Crowsourcing: A Novel Approach to Organizing WiFi Community Networks

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***Abstract*—An operator-assisted crowdsourced WiFi community network can provide high-speed wireless data services in an inexpensive way, by encouraging a set of individual users to form a community and share their private home WiFi access points (APs) with others. Such a novel paradigm has shown great promise in achieving the ubiquitous and full coverage networks. In this paper, we perform a systemic analysis for such a community network, where users are heterogeneous in terms of both the network evaluation and the home location popularity. We formulate the interactions between the network operator and users as a non-cooperative game, and focus on the operator's pricing scheme design and the users' behavior analysis. Specifically, we propose a hybrid pricing scheme combining both the fixed price (e.g., the monthly fee) and the usage-based price (proportional to the WiFi connection time) for AP sharing among users. After analyzing users' best response towards the given pricing scheme, we characterize the dynamic changes of the membership distribution over time and indicate the market equilibrium. Simulation results show that under the different pricing schemes and different roaming qualities, the equilibrium social welfare can be increased to 137% to 147%, comparing with the tradition non-crowdsourced system.**

I. Introduction

A. Background and Motivations

With the rapid proliferation of wireless mobile equipments and wireless technology, the demand for mobile data traffic is increasing at an explosive rate. According to the newest report of Cisco [1], global mobile data traffic has grew 63% in 2016, and is expected to grow at an anticipated annual growth rate of 47% from 2016 to 2021. However, the capacity of cellular network grows much slower, and has fallen far behind of the increase of mobile data traffic. Hence, new networking schemes are developing rapidly in order to meet the increasing network demand, among which an important one is WiFi network. WiFi has been viewed as one of the most promising technologies for wireless networks due to factors such as simple deployment, easy management, and high transmission rate [2]. However, the coverage of a single WiFi access point (AP) is very small, e.g., tens of meters indoors and hundreds of meters outside. Thus, deploying a large-coverage WiFi network is very challenging and expensive for a single operator. To this end, the crowdsourced WiFi community network arises as a promising way.

Crowdsourced WiFi community network has arisen as a novel architecture to solve the shortage of WiFi network. The key idea is to encourage a set of individual users, who own private home WiFi APs, to form a community and share their private APs with each other. By fully utilizing the exiting APs and resource in the community, a large coverage WiFi network can be achieved with a low cost. A famous commercial example of crowdsourced WiFi community network is FON [3], the world-largest WiFi community, which consists of 21 millions private APs all over the word. Notably, in such a community, a user can not only share his AP with others, but also enjoy others' APs when roaming outside. Obviously, the success of such a reciprocal community network largely depends on the active participation and contribution of users whose home APs are in popular locations. Hence, a comprehensive analysis of pricing and reward scheme design is critical and necessary for such a new WiFi network architecture.

In this paper, we focus on studying the incentive issues in such a crowdsourced WiFi community network. Several researches have been carried out to address such issues, including user motivation analysis [4], [5], user behavior analysis [6]–[8], and pricing scheme design [9]–[13]. However, the existing models do not consider the impact of home geographic locations on users' decisions-making, e.g., a user with more popular home location may be more likely to join the community as he can serve more users in the community. In this work, we consider a more realistic network model, where users are characterized not only by the home location factors but also by network evaluation factors. Moreover, we propose a more general and flexible pricing scheme for the operator, which combines both the fixed price and the usage-based price. Note that such a hybrid pricing scheme has not been considered before in the existing literature.

B. Model and Contributions

In this work, we consider a crowdsourced WiFi community network facilitated by a non-profit network operator, whose basic goal is to build a ubiquitous and large coverage network for the community. The network consists of a set of users, and each of them owns a private residential WiFi AP associated with a particular home location. We assume that the operator offers four membership types for users to choose:

- As a *Contributor*, a user needs to contribute to the community, by sharing his home AP with others (beneficiaries) in the community;
- As a *Beneficiary*, a user can benefits from the community, by connecting to the APs of others (contributors) in the community;

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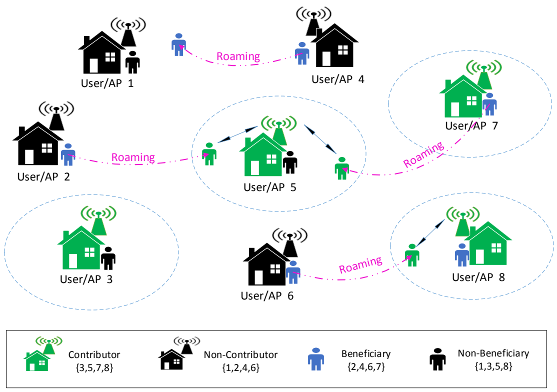

Fig. 1: Crowdsourced WiFi Network Model. Users {3,5,7,8} with green home APs are contributors, and users {2,4,6,7,8} in blue color are beneficiaries.

- As a *Hybrid Contributor and Beneficiary*, a user acts as contributor and beneficiary together, and has double privileges of them: contributing to the community (by sharing his AP with others) and benefitting from the community (by enjoying APs of others);
- As an *alien*, a user acts as neither contributor nor beneficiary, that is, he does not join such a crowdsourced WiFi community network.

Given the membership choices provided by the operator, each user decides whether to join the community, and if so, which membership type is the best choice for him. Different from the existing models (e.g., [6], [7], [11]), we allow users to do the separate decisions on contributing to the community and benefiting from the community. Thus, our model provides more flexibility for users.

Figure 1 illustrates an example of such an crowdsourced WiFi community network. Users $\{3,5,7,8\}$ with green home APs are contributors; users $\{2,4,6,7,8\}$ in blue color are beneficiaries; users $\{7,8\}$ belongs to both sets, hence he is a hybrid contributor and beneficiary; while user 1 belongs neither of these two sets, hence he is an alien. In this example, users $\{2,7\}$ connect to the network provided by user 5, user 6 connects to the network provided by user 8, users $\{1,3,5,8\}$ stay at home and connect to their own APs, while user 4 cannot connect to the network at this moment as there is no AP around him.

In such a crowdsourced WiFi community network, we are interested in solving the following three problems:

1) How should the network operator design the pricing scheme to encourage the active participation of users?
2) What are the best choices of users, and how the membership distribution dynamically evolves over time?
3) What is the equilibrium of the system evolution, and whether it is stable?

Obviously, the first question corresponds to a pricing mechanism design problem, the second and the third correspond to game theoretical analysis.

To study the above problems, we first consider a generic hybrid pricing scheme, which combines both the fixed wholesale pricing scheme (similar as the monthly fee) and the usage-based pricing scheme (proportional to the WiFi connection time). Then, given the hybrid pricing scheme, each user chooses the best membership type (i.e., contributor, beneficiary, hybrid, or alien) to maximize his payoff. When a user chooses to be a contributor to share his AP with others, he can gain certain rewards (paid by those users who access his AP). When a user chooses to be a beneficiary to enjoy the APs of others, he needs to pay certain payments (to those users who share their APs with him).

Such a reward and pricing scheme is used to guarantee the active participation of users and maintain the sustainable development of the community. It is notable that the behaviors of users are coupled. For example, with more users choosing to be contributors, a user may be more likely to be a beneficiary due to an increasing chance of accessing the APs when roaming. Similarly, with more users choosing to be beneficiaries, a user may be more likely to be a contributor as he can gain more rewards from more beneficiaries.

In summary, the main contributions of our work are summarized as follows:

- *Novel Model:* To our best knowledge, this is the first study of a WiFi community network where users are characterized not only by home location factors but also by network evaluation factors. Besides, we decouple the use decision regarding contributing to the community and benefitting from the community, which is more flexible and extends the existing models.
- *System Equilibrium Analysis:* We analyze the user behavior dynamics and system evolution systematically, and characterize the system equilibrium by using evolutionary game theory. We further study how the equilibrium changes with the different pricing schemes provided by the operator, which is important for the operator to design a desired pricing scheme.
- *Practical Insights:* Our results show that with different pricing schemes and roaming qualities, the equilibrium social welfare achieved in such a crowdsourced community network can be increased by different values, comparing with that achieved in the non-crowdsourced system. Thus, it is essential for the operator to design a proper pricing scheme to drive the community into a more desired state.

The rest of the paper is organized as bellow. In Section II, we present the system model. In Section III, we formulate users' membership decision game and analyze the game equilibrium. In Section IV, we present the simulation of our work. In section V, we make a conclusion of our work.

II. System Model

A. Network Model

We consider an operator-assisted crowdsourced WiFi community network, with a set of $\mathcal{I} \triangleq \{1,...,I\}$ individual users. Each user owns a private WiFi AP associated with a special home location l_i, where the user can connect to the network directly. Those APs are distributed in different corners of the community, some are in popular positions (such as central parks), while some are in unpopular positions (such as remote towns). In our model, users are mobile and can roam randomly. ρ_i is a parameter associated with a special home AP location, which represent the average probability of other users roaming around here. A higher ρ_i means a larger number of users roaming around here, hence, such a parameter perfectly captures the popularity of each home position. Besides, each user is associated with a network evaluation parameter θ_i, characterizing his valuation for one unit of utility achieved. Reasonably, a user with a larger θ_i implies network is more valuable for him.

We assume that the operator offers four membership types for users to choose (i.e., contributor, beneficiary, hybrid contributor and beneficiary, and alien), and each of them corresponds to a unique payoff function. Each user can decide whether or not to join such a crowdsourced WiFi community, and if so, which membership type is the best choice. Specifically as a contributor, he needs to share his private AP with others, which may cause a degradation of his own service (at home). Thus, in order to encourage users to contribute to the community, it is necessary to offer certain reward for them. Instead, if a user chooses to be a beneficiary, he can connect to the network by the APs of others when roaming, what we can say is that he benefits from the community. Thus, it is reasonable to charge certain payments from beneficiaries to cover the potential monetary rewards for contributors. Moreover, when a user is not interested in such a community and choose to bean alien, he does not obtain any rewards or need to offer any payments, as he does not contribute to or benefit from the community.

Such reward and payment scheme is determined by the community network operator at the beginning, according to the objective of achieving the maximization of the social welfare. Note that users' decisions not only depend on the pricing scheme provided by the operator, but also on others' decisions(as users' behaviors are coupled). For clearly, we summarize the properties of these user types in Table I.

In the example of Figure 1, green house corresponds to contributor and blue people corresponds to beneficiary, when you see the green house and blue people together, it represents a hybrid contributor and beneficiary, and if none of them, that means an alien. Users $\{3, 5, 8\}$ choose to be contributors only, which means they only want to contribute to the community, but not want to access to others' APs. Users $\{2, 4, 6\}$ choose to be beneficiaries only, this is, they only want to benefit from the community, but not want to share his AP with others. While, user 7 chooses to be a hybrid contributor and beneficiary, which means that he both wants to contribute to the community and benefit from the community. User 1 may be not interested in such a crowdsourced WiFi community network and choose to be an alien.

For analytical convenience, we induce a virtual time-slotted system, where a whole time is divided into a lot of time slots, each of them has the same length and is normalized to be 1. The actually length of it could be a long period such as a day, a week or a month, which depends on the concrete networks and scenarios. In order not to lose the generality, we normalize each time slot to be 1. We assume that each user can change his decision at the beginning of each time slot, depending on the previous state of the system. Such an assumption is mainly used to obtain the closest result to a realistic scene, which has no impact on our model and analysis.

TABLE I: A Summary of Four User Types

User Type	Share his AP	Access other APs	Offer payments	Obtain rewards
Contributor	Yes	No	No	Yes
Beneficiary	No	Yes	Yes	No
Hybrid	Yes	Yes	Yes	Yes
Alien	No	No	No	No

B. Network Operator Model

As mentioned earlier, we consider a non-profit operator, whose basic goals are to build a ubiquitous, large coverage network for community and maximize the social welfare of the community. Such an operator can be acted by acted by government, individuals, or social organizations.

In this work, the network operator proposes a hybrid pricing scheme for users, which combines both wholesale pricing scheme and usage-based pricing scheme, both widely used in reality. When a user choose to be a beneficiary, he will first be charged with a fixed price p (similar to a monthly fee), and then when he connect to the APs of others, he will also be charged with an another fee according to the access usage u_i of the network based on a usage-based price π.[1] While in order to keep system balance, when a user chooses to be a contributor and shares his AP with others, he will receive certain reward too, which consists of two parts: the first part r is an average value, which comes from the average distribution of the sum of the fixed fee paid by the beneficiaries, and the second part is related to the total service volume s_i he offers.[2] A user with a more popular home position (higher ρ_i) has a bigger service volume of others, which means he may obtain more rewards by sharing his AP. Hence, the payments of a beneficiary and the rewards of a contributor can be summarized as follow:

$$P_i = p + u_i \cdot \pi, \qquad \text{for a beneficiary.} \tag{1}$$

$$R_i = r + s_i \cdot \pi, \qquad \text{for a contributor.} \tag{2}$$

$$\text{where:} \qquad r = p \cdot \frac{|\mathcal{I}_{\text{B}}|}{|\mathcal{I}_{\text{C}}|}, \tag{3}$$

P_i represents the whole payment of a beneficiary i, R_i represent the whole rewards of a contributor i, $\mathcal{I}_{\text{C}}$ and $\mathcal{I}_{\text{B}}$ represent the set of contributors and beneficiaries, respectively, while $|\mathcal{I}_{\text{C}}|$ and $|\mathcal{I}_{\text{B}}|$ correspond to the total number of them.

[1]The access usage u_i will be deduced in the next subsection II-C

[2]The service volume s_i will be deduced in the next subsection II-C

We define $\mathcal{I}_{\rm H} \triangleq \mathcal{I}_{\rm C} \bigcap \mathcal{I}_{\rm B}$ as the set of hybrid contributors and beneficiaries, and $\mathcal{I}_{\rm A} \triangleq \mathcal{I}/(\mathcal{I}_{\rm C} \bigcup \mathcal{I}_{\rm B})$ as the set of aliens.

It is obvious that our pricing scheme includes both the pure wholesale pricing scheme (with $p = 0$) and pure usage-based pricing scheme (with $\pi = 0$) as the special cases. Thus, the pricing scheme of our work is a very general form which makes our charging way more adjustable to be used in different scenarios.

C. User Model

In our model, each user has a private AP in his home and those APs are distributed in different corner of the community. Each AP corresponds to a parameter ρ_i, characterizing the average probability of other users roaming to here, a higher ρ_i stands for a more popular home location. Users move randomly, when a user stays at home, he can access the network directly through his own AP, and when roaming outside he can connect to the network trough others' APs. We denote $d_{\rm H} \in [0, 1]$ and $Q_{\rm H}$ as the average access time and the average access quality of one user at his own AP in each time slot, respectively, and denote $d_{\rm L} \in [0, 1]$ and $Q_{\rm L}$ as the average access time and the average access quality of one user at other $I-1$ users' APs when roaming, respectively. Normally, a user can get a higher access quality of network through his own AP than through others' APs, due to several factors such as the greater signals, strong priority, and the less competitions, thus, $Q_{\rm H} > Q_{\rm L} > 0$. However, when a user chooses to a contributor sharing his AP others, his own access quality at home will be given a discount δ and changes to be $\delta \cdot Q_{\rm H}$, as the signal becoming weak due to others access. It is notable that we consider a homogeneous model, where users have same network connection time and access quality.[3] When a user connects to the network successfully, he can obtain certain utility. Each user has a evaluation $\theta_i \in [0, 1]$ for network. A higher θ_i means a more valuable network for him.

Obviously, a user can be fully characterized by the two parameters: ρ_i and θ_i, as the other parameters are all same for each user. Note that different users may have different ρ_i and θ_i, which are independent and identically distributed (i.i.d) in [0,1], according to the $f(\rho)$ and $f(\theta)$, respectively. Such a distribution can be arbitrary,[4] for simplicity, we assume it follows uniform distribution with a joint probability density function $f_{th}(\rho, \theta) = \frac{1}{|\mathcal{I}|}$.

In our model, users with the same value of ρ_i and the same value of θ_i will have the same membership choice. Namely, we define $x_i \in \{C, B, H, A\}$ as the decision of user i, where:

- C: Join the community as a contributor;
- B: Join the community as a beneficiary;
- H: Join the community as a hybrid contributor and beneficiary;
- A: Do not join the community and act as an alien.

[3]Note that our model can be applied to the heterogeneous situation, where different users may have different access time and different access quality.

[4]Our work can be extended to more complex situations where users follow uniform distribution, Gauss distribution, or geometric distribution and so on.

The payoff of each user is defined as the sum of the utility he achieves from the network, the rewards obtained by sharing his AP with others ,and the payment he pays when connecting to othersAPs. For convenience, we denote the payoff of each user i as $v_i(x_i)$, and the objective of each user is to choose the best membership type to maximize his payoff. Thus, we compute the payoff of each membership types as follows:

*1) **Contributor**:* When a user i chooses to be a contributor ($x_i = C$), his home utility will give a discount after sharing his AP with other, and meanwhile he can obtain certain monetary rewards according to the pricing scheme mentioned above. Thus, the payoff of a contributor i is defined as the sum of the degraded utility achieved from his own AP and the rewards offered by beneficiaries:

$$v_i(x_i) = \theta_i \cdot d_{\rm H} \cdot Q_{\rm H} \cdot \delta + r + |\mathcal{I}_{\rm B}/\{i\}| \cdot d_{\rm L} \cdot \rho_i \cdot \pi, \qquad (4)$$

where $|\mathcal{I}_{\rm B}/\{i\}|$ denotes the total number of other beneficiaries except i (if he is a beneficiary too),[5] $s_i = |\mathcal{I}_{\rm B}/\{i\}| \cdot d_{\rm L} \cdot \rho_i$ denotes the contributor i's total service volume that he offers for the beneficiaries, by which, we can see that a higher ρ_i results in a higher s_i, hence a higher payoff of the contributor i.

*2) **Beneficiary**:* When a user i chooses to be a beneficiary ($x_i = B$), he can connect to others' APs when roaming by offering certain payments for contributors. Thus, the payoff of a beneficiary i is defined as the sum of the whole utility achieved from his own AP, the utility achieved from others' APs, and the payments offered for the contributors:

$$\begin{aligned} v_i(x_i) = \theta_i \cdot d_{\rm H} \cdot Q_{\rm H} + \theta_i \cdot d_{\rm L} \cdot \sum_{j \in \mathcal{I}_{\rm C}/\{i\}} \rho_j \cdot Q_{\rm L} \\ -p - d_L \cdot \sum_{j \in Ic/\{i\}} \rho_j \cdot \pi, \end{aligned} \qquad (5)$$

where $|\mathcal{I}_{\rm C}/\{i\}|$ denotes the total number of other contributors except i (if he is a contributor too),[6] $u_i = d_L \cdot \sum_{j \in Ic/\{i\}} \rho_j$ denotes a beneficiary i's access usage of others' APs, which is determined by $\mathcal{I}_{\rm C}$.

*3) **Hybrid Contributor and Beneficiary**:* when user i chooses to be a hybrid contributor and beneficiary ($x_i = H$), he will play the double roles of a contributor and a beneficiary in the community. Thus, his payoff can be calculated by summing the degraded utility achieved from his own AP, the utility achieved from others' APs, the rewards obtained form other beneficiaries and the payments paid to other contributors:

$$\begin{aligned} v_i(x_i) = \theta_i \cdot d_{\rm H} \cdot Q_{\rm H} \cdot \delta + \theta_i \cdot d_{\rm L} \cdot \sum_{j \in \mathcal{I}_{\rm C}/\{i\}} \rho_j \cdot Q_{\rm L} - p \\ f - \pi \cdot dl \cdot \sum_{j \in \mathcal{I}_{\rm C}/\{i\}} \rho_j + r + |\mathcal{I}_{\rm B}/\{i\}| \cdot d_{\rm L} \cdot \rho_i \cdot \pi, \end{aligned} \qquad (6)$$

[5]Note that when $I \to \infty$, the impact of a single user's decision on the system can be ignore, hence $|\mathcal{I}_{\rm B}/\{i\}| \approx |\mathcal{I}_{\rm B}|$

[6]Similarity, $|\mathcal{I}_{\rm C}/\{i\}| \approx |\mathcal{I}_{\rm C}|$ when the system is large enough

*4) **Aliens**:* when a user i chooses to be an alien ($x_i = A$), he does not join the community, which means he neither obtains the rewards from the beneficiaries nor offers payments for contributors. Hence, the payoff of him is just the utility achieved at home:

$$v_i(x_i) = \theta_i \cdot d_{\text{H}} \cdot Q_{\text{H}}, \tag{7}$$

D. Problem Formulation

In our work, we model such a crowdsourced WiFi community network as a non-cooperative game, and formulate the interactions between the operator and users as following order. First, the operator announces fixed price p, and usage-based price π. Second, with the given p and π, each user decides whether to join the community and which membership type is the best choice for him. Thus, we are interested in following three problems: 1) How should the network operator design the pricing scheme to stimulate the active participation of users? 2) Which membership type are best choices for users, and how the membership distribution evolve over time? 3) What is the final equilibrium of the system? In the following work, we will study those issues systematically.

III. Game Equilibrium Analysis

In this section, we study the equilibrium of the non-cooperative game. We first introduce several concepts about evolutionary game, and then analyze the best decision of users' best decisions in such a network model, finally we study the stable equilibrium of the system.

A. Introduction of Important Concepts

*1) **Non-cooperative Game**:* There are two kinds of game models [14]: cooperative game and non-cooperative game. The former kind emphasizes collectivism and collective rationality, while the latter kind emphasizes individual rationality and individual optimal decision. The difference between them lies in whether game players can achieve a binding agreement to achieve the common goals in the process of game, and if not, it is called-non cooperative game. Game theory has been widely used in wireless and networking problems (see, e.g., [15]), including dynamic spectrum sharing and secondary spectrum trading [16]–[19], mobile crowdsensing and crowdsourcing [20]–[24], and mobile data offloading [25]–[28]. In our model, users in the community are game players, the objective of each user is making his best choice to maximize his own payoff, which is self-centered, thus, what we consider is a non-cooperate game. Moreover we consider a large network model, e.g., $I \to \infty$, where a single user's decision has little impact on other users or the system, however, a singer user's decision is largely depending on others' decisions.

*2) **Nash Equilibrium**:* Known as non-cooperative game equilibrium, it is an important term in game theory. It is named after John Nash and characterizes the finally evolution of the system. In our model, each user can make their best decisions to maximize their payoff in each time slot, those decisions may change dynamically and finally come to a balanced situation. In the Nash equilibrium, none of the players has the motivation unilaterally change his strategy, thus the sets of each membership type keep unchanged. Such a Nash equilibrium theory has laid the basic foundation of modern mainstream game theory and economic theory, and has become the main analyzing objects of our work.

*3) **Membership Distribution**:* In our model, each membership type corresponds to a set of users, we combine those different segments as a membership distribution denoted by $(\mathcal{I}_{\text{C}}, \mathcal{I}_{\text{B}}, \mathcal{I}_{\text{H}}, \mathcal{I}_{\text{A}})$. The parameters inside it represent the set of users choosing to be contributor, beneficiary, hybrid, and Aliens, respectively. It is obvious such a population profile can fully characterize the membership decision of all users. Users will remake decisions in a new time slot depending on the previous state of the system, therefore, we can use such a membership distribution to characterize the dynamic changes of the whole system.

B. User Best Decision Analysis

Now we study the users' best membership decision towards the given hybrid pricing scheme(fixed price p and usage-based price π) in the non-cooperative game. We assume that the exiting membership distribution is $(\mathcal{I}_{\text{C}}, \mathcal{I}_{\text{B}}, \mathcal{I}_{\text{H}}, \mathcal{I}_{\text{A}})$.

Lemma 1. *A user with type (ρ_i, θ_i) will choose to be a contributor ($x_i = C$), if and only if his payoff as a contributor after bearing a utility losses at home (for sharing his AP with others) is still larger than the payoff as an alien. By* (4) *and* (7)*, We can have the following condition:*

$$\theta_i \cdot d_{\text{H}} \cdot Q_{\text{H}} \cdot \delta + p \cdot \frac{|\mathcal{I}_{\text{B}}|}{|\mathcal{I}_{\text{C}}|} + |\mathcal{I}_{\text{B}}/\{i\}| \cdot d_{\text{L}} \cdot \rho_i \cdot \pi \geq \theta_i \cdot d_{\text{L}} \cdot Q_{\text{H}}, \tag{8}$$

which equals to:

$$p \cdot \frac{|\mathcal{I}_{\text{B}}|}{|\mathcal{I}_{\text{C}}|} + |\mathcal{I}_{\text{B}}/\{i\}| \cdot d_{\text{L}} \cdot \rho_i \cdot \pi \geq \theta_i \cdot d_{\text{H}} \cdot Q_{\text{H}} \cdot (1 - \delta). \tag{9}$$

The left part of the equation represents the benefits obtained by sharing and the right part represents the losses of utility at home, hence we can say when the rewards obtained is larger than the losses, a user will choose to be a contributor and share his AP with others.

Lemma 2. *A user with type (ρ_i, θ_i) will choose to be a beneficiary ($x_i = B$), if and only if his sum payoff as a beneficiary after offering certain payment for the contributors is still larger than the payoff as an alien. By* (5) *and* (7)*, We have the following condition:*

$$\begin{aligned} \theta_i \cdot d_{\text{H}} \cdot Q_H + \theta_i \cdot d_{\text{L}} \cdot \sum_{j \in \mathcal{I}_{\text{C}}/\{i\}} \rho_j \cdot Q_{\text{L}} - p \\ -\pi \cdot d_{\text{L}} \cdot \sum_{j \in \mathcal{I}_{\text{C}}/\{i\}} \rho_j \geq \theta_i \cdot d_{\text{H}} \cdot Q_{\text{H}}, \end{aligned} \tag{10}$$

which equals to:

$$\theta_i \cdot d_{\text{L}} \cdot \sum_{j \in \mathcal{I}_{\text{C}}/\{i\}} \rho_j \cdot Q_{\text{L}} \geq p + \pi \cdot d_{\text{L}} \cdot \sum_{j \in \mathcal{I}_{\text{C}}/\{i\}} \rho_j. \tag{11}$$

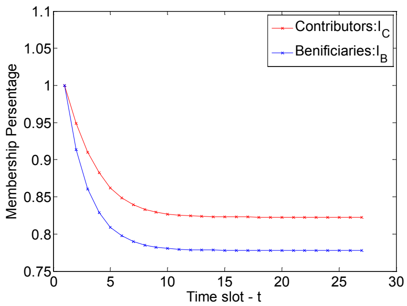

Fig. 2: Dynamics of contributors $\mathcal{I}_{\text{C}}$ and beneficiaries $\mathcal{I}_{\text{B}}$

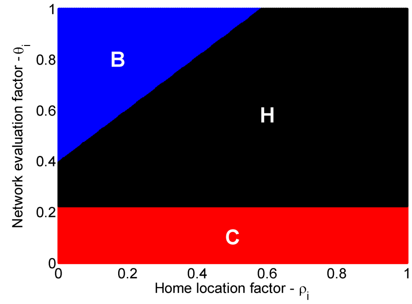

Fig. 3: Membership distribution $(\mathcal{I}_{\text{C}}, \mathcal{I}_{\text{B}}, \mathcal{I}_{\text{H}}, \mathcal{I}_{\text{A}})$ under equilibrium, where $\mathcal{I}_{\text{A}} = 0$, all users join the community

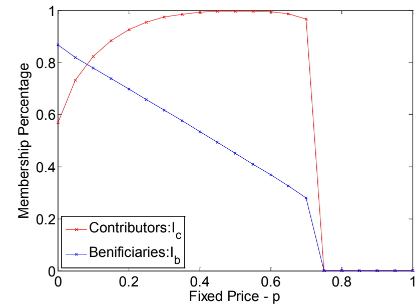

Fig. 4: The percentage of $\mathcal{I}_{\text{C}}$ and $\mathcal{I}_{\text{B}}$ under equilibrium vs fixed price p $(\pi = 0.4)$

The left part of the equation represents the utility achieved from others' APs when roaming, the right part represents the total payment for contributors, thus if the benefit achieved when roaming is larger than the payment, a user will choose to be a beneficiary and enjoy the APs of others.

Lemma 3. *If the conditions* (9) *and* (11) *are both satisfied, the user will choose to be a hybrid contributor and beneficiary* $(x_i = H)$*. However, if none of* (9) *and* (11) *is satisfied, the user will choose to be an alien* $(x_i = A)$.

Obviously, under users' best decisions, a new state of the membership distribution can be driven from the last state, we denote the newly state as $(\mathcal{I}'_{\text{C}}, \mathcal{I}'_{\text{B}}, \mathcal{I}'_{\text{H}}, \mathcal{I}'_{\text{A}})$.

C. System Equilibrium Analysis

From the above analysis, we know that users can remake membership decisions in each new time slot, depending on the exciting state of the system. Thus, the membership distribution of the system will dynamically change over time until reaching to a balanced situation (Nash equilibrium), where no user has the motivation to change his decision. Now we study such a dynamic changes course, and analyze the equilibrium of it. We first define many equal time slots (t = 1, 2, 3, ...) and each is normalized to be 1. We assume that each user can change his decision in a new time slot. Without losing the generality, we denote $(\mathcal{I}^t_{\text{C}}, \mathcal{I}^t_{\text{B}}, \mathcal{I}^t_{\text{H}}, \mathcal{I}^t_{\text{A}})$ as the exiting membership distribution at the time slot t, and the new distribution driven from the last state in slot $t+1$ can be recorded as $(\mathcal{I}^{t+1}_{\text{C}}, \mathcal{I}^{t+1}_{\text{B}}, \mathcal{I}^{t+1}_{\text{H}}, \mathcal{I}^{t+1}_{\text{A}})$. According to the definition of equilibrium, we have the following propositions:

Proposition 1. *A membership distribution is a Nash equilibrium, if and only if:*

$$(\mathcal{I}^t_{\text{C}}, \mathcal{I}^t_{\text{B}}, \mathcal{I}^t_{\text{H}}, \mathcal{I}^t_{\text{A}}) = (\mathcal{I}^{t+1}_{\text{C}}, \mathcal{I}^{t+1}_{\text{B}}, \mathcal{I}^{t+1}_{\text{H}}, \mathcal{I}^{t+1}_{\text{A}}). \tag{12}$$

However, by verification, the system may not converge under the pure best decisions of users mentioned above. In this end, to guarantee the existence of the equilibrium, we propose a more general user response, where users maybe lazy to change their previous membership decisions in each time slot, so we give such response a probability λ, which means the membership distribution will keep the exiting state with the probability λ, and change to a new state with the probability $1-\lambda$. Then,

$$\begin{aligned}(\mathcal{I}^{t+1}_{\text{C}}, \mathcal{I}^{t+1}_{\text{B}}, \mathcal{I}^{t+1}_{\text{H}}, \mathcal{I}^{t+1}_{\text{A}}) &= \lambda \cdot (\mathcal{I}^t_{\text{C}}, \mathcal{I}^t_{\text{B}}, \mathcal{I}^t_{\text{H}}, \mathcal{I}^t_{\text{A}}) \\ &+ (1-\lambda) \cdot (\mathcal{I}'_{\text{C}}, \mathcal{I}'_{\text{B}}, \mathcal{I}'_{\text{H}}, \mathcal{I}'_{\text{A}}).\end{aligned} \tag{13}$$

Clearly, $\lambda = 0$ corresponds to the pure best response of users, $\lambda = 1$ corresponds to the unresponsive users, while $\lambda \in (0, 1)$ corresponds to users who remake their decisions with probability $1-\lambda$. Note that a smoothed dynamic changes course (by setting a high λ) will guarantee that the dynamics (13) converges to the unique equilibrium at a low speed. Such a method has been widely used in the game analysis, which makes our analysis more smooth and have no effect on the final result.

IV. Simulations

In this section, we perform simulation results obtained by update iteration algorithm with the following parameters: $\delta = 0.9$, $d_{\text{H}} = 0.6$, $Q_{\text{H}} = 4$, $d_{\text{L}} = 0.8$, $Q_{\text{L}} = 3.0$.

Firstly, we illustrates how the percentage of contributors $\mathcal{I}_{\text{C}}$ and beneficiaries $\mathcal{I}_{\text{B}}$ int the system dynamically change over time and indicated the final membership distribution of the system under a given network setting. Figure 2 shows that the percentage of contributors (denoted by the red curve) and the percentage of beneficiaries (denoted by the blue curve) will finally converge to the Nush equilibrium (where $\mathcal{I}_{\text{C}} = 82.3\%$, $\mathcal{I}_{\text{B}} = 77.8\%$) after several iterations. We further indicate the membership distribution under equilibrium in Figure 3, where red areas represent users choosing to be contributor only, blue areas represent users choosing to be beneficiaries only, while the black areas represent users choosing to be both contributors and beneficiaries. Thus, the sum of red areas and black areas represent all contributors shares in the system, the sum of blue areas and black areas represent all the beneficiaries shares in the system.

From Figure 2, we can see that by giving a good initial state, the system will finally evolve to a high level where a high portion of users choose to join the community as contributors sharing their APs with others, and a medium portion of user choose to join the community as beneficiaries enjoying others'

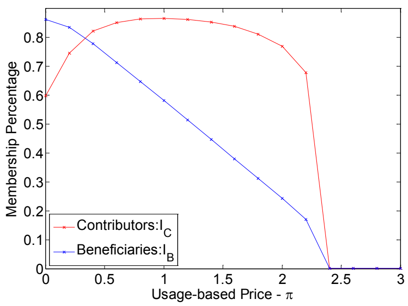

Fig. 5: The percentage of $\mathcal{I}_{\mathrm{C}}$ and $\mathcal{I}_{\mathrm{B}}$ under equilibrium vs usage-based price π $(p = 0.1)$

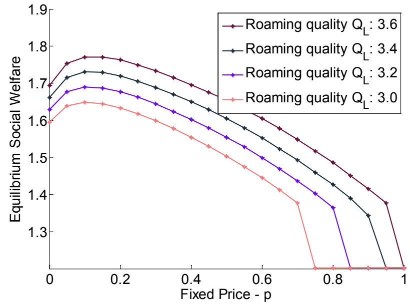

Fig. 6: Social welfare achieved under equilibrium vs fixed price p $(\pi = 0.4)$

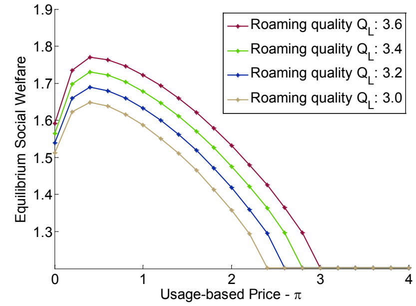

Fig. 7: Social welfare achieved under equilibrium vs usage-based price π $(p = 0.1)$

APs. From Figure 3, we can see that users with higher home location factor ρ_i and lower network evaluation θ_i would more likely choose to be contributors. This corresponds to a situation where users own popular home locations but have little connection demand, hence, they would rather share it with other in exchange for some rewards. We also can see that as long as the network evaluation θ_i is larger than a critical value, all users would choose to be beneficiaries due to the higher connection demand for the networks, by choosing to be beneficiaries, they can obtain more connection chances when roaming. We can conclude that users' decisions on whether choosing to be beneficiaries have nothing to do with home location factors ρ_i but only determined by network evaluation factors θ_i, while users' decision on whether to be contributors are effected by both of them.

Secondly, we study how different pricing schemes affect the equilibrium. Figure 4 shows how the system equilibrium changes with the p under a particular π (unchanged), and Figure 5 indicates how the system equilibrium change with the π under a particular p (unchanged). From the both of them, we can see that with the increasing of p or π, the percentage of beneficiaries $\mathcal{I}_{\mathrm{B}}$ always decreases (as the payments offered is increased), while, the percentage of contributors $\mathcal{I}_{\mathrm{C}}$ first increases (due to the reward obtained is raised) and then decreased (as $\mathcal{I}_{\mathrm{B}}$ is decreasing). However, when the p or π larger than a critical value, e.g., $p = 0.75$ in Figure 4, $\pi = 2.4$ in Figure 5, the system will collapse, where no user will join to the community as beneficiary (due to the high payments) as well as contributor (as no beneficiary in the community). This implies that a smaller pricing scheme is more acceptable for users to join the community.

Thirdly, we drive how different pricing schemes affect the social welfare achieved in the equilibrium state under different roaming quality (decreases from 3.6 to 3.0 with a step of 0.2), where the social welfare is defined as the sum payoff of all users. Figure 6 shows how the equilibrium social welfare changes with the p under a particular π (unchanged) and Figure 7 shows how the equilibrium social welfare changes with the π under a particular p (unchanged). From both figures, we can see that with the increasing of price p or π, the social welfare will first increase, then decrease. and finally drop to a stable value, e.g., 1.2 When the price p or π is larger than a critical value, which perfectly coincides with Figure 4 and Figure 5. The first uptrend is mainly due to raised roaming utility obtained from the increasing percentage of the contributors $\mathcal{I}_{\mathrm{C}}$. The second downtrend is mainly caused by the decline in both the percentage of both contributors $\mathcal{I}_{\mathrm{C}}$ and beneficiaries $\mathcal{I}_{\mathrm{B}}$. While the last stable value corresponds to the social welfare achieved in a traditional non-crowdsourced system, where no users join the community and all act as aliens. Moreover, comparing different curves in each subfigure, we can further see that by setting different roaming quality, the equilibrium social welfare can be increased to 137% to 147% comparing with the traditional non-crowdsourced system, the higher the roaming quality is, the big the equilibrium social welfare is, besides, the critical values are gradually increased under a increasing roaming quality, which implies that a higher roaming quality can achieve a better crowdsourced WiFi community network.

V. Conclusion

In this paper, we study a more general crowdsourced WiFi community network model, where users are characterized not only by the home location factors but also by the network evaluation factors. Different with the existing models, we provide a hybrid pricing scheme containing both wholesale pricing scheme and usage-based pricing scheme, which is more adjustable and can be better used to realistic life. Under the given pricing scheme, each user can remake his best membership decision in a new time slot. We characterize how the membership distribution dynamically evolves and analyze the equilibrium of it. Specially, to guarantee the existence of the equilibrium we propose a general user response, where each user changes his decision with a special probability. At the end of our work, we study the social welfare achieved under the equilibrium state in our model, and compare it with the traditional non-crowdsourced network. In conclusion, our work can serve as a very important reference for the future research.

There are several more interesting directions for the future research. First, we can consider a profitable operator, who aims at pursuing the best payoff of him, and then study our work again. Second, we can consider a more realistic situation where users are heterogeneous, which means different users may have different network connection quality or connection time. Third we can consider a more complex situation where the set of users follows Gauss distribution, Poisson distribution, Geometric distribution, and so on. Our work lays a hard foundation for all of those directions.

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