Minimum-Energy Link Scheduling for Emptying Wireless Networks

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Abstract We consider a wireless network consisting of source-destination pairs, in which each source is required to transmit a given bit volume to its destination. The goal is for all the sources to transmit the given bit volumes, under a time constraint, so that the total transmission energy is minimized. Our approach is the joint optimization of link scheduling and power control for minimum energy. We show that TDMA scheduling is appropriate for this goal, in the sense that TDMA is asymptotically optimal when the time constraint approaches infinity. When the time constraint is strictly bounded, we show that TDMA is also optimal for the case of equal channel gains.

1 Introduction

Energy, whose production and consumption can significantly impact the environment, is a crucial resource in communication and networking. Here we study the energy performance for a wireless network in which sources are to transmit given bit volumes to their destinations under a time constraint. Assume that no additional data is generated. Thus, a source idles after it finishes transmitting the required bit volume. We aim to develop techniques for accomplishing the transmissions so that the total transmission energy expenditure (summed over all transmitting nodes) is minimized. Our transmission problem can also be considered as "emptying" the network, subject to a time constraint, while using minimum transmission energy.

Our approach to solving the problem of minimum-energy transmission for emptying the wireless network is via joint optimization of link scheduling and power control. A transmission schedule specifies a set of time intervals such that in each time interval an appropriate group of sources is chosen to transmit simultaneously. The goal of power control is to choose a transmission power level (and corresponding data rate) for each source. Note that the transmission rate for each source depends on many factors such as power, constraints (on bit volume delivery and transmission time), channel attenuation, receiver noise, interference, as well as encoding-decoding method. Thus, the joint optimization of link scheduling and power control for minimizing energy, subject to various constraints, in general is a complex problem.

Different forms of scheduling exist for many communication and networking problems [1, 7, 9]. In particular, [1] considers optimal scheduling for emptying wireless networks in minimum time, with fixed rates and without power control. The resulting model can then be formulated as linear programming optimization, which can be dealt with by standard linear programming methods (e.g., the simplex algorithm [3]). In contrast, here we deal with minimum-energy scheduling and allow for power and rate control, resulting in a non-linear optimization problem.

We model the wireless network as a K-user interference channel, affected by additive white Gaussian noise (AWGN) (Fig. 1). Model extension to include features such as channel fading, MIMO, cognition, and cooperation is reserved for future studies. Recall that the goal is for all the sources, under a time constraint, to transmit the given bit volumes so that the network transmission energy is minimized. We show that, although simultaneous transmission may be required for maximum bit rates, time division multiple access (TDMA), which avoids simultaneous transmissions, is effective for saving energy. Here TDMA refers to a medium access scheme in which only one node can transmit at a time, but the length of the time allocated to each node depends on factors such as the bit volume to be transmitted, channel gains, and time constraint.

We first show that TDMA is asymptotically optimal when the time constraint approaches infinity (Theorem 1). We then specify the optimal TDMA scheduling for the case of finite time constraint. We also show that transmission times should extend as long as possible, i.e., the inequality time constraint becomes the equality time constraint for minimum energy (Theorems 2 and 3). Schedules that are based on treating interference as noise (TIN) can be effective by allowing certain groups of sources to transmit simultaneously, as long as the levels of signal, noise, and interference of the group satisfy some conditions [7, 9]. In the TIN model, the combined effect of other-user interference is modeled as AWGN with power equal to the sum of the received powers of the interfering users. We then show that TDMA is optimal when the channel gains are equal, i.e., non-TDMA approaches (such as TIN-based scheduling) can not consume less energy than TDMA (Theorem 4).

Although TDMA is a simple concept and widely used in practice, we show that the determination of optimal TDMA scheduling is not simple when TDMA is designed to take into account the joint optimization with constraints on time and bit volume delivery. In fact, the TDMA optimization with constraints turns out to be non-linear, and yields no closed form solutions in general.

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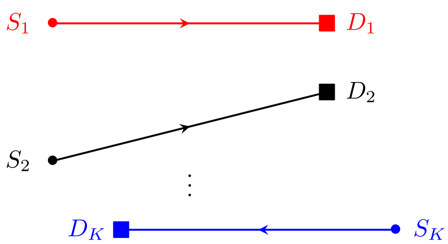

Fig. 1 A K-user interference channel.

2 System Model

We study a stationary network with K links (S_k, D_k), where S_k is the source and D_k is its destination, $k = 1, 2, \ldots, K$. Source S_k is to transmit V_k bits to D_k. Thus, the total network bit volume to be transmitted is

$$V_t = \sum_{k=1}^{K} V_k$$

and the total network energy used for transmitting V_t bits is

$$E_t = \sum_{k=1}^{K} E_k$$

where E_k is the transmission energy used by source S_k for transmitting V_k bits to destination D_k. The network operates on a single frequency band. Thus, mutual interference exists among simultaneous transmissions.

The transmissions of sources $S_1, S_2, \ldots, S_K$ are coordinated via scheduling, defined as follows. Let G be the family of all non-empty subsets of $\{1, 2, \ldots, K\}$. For $g \in G$, let $t_g > 0$ be the time duration in which all sources in $\{S_k : k \in g\}$ transmit simultaneously. A member $g \in G$ is also called a transmission group, in the sense that all sources in $\{S_k : k \in g\}$ transmit simultaneously as a group during time t_g. For $g \in G$, let $g_S = \{S_k : k \in g\}$. Note that $k \in g$ iff $S_k \in g_S$. A transmission schedule is a set $\{t_g > 0 : g \in G\}$. For example, the TDMA schedule is specified by $\{t_{\{1\}}, t_{\{2\}}, \ldots, t_{\{K\}}\}$, where $t_{\{k\}}$ is the time duration in which source S_k transmits alone. Another example is the "all-at-once" schedule $\{t_{\{1,2,\ldots,K\}}\}$, where $t_{\{1,2,\ldots,K\}}$ is the time duration in which all sources transmit simultaneously. Note that TDMA avoids other-user interference, while all-at-once scheduling can result in high levels of other-user interference.

We aim to find an optimal centralized transmission schedule and power assignment so that the total network bit volume V_t is transmitted within a time constraint, and the total network transmission energy E_t is minimized. The transmission energy is specified in terms of power and time. For $k \in g$, let $p(k, g)$ be the transmission power assigned to source S_k, when it transmits simultaneously with other sources in $g_S = \{S_k : k \in g\}$. For the transmission group g and $k \in g$, let $r(k, g)$ be the bit rate for link (S_k, D_k). In general, $r(k, g)$ also depends on other factors such as the time constraint, transmission power, channel bandwidth, and noise, e.g., see (3) and (4) below.

Consider a transmission schedule $\{t_g > 0 : g \in G\}$. The total energy used by S_k for transmitting V_k bits is $E_k = \sum_{g \in G} p(k, g) t_g$. The total energy used by the schedule is then $E_t = \sum_{k=1}^{K} E_k = \sum_{g \in G} P_g t_g$, where $P_g = \sum_{k \in g} p(k, g)$ is the transmission power used by group g. The bit volume transmitted by S_k during time t_g is $r(k, g) t_g$, and the bit volume transmitted by S_k during the entire schedule is $\sum_{g \in G} r(k, g) t_g$. Recall that the goal is to find an optimal schedule and power assignment for transmitting all the bits $V_1, V_2, \ldots, V_K$ with minimum total energy, under a time constraint T. Thus, our minimum-energy problem can be stated as

Minimize $\sum_{g} P_g t_g$

subject to

$$\sum_{g} r(k, g) t_g \geq V_k \qquad k = 1, 2, \ldots, K \tag{1}$$

$$\sum_{g} t_g \leq T \tag{2}$$

$$t_g > 0$$

A feasible solution for this problem exists only for appropriate values of system parameters. For example, when T is sufficiently small and V_k is sufficiently large for some k, the optimization problem has a feasible solution (i.e., the total bit volume V_t is delivered within the time constraint T) only if the transmission power used by source S_k is sufficiently large. In our model, $T > 0$ and $V_k > 0$ in (1) and (2) can be arbitrary, and we impose no restrictions on the minimum and maximum values of the transmission power for each source. As seen later, with power control, TDMA is a feasible solution for all $T > 0$ and $V_k < \infty, k = 1, 2, \ldots, K$.

Closely related to the energy performance is the notion of energy efficiency, which reflects the number of bits transmitted per unit energy. We consider the following two metrics for energy efficiency: total energy efficiency and average energy efficiency, defined respectively by

$$e_t = \frac{V_t}{E_t} = \frac{\sum_{k=1}^{K} V_k}{\sum_{k=1}^{K} E_k}$$

$$e_a = \frac{1}{K} \sum_{k=1}^{K} \frac{V_k}{E_k}$$

Let h_{ij} be the channel gain between source S_i and destination D_j, i.e., if S_i transmits with power $p(i, g)$, then D_j receives power $h_{ij} p(i, g)$. We assume that the channel is band limited to W and is affected by AWGN with power spectral density N_0.

Consider a general schedule $\{t_g > 0 : g \in G\}$. When g is a singleton group, i.e., $g = \{k\}$, then the maximum bit rate for source S_k is given by the Shannon capacity

$$r(k, \{k\}) = W \log_2 \left(1 + \frac{h_{kk} p(k, \{k\})}{W N_0}\right) \tag{3}$$

which can be achieved in the limit sense, i.e., with infinite time delay and computational complexity. When g has more than one member, i.e., $|g| > 1$, then this transmission group forms an interference channel, whose capacity rate region is unknown in general [4, 5, 6, 8]. A simple approximation for rates is the use of the Shannon capacity formula by considering other-user interference as AWGN, i.e., the transmission rate of source S_i is given by

$$r(i,g) = W\log_2\left(1 + \frac{h_{ii}p(i,g)}{\sum_{j\in g\setminus\{i\}} h_{ji}p(j,g) + WN_0}\right) \quad (4)$$

where g is the group of sources that transmit simultaneously, $i \in g$, and h_{ij} is the channel gain between source S_i and destination D_j.

For notational convenience, we define $t_k = t_{\{k\}}$, $p(k,k) = p(k,\{k\})$ and $r(k,k) = r(k,\{k\})$ when the transmission group is a singleton set, i.e., $g = \{k\}$.

3 Minimum-Energy Transmission Scheduling

Now assume that the time constraint is removed, i.e., let $T = \infty$. The case of $T < \infty$ is considered in the next section. Theorem 1 below shows that simple TDMA scheduling, with infinite time duration and vanishing transmission power and rate for each source, optimizes the total energy and energy efficiency. This result serves as a benchmark for performance comparison, and provides an approximation for the case of finite but sufficiently large time constraint.

Theorem 1 Let $N_0(k)$ be the power spectral density for the receiver noise at destination D_k. TDMA scheduling, with vanishing transmission power for each source, minimizes total energy and maximizes energy efficiency. The minimum total energy, maximum total energy efficiency, and maximum average energy efficiency are given, respectively, by

$$E_t = \ln 2 \sum_{k=1}^{K} \frac{N_0(k)V_k}{h_{kk}}$$

$$e_t = \frac{\sum_{k=1}^{K} V_k}{\ln 2 \sum_{k=1}^{K} \frac{N_0(k)V_k}{h_{kk}}}$$

$$e_a = \frac{1}{K\ln 2}\sum_{k=1}^{K} \frac{h_{kk}}{N_0(k)}$$

When source S_k's transmission power $p(k,k) \to 0$, we have rate $r(k,k) \to 0$, time duration $t_k \to \infty$. Further, when $p(k,k) \to 0$ and $p(i,i) \to 0$, we have

$$\frac{t_k}{t_i} \to \frac{N_0(k)V_k h_{ii}}{N_0(i)V_i h_{kk}}$$

Proof Consider an arbitrary schedule $\{t_g > 0 : g \in G\}$. We have

$$W\log_2\left(1 + \frac{h_{kk}p(k,g)}{WN_0(k)}\right) \geq r(k,g) = \frac{v(k,g)}{t_g}$$

which implies that

$$p(k,g) \geq \frac{WN_0(k)}{h_{kk}}\left(e^{\frac{v(k,g)\ln 2}{Wt_g}} - 1\right)$$

By using $e^x - 1 \geq x$, we then have

$$p(k,g) \geq \frac{WN_0(k)}{h_{kk}}\frac{v(k,g)\ln 2}{Wt_g} = \frac{N_0(k)v(k,g)\ln 2}{h_{kk}t_g}$$

The total energy used by S_k for transmitting V_k bits is $E_k = \sum_g p(k,g)t_g$, which is bounded by

$$E_k \geq \frac{N_0(k)}{h_{kk}}\sum_g v(k,g)\ln 2 = \frac{N_0(k)}{h_{kk}}V_k \ln 2$$

which implies that the total network energy is bounded by

$$E_t = \sum_{k=1}^{K} E_k \geq \ln 2 \sum_{k=1}^{K} \frac{N_0(k)V_k}{h_{kk}}$$

We then have

$$e_t = \frac{\sum_{k=1}^{K} V_k}{\sum_{k=1}^{K} E_k} \leq \frac{\sum_{k=1}^{K} V_k}{\ln 2 \sum_{k=1}^{K} \frac{N_0(k)V_k}{h_{kk}}}$$

Because $E_k \geq N_0(k)V_k \ln 2/h_{kk}$, we have $\frac{V_k}{E_k} \leq \frac{h_{kk}}{N_0(k)\ln 2}$, which implies that

$$e_a = \frac{1}{K}\sum_{k=1}^{K}\frac{V_k}{E_k} \leq \frac{1}{\ln 2}\frac{1}{K}\sum_{k=1}^{K}\frac{h_{kk}}{N_0(k)}$$

Now consider TDMA scheduling, i.e., each group has the form $g = \{k\}$. For the transmission of source S_k, we let $t_k = V_k/r(k,k)$, where $r(k,k) = W\log_2\left(1 + \frac{h_{kk}p(k,k)}{N_0(k)W}\right)$. Then

$$E_k = p(k,k)t_k = \frac{p(k,k)V_k}{r(k,k)}$$

$$\begin{aligned}
\lim_{p(k,k)\to 0} E_k &= V_k \lim_{p(k,k)\to 0}\frac{p(k,k)}{r(k,k)} \\
&= V_k \lim_{p(k,k)\to 0}\frac{p(k,k)}{W\log_2\left(1 + \frac{h_{kk}p(k,k)}{N_0(k)W}\right)} \\
&= \frac{V_k}{h_{kk}}N_0(k)\ln 2
\end{aligned}$$

Thus, when $p(k,k) \to 0$, $k = 1,2,\ldots,K$, we have $r(k,k) \to 0$, $t_k \to \infty$, $E_t \to \ln 2\sum_{k=1}^{K}\frac{N_0(k)V_k}{h_{kk}}$, $e_t \to \frac{1}{\ln 2}\frac{\sum_{k=1}^{K} V_k}{\sum_{k=1}^{K}\frac{N_0(k)V_k}{h_{kk}}}$, and $e_a \to \frac{1}{\ln 2}\frac{1}{K}\sum_{k=1}^{K}\frac{h_{kk}}{N_0(k)}$. Note that

$$\frac{t_k}{t_i} = \frac{V_k}{V_i}\frac{r(i,i)}{r(k,k)} = \frac{V_k}{V_i}\frac{\log_2\left(1 + \frac{h_{ii}p(i,i)}{N_0(i)W}\right)}{\log_2\left(1 + \frac{h_{kk}p(k,k)}{N_0(k)W}\right)}$$

When $p(k,k), p(i,i) \to 0$, we then have

$$\frac{t_k}{t_i} \to \frac{V_k}{V_i}\frac{h_{ii}}{h_{kk}}\frac{N_0(k)}{N_0(i)} \qquad \square$$

The minimum energy per bit for the case of $K = 2$ is studied in [2], where TDMA is shown to be optimal in the low-power regime. Thus, Theorem 1 is not surprising, because the sources can transmit with vanishing power levels and rates when the time constraint is removed, resulting in minimum energy usage.

An alternative low-power performance measure to the minimum energy per bit is the slope of the spectral efficiency versus E_b/N_0 curve, i.e., the growth of the achievable rates with the energy per bit [2]. It can be shown that, in the low-power regime, TDMA is suboptimal in terms of this slope [2].

The next section addresses the case of finite T, for which the low-power analysis may not be applicable. For the rest of this paper, we let $N_0(k) = N_0$, $1 \le k \le K$.

4 Minimum-Energy Transmission Scheduling with Time Constraints

Recall that we wish to find a transmission schedule and power assignment so that the network transmission energy is minimized. Theorem 1, which applies to the case of $T = \infty$, shows that TDMA is optimal under arbitrary bit rates, power assignment, receiver noise levels, and channel gains. This section addresses the case of $T < \infty$.

Theorem 1 suggests that TDMA can be nearly optimal among all possible schedules when the time constraint T is finite but sufficiently large. We now derive the TDMA schedule that is optimal among all TDMA schedules for finite T. Theorem 2 below shows that in principle the optimal TDMA schedule can be determined by solving a system of one linear equation and $K-1$ non-linear equations. In general, the solutions can only be approximated by numerical methods, i.e., no closed-form solutions are available.

We show in Theorem 3 that the optimal TDMA schedule can be exactly determined when the channel gains between all source-destination pairs are the same, i.e., $h_{ii} = h_{kk}$, for $1 \le k, i \le K$. We then show in Theorem 4 that TDMA is optimal among all possible schedules for the case of equal channel gains, i.e., $h_{ij} = h$ for $1 \le i, j \le K$.

Theorem 2 Among the TDMA schedules for sources $S_1, S_2, \ldots, S_K$ to transmit $V_t = \sum_{k=1}^{K} V_k$ bits in time T, the TDMA schedule $\{t_1, t_2, \ldots, t_K\}$ minimizes the energy, where t_k, $1 \le k \le K$, are the solutions of the following K equations

$$\sum_{k=1}^{K} t_k = T$$

$$\frac{2^{\frac{V_k}{Wt_k}}}{h_{kk}}\left(1 - \ln 2 \frac{V_k}{Wt_k}\right) - \frac{1}{h_{kk}} = \frac{2^{\frac{V_1}{Wt_1}}}{h_{11}}\left(1 - \ln 2 \frac{V_1}{Wt_1}\right) - \frac{1}{h_{11}}$$

for $2 \le k \le K$. In particular, source S_k transmits with the following power and rate

$$p(k,k) = \frac{WN_0}{h_{kk}}\left(2^{\frac{V_k}{Wt_k}} - 1\right)$$

$$r(k,k) = \frac{V_k}{t_k}$$

The minimum transmission energy for transmitting V_t bits in time $T = \sum_{k=1}^{K} t_k$ is

$$E_t = \sum_{k=1}^{K} \frac{WN_0}{h_{kk}}\left(2^{\frac{V_k}{Wt_k}} - 1\right) t_k$$

Proof The goal is to find the optimal TDMA schedule for minimizing the energy

$$E_t = \sum_{i=1}^{K} p(i,i) t_i \tag{5}$$

subject to the constraints

$$u = T - \sum_{i=1}^{K} t_i \ge 0$$

$$z(i) = r(i,i)t_i - V_i \ge 0, \quad 1 \le i \le K$$

where the rate is given by the Shannon formula

$$r(i,i) = W \log_2\left(1 + \frac{h_{ii}p(i,i)}{WN_0}\right) = \frac{W}{\ln 2} \ln\left(1 + \frac{h_{ii}p(i,i)}{WN_0}\right)$$

The optimization problem with inequality constraints can be solved by the Karush-Kuhn-Tucker (KKT) method [3]. Thus, we form the augmented function

$$L = E_t + \sum_{i=1}^{K} \mu_i z(i) + \mu_{K+1} u$$

where μ_i, $1 \le i \le K+1$, are KKT multipliers.

For $1 \le k \le K$, we have

$$\frac{\partial L}{\partial p(k,k)} = \frac{\partial E_t}{\partial p(k,k)} + \sum_{i=1}^{K} \mu_i \frac{\partial z(i)}{\partial p(k,k)} + \mu_{K+1} \frac{\partial u}{\partial p(k,k)}$$

where

- $\frac{\partial E_t}{\partial p(k,k)} = t_k$
- $\frac{\partial z(k)}{\partial p(k,k)} = \frac{h_{kk}Wt_k}{\ln 2[WN_0 + h_{kk}p(k,k)]}$, $\frac{\partial z(i)}{\partial p(k,k)} = 0$ if $i \ne k$
- $\frac{\partial u}{\partial p(k,k)} = 0$

Thus,

$$\frac{\partial L}{\partial p(k,k)} = t_k + \mu_k \frac{h_{kk}Wt_k}{\ln 2[WN_0 + h_{kk}p(k,k)]}$$

We also have

$$\frac{\partial L}{\partial t_k} = \frac{\partial E_t}{\partial t_k} + \sum_{i=1}^{K} \mu_i \frac{\partial z(i)}{\partial t_k} + \mu_{K+1} \frac{\partial u}{\partial t_k}$$

where

- $\frac{\partial E_t}{\partial t_k} = p(k,k)$
- $\frac{\partial z(k)}{\partial t_k} = \frac{W}{\ln 2} \ln\left(1 + \frac{h_{kk}p(k,k)}{WN_0}\right)$, $\frac{\partial z(i)}{\partial t_k} = 0$ if $i \ne k$
- $\frac{\partial u}{\partial t_k} = -1$

Thus,

$$\frac{\partial L}{\partial t_k} = p(k,k) + \mu_k r(k,k) - \mu_{K+1}$$

The KKT multiplier theorem provides the following necessary conditions [3]: $\mu_i \le 0$, $\frac{\partial L}{\partial p(k,k)} = 0$, $\frac{\partial L}{\partial t_k} = 0$, for $1 \le i \le K+1$ and $1 \le k \le K$, and

$$\sum_{k=1}^{K} \mu_k \left[r(k,k)t_k - V_k\right] + \mu_{K+1}\left(T - \sum_{k=1}^{K} t_k\right) = 0 \tag{6}$$

From $\frac{\partial L}{\partial p(k,k)} = \frac{\partial L}{\partial t_k} = 0$, we have

$$t_k + \mu_k \frac{W}{\ln 2} \frac{h_{kk}t_k}{[WN_0 + h_{kk}p(k,k)]} = 0 \tag{7}$$

$$p(k,k) + \mu_k r(k,k) - \mu_{K+1} = 0 \tag{8}$$

for $1 \le k \le K$.

Because $t_k > 0$, (7) implies that

$$\mu_k = -\frac{\ln 2[WN_0 + h_{kk}p(k,k)]}{h_{kk}W} \tag{9}$$

i.e., $\mu_k < 0$, $1 \le k \le K$.

Suppose that $\mu_{K+1} = 0$. Then (8) and (9) imply that

$$\begin{aligned} p(k,k) &= -\mu_k r(k,k) \\ &= \frac{\ln 2[WN_0 + h_{kk}p(k,k)]}{h_{kk}W} W \log_2\left(1 + \frac{h_{kk}p(k,k)}{WN_0}\right) \end{aligned}$$

which implies that $p(k,k) = 0$, which does not correspond to a feasible solution. Thus, we also have $\mu_{K+1} < 0$. Using (6) with $u \ge 0$, $z(k) \ge 0$, $\mu_k < 0$, $1 \le k \le K$, and $\mu_{K+1} < 0$, we have

$$r(k,k) = \frac{V_k}{t_k} \tag{10}$$

$$\sum_{k=1}^{K} t_k = T \tag{11}$$

From (8) and (9), we then have

$$p(k,k) - \frac{\ln 2}{h_{kk}W}[WN_0 + h_{kk}p(k,k)]\frac{V_k}{t_k} = \mu_{K+1}$$

which can be rewritten as

$$p(k,k)\left(1 - \frac{\ln 2}{W}\frac{V_k}{t_k}\right) - \frac{N_0 \ln 2 V_k}{h_{kk}t_k} = \mu_{K+1}$$

From $r(k,k) = W\log_2\left(1 + \frac{h_{kk}p(k,k)}{WN_0}\right) = \frac{V_k}{t_k}$, we have

$$p(k,k) = \frac{WN_0}{h_{kk}}\left(2^{\frac{V_k}{Wt_k}} - 1\right) \tag{12}$$

We then have

$$\frac{WN_0}{h_{kk}}\left(2^{\frac{V_k}{Wt_k}} - 1\right)\left(1 - \frac{\ln 2}{W}\frac{V_k}{t_k}\right) - \frac{N_0 \ln 2 V_k}{h_{kk}t_k} = \mu_{K+1}$$

which can be simplified to become

$$\frac{2^{\frac{V_k}{Wt_k}}}{h_{kk}}\left(1 - \ln 2\frac{V_k}{Wt_k}\right) - \frac{1}{h_{kk}} = \frac{\mu_{K+1}}{WN_0}$$

Thus, for $2 \le k \le K$,

$$\frac{2^{\frac{V_k}{Wt_k}}}{h_{kk}}\left(1 - \ln 2\frac{V_k}{Wt_k}\right) - \frac{1}{h_{kk}} = \frac{2^{\frac{V_1}{Wt_1}}}{h_{11}}\left(1 - \ln 2\frac{V_1}{Wt_1}\right) - \frac{1}{h_{11}} \tag{13}$$

Note that (11) and (13) form a system of K equations with K unknowns t_k, $1 \le k \le K$.

For $1 \le k \le K$ and $t \in (0,T)$, define

$$f_k(t) = \frac{2^{\frac{V_k}{Wt_k}}}{h_{kk}}\left(1 - \ln 2\frac{V_k}{Wt_k}\right) - \frac{1}{h_{kk}}$$

It can then be shown that f_k is continuous and strictly increasing, i.e., f_k is bijective. Thus, the solution $(t_1, t_2, \ldots, t_K)$ obtained from (11) and (13) is unique, which implies that $(t_1, t_2, \ldots, t_K)$ is optimal. After t_k, $1 \le k \le K$, are determined, the power levels, rates, and total energy are determined from (12), (10), and (5). □

Example 1 Consider a network of two links, i.e., $K = 2$. Further, assume that $T = 1$, $W = 10^6$, $V_1 = 10^7$, $V_2 = 10^8$, $h_{11} = 0.01$, $h_{22} = 0.09$. Using Theorem 2, it can then be shown by numerical calculation that $t_1 \approx 0.09331$ and $t_2 = T - t_1 \approx 0.90669$. Note that $V_t = V_1 + V_2 = 1.1 \times 10^8$. Thus, $t_1 \approx V_1T/V_t = 1/11 = 0.09091$, and $t_2 \approx V_2T/V_t = 1/1.1 = 0.90909$. In this example we then have

$$t_k \approx \frac{V_k}{V_t}T \tag{14}$$

We will show later that approximation (14) becomes exact when $h_{11} = h_{22}$. □

We next assume that $h_{kk} = h$, which is the case, for example, when the source and destination in each pair are separated by (exactly or approximately) the same distance. We then show that the transmission time, power, rate, and energy can be expressed in closed mathematical formulas.

Theorem 3 Assume that $h_{kk} = h$, $1 \le k \le K$. Among the TDMA schedules for sources $S_1, S_2, \ldots, S_K$ to transmit $V_t = \sum_{k=1}^{K} V_k$ bits in time T, the TDMA schedule $\{t_1, t_2, \ldots, t_K\}$ minimizes the energy, where $t_k = \frac{V_k}{V_t}T$, $1 \le k \le K$. In particular, source S_k transmits with power $p(k,k) = \frac{WN_0}{h}\left(2^{\frac{V_t}{WT}} - 1\right)$ and rate $r(k,k) = \frac{V_t}{T}$. The minimum transmission energy for transmitting V_t bits in time T is

$$E_t = \frac{TWN_0}{h}\left(2^{\frac{V_t}{WT}} - 1\right)$$

Proof From Theorem 2, the optimal TDMA schedule is $\{t_1, t_2, \ldots, t_K\}$, where t_k, $1 \le k \le K$, are the solutions of the following K equations

$$\sum_{k=1}^{K} t_k = T$$

$$\frac{2^{\frac{V_k}{Wt_k}}}{h_{kk}}\left(1 - \ln 2\frac{V_k}{Wt_k}\right) - \frac{1}{h_{kk}} = \frac{2^{\frac{V_i}{Wt_i}}}{h_{ii}}\left(1 - \ln 2\frac{V_i}{Wt_i}\right) - \frac{1}{h_{ii}}$$

for $1 \le k, i \le K$, $k \ne i$. For the case of $h_{kk} = h$, we have

$$2^{\frac{V_k}{Wt_k}}\left(1 - \ln 2\frac{V_k}{Wt_k}\right) = 2^{\frac{V_i}{Wt_i}}\left(1 - \ln 2\frac{V_i}{Wt_i}\right)$$

which implies that $\frac{V_k}{t_k} = \frac{V_i}{t_i}$, for $1 \le i,k \le K$. From $t_k = \frac{t_i}{V_i}V_k$, $1 \le i,k \le K$, and $\sum_{k=1}^{K} t_k = T$, we have $\frac{t_i}{V_i}\sum_{k=1}^{K} V_k = T$. Thus, $t_i = \frac{V_i}{\sum_{k=1}^{K} V_k}T = \frac{V_i}{V_t}T$, $1 \le i \le K$. We then have

$$r(k,k) = \frac{V_k}{t_k} = \frac{V_t}{T}$$

$$p(k,k) = \frac{WN_0}{h}\left(2^{\frac{V_k}{Wt_k}} - 1\right) = \frac{WN_0}{h}\left(2^{\frac{V_t}{WT}} - 1\right)$$

$$\begin{aligned} E_t &= \sum_{k=1}^{K} p(k,k)t_k \\ &= \frac{WN_0}{h}\left(2^{\frac{V_t}{WT}} - 1\right)\sum_{k=1}^{K} t_k \\ &= \frac{WN_0}{h}\left(2^{\frac{V_t}{WT}} - 1\right)T \end{aligned}$$

□

Let $E_t(T)$ be the network transmission energy under time constraint T. Assume that $h_{kk} = h$, $1 \le k \le K$. From Theorems 1 and 3, we have $E_t(\infty) = N_0 \ln 2 \sum_{k=1}^{K} \frac{V_k}{h}$ and $E_t(T) = \frac{TWN_0}{h}\left(2^{\frac{V_t}{WT}} - 1\right)$ for $T < \infty$. To compare the energy $E_t(T)$ under time constraint T with the minimum energy $E_t(\infty)$, we consider the ratio

$$\frac{E_t(\infty)}{E_t(T)} = \frac{V_t \ln 2}{WT\left(2^{\frac{V_t}{WT}} - 1\right)} \tag{15}$$

Fig. 2 shows (15) as function of WT for $V_t \in \{10^9, 10^{10}\}$. For a given value of channel bandwidth $W > 0$, $E_t(T)$ decreases as T increases, and approaches the minimum energy $E_t(\infty)$ as $T \to \infty$. As expected, the case of $V_t = 10^9$ approaches the limiting value faster than the case of $V_t = 10^{10}$.

The next result shows that TDMA is optimal when the channel gains are equal, i.e., non-TDMA scheduling can not consume less energy than TDMA.

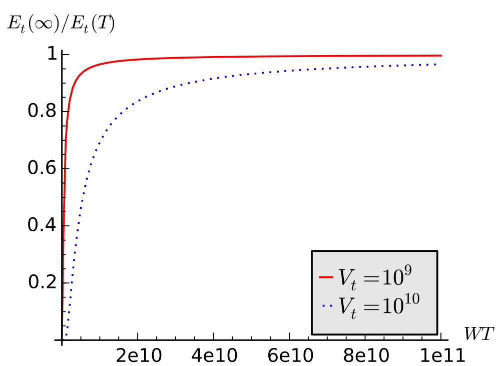

Fig. 2 Evolution of transmission energy as function of WT, where T is the transmission time constraint, W is the channel bandwidth, and V_t is the total bit volume to be delivered to all destinations.

Theorem 4 Assume that $h_{ij} = h$, $1 \le i, j \le K$. Minimum-energy transmission, subject to constraints on finite time and bit volume delivery, can be obtained by TDMA scheduling.

Proof Consider any non-TDMA transmission schedule $\{t_g > 0 : g \in G\}$. Let g be any non-singleton group, i.e., $|g| > 1$. From $h_{ij} = h$, $1 \le i, j \le K$, it can be shown that

$$\sum_{i \in g} r(i, g) \le W \log_2\left(1 + \frac{\sum_{i \in g} p(i, g)}{b}\right)$$

where $b = \frac{WN_0}{h}$ (see [6, 8]).

Let v_i be the bit volume transmitted by source S_i during the time t_g, i.e., $r(i, g) = \frac{v_i}{t_g}$. We then have

$$\frac{1}{t_g}\sum_{i \in g} v_i \le W \log_2\left(1 + \frac{\sum_{i \in g} p(i, g)}{b}\right)$$

which can be shown to be equivalent to

$$\sum_{i \in g} p(i, g) \ge b\left(2^{\frac{1}{Wt_g}\sum_{i \in g} v_i} - 1\right)$$

Now consider the optimal TDMA schedule for the same sources in group g, in which source S_i transmits v_i bits, $i \in g$. Here we require that $\sum_{i \in g} v_i$ bits be transmitted within the time constraint t_g. Using Theorem 3, the total power for the TDMA schedule is

$$\begin{aligned} P_{\text{TDMA}} &= \frac{WN_0}{h}\left(2^{\frac{1}{Wt_g}\sum_{i \in g} v_i} - 1\right) \\ &= b\left(2^{\frac{1}{Wt_g}\sum_{i \in g} v_i} - 1\right) \\ &\le \sum_{i \in g} p(i, g) = P_g \end{aligned}$$

We then have $P_{\text{TDMA}} t_g \le P_g t_g$, i.e., the energy used by the TDMA schedule (for the same group g to transmit $\sum_{i \in g} v_i$ bits within the time constraint t_g) does not exceed the energy used the non-TDMA schedule. Thus, any schedule that includes a non-singleton group can not outperform TDMA, i.e., TDMA is optimal. □

5 Summary

This paper addresses the problem of emptying a wireless network of a prespecified bit volume, with the goal of minimizing transmission energy. Our approach is the joint optimization of transmission scheduling and power control, while addressing both infinite and finite time horizons. As shown in the paper, TDMA scheduling with power control can be effective for saving energy, especially when the transmission time is sufficiently large. In particular, TDMA is asymptotically optimal, when the transmission time constraint approaches infinity. When the time constraint is strictly bounded, TDMA is also optimal for the case of equal channel gains.

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