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# SOME TOUGHNESS RESULTS IN INDEPENDENT DOMINATION CRITICAL GRAPHS

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**Abstract**

A subset $S$ of $V(G)$ is an independent dominating set of $G$ if $S$ is independent and each vertex of $G$ is either in $S$ or adjacent to some vertex of $S$. Let $i(G)$ denote the minimum cardinality of an independent dominating set of $G$. A graph $G$ is $k$-$i$-critical if $i(G) = k$, but $i(G + uv) < k$ for any pair of non-adjacent vertices $u$ and $v$ of $G$. In this paper, we establish that if $G$ is a connected 3-$i$-critical graph and $S$ is a vertex cutset of $G$ with $|S| \geq 3$, then $\omega(G-S) \leq \frac{1+\sqrt{8|S|+1}}{2}$, improving a result proved by Ao [3], where $\omega(G-S)$ denotes the number of components of $G-S$. We also provide a characterization of the connected 3-$i$-critical graphs $G$ attaining the maximum number of $\omega(G - S)$ when $S$ is a minimum cutset of size 2 or 3.

**Keywords:** domination critical, toughness.

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