# Multi-Agent Deep Reinforcement Learning based Power Control for Large Energy Harvesting Networks

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Abstract—The goal in this work is to design online power control policies for large energy harvesting (EH) networks where, due to large energy overhead involved in the exchange of state information among the nodes, it is infeasible to use a centralized policy. Furthermore, typical applications of EH networks concern the scenario where the statistical information, about both the EH process and the wireless channel, is not available. In order to address these challenges, we propose a mean-field multiagent deep reinforcement learning framework. The proposed approach enables the nodes to learn online power control policies in a fully distributed fashion, i.e., it does not require the nodes to exchange the information about their states. Using the underlying structure of the problem, we analytically establish the convergence of the proposed scheme. In particular, we show that the policies obtained using the proposed approach converge to the 'stationary' Nash equilibrium. Our simulation results illustrate the efficacy of the power control policies, learned through the proposed approach. In particular, the mean-field multi-agent reinforcement learning scheme achieves a performance close to the state-of-the-art centralized policies which operate using the information about the state of whole network.

## I. INTRODUCTION

Internet-of-things (IoT) [1] networks connect a large number of low power sensors whose lifespan is typically limited by the amount of the energy that can be stored in their batteries. The advent of the energy harvesting (EH) technology [2] promises to prolong the lifespan of IoT networks, by enabling the nodes to operate using the energy harvested from environmental sources, e.g., solar, wind, etc. On the other hand, this poses new constraints in the way energy is to be managed. An EH node (EHN) operates under the energy neutrality constraint (ENC) which requires that the total energy consumed by the node, up to any point in time, can not exceed the total amount of energy harvested by the node, until that point. This constraint is particularly challenging due to the random nature of the EH process. In addition, at a given instant, an EHN can only store an amount of energy equal to the size of its battery capacity, which further adds to the complexity of the energy management problem in EH networks. As a result, a major and challenging issue in EH-based IoT systems is to devise power

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control policies to maximize the communication performance, while ensuring the operation under ENC.

Furthermore, when only the causal information about energy arrivals and channel states [3] is available the power control policies are termed as online policies. The online policy design problem is essentially a stochastic control problem which, upon discretizing the state space (battery state and channel gains), can be formulated as a Markov decision process (MDP) [4] and can be solved numerically to obtain an optimal online policy. However, the MDP based approach requires perfect knowledge of the statistics of the EH process and wireless channels, which are difficult to know in practice. In order to address this drawback, the framework of reinforcement learning (RL) [5]–[9] or that of Lyapunov optimization [10], [11] have been proposed to approximate the optimal solution. All of these previous works take a centralized approach which makes it infeasible to use them for large networks, where the presence of a large number of nodes causes inevitable feedback overheads, as well as, more importantly, a huge complexity issue. Indeed, the numerical techniques available to solve MDP problems suffer from the so-called "curse-of-dimensionality", making them computationally intractable for the large EH networks.

Therefore, for the large EH networks, in absence of any a-priori knowledge about the statistics of the EH process and the channel, it is essential to develop new techniques for distributed learning of online policies, which do not require any information exchange among the nodes. Distributed approaches for online power control of EH networks has been recently considered in only a handful of works [12]-[15]. In [12], the authors use a distributed Q-learning algorithm where each node independently learns its individual Q-function. However, no convergence guarantee is provided for the proposed method. Note that, in general, when multiple nodes individually use a reinforcement learning algorithm to learn the optimal policy, the convergence is not assured. This is because in such a scenario each individual node experiences an inherently non-stationary environment [16]. The authors in [13] developed a distributed solution to minimize the communication delay in EH-based large networks, assuming the information about the statistics of the EH process and of the wireless channel is known. Interestingly, the interactions among the devices are modeled leveraging the mean-field game theory, a framework specifically conceived to analyze the evolution of the systems composed of a very large number of distributed decision-makers [17], [18]. A multi-agent reinforcement learning (MARL) approach is considered in [14], where an online policy for sum-rate maximization is developed. However, there it is assumed that the system global state is available at each node, which makes the approach from [14] inapplicable to large networks, due to the extensive signaling required to make the system global state available to all network nodes. In [15], a two-hop EH network is considered, and a MARL-based algorithm with guaranteed convergence is proposed to minimize the communication delay.

The objective of this work is to develop a mechanism to learn the optimal online power control policies for fadingimpaired multiple access channel (MAC) with a large number of EH transmitters, in a distributed fashion. The design of distributed power control policies for EH AWGN MAC is presented in [19], [20]. In [19], the authors presented fixedfraction based policies which achieve a throughput within a constant gap from the optimal throughput. The policies presented in [20] achieve the optimal throughput, asymptotically, and operate in a fully distributed fashion. For EH fading MAC, the authors in [21] presented deep neural network (DNN) based online policies which operate in a centralized fashion, and achieve a near-optimal throughput. However, the design of distributed online policies for EH fading MAC is not available in the literature. In this context, we make the following main contributions:

- We model the problem of throughput maximization for EH MAC as a discrete-time mean-field game. Further, exploiting the structure of the problem, we show that the mean-field game has a *unique* stationary solution.
- Next, we propose to use deep reinforcement learning at each individual node to learn the stationary solution of the mean-field game. Under the proposed scheme the nodes learn the optimal power control in a completely distributed fashion, without any apriori knowledge about the statistics of the EH process and propagation channel.
- Our simulation results illustrate that the throughput achieved by the online power control policies learned using the proposed mean-field MARL (MF-MARL) framework is close to the throughput obtained using the stateof-the-art online policies which operate in a centralized fashion.

In the following section, we describe our system model.

# II. SYSTEM MODEL AND PROBLEM FORMULATION

We consider a time-slotted EH network where a large number of *identical* EHNs transmit their data over a block fading channel to an AP which is connected to the electric mains. The set of transmitters is denoted by  $\mathcal{K} \triangleq \{1,2,\ldots,K\}$ , where  $K\gg 1$  denotes the number of EHNs. In the  $n^{\text{th}}$  slot, the fading channel gain between the  $k^{\text{th}}$  transmitter and the AP is denoted by  $g_n^k\in \mathbf{G}_k$ . In each slot, the channel between

<sup>1</sup>For any symbol in the paper, the superscript and subscript represent the node index and the slot index, respectively, and if only the subscript is present then it denote either the node index or the slot index, depending on the context.

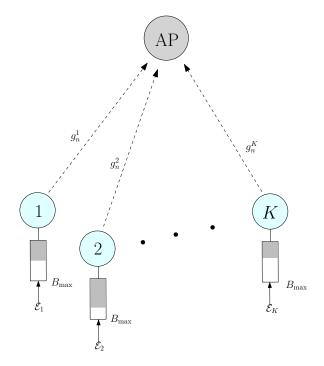


Fig. 1. System model for the EH fading multiple access network. The EH process and battery size at the  $k^{\rm th}$  node are denoted by  $\mathcal{E}_k$  and  $B_{\rm max}$ , respectively. The nodes transmit their data to an AP over a fading channel. The complex channel gain from the transmitter k to the AP, in the  $n^{\rm th}$  slot, is denoted by  $g_n^k$ .

any transmitter and the AP remains constant for the entire slot duration, and changes at the end of the slot. We assume that the wireless channels between the nodes and the AP,  $G_k$ 's, are identically distributed.

In a slot, the  $k^{\text{th}}$  node harvests energy according to a general stationary and ergodic harvesting process  $f_{\mathcal{E}_k}(e_k)$ , where the random variable  $\mathcal{E}_k$  denotes the amount of energy harvested by the  $k^{\text{th}}$  transmitter and  $e_k$  denotes a realization of  $\mathcal{E}_k$ . We assume that the harvesting processes  $\{\mathcal{E}_k\}_{k\in\mathcal{K}}$  are identically distributed across the individual nodes, but not necessarily independent. At each node, the harvested energy is stored in a perfectly efficient, finite capacity battery of size  $B_{\text{max}}$ . Further, only causal and local information is available, i.e., each node knows only its own energy arrivals and the channel states to the AP in the current and all the previous time slots. In particular, no node has information about the battery and the channel state of the other nodes in the network. Also, no information is available about the distribution of the EH process and of the wireless channels at any node.

Let  $p_n^k \leq P_{\max}$  denote the transmit energy used by the  $k^{\text{th}}$  transmitter in the  $n^{\text{th}}$  slot, where  $P_{\max}$  denotes the maximum transmit energy which is determined by the RF front end. Further,  $\mathcal{P}_n \triangleq \{p_n^k\}_{k=1}^K$  denotes the set of transmit energies used in the  $n^{\text{th}}$  slot, by all the transmitters. The battery at the  $k^{\text{th}}$  node evolves as

$$B_{n+1}^k = \min\{[B_n^k + e_n^k - p_n^k]^+, B_{\text{max}}\},\tag{1}$$

where  $1 \le k \le K$ , and  $[x]^+ \triangleq \max\{0, x\}$ . In the above,  $B_n^k$ 

and  $e_n^k$  denote the battery level and the energy harvested by the  $k^{\rm th}$  node, respectively, at the start of the  $n^{\rm th}$  slot. An upper bound on the successful transmission rate of the EH MAC over N slots is given by [22], [23]

$$\mathcal{T}(\mathcal{P}) = \sum_{n=1}^{N} \log \left( 1 + \sum_{k \in \mathcal{K}} p_n^k g_n^k \right), \tag{2}$$

where  $\mathcal{P} \triangleq \{\mathcal{P}_n | 1 \leq n \leq N\}$ . Note that, the above upper bound can be achieved by transmitting independent and identically distributed (i.i.d.) Gaussian signals. In (2), for simplicity and without loss of generality, we set the power spectral density of the AWGN at the receiver as unity.

In the absence of information about the statistics of the EH process and the channel, our goal in this work is to learn an online energy management policy at each node to maximize the time-averaged sum throughput. The optimization problem can be expressed as follows

$$\max_{\{\mathcal{P}\}} \liminf_{N \to \infty} \frac{1}{N} \mathcal{T}(\mathcal{P}), \tag{3a}$$
s.t.  $0 \le p_n^k \le \min\{B_n^k, P_{\max}\}, \tag{3b}$ 

$$s.t. \ 0 \le p_n^k \le \min\{B_n^k, P_{\max}\},\tag{3b}$$

for all n and  $1 \le k \le K$ . Constraint (3b) captures the fact that the maximum energy a node can use in the  $n^{th}$  slot is limited by the minimum between the amount of energy available in the battery,  $B_n^k$ , and the maximum allowed transmit energy  $P_{\text{max}}$ . Note that, since the information about the random energy arrivals and the channel is only causally available and for each node the battery evolves in a Markovian fashion, according to (1), the optimization problem (3) is essentially a stochastic control problem which, upon discretization of the state space, could be formulated as a Markov decision process (MDP). However, obtaining an MDP based solution for the considered setting poses at least three major challenges:

- Infeasible complexity, since in the considered setup a large number of nodes K is present in the network.
- Considerably large feedback overhead, since global information about the battery and channel states of each node would be needed for the operation of the policy.
- Finally, solving the MDP also requires statistical information about the EH process and the wireless channel, which is often difficult to obtain and indeed is not assumed in this work.

For these reasons, the goal of this work is to develop a framework to learn online power control policies in a distributed fashion, i.e., each node learns the optimal online power control policy without requiring to know the battery and channel states, and actions of the other nodes. In the following sections, we develop a provably convergent multi-agent reinforcement

<sup>2</sup>We note that, in a scenario when all the EHNs simultaneously transmit their data, the cumulative signal-to-noise ratio (SNR) term in (2),  $\sum_{k \in \mathcal{K}} p_n^k g_n^k$ , grows with the number of users in the network. In practice, this problem can be circumvented by ensuring that the transmit power of EHNs scales down in inverse proportion to the number of users, i.e.,  $O\left(\frac{1}{K}\right)$ , as when the number of users increases the power per user must decrease in order to ensure that the total energy in the network stays finite.

learning approach exploiting the tools of deep reinforcement learning and mean-field games.

## III. MEAN-FIELD GAME TO MAXIMIZE THE SUM THROUGHPUT

In this section, first we model the sum throughput maximization problem in (3) as a discrete time, finite state meanfield game [24]. Next, we present preliminaries on the discretetime mean-field games and list the key results which are useful in showing the convergence of the proposed approach to the stationary solution of the mean-field game.

#### A. Throughput Maximization Game

The throughput maximization game  $\mathcal{G}_T \triangleq \{\mathcal{K}, \mathcal{S}, \mathcal{P}, \mathcal{R}\}$ consists of:

- The set of players  $\mathcal{K} \triangleq \{1, 2, \dots, K\}$ , each one corresponding to a unique EH transmitter, where K >> 1;
- The state space of all the players  $S \triangleq \times_{k \in \mathcal{K}} S^k$ , with  $S^k$  denoting the space of all the states  $s^k$  for the  $k^{\text{th}}$  transmitter and  $|S^k| \triangleq d$ . Also, let  $s_n^k \triangleq (B_n^k, g_n^k, e_n^k)$  denote the state of the  $k^{\text{th}}$  transmitter in the  $n^{\text{th}}$  slot, where  $B_n^k, g_n^k$ , and  $e_n^k$  are discrete-valued; The set of policies of all the nodes  $\mathcal{P} \triangleq \{\mathcal{P}^k\}_{k \in \mathcal{K}}$ , where
- $\mathcal{P}^k$  denotes the policy of the  $k^{\text{th}}$  node;
- ullet The set of reward functions of all the nodes  $\mathcal{R} \ riangleq$  $\{\mathcal{R}_k\}_{k\in\mathcal{K}}$ , where  $\mathcal{R}_k$  is the reward function of node k.

Note that, since all the transmitters are identical, the state space of individual nodes,  $S^k$ , is the same set for all k = 1, ..., K. In the  $n^{\text{th}}$  time slot, the  $k^{\text{th}}$  node uses  $p_n^k$  amount of energy, prescribed by its policy  $\mathcal{P}^k$ , and collects a reward according to its reward function  $\mathcal{R}_k$  and evolves from one state to another.

Under the mean field hypothesis [24], the reward obtained by a given node depends on the other nodes only through the distribution of all the nodes across the states. Let  $\pi_n \triangleq$  $(\pi_n^1,\dots,\pi_n^d)$  denote the distribution of all the nodes across the states in the  $n^{\rm th}$  slot, where  $\pi_n^i$  denotes the fraction of nodes in the  $i^{th}$  state. Thus, in the  $n^{th}$  slot the reward obtained by the  $k^{\text{th}}$  node can be expressed as

$$\mathcal{R}_{k}(\boldsymbol{\pi}_{n}, p_{n}^{k}) = \log\left(1 + p_{n}^{k} + \sum_{i=1}^{d} (K - 1)\pi_{n}^{i} p_{i} g_{i}\right)$$

$$= \log\left(1 + \sum_{i=1}^{d} K \pi_{n}^{i} p_{i} g_{i}\right), \tag{4}$$

where  $g_i$  is the wireless channel gain between the nodes in the  $i^{\text{th}}$  state and the AP, and  $p_i \in \mathcal{A}_p \triangleq \{0, p_{\min}, \dots, P_{\max}\}$ denotes the energy level used for transmission by the nodes in the  $i^{th}$  state. Here,  $p_{min}$  denotes the minimum energy required for transmission. Note that, (4) is written using the fact that under the mean-field hypothesis all the nodes are identical and hence use the same policy which also implies that the reward function,  $\mathcal{R}_k(\cdot,\cdot)$ , is identical for all the nodes. Hence, to simplify the notations, in the ensuing discussion we omit the node index k. Also, (4) implicitly assumes that all nodes in state i use the energy  $p_i$  which is motivated<sup>3</sup> by the fact that for an MDP with finite state and action sets the optimal policy is a Markov deterministic policy [25, Thm. 8.4.7], i.e., in a slot the optimal transmit energy for a node depends only on its current state.

In the  $n^{\text{th}}$  slot, when a node in state  $s_n \in \mathcal{S}$  transmits using energy  $p_{s_n}$  the system evolves as

$$\pi_{n+1}^{j} = \sum_{i} \pi_{n}^{i} P_{ij}^{n} (p_{i}), \qquad (5)$$

where  $P_{ij}^n(\cdot)$  denotes the probability in the slot n that a node in state i transits to state j, and depends on,  $p_i$ , the energy used for transmission by the node in the  $i^{\text{th}}$  state<sup>4</sup>. In addition,  $P_{ij}^n(\cdot)$  is determined by the statistics of the EH process and the wireless channel. In a given slot, all the nodes obtain a reward,  $\mathcal{R}\left(\pi_n,\mathcal{P}\right)$ , equal to the total number of bits successfully decoded in that slot, by the AP.

For a given node, starting from the  $n^{\text{th}}$  slot the expected sum-throughput obtained by following a policy  $\mathcal P$  can be expressed as

$$V_n(\boldsymbol{\pi}_n, \mathcal{P}) = \mathcal{R}\left(\boldsymbol{\pi}_n, \mathcal{P}\right) + V_{n+1}\left(\boldsymbol{\pi}_{n+1}, \mathcal{P}\right), \tag{6}$$

where  $V_{n+1}(\pmb{\pi}_{n+1},\mathcal{P})$  denotes the expected throughput obtained by following a policy  $\mathcal{P}$  starting from the slot n+1, when in the  $(n+1)^{th}$  slot the distribution of the nodes across the states is given by  $\pi_{n+1}$ . In the rest of the paper  $V(\cdot,\cdot)$  is also termed as the value function. In the above, similar to an MDP [25], (6) is written using the fact that the expected sumthroughput obtained by following a policy  $\mathcal{P}$ , starting from the time slot n, is equal to the sum of the expected sum-throughput obtained in the slot n and the slot n+1 onward. Note that, under the mean-field hypothesis, the expected sum-throughput in (6) is identical for all the nodes and due to special structure of the reward function, the value function of each node  $V(\cdot, \cdot)$ only depends on the distribution of the nodes across the states,  $\pi_n$ , not on the state of the individual nodes. Hence, (6) does not include a superscript/subscript to denote the node index. In the following, we present preliminaries on the discrete-time finite state mean field games.

## B. Preliminaries: discrete-time finite state Mean-field games

In the following, we define the notions of Nash equilibrium, stationary solution, and briefly summarize the key results used for proving the convergence of the proposed MARL algorithm. For a detailed exposition on the discrete-time finite state mean-field games we refer the readers to [24].

**Definition 1** (Nash maximizer). For a fixed probability vector  $\pi_n$ , a policy  $\mathcal{P}^*$  is said to be a Nash maximizer if and only if

$$V_n(\boldsymbol{\pi}_n, \mathcal{P}) \leq V_n(\boldsymbol{\pi}_n, \mathcal{P}^*)$$
, for all policies  $\mathcal{P}$ .

Next, for a discrete-time finite state mean-field game, we define the notions of a solution and stationary solution.

**Definition 2** (Solution of a mean-field game). Suppose that for each  $\pi_n$  there exists a Nash maximizer  $\mathcal{P}^*$ . Then a sequence of tuples  $\{(\pi_n, V_n) \text{ for } n \in \mathbb{N}\}$  is a solution of the mean field game if for each  $n \in \mathbb{N}$  it satisfies (5) and (6) for some Nash maximizer of  $V_n$ .

**Definition 3** (Stationary solution). Let  $\mathcal{G}_{\pi}$  and  $\mathcal{K}_{V}$  be defined as  $\mathcal{G}_{\pi_{n}}(V_{n+1}) = V_{n}(\pi_{n}, \mathcal{P})$ , and  $\mathcal{K}_{V_{n}}(\pi_{n}) = \pi_{n+1}$ . A pair of tuple  $(\tilde{\pi}, \tilde{V})$  is said to be a stationary solution if and only if

$$\mathcal{G}_{\tilde{\pi}}(\tilde{V}) = \tilde{V} \text{ and } \mathcal{K}_{\tilde{V}}(\tilde{\pi}) = \tilde{\pi}.$$

Note that, the operators  $\mathcal{K}_{V_n}(\cdot)$  and  $\mathcal{G}_{\pi_n}(\cdot)$  are compact representations of (5) and (6), respectively, which are essentially discrete time counterparts of Hamilton-Jacobi-Bellman and Fokker-Planck equations. The stationary solution of a mean-field game,  $(\tilde{\pi}, \tilde{V})$ , is a fixed-point of operators  $\mathcal{G}_{\pi}$  and  $\mathcal{K}_V$ . Next, we list the results which identifies the conditions under which a stationary solution exists. We omit the proofs for brevity. These results are later used for proving the convergence of our mean-field MARL (MF-MARL) algorithm to the stationary solution.

**Theorem 1** (Uniqueness of Nash maximizer (Theorem 2 [24])). Let  $f_i(\mathcal{P}) = \frac{\partial V(\pi,\mathcal{P})}{\partial p_i}$  where  $p_i \in [0,P_{\max}]$  for all  $1 \leq i \leq d$ . If the value function  $V_n$  is convex and continuous with respect to  $p_i$ , and  $f_i$  is strictly diagonally convex, i.e., it satisfies

$$\sum_{i=1}^{d} (p_i^1 - p_i^2)(f_i(\mathcal{P}^1) - f_i(\mathcal{P}^2)) > 0, \tag{7}$$

then there exists a unique policy which is a Nash maximizer for the value function V. Here,  $p_i^1$  and  $p_i^2$  denote the actions prescribed in the  $i^{th}$  state by two arbitrary policies  $\mathcal{P}^1$  and  $\mathcal{P}^2$ , respectively.

The following result shows that if the reward function is monotonic with respect to both the variables,  $\pi$  and  $p_i$ , then the mean-field game admits a unique stationary solution.

**Theorem 2** (Uniqueness of stationary solution (Proposition 4.3.1, [26])). Let the value function be a continuous function with respect to its both arguments, and also assume that there exists a unique Nash maximizer  $\mathcal{P}_n$  for all  $n \in \{0, 1, 2, \dots\}$ . Further, let the reward function be monotone with respect to the distribution  $\pi$ , i.e.

$$\sum_{i=1}^{d} (\pi_i^2 - \pi_i^1) (\mathcal{R}_i(\mathcal{P}^1, \pi^2) - \mathcal{R}_i(\mathcal{P}, \pi^1)) \ge 0, \tag{8}$$

then there exists a unique solution for the mean-field game. In the above,  $\mathcal{R}_i(\cdot,\cdot)$  denotes the reward obtained by the nodes in the  $i^{th}$  state.

In the following, we establish that the mean-field game  $G_T$  admits a unique stationary solution.

<sup>&</sup>lt;sup>3</sup>Note that, for stochastic games with partial observability, an optimal policy could be history dependent.

 $<sup>^4</sup>$ Note that, in a general mean-field game the transition probabilities  $P^n_{ij}$  may also depend on the actions of the other players.

C. Unique Stationary Solution for  $\mathcal{G}_T$ 

Theorem 3. The throughput maximization mean-field game  $\mathcal{G}_T$  has a unique stationary solution.

In the next section, we present an algorithm to learn the stationary solution of the mean-field game  $\mathcal{G}_T$ , as well as the corresponding Nash maximizer power control policy. The proposed approach uses reinforcement learning for this purpose and is termed as MF-MARL approach.

# IV. MF-MARL APPROACH TO POWER CONTROL

In this section, we present our mean-field MARL approach to learn online power control policies which maximize the throughput of an EH fading MAC with large number of users. We show that the proposed approach enables the distributed learning of the power control policies which eventually converge to the stationary Nash equilibrium. The proposed MF-MARL algorithm to obtain the online policies exploits the fact that a discrete time finite state mean field game has the fictitious play property (FPP) [26]. The FPP for a discrete time mean field game is described in the following. Let m denote the iteration index and  $\bar{\pi}_1$  denote an arbitrary probability vector representing the initial distribution of the nodes across the states. Let

$$\mathcal{P}_{m}^{*} \triangleq \arg\max_{\mathcal{P}} V_{m}\left(\bar{\boldsymbol{\pi}}_{m}, \mathcal{P}\right), \tag{9}$$

$$\boldsymbol{\pi}_{m+1} = \mathcal{K}_{V_m(\mathcal{P}_m^*)}(\boldsymbol{\pi}_m),\tag{10}$$

$$\pi_{m+1} = \mathcal{K}_{V_m(\mathcal{P}_m^*)}(\pi_m), \tag{10}$$
and 
$$\bar{\pi}_{m+1} = \frac{m}{m+1}\bar{\pi}_m + \frac{1}{m+1}\pi_{m+1}. \tag{11}$$

The procedure described by (9), (10) and (11) is called the fictitious play procedure. As described in (9), at the  $m^{th}$ iteration, a node attempts to learn the Nash maximizer, given that its belief about the distribution of the nodes across the states is  $\bar{\pi}_m$ . Further, at each iteration of the fictitious play procedure the belief about the distribution is updated using (10) and (11). A discrete-time mean field game is said to have the FPP if and only if the procedure described by (9), (10) and (11) converges. The following result provides the conditions under which the fictitious play procedure converges to the unique solution of the discrete-time mean field game.

Theorem 4 (Convergence of FPP to unique stationary solution (Theorem 4.3.2 [26])). Let  $(\pi_m, V_m)$  denote the sequence generated through the FPP. If a mean-field game has the unique Nash maximizer at each stage of the game, and the reward function is continuous and monotone with respect to probability vector  $\boldsymbol{\pi}$ , then the sequence  $(\boldsymbol{\pi}_m, V_m)$  converges to  $(\tilde{\boldsymbol{\pi}}, V)$ , the unique stationary solution of the mean-field game.

For the throughput maximizing mean-field game  $\mathcal{G}_T$ , the convergence of the FPP to the stationary solution of the game directly follows from the above result and the Theorem 3. As a consequence of this result, the stationary solution of **Algorithm 1**: MF-MARL approach to learn online policies

**Initialize**:  $\bar{\pi}_1$  to a valid probability vector,  $\epsilon_1$ ,  $\tilde{\epsilon}$ ,  $N_1$  and  $m \leftarrow 0, n \leftarrow 0$ . Here,  $N_1$  denotes the length of a Q-learning episode.

do

- 1) Set  $m \leftarrow m + 1$ ; at each node execute Q-learning algorithm to learn Nash maximizer  $\mathcal{P}_m^k$ \*
- 2) In the  $n^{\text{th}}$  time-slot,  $n \leq N_1$ , of Q-learning episode the AP estimate  $\pi_{m_n}$ .
- 3)  $n \leftarrow n+1$ . If  $\|\boldsymbol{\pi}_{m_n} \boldsymbol{\pi}_m\|_2 \ge \epsilon_1$  or n > N, broadcast  $\pi_{m+1} = \pi_{m_{n+1}}$ ; else go to step 2.
- 4) Update  $\bar{\pi}_{m+1}$  using (11) and  $n \leftarrow 0$ .

while  $\|\bar{\boldsymbol{\pi}}_{m+1} - \bar{\boldsymbol{\pi}}_m\|_2 \leq \tilde{\epsilon}$ .

Output: The near-optimal policies and distribution are given by  $\mathcal{P}^*$  and  $\tilde{\pi}$ , respectively.

the game  $\mathcal{G}_T$  can be learned through the fictitious play procedure, provided the Nash maximizer can be found at each iteration of the fictitious play procedure, and the belief about the distribution is updated correspondingly. The MF-MARL proposes to use the reinforcement learning to learn the Nash maximizer at each iteration, i.e., for a given belief distribution  $\bar{\pi}$  each node uses a reinforcement learning algorithm to learn the Nash maximizer. The proposed MF-MARL approach is described in Algorithm 1.

In order to implement the step 2 of the algorithm, the AP builds an estimate of  $\pi_{m_n}$ , and periodically broadcasts it to the entire network. In the simulations, presented in Sec. V, we use the empirical distribution as an estimate of  $\pi_{m_n}$ . Also, in order to implement the Q-learning algorithm each node requires the information about reward, i.e., the sum-throughput, obtained in each slot. Since the reward function is same across the nodes, this could be accomplished by using the belief about the distribution. In particular, each node uses its own policy and the belief about the distribution to build an estimate of the reward obtained in each slot. Alternatively, in each slot the AP can directly broadcast the total number of bits successfully decoded by it. The latter method obviates the need to maintain a belief about the distribution of the nodes, albeit at the cost of a higher feedback overhead. The latter method is essentially cooperative multi-agent Q-learning [27] where nodes attempt to maximize a common reward function. In our simulations it is observed that the proposed MF-MARL based approach performs marginally better than the cooperative multi-agent Q-learning method.

At each node, we implement the reinforcement learning algorithm using the deep Q-learning [28] method where the Q-function is approximated using a DNN. In order to learn the Q-function, the DNN is successively trained using the problem data. This approach of using a DNN to learn Q-function has the following advantages: (i) it obviates the need to discretize the state space, as the Q-function approximation learned using the DNN is continuous over the state space, whereas in conventional approach it is learned for discrete state-action

Mean	Centralized Policy	MF-MARL policy	MF-MARL policy	Cooperative Q-learning	Cooperative Q-learning
(m)	(RPS in nats)	(RPS in nats)	(Percentage )	(RPS in nats)	(Percentage )
4	3.1498	2.9371	93.24%	2.9354	93.19%
5	3.3107	3.1353	94.70%	3.0046	90.75%
6	3.4410	3.2896	95.60%	3.1852	92.56%
7	3.5102	3.3038	94.11%	3.2417	92.35%
8	3.6146	3.3903	93.79%	3.3064	91.47%
9	3.6166	3.4799	96.22%	3.4528	93.90%

pairs, (ii) it is inherently faster, compared to the conventional approach of implementing the Q-learning. This is because for a given state the Q-function corresponding to all the actions is learned simultaneously. We also note that in the first and second step of Algorithm 1, the use of Q-learning could be replaced by any other variant of reinforcement learning schemes, e.g., actor-critic algorithm. In the following section, we present the simulation results.

#### V. SIMULATION RESULTS

We consider an EH fading MAC with K = 5 EH transmitters where each EHN harvests energy according to a nonnegative truncated Gaussian distribution with mean m and variance v = 3.5, independently of the other nodes. The size of the battery at each transmitter is  $B_{\text{max}} = 20$  and the maximum amount of energy allowed to be used for transmission in a slot is  $P_{\rm max}=15$ . Note that, the unit of energy is  $10^{-2}$  J. We benchmark the performance of the proposed MF-MARL and cooperative Q-learning approach against the state-of-the-art DNN based centralized online policy [21]. In the centralized scheme the online policy is learned by training a DNN using the data obtained by jointly optimal offline policies proposed in [22]. In contrast to online policies, the offline policies are designed assuming non-causal information about the energy arrivals and the channel states over the entire time-horizon. It is observed in [21] that the throughput achieved by the DNN based centralized online policy is more than 90% of the throughput achieved by the optimal offline policies [22]. At each node, the deep Q network consists of 10 hidden layers and one input and output layer. The input layer contains 3 neurons, while the number of neurons in the output layer is equal to |A| = 150, where  $A = \{0, 0.1, 0.2, ..., 15\}$ . The first, third, fifth, seventh, and ninth hidden layer consists of 60, 58, 56, 54, and 52 neurons, respectively. The number of neurons in each even indexed hidden layer remains same as in the preceding odd indexed hidden layer. At each layer, except the output layer, the rectified linear unit (ReLu) is used as an activation function. The output layer uses a linear activation function. The deep Q-learning algorithm uses  $\gamma = 0.99$ , and uses the exploration probability  $\epsilon_{\rm max}=1$  at the start which decays to  $\epsilon_{\min} = 0.01$  with a decay factor equal to 0.995. The replay memory of length 2000 is used. In Algorithm 1, we use  $\epsilon_1=0.01,\ \tilde{\epsilon}=0.001,\ \text{and}\ N_1=1000.$  The performance is

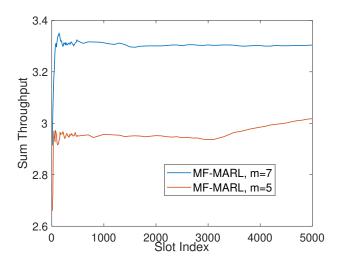


Fig. 2. Convergence of the MF-MARL algorithm. For m=7, within first 1000 slots the sum-throughput obtained by the MF-MARL approaches roughly 99% of the value shown in Table I. On the other hand, for m=5, it takes approximately 5000 slots to achieve a sum-throughput which is within 97% of the value attained finally.

evaluated by averaging the sum-throughput obtained over  $10^5\,\mathrm{slots}.$ 

As observed from the results presented in Table I, the policies obtained using the proposed MF-MARL based approach achieve the sum-throughput which is close to the throughput achieved by the DNN-based centralized policies. However, in contrast to the proposed approaches, the centralized online policy requires the information about the state of all the nodes in the network. Note that, in order to implement MF-MARL (or deep Q-learning), the actions space, A, has to be quantized which leads to the loss in the throughput, relative to the centralized scheme where the output transmit powers are continuous. We observe that the proposed MF-MARL based approach performs marginally better than the cooperative multi-agent Q-learning based scheme. However, in contrast to cooperative multi-agent Q-learning approach, the MF-MARL based procedure requires significantly less feedback. Also, it is interesting to note that the proposed MF-MARL algorithm achieves near-optimal throughput even for a network with small number of nodes. Further, the result in Fig. 2 show the throughput achieved by our MF-MARL algorithm as a function of slot index. It is interesting to observe that the MF-MARL algorithm converges very fast, i.e., for m=7 and m=5 the obtained throughput stabilizes within first 1000 and 5000 slots, respectively. A similar trend is observed for cooperative Q-learning also.

## VI. CONCLUSIONS

In this work, we proposed a mean-field multi-agent reinforcement learning based framework to learn online power control policies for large EH networks. The proposed approach enables the nodes to learn the online power control policies in a completely distributed fashion. Moreover, using the structure of the underlying problem, we analytically showed that the learning process converges to the unique stationary solution of the mean-field game. Our simulation results corroborated the theoretical findings. The future work could involve characterizing the convergence speed of the proposed MF-MARL algorithm. The proposed MF-MARL framework is useful for optimization of large wireless networks.

#### APPENDIX

*Proof.* The proof follows directly from the result in Theorem 2, provided there exists a unique Nash maximizer and the reward function is monotone in variable  $\pi$ . The uniqueness of Nash maximizer can be established using the result in Theorem 1. It is easy to verify that the reward and value function of the game  $\mathcal{G}_T$  satisfies the strictly diagonally concavity property. In order to complete the proof, we just need to show that the reward function is monotone with parameter  $\pi$ , i.e.,

$$\sum_{i=1}^{d} (\pi_i^2 - \pi_i^1) (\mathcal{R}_i(\mathcal{P}, \pi^2) - \mathcal{R}_i(\mathcal{P}, \pi^1)) \ge 0.$$
 (12)

The proof follows by noting the fact that since the reward obtained by a node does not depend on the state of the node, i.e.,  $\mathcal{R}_i(\mathcal{P},\pi^2)=\mathcal{R}(\mathcal{P},\pi^2)$ . Hence, the RHS in the above can be expressed as  $(\mathcal{R}(\mathcal{P},\pi^2)-\mathcal{R}(\mathcal{P},\pi^1))\left(\sum_{i=1}^d\pi_i^1-\sum_{i=1}^d\pi_i^1\right)=0$ .

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