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THE 1, 2, 3-CONJECTURE AND 1, 2-CONJECTURE FOR SPARSE GRAPHS

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Abstract

The $1, 2, 3$-Conjecture states that the edges of a graph without isolated edges can be labeled from $\{1, 2, 3\}$ so that the sums of labels at adjacent vertices are distinct. The $1, 2$-Conjecture states that if vertices also receive labels and the vertex label is added to the sum of its incident edge labels, then adjacent vertices can be distinguished using only $\{1, 2\}$. We show that various configurations cannot occur in minimal counterexamples to these conjectures. Discharging then confirms the conjectures for graphs with maximum average degree less than $8/3$. The conjectures are already confirmed for larger families, but the structure theorems and reducibility results are of independent interest.

Keywords: $1, 2, 3$-Conjecture, $1, 2$-Conjecture, reducible configuration, discharging method.

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