

## DECOMPOSITIONS OF COMPLETE BIPARTITE GRAPHS AND COMPLETE GRAPHS INTO PATHS, STARS, AND CYCLES WITH FOUR EDGES EACH

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### Abstract

Let  $G$  be either a complete graph of odd order or a complete bipartite graph in which each vertex partition has an even number of vertices. In this paper, we determine the set of triples  $(p, q, r)$ , with  $p, q, r > 0$ , for which there exists a decomposition of  $G$  into  $p$  paths,  $q$  stars, and  $r$  cycles, each of which has 4 edges.

**Keywords:** complete graph, complete bipartite graph, path, star, cycle, decomposition.

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