Joint Optimization of Relaying Rate and Energy Consumption for Cooperative Mobile Edge Computing

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***Abstract*—Mobile edge computing networks with energy constrained users, which do not have access to licensed spectrum, might face difficulty in completing delay sensitive computational tasks due to lack of proper offloading spectrum. This paper considers a mobile edge computing scenario in the context of cooperative communication, where an user (with no access to licensed spectrum) relays a licensed spectrum holder's data to get access to the licensed spectrum for computation task offloading. We consider the relay to have a heavy computational task to complete by a given time duration. However, due to less computation power, the relay might not be able to meet the timeline. In this regard, the relay might offload partial computational task over the licensed spectrum to a more computationally powerful node, e.g., mobile edge computing server. The licensed spectrum holder might be interested in maximizing the relaying rate; whereas, the relay might want to minimize the total energy consumption for the task computation. We consider the relay partially offloading it's computational task and formulate an optimization problem capturing two different interests for the licensed spectrum holder and the relay. In the optimization problem, we consider relevant constraints to make it more general. The optimization problem is observed to be non-convex. However, after analysis, we propose a less complex iterative algorithm. We observe that joint optimization provides a better trade-off between the relaying rate and the energy consumption at the WU, which is not observed for individual relaying rate maximization and energy consumption minimization.**

***Keywords*—Mobile edge computing, Relaying, Partial offloading, Optimization**

I. Introduction

Computational complexity of wireless users (WUs) is rising day by day with the advent of different technologies, such as, facial recognition, augmented reality, virtual reality, highly interactive online gaming, internet of things (IoT), etc. Most of these applications require low latency, which might not be attained at WUs with limited computation capability. WUs might take help of a centralized cloud computing facility by offloading computation task for remote computation [1]. However, offloading to a distant cloud computing facility might not be beneficial in terms of latency.

In recent years, mobile edge computing (MEC) has gained significant attention as WUs might offload computation task to the MEC servers in close proximity for remote computation, which might reduce the latency substantially compared to offloading for a distant cloud computing facility. MEC has been proposed by Europeans Telecommunications Standards Institute (ETSI) [2], which aims in improving WUs' experience with latency responses. In [3], we find two different types of task offloading modes, i.e., binary and partial. WUs either offload total computational task or do local computation in binary offloading mode. In partial offloading mode, WUs offload a fraction of computational task to the MEC server. WUs of networks, e.g., adhoc, internet of things (IoT), which do not have dedicated licensed spectrum, may offload their computational tasks over unlicensed spectrum. However, WUs might face uncontrolled interference during offloading over unlicensed spectrum and hence might not satisfy the delay requirement.

To avoid uncontrolled interference, WUs might get access to licensed spectrum by spectrum sharing, which has gained widespread attention in different communication networks like 5G [4], IoT [5], etc. In literature, we find different spectrum sharing methods, e.g., opportunistic [6] and cooperative relaying [7]. In the context of MEC, spectrum sharing might be relevant for wireless networks (e.g., adhoc, IoT), which might not have any dedicated spectrum for computation task offloading. We consider a MEC network in conjunction with cooperative relaying based spectrum sharing in this paper. Related literature includes [7], where authors have considered two separate optimization problems for finding relaying and MEC parameters. Though authors of [8]–[11] have not considered spectrum sharing, they have considered cooperative relaying in the context of MEC. In [8], [9], authors have considered energy minimization problem; whereas, authors of [10], [11] have considered minimization of weighted sum of energy and latency. Out of [7]–[11], authors of [10], [11] have considered partial offloading in their work.

In this paper, we consider a WU with low computation power and high computational task. The WU takes help of a MEC server to complete the computational task by a firm timeline. As has been discussed earlier, in order to reduce computation latency, the WU shares a licensed spectrum. Typically, the wireless network whose spectrum is shared, is denoted by primary network in literature [7]. Following the same convention, we denote the transmitter and the corresponding receiver of the primary network by primary transmitter (PT) and primary receiver (PR), respectively. The WU follows decode and forward (DF) relaying principle while forwarding the PT's data to the PR to get access of the primary network's licensed spectrum. We optimize weighted sum of relaying rate and the WU's energy consumption. As spectrum sharing has not been considered in [8]–[11], PT's rate maximization is not relevant there. Though authors of [7] have considered spectrum sharing, they have not considered joint optimization

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of relaying rate and energy consumption. Precisely, we can outline contributions of this paper as:

- We consider joint optimization of the PT's rate and the WU's energy consumption in this paper while considering weighted sum of these two metrics. The PT is guaranteed rate above some predefined threshold; whereas, a hard timeline is considered in constraint to capture latency sensitivity of the WU's application. To the best of our knowledge, this form of optimization problem has not been considered earlier for MEC.
- We also perform comparative study with two other optimization scenarios, e.g., relaying rate maximization and energy consumption reduction. We observe that joint optimization provides better utility value (i.e., total utility constructed by the relaying rate and energy consumption) while providing nice trade-off between the relaying rate and the energy consumption. We observe that compared to other two optimization scenarios, joint optimization helps in saving significant amount of energy at the relay at the cost of negligible loss in relaying rate.

II. System Model

In Fig. 1, we show logical diagram of our system model, where we consider that the WU has high comptational task [1] to complete by a given time duration. However, due to low computation power, the WU might not be able to finish the task by the given time duration. The WU offloads partial amount of the task to the MEC server using the PT's licensed spectrum. The WU gets access to the licensed spectrum in remuneration of relaying the PT's data to the PR. The WU has two separate hardware blocks for transceiver and computation, as shown in Fig. 1. Channel gains between PT-PR, PT-WU, WU-PR, and WU-MEC server are denoted by h_{1-4}, respectively. We consider partial offloading in this paper.

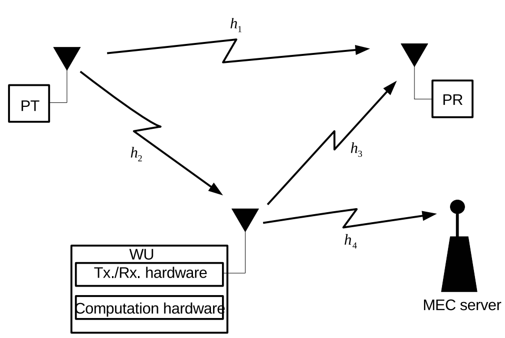

Fig. 1. Relaying based MEC network

We assume both the WU and the PT operate in a slotted way over time and follow a synchronized slot structure. During each slot, time frame duration is considered to be T. The WU performs local computing throughout a frame duration, which does not hamper in the relaying process due to separate hardware blocks for computation and communication. We assume the WU locally computes $\beta (0 \leq \beta \leq 1)$ fraction of the task. Based on activities at the PT, PR, WU, and MEC server transcievers, we divide a frame duration into three phases, which is shown in Fig. 2:

- **Phase-1:** The PT broadcasts data to the PR and the WU for $\tau_r/2$ duration.
- **Phase-2:** The WU relays the PT's data to the PR for $\tau_r/2$ duration.[2]
- **Phase-3:** In Phase-3, three different activities take place over the duration $(T-\tau_r)$, i.e., the WU offloads $(1-\beta)$ fraction of the task to the MEC server. The MEC server computes the data and feedbacks the result to the WU.

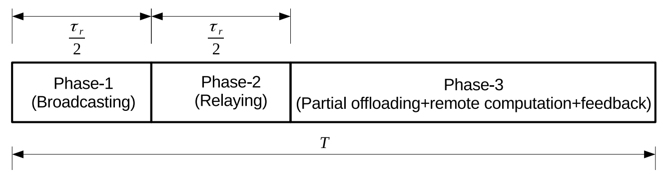

Fig. 2. Different phases based on transceivers activities at the PT, PR, WU, and MEC server

Possible application of the system model: We might correspond the WU to a sensor node in a narrow band-internet of things (NB-IoT) network, which might have a heavy computational task to complete by a particular time duration [12]. Due to the computational power limitation, the WU takes help of a MEC server. The WU might share a licensed spectrum with corresponding licensed users following in-band operation [5], in return cooperating licensed users in their communications. The WU communicates with the MEC server using the licensed spectrum.

In following subsections, we discuss about primary network's rate and different relevant parameters related to both local as well as remote computation.

A. PT's rate

We consider DF relaying in this work. Following [13], [14], we can write the PT's rate as:

$$R_P = \frac{\tau_r B_w}{2} \log_2 \{1 + \min(\text{SNR}_{1+3}, \text{SNR}_2)\} \quad (1)$$

where B_w is the bandwidth of the licensed spectrum and

$$\text{SNR}_{1+3} = \frac{P_{\text{PT}} h_1 d_1^{-\alpha}}{\sigma^2} + \frac{P_{\text{WU},1} h_3 d_3^{-\alpha}}{\sigma^2}, \text{SNR}_2 = \frac{P_{\text{PT}} h_2 d_2^{-\alpha}}{\sigma^2}.$$

P_{PT} and $P_{\text{WU},1}$ are the PT's transmission and the WU's relaying powers, respectively. d_1, d_2, and d_3 denote distances between PT-PR, PT-WU, and WU-PR, respectively. α is the path loss exponent and σ^2 denotes total noise power at the receiver (we consider identical noise power at all receivers). We consider block fading in this paper, such that, h_{1-4} remains same throughout a slot duration. We mainly focus on the case where the PT gets advantage due to relaying compared to direct transmission, i.e., $h_1 d_1^{-\alpha} < h_2 d_2^{-\alpha}$.

[1]We use the word 'task' in place of 'computational task'

[2]Different time durations for Phase-1 and 2 might be considered, which brings more variables in the optimization problem. We leave this for future work.

B. Local computation

We characterize the WU by a positive parameter tuple $\langle Q, C_{\text{WU}}, f_{\text{WU}}, T\rangle$, where Q denotes number of bits with which the task at the WU is formed, C_{WU} is the number of CPU cycles required to compute a bit at the WU, f_{WU} is the WU's CPU frequency (CPU cycles/second), and T is the time duration by which the WU needs to finish its task. We assume that the WU adopts dynamic frequency and voltage scaling (DVFS) technique to step up or down the CPU frequency. The WU's computation capability is limited by a maximum value for CPU frequency $f_{\text{WU}}^{\max}$. We discuss about local computation time and energy consumption below.

1) Associated time: The WU might not be able to complete the task by stipulated time duration due to computation capability limitation. We assume the WU chooses to compute β fraction of the task, such that, the required time to complete the task becomes:

$$t_{\text{WU}}^{(c)} = \frac{\beta Q C_{\text{WU}}}{f_{\text{WU}}}. \tag{2}$$

2) Associated energy consumption: Due to the computation procedure, the WU depletes energy. Total energy consumption due to computation has three different factors [15], i.e., dynamic, short circuit, and leakage energy consumption. As dynamic energy consumption dominates over the other two factors, we do our further analysis based on the dynamic energy consumption during computation. According to [16], we can write the energy consmption at the WU due to computation as:

$$E_{\text{WU}}^{(\text{c})} = k_{\text{WU}} \beta Q C_{\text{WU}} f_{\text{WU}}^2 \tag{3}$$

where k_{WU} is a constant which is related to the hardware architeture at the WU.

C. Remote computation

The WU offloads $(1-\beta)$ fraction of the task to the MEC server for remote computation. We characterize the MEC server by $\langle C_{\text{MEC}}, f_{\text{MEC}}\rangle$, where C_{MEC} is the number of CPU cycles to compute a bit at the MEC server and f_{MEC} is the MEC server's CPU frequency. Following, we discuss about time duration and energy consumption associated with the remote computation procedure.

1) Associated time: We consider the total time duration, i.e., the instant of data offloading from the WU to the instant of receiving feedback at the WU from the MEC, to compute the total time duration for remote computation procedure. The WU offloads task to the MEC server for duration:

$$t_{\text{WU}}^{(\text{o})} = \frac{(1-\beta)Q}{B_w \log_2\left\{1 + \frac{P_{\text{WU},2} h_4 d_4^{-\alpha}}{\sigma^2}\right\}}, \tag{4}$$

where $P_{\text{WU},2}$ is the transmission power at the WU during offloading. h_4 and d_4 are channel gain and distance between WU-MEC, respectively. The MEC server computes $(1-\beta)Q$ bits for following time duration:

$$t_{\text{MEC}}^{(\text{o})} = \frac{(1-\beta)Q C_{\text{MEC}}}{f_{\text{MEC}}}. \tag{5}$$

We assume that the MEC outputs $\gamma(\gamma < 1)$ fraction of input data, such that, under the assumption of reciprocal channel gains between WU-MEC and MEC-WU, we can write the feedback time duration as:

$$t_f^{(\text{o})} = \frac{\gamma Q(1-\beta)}{B_w \log_2\left\{1 + \frac{P_{\text{MEC}} h_4 d_4^{-\alpha}}{\sigma^2}\right\}}. \tag{6}$$

We can write the total time duration to get back the remotely computed data at the WU as:

$$t_r^{(\text{o})} = t_{\text{WU}}^{(\text{o})} + t_{\text{MEC}}^{(\text{o})} + t_f^{(\text{o})}. \tag{7}$$

2) Associated energy consumption: Like the time calculation, we calculate associated energy consumption at the WU for the remote computation procedure, which is mainly for data offloading:

$$E_{\text{WU}}^{(\text{o})} = P_{\text{WU},2} t_{\text{WU}}^{(\text{o})}. \tag{8}$$

Under the assumption of small amount of feedback data from the MEC server after computation, we neglect the energy associated with the reception of feedback at the WU [9].

III. System optimization

In our system model, there are two different networks, that have two different interests. For the primary network, the PT will try to maximize the relaying rate; whereas, in the other network, the WU will try to complete it's task by stipulated time while minimizing required resource, e.g., consumed energy. Two different optimization problems emerge for two networks. In this paper, we consider single weighted objective function which considers both primary network's relaying rate and other network's energy consumption into account, i.e.,

$$\begin{aligned} &U(\beta, \tau_r, f_{\text{WU}}, P_{\text{WU},1}, P_{\text{WU},2}) \\ &= R_p - \psi\left[P_{\text{WU},1}\frac{\tau_r}{2} + E_{\text{WU}}^{(\text{c})} + E_{\text{WU}}^{(\text{o})}\right], \end{aligned} \tag{9}$$

where $\psi(\psi > 0)$ (in bits/Joule) is a weight to make the objective function's unit in bits. It can be observed from (9) that we consider the energy consumption at the WU for relaying the PT's data.

Please note that none of [7]–[11] has considered such weighted objective function. We write our optimization problem as follows:

$$P1: \underset{\beta, \tau_r, f_{\text{WU}}, P_{\text{WU},1}, P_{\text{WU},2}}{\text{maximize}} \quad U(\beta, \tau_r, f_{\text{WU}}, P_{\text{WU},1}, P_{\text{WU},2})$$

subject to:

$$R_p \geq R_L \tag{10a}$$
$$t_{\text{WU}}^{(\text{c})} \leq T \tag{10b}$$
$$\tau_r + t_r^{(\text{o})} \leq T \tag{10c}$$
$$0 \leq P_{\text{WU},1} \leq P_{\max} \tag{10d}$$
$$0 \leq P_{\text{WU},2} \leq P_{\max} \tag{10e}$$
$$P_{\text{WU},1} + P_{\text{WU},2} \leq P_{\text{avg}} \tag{10f}$$
$$0 \leq f_{\text{WU}} \leq f_{\text{WU}}^{\max} \tag{10g}$$
$$0 \leq \beta \leq 1. \tag{10h}$$

We consider a constraint on minimun required relaying rate for the primary network in (10a). The WU locally computes β fraction of the task, which should be completed by the time duration T as has been given in (10b). Task offloading, remote computation at the MEC server, and feedback from the MEC server to the WU should be completed by the time duration T, which is represented in (10c). The WU's instantaneous transmission power constraints are taken into account in (10d) and (10e); whereas, (10f) is for the power budget constraint at the WU. (10g) and (10h) are constraints on the WU's CPU frequency and fraction of the computed task at the WU.

A. Feasibility check for the optimization problem $P1$

We observe that the optimization problem $P1$ might not be feasible all time for given values of different system parameters. From (10b), we get maximum value for β as:

$$\beta_{\text{max}} = \min\left\{\frac{Tf_{\text{WU}}^{max}}{QC_{\text{WU}}}, 1\right\}. \tag{11}$$

We put $\beta = \beta_{\text{max}}$ as received in (11), and maximum value for $P_{\text{WU,2}}$, i.e., $P_{\text{WU,2}} = P_{\text{max}}$, in (10c), to get highest possible value for τ_r as:

$$\tau_r^{\text{max}} = T - t_r(\beta = \beta_{\text{max}}, P_{\text{WU,2}} = P_{\text{max}}), \tag{12}$$

where $t_r(\beta = \beta_{\text{max}}, P_{\text{WU,2}} = P_{\text{max}})$ denotes the value for t_r considering $\beta = \beta_{\text{max}}$ and $P_{\text{WU,2}} = P_{\text{max}}$.

In a similar way, we can evaluate lowest possible value for τ_r from (10a). For that, we consider $P_{\text{WU,1}} = P_{\text{max}}$ while evaluating SNR_{1+3} in (1), such that:

$$\tau_r^{\text{min}} = \frac{2R_L}{B_w \log_2\left[1 + \min\left\{\text{SNR}_{1+3}(P_{\text{WU,1}} = P_{\text{max}}), \text{SNR}_2\right\}\right]}. \tag{13}$$

From (12) and (13), we can conclude that the optimization problem becomes infeasible for $\tau_r^{\text{min}} > \tau_r^{\text{max}}$.

B. Solution process for the optimization problem $P1$

In this sub-section, we discuss about the solution procedure of the optimization problem $P1$ when it meets the feasible condition as has been discussed in the previous sub-section. From expressions of different terms of the utility function of the optimization problem $P1$ as given in (3) and (8), it can be observed that there are multiplicative terms (i.e., multiplication of two different optimization variables). Therefore, the objective function of the optimization problem $P1$ does not follow the convexity property, which makes the optimization problem $P1$ non-convex. We analyse the optimization problem and try to devise an efficient way to solve it.

It can be observed that $E_{\text{WU}}^{(\text{c})}$ is a monotonically increasing function of f_{WU}. As we are trying to minimize the term $E_{\text{WU}}^{(\text{c})}$ in optimization problem $P1$, from the constraint as given in (10b), we can say that for given value of β, optimal value for f_{WU} is received for equality, i.e.,

$$f_{\text{WU}}(\beta) = \beta QC_{\text{WU}}/T. \tag{14}$$

Hence, we can modify the term $E_{\text{WU}}^{(\text{c})}$ in optimization problem $P1$ by considering $f_{\text{WU}} = f_{\text{WU}}(\beta)$:

$$E_{\text{WU}}^{(\text{c})}(\beta) = \frac{k_{\text{WU}}(\beta QC_{\text{WU}})^3}{T^2}. \tag{15}$$

Therefore, we can represent the modified objective function as independent of f_{WU}:

$$\begin{aligned} &U(\beta, \tau_r, P_{\text{WU,1}}, P_{\text{WU,2}}) \\ &= R_p - \psi\left[P_{\text{WU,1}}\frac{\tau_r}{2} + E_{\text{WU}}^{(\text{c})}(\beta) + E_{\text{WU}}^{(\text{o})}\right]. \end{aligned} \tag{16}$$

Modified optimization problem becomes:

$$\begin{aligned} P2: \quad & \underset{\beta, \tau_r, P_{\text{WU,1}}, P_{\text{WU,2}}}{\text{maximize}} \; U(\beta, \tau_r, P_{\text{WU,1}}, P_{\text{WU,2}}) \\ \text{subject to:} \quad & (10\text{a}) - (10\text{f}) \\ & 0 \le \beta \le \beta_{\text{max}}. \end{aligned} \tag{17}$$

From the optimization problem $P2$, it can be observed that it is still non-convex due to multiplicative terms in the objective function. From our analysis, we present following proposition.

Proposition 3.1: For given feasible values of $\beta = \overline{\beta}$ and $\tau_r = \overline{\tau}_r$, we can derive optimal values for $P_{\text{WU,1}}$ and $P_{\text{WU,2}}$ as given in (18) and (19), respectively. Following notations are used in expressions for both power levels at the WU:

$$\begin{aligned} x &\triangleq \frac{(1-\overline{\beta})Q}{B_w\left[T - \overline{\tau}_r - t_{\text{MEC}}(\overline{\beta}) - t_f(\overline{\beta})\right]} \\ P_1 &\triangleq \left\{P_{\text{WU,1}} | B_w\frac{\overline{\tau}_r}{2}\log_2(1 + \text{SNR}_{1+3}) = R_L\right\} \\ P_2 &\triangleq P_{avg} - P_{\text{WU,2}}(\overline{\beta}, \overline{\tau}_r) \\ P_3 &\triangleq \{P_{\text{WU,1}} | \text{SNR}_{1+3} = \text{SNR}_2 \text{ in (1)}\} \\ P_i &\triangleq \left\{P_{\text{WU,1}} | \frac{\partial(R_p(\overline{\tau}_r) - \psi P_{\text{WU,1}})}{\partial P_{\text{WU,1}}} = 0\right\}, \end{aligned}$$

where $t_{\text{MEC}}(\overline{\beta})$ and $t_f(\overline{\beta})$ are received from (5) and (6), respectively for $\beta = \overline{\beta}$. Please note that for $P_2 < 0$, the optimization problem becomes infeasible for given $\overline{\beta}$ and $\overline{\tau}_r$.

$$\begin{aligned} &P_{\text{WU,1}}^*(\overline{\beta}, \overline{\tau}_r) \\ &= \begin{cases} \max\{P_1, 0\} & P_i < 0 \\ \min\{P_i, P_{\text{max}}, P_2, P_3\} & P_i > 0 \end{cases} \end{aligned} \tag{18}$$

$$P_{\text{WU,2}}^*(\overline{\beta}, \overline{\tau}_r) = \frac{2^x - 1}{h_4 d_4^{-\alpha}/\sigma^2}. \tag{19}$$

Proof: For $\beta = \overline{\beta}$ and $\tau_r = \overline{\tau}_r$, we can write the objective function as:

$$\begin{aligned} &U(\overline{\beta}, \overline{\tau}_r, f_{\text{WU}}, P_{\text{WU,1}}, P_{\text{WU,2}}) \\ &= R_p(\overline{\tau}_r) - \psi\left[\frac{\overline{\tau}_r}{2}P_{\text{WU,1}} + E_{\text{WU}}^{(\text{o})}(\overline{\beta}) + E_{\text{WU}}^{(\text{c})}(\overline{\beta})\right], \end{aligned} \tag{20}$$

where $R_p(\overline{\tau}_r)$ is received for $\tau_r = \overline{\tau}_r$ in (1), $E_{\text{WU}}^{(\text{o})}(\overline{\beta})$ is received for $\beta = \overline{\beta}$ in (8).

We can observe that the utility function has separate terms for $P_{\text{WU,1}}$ and $P_{\text{WU,2}}$. We analyse the utility function for $P_{\text{WU,2}}$ from the first order derivative of $U(\overline{\beta}, \overline{\tau}_r, f_{\text{WU}}, P_{\text{WU,1}}, P_{\text{WU,2}})$ with respect to $P_{\text{WU,2}}$, i.e.,

$$\frac{C_1\left[\ln(1 + C_2P_{\text{WU,2}}) - \frac{C_2P_{\text{WU,2}}}{1+C_2P_{\text{WU,2}}}\right]}{\{\log(1 + C_2P_{\text{WU,2}})\}^2}, \tag{21}$$

where $C_1 \triangleq \frac{\psi\gamma(1-\overline{\beta})Q}{B_w\log_2(e)}$ and $C_2 \triangleq \frac{h_4 d_4^{-\alpha}}{\sigma^2}$. From (21), we can say that the term is always positive for feasible values of $P_{\text{WU,2}}$, which can be easily proved by taking derivative of the numerator. Therefore, we can conclude the utility function, i.e., $U(\overline{\beta}, \overline{\tau}_r, f_{\text{WU}}, P_{\text{WU,1}}, P_{\text{WU,2}})$, is a monotonically decreasing function for $P_{\text{WU,2}}$. As we are trying to maximize $U(\overline{\beta}, \overline{\tau}_r, P_{\text{WU,1}}, P_{\text{WU,2}})$, we should consider lowest feasible value for $P_{\text{WU,2}}$, which we get from (10c), i.e., $P_{\text{WU,2}} \geq$

$\frac{2^x-1}{h_4 d_4^{-\alpha}/\sigma^2}$, where x has been defined in the statement of this proposition. Also it can be observed from (10f) that the feasible range for $P_{\text{WU},1}$ is maximum for $\overline{\tau}_r$ and $\overline{\beta}$ when we consider $P_{\text{WU},2} = \frac{2^x-1}{h_4 d_4^{-\alpha}/\sigma^2}$. Therefore, from our discussions, we can conclude that the optimal value for $P_{\text{WU},2}$ becomes $P^*_{\text{WU},2}(\overline{\beta}, \overline{\tau}_r) = \frac{2^x-1}{h_4 d_4^{-\alpha}/\sigma^2}$.

Now, we find out optimal value for $P_{\text{WU},1}$. From the expression of $U(\overline{\beta}, \overline{\tau}_r, P_{\text{WU},1}, P_{\text{WU},2})$ as given in (20), it can be observed that $U(\overline{\beta}, \overline{\tau}_r, P_{\text{WU},1}, P_{\text{WU},2})$ is a concave function of $P_{\text{WU},1}$ with inflection point at P_i as defined in the statement of this proposition. However, based on values for ψ, we might get $P_i < 0$, under which $U(\overline{\beta}, \overline{\tau}_r, P_{\text{WU},1}, P_{\text{WU},2})$ monotonically decreases for $P_{\text{WU},1} \geq 0$. Therefore, for $P_i < 0$, we should consider $P_{\text{WU},1} = P_1$, which holds equality in (10a). However, for $P_i > 0$, we need to keep another fact in mind, i.e., the relaying rate depends on the minimum value among two SNRs, i.e., SNR_{1+3} and SNR_2 in (1). The relaying rate does not increase while increasing $P_{\text{WU},1}$ beyond P_3 (as defined in the statement of this proposition); whereas, the energy consumption at the WU for relaying increases with $P_{\text{WU},1}$. Therefore, we should consider $P^*_{\text{WU},1}(\overline{\beta}, \overline{\tau}_r) = \min\{P_i, P_{\max}, P_2, P_3\}$. ■

Now, we analyse the optimization problem for given feasible values of $P_{\text{WU},1} = \overline{P}_{\text{WU},1}$ and $P_{\text{WU},2} = \overline{P}_{\text{WU},2}$. We define following notations, which we use in further discussions:

$$\begin{aligned}
&U(\beta, \tau_r, \overline{P}_{\text{WU},1}, \overline{P}_{\text{WU},1}) \\
&= \tau_r\Big[\frac{B_w}{2}\log_2\{1+\min(\text{SNR}_{1+3}(\overline{P}_{\text{WU},1}), \text{SNR}_2)\} - \\
&\qquad \frac{\psi \overline{P}_{\text{WU},1}}{2}\Big] - \psi[E^{(\text{o})}_{\text{WU}}(\overline{P}_{\text{WU},2}) + E^{(\text{c})}_{\text{WU}}(\overline{\beta})] \\
&a \triangleq T - \frac{Q}{B_w \log_2\left\{1+\frac{\overline{P}_{\text{WU},2} h_4 d_4^{-\alpha}}{\sigma^2}\right\}} \\
&- \frac{\gamma Q}{B_w \log_2\left\{1+\frac{P_{\text{MEC}} h_4 d_4^{-\alpha}}{\sigma^2}\right\}} - \frac{Q C_{\text{MEC}}}{f_{\text{MEC}}} \\
&b \triangleq a/(T-a),
\end{aligned}$$

where $E^{(\text{o})}_{\text{WU}}(\overline{P}_{\text{WU},2})$ is received from (8) for $P_{\text{WU},2} = \overline{P}_{\text{WU},2}$. For $P_{\text{WU},1} = \overline{P}_{\text{WU},1}$ and $P_{\text{WU},2} = \overline{P}_{\text{WU},2}$, we can write the corresponding optimization problem as:

$$P3: \quad \underset{\beta,\tau_r}{\text{maximize}} \quad U(\beta, \tau_r, \overline{P}_{\text{WU},1}, \overline{P}_{\text{WU},2})$$

$$\text{subject to: } \tau_r \geq \tau_r^{\text{LB}} \tag{22a}$$

$$0 \leq \beta \leq \beta_{\max} \tag{22b}$$

$$\frac{\tau_r}{a} - \frac{\beta}{b} \leq 1, \tag{22c}$$

where $\tau_r^{\text{LB}} = \frac{2R_L}{B_w \log_2\{1+\min(\text{SNR}_{1+3}(\overline{P}_{\text{WU},1}), \text{SNR}_2)\}}$, we derive (22a) and (22c) from (10a) and (10c), respectively while considering $P_{\text{WU},1} = \overline{P}_{\text{WU},1}, P_{\text{WU},2} = \overline{P}_{\text{WU},2}$.

From the expression of the objective function in the optimization problem $P3$ as given above, it can be observed that it consists of linear and non-linear terms for τ_r and β, respectively. Moreover, we also observe that the optimization problem $P3$ is concave over τ_r and β. From our analysis, we present following proposition.

Proposition 3.2: We define the term $C_3 \triangleq \frac{B_w}{2}\log_2\{1+\min(\text{SNR}_{1+3}(\overline{P}_{\text{WU},1}), \text{SNR}_2)\} - \frac{\psi \overline{P}_{\text{WU},1}}{2}$, based on which we can find out optimal values for β and τ_r for the optimization problem $P3$ as given below. We also define following notations, which are used in this proposition:

$$\beta_l \triangleq \left[\frac{\tau_r^{\text{LB}}}{a} - 1\right] b, \beta_i \quad \triangleq \left\{\beta \Big| \frac{\partial U(\beta, \tau_r, \overline{P}_{\text{WU},1}, \overline{P}_{\text{WU},2})}{\partial \beta} = 0\right\}$$

$\underline{For: C_3 > 0}$

Optimal $\beta^*(\overline{P}_{\text{WU},1}, \overline{P}_{\text{WU},2})$ and $\tau_r^*(\overline{P}_{\text{WU},1}, \overline{P}_{\text{WU},2})$ lies on the straight line, i.e., $\frac{\tau_r}{a} - \frac{\beta}{b} = 1$, which we get after solving the dual optimization problem for $P3$, i.e.,

$$\underset{\boldsymbol{\lambda}}{\text{minimize}} \ \underset{\beta,\tau_r}{\text{maximize}} \ \mathcal{L}(\beta, \tau_r, \boldsymbol{\lambda}) \tag{23}$$

where $\mathcal{L}(\beta, \tau_r, \lambda) = U(\beta, \tau_r, \overline{P}_{\text{WU},1}, \overline{P}_{\text{WU},2}) + \lambda_1(\tau_r - \tau_r^{\text{LB}}) - \lambda_2(\beta - \beta_{\max}) - \lambda_3(\frac{\tau_r}{a} - \frac{\beta}{b} - 1)$, is Lagrangian function for the optimization problem $P3$ and $\boldsymbol{\lambda} = [\lambda_1, \lambda_2, \lambda_3]$ are dual variables for (22a)-(22c), respectively.

$\underline{For: C_3 < 0}$

$$\beta^*(\overline{P}_{\text{WU},1}, \overline{P}_{\text{WU},2}) = \begin{cases} \beta_{\max}, & \beta_i > \beta_{\max} \\ \max\{\beta_i, \beta_l\}, & \beta_i < \beta_{\max} \end{cases} \tag{24}$$

$$\tau_r^*(\overline{P}_{\text{WU},1}, \overline{P}_{\text{WU},2}) = \tau_r^{\text{LB}}. \tag{25}$$

Proof: From the expression of the utility function of the optimization problem $P3$ as given above, it can be observed that the utility function is linear over τ_r and non-linear over β. Moreover, the objective function is concave over β. It is intuitive that the optimal value for τ_r depends on the coefficient of τ_r in the objective function of the optimization problem $P3$, i.e., C_3 as has been defined in this proposition. Hence the optimal value for β depends on the value of C_3.

For $C_3 > 0$, we solve the dual optimization problem for $P3$ using following Karush-Kuhn-Tucker (KKT) conditions [17]:

$$\frac{\partial \mathcal{L}(\beta, \tau_r, \lambda)}{\partial \tau_r} = \frac{\partial \mathcal{L}(\beta, \tau_r, \lambda)}{\partial \beta} = 0, \lambda_{1-3} \geq 0,$$

$$\lambda_1(\tau_r - \tau_r^{\text{LB}}) = \lambda_2(\beta - \beta_{\max}) = \lambda_3(\frac{\tau_r}{a} - \frac{\beta}{b} - 1) = 0.$$

We check that $\lambda_3 \neq 0$, otherwise, it violates the condition $\frac{\mathcal{L}(\beta,\tau_r,\lambda)}{\partial \tau_r} = 0$. This proves that $\frac{\tau_r}{a} - \frac{\beta}{b} = 1$ is the optimal condition. We solve above mentioned KKT conditions to get $\beta^*(\overline{P}_{\text{WU},1}, \overline{P}_{\text{WU},2})$ and $\tau_r^*(\overline{P}_{\text{WU},1}, \overline{P}_{\text{WU},2})$ for $C_3 > 0$. Due to brevity, we choose to omit further solution procedures.

For $C_3 < 0$, objective function reduces while increasing the value for τ_r. Therefore, we choose the lowest possible value for τ_r for $C_3 < 0$ in (25). As the objective function is concave over β, we can derive the inflection point as β_i as has been defined in the definition of this proposition. Based on the feasible range, we derive the optimal value for β in (24). ■

It is to be noted that both optimization problems, i.e., for fixed β, τ_r and for fixed $P_{\text{WU},1}, P_{\text{WU},2}$, are solved optimally. We follow iterative algorithm as given in Algorithm 1. Please note that as we are optimally evaluating parameters' values in each iteration, the utility value improves in each iteration [18], i.e., $U(\beta^j, \tau_r^j, P^{j+1}_{\text{WU},1}, P^{j+1}_{\text{WU},2}) > U(\beta^j, \tau_r^j, P^j_{\text{WU},1}, P^j_{\text{WU},2})$ and $U(\beta^{j+1}, \tau_r^{j+1}, P^j_{\text{WU},1}, P^j_{\text{WU},2}) > U(\beta^j, \tau_r^j, P^j_{\text{WU},1}, P^j_{\text{WU},2})$.

Algorithm 1: Evaluation of different parameters of the optimization problem $P2$

Data: Consider $j \leftarrow 0$, $\beta^j \leftarrow \overline{\beta}$ and $\tau_r^j \leftarrow \overline{\tau}_r$.
repeat
  Evaluate $P_{\text{WU,1}}(\beta^j, \tau_r^j)$ and $P_{\text{WU,2}}(\beta^j, \tau_r^j)$ following Proposition 3.1;
  $P_{\text{WU,1}}^{j+1} \leftarrow P_{\text{WU,1}}(\beta^j, \tau_r^j)$, $P_{\text{WU,2}}^{j+1} \leftarrow P_{\text{WU,2}}(\beta^j, \tau_r^j)$;
  Evaluate $\beta(P_{\text{WU,1}}^{j+1}, P_{\text{WU,2}}^{j+1})$ and $\tau_r(P_{\text{WU,1}}^{j+1}, P_{\text{WU,2}}^{j+1})$;
  $\beta^{j+1} \leftarrow \beta(P_{\text{WU,1}}^{j+1}, P_{\text{WU,2}}^{j+1})$, $\tau_r^{j+1} \leftarrow \tau_r(P_{\text{WU,1}}^{j+1}, P_{\text{WU,2}}^{j+1})$;
  $j \leftarrow j+1$;
until $\beta^j == \beta^{j-1}, \tau_r^j == \tau_r^{j-1}, P_{WU,1}^j == P_{WU,1}^{j-1}, P_{WU,2}^j == P_{WU,2}^{j-1}$;
Result: $\beta^j, \tau_r^j, P_{\text{WU,1}}^j, P_{\text{WU,2}}^j$

IV. Results and Discussions

In this section, we try to analyse the effect of different parameters. We consider a scenario, where the WU lies on the straight line connecting the PT and the PR. Moreover, the PR and the MEC server are located at a same place, which means $d_3 = d_4 = d_1 - d_2$. In Fig. 3, we provide a logical diagram for the simulation scenario, which we consider in this paper. In Table I, we show different parameters' values which we

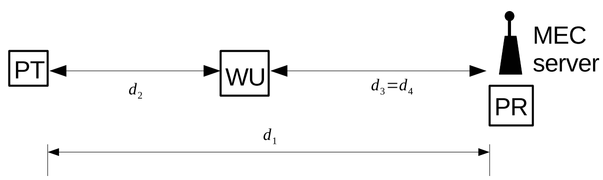

Fig. 3. Logical diagram for simulation scenario

consider in generating our results. Among other variables, we consider channel gains h_{1-4} follow exponential distributions with unit mean.

TABLE I. Different parameters' values

Parameters	Values	Parameters	Values
T	0.1 sec.	k_{WU}	10^{-26}
B_w	15 KHz.	C_{WU}	10^5
σ^2	-132.24 dBm	k_{MEC}	10^{-26}
P_{PT}	43 dBm	C_{MEC}	10^5
P_{max}	30 dBm	f_{max}	500 MHz.
P_{avg}	43 dBm	f_{MEC}	10 GHz.
γ	.01	d_1	100 m.

A. Convergence analysis of of iterative algorithm

In Fig. 4, we show how the total utility value as has been defined in (9), changes with number of iterations in Algorithm 1. We consider $R_L = Q = 1000$ bits, $\psi = 1, d_3 = 50$ meters, and $\alpha = 4$. We observe that Algorithm 1 converges in finite iterations.

B. Effect of different parameters on relaying rate and energy consumption

In Fig. 5, for $\psi = 1$, We plot the relaying rate for three different values of $\alpha = 3, 3.5, 4$. It is observed that for a fixed value of α, the relaying rate increases with the distance between the PT and the WU upto some point and

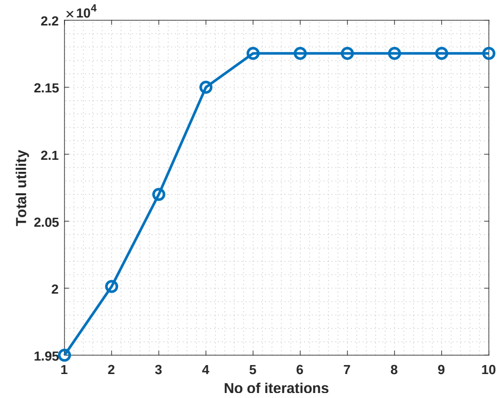

Fig. 4. Convergence analysis of Algorithm 1

then it decreases. As the distance between the PT and the WU increases, SNR values, i.e., SNR_{1+3} and SNR_2 in (1), increases and decreases, respectively. The relaying rate increases until we get $\text{SNR}_{1+3} = \text{SNR}_2$ and for $\text{SNR}_2 < \text{SNR}_{1+3}$, the relaying rate decreases again. As we increase the value for α, for a given value of d_2, both SNR values reduce, which effectively leads to a lower relaying rate as shown in Fig. 5.

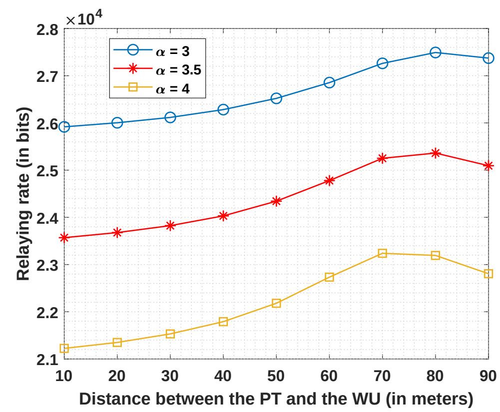

Fig. 5. Relaying rate for varying $d_2(\psi = 1, R_L = 5000$ bits,$Q = 1000$ bits)

In Fig. 6, for $\psi = 9 \times 10^4$, we plot different energy consumption values, i.e., due to relaying, offloading, local computing, at the WU while varing the distance between the PT and the WU, for $\alpha = 4$ and 4.2. As d_2 increases, relaying power increases until we get $SNR_{1+3} = SNR_2$. For $SNR_2 < SNR_{1+3}$, relaying power decreases as more relaying power does not fetch more relaying rate any more. As value for α increases for fixed value of d_2, values for P_i and P_3 in Proposition 3.1 reduce, which reduces the value for $P_{\text{WU,1}}$ and hence the relaying energy consumption reduces. In Fig. 6, this analogy has been captured in relaying power plot for varying d_2. However, for offloading, we observe that offloading energy consumption reduces and increases for increasing d_2 and α, respectively. As the WU approaches near to the MEC server (i.e., d_2 increases), the offloading energy reduces; whereas, for higher values for α, the WU depletes more energy to offload the data. We observe that the energy consumption due to local computing at the WU increases and reduces as we increase d_2 and α, respectively. As d_2 increases, the propagation loss in the wireless link between PT-WU becomes high. Therefore, more relaying time

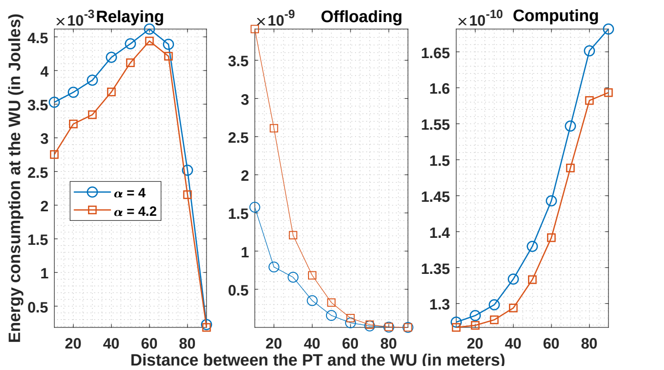

Fig. 6. Different energy consumptions at the WU for varying $d_2(\psi = 9 \times 10^5, R_L = 5000$ bits,$Q = 1000$ bits)

is required to optimize the relaying rate. More relaying time reduces offloading time duration at the WU, which means more local computation at the WU. This fact helps us describing the increasing nature of computation energy at the WU for d_2 in Fig. 6. As the value for α increases, upper cap on the relaying rate decreases, which has been discussed earlier for Fig. 5. Hence, relaying time reduces for increasing α, which gives more time to the WU to offload data. Due to this fact, computation energy at the WU reduces for increasing α.

C. *Feasibility of $P1$ for different values of R_L and T*

In Fig. 7, we plot optimized relaying rate while varying R_L and T. We also consider three different values for $\psi = 1, 10^5$, and 10^6. In the left hand plot of Fig. 7, it can be observed that the relaying rate is far above the constraint value (i.e., R_L) for lower values of ψ. However, as we increase the value for ψ, the relaying rate decreases. As we increase the value for ψ, both relaying power and duration at the WU, i.e., $P_{\text{WU},1}$ and $\tau_r/2$, respectively, reduces, which in turn reduces the relaying rate. It can be also observed that relaying rates for different values of ψ, become zero at and after $R_L = 7200$ bits, which is due to the infeasibility of the considered optimization problem. The infeasibility condition does not depend on the value of ψ (can be observed from (12) and (13)). In the right hand plot of Fig. 7, we consider $R_L = 5000$ bits. It can be observed that the relaying rate increases as we increase the time frame duration, which is intuitive. For lower values of T, the optimization problem is infeasible, as the WU can not complete the offloading and hence the computational task by the given time duration.

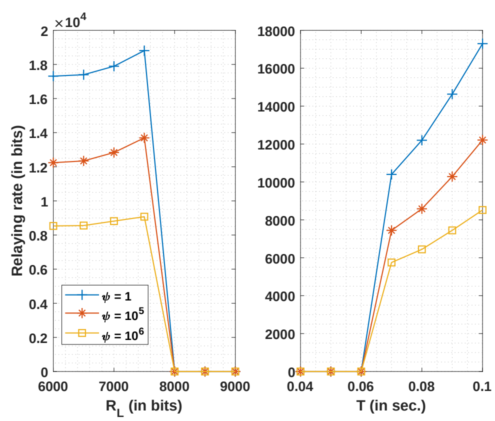

Fig. 7. Relaying rate for varying R_L,$T(d_2 = 10m., \alpha = 4.2, Q = 1000$ bits)

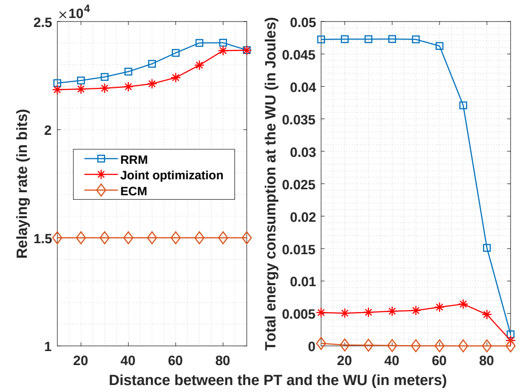

Fig. 8. Comparison between Joint optimization, RRM, and ECM in terms of relaying rate and energy consumption for varying $d_2(R_L = 15000$ bits, $Q = 1000$ bits, $\alpha = 3.8$, and $\psi = 10^5)$

D. *Comparative analysis*

Joint optimization of relaying rate and energy consumption depends on the value for ψ. We compare the joint optimization with two separate optimization problems, i.e., relaying rate maximization (RRM) and energy consumption minimization (ECM), respectively to better understand the trade-off between relaying rate and energy consumption.

We present the RRM problem as follows:

$$P5: \underset{\beta=\beta_{\max},\tau_r,f_{\text{WU}}=f_{\text{WU}}^{\max},P_{\text{WU},1},P_{\text{WU},2}}{\text{maximize}} \quad R_p$$
$$\text{subject to:} \quad (10a), (10c) - (10f). \tag{26a}$$

As the relaying rate is independent of β and f_{WU}, we have fixed those two parameters at their maximum values. We perform the optimization problem $P5$ over τ_r, $P_{\text{WU},1}$, and $P_{\text{WU},2}$. In order to solve the optimization problem, we make search operation over τ_r in the range of $\tau_r^{\min}$ to $\tau_r^{\max}$ and find values for $P_{\text{WU},1}$ and $P_{\text{WU},2}$ for each value of τ_r.

In ECM problem, we minimize the energy consumption at the WU while considering all other constraints like the optimization problem $P1$:

$$P6: \underset{\beta,\tau_r,f_{\text{WU}},P_{\text{WU},1},P_{\text{WU},2}}{\text{minimize}} \quad P_{\text{WU},1}\frac{\tau_r}{2} + E_{\text{WU}}^{(\text{c})} + E_{\text{WU}}^{(\text{o})}$$
$$\text{subject to:} \quad (10a) - (10h). \tag{27a}$$

As discussed earlier, we consider $f_{\text{WU}} = \beta Q C_{\text{WU}}/T$ due to it's optimality and modify the constraint as given in (10h) by $0 \leq \beta \leq \frac{T f_{\text{WU}}^{\max}}{R C_{\text{WU}}}$. We perform search operation over $\tau_r^{\min} \leq \tau_r \leq \tau_r^{\max}$ and $0 \leq \beta \leq \beta_{\max}$ and find values for $P_{\text{WU},1}$ and $P_{\text{WU},2}$ for each value of τ_r and β. In Figs. 8 and 9, we compare our proposed joint optimization with other two optimization problems, i.e., RRM and ECM, as has been discussed above. In Fig. 8, RRM provides better relaying rate and the WU consumes less energy for ECM, which goes according to our intuitions. For certain parameters' values, in Fig. 6, we observe that the energy consumption at the WU is dominated by the relaying energy for the joint optimization, which helps us in understanding the nature of total energy consumption at the WU for joint optimization in Fig. 8. For

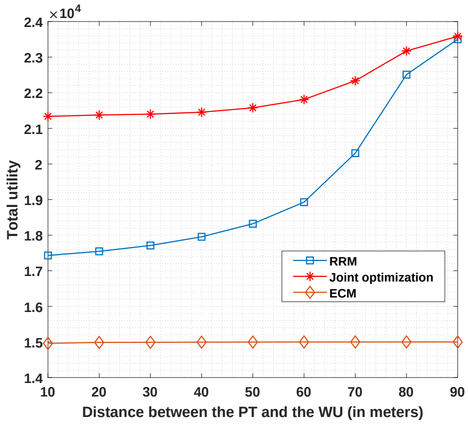

Fig. 9. Comparison between Joint optimization, RRM, and ECM in terms of total utility for varying $d_2 (R_L = 15000$ bits, $Q = 1000$ bits, $\alpha = 3.8$, and $\psi = 10^5)$

RRM, the WU uses maximum power in relaying for lower values of d_2, which reduces gradually with d_2. In Fig. 8, we observe lower gap in relaying rate for joint optimization and RRM; however, the gap in energy consumption is substantially high for RRM and joint optimization for a large range of d_2. From Fig. 8, we observe that joint optimization provides a nice trade-off between the relaying rate and the energy consumption compared to rate maximization and energy minimization.

In Fig. 9, we compare joint optimization with the other two optimization problems in terms of total utility as has been defined in (9). It can be observed that joint optimization outperforms the other two, which goes according to our intuition. As d_2 increases, relaying rate and energy consumption at the WU comes closer for joint optimization and rate maximization (can be observed from Fig. 8), which is the reason behind the lower gap in total utility values for joint optimization and rate maximization for higher d_2. Please note that for rate maximization, we perform search over τ_r to find optimal value.

V. Conclusion

In this paper, we have studied MEC in the context of cooperative relaying. The WU, who takes help of the MEC server to complete a computational task by a particular time duration, relays (i.e., DF relaying) the PT's data to the PR to get access of the high bandwidth licensed spectrum. The WU offloads partial computational task to the MEC server using the high bandwidth licensed spectrum. We have formulated an optimization problem where we jointly maximize the relaying rate and minimize the WU's energy consumption. We make our optimization problem more general while considering constraints on the relaying rate, transmission power at the WU, and computational task completion time duration. We analyse the relaying rate and energy consumptions at the WU for different parameters. We also perform comparative analysis of our proposed joint optimization problem with two individual optimization problems, i.e., relaying rate maximization and energy consumption minimization. We observe that joint optimization problem provides better trade-off between the relaying rate and energy consumption compared to other two. Moreover, joint optimization also performs better in terms of total utility value. We have considered single WU in this paper. It would be interesting to analyse the problem for multiple WUs with heterogeneous delay requirements.

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