Stability Analysis for Energy Harvesting Sources over Time Varying Wireless Channels with Relays

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***Abstract*—In this paper, we propose and analyze transmission strategies for a source node with energy harvesting capability. The source node is able to either transmit directly to the destination or transmit with the help of a relay node through a network-level cooperation protocol. The relay node also has energy harvesting capability. The channels between different nodes are time varying. The packets arrival to the source and the energy arrival to the source and the relay are modeled by discrete-time stochastic processes. We derive the maximum stable throughput rate of the source for different transmission strategies in closed-form. The proposed strategies exploit the channel state information (CSI). We derive the stability conditions also in the case of imperfect channel measurements. Finally, we propose and analyze a modified relaying strategy which enhances the performance when the relay has a low energy arrival rate.**

I. Introduction

Energy harvesting enables wireless nodes to be recharged by the surrounding environment. Energy harvesting in wireless networks has been enabled because of the developments in hardware design. Examples of energy harvesting techniques can be found in [1], [2], [3]. Also, examples of energy harvesting wireless networks are sensor networks [4] and wireless cellular networks with devices which need to operate for long time [5]. In our work, we deal with the stochastic nature of the energy harvesting process without considering the harvesting technique.

When dealing with nodes powered by non-rechargeable batteries, the common objectives are short term such as maximizing the lifetime of the network [6], [7]. The harvesting capability enables us to consider different performance measures such as the throughput and the stability of the network [8].

On the other hand, cooperative diversity enables single antenna users to benefit from the spatial diversity by delivering data with the help of relay nodes. Numerous works have been done to analyze cooperative diversity at the physical layer based on information theoretic considerations [9], [10]. It has also been shown that cooperation can be applied at the network layer. In [11], a network-level cooperation protocol has been used to increase the stable throughput region for the uplink of a wireless network. Also in [12], a network level cooperation protocol has been exploited to enhance the performance in a multicasting scenario.

The time varying nature of the wireless channels leads to decrease in the reliability of transmission over these channels. The availability of instantaneous channel state information of links plays an important role in enhancing the performance of the wireless networks [13], [14]. In this paper, we introduce transmission strategies which exploit the knowledge of the channel between the source and the destination to increase the maximum stable throughput of the source.

Cooperative diversity in energy harvesting networks at the physical layer has been considered before in a number of works as in [15], [16]. Also, the problem of power optimization for energy harvesting networks with network-level cooperation has been discussed in [17]. The authors have derived the maximum stable throughput rate for a network consisting of a source, a relay and a destination. The relaying strategy is Time Division Multiple Access (TDMA). In this strategy, the odd time slots are assigned to the source transmissions and the even time slots are assigned to the relay transmissions. This strategy has low channel utilization because of the fixed assignment of the time slots. As a result, it has been shown in [17] that the direct transmission has higher stable throughput than this relaying scheme for high energy arrival rates. In our work, we propose a relaying scheme which has higher channel utilization than the relaying scheme in [17].

In this paper, we investigate the impact of energy harvesting capability on the stable throughput rate of a source node. We start by calculating the stable throughput of the source while transmitting to the destination directly over a time varying channel. The channel is modeled by a two-state discrete-time process. The packets and energy arrivals into the source are modeled by discrete-time stochastic processes. Also, we derive the maximum stable throughput rate of a source node which is helped by a relay node through a network-level cooperation protocol. The relay also has energy harvesting capability. Due to the stochastic nature of the data arrivals to the source, we propose a strategy in which the relay transmits during the idle periods of the source to efficiently utilize the channel. The proposed transmission strategies exploit the knowledge of the CSI of the channel between the source and the destination. The source transmits with probability 1 when the channel is in the good state if its energy queue is not empty, but it randomly transmits with a certain probability if the channel is in the poor state. We calculate the optimal value of this probability. Also, we propose a modified relaying technique to enhance the performance especially when the energy arrival rate to the relay node is relatively small. Also, we derive the stable throughput rate of the source when its decision depends on

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imperfect channel measurements.

The study of a simple model consisting of only a source, a relay and a destination is both instructive and necessary. It reveals insights at the conceptual level about the effects of cooperative relaying and exploiting channel information on the stability of energy harvesting networks. More work needs to be done to exploit the results of this work in more realistic systems. Also, energy harvesting capability and channel knowledge can much affect the dynamic behavior of the proposed system but it is out of this paper scope.

II. System Model

We consider a network which consists of a source node, a relay node, and a destination node as shown in figure 1. Each of the source and the relay has an infinite data queue for storing fixed length packets. These queues are denoted by Q_S and Q_R respectively. We assume that the source has its own traffic while the relay does not have its own traffic and is used only for cooperation. The data arrival to the source data queue is modeled by a Bernoulli process. Also, each of the source and the relay has an infinite energy queue. These queues are denoted by E_S and E_R respectively. The usage of infinite queues is a reasonable approximation when the data queues are large enough compared to the packet size and the energy queues are large enough compared to the energy unit [18]. All nodes are half-duplex and thus they can not transmit and receive simultaneously. Time is assumed to be slotted such that each packet transmission takes one time slot. Transmission of a data packet from a node requires using a single unit of energy from the corresponding energy queue. For simplicity, we assume that the energy consumption in a node is due to transmission only and therefore the processing and reception energy are considered to be negligible. Each of the source and the relay can acquire a single unit of energy at each time slot with probabilities q_S and q_R respectively that the energy arrival processes are modeled by Bernoulli processes.

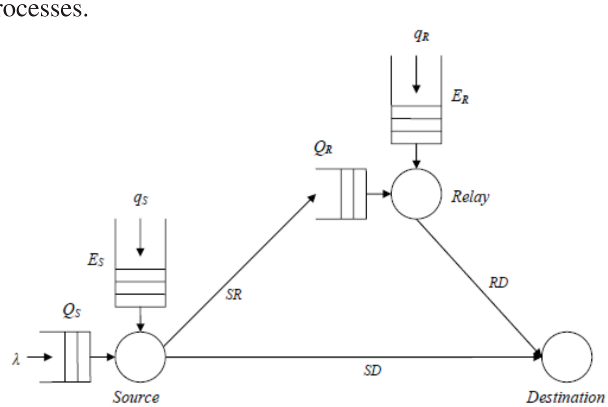

Fig. 1. System Model

All the channels, which are denoted by SD, SR and RD, are modeled by independent two-state discrete-time processes. The channels are also independent of the packet generation process and the energy harvesting at the source and the relay. Each channel state corresponds to a degree of channel connectivity. State 1 corresponds to good connectivity while state 0 corresponds to poor connectivity. The quality of the channels is represented by the success probability of a packet. The packet success probabilities are denoted by $f_{SD,i}$, $f_{SR,i}$ and $f_{RD,i}$ when the corresponding channels are in state $i = 0, 1$. These success probabilities are determined by the system physical parameters such as transmission power, modulation scheme, coding scheme and targeted bit-error rate. We assume that each channel remains fixed for a time slot and is able to move into another state in the next slot. The steady state probabilities for the channels to be in state $i = 0, 1$ are $\pi_{SD,i}$, $\pi_{SR,i}$ and $\pi_{RD,i}$ respectively.

In [19], Loynes' theorem states that if the arrival and service processes at a queue are jointly stationary, then the queue is stable if the average arrival rate is less than the average service rate. Throughout the paper, we denote the average arrival rate at the source data queue by λ. The average arrival rate to the relay data queue is denoted by λ_R. The average service rate of the source data queue in the case of no relaying is denoted by μ_S^{NR}. The average service rate of the source data queue in the case of cooperative relaying is denoted by μ_S^{CR}. Also, the average service rate of the relay data queue is denoted by μ_R.

III. Network Protocols

In this section, we present two transmission protocols for delivering the packets from the source to the destination either directly with no relaying or by allowing the relay to help.

A. No Relaying

In this case, the system consists only of the source and the destination. The packets can reach the destination through the channel SD. The source can transmit only when both its energy queue and its data queue are not empty. The channel SD state is known at the source at the time of transmission and it is throughput-optimal for the source to transmit with probability 1 when the channel is in state 1. Thus, the transmission strategy when the data queue is not empty is described as follows: if the source energy queue is not empty and the channel SD is in state 1, the source is going to transmit. Also, if the source energy queue is not empty and the channel SD is in state 0, the source is going to transmit with some probability p_0. The packet is released from the source data queue if it is successfully received by the destination; otherwise it remains at the source data queue for retransmission. The feedback to the source is in the form of Acknowledgment or Negative-Acknowledgment. In this mechanism, a short-length error-free packets are broadcasted by the destination over a separate channel to inform the network users about the reception status.

The probability p_0 controls the utilization of the channel when the channel is in state 0. Increasing p_0 leads to one of the following two effects. First, It may increase the energy used when the channel is in state 0 by decreasing the energy used when the channel is in state 1. This leads to increase of the joint probability of the channel to be in state 1 and the source energy queue to be empty which affects the performance negatively. Second, increasing p_0 may increase the energy used when the channel is in state 0 by exploiting unused harvested energy without affecting the amount of energy used when the channel is in state 1. This effect improves the system performance.

B. Cooperation with the Relay

The source transmits its traffic with the help of the relay. At a time slot, the source is able to transmit if both its energy queue and its data queue are not empty. It transmits with probability 1 when the channel SD is in state 1 and with probability p_0 when the channel is in state 0. If the packet is successfully received by the destination or by the relay, it is released from the source data queue; otherwise it is kept in the source data queue for retransmission. The retransmission scheme is the same as mentioned in the last subsection. At the beginning of every time slot, the relay senses the channel. We assume perfect sensing by the relay for the source transmissions. If the source is not transmitting, the relay uses these idle time slots to transmit the packets in its data queue to the destination when its energy queue is not empty. Hence, no explicit channel resources are assigned to the relay. A packet is released from the relay data queue if it is successfully received by the destination; otherwise it is kept for retransmission.

In this protocol, we let the source transmissions depend only on the state of the channel SD that the source transmission control protocol is the same as the case of no relaying. That allows us to illustrate the effect of relaying on the stability condition of the source. The proposed system can have better performance by allowing a different transmission protocol at the source in which the source considers both the channels SD and SR. Also, the relay can consider the channel RD while transmitting to the destination. Including this transmission control protocol in the analysis is straightforward but is not included for brevity.

IV. Stable Throughput Analysis

In this section, we derive the maximum stable throughput rate of the source for the proposed transmission protocols.

A. No Relaying

In order to calculate the maximum stable throughput rate for the source data queue, we have to consider the maximum service rate for the source energy queue which is the rate of which the source node attempts to transmit. Each transmission attempt uses a single unit energy. As a result, the energy departure process is modeled by a Bernoulli process. Therefore, the source energy queue forms a discrete-time M/M/1 system. The transmission attempt rate equals $\pi_{SD,1} + \pi_{SD,0}p_0$. The arrival rate of the energy to the source is q_S. If the energy arrival rate to the source is larger than the transmission attempting rate, the number of energy units in the energy queue approaches infinity almost surely. Therefore, the probability of the energy queue to be empty is zero. On the other hand, if the energy arrival rate to the source node is smaller than or equal to the transmission attempting rate, it follows from [20] for discrete-time M/M/1 system that the probability of energy queue to be not empty is the ratio between the energy arrival rate and the transmission attempting rate. As a result, the probability of the energy queue to be not empty is written as follows

$$Pr[E_S \neq 0] = \frac{\min(q_S, \pi_{SD,1} + \pi_{SD,0}p_0)}{\pi_{SD,1} + \pi_{SD,0}p_0} \quad (1)$$

The probability of a packet to be delivered, given that the source is able to transmit, is $\pi_{SD,1}f_{SD,1}+\pi_{SD,0}p_0f_{SD,0}$. The source data queue service rate is the product of the success probability given that the source is able to transmit by the probability that the energy queue is not empty. The stability condition for the source data queue, when relaying is not used, is $\lambda < \mu_S^{NR}$ which can be written as

$$\lambda < Pr[E_S \neq 0](\pi_{SD,1}f_{SD,1} + \pi_{SD,0}p_0f_{SD,0}) \quad (2)$$

In the case of no availability of CSI at the source, it transmits with probability 1 when the energy queue is not empty. The expression of the stability condition can be evaluated by setting p_0 to be 1. The stability condition can be stated as follows

$$\lambda < q_S(\pi_{SD,1}f_{SD,1} + \pi_{SD,0}f_{SD,0}) \quad (3)$$

B. Cooperation with the Relay

In this protocol, the system is stable if both the source data queue and the relay data queue are stable. In the following subsections, we derive the stability conditions for each queue separately.

1) Source Data Queue: The maximum data arrival rate which maintains the stability of the source data queue is limited by its service rate. A packet at the source is served if it is successfully delivered to the relay or the destination. The service rate of the source queue is found to be

$$\begin{aligned}\mu_S^{CR} =& \\ Pr[E_S \neq 0][&\pi_{SD,1}(\pi_{SR,1}[1-(1-f_{SD,1})(1-f_{SR,1})] \\ &+ \pi_{SR,0}[1-(1-f_{SD,1})(1-f_{SR,0})]) \\ &+ \pi_{SD,0}p_0(\pi_{SR,1}[1-(1-f_{SD,0})(1-f_{SR,1})] \\ &+ \pi_{SR,0}[1-(1-f_{SD,0})(1-f_{SR,0})])]\end{aligned} \quad (4)$$

2) Relay Data Queue: We start by calculating the probability that the channel is occupied by the source transmissions and this probability is denoted by ρ_S. As the source data queue forms a discrete-time M/M/1 system and assuming that the source data queue is stable, it follows from [20] that the probability ρ_S is calculated as follows

$$\rho_S = \frac{\lambda Pr[E_S \neq 0]}{\mu_S^{CR}} \quad (5)$$

The arrival rate for the relay data queue is the probability that a packet is received by the relay at any given time slot. It is calculated as follows

$$\lambda_R = \rho_S Pr[\text{Packet received by relay only}] \quad (6)$$

The Pr[Packet received by relay only] is denoted by P_R and its value is calculated as follows

$$P_R = \pi_{SD,1}(\pi_{SR,1}f_{SR,1}(1-f_{SD,1})+\pi_{SR,0}f_{SR,0}(1-f_{SD,1})) + \pi_{SD,0}p_0(\pi_{SR,1}f_{SR,1}(1-f_{SD,0})+\pi_{SR,0}f_{SR,0}(1-f_{SD,0})) \quad (7)$$

Also, we denote the probability that a packet is received by either the relay or the destination by P_E and we calculate its value as follows

$$\begin{aligned} P_E = \pi_{SD,1}(&\pi_{SR,1}[1-(1-f_{SD,1})(1-f_{SR,1})] \\ &+ \pi_{SR,0}[1-(1-f_{SD,1})(1-f_{SR,0})]) \\ &+ \pi_{SD,0}p_0(\pi_{SR,1}[1-(1-f_{SD,0})(1-f_{SR,1})] \\ &+ \pi_{SR,0}[1-(1-f_{SD,0})(1-f_{SR,0})]) \end{aligned} \quad (8)$$

The expression of λ_R can be rewritten as follows

$$\lambda_R = \lambda\frac{P_R}{P_E} \quad (9)$$

Then, the service rate of the relay data queue equals

$$\mu_R = (\pi_{RD,1}f_{RD,1}+\pi_{RD,0}f_{RD,0})\min(q_R, 1-\rho_S) \quad (10)$$

The complete derivation of the expression of the service rate of the relay data queue is found in Appendix A.

3) Stability Conditions: To ensure that the system is stable, both source and relay data queues have to be stable. As a result, both the conditions $\lambda < \mu_S^{CR}$ and $\lambda_R < \mu_R$ should be satisfied. By substituting using equation (9) in the second condition, it is written as

$$\lambda < \frac{P_E}{P_R}(\pi_{RD,1}f_{RD,1}+\pi_{RD,0}f_{RD,0})\min(q_R, 1-\rho_S) \quad (11)$$

Note that the right hand side of the inequality is still function of λ. By combining the conditions on λ, we get the general expression for the maximum stable throughput as follows

$$\lambda < \min(\mu_S^{CR}, \frac{P_E}{P_R}(\pi_{RD,1}f_{RD,1}+\pi_{RD,0}f_{RD,0})q_R, \frac{P_E(\pi_{RD,1}f_{RD,1}+\pi_{RD,0}f_{RD,0})}{P_R+(\pi_{RD,1}f_{RD,1}+\pi_{RD,0}f_{RD,0})}) \quad (12)$$

In the case of no availability of CSI at the source node, the expression of the stability condition is calculated by setting p_0 to be 1.

The same analysis is still valid when the energy arrival processes and the data arrival process are modeled by Poisson processes. In this case, the energy queues and the source data queue form M/G/1 systems.

V. Imperfect Channel Measurement

In this section, we study the effect of channel uncertainty on the stable throughput of the source for the proposed transmission strategies. The measured channel is the channel SD. We denote the probability of measuring the channel to be in state 1 given that the channel is in state 0 by $p_{1|0}$ and the probability of measuring the channel to be in state 0 given that the channel is in state 1 by $p_{0|1}$. Also, we denote the steady state probability of the channel SD to be measured in state 1 and 0 by $\hat{\pi}_{SD,1}$ and $\hat{\pi}_{SD,0}$ respectively. The expressions of the steady state probabilities are

$$\hat{\pi}_{SD,1} = \pi_{SD,1}(1-p_{0|1})+\pi_{SD,0}p_{1|0} \quad (13)$$

$$\hat{\pi}_{SD,0} = \pi_{SD,1}p_{0|1}+\pi_{SD,0}(1-p_{1|0}) \quad (14)$$

A. No Relaying

In this case, the source transmits with probability 1 when the channel is measured to be in state 1. It transmits with probability p_0 when the channel is measured to be in state 0. As a result, the source energy queue service rate is $\hat{\pi}_{SD,1} + \hat{\pi}_{SD,0}p_0$. Thus, the probability of the source energy queue to be not empty is written as follows

$$Pr[E_S \neq 0] = \frac{\min(q_S, \hat{\pi}_{SD,1}+\hat{\pi}_{SD,0}p_0)}{\hat{\pi}_{SD,1}+\hat{\pi}_{SD,0}p_0} \quad (15)$$

The probability of a packet to be successfully received by the destination given that the source is able to transmit equals $(\pi_{SD,1}f_{SD,1}[(1-p_{0|1})+p_0p_{0|1}]+\pi_{SD,0}f_{SD,0}[p_{1|0}+(1-p_{1|0})p_0])$. Hence, the stability condition for the source data queue is written as follows

$$\lambda < \frac{\min(q_S, \hat{\pi}_{SD,1}+\hat{\pi}_{SD,0}p_0)}{\hat{\pi}_{SD,1}+\hat{\pi}_{SD,0}p_0} (\pi_{SD,1}f_{SD,1}[(1-p_{0|1})+p_0p_{0|1}] + \pi_{SD,0}f_{SD,0}[p_{1|0}+(1-p_{1|0})p_0]) \quad (16)$$

B. Cooperation with the relay

In this case, the service rate of the source data queue is affected by the erroneous channel measurements. The service rate can be written as follows

$$\begin{aligned} \mu_S^{CR} = &Pr[E_S \neq 0] \\ &[\pi_{SD,1}[(1-p_{0|1})+p_0p_{0|1}](\pi_{SR,1}[1-(1-f_{SD,1})(1-f_{SR,1})] \\ &+ \pi_{SR,0}[1-(1-f_{SD,1})(1-f_{SR,0})]) \\ &+\pi_{SD,0}[p_{1|0}+(1-p_{1|0})p_0](\pi_{SR,1}[1-(1-f_{SD,0})(1-f_{SR,1})] \\ &+ \pi_{SR,0}[1-(1-f_{SD,0})(1-f_{SR,0})])] \end{aligned} \quad (17)$$

As a result, the probability of the channel to be occupied by the source transmissions is updated by using the updated values of both μ_S^{CR} and $Pr[E_S \neq 0]$. Also, the values of P_R and P_E are updated because of the uncertainty of the channel measurements. The values are calculated as follows

$$\begin{aligned} P_E = &\pi_{SD,1}[(1-p_{0|1})+p_0p_{0|1}](\pi_{SR,1}[1-(1-f_{SD,1})(1-f_{SR,1})] \\ &+ \pi_{SR,0}[1-(1-f_{SD,1})(1-f_{SR,0})]) \\ &+\pi_{SD,0}[p_{1|0}+(1-p_{1|0})p_0](\pi_{SR,1}[1-(1-f_{SD,0})(1-f_{SR,1})] \\ &+ \pi_{SR,0}[1-(1-f_{SD,0})(1-f_{SR,0})]) \end{aligned} \quad (18)$$

$$P_R = \pi_{SD,1}[(1-p_{0|1}) + p_0 p_{0|1}](\pi_{SR,1} f_{SR,1}(1-f_{SD,1}) + \pi_{SR,0} f_{SR,0}(1-f_{SD,1})) + \pi_{SD,0}[p_{1|0} + (1-p_{1|0})p_0](\pi_{SR,1} f_{SR,1}(1-f_{SD,0}) + \pi_{SR,0} f_{SR,0}(1-f_{SD,0})) \quad (19)$$

The expressions for λ_R and μ_R remain the same as in equations (9) and (10) but the values of ρ_S, P_E and P_R are updated as shown above. As a result, the stability condition is the same as in equation (12) using the updated values of the parameters.

VI. Transmission Optimization

In this section, we evaluate the value of the parameter p_0 to maximize the maximum stable throughput rate for different protocols which is denoted by λ_{max}. The value of p_0 belongs to [0,1].

A. No Relaying

We have derived the stability condition in this case to have the expression in equation (2). We are going to consider two cases depending on the system parameters.

1) $\pi_{SD,1} > q_S$: The value of $\pi_{SD,0}p_0$ is always greater than or equal to 0. Then, we can rewrite the expression of λ_{max} as

$$\lambda_{max} = \frac{q_S(\pi_{SD,1} f_{SD,1} + \pi_{SD,0} p_0 f_{SD,0})}{\pi_{SD,1} + \pi_{SD,0} p_0} \quad (20)$$

This value as a function of p_0 is found to be a decreasing function of p_0 by calculating its first derivative. The first derivative is always negative for any value of p_0. As a result, the optimal value of p_0 is 0.

2) $\pi_{SD,1} \leq q_S$: In this case, we can rewrite the expression of λ_{max} as follows

$$\lambda_{max} = \begin{cases} \pi_{SD,1} f_{SD,1} + \pi_{SD,0} p_0 f_{SD,0}, & \text{if } p_0 \leq \frac{q_S - \pi_{SD,1}}{\pi_{SD,0}} \\ \frac{q_S(\pi_{SD,1} f_{SD,1} + \pi_{SD,0} p_0 f_{SD,0})}{\pi_{SD,1} + \pi_{SD,0} p_0}, & \text{if } p_0 > \frac{q_S - \pi_{SD,1}}{\pi_{SD,0}} \end{cases} \quad (21)$$

The first expression is an increasing function of p_0. The second one is a decreasing function of p_0. The optimal value of p_0 equals $(q_S - \pi_{SD,1})/\pi_{SD,0}$.

From these results, we can write the general expression for the optimal value of p_0 as follows

$$p_0^* = \max(0, \frac{q_S - \pi_{SD,1}}{\pi_{SD,0}}) \quad (22)$$

B. Transmission with Relaying

The optimal value of p_0 is the solution of the problem

$$p_0^* = \arg\max_{p_0}(\min\{f_1(p_0), f_2(p_0), f_3(p_0)\}) \quad (23)$$

where the values of $f_1(p_0)$, $f_2(p_0)$ and $f_3(p_0)$ are obtained from equation (12). It can be shown that $f_2(p_0)$ and $f_3(p_0)$ are decreasing functions by calculating the first derivative of each of the functions and showing that it is always negative. Also, if $\pi_{SD,1} > q_S$, we can show that $f_1(p_0)$ is a decreasing function in p_0. Then, the optimal value of p_0 should be 0.

On the other hand, we consider the case when $\pi_{SD,1} \leq q_S$ in which $f_1(p_0)$ is an increasing function in p_0 for p_0 belongs to $[0, (q_S - \pi_{SD,1})/\pi_{SD,0}]$ and a decreasing function in p_0 for p_0 belongs to $[(q_S - \pi_{SD,1})/\pi_{SD,0}, 1]$. We denote the increasing part of $f_1(p_0)$ by $f_{11}(p_0)$ which has the same expression as P_E.

We calculate the intersection points between $f_{11}(p_0)$ and both $f_2(p_0)$ and $f_3(p_0)$. We denote these points by PI_{12} and PI_{13} respectively. We calculate these values as follows

$$PI_{12} = \frac{(\pi_{RD,1} f_{RD,1} + \pi_{RD,0} f_{RD,0}) q_R}{\pi_{SD,0}(\pi_{SR,1} f_{SR,1}(1-f_{SD,0}) + \pi_{SR,0} f_{SR,0}(1-f_{SD,0}))} - \frac{\pi_{SD,1}(\pi_{SR,1} f_{SR,1}(1-f_{SD,1}) + \pi_{SR,0} f_{SR,0}(1-f_{SD,1}))}{\pi_{SD,0}(\pi_{SR,1} f_{SR,1}(1-f_{SD,0}) + \pi_{SR,0} f_{SR,0}(1-f_{SD,0}))} \quad (24)$$

$$PI_{13} = \frac{1}{H}[(\pi_{RD,1} f_{RD,1} + \pi_{RD,0} f_{RD,0})(1-\pi_{SD,1}) - \pi_{SD,1}(\pi_{SR,1} f_{SR,1}(1-f_{SD,1}) + \pi_{SR,0} f_{SR,0}(1-f_{SD,1}))] \quad (25)$$

where $H = \pi_{SD,0}(\pi_{RD,1} f_{RD,1} + \pi_{RD,0} f_{RD,0}) + \pi_{SD,0}((\pi_{SR,1} f_{SR,1}(1-f_{SD,0}) + \pi_{SR,0} f_{SR,0}(1-f_{SD,0})))$. We consider three cases for the values of these intersection points:

1) At least one point is less than 0: In this case the function $\min(f_1(p_0), f_2(p_0), f_3(p_0))$ is a decreasing function in p_0 for p_0 belongs to [0,1]. As a result, the optimal value of p_0 is 0.

2) At least one point belongs to $[0, (q_S - \pi_{SD,1})/\pi_{SD,0}]$ and no point less than 0: In this case, the function $\min(f_1(p_0), f_2(p_0), f_3(p_0))$ is increasing till the first intersection point and then it is decreasing. As a result, the optimal value of p_0 is $\min(PI_{12}, PI_{13})$.

3) Both points are larger than $(q_S - \pi_{SD,1})/\pi_{SD,0}$: In this case, the function $\min(f_1(p_0), f_2(p_0), f_3(p_0))$ is increasing till $(q_S - \pi_{SD,1})/\pi_{SD,0}$ and then it is decreasing. As a result, the optimal value of p_0 is $(q_S - \pi_{SD,1})/\pi_{SD,0}$.

Thus, we can generally write the optimal value p_0 as follows

$$p_0^* = \max(0, \min(PI_{12}, PI_{13}, \frac{q_S - \pi_{SD,1}}{\pi_{SD,0}})) \quad (26)$$

VII. Modified Relaying

Due to the random availability of energy at the relay, it is not always beneficial to allow all packets to be relayed. If q_R has a small value such that the maximum stable throughput rate is the second term in the equation (12) which is proportional to q_R, then for certain values of q_R, relaying can lead to a lower stable throughput than direct transmission. To improve the performance of the network, we propose a strategy in which only a proportion of the source packets are to be relayed. The remaining packets are obliged to be transmitted to the destination through the direct link only. Let r be the proportion of the source data packets which are going to be relayed.

A. Source Data Queue

The service rate for the proportion r of the packets is μ_S^{CR} and the service rate for the remaining packets is μ_S^{NR}. Then, the average service rate of the source data queue is calculated as follows

$$\mu_S^M = r\mu_S^{CR} + (1-r)\mu_S^{NR} \quad (27)$$

where μ_S^{NR} is calculated using equation (2) and μ_S^{CR} is calculated using equation (4).

B. Relay Data Queue

Using the same derivation of equation (9), the arrival rate to the relay λ_R^M is calculated as follows

$$\lambda_R^M = r\lambda \frac{P_R}{P_E} \quad (28)$$

Let ρ_S^M be the probability that the channel is occupied by the source. As a result, ρ_S^M is calculated as follows

$$\rho_S^M = \frac{\lambda Pr[E_S \neq 0]}{\mu_S^M} \quad (29)$$

Thus, the service rate of the relay data queue is shown as follows

$$\mu_R^M = (\pi_{RD,1} f_{RD,1} + \pi_{RD,0} f_{RD,0}) \min(q_R, 1-\rho_S^M) \quad (30)$$

C. Stability Conditions

For system stability, both source and relay queues have to be stable. As a result, both the conditions $\lambda < \mu_S^M$ and $\lambda_R^M < \mu_R^M$ should be satisfied. The condition $\lambda_R^M < \mu_R^M$ can be written as follows

$$\lambda < \frac{1}{r}\frac{P_E}{P_R}(\pi_{RD,1} f_{RD,1} + \pi_{RD,0} f_{RD,0}) \min(q_R, 1-\rho_S^M) \quad (31)$$

The right hand side of the inequality is still function of λ. Let α be the value of ρ_S^M/λ which does not depend on λ. By combining the conditions on λ, we get the general expression for the maximum stable throughput as follows

$$\lambda < \min(\mu_S^M, \frac{1}{r}\frac{P_E}{P_R}(\pi_{RD,1} f_{RD,1} + \pi_{RD,0} f_{RD,0}) q_R, \frac{P_E(\pi_{RD,1} f_{RD,1} + \pi_{RD,0} f_{RD,0})}{rP_R + (\pi_{RD,1} f_{RD,1} + \pi_{RD,0} f_{RD,0})P_E \alpha}) \quad (32)$$

VIII. Numerical Results

In this section, we present numerical results to illustrate the previous theoretical development. We illustrate the effects of different system parameters on the maximum stable throughput of the proposed transmission protocols. In the following results, we fix the channels success probabilities to be $f_{SD,1} = 0.4$, $f_{SD,0} = 0.1$, $f_{SR,1} = 0.8$, $f_{SR,0} = 0.2$, $f_{RD,1} = 0.8$ and $f_{RD,1} = 0.2$. Also, we let the channels distributions be identical such that $\pi_{SD,1} = \pi_{SR,1} = \pi_{RD,1} = \pi_1$ and $\pi_{SD,0} = \pi_{SR,0} = \pi_{RD,0} = \pi_0$. We denote the system with no relaying capability by "No Relaying". Also, we denote the system in which full cooperative relaying is exploited by "With Relaying".

In figure 2, we show the maximum stable throughput of the two proposed network protocols against the probability

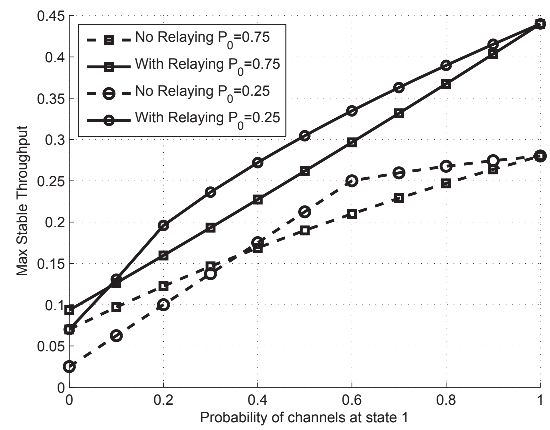

Fig. 2. Maximum stable throughput against π_1

of the channels to be in state 1. We fix the system parameters $q_S = 0.7$ and $q_R = 0.3$. The results are for p_0 with the values 0.25 and 0.75. For small values of π_1, the performance of the system is better for larger p_0 because it is better for the source to make more transmission attempts during the time slots in which the channel is in state 0. For large values of π_1, the performance of the system is better for smaller p_0 because the source should not waste much of its energy in transmission during the time slots in which the channel is in state 0.

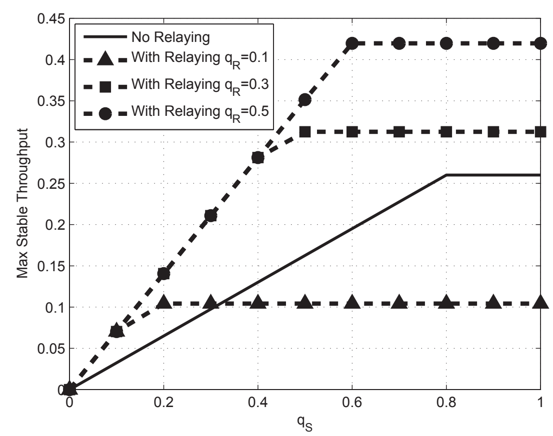

Fig. 3. Maximum stable throughput against q_S

In figure 3, we show the maximum stable throughput of the two proposed network protocols against the energy arrival rate to the source energy queue. We fix the system parameters $p_0 = 0.5$ and $\pi_1 = 0.6$. The results are for q_R with the values 0.1, 0.3 and 0.5. For the case $q_R = 0.1$, the maximum stable throughput of the cooperative relaying protocol becomes less than the throughput of the protocol with no relaying. That is because the channel SR has higher success probability than the channel SD. Then, most of the source packets are forwarded to the relay. Also due to limited energy at the relay and to maintain the stability of the relay data queue, the maximum stable throughput of the system is lowered.

In figure 4, we show the maximum stable throughput of the two proposed network protocols against the probability to attempt transmission while the channel in state 0. We fix the system parameters $q_S = 0.7$ and $q_R = 0.3$. The results are for π_1 with the values 0.6, 0.5 and 0.4. This figure shows the effect of exploiting the knowledge of the CSI of the

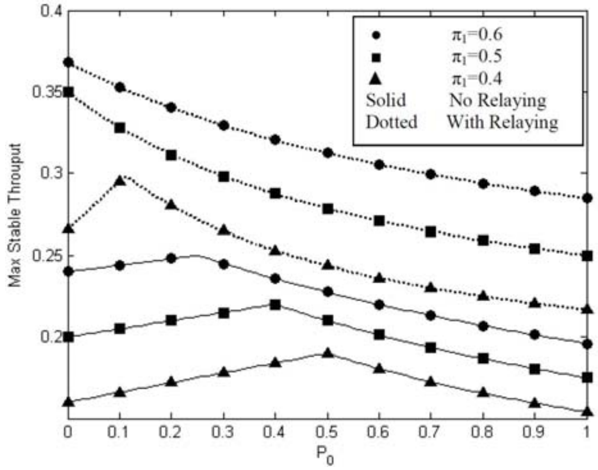

Fig. 4. Maximum stable throughput against p_0

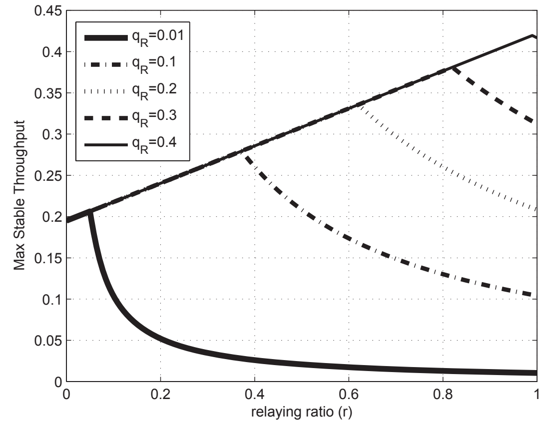

Fig. 6. Maximum stable throughput against r

channel between the source and destination. The performance when no CSI available is equivalent to the performance of the system with p_0 equals 1. For any value of π_1, the system is able to have higher stable throughput using the knowledge of the CSI than the system with no CSI at the source by selecting a suitable p_0.

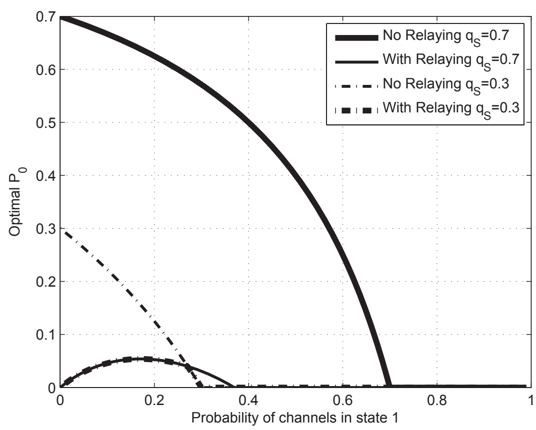

Fig. 5. The value of p_0^* against π_1

In figure 5, we show the optimal transmission probability with the channel SD in state 0 against π_1. We fix $q_R = 0.3$. The results are for q_S with the values 0.3 and 0.7. The figure shows that the optimal p_0 takes small value when q_S is low as energy is better to be used when the channel in its good state. Also when the probability of the channel to be in state 1 is high, the optimal value of p_0 equals 0 as there will be no need to transmit while the channel is in state 0. In the case of no relaying, p_0 takes larger values than the case of cooperative relaying because there is no benefit for leaving the channel idle while there is unused energy at the source. For the cooperative relaying, keeping the channel idle allows the relay to transmit which can be more beneficial than allowing the source to transmit with the channel SD at state 0.

In figure 6, we show the maximum stable throughput of the modified relaying protocol against r. We fix the system parameters $q_S = 0.6$, $p_0 = 0.5$ and $\pi_1 = 0.6$. The results are for q_R with the values 0.01, 0.1, 0.2, 0.3 and 0.4. The figure shows the effect of changing r on the stable throughput of the source. Also, the optimal value of r is proportional to q_R such that the optimal relaying ratio is large when q_R is large.

IX. Conclusion

In this paper, we have proposed and analyzed protocols for transmission from a source that has energy harvesting capability. We have considered the case in which a relay is used to help the source transmissions. The relay also has energy harvesting capability. The proposed protocol allows the relay to use the idle time slots of the source and hence avoids allocating any explicit resources to the relay. Our analysis shows that cooperation increases the maximum stable throughput rate in most cases except when the energy harvesting rate of the relay is small. The proposed strategy exploits the knowledge of the CSI of the channel between the source and the destination such that the source transmits with probability 1 if the channel is in state 1 and transmits with a certain probability if the channel is in state 0. The optimal probability has also been calculated. The effect of imperfect channel measurements has been considered. Then, a modified relaying scheme has been introduced. In this scheme, some of the source data packets are prevented from being relayed to improve the performance when the energy harvesting rate at the relay is relatively small.

Appendix A
Derivation of the Service Rate for the Relay Data Queue for Transmission Protocol with Relaying

We are going to calculate the service rate of the relay data queue. Let p_{RD} be the probability that a packet received by the destination due to a relay transmission. The packet is to be decoded successfully when the relay is able to transmit and the channel RD is not in outage. The relay is able to transmit when the relay energy queue is not empty. The value of P_{RD} is calculated as follows

$$P_{RD} = Pr[E_R \neq 0](\pi_{RD,1} f_{RD,1} + \pi_{RD,0} f_{RD,0}) \quad (33)$$

The relay energy queue forms a discrete-time M/M/1 system for the same reasoning as the source energy queue. The service rate of the relay energy queue is the rate of attempting transmission of the relay node. The transmission attempting rate equals $(1-\rho_S)$. The arrival rate of energy to the relay is q_R. Also, if the energy arrival rate of the relay node is larger than the transmission attempting rate, the number of energy units in the queue approaches infinity almost surely. Therefore, the probability of the energy queue to be empty is zero. On the other hand, if the energy arrival rate of the relay node is smaller than or equal to the transmission attempting rate, the probability of energy queue to be not empty is the ratio between the energy arrival rate and the transmission attempting rate. As a result, the probability of the energy queue to be not empty is written as follows

$$Pr[E_R \neq 0] = \frac{\min(q_R, 1-\rho_S)}{1-\rho_S} \quad (34)$$

Let T_R be the number of time slots needed for the relay to serve a packet in the relay data queue assuming that the relay continuously transmits. Then, T_R has a geometric probability distribution as follows

$$Pr[T_R = k] = P_{RD}(1-P_{RD})^{k-1} \quad (35)$$

Then, the expected value of the number of time slots needed till the packet is decoded correctly by the destination, assuming that the relay continuously transmits, is shown to be

$$E[T_R] = \frac{1}{P_{RD}} \quad (36)$$

Let v_1,v_2,.....be a sequence of random variables. The random variable v_i represents the number of successive time slots in which the source is going to be busy before the i^{th} relay retransmission. This sequence represents an i.i.d sequence. The probability of the source to be busy is ρ_S. Then, the number of successive time slots, in which the source is busy, follows a geometric distribution as follows

$$Pr[v = k] = \rho_S^k(1-\rho_S) \quad (37)$$

The expected value of the number of successive time slots, in which the source is busy, is calculated as follows

$$E[v] = \frac{\rho_S}{(1-\rho_S)} \quad (38)$$

Let T be the number of time slots needed for the relay to get served including those in which the source will be transmitting, then we have

$$T = T_R + \sum_{i=1}^{T_R} v_i \quad (39)$$

This expression results from that the i^{th} transmission of the T_R relay transmissions is followed by busy period of length v_i. Then, the expected value of the number of time slots needed for the relay to get served, including those in which the source will be transmitting, is calculated as follows

$$E[T] = E[T_R](1+E[v]) = \frac{E[T_R]}{(1-\rho_S)} \quad (40)$$

Thus, the service rate of the relay data queue is shown as follows

$$\begin{aligned}\mu_R &= \frac{1}{E[T]} \\ &= (\pi_{RD,1}f_{RD,1} + \pi_{RD,0}f_{RD,0})\min(q_R, 1-\rho_S) \quad (41)\end{aligned}$$

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