

THE $\{-2, -1\}$ -SELFDUAL AND DECOMPOSABLE TOURNAMENTS

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Abstract

We only consider finite tournaments. The dual of a tournament is obtained by reversing all the arcs. A tournament is *selfdual* if it is isomorphic to its dual. Given a tournament T , a subset X of $V(T)$ is a *module* of T if each vertex outside X dominates all the elements of X or is dominated by all the elements of X . A tournament T is *decomposable* if it admits a module X such that $1 < |X| < |V(T)|$.

We characterize the decomposable tournaments whose subtournaments obtained by removing one or two vertices are selfdual. We deduce the following result. Let T be a non decomposable tournament. If the subtournaments of T obtained by removing two or three vertices are selfdual, then the subtournaments of T obtained by removing a single vertex are not decomposable. Lastly, we provide two applications to tournaments reconstruction.

Keywords: tournament, decomposable, selfdual.

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REFERENCES

- [1] M. Achour, Y. Boudabbous and A. Boussaïri, *The $\{-3\}$ -reconstruction and the $\{-3\}$ -self duality of tournaments*, *Ars Combin.* **122** (2015) 355–377.
- [2] M. Basso-Gerbelli and P. Ille, *La reconstruction des relations définies par interdits*, *C. R. Acad. Sci. Paris, Sér. I Math.* **316** (1993) 1229–1234.
- [3] H. Belkhechine, I. Boudabbous and J. Dammak, *Morphologie des tournois (-1) -critiques*, *C. R. Acad. Sci. Paris, Sér. I Math.* **345** (2007) 663–666.
doi:10.1016/j.crma.2007.11.006
- [4] A. Bondy and R.L. Hemminger, *Graph reconstruction, a survey*, *J. Graph Theory* **1** (1977) 227–268.
doi:10.1002/jgt.3190010306
- [5] H. Bouchaala, *Sur la répartition des diamants dans un tournoi*, *C. R. Acad. Sci. Paris, Sér. I Math.* **338** (2004) 109–112.
doi:10.1016/j.crma.2003.11.018
- [6] H. Bouchaala and Y. Boudabbous, *La $\{-k\}$ -autodualité des sommes lexicographiques finies de tournois suivant un 3-cycle ou un tournoi critique*, *Ars Combin.* **81** (2006) 33–64.
- [7] Y. Boudabbous, J. Dammak and P. Ille, *Indecomposability and duality of tournaments*, *Discrete Math.* **223** (2000) 55–82.
doi:10.1016/S0012-365X(00)00040-6
- [8] Y. Boudabbous and A. Boussaïri, *Reconstruction des tournois et dualité*, *C. R. Acad. Sci. Paris, Sér. I Math.* **320** (1995) 397–400.
- [9] Y. Boudabbous and P. Ille, *Indecomposability graph and critical vertices of an indecomposable graph*, *Discrete Math.* **309** (2009) 2839–2846.
doi:10.1016/j.disc.2008.07.015
- [10] Y. Boudabbous and P. Ille, *Cut-primitive directed graphs versus clan-primitive directed graphs*, *Adv. Pure Appl. Math.* **1** (2010) 223–231.
doi:10.1515/apam.2010.013
- [11] A. Boussaïri, *Décomposabilité, dualité et groupes finis en théorie des relations* (Ph.D. Thesis, Université Claude Bernard, Lyon I, 1995).
- [12] A. Boussaïri, P. Ille, G. Lopez and S. Thomassé, *The C_3 -structure of the tournaments*, *Discrete Math.* **277** (2004) 29–43.
doi:10.1016/S0012-365X(03)00244-9
- [13] A. Cournier and M. Habib, *A new linear algorithm for modular decomposition*, in: *Trees in Algebra and Programming*, S. Tison (Ed(s)), (Springer, 1994) 68–84.
doi:10.1007/BFb0017474
- [14] A. Ehrenfeucht, T. Harju and G. Rozenberg, *The Theory of 2-Structures, A Framework for Decomposition and Transformation of Graphs* (World Scientific, 1999).
doi:10.1142/4197

- [15] W.J.R. Eplett, *Self-converse tournaments*, *Canad. Math. Bull.* **22** (1979) 23–27.
doi:10.4153/CMB-1979-004-6
- [16] R. Fraïssé, *Theory of Relations*, Revised Edition (North-Holland, 2000).
- [17] T. Gallai, *Transitiv orientierbare Graphen*, *Acta Math. Acad. Sci. Hungar.* **18** (1967) 25–66.
doi:10.1007/BF02020961
- [18] F. Harary and E. Palmer, *On the problem of reconstructing a tournament from subtournaments*, *Monatsh. Math.* **71** (1967) 14–23.
doi:10.1007/BF01299955
- [19] P. Ille, *La reconstruction des relations binaires*, *C. R. Acad. Sci. Paris, Sér. I Math.* **306** (1988) 635–638.
- [20] P. Ille, *Recognition problem in reconstruction for decomposable relations*, in: *Finite and Infinite Combinatorics in Sets and Logic*, B. Sands, N. Sauer and R. Woodrow (Ed(s)), (Kluwer Academic Publishers, 1993) 189–198.
doi:10.1007/978-94-011-2080-7_13
- [21] W.M. Kantor, *Automorphism groups of designs*, *Math. Z.* **109** (1969) 246–252.
doi:10.1007/BF01111409
- [22] W.M. Kantor, *Automorphism groups of designs*, *Math. Z.* **109** (1969) 246–252.
doi:10.1007/BF01111409
- [23] G. Lopez, *Deux résultats concernant la détermination d’une relation par les types d’isomorphie de ses restrictions*, *C. R. Acad. Sci. Paris, Sér. A-B* **274** (1972) 1525–1528.
- [24] G. Lopez, *L’indéformabilité des relations et multirelations binaires*, *Z. Math. Logik Grundlag. Math.* **24** (1978) 303–317.
doi:10.1002/malq.19780241905
- [25] G. Lopez and C. Rauzy, *Reconstruction of binary relations from their restrictions of cardinality 2, 3, 4 and $(n - 1)$, II*, *Z. Math. Logik Grundlag. Math.* **38** (1992) 157–168.
doi:10.1002/malq.19920380111
- [26] F. Maffray and M. Preissmann, *A translation of Tibor Gallai’s paper: Transitiv orientierbare Graphen*, in: *Perfect Graphs*, J.L. Ramirez-Alfonsin and B.A. Reed (Ed(s)), (Wiley, 2001) 25–66.
- [27] J.W. Moon, *Tournaments whose subtournaments are irreducible or transitive*, *Canad. Math. Bull.* **22** (1979) 75–79.
doi:10.4153/CMB-1979-010-7
- [28] M. Pouzet, *Application d’une propriété combinatoire des parties d’un ensemble aux groupes et aux relations*, *Math. Z.* **150** (1976) 117–134.
doi:10.1007/BF01215230
- [29] K.B. Reid and C. Thomassen, *Strongly self-complementarity and hereditarily isomorphic tournaments*, *Monatsh. Math.* **81** (1976) 291–304.
doi:10.1007/BF01387756

- [30] M.Y. Sayar, *Partially critical indecomposable tournaments and partially critical supports*, Contrib. Discrete Math. **6** (2011) 52–76.
- [31] J.H. Schmerl and W.T. Trotter, *Critically indecomposable partially ordered sets, graphs, tournaments and other binary relational structures*, Discrete Math. **113** (1993) 191–205.
doi:10.1016/0012-365X(93)90516-V
- [32] J. Spinrad, *P_4 -trees and substitution decomposition*, Discrete Appl. Math. **39** (1992) 263–291.
doi:10.1016/0166-218X(92)90180-I
- [33] P.K. Stockmeyer, *The falsity of the reconstruction conjecture for tournaments*, J. Graph Theory **1** (1977) 19–25.
doi:10.1002/jgt.3190010108
- [34] S.M. Ulam, *A Collection of Mathematical Problems* (Intersciences Publishers, 1960).

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