Cooperative Wi-Fi Deployment: A One-to-Many Bargaining Framework

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***Abstract*—In this paper, we study the cooperative Wi-Fi deployment problem, where the mobile network operator (MNO) cooperates with some venue owners (VOs) to deploy public Wi-Fi networks. The MNO negotiates with the VOs to determine where to deploy Wi-Fi and how much to pay. The MNO's objective is to maximize its payoff, which depends on the payments to VOs, the benefits due to data offloading and mobile advertising, and the costs due to deploying and operating Wi-Fi. We analyze the interactions among the MNO and VOs under the *one-to-many bargaining* framework, where the MNO bargains with VOs sequentially, taking into account the *externalities* among different steps of bargaining. We apply the Nash bargaining theory to analyze the cases with *exogenous* and *endogenous* bargaining sequences. For the former case, the bargaining sequence is predetermined, and we apply backward induction to compute the optimal bargaining solution related to the cooperation decisions and payments. For the latter case, the MNO can decide the bargaining sequence to maximize its payoff. We explore the structural property of the one-to-many bargaining, and design an *Optimal VO Bargaining Sequencing* (OVBS) algorithm that computes the optimal bargaining sequence. More precisely, we categorize VOs into three types based on the impact of the Wi-Fi deployment at their venues, and show that it is optimal for the MNO to bargain with these three types of VOs sequentially. Numerical results show that the optimal bargaining sequence improves the MNO's payoff over the random and worst bargaining sequences by up to 14.7% and 45.8%, respectively.**

I. Introduction

A. Motivation

The global cellular network has witnessed an explosive growth of mobile data traffic, hence *mobile network operators* (MNOs) are seeking innovative approaches to reduce network congestion and improve users' experience. With the recent technology developments and standardization efforts (*e.g.*, Hotspot 2.0 and ANDSF), Wi-Fi data offloading has emerged as an important approach to alleviate cellular congestion. A recent study [1] showed that Wi-Fi has offloaded 65% of total mobile traffic in the major cities in Korea. The annual global Wi-Fi deployment rate is expected to increase to 10.5 million in 2018 [2].

Instead of building their own Wi-Fi hotspots, many MNOs have been collaborating with *venue owners* (VOs), which are the owners of public places such as shopping malls and stadiums, on hotspot installment [2]. Since a large volume of cellular data traffic is generated from these crowded public places, MNOs are especially interested in deploying hotspots at these venues to relieve the traffic congestion. With the location information provided by Wi-Fi hotspots, MNOs can also earn profits by delivering context-aware mobile advertisements to mobile users[1]. Meanwhile, VOs also welcome the MNOs' help in building the carrier-grade Wi-Fi, which usually provides a higher capacity and better integration with the cellular network than a regular Wi-Fi [3], hence significantly enhances the mobile users' experience and attracts more visitors to those Wi-Fi available venues. Moreover, the carrier-grade Wi-Fi can help both MNOs and VOs collect visitor analytics, provide location-based service, and promote products or activities [2], [3]. Therefore, both MNOs and VOs benefit from the Wi-Fi deployment and have incentives to provide Wi-Fi service cooperatively. An existing example is the cooperative deployment by AT&T (as the MNO) and Starbucks (as the VO) in the U.S. [4]. Although such cooperation is increasingly popular, the detailed economic interactions among MNOs and VOs still have not been sufficiently explored and understood by the existing literatures. This motivates us to extensively analyze both MNOs and VOs' strategies in the cooperative Wi-Fi deployment in this paper.

B. Our Work

We consider a case where both MNOs and VOs have considerable market power, in which case we study the cooperative Wi-Fi deployment problem under the one-to-many bargaining framework[2]. Specifically, a monopoly MNO bargains with multiple VOs in sequence[3], *i.e.*, at each step the MNO bargains with only one VO for deploying Wi-Fi at the corresponding venue. We analyze the bargaining solution of each step, including the cooperation decision and payment, by using the

[1]Although the MNO can also deliver advertisements through the cellular network, users are much more receptive to advertising through Wi-Fi due to their voluntary use of Wi-Fi [3]. Furthermore, Wi-Fi usually provides more accurate user localization, and is more suitable for supporting multimedia advertisement due to the high data rate.

[2]The case where different sides have unbalanced market power can be studied in the same framework as in this paper, using the asymmetric Nash bargaining formulation [5].

[3]More precisely, the one-to-many bargaining contains several types. The most common type is the one-to-many bargaining with a *sequential* bargaining protocol. Another type is the one-to-many bargaining with a *concurrent* bargaining protocol, where the buyer bargains with multiple sellers concurrently [6]. In practice, conducting the concurrent bargaining is much more difficult than the sequential bargaining, as it requires the evaluation of simultaneous responses of all bargainers. In this paper, we focus on the sequential bargaining protocol in the one-to-many bargaining.

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Nash bargaining theory [7]. Since the MNO's willingness to deploy new hotspots decreases as the number of deployed hotspots increases, the cooperation between the MNO and a particular VO imposes *negative externality* to the bargaining among the MNO and other VOs. Such an externality significantly complicates the analysis. There are very few literatures studying the one-to-many bargaining, especially under the Nash bargaining theory. Our work provides a systematic study on this problem.

In the first part of this paper, we study the scenario where the MNO bargains with VOs sequentially according to a predetermined bargaining sequence (*i.e.*, *exogenous* bargaining sequence). We take into account the *data offloading benefit*, *Wi-Fi operation cost*, *advertising profit*, and *business revenue* of the MNO and VOs, and answer the following questions: (1) *Which VOs should the MNO cooperate with?* (2) *How much should the MNO pay these VOs?* We apply backward induction to compute the optimal bargaining solution.

In the second part of this paper, we study the scenario where the MNO first determines the bargaining sequence (*i.e.*, *endogenous* bargaining sequence) and then bargains with VOs accordingly. We want to answer the following key question: *Under what bargaining sequence can the MNO maximize its payoff?* Based on the analysis in the first part, we can compute the MNO's payoff once given a particular bargaining sequence. However, due to the complex structure of the one-to-many bargaining, we often cannot obtain the closed-form solution of such a payoff. Therefore, it is very complicated to directly compare the MNO's payoffs under all possible bargaining sequences and pick the optimal one.

To tackle the high complexity of the optimal sequencing problem, we first prove an important structural property of the one-to-many bargaining. More precisely, we categorize VOs into three types based on the impact of the Wi-Fi deployment at their venues. We can show that there exists a group of optimal bargaining sequences, under which the MNO bargains with these three types of VOs sequentially. As a result, we design an *Optimal VO Bargaining Sequencing* (OVBS) algorithm that searches for the optimal bargaining sequence from a significantly reduced set. In fact, the structural property we prove in this paper is general, and is valid for many other one-to-many bargaining problems. We further characterize two special system settings, where we can explicitly determine the optimal sequence without running OVBS.

The main contributions of this paper are as follows:

- *Systematic study of the one-to-many bargaining*: To the best of our knowledge, this is the first paper studying the one-to-many bargaining with both *exogenous* and *endogenous* bargaining sequences under the Nash bargaining theory. Most results in this paper are general enough to be applied in other one-to-many bargaining problems.
- *Algorithm design of the optimal bargaining sequence search*: We first highlight the fact that the bargaining sequence may significantly affect the solution to the one-to-many bargaining. Then we prove an important structural property for the one-to-many bargaining, and design a low-complexity *Optimal VO Bargaining Sequencing* (OVBS) algorithm to compute the optimal sequence.
- *Modeling and analysis of the cooperative Wi-Fi deployment*: To the best of our knowledge, this is the first paper studying the economic interactions among MNOs and VOs in terms of the cooperative Wi-Fi deployment. We consider the negative externalities among different steps of negotiation, and investigate the optimal sequencing strategy for the MNO. Numerical results show that the optimal bargaining sequence improves the MNO's payoff over the random and worst bargaining sequences by up to 14.7% and 45.8%, respectively.

C. Literature Review

There are a few literatures studying the public Wi-Fi deployment problem. Zheng *et al.* in [8] proposed Wi-Fi deployment algorithms which provide the worst-case guarantee to the interconnection gap for vehicular Internet access. Wang *et al.* in [9] exploited users' mobility patterns to deploy Wi-Fi access points, aiming at maximizing the continuous Wi-Fi coverage for mobile users. However, none of these works studied the economic issues in the Wi-Fi deployment.

In terms of the *one-to-many bargaining*, the most relevant works are [6], [10]. Both papers studied the one-to-many bargaining under the Nash bargaining theory. However, since they did not consider the *cooperation cost*, their conclusion was that the bargaining sequence does not affect the buyer's payoff, and their analysis was limited to the one-to-many bargaining with *exogenous* sequence. In our work, we take into account the *cooperation cost* (*e.g.*, Wi-Fi installment and operation cost), which complicates the one-to-many bargaining with *exogenous* sequence. Such consideration also motivates us to study the one-to-many bargaining with *endogenous* sequence. References [11]–[13] studied the one-to-many bargaining under the strategic game theory. In their problems, the buyer bargains with multiple sellers on a joint project that requires the cooperation of all participants. It is different from our problem, as here the MNO may only cooperate with a subset of the VOs on the Wi-Fi deployment.

II. System Model

A. Basic Settings

We consider one mobile network operator (MNO), who operates multiple macrocells and bargains with venue owners (VOs) to deploy Wi-Fi access points. We assume each venue (such as a cafe) is covered by at most one cellular macrocell. Since deploying Wi-Fi at a particular venue only offloads traffic for the corresponding macrocell and does not benefit other macrocells, the MNO can consider the Wi-Fi deployments for different macrocells separately. Without loss of generality, we study the MNO's strategy within one macrocell.

We consider a set $\mathcal{N} \triangleq \{1, 2, \ldots, N\}$ of VOs, whose venues are non-overlapping but covered by the same macrocell. Each VO $n \in \mathcal{N}$ is described by a tuple (X_n, R_n, C_n, A_n). Specifically, (1) $X_n \geq 0$ denotes the expected amount of offloaded macrocell traffic when Wi-Fi is deployed at venue n[4]; (2) $R_n \geq 0$ denotes the extra revenue that Wi-Fi creates for VO n's business (*e.g.*, via attracting more customers and collecting customer analytics); (3) $C_n \geq 0$ denotes the total cost for the MNO to deploy and operate Wi-Fi at venue n, including the installment fee, management cost, and backhaul cost[5]; (4) $A_n \geq 0$ denotes the advertising profit to the MNO[6] when Wi-Fi is deployed at venue n. We assume that both the MNO and VO n have the complete information of (X_n, R_n, C_n, A_n)[7].

B. MNO's Payoff, VO's Payoff and Social Welfare

We use $b_n \in \{0,1\}$ to denote the bargaining outcome between the MNO and VO n: $b_n = 1$ if they agree on the Wi-Fi deployment, and $b_n = 0$ otherwise. We use $p_n \in \mathbb{R}$ to denote the MNO's payment[8] to VO n. Later we will see, under the Nash bargaining solution, $p_n = 0$ whenever $b_n = 0$, *i.e.,* there is no transfer if no agreement is reached.

To simplify the notations, we define $\boldsymbol{b}_n \triangleq (b_1, b_2, \ldots, b_n)$ and $\boldsymbol{p}_n \triangleq (p_1, p_2, \ldots, p_n)$ as the bargaining outcomes and payments between the MNO and the *first* $n \in \mathcal{N}$ VOs.

The *MNO's payoff* depends on the offloading benefit, advertising profit, Wi-Fi deployment and operation cost, and its payment to VOs. Based on $\boldsymbol{b}_N$ and $\boldsymbol{p}_N$, the MNO's payoff is

$$U(\boldsymbol{b}_N, \boldsymbol{p}_N) = f\left(\sum_{n=1}^{N} b_n X_n\right) + \sum_{n=1}^{N} b_n (A_n - C_n) - \sum_{n=1}^{N} p_n. \quad (1)$$

Here $f(\cdot)$ is a non-negative, increasing, and strictly concave function, which characterizes the offloading benefit of the MNO [6]. Naturally, we have $f(0) = 0$.

VO n's payoff depends on the revenue directly brought by Wi-Fi and the MNO's payment as follows,

$$V_n(b_n, p_n) = b_n R_n + p_n. \quad (2)$$

The *social welfare* is the aggregate payoff of the MNO and all VOs:

$$\begin{aligned} \Psi(\boldsymbol{b}_N) &= U(\boldsymbol{b}_N, \boldsymbol{p}_N) + \sum_{n=1}^{N} V_n(b_n, p_n) \\ &= f\left(\sum_{n=1}^{N} b_n X_n\right) + \sum_{n=1}^{N} b_n Q_n, \end{aligned} \quad (3)$$

where for each VO $n \in \mathcal{N}$ we define

$$Q_n \triangleq R_n + A_n - C_n. \quad (4)$$

Here Q_n captures the increase in social welfare by deploying Wi-Fi at venue n, excluding the data offloading effect. Since the payment terms are cancelled out in (3), the social welfare only depends on the bargaining outcomes between the MNO and N VOs, *i.e.,* $\boldsymbol{b}_N = (b_1, b_2, \ldots, b_N)$.

[4]To simplify the description, we use venue n to refer to VO n's venue.

[5]In practice, some VOs undertake the backhaul cost for the MNO. This can be easily incorporated into our analysis by properly redefining R_n and C_n.

[6]Sometimes VOs promote their products via Wi-Fi, and we include the corresponding advertising profit in R_n.

[7]In our future work, we will analyze the situation where the MNO has limited information on these parameters.

[8]Notice that p_n may be negative, in which case VO n pays the MNO as deploying Wi-Fi is more beneficial to it than the MNO.

III. One-To-One Bargaining

We first study a special case where there is only one VO, *i.e.,* $|\mathcal{N}| = 1$. We analyze the one-to-one bargaining under the Nash bargaining theory, which helps us to better understand the more general results in the later sections.

According to [7], the Nash bargaining solution (NBS) of the one-to-one bargaining solves the following problem:

$$\begin{aligned} &\max \ (U(b_1, p_1) - U(0,0)) \cdot (V_1(b_1, p_1) - V_1(0,0)) \\ &\text{s.t.} \ U(b_1, p_1) - U(0,0) \geq 0, V_1(b_1, p_1) - V_1(0,0) \geq 0, \\ &\text{var.} \quad b_1 \in \{0,1\}, p_1 \in \mathbb{R}, \end{aligned} \quad (5)$$

where $U(0,0)$ and $V_1(0,0)$ are the disagreement points of the MNO and VO 1, *i.e.,* their payoffs when no agreement is reached. The NBS essentially maximizes the product of the MNO and VO 1's payoff gains over their disagreement points. Intuitively, with a higher disagreement point, the MNO (or the VO) can obtain a larger payoff under the NBS.

According to (1) and (2), $U(0,0) = V_1(0,0) = 0$. We further define $\pi_1 \triangleq V_1(b_1, p_1)$ as the payoff of VO 1. This enables us to rewrite problem (5) with respect to π_1 and $\Psi(b_1)$:

$$\begin{aligned} &\max \quad (\Psi(b_1) - \pi_1) \cdot \pi_1 \\ &\text{s.t.} \quad \Psi(b_1) - \pi_1 \geq 0, \pi_1 \geq 0, \\ &\text{var.} \quad b_1 \in \{0,1\}, \pi_1 \in \mathbb{R}. \end{aligned} \quad (6)$$

Problems (5) and (6) are equivalent. That is to say, bargaining on (b_1, p_1) is equivalent to bargaining on (b_1, π_1). Given any bargaining solution in terms of (b_1, π_1), we can compute the equivalent bargaining solution in terms of (b_1, p_1) as $(b_1, p_1) = (b_1, \pi_1 - b_1 R_1)$.

The closed-form solution to (6) is (see our technical report [14] for the detailed proof)

$$(b_1^*, \pi_1^*) = \begin{cases} \left(1, \frac{1}{2}\Psi(1)\right), & \text{if } \Psi(1) \geq 0, \\ (0,0), & \text{otherwise}, \end{cases} \quad (7)$$

where the social welfare $\Psi(1) = f(X_1) + Q_1$ is defined in (3). Result (7) indicates that if reaching agreement increases the social welfare, *i.e.,* $\Psi(1) \geq \Psi(0) = 0$, the MNO will deploy Wi-Fi at venue 1 and equally share the generated social welfare with VO 1; otherwise no Wi-Fi will be deployed and both the MNO and VO 1 will obtain zero payoff.

IV. One-to-Many Bargaining with Exogenous Sequence

In this section, we study the case where the MNO bargains with N VOs sequentially under a fixed sequence. We illustrate the bargaining protocol in Figure 1. At each step, the MNO bargains with one VO $n \in \mathcal{N}$ on (b_n, p_n).

We define π_n as VO $n \in \mathcal{N}$'s payoff. As shown in Section III, bargaining on (b_n, p_n) and bargaining on (b_n, π_n) are equivalent. Therefore, in Sections IV and V, we present the NBS in the form of (b_n, π_n) to simplify the notations. Like $\boldsymbol{b}_n$ and $\boldsymbol{p}_n$, we use $\boldsymbol{\pi}_n \triangleq (\pi_1, \pi_2, \ldots, \pi_n)$ to denote the payoffs of the *first* n VOs.

Without loss of generality, we assume that the bargaining sequence follows $1, 2, \ldots, N$, *i.e.,* at step n, the MNO bargains with VO n. We analyze the problem by backward induction.

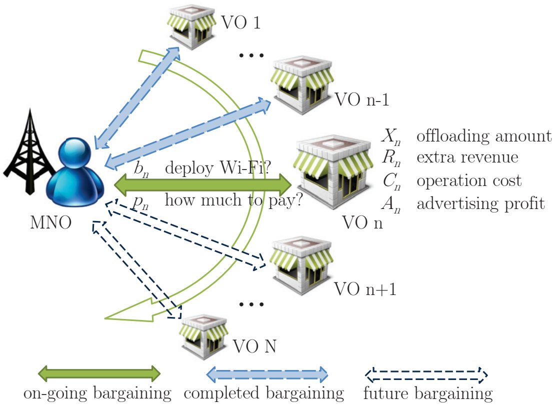

Fig. 1: Bargaining Protocol.

Step N: Suppose that the MNO has already bargained with VO $1, \ldots, N-1$, and has reached $\boldsymbol{b}_{N-1}$ and $\boldsymbol{\pi}_{N-1}$. It now bargains with VO N.

The MNO and VO N's disagreement points are

$$U_N^0 = \Psi\left((\boldsymbol{b}_{N-1}, 0)\right) - \sum_{n=1}^{N-1} \pi_n,$$
$$V_N^0 = 0.$$

Here $\Psi\left((\boldsymbol{b}_{N-1}, 0)\right)$ is the social welfare when the bargaining outcomes of all N steps are given as $(\boldsymbol{b}_{N-1}, 0)$, *i.e.*, assuming that no agreement is reached in step N. We obtain U_N^0 by subtracting VOs' payoffs from the social welfare. VO N has a zero disagreement point if not reaching agreement with the MNO.

If they reach (b_N, π_N) in step N, their payoffs are

$$U_N^1 = \Psi\left((\boldsymbol{b}_{N-1}, b_N)\right) - \sum_{n=1}^{N-1} \pi_n - \pi_N,$$
$$V_N^1 = \pi_N.$$

Here $\Psi\left((\boldsymbol{b}_{N-1}, b_N)\right)$ is the social welfare when the bargaining outcomes are given as $(\boldsymbol{b}_{N-1}, b_N)$.

Hence, the Nash bargaining problem at step N is:

$$\begin{aligned} \max \quad & \left(U_N^1 - U_N^0\right) \cdot \left(V_N^1 - V_N^0\right) \\ \text{s.t.} \quad & U_N^1 - U_N^0 \geq 0, V_N^1 - V_N^0 \geq 0, \\ \text{var.} \quad & b_N \in \{0,1\}, \pi_N \in \mathbb{R}. \end{aligned} \tag{8}$$

We solve (8) and obtain the NBS for step N:

$$(b_N^*, \pi_N^*) = \begin{cases} \left(1, \frac{1}{2}\Delta_N\left(\boldsymbol{b}_{N-1}\right)\right), & \text{if } \Delta_N\left(\boldsymbol{b}_{N-1}\right) \geq 0, \\ (0,0), & \text{otherwise,} \end{cases} \tag{9}$$

where we define

$$\Delta_N\left(\boldsymbol{b}_{N-1}\right) \triangleq \Psi\left((\boldsymbol{b}_{N-1}, 1)\right) - \Psi\left((\boldsymbol{b}_{N-1}, 0)\right). \tag{10}$$

Here $\Delta_N\left(\boldsymbol{b}_{N-1}\right)$ can be understood as follows: if we treat the MNO and VO N as a coalition, $\Delta_N\left(\boldsymbol{b}_{N-1}\right)$ describes the increase in the coalition's payoff by deploying Wi-Fi at venue N. If and only if such a value is non-negative, the MNO and VO N will reach agreement and equally share the generated revenue; otherwise no agreement is reached. This is similar as the one-to-one bargaining in Section III.

We can also understand $\Delta_N\left(\boldsymbol{b}_{N-1}\right)$ as the increase in social welfare by deploying Wi-Fi at venue N. This is because VO N is the last one that the MNO bargains with. For a general bargaining step $n \in \mathcal{N}$, we will later show that $\Delta_n\left(\boldsymbol{b}_{n-1}\right)$ is generally *not* equal to the increase in social welfare by deploying Wi-Fi at venue n.

Based on (9), (b_N^*, π_N^*) depends on vector $\boldsymbol{b}_{N-1}$ but is independent of vector $\boldsymbol{\pi}_{N-1}$. This means that the NBS for step N only depends on the first $N-1$ steps' bargaining outcomes, and not on the payments between the MNO and VOs. In what follows, we use $b_N^*\left(\boldsymbol{b}_{N-1}\right)$ and $\pi_N^*\left(\boldsymbol{b}_{N-1}\right)$ to indicate such dependence.

Step $N-1$: Suppose that the MNO has already bargained with VO $1, \ldots, N-2$, and has reached $\boldsymbol{b}_{N-2}$ and $\boldsymbol{\pi}_{N-2}$. It now bargains with VO $N-1$.

The MNO and VO $N-1$'s disagreement points are

$$U_{N-1}^0 = \Psi\left((\boldsymbol{b}_{N-2}, 0, b_N^*\left((\boldsymbol{b}_{N-2}, 0)\right))\right) - \sum_{n=1}^{N-2} \pi_n - \pi_N^*\left((\boldsymbol{b}_{N-2}, 0)\right),$$
$$V_{N-1}^0 = 0.$$

We obtain U_{N-1}^0 by subtracting VOs' payoffs from the social welfare. Here $b_N^*\left((\boldsymbol{b}_{N-2}, 0)\right)$ and $\pi_N^*\left((\boldsymbol{b}_{N-2}, 0)\right)$ together correspond to the NBS for step N when the bargaining outcomes of the first $N-1$ steps are $(\boldsymbol{b}_{N-2}, 0)$, as computed by (9).

If they reach (b_{N-1}, π_{N-1}) in step $N-1$, their payoffs are

$$\begin{aligned} U_{N-1}^1 = & \Psi\left((\boldsymbol{b}_{N-2}, b_{N-1}, b_N^*\left((\boldsymbol{b}_{N-2}, b_{N-1})\right))\right) \\ & - \sum_{n=1}^{N-2} \pi_n - \pi_{N-1} - \pi_N^*\left((\boldsymbol{b}_{N-2}, b_{N-1})\right), \end{aligned}$$
$$V_{N-1}^1 = \pi_{N-1}.$$

Here $b_N^*\left((\boldsymbol{b}_{N-2}, b_{N-1})\right)$ and $\pi_N^*\left((\boldsymbol{b}_{N-2}, b_{N-1})\right)$ are also determined by (9).

Hence, the Nash bargaining problem at step $N-1$ is:

$$\begin{aligned} \max \quad & \left(U_{N-1}^1 - U_{N-1}^0\right) \cdot \left(V_{N-1}^1 - V_{N-1}^0\right) \\ \text{s.t.} \quad & U_{N-1}^1 - U_{N-1}^0 \geq 0, V_{N-1}^1 - V_{N-1}^0 \geq 0, \\ \text{var.} \quad & b_{N-1} \in \{0,1\}, \pi_{N-1} \in \mathbb{R}. \end{aligned} \tag{11}$$

We solve (11) and obtain the NBS for step $N-1$:

$$(b_{N-1}^*, \pi_{N-1}^*) = \begin{cases} \left(1, \frac{1}{2}\Delta_{N-1}\left(\boldsymbol{b}_{N-2}\right)\right), & \text{if } \Delta_{N-1}\left(\boldsymbol{b}_{N-2}\right) \geq 0, \\ (0,0), & \text{otherwise,} \end{cases} \tag{12}$$

where we define

$$\begin{aligned} \Delta_{N-1}\left(\boldsymbol{b}_{N-2}\right) \triangleq & \Psi\left((\boldsymbol{b}_{N-2}, 1, b_N^*\left((\boldsymbol{b}_{N-2}, 1)\right))\right) - \pi_N^*\left((\boldsymbol{b}_{N-2}, 1)\right) \\ & - \Psi\left((\boldsymbol{b}_{N-2}, 0, b_N^*\left((\boldsymbol{b}_{N-2}, 0)\right))\right) + \pi_N^*\left((\boldsymbol{b}_{N-2}, 0)\right). \end{aligned} \tag{13}$$

If we treat the MNO and VO $N-1$ as a coalition, then $\Delta_{N-1}\left(\boldsymbol{b}_{N-2}\right)$ describes the increase in the coalition's payoff by deploying Wi-Fi at venue $N-1$, taking into account VO N's response. Result (12) shows that $\left(b_{N-1}^*, \pi_{N-1}^*\right)$ is the function of vector $\boldsymbol{b}_{N-2}$.

Step $\boldsymbol{k}$, $k \in \{1, 2, \ldots, N-2\}$: Suppose that the MNO has bargained with VO $1, \ldots, k-1$, and has reached $\boldsymbol{b}_{k-1}$ and $\boldsymbol{\pi}_{k-1}$. It now bargains with VO k. In order to save space, we leave the detailed analysis for step k in [14]. Like step N and N–1, we can express U_k^0, V_k^0, U_k^1, and V_k^1 by utilizing the NBS $\left(b_{k+1}^*, \pi_{k+1}^*\right), \ldots, \left(b_N^*, \pi_N^*\right)$ obtained in step $k+1, \ldots, N$. Then we solve the Nash bargaining problem for step k and obtain the following NBS:

$$\left(b_k^*, \pi_k^*\right) = \begin{cases} \left(1, \frac{1}{2}\Delta_k\left(\boldsymbol{b}_{k-1}\right)\right), & \text{if } \Delta_k\left(\boldsymbol{b}_{k-1}\right) \geq 0, \\ (0,0), & \text{otherwise}, \end{cases} \tag{14}$$

where we define

$$\begin{aligned} &\Delta_k\left(\boldsymbol{b}_{k-1}\right) \triangleq \\ &\Psi\left(\left(\boldsymbol{b}_{k-1}, 1, b_{k+1}^*\left(\left(\boldsymbol{b}_{k-1}, 1\right)\right), \ldots, b_N^*\left(\left(\boldsymbol{b}_{k-1}, 1, \ldots\right)\right)\right)\right) \\ &-\pi_{k+1}^*\left(\left(\boldsymbol{b}_{k-1}, 1\right)\right) - \ldots - \pi_N^*\left(\left(\boldsymbol{b}_{k-1}, 1, \ldots\right)\right) \\ &-\Psi\left(\left(\boldsymbol{b}_{k-1}, 0, b_{k+1}^*\left(\left(\boldsymbol{b}_{k-1}, 0\right)\right), \ldots, b_N^*\left(\left(\boldsymbol{b}_{k-1}, 0, \ldots\right)\right)\right)\right) \\ &+\pi_{k+1}^*\left(\left(\boldsymbol{b}_{k-1}, 0\right)\right) + \ldots + \pi_N^*\left(\left(\boldsymbol{b}_{k-1}, 0, \ldots\right)\right). \end{aligned} \tag{15}$$

After applying backward induction[9] to the analysis from step N to 1, we can eventually obtain the bargaining outcomes in all steps and all VOs' payoffs. We denote them[10] by $\hat{\boldsymbol{b}}_N = \left(\hat{b}_1, \ldots, \hat{b}_N\right)$ and $\hat{\boldsymbol{\pi}}_N = \left(\hat{\pi}_1, \ldots, \hat{\pi}_N\right)$. Based on $\hat{\boldsymbol{b}}_N$ and $\hat{\boldsymbol{\pi}}_N$, we can easily compute the MNO's eventual payoff as

$$\begin{aligned} U_0 &= \Psi\left(\hat{\boldsymbol{b}}_N\right) - \sum_{n=1}^{N} \hat{\pi}_n \\ &= f\left(\sum_{n=1}^{N} \hat{b}_n X_n\right) + \sum_{n=1}^{N} \hat{b}_n Q_n - \sum_{n=1}^{N} \hat{\pi}_n. \end{aligned} \tag{16}$$

V. One-to-Many Bargaining with Endogenous Sequence

In this section, we study the one-to-many bargaining with endogenous sequence, *i.e.*, the bargaining sequence is selected by the MNO to maximize its payoff.

The examples in Figure 2 illustrate that the bargaining sequence can significantly affect the bargaining solution and the MNO's payoff. By exchanging the bargaining positions of the two VOs (*red* and *white*), the MNO improves its payoff from 0.875 to 1 (the detailed analysis is given in [14]).

A. Optimal Sequencing Problem

We use $\boldsymbol{l} = (l_1, l_2, \ldots, l_N)$ to denote the bargaining sequence, *i.e.*, the MNO bargains with VO $l_n \in \mathcal{N}$ at step n. We further define $\mathcal{L}$ as the set of all possible bargaining sequences:

$$\mathcal{L} \triangleq \left\{\boldsymbol{l} : l_i, l_j \in \mathcal{N} \text{ and } l_i \neq l_j, \forall i \neq j, i, j \in \mathcal{N}\right\}.$$

We use $U_0^{\boldsymbol{l}}$ to denote the MNO's payoff under bargaining sequence $\boldsymbol{l} \in \mathcal{L}$. The MNO's optimal sequencing problem is

$$\max_{\boldsymbol{l} \in \mathcal{L}} U_0^{\boldsymbol{l}}, \tag{17}$$

i.e., choosing the optimal sequence $\boldsymbol{l}^*$ to maximize its payoff.

To solve (17), the straightforward method is to compute the MNO's payoff for each $\boldsymbol{l} \in \mathcal{L}$ and determine $\boldsymbol{l}^*$ accordingly.

[9] We summarize this backward induction by a recursive algorithm in [14].

[10] Recall that, bargaining on $(\boldsymbol{b}_N, \boldsymbol{\pi}_N)$ and $(\boldsymbol{b}_N, \boldsymbol{p}_N)$ are equivalent, so we can easily obtain $\hat{\boldsymbol{p}}_N = (\hat{p}_1, \ldots, \hat{p}_N)$ by $\hat{p}_n = \hat{\pi}_n - \hat{b}_n R_n$.

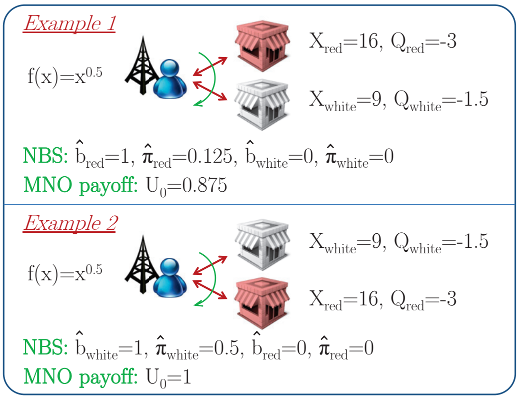

Fig. 2: The impact of the bargaining sequence on the MNO's payoff.

Since $|\mathcal{L}| = N!$, the computational complexity of this method is high. In Section V-B, we prove an important structural property for the one-to-many bargaining, which allows us to design an *Optimal VO Bargaining Sequencing* (OVBS) algorithm with a significantly lower complexity. In Sections V-C and V-D, we study two special cases where we can explicitly determine $\boldsymbol{l}^*$ without running OVBS.

B. Structural Property and OVBS Algorithm

Recall the definition of Q_n in (4), based on which we can categorize VOs into three types:

Definition 1. *VO $n \in \mathcal{N}$ belongs to*
(1) type 1, *if $Q_n \geq 0$;*
(2) type 2, *if $Q_n < 0$ and $f(X_n) + Q_n \geq 0$;*
(3) type 3, *if $Q_n < 0$ and $f(X_n) + Q_n < 0$.*

Based on the definition of the social welfare (3), such categorization can be understood as follows:

- For type 1 VO n, its cooperation with the MNO does not decrease the social welfare, *i.e.*, $\Psi\left((b_1, \ldots, b_{n-1}, 1, b_{n+1}, \ldots, b_N)\right) \geq \Psi\left((b_1, \ldots, b_{n-1}, 0, b_{n+1}, \ldots, b_N)\right)$ for all $(b_1, \ldots, b_{n-1}, b_{n+1}, \ldots, b_N)$;
- For type 2 VO n, its cooperation with the MNO may or may not decrease the social welfare, which depends on other VOs' parameters and bargaining positions;
- For type 3 VO n, its cooperation with the MNO decreases the social welfare, *i.e.*, $\Psi\left((b_1, \ldots, b_{n-1}, 1, b_{n+1}, \ldots, b_N)\right) < \Psi\left((b_1, \ldots, b_{n-1}, 0, b_{n+1}, \ldots, b_N)\right)$ for all $(b_1, \ldots, b_{n-1}, b_{n+1}, \ldots, b_N)$.

We assume that the number of each type of VOs is N_1, N_2, and N_3, respectively, with $N_1 + N_2 + N_3 = N$. Then we have the following lemmas[11].

Lemma 1. *The MNO will always cooperate with a type* 1 *VO, regardless of the VO's position in the bargaining sequence.*

Lemma 2. *The MNO will never cooperate with a type* 3 *VO, regardless of the VO's position in the bargaining sequence.*

Lemma 3. *If the bargaining sequence follows* $1, 2, \ldots, N$, *and VO k belongs to type* 1, *where* $k \in \{2, 3, \ldots, N\}$, *the*

[11] The detailed proofs of all lemmas and theorems are given in [14].

Algorithm 1 *Optimal VO Bargaining Sequencing* (OVBS)

Phase 1: Construct the reduced set $\mathcal{L}^{RE}$
Order all type 1 VOs arbitrarily, and denote the sequence by a vector $\boldsymbol{h}^1 = \left(h_1^1, h_2^1 \ldots, h_{N_1}^1\right)$;
Order all type 3 VOs arbitrarily, and denote the sequence by a vector $\boldsymbol{h}^3 = \left(h_1^3, h_2^3 \ldots, h_{N_3}^3\right)$;
Denote the set of all permutations of type 2 VOs by set $\mathcal{H}^2$. Each permutation is denoted by a vector $\boldsymbol{h}^2 = \left(h_1^2, h_2^2 \ldots, h_{N_2}^2\right) \in \mathcal{H}^2$.
Pick every $\boldsymbol{h}^2 \in \mathcal{H}^2$ and construct the corresponding total sequencing by $\boldsymbol{l} = \left(\boldsymbol{h}^1, \boldsymbol{h}^2, \boldsymbol{h}^3\right)$. Denote the set of all such $\boldsymbol{l}$s as $\mathcal{L}^{RE}$.
Phase 2: Search the optimal sequence
Apply the backward induction and (16) in Section IV to compute $U_0^{\boldsymbol{l}}$ for each $\boldsymbol{l} \in \mathcal{L}^{RE}$ and return $\boldsymbol{l}^{RE} = \underset{\boldsymbol{l} \in \mathcal{L}^{RE}}{\arg\max} U_0^{\boldsymbol{l}}$.

MNO's payoff does not decrease after exchanging VO $k-1$ and k's bargaining positions.

Lemma 3 shows that bargaining with a type 1 VO before any other VO will not decrease the MNO's payoff. Now we are ready to state our main theorem, which describes the structural property of the optimal bargaining sequence.

Theorem 1. *There exists a non-empty set of optimal bargaining sequences $\mathcal{L}^* \subseteq \mathcal{L}$, such that any $\boldsymbol{l} \in \mathcal{L}^*$ satisfies both of the following two conditions*[12]*:*

(1) VO $l_1, l_2, \ldots, l_{N_1}$ are of type 1;

(2) VO $l_{N_1+N_2+1}, l_{N_1+N_2+2}, \ldots, l_N$ are of type 3.

For any optimal sequence $\boldsymbol{l} \in \mathcal{L}^*$,

(1) if the MNO interchanges the bargaining positions of any two type 1 VOs, the MNO's payoff will not change;

(2) if the MNO interchanges the bargaining positions of any two type 3 VOs, the MNO's payoff will not change.

Notice that, there may exist some optimal bargaining sequences that are not in set $\mathcal{L}^*$. Since our focus is to maximize the MNO's payoff by a properly chosen sequence, we will focus on set $\mathcal{L}^*$ in this paper.

Based on Theorem 1, we propose an *Optimal VO Bargaining Sequencing* (OVBS) algorithm (*i.e.*, Algorithm 1), which solves the optimal sequencing problem (17) as follows.

Theorem 2. *The sequence $\boldsymbol{l}^{RE}$ obtained by OVBS lies in set $\mathcal{L}^*$. In other words, $\boldsymbol{l}^{RE}$ is one of the optimal bargaining sequences for problem (17).*

The basic idea of OVBS is to utilize Theorem 1 to reduce the searching space of $\boldsymbol{l}^*$ from set $\mathcal{L}$ to a new constructed set $\mathcal{L}^{RE}$. Since $|\mathcal{L}| = N!$ and $\left|\mathcal{L}^{RE}\right| = N_2!$, the complexity of determining $\boldsymbol{l}^*$ is significantly reduced.

To summarize, the optimal sequence determined by OVBS has the following features:

[12]Naturally, VO $l_{N_1+1}, l_{N_1+2}, \ldots, l_{N_1+N_2}$ are of type 2 when these two conditions are satisfied.

Fig. 3: Structure of the Optimal Bargaining Sequence under OVBS.

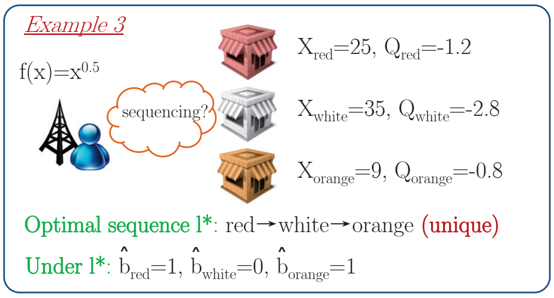

Fig. 4: Counter-Intuitive Sequencing for Type 2 VOs

(a) The MNO bargains with type 1, type 2, and type 3 VOs sequentially (Theorem 1); **(b)** The MNO will cooperate with all type 1 VOs (Lemma 1); **(c)** The MNO will not cooperate with any type 3 VO (Lemma 2); **(d)** Interchanging any two type 1 VOs' positions will not change the MNO's payoff (Theorem 1); **(e)** Interchanging any two type 3 VOs' positions will not change the MNO's payoff (Theorem 1).

We illustrate the optimal sequence's structure in Figure 3.

The optimal sequencing problem among type 2 VOs is very complicated. To see this, consider the illustrative example in Figure 4 where all VOs are of type 2. The MNO's unique optimal sequencing strategy is *red*, *white*, and *orange*, and the MNO will only cooperate with VOs *red* and *orange*. In other words, it is optimal for the MNO in this example to bargain with someone (VO *white*) that it will not cooperate with ahead of someone (VO *orange*) that it will cooperate with. The reason for this counter-intuitive result is that such strategy helps the MNO earn more profit in the first step of bargaining (with VO *red*). This example shows that sequencing type 2 VOs is challenging, and it is difficult to further reduce the searching space $\mathcal{L}^{RE}$.

C. Special Case 1: Only Type 1 VOs

We next study a special case where all VOs are of type 1, *i.e.*, $Q_n \geq 0$ for all $n \in \mathcal{N}$. In this case, we not only know that any bargaining sequence is optimal (based on Theorem 1), but also can obtain the closed-form solution of the MNO's payoff as follows.

Theorem 3. *If all VOs are of type 1, the MNO's payoff is independent of the bargaining sequence $\boldsymbol{l}$ and is given as:*

$$U_0 = \frac{1}{2^N} \sum_{\boldsymbol{b}_N \in \mathcal{B}} \Psi\left(\boldsymbol{b}_N\right), \tag{18}$$

where $\mathcal{B} \triangleq \{(b_1, b_2, \ldots, b_N) : b_n \in \{0,1\}, \forall n \in \mathcal{N}\}$.

Mathematically, the MNO's payoff in (18) can be viewed as the expected social welfare under such a scenario, where the MNO cooperates with each VO with a probability of 0.5.

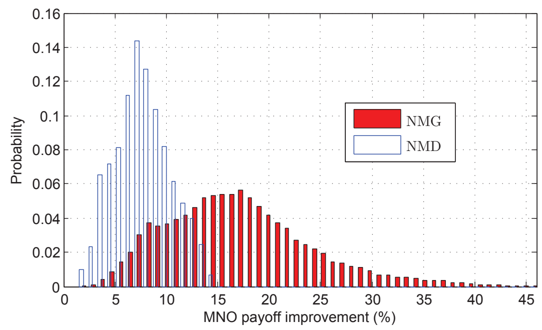

Fig. 5: Optimal Sequence VS Worst/Random Sequence.

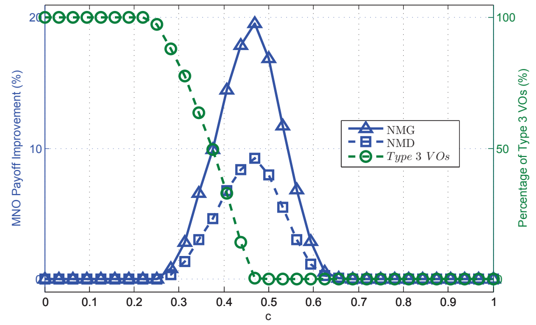

Fig. 6: Influence of the Offloading Benefit Function $f(\cdot)$.

This observation is consistent with [6], [10]. In fact, [6], [10] studied the one-to-many bargaining without cooperation cost. Hence, the buyer would definitely cooperate with all sellers. That corresponds to the special case we study in this subsection, *i.e.,* all VOs are of type 1. In this case, the bargaining sequence does not affect the buyer's payoff, so [6], [10] only studied the one-to-many bargaining with exogenous sequence. Our work in Sections IV and V considers a more general case where the buyer (MNO) may not necessarily cooperate with sellers (VOs), and provides a deeper understanding on the one-to-many bargaining with both exogenous and endogenous sequences.

D. Special Case 2: Sortable VOs

In this subsection, we study another special case where all VOs are *sortable*, which is defined in the following.

Definition 2. *A set $\mathcal{N}$ of VOs is sortable if and only if for any $i, j \in \mathcal{N}$, we have $(X_i - X_j)(Q_i - Q_j) \geq 0$.*

When a set of VOs are sortable[13], we can sort them based on (X_n, Q_n). The following theorem shows that this simple sorting generates the optimal bargaining sequence.

Theorem 4. *If all VOs are sortable, we can construct a sequence $\boldsymbol{l}$ such that $X_{l_n} \geq X_{l_{n+1}}$, $Q_{l_n} \geq Q_{l_{n+1}}$, $\forall n \in \{1, 2, \ldots, N-1\}$. Furthermore:*

(1) $\boldsymbol{l}$ is the optimal bargaining sequence;

(2) Under $\boldsymbol{l}$, the MNO will and only will cooperate with the first k VOs, i.e., VO $l_1, l_2, \ldots, l_k$, where $k \in \{0\} \cup \mathcal{N}$ is uniquely determined by the following inequalities:

$$f\left(\sum_{n=l_1}^{l_{k-1}} X_n + X_{l_k}\right) - f\left(\sum_{n=l_1}^{l_{k-1}} X_n\right) + Q_{l_k} \geq 0, \quad (19)$$

$$f\left(\sum_{n=l_1}^{l_k} X_n + X_{l_{k+1}}\right) - f\left(\sum_{n=l_1}^{l_k} X_n\right) + Q_{l_{k+1}} < 0. \quad (20)$$

That is to say, when all VOs are sortable, we can explicitly determine the optimal bargaining sequence and identify those VOs that the MNO will cooperate with.

[13]In the extreme case where all VOs are homogeneous in X_n or Q_n, they are sortable according to Definition 2.

VI. Numerical Results

In this section, we evaluate the performance of the optimal sequencing and study the impact of different parameters on the MNO's payoff.

A. Performance of Optimal Sequencing

First we define some criteria to capture the performance gap between different sequencing strategies. For a set $\mathcal{N}$ of VOs and the corresponding set $\mathcal{L}$ of bargaining sequences, we define the MNO's maximum, minimum, and average payoff as follows:

$$U_0^{\max} \triangleq \max_{\boldsymbol{l} \in \mathcal{L}} U_0^{\boldsymbol{l}}, \ U_0^{\min} \triangleq \min_{\boldsymbol{l} \in \mathcal{L}} U_0^{\boldsymbol{l}}, \ U_0^{\text{ave}} \triangleq \frac{1}{|\mathcal{L}|} \sum_{\boldsymbol{l} \in \mathcal{L}} U_0^{\boldsymbol{l}}.$$

Hence, $U_0^{\max}$, $U_0^{\min}$, and U_0^{ave} measure the MNO's payoff under the optimal sequence, worst sequence, and random sequence, respectively. Then we define the normalized maximum gap (NMG) and the normalized maximum deviation (NMD):

$$\text{NMG} \triangleq \frac{U_0^{\max} - U_0^{\min}}{U_0^{\min}}, \ \text{NMD} \triangleq \frac{U_0^{\max} - U_0^{ave}}{U_0^{ave}}.$$

NMG and NMD capture the performance improvement of the optimal sequence over the worst sequence and the random sequence, respectively.

1) Distribution of NMG and NMD: We choose $f(x) = x^{1/2}$, $|\mathcal{N}| = 5$, and assume that each VO n's (X_n, Q_n) follows the same uniform distribution ($X_n \sim U[80, 110]$ and $Q_n \sim U[-8, -3]$). We run the experiment 30,000 times, and record the probability distribution of NMG and NMD in Figure 5. We conclude that, (1) compared with the worst sequence, the optimal sequence improves the MNO's payoff by 16.9% on average and by 45.8% in the extreme case; (2) compared with the random sequence, the optimal sequence improves the MNO's payoff by 7.8% on average and by 14.7% in the extreme case.

2) Influence of the offloading benefit: We further investigate the influence of the concavity of function $f(\cdot)$ on the bargaining result. We still assume the uniformly distributed (X_n, Q_n) ($X_n \sim U[80, 110]$ and $Q_n \sim U[-8, -3]$) and set $|\mathcal{N}| = 4$. Furthermore, we assume $f(x) = x^c$ and choose c from 0 to 1, where a smaller c means a more concave function. For each c, we run the experiment 3,000 times and compute the expected NMG and NMD, as shown in Figure 6. From Definition 1,

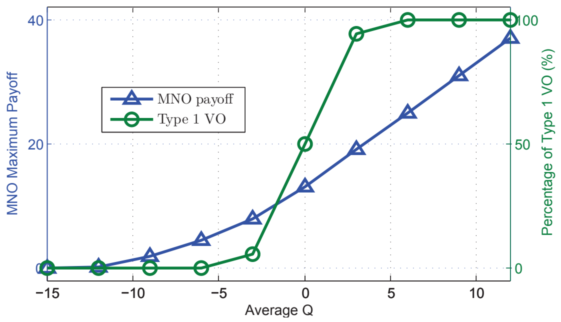

Fig. 7: Influence of $\mathbb{E}\{Q_n\}$.

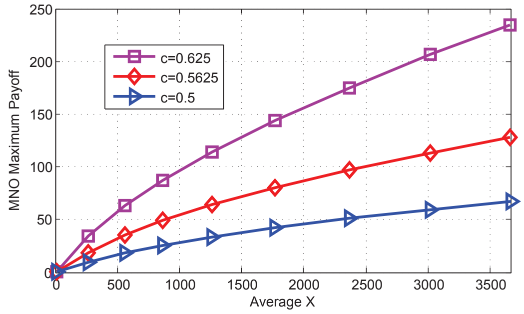

Fig. 8: Influence of $\mathbb{E}\{X_n\}$.

the percentage of type 3 VOs changes according to function $f(\cdot)$. Hence, in Figure 6 we also plot the expected percentage of type 3 VOs against c. We observe that both NMG and NMD first increase and then decrease. This is because when c is small, the offloading benefit for the MNO is small and most VOs are of type 3. Based on Lemma 2, the MNO never cooperates with these type 3 VOs. Hence, for small c, the optimal sequencing does not significantly improve the MNO's payoff. As c increases to 0.47, the expected percentage of type 3 VOs decreases to zero, and both NMG and NMD reach their peak values. When c continues to increase, function $f(\cdot)$ becomes less concave and the externalities among different steps of bargaining become weaker. As a result, the advantage of the optimal sequencing reduces and both NMG and NMD decrease.

B. MNO's Payoff

In this subsection, we simulate the impact of different parameters on the MNO's maximum payoff, *i.e.*, U_0^{max}.

1) Influence of Q_n: In Figure 7, we set $|\mathcal{N}| = 4$, $f(x) = x^{1/2}$, and plot U_0^{max} against the mean of the random variable Q_n. We find U_0^{max} increases with $\mathbb{E}\{Q_n\}$, because large $\mathbb{E}\{Q_n\}$ implies a large benefit (or a small cost) of deploying Wi-Fi network, and the MNO can earn more profit from the cooperative Wi-Fi deployment. Furthermore, we find U_0^{max} eventually linearly increases when $\mathbb{E}\{Q_n\} \geq 6$. To explain this, we also show the percentage of type 1 VOs in Figure 7. As $\mathbb{E}\{Q_n\}$ increases, the percentage of type 1 VOs approaches 100%. Based on (3) and (18), when all VOs are of type 1, we have

$$U_0^{\max} = \frac{1}{2^N} \sum_{\boldsymbol{b}_N \in \mathcal{B}} f\left(\sum_{n=1}^{N} b_n X_n\right) + \frac{1}{2}\sum_{n=1}^{N} Q_n, \tag{21}$$

where $\mathcal{B}$ is defined in Theorem 3. Hence, U_0^{max} linearly increases with $\mathbb{E}\{Q_n\}$, and the slope of the curve is $N/2$.

2) Influence of X_n: In Figure 8, we set $|\mathcal{N}| = 4$, $f(x) = x^c$, and plot U_0^{max} against the mean of the random variable X_n under different values of c. We observe that U_0^{max} concavely increases with $\mathbb{E}\{X_n\}$. Based on (1), such a concavity is due to the concave offloading benefit function $f(\cdot)$. Since a larger c corresponds to a less concave function $f(\cdot)$, we observe that, in Figure 8, an increase of c leads to a decrease of the concavity of the U_0^{max} curve.

VII. Conclusion

In this paper, we investigated the economic interactions among the MNO and VOs in the cooperative Wi-Fi deployment. We analyzed the problem under the one-to-many bargaining framework with both exogenous and endogenous sequences. For the former case, we applied backward induction to compute the bargaining results in terms of the cooperation decisions and payments for a given bargaining sequence. For the latter case, we proposed the OVBS algorithm that searches for the optimal bargaining sequence by leveraging the structural property. Numerical results showed that the optimal bargaining sequence improves the MNO's payoff as compared with the random and worst bargaining sequences. In our future work, we will further analyze the optimal bargaining strategy when the MNO has limited information of the VOs.

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