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INDEPENDENCE NUMBER, CONNECTIVITY AND ALL FRACTIONAL (a, b, k)-CRITICAL GRAPHS

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Abstract

Let G be a graph and a, b and k be nonnegative integers with $1 \leq a \leq b$. A graph G is defined as *all fractional* (a, b, k)*-critical* if after deleting any k vertices of G, the remaining graph has all fractional $[a, b]$-factors. In this paper, we prove that if $\kappa(G) \geq \max\left\{\frac{(b+1)^2+2k}{2}, \frac{(b+1)^2\alpha(G)+4ak}{4a}\right\}$, then G is all fractional (a, b, k)-critical. If $k = 0$, we improve the result given in [Filomat 29 (2015) 757–761]. Moreover, we show that this result is best possible in some sense.

Keywords: independence number, connectivity, fractional $[a, b]$-factor, fractional (a, b, k)-critical graph, all fractional (a, b, k)-critical graph.

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