procedure to reduce bias in the estimation of spatial causal effects. We now discuss the challenges of each step.

Spatial and Temporal Dependence. Spatial or temporal autocorrelation in covariates is an important factor that must be considered in the S/STM. A similarity index based on spatial covariates of the spatial units may also result in a multivariate spatial process. The spatial dependence in the covariates can be effective in measuring the value of the similarity index. Failing to consider spatial or temporal dependence in the similarity measurement and pairing steps may result in imbalanced and biased matches.

Covariate Prioritisation. Covariate prioritization (CP) is a procedure that enables bringing the qualitative domain expertise of experts to the process of matching and thus minimizing the imbalance. An exact matching method based on categorical covariates (one of the simplest methods for CP) and the caliper matching methods (predefined threshold on a covariate) are both established methods of CP [7]. CP based on expert input can effectively contribute to the matching process and causal effect inference, yet caution is required to avoid bringing human bias into the matching process.

Similarity Measurement of Spatial Units. The effect of the spatial structure of covariates on the value of similarity indices is a challenge for S/STM, as is the effect of time dependence when matching spatial panel data. We defined the spatial similarity with Equation 2.2 as a multivariate spatial process that may be affected by spatial dependence. PS has been suggested as a similarity index for spatial units in a matching procedure [2, 11]. However, King and Nielsen [8] recently showed that PS may lead to matching imbalance, known as the Propensity Score Paradox. Mahalanobis distance has also been applied to quantify the similarity between spatial units, yet it is problematic when applied on units with a large number of covariates that are not normally distributed [5]. Hence, specialized similarity indices should be developed for spatial and spatiotemporal similarity quantification.

Spatial/Spatiotemporal Pairing. Following similarity measurement, the process of finding a suitable match (i.e., pairing) remains one of the main hurdles of the overall process for S/STM. Current pairing methods in S/STM include Nearest Neighbour search, caliper methods, and threshold distance-based search. Matching pairs only based on distance or search radius threshold between the spatial units is not optimal, as it neglects spatial/temporal autocorrelation and heterogeneity. Such simplistic pairing processes may lead to incorrect matches and, consequently, incorrect estimates of causal effects. An integrated pair search method is required for S/STM, including the multidimensional consideration of similarity across time and geographical distance and neighbourhood evaluation. As shown in Equation 2.2, after measuring similarity, the definition of a suitable spatial function W is essential for a match with low imbalance.

Balance Checking. Ultimately, the most influential factor in the outcomes of causal analysis is having a balanced set of treated and control group members after matching. Therefore, balance checking is a critical validation step that enables to have unbiased causal inference. Equation 2.2 shows the quantification of imbalance for a given matching model. The goal is to achieve an imbalance close to the imbalance of a theoretical exact matching process. Typically, a threshold value of imbalance is set for the imbalance index. Metrics for assessing imbalance in the matching process include standardized mean differences [4], and variance ratios [14]. For spatial data, new metrics may be needed enabling to better assess the balance in the matching process on S/STM data, with special consideration for the characteristics of static and dynamic spatial data.

Spatial/Spatiotemporal Structure of Outcome Variables. Common spatial data generation processes include the General Nesting Spatial Model, Spatial Autoregressive Combined Model, Spatial Durbin Model, Spatial Durbin Error Model, Spatial Autoregressive Model, Spatial Lag of X Model, Spatial Error Model, and Ordinary Least Squares Model [6]. In the case of dynamic data, the spatiotemporal data generation processes we may consider the General Nesting Spatiotemporal Model, Vector Autoregressive Model [17], Spatiotemporal Autoregressive Combined Model, Spatiotemporal Durbin Model, Spatiotemporal Durbin Error Model, Spatiotemporal Autoregressive Model, Spatiotemporal Lag of X Model, and Spatiotemporal Error Model [3]. The nuanced selection of the right model applicable to the data generation process will allow for better quantification of the causal effect, which is the final step in our framework.

Estimation of Direct and Indirect causal Effects. The direct and indirect treatment effects resulting from spatial dependence between units must be considered to quantify the true effect of treatment accurately (e.g., a policy intervention applied to different spatial units, for instance, administrative regions). The indirect effects occur when treated spatial units are adjacent to untreated spatial units. In Equations 1 and 2, the first and second terms of the causal effect component relate to the direct and indirect effects, respectively. After matching, an assessment of the structure of the underlying data generation process is needed before the quantification of causal effects.

3 Conclusion

We discussed the challenges of the spatial and spatiotemporal data matching process, a critical analytical step in causal analysis. We proposed an outline of a framework for spatial matching. We discussed why spatial dependence and spatial heterogeneity challenge the matching process on spatial data. The effects of temporal autocorrelation in panel spatial data further complicate matching. We discussed, in particular, the issue of imbalance in matching results, including when applying similarity measurement methods that do not explicitly consider the spatio-temporal structure in the matching process (e.g., PS or MD matching), failing to capture the effects of spatial dependence and heterogeneity. We reflected upon the need to explore nuanced unit similarity measurement in space. We next will address the process of supporting analysts through this matching framework computationally, aiming to investigate whether the interpretation of the structures in the data may be automated to the extent that analysts can be guided through the process.

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