A Mechanism for Price Differentiation and Slicing in Wireless Networks

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***Abstract*—Slicing has been introduced in 5G networks in order to deliver the higher degree of flexility and scalability required by future services. Slice tenants such as virtual wireless operators, service providers or smart-city services will be able to book a share of the infrastructure, possibly including storage, computing capacity and link bandwidth. However, 5G slicing is attractive for infrastructure providers as long as they are able to generate revenues, while at once satisfying the tenants' competing and variable demands and coping with resources availability.**

This work proposes a flexible mechanism based on a multi-bidding scheme for 5G slice allocation. It is able to attain desirable fairness and efficiency figures in order to serve slice tenants and associated mobile users. Built on the notion of normalised Nash equilibrium, it is also provably overbooking-free even though the players' bids are oblivious to infrastructure resources constraints. Also, it is compatible with standard radio access schedulers used in modern mobile networks.

Finally, a practical algorithm is proposed to drive the system to the socially-optimal operating point via an online procedure rooted on a primal-dual distributed algorithm. Numerical simulations confirm the viability of the mechanism in terms of efficiency and fairness.

***Index Terms*—Game theory, Kelly mechanism, normalised equilibrium, primal-dual algorithms, wireless Network slicing, resource allocation.**

I. Introduction

In the emerging 5G technology, slicing allows mobile network operators (MNO) to offer differentiated services to their customers using shared resource pools. A slice, in this context, is a share of the mobile network operator infrastructure obtained via Software-Defined Networking (SDN) and Network Function Virtualization (NFV) technologies. A slice forms a logical network on top of the physical one [1], [2]. Evolving from previous mobile technology, the 5G core network architecture integrates data-centers into their architectures to support network function virtualisation and computation offloading. Thus, a slice will typically encompass different resource types, such as radio access capacity, edge storage memory and computing power available within the MNO infrastructure [1].

Slicing techniques entered the standardisation phase recently [3] so that specifications 5G system's slicing architecture and requirements are now available. Some technical aspects such as slice insulation and fair slice allocation are still is a key challenge to upgrade LTE technology towards 5G, with large effort by the research community to overcome such technical issues [4], [5], [6], [7], [8]. Using slice insulation, virtual private networks will be shipped on top of the existing mobile network infrastructure with dedicated customer support. Thus, new emerging service providers will demand a slice to offer dedicated services to their customers on top of the MNO's infrastructure, e.g., for real time gaming, multimedia applications, social networks, etc.

Ultimately, slicing will deeply change the business model in the mobile communication industry [9], [10] and a crucial aspect is how to jointly price and share resources assigned simultaneously across slices. Mechanisms to price and share resources have appeared in literature [11], [12], [13], [14], assuming customers would demand several resources at once using vectors of bids and so specify their demands.

However, compared to standard settings, e.g., in cloud computing [11], [12], slicing in 5G networks has key differences. First, in cloud computing the pool of resources is often overprovisioned, whereas in 5G scarce radio resources are critical for QoE and requires careful resources allocation. Second, mobile networks are traditionally designed for fair resource sharing, since near far effects and fading induce very different conditions across a deployment in the same cell and across cells. Third, load conditions across a 5G deployment may be at once dynamic and heterogeneous. Finally, joint slice allocation and pricing schemes need to adhere to Service Level Agreement (SLA), which are de facto mandatory in the telecommunication industry [15]. This heavily discourages overbooking as a viable option for MNOs willing to increase their revenues by adopting new slicing technology.

In this paper, we propose a new theoretical framework for pricing slices of resources, based on the Kelly mechanism and the concept of the normalized Nash equilibrium. The basic Kelly mechanism is a bidding mechanism where slice tenants submit an individual bid to the resource owner to obtain an amount of resource. They receive a fraction of the whole resource proportionally to the received bids, and they pay depending on how much they bid. Thus, tenants bid strategically to obtain a share of a single resources [16], [17], [18]. We shall consider a multi-bid version of the Kelly mechanism, where the MNO exposes to tenants a vector of prices per resource. Multi-bid auctions are a main line of research in cloud computing, where clients compete to purchase bundles of cloud resources [11], [12]. The case for using the Kelly mechanism in 5G networks comes from the fact that it offers high flexibility: it applies to bundled resources, i.e., pre-defined blocks of computing and communication resources

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TABLE I
MAIN NOTATIONS USED THROUGHOUT THE PAPER

Symbol	*Meaning*
$\mathcal{C} := \{1,\dots,C\}$	set of base stations
$\mathcal{S} := \{1,\dots,S\}$	set of slices (tenants)
N_c^s	number of active users on slice s at base station c
γ_c^s	unitary price for bids of slice s at base station c
B_c	total available bandwidth at base station c
b_c^s	the bandwidth allocated to slice s at base station c
V_c^s	the benefit function of slice s at base station c
$\mathbf{V}^s$	the total benefit function of slice s
α_s	alpha fair scheduling parameter for slice s
h_u	channel state of user u
$\mathbf{h}$	the channel state vector for all users
r_u	the rate attained by a user u
p_u	transmission power of user u
x_c^s	bid of slice s for bandwidth at base station c
$x^s = (x_1^s \dots x_C^s)$	vector of bids of player s across base stations
$x_c^{-s} = \sum_{s' \neq s} x_c^{s'}$	bid of all other players but s on base station c
β_n	step size for the learning algorithm

in the form of virtual machines or containers. But it also applies well to elastic radio resources, where it is customary to use utility-based schedulers such as the Proportional Fair Scheduler (PFS) [19].

It is important to notice that, while bidding serves very well the purpose to generate resources demands at the tenants' side, from the MNO's point of view, overbooking may represent the major risk when using a competitive bidding mechanism. In fact, the aggregate behavior of tenants in general will not comply to the MNO's system capacity constraints.

Main contribution. We part from standard propositions of joint pricing and resources allocation in mobile networks in literature. We provide an explicit theoretical connection between price definition, bidding mechanism and coupled constraints across slices. Such fundamental problem can be solved by rooting the pricing scheme in the theory of normalised Nash equilibria, according to the seminal work of Rosen [20]. We solve the problem by cascading two coupled games, namely, the Shadow Pricing Game and the Allocation Game. In a fashion which echoes the original ideas of Kelly on shadow prices for multicommodity flow optimisation [21], the Shadow Pricing Game let the MNO settle the price vector via a uniquely determined normalised Nash equilibrium. The resulting price vector induces a Nash equilibrium in the Allocation Game respecting the resources constraint and thus provably overbooking free. Finally, we show that the price vector can be designed to attain the social optimum for the game. In order to render the mechanism practically viable, we provide an online learning procedure based on a primal-dual distributed algorithm able to drive the system to the target socially optimal equilibrium and requiring at each step to disclose solely the price and the bid vectors generated at each step.

II. OPTIMISATION FRAMEWORK

Let a single MNO having a set of base stations $\mathcal{C}$ shared by a set $\mathcal{S}$ of tenants that need physical network resources in order to serve their users. This can be the case of an application provider serving several customers in mobility. The 5G paradigm envisions for MNO resources to be heterogeneous and include not only standard radio resources such as PRBs, but also storage, CPU and backhauling. The MNO assigns to each tenant a slice of resources, and we assume that each tenant proposes a service covered by all base stations in $\mathcal{C}$. Each tenant's users generate demands, and such demands will inevitably depend on their specific location, thus inducing different slice-dependent demand at each base station.

In this section, we confine the discussion to a RAN version of the the slicing problem, where the MNO schedules wireless resources, namely downlink PRBs among multiple tenants. While the RAN resources allocation problem is a known and well studied one, heterogeneity of traffic demands across tenants and cells captures the main features of slice resources allocation, including fairness issues. The case of multi-resources allocation, spanning other type of infrastructure resources beyond PRBs is an immediate extension of the scheme presented for RAN resources.

Let each slice tenant s be associated with users presence vector $\mathbf{N^s} = (N_1^s, N_2^s.., N_C^s)$ where C is the total number of cells and N_c^s is the number of active users on slice s at base station $c \in \mathcal{C}$. Here a base station is modelled as a finite resource shared by its associated users. We observe that the number of active users associated to the same base station vary across slices, and vary across base stations also for same slice.

First let us consider some fixed channel condition at all users and at all slices, and let r_u be the rate attained by a user u in slice s at cell c. The slice benefit function

$$V_c^s(b_c^s) := \sum_{u=1}^{N_c^s} f_s\left(r_u(b_c^s)\right) \tag{1}$$

where b_c^s is the amount of resources (bandwidth) allocated to slice s at base station c and under α-fair scheduling it holds

$$f_s(r_u) = \begin{cases} \frac{(r_u)^{1-\alpha_s}}{(1-\alpha_s)} & \text{if } \alpha_s \neq 1 \\ \log(r_u) & \text{if } \alpha_s = 1 \end{cases} \tag{2}$$

The meaning of (2) is that, when slice c has received capacity b_c^s, user u of slice s associated to base station c receives a rate which is the α-fair share attained with his peer users on the same slice. The average rate r_u of any user u is determined by the scheduling policy and by all the specific techniques used at physical layer and MAC layer, such as modulation, coding, scheduling, etc.

In the case when the channel per user varies over time, let $b_c^s \log(1 + \frac{p_u h_u}{N_0})$ the instantaneous rate when tenant's user u is scheduled, at transmission power p_u, noise power N_0 and under channel state h_u, where $\mathcal{H}_u$ is the finite set of possible channel states of user u. Vector $\mathbf{h} = (h_1, \dots, h_{N_c^s})$ is thus the channel state vector for all users in cell s. Users of slice s are served under some scheduling policy $\Pi(\cdot|\cdot)$ at cell c, which depends on the past and present users' channel state; at each time-slot, the slice scheduler then allocates the channel to a tagged user u in cell c with probability $\Pi(u|\mathbf{h})$. The average rate achieved by user u under policy Π is

$$r_u = g_u(b_c^s, \Pi(u|\mathbf{h})) := \mathbb{E}_{\mathbf{h}}\left[b_c^s \log(1 + h_u \frac{p_u}{N_0})\Pi(u|\mathbf{h})\right] \tag{3}$$

where the expectation is taken with respect to the channel distribution. We observe that, irrespective of the actual

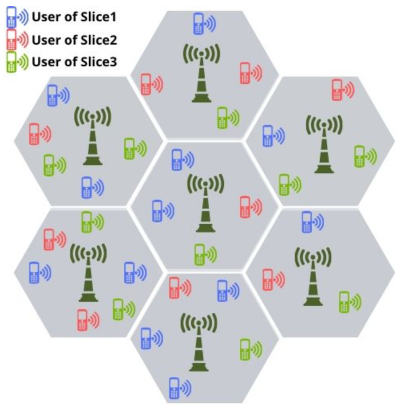

Fig. 1. Resource Slicing in 5G Networks: slices are assigned radio resources on a per-cell basis.

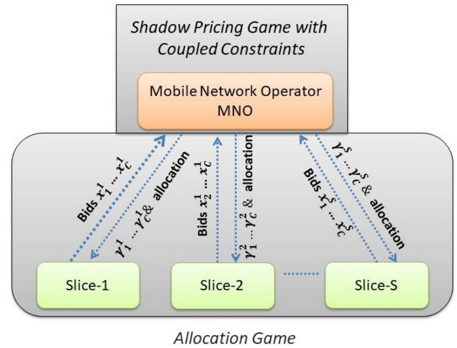

Fig. 2. The MNO uses the Shadow Pricing Game (top) to generate the price vector, whereas the result of the Allocation Game (bottom) decides the actual resource slicing based on the tenants bids.

scheduling policy, the average rate for a tagged user u is linear in the slice bandwidth b_c^s at cell c. Once we fixed $k_u^* = (\mathbb{E}_h [\log(1 + h_u \Lambda_0)\Pi(u|\mathbf{h})])$, the total benefit function for slice s writes as

$$V_c^s = \sum_{u=1}^{N_c^s} f_s\left(k_u^* \, b_c^s\right)$$

which is again an increasing concave function of the allocated bandwidth per slice.

The classical optimisation framework for the MNO prescribes to provide efficient yet fair allocation for all users belonging to the same slice according to slices' load. Since scheduling is performed per cell, however, it is necessary for the resources allocation to be fair – within the same slice – also across users associated to different base stations. Such a trade-off between efficiency and fairness can be captured by formulating the utility of a given slice as:

$$\mathbf{V}^s(\mathbf{b^s}) = \sum_{c \in \mathcal{C}} V_c^s(b_c^s) \tag{4}$$

For the sake of discussion, we shall assume that the number of users is fixed. Applied at the cell level, utility (4), is able to express the customary trade-off between efficiency and fairness among users associated to a tagged slice service. However, it also allows to achieve such a trade-off horizontally, that is across cells. For $\alpha = 1$, for instance, the customary log-based proportional-fair utility will severely penalise serving high throughput in a lightly loaded cell while starving slice users in another hot-spot cell.

The main objective of the MNO is to maximise the total utility of slices, leading to the following 5G resource allocation problem

$$\begin{aligned} P: \qquad & \underset{\mathbf{b}}{\text{maximize}} && \sum_{s \in \mathcal{S}} \mathbf{V}^s(\mathbf{b^s}) \\ & \text{subject to} && b_c^s \geq 0 \\ & && \sum_{s\mathcal{S}} b_c^s \leq B_c, \quad \forall c \in \mathcal{C} \end{aligned}$$

where B_c is the total bandwidth available at base station c.

This optimisation problem can be solved by well-known convex optimisation methods. However, such a centralised resource allocation scheme, while addressing properly the MNO constraints on the resources allocation, lacks of scalability, and may lead to excessive communication overhead when the number of slices increases. Furthermore, it is known that such solutions are rarely viable when under dynamic network conditions. Thus, while (P) still provides reference performance figure for our slicing problem, we shall focus on a decentralised scheme where the resources allocation decision is mediated by a pricing scheme pivoting on the Kelly mechanism.

III. Kelly Mechanism Based Resource Allocation

In this section, we design a bidding scheme solving problem (P). The curse of bidding-based schemes is that typically they cannot rule out the possibility of resources overbooking. Indeed, it is unrealistic to assume that, under a competitive pricing scheme, tenants would jointly account for the per-cell *coupled constraint* $\sum_s b_c^s \leq B_c$, appearing in (P).

The scheme we propose, conversely, is provably overbooking-free even though the slice bids are oblivious to infrastructure resources constraints. Our design is based on two coupled games, entangled by the same Nash equilibrium: first, a virtual game, namely the *Shadow Pricing Game*, which generates the vector of resource prices, and for which there exist a unique normalised Nash equilibrium, and second the *Allocation Game* where – based on the so-determined prices – the MNO rules a multi- dimensional Kelly mechanism where each tenant acquires a slice of resource in each cell $c \in \mathcal{C}$ at a price.

Overall, the proposed mechanism can be seen as the cascade of the following two items (see Fig. 2).

- The MNO settles the price obtained by Shadow Game in a way to respect the coupled constrains of resources;
- The prices are announced to tenants in the Allocation game to obtain the Nash equilibrium that respects the resource constraints on resources.
- A specific price will be designed by the MNO to attain the social optimum for the game.

We observe that the proposed scheme requires indeed full information at the MNO side on the tenants' valuations. Thus, by itself, it does not represent a feasible scheme. We shall relax such demanding request with a learning procedure able to drive the system to the Nash equilibrium resulting from the aforementioned cascade.

A. Shadow Pricing Game

The Shadow Pricing Game is a virtual game where the tenants compete for resource access and do not share information on the amount of resource they ask for. The result of this virtual game which matters to the MNO is the resulting price vector. In fact, the seminal work of Rosen [20] ensures the existence of a unique equilibrium vector of this game in the form of a normalised Nash equilibrium, a concept which is pivotal in this paper. Such equilibrium is given by the concatenation of a bid vector and a vector of multipliers. The latter are actually the price vector we are interested in.

This price definition has indeed an algorithmic flavour, reflected in the scheme in Fig. 2: before posting the unit prices for resources, the MNO determines the price vector as the solution of the virtual game. In other words, the Shadow Pricing Game is a virtual game which solves for the optimal price as the signal by which the MNO can drive the Allocation Game to a feasible equilibrium with respect to the capacity constraints in (P).

In the virtual game, each tenant tries to maximise her benefit while obeying the coupled constraints

$$\sum_{s\in\mathcal{S}} b_c^s \le B_c, \quad \forall c \in \mathcal{C}. \tag{5}$$

Thus, the decision problem for a tagged tenant s writes as

$$Q_s \qquad \underset{b^s\ge 0}{\text{maximize}} \quad \mathbf{V}^s(\mathbf{b^s}, \mathbf{b^{-s}})$$
$$\text{subject to} \quad \sum_{s\in\mathcal{S}} b_c^s \le B_c, \quad \forall c \in \mathcal{C}.$$

The system $\{Q_1, \dots, Q_S\}$ represents the formalisation of the Shadow Pricing Game: the notion of an equilibrium for such a continuous game requires to account for the presence of constraints, that is

Definition 1. *A strategy $\mathbf{b}^* = (\mathbf{b^1}^*, \dots, \mathbf{b^S}^*)$ is called Nash Equilibrium for game $\{Q_1, \dots, Q_S\}$ if*

$$\mathbf{V^s}(\mathbf{b^{s*}}, \mathbf{b^{-s*}}) \ge \mathbf{V^s}(\mathbf{b^s}, \mathbf{b^{-s*}}) \tag{6}$$

for all $s \in \mathcal{S}$, $b_c^s \ge 0$ and $\sum_{s\in\mathcal{S}} b_c^s \le B_c, \forall c \in \mathcal{C}$.

Here, with standard notation, $(\mathbf{b^s}, \mathbf{b^{-s*}})$ refers to the multistrategy vector whose s-th element equals $\mathbf{b^s}$ and all other strategy vectors equal $\mathbf{b^{-s*}}$.

We should observe that in game with coupled constraints, the equilibrium is, in general, non unique. Actually, by inspection we note that the Shadow Pricing Game has an infinite number of equilibria. Conversely, it is the *normalized* Nash equilibrium that, under specific assumptions, results to be unique. Its definition requires to introduce some further notation.

Because of concavity in players' own strategy [20], a multistrategy vector $\mathbf{b}^*$ is a Nash Equilibrium for the Shadow Pricing Game if and only if it satisfies simultaneously the Karush–Kuhn–Tucker (KKT) conditions, which are: $\forall c \in \mathcal{C}, \forall s \in \mathcal{S}$

$$\frac{\partial \mathbf{V^s}(\mathbf{b}^*)}{\partial b_c^s} - \lambda_c^s + \xi_c^s = 0 \tag{7a}$$

$$\lambda_c^s \left(\sum_{s'\in\mathcal{S}} b_c^{s'*} - B_c \right) = 0 \tag{7b}$$

$$\xi_c^s b_c^{s*} = 0 \tag{7c}$$

$$\lambda_c^s \ge 0,\ \xi_c^s \ge 0. \tag{7d}$$

Definition 2. *A r-normalized equilibrium point is such that there exists $\lambda_c > 0$ associated to each base station so that for all customers $\lambda_c^s = \lambda_c / r_c^s$, for a suitable vector of nonnegative vector of coefficients r.*

The important property of normalised Nash equilibria we are leveraging in the rest of the discussion is in the following

Theorem 1 ([20], Thm. 3)**.** *There exists a unique r-normalized equilibrium point for the Shadow Pricing Game for every specified $r > 0$*

While the Pricing Game in practice may not be practically viable (indeed it is not reasonable to expect players to respect the aggregate constraint in calculating their best response), the development in this section has showed how to map the Pricing Game onto the Allocation Game

B. Allocation Game

Once the MNO obtained the vector prices, the actual game is an auction-based bandwidth allocation mechanism, in which each slice tenant s submits bid vector $x^s = (x_1^s, \dots, x_C^s)$, one bid for each one of the C base stations. Bid x_c^s represents the amount of bandwidth demanded for slice s at base station c.

The MNO collects all bids for each base station and assigns to each slice s, a fraction of each base station corresponding to the ratio he attained given the bids received for that base station, namely the quantity

$$b_c^s := B_c \frac{x_c^s}{\sum_{s'\in\mathcal{S}} x_c^{s'}}, \tag{8}$$

where B_c represents the total bandwidth available at base station c. As the valuation of each slice is function of bandwidth received by it, from (1) we write the valuation of slice s as

$$\mathbf{V}^s(\mathbf{b}^s) = \sum_{c\in\mathcal{C}} V_c^s \left(\frac{x_c^s}{\sum_{s'\in\mathcal{S}} x_c^{s'}} B_c \right). \tag{9}$$

For each slice s, $\mathbf{V}^s$ is an increasing function in b_c^s: without any payment slice tenants will always bid as much as possible in order to increase their own benefit. However, after submitting the bids, each customer pays to the MNO the cumulative sum of prices for the bids she made. More precisely, let γ_s^c be the unit cost for bidding for one resource unit (e.g., one PRB) at base station c for slice s. Then, each slice tenant s pays $\gamma_c^s x_c^s$ for the resources obtained at base station c.

In turn, the utility of a MNO customer is defined as the difference between the overall benefit obtained by using the portion of bandwidth at different base station and the total cost to pay for using them:

$$\mathbf{U^s}(\mathbf{x}^s, \mathbf{x}^{-s}, \gamma^s) = \sum_{c \in \mathcal{C}} V_c^s \left(\frac{x_c^s}{\sum_{s' \in \mathcal{S}} x_c^{s'}} B_c \right) - \gamma_c^s x_c^s. \quad (10)$$

The tenants are rational players and bid for PRBs so as to optimise their utility (10). Thus, the decision problem of each slice $s \in \mathcal{S}$ is to find the optimal x^s optimizing its own utility:

$$P_s: \quad \underset{\mathbf{x}^s}{\text{maximize}} \quad \mathbf{U^s}(\mathbf{x}^s, \mathbf{x}^{-s}, \gamma^s)$$
$$\text{subject to} \quad x_c^s \geq 0, \forall c \in \mathcal{C}$$

Then, the slicing allocation problem can be interpreted as a competitive game where players, i.e. the customers compete to acquire bandwidth for their own slice in order to increase their utility. The standard notation $\{P_1 \dots P_S\}$ describes formally the *bandwidth allocation game*.

For this game we can consider the standard notion of a Nash equilibrium, where we do not have coupled constraints, that is a multistrategy $x^* = (x^{1^*}, \dots, x^{S^*})$ where for all players $s \in \mathcal{S}$, $U_s(x^{s*}, x^{-s^*}) \geq U_s(x^s, x^{-s^*})$ with $x_c^s \geq 0, \forall c \in \mathcal{C}$. In particular, it is known that in the single resource case

Theorem 2 ([16])**.** *The Kelly mechanism has a unique Nash equilibrium.*

Clearly, the uniqueness result extends immediately to the Allocation Game since the resources are orthogonal[1].

The next result is the central result of this paper, since it provides the connection between the equilibria of the two games:

Theorem 3. *Every r-normalised Nash equilibrium of the Shadow Pricing Game with shadow prices λ_c is a Nash equilibrium for the corresponding Allocation Game with $\gamma_c^s = \frac{b_c^{-s}}{B_c} \lambda_c^s$.*

Proof. Let us consider a normalised Nash equilibrium $\bar{b}$ of the Shadow Pricing Game. From (7c), we have that necessarily

$$\forall s \in \mathcal{S}, \forall c \in \mathcal{C}, \frac{\partial V_s}{\partial b_c^s}(\bar{b}) = \lambda_c^s - \xi_c^s.$$

If we replace $\lambda_c^s = \frac{B_c}{b_c^{-s}} \gamma_c^s$ and $\eta_c^s = \frac{B_c}{b_c^{-s}} \xi_c^s$ for all $c \in \mathcal{C}$ we get

$$\frac{\partial V_s(\overline{x})}{\partial x_c^s} \frac{b_c^{-s}}{B_c} = \frac{\partial U_s}{\partial x_c^s} = \gamma_c^s - \eta_c^s.$$

Thus

$$\begin{cases} \frac{\partial U_s}{\partial x_c^s}(\overline{x}) & \leq 0 \text{ if } x_c^s = 0 \\ \frac{\partial U_s}{\partial x_c^s}(\overline{x}) & = 0 \text{ if } x_c^s > 0 \end{cases}$$

Thus $\overline{x}$ satisfies the KKT conditions of the optimization problem associated to the Allocation Game. Since function U_s is concave with respect to variable x^s and the constraints are linear, they are also sufficient and thus $\overline{x}$ is a Nash equilibrium of the Allocation Game. □

[1]The important case when tenants have a total bidding budget is left as part of future works.

IV. Social Optimal Pricing

In this section we will show that the proposed mechanism is able to attain the social optimum. This mechanism is based on a simple pricing can force slices to choose an equilibrium (respecting the resources coupled constraint) that coincide with the optimal solution of (P). Using the cascade of both the Shadow Pricing game and the Allocation game, the pricing and allocation are performed in a distributed manner with no need to exchange per-bandwidth allocation information.

Let us recall the original problem introduced in Sec. (III), where the MNO's goal is to solve problem (P). Concavity of the objective function ensures that there exists a unique allocation $\mathbf{b}^*$ which maximizes the objective function. Let now consider the Lagrangian associated to problem (P): it writes $L(b, \mu, \nu) = \sum_s (V(\mathbf{b}^s) - \sum_c \mu_c (\sum_s b_c^s - B_c) - \sum_c \nu_c^s b_c^s)$.

Since the problem is feasible and constraints are affine, KKT conditions for (P) are necessary and sufficient for optimality of a solution $(\mathbf{b}^*, \mu^*, \nu^*)$ such that $\forall s \in \mathcal{S} \; \forall c \in \mathcal{C}$

$$\frac{\partial V_s(b^*)}{\partial b_c^s} - \mu_c^* + {\nu^*}_c^s = 0 \quad \text{(11a)}$$

$$\mu_c^* \left(\sum_{s' \in \mathcal{S}} {b^*}_c^{s'} - B_c \right) = 0 \quad \text{(11b)}$$

$${\nu^*}_c^s {b^*}_c^s = 0, {\mu^*}_c^s \geq 0, \; {\nu^*}_c^s \geq 0 \quad \text{(11c)}$$

Where $\mu = (\mu_1, \dots, \mu_C)$ are the C Lagrange multipliers for the cells capacity constraints.

Now, if we consider the r-normalized Nash equilibrium for shadow pricing game with $r_c^s = 1$, for $\forall s \in \mathcal{S}$ and $\forall c \in \mathcal{C}$, we obtain $\lambda_c^1 = \dots = \lambda_c^S = \lambda_c$ for all $c \in \mathcal{C}$ in the KKT conditions (7c). For $\mu_c = \lambda_c$ the conditions (7c) and (11) are equivalent and as we have already proved uniqueness of r-normalized Nash equilibrium in Thm. 1, it holds $\mu^* = \lambda^*$.

But then, from Thm. 3 we obtain also that the relation between the Allocation Game and the Shadow Pricing game

$$\gamma_c^s = \frac{b_c^{-s}}{B_c} \lambda_c^* \quad (12)$$

settles the Allocation Game on the social optimum.

Theorem 4. *The normalised Nash equilibrium attained for $r_c^s = 1$, $\forall s \in \mathcal{S}$, $\forall c \in \mathcal{C}$ by the Shadow Pricing Game attains the social optimum for problem (P), and so does the Allocation Game under prices γ_c^s as in* (12).

V. Learning and System Stability

We have already seen in the previous section that the proposed mechanism has a unique equilibrium for any price vector decided by the infrastructure owner. However, while the MNO can use the price vector as a signal to drive the system to a socially optimum operating point, the game formulation of the mechanism has scarce practical relevance. In fact, since the tenants' valuation of resources is typically unknown to the MNO. To this respect, we propose a learning algorithm to converge iteratively to the target equilibrium in a distributed fashion. In the proposed solution, the only signal exchanged between the MNO and the tenants at each step are the bid vector and the price vector.

We use the dual averaging or mirror–descent method discussed in [22], [23]. However, those works only considered orthogonal constraints. Thus, we have adapted the original algorithm to tackle the coupled constrained setting. The idea behind the mirror descent is each player estimates his gradient and takes steps along the gradient in dual space (where the gradient lives). The aggregated of the s-th player's gradient steps is updated according to equation

$$y^s(n+1) = \left[y^s(n) + \beta_n \; \nabla_{x^s} U_s(x^s(n), x^{-s}(n), \gamma^s(n))\right] \tag{$\mathcal{A}1$}$$

In the above equation y^s is an auxiliary variable which accumulates the discounted gradient and β_n is a standard step size, where $\sum_{n=0}^{\infty} \beta_n = +\infty$ and $\sum_{n=0}^{\infty} \beta_n^2 < +\infty$. Every player s uses his own updated output value y^s to take next action. The technique takes the name mirror–descent because each player s "mirrors back" the variable y^s to his action space $\mathcal{X}_s$ according to the mapping

$$x^s(n+1) = \underset{x^s \in \mathcal{X}_s}{\operatorname{argmax}} \left\{ \langle y^s(n), x^s(n) \rangle - h_s(x^s(n)) \right\} \tag{$\mathcal{A}2$}$$

Here, $h_s(x)$ is regularizer, according to definition 3.1 in [22], or rather a penalty function over the feasible action set $\mathcal{X}_s$. Penalty $h_s(x)$ permits convergence within the interior of the domain set, that is, the feasible multistrategy set. In our case we use entropic regularization, also known as Gibbs entropy function; it takes the form, $\forall c \in \mathcal{C}$

$$h^s_c(x^s) = x^s_c \log(x^s_c) + (1 - x^s_c) \log(1 - x^s_c)$$

over domain $\mathcal{X}^s_c = \{x : 0 \le x^s_c \le 1\}$. The advantage of this formulation is that it can be easily scaled to original constraints. Furthermore, by applying KKT conditions to maximization problem ($\mathcal{A}2$), after some calculations it produces the exponential mapping for all $s \in \mathcal{S}$ and $c \in \mathcal{C}$:

$$x^s_c(n+1) = \frac{B_c \exp(y^s_c(n))}{1 + \exp(y^s_c(n)))} \tag{13}$$

The one above is similar to well-know Logit map, where player gives the weights to different resources depending on exponential of aggregated gradients. The players take the actions (in our case the bids) independently of each other, which could results in violation of the resource capacity constraints. In order to handle this problem, the MNO updates the prices in such a way that, the players are forced to obey coupled constraints. The prices appear as Lagrange multipliers for coupled constraints (capacity constraints). As similar to the players, she takes the step along the negative gradient of Lagrangian and updates the price per resource:

$$\lambda^c(n+1) = \max\left(0, \lambda^c(n) + \beta_n \Big(\sum_{s' \in \mathcal{S}} x^{s'}_c - B_c \Big)\right) \tag{14}$$

This updated value of Lagrangian multipliers act as new prices for all tenants and resources, that is

$$\gamma^s_c(n) = \frac{x^{-s}_c}{B_c} \lambda_c(n)$$

If all the players and MNO simultaneously take action as per the designed algorithm, the proposed algorithm converges to the unique Normalized Nash equilibrium (x^*, λ^*) of the Pricing Game. Moreover, if we fixed the prices γ and players are allowed to play only according to the algorithm, the designed algorithm converges to unique Nash equilibrium of the Allocation Game. Note that the update rule of the MNO corresponds to the choice of vector r such that $r_c = 1$ for all $c \in \mathcal{C}$. To this respect, we provided a formulation where the r-normalised Nash equilibrium corresponds to the social optimum. In general, it is possible to set the coefficients of vector r at will. Clearly, the corresponding solution will not be converging to the social optimum, but this provides some space for tenants' prioritisation, which we leave as part of future works.

Algorithm 1 On-line Distributed Learning Algorithm

Require: $\sum_{n=0}^{\infty} \beta_n = \infty, \beta_n \to 0$ as $n \to \infty$
1: **repeat** at time step $n = 1, 2, \ldots,$
2:   **for each** player $s \in \mathcal{S}$
3:     Observe gradient of utility and update
4:     ${y'}^s \leftarrow y^s + \beta_n \nabla_{x^s} U_s(x^s, x^{-s}, \gamma)$
5:   **end for**
6:   **for each** player $s \in \mathcal{S}$
7:     **for each** resource $c \in \mathcal{C}$
8:       Play ${x'}^s_c \leftarrow \frac{B_c \exp(y^s_c)}{1+\exp(y^s_c)}$
9:     **end for**
10:   **end for**
11:   **for each** resource $c \in \mathcal{C}$ update the price
12:     ${\lambda'}_c \leftarrow \max\left[0, \lambda_c + \frac{\beta_n}{K}\left(\sum_{s \in \mathcal{S}} x^s_c - B_c\right)\right]$
13:   **end for**
14:   **for each** player $s \in \mathcal{S}, \forall c \in \mathcal{C}$
15:     $\gamma^s_c \leftarrow \frac{x^{-s}_c}{B_c} \lambda_c$
16:   **end for**
17: **until** $\|(x', \lambda') - (x, \lambda)\| \le \epsilon$

Theorem 5. *If algorithm satisfies the required conditions for step size sequence, $\sum_{n=0}^{\infty} \beta_n = \infty, \beta_n \to 0$ as $n \to \infty$ then for sufficiently large K Algorithm 1 converges to the unique Normalized Nash equilibrium of the Shadow Pricing Game.*

Proof. In [24] authors had already proved convergence of algorithm as similar to ours for the single resource Allocation Game. We will use the same technique as discussed in Theorem 2 [24] to prove convergence of the Algorithm 1 to Normalized Nash equilibrium of the Pricing Game. Now to show convergence of the Algorithm we will first show the asymptotic stability of mean dynamics of the algorithm. We write the continuous-time equivalent from steps 4, 8 and 12 of the algorithm. For simplicity of exposition, we consider here the single user case, since the general case follows immediately:

$$\dot{y}^s_c = \frac{\partial V_s(x)}{\partial x^s_c} - \gamma^s_c \tag{15}$$

$$x^s_c = \frac{\exp y^s_c}{1 + \exp y^s_c} \tag{16}$$

$$\dot{\lambda}_c = \left(\sum_{s'} x^{s'}_c - B_c \right) \tag{17}$$

Taking derivative of (16) and replacing in (15) gives

$$\dot{x}^s_c = x^s_c(1 - x^s_c)\left(\frac{\partial V_s(x)}{\partial x^s_c} - \gamma^s_c\right) \tag{18}$$

$$\dot{\lambda}_c = \left(\sum_{s'} x^{s'}_c - B_c\right) \tag{19}$$

Now to show stability of dynamics, let consider Lyapunov function

$$\mathcal{L}(x,\lambda) = \sum_s \left[{x^*}^s_c \log \frac{{x^*}^s_c}{x^s_c} - (1 - {x^*}^s_c) \log \frac{1 - {x^*}^s_c}{1 - x^s_c} \right] + \frac{1}{2}\|\lambda_c - \lambda^*_c\|^2 \tag{20}$$

Taking derivative gives

$$\dot{\mathcal{L}} = \sum_s \frac{x^c_{-s} B_c}{(\sum_{s'} x^{s'}_c)^2} \left[\frac{\partial V_s(y^s_c)}{\partial y^s_c}(x^s_c - {x^*}^s_c) - \lambda^*((x^s_c - {x^*}^s_c)) \right] \tag{21}$$

where at the equilibrium $\lambda^* = \frac{\partial V_s({y^*}^s_c)}{\partial y^s_c}\ \forall s \in \mathcal{S}$.

If the pseudo gradient $(\frac{\partial V_1(x_c)}{\partial x^1_c} \dots \frac{\partial V_S(x_c)}{\partial x^S_c)})$ is diagonally strict concave (DSC) (see[20]), then $\dot{\mathcal{L}}$ is negative. But in our case DSC doesn't hold in general. However it holds for some neighborhood around the Nash equilibrium x^*, therefore (21) is negative for some $B_c = B'_c$ which is in neighborhood of $\sum_{s'} x^s$. In order to overcome this problem, we scale down the step length for Lagrange multiplier update rule by some sufficiently large constant K which makes (21) negative, thus rendering the dynamics asymptotically stable. The rest of proof follows from the Theorem 2 [24]. □

VI. Numerical Experiments

In this section we will provide numerical results to demonstrate the behaviour of the proposed mechanism. For the numerical experiment we considered a system with three slices $\mathcal{S} = \{1,2,3\}$ and two base stations $\mathcal{C} = \{1,2\}$. Tenants of slices $1,2$ and 3 have $N^1_1 = 3$, $N^2_1 = 5$ and $N^3_1 = 2$ users, respectively, associated at base station 1. At base station 2 they have $N^1_2 = 2$, $N^2_2 = 4$ and $N^3_2 = 6$ users, respectively. The available bandwidth at each base station is 30 MHz and we assume that the SNR of each user lies in the range between 30 and 75 dBs. Every slice uses some scheduling policy to assign the acquired bandwidth among its users: for the purpose of numerical illustration we assume that each slice is served using per-slice proportional fair scheduling.

The distributed learning Algorithm-1 is employed in order to determine the socially optimal Nash Equilibrium. Plots (a) and (b) in Fig. 3 show the converging dynamics of the bandwidth bids vector. As seen there, it stabilises at the target Nash equilibrium for both base stations 1 and 2. The distribution of bandwidth allocation at Nash equilibrium is shown in bar graph (g). As it can be clearly seen, the allocation of bandwidth at both base stations is consistent with the number of user per slices. In fact, at base station 2, slice 2 has more users compared to the other two slices; as expected, it attains hence a larger share of the available bandwidth. The target allocation has been achieved by using the pricing vector which is shown in the plot (c) and (d) of Fig.3. In those graphs we observe the convergence of prices per slice and per base station. The prices charged by MNO for each slice are inversely proportional to number of the users. Finally, bar graphs (e) and (f) illustrate the throughput achieved per user under the resulting bandwidth allocation; the graphs indicate a mild throughput variation across the users within a slice, a result consistent with the use of PFS at slice level.

VII. Conclusions

In this paper, we have considered a scenario where customers compete to obtain a slice of resource in 5G networks. We employ a mechanism based on a multi-bid Kelly mechanism, using as price vector the one resulting from the normalised Nash equilibrium which solves a dual game under coupled constraints. The solution of the game is obtained via an online learning mechanism which ultimately converges to the social optimum. The key technical challenge overcome by the proposed bidding mechanism is to account for the coupled constraints dictated by the available infrastructure resources. This renders the proposed one an interesting candidate mechanism for pricing slicing in 5G networks. In fact, to the best of the authors' knowledge, no suitable learning mechanism is known for Nash equilibria under the coupled resources constraints which are central in 5G resources slicing.

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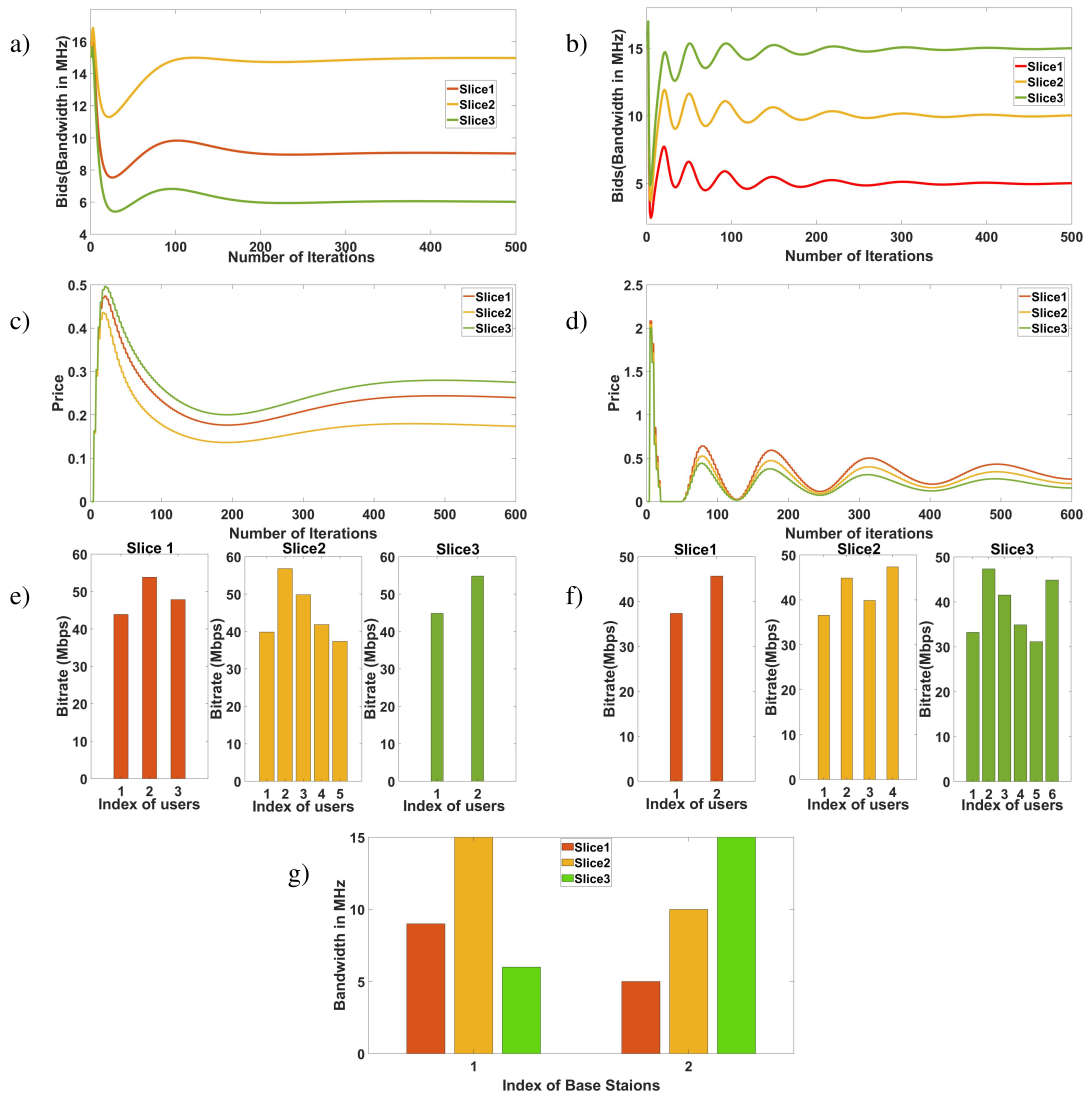

Fig. 3. (a and b) convergence of the Algorithm to the socially optimal Nash Equilibrium at base station 1 and at base station 2 (c and d); convergence of price vectors for base station 1 and base station 2; (e and f) throughput achieved by the users in base station 1, and base station 2; g) allocation of bandwidth among the slices at base station 1 and 2.

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