Experimental Comparison of Linear Prediction Schemes for Multi-User MIMO Systems

Toshiki Tanaka*, Hidekazu Murata*, Susumu Yoshida*, Koji Yamamoto*,
Daisuke Umehara†, Satoshi Denno‡, Masahiro Morikura*
*Graduate School of Informatics, Kyoto University, Yoshida-hommachi, Sakyo-ku, Kyoto 606-8501, Japan
Email: contact-h25e@hanase.kuee.kyoto-u.ac.jp
†Graduate School of Science and Technology, Kyoto Institute of Technology, Matsugasaki, Sakyo-ku, Kyoto 606-8585, Japan
‡Graduate School of Natural Science and Technology, Okayama University
3-1-1, Tsushima-Naka, Kita-ku, Okayama 700-8530, Japan

***Abstract*—In a multi-user multiple-input multiple-output (MU-MIMO) system, zero-forcing precoding can completely eliminates inter-user interference when the perfect channel state information is available at the base station. However, in time-varying channels, the transmission performance of MU-MIMO systems is severely degraded due to inter-user interference. In this paper, channel prediction is employed for channel tracking, and in-lab experiments are carried out to confirm its performance. The bit error rate (BER) performance is measured and compared with the computer simulation results. Both in-lab experiments and the computer simulations show that channel prediction improves the BER performance.**

I. Introduction

Multiple-input multiple-output (MIMO) techniques are employed in the current and future wireless communication systems. This technique utilizes multiple antennas at a base station (BS) and a mobile station (MS). Multiple antennas can provide spatial diversity and spatial multiplexing effects.

Multi-user MIMO (MU-MIMO) techniques have been studied because it can achieve capacity enhancement even for a mobile station having few antennas [1]. In MU-MIMO systems, the computational complexity required for mobile stations can be reduced with precoding at the BS. For precoding, downlink channel state information (CSI) is required. Downlink channel estimation methods have been studied in [2] [3]. In this paper, a simple two-way channel estimation method [4] [5] is used. Measurement experiments of MU-MIMO systems have been reported [6] [7] [8].

However, time difference between channel estimation and precoding results in performance degradation in time-varying channels. Channel prediction is employed to compensate for this degradation [9] [10]. One of channel prediction techniques is linear prediction. Performance evaluation of MU-MIMO systems with linear extrapolation using measurement data have been reported [11].

In this paper, the system performance of a MU-MIMO system with the linear prediction is evaluated in term of bit error rate (BER). The sample matrix inversion (SMI) algorithm and the recursive least squares (RLS) algorithm are employed in order to calculate the tap weight vector of linear predictor. In-lab experiments are conducted using a fading emulator.

II. System model

We considered a MU-MIMO transmission system. In this paper, we assume a BS is equipped with M antennas and N MSs are each equipped with a single antenna. In MU-MIMO systems, the received signal $\boldsymbol{y}(t) \in \mathbb{C}^{N\times 1}$ at time t is given with the transmit signal $\boldsymbol{m}(t) \in \mathbb{C}^{M\times 1}$ at time t by

$$\boldsymbol{y}(t) = \boldsymbol{H}(t)\boldsymbol{m}(t) + \boldsymbol{n}(t), \tag{1}$$

where $\boldsymbol{H}(t) \in \mathbb{C}^{N\times M}$ is the channel matrix at time t. One way of dealing with inter-user interference is ZF precoding. Psuedoinverse of $\boldsymbol{H}(t)$ with $\boldsymbol{m}(t)$ at a transmitter results in $\boldsymbol{y}(t) = \boldsymbol{m}(t) + \boldsymbol{n}(t)$ at the receiver. When $\boldsymbol{H}(t)$ is known at the transmitter, ZF precoding imposes the constraint that all interference terms be zero.

The problem is the difference between the estimated channel matrix $\boldsymbol{H}(n)$ and the $\boldsymbol{H}(n+\tau)$, where τ is the time difference between estimation and transmission. In this paper, $\hat{\boldsymbol{H}}(n+1)$ is obtained by the linear prediction, and then the channel matrix $\boldsymbol{H}(n+\tau)$ for precoding is calculated based on $\boldsymbol{H}(n-1)$, $\boldsymbol{H}(n)$, and $\hat{\boldsymbol{H}}(n+1)$.

A. Channel Estimation

In the MU-MIMO transmission system under consideration, the BS is able to find the knowledge of the downlink channel based on uplink channel coefficients and round-trip channel coefficients by the two-way estimation method (echo-MIMO) [4] [5]. This method can reduce the required computational complexity of the MS because the signal processing for channel estimation is performed at the BS. To know both uplink and round-trip channel coefficients, the BS sends the training signal, and then MSs repeat and send its received signal back to the BS [4] described in Fig. 1.

The BS equipped with M antennas sends a round-trip training signal to N MSs. A round-trip training signal $\boldsymbol{y}$ at MSs is given by

$$\boldsymbol{y} \quad = \quad \boldsymbol{H}_{\mathrm{down}}\boldsymbol{m} + \boldsymbol{n}_{\mathrm{MS}}, \tag{2}$$

where $\boldsymbol{m}$ is a round-trip training signal at the BS. Training signals transmitted from antennas are orthogonal. MSs send

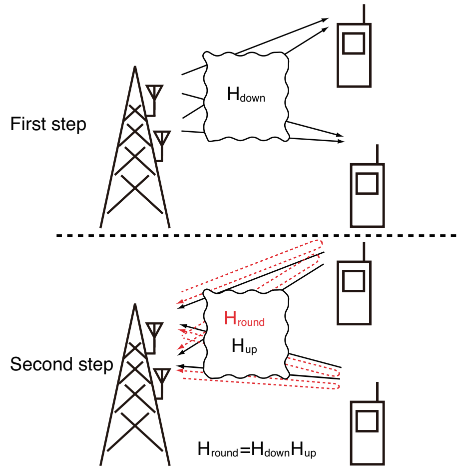

Fig. 1. The figure shows two-way channel estimation. First, a BS sends a training signal for round-trip to MSs. Second, MSs send back the received signal and a training signal to the BS. The BS receives signals from MSs. We can estimate $\boldsymbol{H}_{\text{round}}$ by a signal for round-trip and $\boldsymbol{H}_{\text{up}}$ by a training signal from MSs.

the received signal $\boldsymbol{y}$ together with a training signal $\boldsymbol{m}$ back to the BS. Then the BS receives signals $\hat{\boldsymbol{y}}$ and $\boldsymbol{y}'$ from MSs given by

$$\begin{aligned} \hat{\boldsymbol{y}} &= \boldsymbol{H}_{\text{up}}\boldsymbol{y} + \boldsymbol{n}_{\text{BS}} \\ &= \boldsymbol{H}_{\text{round}}\boldsymbol{m} + \hat{\boldsymbol{n}} \quad (3) \\ \boldsymbol{y}' &= \boldsymbol{H}_{\text{up}}\boldsymbol{m} + \boldsymbol{n}_{\text{BS}}, \quad (4) \end{aligned}$$

where $\boldsymbol{H}_{\text{round}} = \boldsymbol{H}_{\text{up}}\boldsymbol{H}_{\text{down}}$ as round-trip channel matrix and $\hat{\boldsymbol{n}} = \boldsymbol{H}_{\text{up}}\boldsymbol{n}_{\text{MS}} + \boldsymbol{n}_{\text{BS}}$ as round-trip noise. At the BS, we use the linear least squares channel estimation method [4][12] for both $\boldsymbol{H}_{\text{up}}$ and $\boldsymbol{H}_{\text{round}}$ given by

$$\boldsymbol{H}'_{\text{up}} = \mathrm{E}[\boldsymbol{y}'\boldsymbol{m}^{\dagger}] \quad (5)$$

$$\boldsymbol{H}'_{\text{round}} = \mathrm{E}[\hat{\boldsymbol{y}}\boldsymbol{m}^{\dagger}], \quad (6)$$

where $[\cdot]^{\dagger}$ denotes pseudo inverse. Then $\boldsymbol{H}'_{\text{down}}$ is given by

$$\boldsymbol{H}'_{\text{down}} = \boldsymbol{H}'^{\dagger}_{\text{up}}\boldsymbol{H}'_{\text{round}}. \quad (7)$$

III. Linear Prediction and Application for ZF Precoding

In this paper, we use SMI algorithm and RLS algorithm for the linear prediction. The linear prediction is based on a k tap transversal filter shown in Fig. 2. Let the tap-input vector $\boldsymbol{x}(i) \in \mathbb{C}^{k\times 1}$ and the tap weight vector $\boldsymbol{w}(n) \in \mathbb{C}^{k\times 1}$. The predictor output $x(n+1)$ is given by

$$x(n+1) = \boldsymbol{w}^{\mathrm{H}}(n)\boldsymbol{x}(n). \quad (8)$$

A. Sample Matrix Inversion

SMI algorithm is based on the Wiener solution. Cost function $L(n)$ at time n is given by

$$L(n) = \sum_{i=n-m}^{n} |e(i)|^2, \quad (9)$$

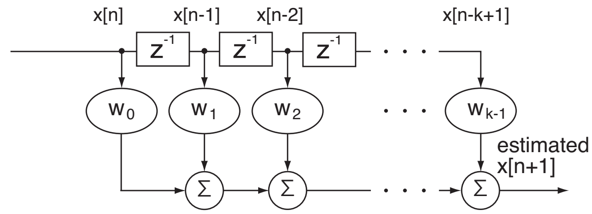

Fig. 2. Transversal filter.

where $e(i)$ is the difference between the desired response $d(i)$ and the predictor output $y(i)$ and m is the number of iterations for correlation estimation. The error signal $e(i)$ is given by

$$\begin{aligned} e(i) &= d(i) - y(i) \\ &= d(i) - \boldsymbol{w}^{\mathrm{H}}(n)\boldsymbol{x}(i), \quad (10) \end{aligned}$$

where $y(i)$ is produced by $\boldsymbol{x}(i)$ and $\boldsymbol{w}(n)$, for which the cost function $L(n)$ attains its minimum value. If $\boldsymbol{R}(n) \in \mathbb{C}^{k\times k}$ is invertible, $\boldsymbol{w}(n)$ is defined by

$$\boldsymbol{w}(n) = \boldsymbol{R}^{-1}(n)\boldsymbol{r}(n), \quad (11)$$

where the correlation matrix $\boldsymbol{R}(n)$ and the cross-correlation vector $\boldsymbol{r}(n) \in \mathbb{C}^{k\times 1}$ are given by

$$\begin{aligned} \boldsymbol{R}(n) &= \sum_{i=n-m}^{n} \boldsymbol{x}(i)\boldsymbol{x}^{\mathrm{H}}(i) \\ \boldsymbol{r}(n) &= \sum_{i=n-m}^{n} \boldsymbol{x}(i)x^*(i+1), \quad (12) \end{aligned}$$

where $[\cdot]^*$ denotes complex conjugation.

B. Recursive Least Squares

RLS algorithm is the method of least squares to develop the recursive algorithm. It introduces a forgetting factor into the definition of the cost function $\hat{L}(n)$ given by

$$\hat{L}(n) = \sum_{i=1}^{n} \lambda^{n-i}|e(i)|^2, \quad (13)$$

where λ is the forgetting factor. If $\hat{\boldsymbol{R}}(n) \in \mathbb{C}^{k\times k}$ is invertible, the tap weight vector $\hat{\boldsymbol{w}}(n) \in \mathbb{C}^{k\times 1}$ is defined by

$$\hat{\boldsymbol{w}}(n) = \hat{\boldsymbol{R}}^{-1}(n)\hat{\boldsymbol{r}}(n), \quad (14)$$

where the correlation matrix $\hat{\boldsymbol{R}}(n)$ and the cross-correlation vector $\hat{\boldsymbol{r}}(n) \in \mathbb{C}^{k\times 1}$ are given by

$$\begin{aligned} \hat{\boldsymbol{R}}(n) &= \sum_{i=1}^{n} \lambda^{n-i}\boldsymbol{x}(i)\boldsymbol{x}^{\mathrm{H}}(i) \\ &= \lambda\hat{\boldsymbol{R}}(n-1) + \boldsymbol{x}(n)\boldsymbol{x}^{\mathrm{H}}(n) \quad (n = 1, 2, \ldots) \quad (15) \\ \hat{\boldsymbol{r}}(n) &= \sum_{i=1}^{n} \lambda^{n-i}\boldsymbol{x}(i)x^*(i+1) \\ &= \lambda\hat{\boldsymbol{r}}(n-1) + \boldsymbol{x}(n)x^*(n+1) \quad (n = 1, 2, \ldots) (16) \end{aligned}$$

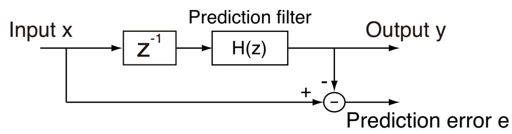

Fig. 3. Transfer function model of linear prediction filter.

The initial value of $\hat{\boldsymbol{R}}(n)$ and $\hat{\boldsymbol{r}}(n)$ are

$$\begin{aligned}\hat{\boldsymbol{R}}(0) &= \delta \boldsymbol{I}\\ \hat{\boldsymbol{r}}(0) &= \mathbf{0},\end{aligned} \tag{17}$$

where δ is a small positive value. In (15), the following equation is held [13]:

$$\hat{\boldsymbol{R}}^{-1}(n) = \lambda^{-1}[\hat{\boldsymbol{R}}^{-1}(n-1) - \boldsymbol{k}(n)\boldsymbol{x}^{\mathrm{H}}(n)\hat{\boldsymbol{R}}^{-1}(n-1)], \tag{18}$$

where the gain vector $\boldsymbol{k}(n) \in \mathbb{C}^{k\times 1}$ is given by

$$\boldsymbol{k}(n) = \frac{\lambda^{-1}\hat{\boldsymbol{R}}^{-1}(n-1)\boldsymbol{x}(n)}{1+\lambda^{-1}\boldsymbol{x}^{\mathrm{H}}(n)\hat{\boldsymbol{R}}^{-1}(n-1)\boldsymbol{x}(n)}. \tag{19}$$

By rearranging (18),

$$\begin{aligned}\boldsymbol{k}(n) &= [\lambda^{-1}\hat{\boldsymbol{R}}^{-1}(n-1) - \lambda^{-1}\boldsymbol{k}(n)\boldsymbol{x}^{\mathrm{H}}(n)\\ &\quad \hat{\boldsymbol{R}}^{-1}(n-1)]\boldsymbol{x}(n)\\ &= \hat{\boldsymbol{R}}^{-1}(n)\boldsymbol{x}(n).\end{aligned} \tag{20}$$

From (15), (18) and (20), we have

$$\begin{aligned}\boldsymbol{w}(n) &= \hat{\boldsymbol{R}}^{-1}(n)\boldsymbol{r}(n)\\ &= \hat{\boldsymbol{R}}^{-1}(n)[\lambda\boldsymbol{r}(n-1) + \boldsymbol{x}(n)x^*(n+1)]\\ &= \boldsymbol{w}(n-1) - \boldsymbol{k}(n)\boldsymbol{x}^{\mathrm{H}}(n)\boldsymbol{w}(n-1)\\ &+ \boldsymbol{k}(n)x^*(n+1).\end{aligned} \tag{21}$$

The update of RLS algorithm is given by (18), (19), (21). Then, we can predict the future value $\hat{x}(n+1)$.

C. Threshold technique

In SMI or RLS algorithm, the estimated correlation matrix becomes imperfect when channel variation is too slow or the number of samples for correlation matrix estimation is small. In this condition, the tap weight vector $\boldsymbol{w}(n) = [w_0, w_1, \ldots, w_{k-1}]^{\mathrm{T}}$ diverges, which results in severe performance degradation. Thus, It is necessary to avoid the unexpected value of the tap weight vector. Introducing a threshold for the tap weight vector coefficient is a simple way to avoid this performance degradation. When the absolute value of w_{k-1} exceeds 1, it is considered that the linear prediction does not work properly and should not be used at time n. Fig. 3 shows a transfer function model of linear

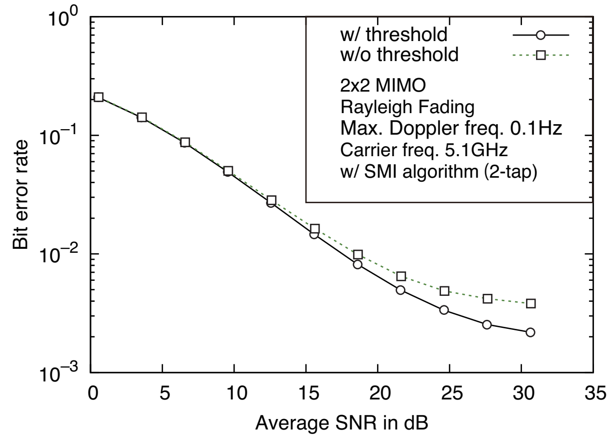

Fig. 4. BER performance curves for 2×2 MU-MIMO with channel prediction in computer simulations. Introducing a threshold of tap weight vector results in that BER performance is improved especially in high SNR region.

prediction. In Fig. 3, prediction error $e(z)$ is given by

$$\begin{aligned}e(z) &= x(z)(1-(z^{-1}H(z))\\ e(z)/x(z) &= 1-(z^{-1}(\sum_{i=0}^{k-1} w_i z^{-i}))\\ &= 1-\sum_{i=0}^{k-1} w_i z^{-i-1}.\end{aligned} \tag{22}$$

The absolute value of w_{k-1} should be kept less than 1. Otherwise, linear prediction does not work because a zero of e is outside unit-circle and this filter works for a zero of e. It is introduced that this system is only stable when absolute value of w_{k-1} is kept less than 1. Fig. 4 shows the impact of the threshold on the BER performance of 2×2 MU-MIMO with linear prediction by computer simulations.

D. Channel prediction for precoding

In time-varying channels, time difference τ between channel estimation and precoding at the BS causes performance degradation in precoding. In this paper, the channel matrix $\boldsymbol{H}(n+\tau)$ is estimated with $\boldsymbol{H}(n-1)$, $\boldsymbol{H}(n)$ and $\hat{\boldsymbol{H}}(n+1)$ which is obtained by the linear prediction. $\hat{\boldsymbol{H}}(n+\tau)$ is given using second-order interpolation by

$$\begin{aligned}\hat{\boldsymbol{H}}(n+\tau) &= \frac{\tau(\tau-1)}{2}\boldsymbol{H}(n-1) - (\tau+1)(\tau-1)\\ &\quad \boldsymbol{H}(n) - \frac{\tau(\tau+1)}{2}\hat{\boldsymbol{H}}(n+1).\end{aligned} \tag{23}$$

IV. In-lab Experiments

The MU-MIMO transmission performance measurements were conducted using a fading emulator. In this paper, a BS equipped with $M = 2$ antennas and $N = 2$ MSs each equipped one antenna are considered. The fading emulator emulates eight channels and these channels are used as a 2×2 MU-MIMO channel. These channels are i.i.d. Rayleigh fading channels and direction-of-arrival distributions of these

Fig. 5. BS equipment.

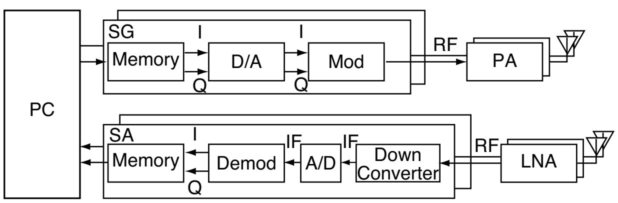

Fig. 6. Block diagram of the BS equipment.

channels are uniform. In experiments, emulated channels are used for both uplink and downlink.

A. Experimental system

1) BS equipment: Fig. 5 shows the BS equipment. Two modular RF signal generators (SGs), two modular RF signal analyzers (SAs), an FPGA board, and a modular PC are contained in the BS equipment. In two SGs or two SAs, the same local oscillator is used. Fig. 6 shows the block diagram of the BS equipment. An SG consists of a 16-bit DAC and a vector modulator. An SA consists of a downconverter and a 16-bit ADC. In SA, the received signal is translated to low IF by the downconverter, and then digitized by the ADC. The digital low IF signal is demodulated and recorded in the onboard memory. For the local oscillator, a 10 MHz reference is used.

2) MSs equipment: Fig. 7 shows MSs equipment. the universal software radio peripheral (USRP) is used to each MS equipment. Fig. 8 shows the block diagram of the MS equipment. It consists of a motherboard and a daughterboard. A motherboard performs A/D and D/A conversion and (de)modulation between baseband and low IF. A daughterboard contains RF front end which performs frequency translation between RF and low IF. Signal processing is done on an external PC through the Gigabit Ethernet interface. The

Fig. 7. MSs equipment (USRP) .

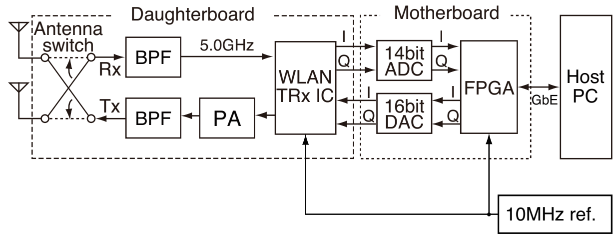

Fig. 8. Block diagram of a MS equipment.

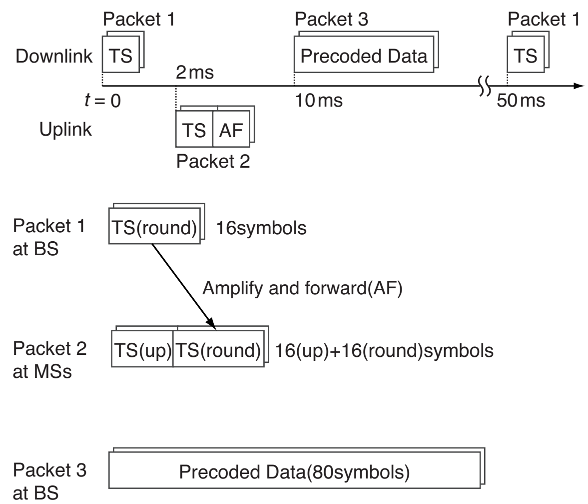

Fig. 9. Packet structure.

same 10 MHz reference as the BS equipment is used and the synchronization between the BS and MSs is established.

3) Packet structure: Fig. 9 shows the packet structure. A frame is constructed by three packets. First, a BS transmits training sequences (TSs) for the round-trip channel at the time $t = 0$. The training sequences transmitted from the BS are orthogonal and transmitted at the same time and frequency. Each MS sends back the received round-trip TS by amplify-and-forward relaying with another TS for uplink channel estimation at $t = 2\,\mathrm{ms}$. Both of the two TSs have 16 symbols. We can estimate round-trip channel matrix $\boldsymbol{H}'_{\mathrm{round}}$ and uplink channel matrix $\boldsymbol{H}'_{\mathrm{up}}$ at the BS. The BS estimates

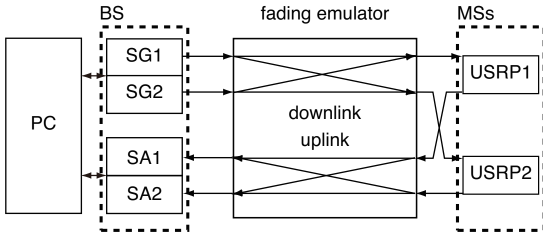

Fig. 10. Experimental setup for in-lab experiments. BS and MSs are connected through the fading emulator.

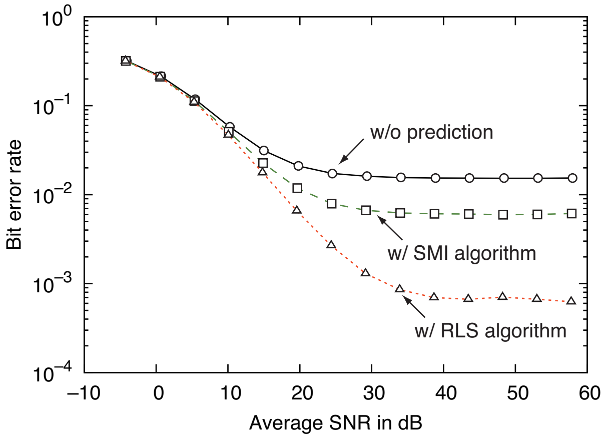

Fig. 11. BER performance curves for 2×2 MU-MIMO with channel prediction in computer simulations.

downlink channel $\boldsymbol{H}'_{\mathrm{down}}$ at $t = 0$ given by (7). A BS predicts downlink channel at $t = 10\,\mathrm{ms}$ using downlink channel state information at previous frames, and transmits 80-symbol-long precoded data at $t = 10\,\mathrm{ms}$. The frame duration is $50\,\mathrm{ms}$ in order to ensure that the signal processing and data recording are finished on time in an external Windows PC.

4) Experimental setup: Fig. 10 shows the experimental setup. The transmit power of TSs is kept constant in the experiments. The peak power of data packets is high because of precoding and the linear prediction. The transmit power is limited by the BS equipment and the transmit power of precoded data packets is normalized by the peak of the transmit power. The number of prediction taps is two because of the computational complexity and slow convergence especially with SMI algorithm. The number of iterations for correlation estimation is 500 for SMI algorithm and forgetting factor λ is 0.995 for RLS algorithm.

B. Experimental results

In this section, we demonstrate the impact of channel prediction on the BER performance of 2×2 MU-MIMO system. Fig. 11 shows the BER performance of computer simulations. Fig. 12 shows the BER performance of in-lab experiments. Table I shows the major experimental parameters. In computer simulations, we consider Rayleigh fading channels based on the Jakes model. The transmit power of TSs in packet 1 and 2 is kept high and constant between in-lab experiments and

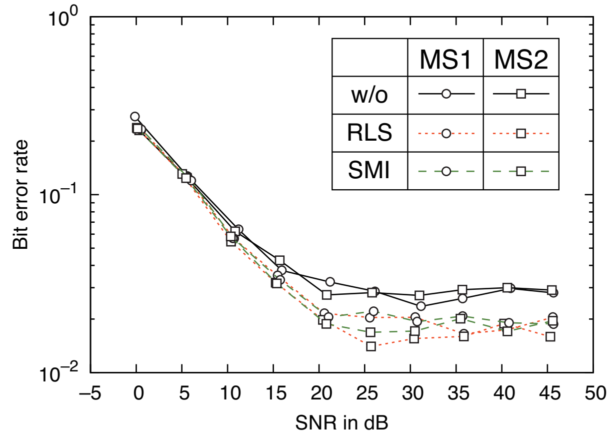

Fig. 12. BER performance curves for 2×2 MU-MIMO with channel prediction in experimental results.

computer simulations. In experiments, the transmit power of TSs is -15 dBm and the AF gain of the training signal at MSs for the round-trip channel is $55\,\mathrm{dB}$. The channel prediction technique enables the BER performance improvement both in in-lab experiments and computer simulations. With the RLS prediction method, the BER performance is greatly improved in computer simulations. In in-lab experiments, there is no significant difference between the BER performance of RLS prediction and that of SMI prediction. The reason is now under investigation, but the phase noise unique to in-lab experiments is the most possible reason. Also, difficulty in estimating the correlation matrix may be due to the phase noise.

V. Conclusion

By using the MU-MIMO experimental system, we evaluated the effects of channel prediction. The in-lab transmission experiments were carried out using a fading emulator. In time-varying channels, channel prediction improves the system performance using both SMI algorithm and RLS algorithm.

Acknowledgment

This work was supported by Strategic Information and Communications R&D Promotion Programme (SCOPE) of the Ministry of Internal Affairs and Communications, Japan.

References

[1] Q. Spencer, C. Peel, A. Swindlehurst, and M. Haardt, "An introduction to the multi-user MIMO downlink," *IEEE Communications Magazine*, vol. 42, no. 10, pp. 60–67, Oct. 2004.

[2] W. J. Hwang, J.-H. Jang, and H.-J. Choi, "An enhanced channel estimation with partial interference cancelation for MU-MIMO system," *Communications Letters, IEEE*, vol. 16, no. 8, pp. 1232–1235, 2012.

[3] B. Zhou, L. Jiang, C. He, L. Zhang, S. Zhao, Z. Yi, and L. Lin, "On ber of tdd multiuser MIMO system with channel estimation error and delay," in *Proc. IEEE VTC 2011-Spring*, May 2011.

[4] L. P. Withers, R. M. Taylor, and D. M. Warme, "Echo-MIMO: A two-way channel training method for matched cooperative beamforming," *IEEE Trans. Sig. Process.*, vol. 56, no. 9, pp. 4419–4432, Sept. 2008.

[5] R. Osawa, H. Murata, K. Yamamoto, and S. Yoshida, "Performance of two-way channel estimation technique for multi-user distributed antenna systems with spatial precoding," in *Proc. IEEE VTC 2009-Fall*, Sept. 2009.

TABLE I
EXPERIMENTAL PARAMETERS.

Parameters	Values
Carrier frequency	5.11 GHz
Number of BS antennas	2
Number of MSs	2
Frame duration	50 ms
Symbol rate	312.5k symbols/s
Modulation	QPSK
Filter	Root roll-off Nyquist ($\alpha = 0.4$)
Emulated channel model	Rayleigh
Maximum Doppler frequency	5 Hz
Number of prediction taps	2
BS parameters	**Values**
ADC/DAC resolution	16 bit
CPU of embedded PC	Core i7 1.73 GHz
Downlink channel estimation	Echo-MIMO
Interpolation method	Lagrange (second-order)
Precoding	Linear (ZF)
Transmit power of TSs	-15 dBm
MS parameters	**Values**
Model	Ettus USRP N210
Daughterboard	XCVR2450
ADC/DAC resolution	14/16 bit

[6] M. Taniguchi, H. Murata, S. Yoshida, K. Yamamoto, D. Umehara, S. Denno, and M. Morikura, “Field experiments of linearly precoded multi-user MIMO system at 5GHz band,” in *Proc. IEEE VTC 2012-Fall*, Sept. 2012.

[7] F. Kaltenberger, D. Gesbert, R. Knopp, and M. Kountouris, “Correlation and capacity of measured multi-user MIMO channels,” in *Proc. IEEE PIMRC 2008*, Sept. 2008.

[8] Y. Kakishima, T. Kawamura, Y. Kishiyama, H. Taoka, and H. Andoh, “Indoor experiments on 4-by-2 multi-user MIMO employing various transmitter antenna arrangements in LTE-advanced downlink,” in *Proc. IEEE VTC 2012-Fall*, Sept. 2012.

[9] K. Kobayashi, T. Ohtsuki, and T. Kaneko, “Mimo systems in the presence of feedback delay,” in *Proc. IEEE ICC 2006*, vol. 9, Jun. 2006.

[10] T. Svantesson and A. Swindlehurst, “A performance bound for prediction of MIMO channels,” *IEEE Trans. Sig. Process.*, vol. 54, no. 2, pp. 520–529, Feb. 2006.

[11] H. P. Bui, Y. Ogawa, T. Nishimura, and T. Ohgane, “Performance evaluation of a multi-user MIMO system with prediction of time-varying indoor channels,” *IEEE Trans. Antennas Propag.*, pp. 371–379, Jan. 2013.

[12] J. Nair and R. V. R. Kumar, “An optimal superimposed training sequence for channel estimation in OFDM systems,” in *Proc. IEEE PIMRC 2008*, Sept. 2008.

[13] S. Haykin, *ADAPTIVE FILTER THEORY*, Pretice Hall, 1991.