On the Optimal ARQ Distribution for Low-Latency Communication over Line-of-Sight Dominated Multi-Hop Networks

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***Abstract*—Multi-hop networks that are dominated by line-of-sight (LOS) wireless channels have gained traction in the recent past owing to the emergence of wireless networks based on unmanned aerial vehicles. One of the challenges in such vehicular networks is to design communication strategies to provide both ultra-reliability and low-latency features. Towards providing ultra-reliability against channel impairments, it is well known that automatic repeat request (ARQ) based decode-and-forward relaying is an effective strategy wherein each transmitter can be allotted an appropriate number of re-transmissions based on the LOS component of its forward link. However, in order to provide low-latency features, it is also known that multiple re-transmissions may not be a favorable choice as the total number of re-transmissions across the relays incurs significant delay in communicating the packet in the end-to-end network. Identifying this conflict introduced by the ARQ protocol, we investigate the optimal allocation of the number of ARQs at each link so as to minimize the packet-drop-probability at the destination subject to a sum constraint on the total number of ARQs allotted to all the nodes in the network. First, we prove a set of necessary and sufficient conditions on the optimal ARQ distribution, and then use these conditions to propose a low-complexity algorithm to solve the problem statement. Through extensive simulation results, we show that the proposed algorithm significantly reduces the computational complexity when compared to exhaustive search and yet recovers the optimal ARQ distribution.**

***Index Terms*—Multi-hop network, low-latency, ultra-reliability, ARQ based protocol, line-of-sight component.**

I. Introduction

Multi-hop wireless networks have found extensive applications [1]–[3] for their ability to extend the coverage area of power-limited radio transmitters. In particular, these networks are known to facilitate long-range communication between a source and a destination node by introducing multiple short-range links thereby guaranteeing high reliability features against impairments introduced by fading channels. Towards achieving high end-to-end reliability over multi-hop networks, a number of protocols have been proposed by carefully analyzing the trade-off between complexity and error performance. On the one hand, the class of Amplify-and-Forward (AF) protocols [4] are known to assist low-complexity processing at the intermediate relays, however, they are also known to boost the accumulated noise witnessed at the destination, which in turn degrades the error performance. On the other hand, the class of Decode-and-Forward (DF) protocols [4] have been shown to deliver high end-to-end reliability provided the decoding strategy at the relays ensures perfect recovery of the packets. One such strategy towards ensuring perfect recovery of packets is the class of automatic repeat request (ARQ) based DF strategies, wherein each intermediate relay node is allotted multiple re-transmissions to combat multipath fading [5].

While the idea of relaying packets through ARQ based DF strategy ensures *ultra-reliability*, the very fact that each intermediate node has to process the packets will incur substantial delay on the packets. In particular, the delay introduced at each relay comprises processing delay (for the decoding and encoding operations) and transmission delay (for multiple re-transmissions), and moreover, the end-to-end delay incurred on the packets is the sum of the delays contributed by all the relays on the path. Although there exists a gamut of contributions on optimizing reliability over multi-hop networks, very few have studied the underlying trade-off between end-to-end reliability and end-to-end delay offered by these protocols [6], [7]. We highlight that a study of this nature is paramount especially when the packets from the source have both ultra-reliability and low-latency constraints, i.e., when the packets at the source node must reach the destination within a given deadline [8]–[10]. Example applications including such constraints are vehicular networks with autonomous driving vehicles, robotic surgery etc. In a nutshell, motivated by facilitating low-latency transmission of packets, we study the trade-off between end-to-end reliability and end-to-end delay in a multi-hop network that employs ARQ based DF strategy. We specifically consider multi-hop networks that are dominated by line-of-sight (LOS) fading channels since such LOS channels usually manifest in applications that need both low-latency and ultra-reliability features, e.g. Unmanned-Aerial-Vehicles (UAVs) [11].

A. Problem Statement

In an ARQ based DF relaying strategy, each relay node is given a certain number of attempts to transmit the packets upon failure to decode by the next node in the chain. As a consequence, if a relay node is unable to correctly decode the packet within the given number of attempts, then the packet is said to be dropped at that link. Since the packet can be dropped at any of the links in the network, one important design objective is to minimize the end-to-end packet-drop-

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TABLE I: Novelty of our work with respect to existing contributions

Reference	Existing contributions	Limitations in comparison with our work
[13]	Achieves high reliability under the constraint of strict latency by using cooperative ARQ scheme. The idea is to vary the reserved time for re-transmission	Not considered LOS channels. Not addressed the optimal distribution of ARQs
[6]	Low-latency and high-reliability communications in the control and non-payload communications (CNPC) link of UAVs. This is a multiple antenna based model	Not addressed the optimal distribution of ARQs
[7]	Uses Gaussian-Chebyshev quadrature (GCQ) to approximate the achievable data rate	Not minimized the latency with respect to ARQs No ARQ scheme considered
[14]	Adaptive HARQ (A-HARQ) scheme is proposed. Re-transmissions are done on better quality sub-bands	LOS channels not considered. Not relevant to address optimal ARQ distribution
[15]	Analyzes packet-drop-probability for energy harvesting nodes with links using ARQ or hybrid-ARQ	No LOS channel. Does not improve the reliability with a given constraint on latency. Optimal distribution of ARQs not studied.

probability (PDP), which is the fraction of packets that do not reach the destination. A straightforward solution to minimize the PDP is to allocate appropriate number of ARQs on each link based on its LOS component. However, we note that ARQs result in significant amount of end-to-end delay on the packet due to multiple rounds of re-transmissions, and this delay, in the worst case, is proportional to the total number of ARQs allotted to all the nodes in the network. Therefore, on packets, which have low-latency constraints, the total number of allotted ARQs must be bounded. As a result of this conflict of jointly providing ultra-reliability and low-latency features, we identify an interesting problem of "*How to allocate ARQs to each link such that the PDP is minimized under the constraint that the total number of ARQs allotted across the relay nodes is bounded by a fixed number?*".

B. Contributions

1) We propose a new problem on distributing the number of re-transmissions across multiple relay nodes in a LOS dominated multi-hop network so as to support low-latency and ultra-reliability features on the underlying packets (See Section II). In particular, given a multi-hop network with potentially distinct LOS components of the links, we formulate an optimization problem of minimizing the PDP under the constraint that the sum of the ARQs across all the links in the network is bounded. First, we show that this optimization problem involves a non-linear objective function with non-negative integer-constraints on the solution. Because of the sum constraint on the total number of ARQs, we show that the size of the search space is bounded. However, we also show that computing the optimal distribution of ARQs through exhaustive search is not feasible to implement in practice. Towards solving this problem with low complexity methods, we prove a set of necessary and sufficient conditions on the optimal solution of the optimization problem (See Section III).

2) At high signal-to-noise-ratio (SNR) values, we observe that the set of necessary and sufficient conditions simplifies to a set of linear equations relating the number of re-transmissions allotted to the N links. Using this special case, we convert the problem of computing the optimal distribution of re-transmissions to an equivalent problem of solving a system of linear equations in $\mathbb{R}^N$, and to another problem of searching a distribution of re-transmissions in the integer search-space that are nearest to the real solution. Through this approach, we show that the search space for finding the optimal distribution of re-transmission can be significantly reduced when compared to the exhaustive search method. Although our approach of formulating an equivalent problem is motivated by high SNR approximation of the necessary and sufficient conditions, we show through simulation results that our algorithm continues to generate a small list size at low and moderate SNR values (See Section IV). Furthermore, we highlight that our algorithm scales well with the number of hops, and importantly provides substantial reduction in complexity when the bound on the total number of ARQs allotted across the relay nodes increases.

Although [6], [7], [13], [14] have studied latency and reliability aspects of multi-hop networks, they have neither addressed LOS environments nor have considered the optimization problem of minimizing the PDP with a constraint on the total number of ARQs. Some specific differences between our work and the prior contributions are listed in Table I.

Notations: We use $x \sim \mathcal{CN}(0, \sigma^2)$ to represent a circularly symmetric complex Gaussian random variable with mean 0 variance σ^2. The set of all complex numbers, real numbers, rational numbers, integers, and positive integers, are respectively denoted by $\mathbb{C}$, $\mathbb{R}$, $\mathbb{Q}$, $\mathbb{Z}$, and $\mathbb{Z}_+$. We use $\iota = \sqrt{-1}$. We use $d(\mathbf{a}, \mathbf{b})$ to represent the Hamming distance between two vectors $\mathbf{a}$ and $\mathbf{b}$.

II. LOS Dominated Multi-Hop Network Model

Consider an N-hop network, as shown in Fig. 1, which includes a source node, a destination node, and a set of $N-1$ relays. We assume that the information bits from the source node are aggregated in the form of packets, and these packets are communicated to the destination in a multi-hop fashion using the $N-1$ intermediate relays. In other words, the multi-hop network consists of N wireless links, wherein the first link corresponds to the channel between the source node and relay R_1, the second link corresponds to the channel between R_1 and R_2, and similarly, the N-th link corresponds to the channel between relay R_{N-1} and the destination node. We assume that the channel between any two successive nodes is characterized by Rician fading with a quasi-static time-interval of L channel

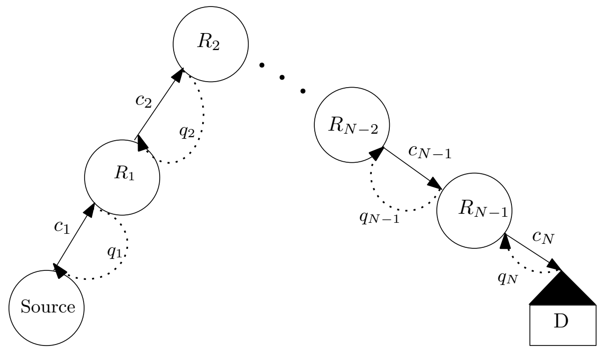

Fig. 1: Depiction of LOS dominated N-hop network, where $0 \leq c_k \leq 1$ represents the LOS component of the k-th link, and $q_k \in \mathbb{Z}_+$ denotes the number of ARQs allotted to the transmitter of the k-th link, for $1 \leq k \leq N$.

uses. In particular, the complex baseband channel of the k-th link, for $1 \leq k \leq N$, is modeled by

$$h_k = \sqrt{\frac{c_k}{2}}(1+\iota) + \sqrt{\frac{(1-c_k)}{2}}g_k,$$

where $0 \leq c_k \leq 1$ captures the LOS component, and $1 - c_k$ is the Non-LOS (NLOS) component such that g_k is distributed as $\mathcal{CN}(0,1)$. In this signal model, the LOS component c_k is a deterministic quantity, thereby ensuring the equality $\mathbb{E}[|h_k|^2] = 1$ irrespective of the value of c_k. As a special case, $c_k = 0$ and $c_k = 1$ capture the well-known Rayleigh and Gaussian channels, respectively, whereas the intermediate values capture different degrees of Rician fading channels. Assuming that the LOS components of the N links can be potentially different, henceforth, throughout the paper, we use the vector $\mathbf{c} = [c_1, c_2, \ldots, c_N]$ to highlight the LOS components of the N-hop network.

Let $\mathcal{C} \subset \mathbb{C}^L$ denote the channel code employed at the source node of rate R bits per channel use, i.e., $R = \frac{1}{L}\log(|\mathcal{C}|)$. Let $\mathbf{x} \in \mathcal{C}$ denote a codeword (henceforth referred to as packets) transmitted by the source node such that $\frac{1}{L}\mathbb{E}[|\mathbf{x}|^2] = 1$, where the expectation is taken over $\mathcal{C}$. When $\mathbf{x}$ is transmitted over the k-th link, for $1 \leq k \leq N$, the corresponding received symbols after L channel uses is given by $\mathbf{y}_k = h_k\mathbf{x} + \mathbf{w}_k \in \mathbb{C}^L$, where $\mathbf{w}_k$ is the additive white Gaussian noise (AWGN) at the receiver of the k-th link, distributed as $\mathcal{CN}(0, \sigma^2\mathbf{I}_L)$. We assume that the receiver of the k-th link has perfect knowledge of h_k, whereas the transmitter of the k-th link does not have the knowledge of h_k. Since the channel realization h_k is random, and the realization remains constant for L channel uses, the instantaneous mutual information of the k-th link may not support the transmission rate. Therefore, the corresponding relay node will be unable to correctly decode the packet when the mutual information offered by the channel is less than R. The probability of such an outage event is given by

$$P_k = \Pr\Big(R > \log_2(1 + |h_k|^2\gamma)\Big) = \mathrm{F}\left(\frac{2^R - 1}{\gamma}\right), \qquad (1)$$

where $\gamma = \frac{1}{\sigma^2}$ is the average signal-to-noise-ratio (SNR) of the k-th link, $\mathrm{F}(x)$ is the cumulative distribution function of $|h_k|^2$, defined as

$$\mathrm{F}\left(\frac{2^R - 1}{\gamma}\right) = 1 - Q_1\left(\sqrt{\frac{2c_k}{(1-c_k)}}, \sqrt{\frac{2(2^R-1)}{\gamma(1-c_k)}}\right),$$

such that $Q_1(\cdot,\cdot)$ is the first-order Marcum-Q function [12].

In this N-hop network model, we assume that communication between any two successive nodes follows the ARQ protocol, i.e., a transmitter node gets an ACK or NACK from the next node in the chain indicating the success or failure of the transmission, respectively. Upon receiving a NACK, the transmitter re-transmits the packet. Let q_k be the maximum number of attempts given to the transmitter of the k-th link. Consolidating the number of attempts given to each link, the ARQ distribution of the multi-hop network is represented by the vector $\mathbf{q} = [q_1, q_2, \ldots, q_N]$. Since we are addressing low-latency applications, we impose the constraint $\sum_{i=1}^{N} q_i = q_{sum}$, for some $q_{sum} \in \mathbb{Z}_+$, which captures an upper bound on the end-to-end delay on the packets.

Note that if a node fails to deliver the packet to the next node within the allotted number of attempts, then the packet is said to be dropped in the network. As the packet can be dropped in any of the links, the packet-drop-probability (PDP) of the N-hop network is given by

$$p_d = \sum_{k=1}^{N} P_k^{q_k}\left(\prod_{i=1}^{k-1}(1 - P_i^{q_i})\right). \qquad (2)$$

When calculating the above expression, we have assumed that the channel realization h_k takes independent realizations across the number of attempts, and the number of ARQs assigned to a transmitter is not known to the other nodes in the network.

A. Formulation of Optimization Problem

For a multi-hop network with LOS vector $\mathbf{c}$ and SNR $\gamma = \frac{1}{\sigma^2}$, we are interested in computing the ARQ distribution $\mathbf{q}$ which minimizes the PDP expression in (2) under the constraint that $\sum_{k=1}^{N} q_k = q_{sum}$, for a given $q_{sum} \in \mathbb{Z}_+$. We present this problem formulation as Problem 1, as shown below. Henceforth, throughout this paper, we refer to the solution of Problem 1 as the optimal ARQ distribution. We highlight that Problem 1 is a non-linear optimization problem with non-negative integer constraints on the solution. Since there is a sum constraint on $\sum_{k=1}^{N} q_k = q_{sum}$, it is straightforward to note that the search space for determining the optimal distribution is bounded. In particular, it can be shown that the number of candidates in the search space is $\binom{q_{sum}-1}{N-1}$. Therefore, with large values of q_{sum} and N, it is not feasible to implement exhaustive search to solve Problem 1. Identifying this limitation of exhaustive search, we obtain analytical results on the structure of the objective function and the underlying constraints so that Problem 1 can be solved with lower complexity than that of exhaustive search.

Problem 1: For a multi-hop network with a given LOS vector $\mathbf{c}$, and a given SNR $\gamma = \frac{1}{\sigma^2}$, solve

$$q_1^*, q_2^*, \ldots q_N^* = \arg \min_{q_1, q_2, \cdots q_N} \; p_d$$
$$\text{subject to } q_k \geq 1,$$
$$q_k \in \mathbb{Z}_+,$$
$$q_1 + q_2 + \ldots + q_N = q_{sum}.$$

III. Sufficient and Necessary Conditions on the Optimal ARQ Distribution

In this section, we present some insights on the expression of PDP, which in turn will be useful in solving Problem 1.

Definition 1: Let $\pi : \mathbb{R}^N \rightarrow \mathbb{R}^N$ denote a permutation operator on an N-dimensional Euclidean space. For an N-hop network with LOS vector $\mathbf{c}$, we define an equivalent multi-hop network with LOS vector $\bar{\mathbf{c}} = \pi(\mathbf{c})$, wherein the wireless channel of the k-th link experiences the LOS component $\bar{\mathbf{c}}(k)$, where $\bar{\mathbf{c}}(k)$ denotes the k-th component of the vector $\bar{\mathbf{c}}$.

Theorem 1: The PDP of an N-hop network with LOS vector $\mathbf{c}$ and ARQ distribution $\mathbf{q}$ is equal to the PDP of an N-hop network with LOS vector $\pi(\mathbf{c})$ and ARQ distribution $\pi(\mathbf{q})$, where π is any permutation operator on $\mathbb{R}^N$.

Proof: We will prove this theorem using the method of induction. For $N = 2$, the PDP can be written as

$$p_d = P_1^{q_1} + P_2^{q_2}(1 - P_1^{q_1}) = P_1^{q_1} + P_2^{q_2} - P_1^{q_1} P_2^{q_2}. \quad (3)$$

By swapping c_1 and c_2, and also q_1 and q_2, we obtain

$$p_d' = P_2^{q_2} + P_1^{q_1}(1 - P_2^{q_2}) = P_2^{q_2} + P_1^{q_1} - P_2^{q_2} P_1^{q_1}. \quad (4)$$

From (3) and (4), it is clear that $p_d = p_d'$. Thus, the statement of the theorem is proved for $N = 2$.

Assume that for $N = k$, swapping any two links will not change the PDP. For $N = k + 1$, we want to prove that swapping any two links will not change the PDP. The PDP expression in such a case can be written as

$$p_d = P_1^{q_1} + P_2^{q_2}(1 - P_1^{q_1}) + \ldots + P_k^{q_k}\Big[\prod_{j=1}^{k-1}(1 - P_j^{q_j})\Big] + P_{k+1}^{q_{k+1}}\Big[\prod_{j=1}^{k}(1 - P_j^{q_j})\Big].$$

By taking $(1 - P_1^{q_1})$ common from the second term onward, we can rewrite the above expression as

$$p_d = P_1^{q_1} + (1 - P_1^{q_1})\Bigg[P_2^{q_2} + P_3^{q_3}(1 - P_2^{q_2}) + \ldots + P_{k+1}^{q_{k+1}}(1 - P_2^{q_2}) \ldots (1 - P_k^{q_k})\Bigg]. \quad (5)$$

It can be seen from (5) that the k terms in the square bracket constitute the PDP of a k-hop network with the LOS vector $[c_2, c_3, \ldots, c_{k+1}]$ and the ARQ distribution $[q_2, q_3, \ldots, q_{k+1}]$. Therefore, by hypothesis of induction, swapping any two links within the set $\{c_2, c_3, \ldots, c_{k+1}\}$ will not change the PDP. It now remains to show that swapping the link with LOS component c_1 with any of the links in the set $\{c_2, c_3, \ldots, c_{k+1}\}$ will not change the PDP. For illustrative purposes, we will show that swapping c_1 with c_{k+1} does not change the PDP, although this approach can be applied to swap c_1 with any of the links in the set $\{c_2, c_3, \ldots, c_{k+1}\}$. Towards swapping c_1 with c_{k+1}, let us first swap c_2 and c_{k+1} using (5), to obtain

$$p_d = P_1^{q_1} + (1 - P_1^{q_1})\Bigg[P_{k+1}^{q_{k+1}} + P_3^{q_3}\Big(1 - P_{k+1}^{q_{k+1}}\Big) + \ldots + P_2^{q_2}(1 - P_{k+1}^{q_{k+1}})\Big(\prod_{j=3}^{k}(1 - P_j^{q_j})\Big)\Bigg].$$

Note that this manipulation does not change the PDP due to the induction step. We further rewrite p_d as

$$p_d = P_1^{q_1} + P_{k+1}^{q_{k+1}}(1 - P_1^{q_1}) + (1 - P_1^{q_1})\Bigg[P_3^{q_3}(1 - P_{k+1}^{q_{k+1}}) + \ldots + P_2^{q_2}(1 - P_{k+1}^{q_{k+1}})\Big(\prod_{j=3}^{k}(1 - P_j^{q_j})\Big)\Bigg]. \quad (6)$$

By swapping the links with LOS components c_1 and c_{k+1} in the above expression, we obtain

$$p_d' = P_{k+1}^{q_{k+1}} + P_1^{q_1}(1 - P_{k+1}^{q_{k+1}}) + \quad (7)$$
$$(1 - P_{k+1}^{q_{k+1}})\Bigg[P_3^{q_3}(1 - P_1^{q_1}) + \ldots + P_2^{q_2}(1 - P_1^{q_1})\Big(\prod_{j=3}^{k}(1 - P_j^{q_j})\Big)\Bigg]. \quad (8)$$

It is straightforward to observe that $p_d = p_d'$. Therefore, for $N = k + 1$, we have shown that swapping any two links will not change the PDP. Finally, since it is well known that a permutation π can be realized through a sequence of swaps, it follows that the PDP of an N-hop network with LOS vector $\mathbf{c}$ and ARQ distribution $\mathbf{q}$ is same as that of the PDP of the network with LOS vector $\pi(\mathbf{c})$ and ARQ distribution $\pi(\mathbf{q})$. ■

The following theorem shows that a link with higher LOS component must not be given more ARQs than the link with lower LOS component.

Theorem 2: With the LOS vector $\mathbf{c}$, let the SNR be such that $P_k < \frac{1}{2}, \forall\, k$. Then the optimal ARQ distribution $\mathbf{q}$ satisfies the property that whenever $c_i \geq c_j$, we have $q_i \leq q_j \;\forall\, i, j$.

Proof: To highlight c_i and c_j, we rewrite $\mathbf{c}$ as $[c_1, c_2, \ldots, c_i, \ldots, c_j, \ldots, c_{N-1}, c_N]$ such that $j > i$. Suppose that $c_j > c_i$, and q_i and q_j respectively denote the number of ARQs given to the i-th link and the j-th link. Furthermore, let us assume that $q_i = q_j = q$. Suppose that we have an additional ARQ with us, and the problem is whether to allot that additional ARQ to the i-th link or the j-link such that the PDP is minimized. Towards solving this problem, let us consider an equivalent multi-hop network with LOS vector $\mathbf{c}' = [c_1, c_2, \ldots, c_{N-1}, \ldots, c_N, \ldots, c_i, c_j]$, wherein $\mathbf{c}'$

is obtained from $\mathbf{c}$ by swapping c_i with c_{N-1} and c_j with c_N. From Theorem 1, we know that the PDP of the multi-hop networks with the LOS vectors $\mathbf{c}$ and $\mathbf{c}'$ are identical. Furthermore, the PDP of the N-hop network with LOS vector $\mathbf{c}'$, is written as

$$\begin{aligned} p_d &= P_1^{q_1} + P_2^{q_2}(1-P_1^{q_1}) + \dots \\ &\quad + \left(P_i^{q_i} + P_j^{q_j}(1-P_i^{q_i})\right) \prod_{k\in[N]\setminus\{i,j\}} (1-P_k^{q_k}). \end{aligned}$$

Note that $P_i^{q_i}$ and $P_j^{q_j}$ appear only in the last term of the above expression. Since the question of allocating the additional ARQ is dependent only on the expression $P_i^{q_i}+P_j^{q_j}(1-P_i^{q_i})$, we henceforth do not use the entire expression for PDP. Additionally, since $q_i = q_j = q$, we obtain one of the following expressions when allocating the additional ARQ,

$$\begin{aligned} A &= P_i^{q+1} + P_j^{q}(1-P_i^{q+1}), \\ B &= P_i^{q} + P_j^{q+1}(1-P_i^{q}). \end{aligned}$$

Since $c_i < c_j$, we know that $P_i > P_j$. To prove the statement of the theorem, we have to show that $A < B$. As $0 < P_i, P_j < 1$, it is clear that $P_i^{q+1} < P_i^q$ and $P_j^{q+1} < P_j^q$. Furthermore, $A - B$ can be calculated as

$$A - B = P_i^q(P_i - 1) + P_j^q(1-P_j) + P_j^q P_i^q (P_j - P_i). \quad (9)$$

Note that the first and the third term in above equation are negative, whereas the second term is positive. Therefore, if the absolute value of the first term is greater than the absolute value of the second term, then $A - B < 0$. In the rest of the proof, we show that $P_i^q(1-P_i) > P_j^q(1-P_j)$, for any $q \geq 1$. With $q = 1$, the above equation can be written as $P_i(1-P_i) > P_j(1-P_j)$. It is straightforward to prove that the above inequality holds if $P_i + P_j < 1$. Thus, the statement of the theorem is proved for $q = 1$. Now, since $P_i > P_j$, note that $\frac{P_i^q}{P_j^q}$ increases as q increases, and therefore, for any $q \in \mathbb{Z}_+$, we have the inequality

$$\frac{P_i^q}{P_j^q}\frac{(1-P_i)}{(1-P_j)} > \frac{P_i}{P_j}\frac{(1-P_i)}{(1-P_j)} > 1. \quad (10)$$

This implies that the magnitude of the first term of (9) is greater than the magnitude of the second term, and therefore, we have $A - B < 0$. This completes the proof. ■

In the following definition, we formally introduce the search space for the optimal ARQ distribution as highlighted in Problem 1.

Definition 2: The search space for the optimal ARQ distribution is denoted by $\mathbb{S} = \{\mathbf{q} \in \mathbb{Z}_+^N \mid \sum_{i=1}^N q_i = q_{sum} \ \& \ q_i \geq 1 \ \forall \ i\}$.

For a given point $\mathbf{q} \in \mathbb{S}$, we define its neighbors in the following definition.

Definition 3: For a given $\mathbf{q} \in \mathbb{S}$, the set of its neighbors is defined as $\mathcal{D}(\mathbf{q}) = \{\bar{\mathbf{q}} \in \mathbb{S} \mid d(\mathbf{q}, \tilde{\mathbf{q}}) = 2\}$, where $d(\mathbf{q}, \bar{\mathbf{q}})$ denotes the number of disagreements between $\mathbf{q}$ and $\bar{\mathbf{q}}$.

Note that for a given $\mathbf{q} \in \mathbb{S}$, we have $|\mathcal{D}(\mathbf{q})| \leq 2\binom{N}{2}$. In the next definition, we formally introduce a local minima of the space $\mathbb{S}$ by evaluating the PDP of the multi-hop network over the points in $\mathbb{S}$.

Definition 4: An ARQ distribution $\mathbf{q}^* \in \mathbb{S}$ is said to be a local minima of $\mathbb{S}$, if it satisfies the condition $p_d(\mathbf{q}^*) \leq p_d(\mathbf{q})$, for every $\mathbf{q} \in \mathcal{D}(\mathbf{q}^*)$, where $p_d(\mathbf{q}^*)$ and $p_d(\mathbf{q})$ represent the PDP evaluated at the distributions $\mathbf{q}^*$ and $\mathbf{q}$, respectively.

Using the above definition, we derive a set of necessary and sufficient conditions on the local minima in the following theorem.

Theorem 3: For an N-hop network with LOS vector $\mathbf{c}$, the ARQ distribution $\mathbf{q}^* = [q_1^*, q_2^*, \dots, q_N^*]$ is a local minima if and only if q_i^* and q_j^* for $i \neq j$ satisfy the following bounds

$$\frac{q_i^*}{(q_j^*-1)} \geq \frac{1}{(q_j^*-1)\log P_i}\log\left(\frac{C_{q_i^*-1}}{C_{q_j^*-2}}\right) + \frac{\log P_j}{\log P_i}, \quad (11)$$

$$\frac{q_i^*-1}{q_j^*} \leq \frac{1}{q_j^*\log P_i}\log\left(\frac{D_{q_i^*-1}}{D_{q_j^*}}\right) + \frac{\log P_j}{\log P_i}, \quad (12)$$

where $C_{q_i^*-1} = \sum_{r=0}^{q_1^*-1} P_i^r$, $C_{q_j^*-2} = \sum_{k=0}^{q_j^*-2} P_j^k$, $D_{q_2^*} = \sum_{k=0}^{q_j^*} P_j^k$ and $D_{q_i^*-1} = \sum_{r=0}^{q_i^*-1} P_i^r$.

Proof: From Definition 3, it is clear that a neighbor of $\mathbf{q}^*$ in the search space $\mathbb{S}$ differs in two positions with respect to $\mathbf{q}^*$. Let us consider two neighbors of $\mathbf{q}^*$ that differ in the i-th and j-th index, where $i \neq j$. Such neighbors are of the form $\tilde{\mathbf{q}}_+ = [q_1^*, q_2^*, \dots, q_i^*+1, \dots, q_j^*-1, \dots, q_N^*]$ and $\tilde{\mathbf{q}}_- = [q_1^*, q_2^*, \dots, q_i^*-1, \dots, q_j^*+1, \dots, q_N^*]$ provided $q_i^* - 1 \geq 1$ and $q_j^* - 1 \geq 1$. From Theorem 1, instead of considering the multi-hop network with LOS vector $\mathbf{c} = [c_1, c_2, \dots, c_i, \dots, c_j, \dots, c_{N-1}, c_N]$, we consider a permuted version of it with the LOS vector $\mathbf{c} = [c_1, c_2, \dots, c_{N-1}, \dots, c_N, \dots, c_i, c_j]$, wherein the i-th link is swapped with $(N-1)$-th link, and the j-th link is swapped with N-th link. Correspondingly, the local minima and its two neighbors are respectively of the form $\mathbf{q}^* = [q_1^*, q_2^*, \dots, q_{N-1}^*, \dots, q_N^*, \dots, q_i^*, q_j^*]$, $\tilde{\mathbf{q}}_+ = [q_1^*, q_2^*, \dots, q_{N-1}^*, \dots, q_N^*, \dots, q_i^*+1, q_j^*-1]$ and $\tilde{\mathbf{q}}_- = [q_1^*, q_2^*, \dots, q_{N-1}^*, \dots, q_N^*, \dots, q_i^*-1, q_j^*+1]$. From the definition of local minima, we have the inequalities

$$p_d(\mathbf{q}^*) \leq p_d(\tilde{\mathbf{q}}_+), \text{ and } p_d(\mathbf{q}^*) \leq p_d(\tilde{\mathbf{q}}_-), \quad (13)$$

where $p_d(\mathbf{q}^*)$, $p_d(\tilde{\mathbf{q}}_+)$ and $p_d(\tilde{\mathbf{q}}_-)$ represent the PDP evaluated at the distributions $\mathbf{q}^*$, $\tilde{\mathbf{q}}_+$, and $\tilde{\mathbf{q}}_-$, respectively. Due to the structure of the PDP and the fact that $\tilde{\mathbf{q}}_+$ and $\tilde{\mathbf{q}}_1$ differ only in the last two positions, it can be shown that the inequalities in (13) are equivalent to

$$P_i^{q_i^*} + P_j^{q_j^*}\left(1 - P_i^{q_i^*}\right) \leq P_i^{q_i^*+1} + P_j^{q_j^*-1}\left(1 - P_i^{q_j^*+1}\right), \quad (14)$$

$$P_i^{q_i^*} + P_j^{q_j^*}\left(1 - P_i^{q_i^*}\right) \leq P_i^{q_i^*-1} + P_j^{q_j^*+1}\left(1 - P_i^{q_j^*-1}\right), \quad (15)$$

respectively. First, let us proceed with (14) to derive a necessary and sufficient condition on q_i^* and q_j^*. After modifications, the inequality in (14) can be rewritten as

$$P_i^{q_i^*}(1-P_i) \ + \ P_j^{q_j^*}(1-P_i^{q_i^*}) - P_j^{q_j^*-1}(1-P_i^{q_i^*+1}) \leq 0.$$

We can further rewrite it as

$$(1-P_i)\left(P_i^{q_i^*}+P_j^{q_j^*}\left(\sum_{r=0}^{q_i^*-1}P_i^r\right)-P_j^{q_j^*-1}\left(\sum_{k=0}^{q_i}P_i^k\right)\right)\leq 0,$$

using the following standard equality,

$$(1-P_i^n)=(1-P_i)(1+P_i+P_i^2+\ldots+P_i^{n-1}). \quad (16)$$

Since $(1-P_i)\geq 0$ is always true, this implies that

$$P_i^{q_i^*}+P_j^{q_j^*}\left(\sum_{r=0}^{q_i^*-1}P_i^r\right)-P_j^{q_j^*-1}\left(\sum_{k=0}^{q_i^*}P_i^k\right)\leq 0.$$

Furthermore, we can rewrite the above inequality as

$$P_i^{q_i^*}\left(1-P_j^{q_j^*-1}\right)-\left(\sum_{r=0}^{q_i^*-1}P_i^r\right)\left(P_j^{q_j^*-1}(1-P_j)\right)\leq 0$$

Expanding $\left(1-P_j^{q_j^*-1}\right)$ and also using the fact that $(1-P_j)\geq 0$, we can write the above inequality as

$$P_i^{q_i^*}\left(\sum_{k=0}^{q_j^*-2}P_j^k\right)-\left(\sum_{r=0}^{q_i^*-1}P_i^r\right)P_j^{q_j^*-1}\leq 0.$$

This further implies that

$$\frac{P_i^{q_i^*}}{P_j^{q_j^*-1}}\leq\left(\frac{\sum_{r=0}^{q_i^*-1}P_i^r}{\sum_{k=0}^{q_j^*-2}P_j^k}\right). \quad (17)$$

With $C_{q_i^*-1}\triangleq\left(\sum_{r=0}^{q_i^*-1}P_i^r\right)$, and $C_{q_j^*-2}\triangleq\left(\sum_{k=0}^{q_j^*-2}P_j^k\right)$, we have,

$$\frac{P_i^{q_i^*}}{P_j^{q_j^*-1}}\leq\frac{C_{q_i^*-1}}{C_{q_j^*-2}}.$$

By taking logarithm on both sides, and subsequently rearranging the terms, we get

$$\frac{q_i^*}{(q_j^*-1)}\geq\frac{1}{(q_j^*-1)\log P_i}\log\left(\frac{C_{q_i^*-1}}{C_{q_j^*-2}}\right)+\frac{\log P_j}{\log P_i}. \quad (18)$$

This completes the proof for the first necessary condition. Although the second necessary condition can be proved along the same lines using (15), we omit the proof due to lack of space in this paper. We highlight that the two conditions in (11) and (12) are also sufficient since the bounds are obtained by rearranging the terms in the condition on local minima. ■

Corollary 1: At high SNR, i.e., when P_k is negligible for each k, we have

$$\frac{q_i^*}{(q_j^*-1)}\geq\frac{\log P_j}{\log P_i}+\epsilon_{i,j}^{(1)}, \quad (19)$$

$$\frac{q_i^*-1}{q_j^*}\leq\frac{\log P_j}{\log P_i}+\epsilon_{i,j}^{(2)}, \quad (20)$$

where $|\epsilon_{i,j}^{(1)}|$ and $|\epsilon_{i,j}^{(2)}|$ are small numbers.

Proof: When P_i and P_j are negligible, the first terms of the right hand side of both (11) and (12) are negligible because $\log\left(\frac{C_{q_i^*-1}}{C_{q_j^*-2}}\right)\approx 0$ and $\log\left(\frac{D_{q_i^*-1}}{D_{q_j^*}}\right)\approx 0$, and $\log(\frac{1}{P_i})>>0$. However, we note that these terms may either be positive or negative depending on the values of $q_i, q_j, P_i,$ and P_j. Therefore, by considering the polarity of these values, we bound the absolute values of $\epsilon_{i,j}^{(1)}$ and $\epsilon_{i,j}^{(2)}$ in the statement of the corollary. ■

Based on the necessary and sufficient conditions derived in Theorem 3, we are ready to synthesize a low complexity algorithm to solve Problem 1.

IV. Low-complexity List-Decoding Algorithm

From Corollary 1, it is straightforward to note that at high SNR values, the necessary and sufficient conditions on the local minima satisfy the bounds in (19) and (20), for every pair i,j such that $i\neq j$. We immediately notice that the following inequality also holds

$$\frac{q_i^*-1}{q_j^*}<\frac{q_i^*}{q_j^*}<\frac{q_i^*}{q_j^*-1}. \quad (21)$$

Using (19), (20), and the strict inequality constraints in (21), we propose a method to choose the ARQ distribution $\mathbf{q}$ in the following proposition.

Proposition 1: If the ARQ distribution $\mathbf{q}$ is chosen such that $\frac{q_i}{q_j}=\frac{\log P_j}{\log P_i}$, for $i\neq j$, then $\mathbf{q}$ is a local minima of the search space at high SNR.

Proof: By choosing $\mathbf{q}$ such that $\frac{q_i}{q_j}=\frac{\log P_j}{\log P_i}$ for $i\neq j$ ensures that the sufficient conditions in (19) and (20) are trivially satisfied when $\epsilon_{i,j}^{(1)}<0$ and $\epsilon_{i,j}^{(2)}>0$. However, when $\epsilon_{i,j}^{(1)}>0$ and $\epsilon_{i,j}^{(2)}<0$, the sufficient conditions in (19) and (20) continue to satisfy, provided the SNR is sufficiently large to bound $|\epsilon_{i,j}^{(1)}|<\frac{q_i^*}{q_j^*-1}-\frac{q_i^*}{q_j^*}$ and $|\epsilon_{i,j}^{(2)}|<\frac{q_i^*}{q_j^*}-\frac{q_i^*-1}{q_j^*}$. ■

Based on the results in Proposition 1, we formulate Problem 2 as a means of solving Problem 1 at high SNR. However, from Problem 2, it is straightforward to note that a solution is not guaranteed since the ratio $\frac{\log P_j}{\log P_i}$, which is computed based on the LOS components and the SNR, need not be in $\mathbb{Q}$. Therefore, we propose to solve Problem 2 without the integer constraints, i.e., to find an ARQ distribution $\mathbf{q}\in\mathbb{R}^N$ satisfying the constraints $\frac{q_i}{q_j}=\frac{\log P_j}{\log P_i}$, for all i,j such that $i\neq j$, and $\sum_{k=1}^N q_k=q_{sum}$.

Problem 2: For a given $\{P_1,P_2,\ldots,P_N\}$, find $q_1,q_2,\ldots q_N$ such that

$$\begin{aligned}\frac{q_i}{q_j}&=\frac{\log P_j}{\log P_i},\ \forall\ i,j \text{ such that } i\neq j,\\ q_k&\geq 1,\ \forall\ k,\\ q_k&\in\mathbb{Z}_+,\ \forall k,\\ q_1+q_2+\ldots+q_N&=q_{sum}.\end{aligned}$$

A. Towards Solving Problem 2 without Integer Constraints

Towards solving Problem 2 without the integer constraints, we define $d_{i,j} \triangleq \frac{\log P_j}{\log P_i}$ for $i \neq j$. With that, the task of solving Problem 2 in $\mathbb{R}^N$ can be viewed as the task of solving the system of linear equations: $\mathbf{A}\mathbf{q}_{real} = \mathbf{b}$, where

$$\mathbf{A} = \begin{bmatrix} 1 & -d_{1,2} & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & -d_{2,3} & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \dots & \dots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \dots & \dots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & \dots & 1 & -d_{N-1,N} \\ 1 & 1 & 1 & \dots & \dots & 1 & 1 \end{bmatrix} \in \mathbb{R}^{N \times N},$$

$\mathbf{q}_{real} = [q_1, q_2, \dots, q_N]^T$ and $\mathbf{b} = [0, 0, \dots, 0, q_{sum}]^T$. Subsequently, a solution in $\mathbb{R}^N$ can be obtained as

$$\mathbf{q}_{real} = \mathbf{A}^{-1}\mathbf{b}, \tag{22}$$

as long as $\mathbf{A}$ is full rank. Although $\mathbf{q}_{real}$ in (22) satisfies the first and the last constraints of Problem 2, it cannot be used in the framework of multi-hop network since its components need not belong to $\mathbb{Z}_+$. In order to force the solution to lie in $\mathbb{Z}_+$, in the next section, we provide an algorithm that searches for ARQ distributions in $\mathbb{S}$ that are nearest to $\mathbf{q}_{real}$.

Remark 1: It is possible to prove by contradiction that $\mathbf{q}_{real}$ cannot have any negative components since $d_{i,j}$ is strictly non-negative for all i, j. If at least one component of $\mathbf{q}_{real}$ is negative, it implies that every component of $\mathbf{q}_{real}$ is negative, and therefore, the sum constraint corresponding to the last row of $\mathbf{A}\mathbf{q}_{real} = \mathbf{b}$ will not be satisfied.

B. List Generation using the Non-Integer Solution

Our approach, as presented in Algorithm 1, is to search for integer solutions in $\mathbb{S}$ that are nearest to $\mathbf{q}_{real}$. In particular, using $\mathbf{q}_{real}$, we obtain an ARQ distribution, denoted by $\tilde{\mathbf{q}} = [\tilde{q}_1, \tilde{q}_2, \dots, \tilde{q}_N] \in \mathbb{Z}^N$, by ceiling every component of $\mathbf{q}_{real}$, i.e., $\tilde{\mathbf{q}} = \lceil \mathbf{q}_{real} \rceil$. Since $\tilde{\mathbf{q}}$ may have zeros in some positions, we provide a brute-force correction by converting those zeros to ones. Subsequently, we compute $\sum_{i=1}^{N} \tilde{q}_i$, to verify the sum constraint. Due to the ceiling operation on each component, $\sum_{i=1}^{N} \tilde{q}_i$ is expected to exceed the sum constraint. Let E denote $\left(\sum_{i=1}^{N} \tilde{q}_i\right) - q_{sum}$. To identify the candidates in $\mathbb{S}$, we choose E positions in $\tilde{\mathbf{q}}$ and subtract one ARQ from each of these positions to make sure that the sum constraint is satisfied. Although, at most $\binom{N}{E}$ vectors in $\mathbb{S}$ can be generated this way, some of the combinations may not be valid due to the results of Theorem 2. Thus, we create a list of ARQ distributions in $\mathbb{S}$ (denoted by $\mathcal{L} \subset \mathbb{S}$) from $\mathbf{q}_{real}$. Finally, we compute the PDP of every ARQ distribution in $\mathcal{L}$, and then choose the one which minimizes the PDP. An illustrative example of our approach is given in Fig. 2 for a 2-hop network.

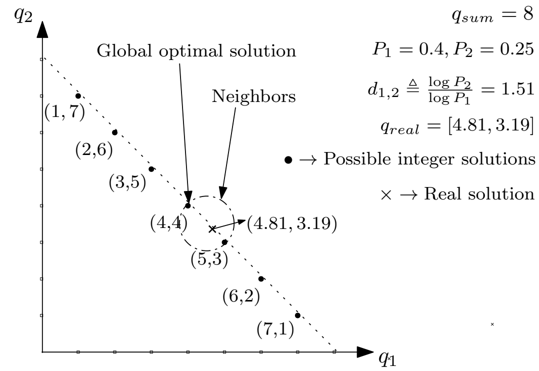

Fig. 2: An illustrative example with $N = 2$: The LOS components and the SNR of the two links are such that $P_1 = 0.4$ and $P_2 = 0.25$. With $q_{sum} = 8$, our approach generates a list consisting 2 ARQ distributions, whereas the size of the search space is 7.

Algorithm 1 List Generation Based Algorithm

Input: $\mathbf{A}$, $\mathbf{b}$, q_{sum}, $\mathbf{c} = [c_1, c_2, \dots, c_N]$

Output: $\mathcal{L} \subset \mathbb{S}$ - List of ARQ distributions in $\mathbb{S}$.

```
Compute q_real = A^{-1} b.
Compute q~ = ⌈q_real⌉.
for i = 1 : N do
    if q~_i = 0 then
        q~_i = q~_i + 1
    end if
end for
Compute E = (Σ_{i=1}^{N} q~_i) − q_sum
L = {q ∈ S | d(q, q~) = E, q_j ≯ q_i for c_i < c_j}.
```

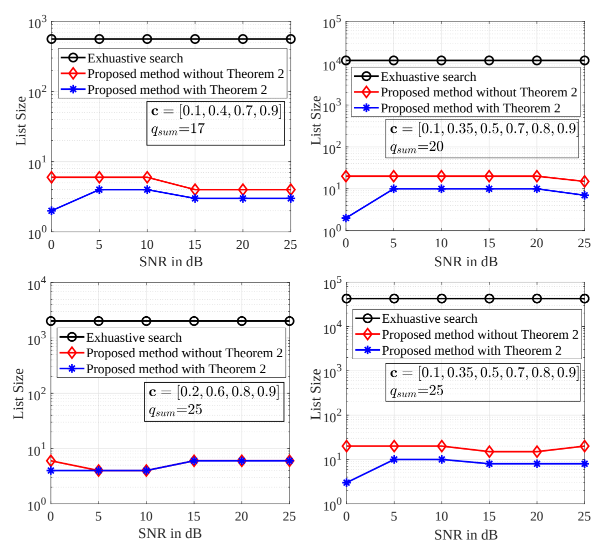

Fig. 3: List size for $N = 4$ and $N = 6$ at $R = 1$.

V. Complexity Analysis and Simulation Results

As highlighted in Section II, the computational complexity for solving Problem 1 through exhaustive search is $\binom{q_{sum}-1}{N-1}$. In contrast, we have used the results from Theorem 3, to first solve a relaxed version of Problem 2 in $\mathbb{R}^n$ (instead of $\mathbb{Z}^n$), and then search for candidates in $\mathbb{S}$ that are nearest to $\mathbf{q}_{real}$. Thus, the computational complexity of our method is dominated by the complexity of solving the system of linear equations, and

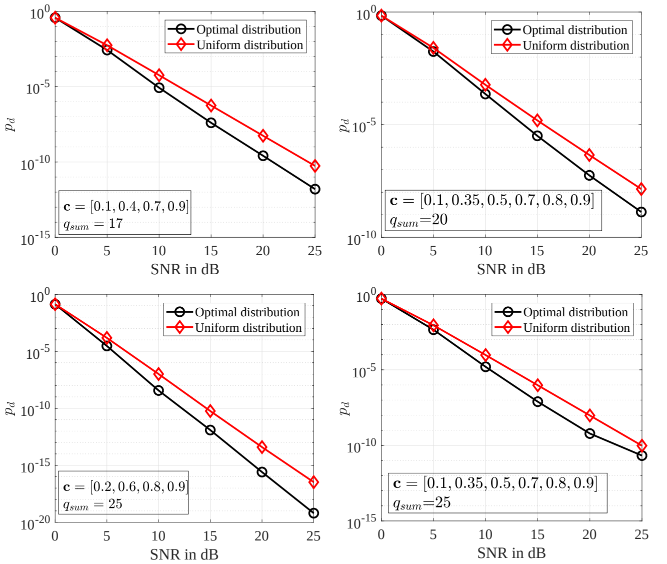

Fig. 4: PDP for $N = 4, 6$ at $R = 1$. With uniform distribution, each link is first allotted $\lfloor \frac{q_{sum}}{N} \rfloor$ ARQs, whereas the remaining ARQs are equally shared by the first q_{sum} mod N links.

that of the algorithm used to generate the list of candidates in $\mathbb{S}$. While the complexity for the former case is $O(N^3)$, the complexity of generating the list is at most $\binom{N}{E}$, where E is the excess number of ARQs after the ceiling operation.

To showcase the difference between the size of the list and that of the search space, we plot them in Fig. 3 for several instantiations of multi-hop networks with $N = 4$ and $N = 6$. In particular, we plot the list size both with and without incorporating the results of Theorem 2. For the former case, when subtracting one ARQ from all possible $\binom{N}{E}$ positions from $\tilde{\mathbf{q}}$, we discard those ARQ distributions which follow the rule $q_i > q_j$ whenever $c_i > c_j$. As a result, we observe that the list size shortens remarkably after incorporating the rule of Theorem 2. Based on the simulation results, we observe that the ARQ distribution which minimizes the PDP from the list $\mathcal{L}$ matches the result of exhaustive search, thereby confirming that our list indeed encapsulates the optimal ARQ distribution of the underlying problem. Although we used high SNR results of Corollary 1 to synthesize the list-decoding method, we observe that the size of the list reduces significantly at low and medium range of SNR values as well. We attribute this behavior to the fact that the parameters $\epsilon_{i,j}^{(1)}$ and $\epsilon_{i,j}^{(2)}$, for $i \neq j$, satisfied $\epsilon_{i,j}^{(1)} < 0$ and $\epsilon_{i,j}^{(2)} > 0$, which in turn ensured that $\mathbf{q}_{real}$ satisfied the sufficient conditions of local minima. Finally, for the parameters considered in Fig. 3, we also plot the corresponding PDP in Fig. 4 so as to highlight the suboptimality of uniform ARQ distribution.

VI. Summary

We have addressed a new framework to reliably communicate low-latency packets over a multi-hop network dominated by line-of-sight channels. We have specifically considered the question of how to distribute a given number of ARQs across the relay nodes in an ARQ based decode-and-forward relaying protocol such that the packet-drop-probability is minimized. To facilitate solving this problem with low-complexity methods, we have derived necessary and sufficient conditions on the optimal distribution of ARQs, and have subsequently used these conditions to propose a list-based enumeration algorithm. Simulation results confirm that the generated list is substantially shorter than that of exhaustive search, thereby rendering our algorithm amenable to implementation in practice.

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