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Generalised fuzzy soft sets

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ABSTRACT

In this paper, we define generalised fuzzy soft sets and study some of their properties. Application of generalised fuzzy soft sets in decision making problem and medical diagnosis problem has been shown.

1. Introduction

Molodtsov initiated the theory of soft sets as a new mathematical tool for dealing with uncertainties which traditional mathematical tools cannot handle. He has shown several applications of this theory in solving many practical problems in economics, engineering, social science, medical science, etc. Later other authors like Maji et al. [1–3] have further studied the theory of soft sets and used this theory to solve some decision making problems. They have also introduced the concept of fuzzy soft set, a more generalised concept, which is a combination of fuzzy set and soft set and studied its properties. In 2007, Aktas and Cagman [4] have introduced the notion of soft groups. Recently Kong et al. [5,6] have applied the soft set theoretic approach in decision making problems. Majumdar and Samanta [7,8] have studied the problem of similarity measurement between soft sets and fuzzy soft sets.

In this paper we have further generalised the concept of fuzzy soft sets as introduced by Maji et al. [1]. In our generalisation of fuzzy soft set, a degree is attached with the parametrization of fuzzy sets while defining a fuzzy soft set. This definition is more realistic as it involves uncertainty in the selection of a fuzzy set corresponding to each value of the parameter. Relations on generalised fuzzy soft sets are defined and their properties are studied and as an application a decision making problem is solved. We have further studied the similarity between two generalised fuzzy soft sets and it has been applied in medical diagnosis.

The organization of this paper is as follows: In Section 2, some preliminary definitions and results are given which will be used in the rest of the paper. In Section 3, a definition of generalised fuzzy soft set is given and some of its properties are studied. In Section 4, relations on generalised fuzzy soft sets are defined and a decision making problem has been solved using this relation. In Section 5, similarity between two generalised fuzzy soft sets has been discussed. An application of this similarity measure in medical diagnosis has been shown in Section 6. Section 7 concludes the paper.

2. Preliminaries

In this section we give few definitions and properties regarding fuzzy soft sets.

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Definition 2.1 (*[9]*). Let U be an initial universal set and let E be a set of parameters. Let $P(U)$ denote the power set of U. A pair (F, E) is called a soft set over U iff F is a mapping given by $F : E \to P(U)$.

Definition 2.2 (*[1]*). Let U be an initial universal set and let E be a set of parameters. Let I^U denote the power set of all fuzzy subsets of U. Let $A \subset E$.

A pair (F, E) is called a fuzzy soft set over U, where F is a mapping given by $F : A \to I^U$.

Example 2.3. As an illustration, consider the following example.

Suppose a fuzzy soft set (F, E) describes attractiveness of the shirts with respect to the given parameters, which the authors are going to wear.

$U = \{x_1, x_2, x_3, x_4, x_5\}$ which is the set of all shirts under consideration. Let I^U be the collection of all fuzzy subsets of U. Also let $E = \{e_1 = \text{“}colorful\text{”}, e_2 = \text{“}bright\text{”}, e_3 = \text{“}cheap\text{”}, e_4 = \text{“}warm\text{”}\}$.

Let

$$F(e_1) = \left\{\frac{x_1}{0.5}, \frac{x_2}{0.9}, \frac{x_3}{0}, \frac{x_4}{0}, \frac{x_5}{0}\right\}, \qquad F(e_2) = \left\{\frac{x_1}{1.0}, \frac{x_2}{0.8}, \frac{x_3}{0.7}, \frac{x_4}{0}, \frac{x_5}{0}\right\},$$
$$F(e_3) = \left\{\frac{x_1}{0}, \frac{x_2}{0}, \frac{x_3}{0}, \frac{x_4}{0.6}, \frac{x_5}{0}\right\}, \qquad F(e_4) = \left\{\frac{x_1}{0}, \frac{x_2}{1.0}, \frac{x_3}{0}, \frac{x_4}{0}, \frac{x_5}{0.3}\right\}.$$

Then the family $\{F(e_i), i = 1, 2, 3, 4\}$ of I^U is a fuzzy soft set (F, E).

Definition 2.4 (*[1]*). For two fuzzy soft sets (F, A) and (G, B) over a common universe U, we say that (F, A) is a fuzzy soft subset of (G, B) if (i) $A \subset B$, (ii) $\forall \varepsilon \in A$, $F(\varepsilon)$ is a fuzzy subset of $G(\varepsilon)$.

Definition 2.5 (*[1] Equality of Two Fuzzy Soft Sets*). Two soft sets (F, A) and (G, B) over a common universe U are said to be fuzzy soft equal if (F, A) is a fuzzy soft subset of (G, B) and (G, B) is a fuzzy soft subset of (F, A).

Definition 2.6 (*[1] Complement of a Fuzzy Soft Set*). The complement of a fuzzy soft set (F, A) is denoted by $(F, A)^C$ and is defined by $(F, A)^C = (F^C, \neg A)$, where $F^C : \neg A \to I^U$ is a mapping given by $F^C(\alpha)$ is a fuzzy complement of $F(\neg\alpha)$, $\forall \alpha \in \neg A$.

Definition 2.7 (*[1] Null Fuzzy Soft Set*). A soft set (F, A) over U is said to be null fuzzy soft set denoted by Φ, if $\forall \varepsilon \in A$, $F(\varepsilon)$ is the null fuzzy set $\bar{0}$ of U, where $\bar{0}(x) = 0\ \forall x \in U$.

Definition 2.8 (*[1] Absolute Fuzzy Soft Set*). A soft set (F, A) over U is said to be absolute fuzzy soft set denoted by $\tilde{A}$, if $\forall \varepsilon \in A$, $F(\varepsilon)$ is the fuzzy set $\bar{1}$ of U, $\bar{1}(x) = 1\ \forall x \in U$.

Definition 2.9 (*[1]*). Union of two fuzzy soft sets (F, A) and (G, B) over a common universe U is a soft set (H, C), where $C = A \cup B$ and which is defined as follows:

$$H(e) = F(e), \quad e \in A - B, = G(e), \quad e \in B - A, = F(e) \cup G(e), \quad e \in A \cap B,\ \forall\, e \in C.$$

We write

$$(H, C) = (F, A)\tilde{\cup}(G, B).$$

Definition 2.10 (*[1]*). Intersection of two fuzzy soft sets (F, A) and (G, B) over a common universe U is a soft set (H, C), where $C = A \cap B$ and which is defined as follows:

$$H(e) = F(e) \cap G(e), \quad \forall\, e \in C.$$

We write

$$(H, C) = (F, A)\tilde{\cap}(G, B).$$

Proposition 2.11 (*[1]*). *The following results hold here.*

(i) $(F, A)\tilde{\cup}(F, A) = (F, A)$,
(ii) $(F, A)\tilde{\cap}(F, A) = (F, A)$,
(iii) $(F, A)\tilde{\cup}\Phi = (F, A)$,
(iv) $(F, A)\tilde{\cap}\Phi = \Phi$,
(v) $(F, A)\tilde{\cup}\tilde{A} = \tilde{A}$,
(vi) $(F, A)\tilde{\cap}\tilde{A} = (F, A)$.

Proposition 2.12 (*[1]*). *The following results hold here.*

(i) $((F, A)\tilde{\cup}(G, B))^C = (F, A)^C\tilde{\cup}(G, B)^C$,
(ii) $((F, A)\tilde{\cap}(G, B))^C = (F, A)^C\tilde{\cap}(G, B)^C$.

Note 2.13. De Morgan's Laws also do not hold here.

Definition 2.14 (*[10]*). A fuzzy soft relation is defined as soft set over the fuzzy power set of the cartesian product of two crisp sets. Let X and Y be two crisp sets and E is the set of parameters, then a function $R : E \to I^{X\times Y}$ is called a fuzzy soft relation.

3. Generalised fuzzy soft sets

In this section we give a modified definition of fuzzy soft sets.

Definition 3.1. Let $U = \{x_1, x_2, \dots, x_n\}$ be the universal set of elements and $E = \{e_1, e_2, \dots, e_m\}$ be the universal set of parameters. The pair (U, E) will be called a soft universe. Let $F : E \to I^U$ and μ be a fuzzy subset of E, i.e. $\mu : E \to I = [0, 1]$, where I^U is the collection of all fuzzy subsets of U. Let F_μ be the mapping $F_\mu : E \to I^U \times I$ be a function defined as follows: $F_\mu(e) = (F(e), \mu(e))$, where $F(e) \in I^U$. Then F_μ is called a generalised fuzzy soft set (GFSS in short) over the soft universe (U, E).

Here for each parameter e_i, $F_\mu(e_i) = (F(e_i), \mu(e_i))$ indicates not only the degree of belongingness of the elements of U in $F(e_i)$ but also the degree of possibility of such belongingness which is represented by $\mu(e_i)$.

Example 3.2. Let $U = \{x_1, x_2, x_3\}$ be a set of three shirts under consideration. Let $E = \{e_1, e_2, e_3\}$ be a set of qualities where $e_1 = \textit{bright}$, $e_2 = \textit{cheap}$, $e_3 = \textit{colorful}$. Let $\mu : E \to I = [0, 1]$ be defined as follows: $\mu(e_1) = 0.1$, $\mu(e_2) = 0.4$, $\mu(e_3) = 0.6$.

We define a function $F_\mu : E \to I^U \times I$ be defined as follows:

$$F_\mu(e_1) = \left(\left\{\frac{x_1}{0.7}, \frac{x_2}{0.4}, \frac{x_3}{0.3}\right\}, 0.1\right), \qquad F_\mu(e_2) = \left(\left\{\frac{x_1}{0.1}, \frac{x_2}{0.2}, \frac{x_3}{0.9}\right\}, 0.4\right),$$
$$F_\mu(e_3) = \left(\left\{\frac{x_1}{0.8}, \frac{x_2}{0.5}, \frac{x_3}{0.2}\right\}, 0.6\right).$$

Then F_μ is a GFSS over (U, E).

In matrix form this can be expressed as $F_\mu = \begin{pmatrix} 0.7 & 0.4 & 0.3 & 0.1 \\ 0.1 & 0.2 & 0.9 & 0.4 \\ 0.8 & 0.5 & 0.2 & 0.6 \end{pmatrix}$, where the ith row vector represent $F_\mu(e_i)$, the ith column vector represent x_i, the last column represent the values of μ and it will be called membership matrix of F_μ.

Definition 3.3. Let F_μ and G_δ be two GFSS over (U, E). Now F_μ is said to be a generalised fuzzy soft subset of G_δ if

(i) μ is a fuzzy subset of δ (ii) $F(e)$ is also a fuzzy subset of $G(e)$, $\forall\, e \in E$.

In this case we write $F_\mu \subseteq G_\delta$.

Example 3.4. Consider the GFSS F_μ over (U, E) given in Example 3.2. Let G_δ be another GFSS over (U, E) defined as follows:

$$G_\delta(e_1) = \left(\left\{\frac{x_1}{0.2}, \frac{x_2}{0.3}, \frac{x_3}{0.1}\right\}, 0.1\right), \qquad G_\mu(e_2) = \left(\left\{\frac{x_1}{0.0}, \frac{x_2}{0.1}, \frac{x_3}{0.7}\right\}, 0.3\right),$$
$$G_\delta(e_3) = \left(\left\{\frac{x_1}{0.7}, \frac{x_2}{0.3}, \frac{x_3}{0.1}\right\}, 0.5\right), \quad \text{where } \delta \in I^E \text{ be defined as above.}$$

Then G_δ is a generalised fuzzy soft subset of F_μ.

Note 3.5 (*[11]*). Let c be an involutive fuzzy complement and g be an increasing generator of c.

Let $*$ and $\circ$ be two binary operations on [0, 1] defined as follows:

$$a * b = g^{-1}(g(a) + g(b) - g(1)) \quad \text{and} \quad a \circ b = g^{-1}(g(a) + g(b)).$$

Then $*$ is a *t-norm* and $\circ$ is a *t-conorm*. Moreover $(*, \circ, c)$ becomes a dual triple.

Henceforth in the rest of the paper we will take such an involutive dual triple to consider the general case.

Definition 3.6. Let F_μ be a GFSS over (U, E). Then the complement of F_μ, denoted by F_μ^c and is defined by $F_\mu^c = G_\delta$, where $\delta(e) = \mu^c(e)$ and $G(e) = F^c(e)$, $\forall\, e \in E$.

Note 3.7. Obviously $(F_\mu^c)^c = F_\mu$ as the fuzzy complement c is involutive in nature.

Definition 3.8. Union of two GFSS F_μ and G_δ, denoted by $F_\mu \tilde{\cup} G_\delta$, is a GFSS H_ν, defined as $H_\nu : E \rightarrow I^U \times I$ such that $H_\nu(e) = (H(e), \nu(e))$, where $H(e) = F(e) \circ G(e)$ and $\nu(e) = \mu(e) \circ \delta(e)$.

Definition 3.9. Intersection of two GFSS F_μ and G_δ, denoted by $F_\mu \tilde{\cap} G_\delta$, is a GFSS H_ν, defined as $H_\nu : E \rightarrow I^U \times I$ such that $H_\nu(e) = (H(e), \nu(e))$, where $H(e) = F(e) * G(e)$ and $\nu(e) = \mu(e) * \delta(e)$.

Example 3.10. Let us consider the generalised fuzzy soft sets F_μ and G_δ defined in Examples 3.2 and 3.4 respectively. Let us define the *t-norm* $*$ on [0, 1] as follows: $a * b = a.b$ and the *t-conorm* $\circ$ on [0, 1] as follows: $a \circ b = a + b - a.b$. Let us also take c as the fuzzy complement i.e. $a^c = 1 - a$. Then $(*, \circ, c)$ forms a involutive dual triple.

Then

$$F_\mu \tilde{\cup} G_\delta = \begin{pmatrix} 0.76 & 0.58 & 0.37 & 0.19 \\ 0.1 & 0.28 & 0.97 & 0.58 \\ 0.94 & 0.65 & 0.28 & 0.80 \end{pmatrix}, \qquad F_\mu \tilde{\cap} G_\delta = \begin{pmatrix} 0.14 & 0.12 & 0.03 & 0.01 \\ 0 & 0.02 & 0.63 & 0.12 \\ 0.56 & 0.15 & 0.02 & 0.3 \end{pmatrix}$$

and

$$G_\mu^c = \begin{pmatrix} 0.8 & 0.7 & 0.9 & 0.9 \\ 1 & 0.9 & 0.3 & 0.7 \\ 0.3 & 0.7 & 0.9 & 0.5 \end{pmatrix}.$$

Definition 3.11. A GFSS is said to be a generalised null fuzzy soft set, denoted by Φ_θ, if $\Phi_\theta : E \rightarrow I^U \times I$ such that $\Phi_\theta(e) = (F(e), \theta(e))$, where $F(e) = \bar{0}\ \forall\, e \in E$ and $\theta(e) = 0\ \forall\, e \in E$.

Definition 3.12. A GFSS is said to be a generalised absolute fuzzy soft set, denoted by $\tilde{A}_\alpha$ if $\tilde{A}_\alpha : E \rightarrow I^U \times I$, where $\tilde{A}_\alpha(e) = (A(e), \alpha(e))$ is defined by $A(e) = \bar{1}\ \forall\, e \in E$, and $\alpha(e) = 1\ \forall\, e \in E$.

Proposition 3.13. *Let F_μ be a GFSS over (U, E), then the following holds:*

(i) *F_μ is a GF soft subset of $F_\mu \tilde{\cup} F_\mu$.*
(ii) *$F_\mu \tilde{\cap} F_\mu$ is a GF soft subset of F_μ.*
(iii) $F_\mu \tilde{\cup} \Phi_\theta = F_\mu$
(iv) $F_\mu \tilde{\cap} \Phi_\theta = \Phi_\theta$
(v) $F_\mu \tilde{\cup} \tilde{A}_\alpha = \tilde{A}_\alpha$
(vi) $F_\mu \tilde{\cap} \tilde{A}_\alpha = F_\mu$

Proof. The results trivially follow from definition. □

Note 3.14. Instead of taking any dual triple as described in Note 3.5, if we take standard fuzzy operations (i.e. max, min and standard complement) then we get equality relation in (i) and (ii) above.

Proposition 3.15. *The following laws also hold here:*(a) $F_\mu \tilde{\cup} F_\mu^c = \tilde{A}_\alpha$ *and* (b) $F_\mu \tilde{\cap} F_\mu^c = \Phi_\theta$.

Note 3.16. The law of excluded middle and the law of contradiction holds here.

Proposition 3.17. *Let F_μ, G_δ and H_λ be any three GFSS over (U, E), then the following holds:*

(i) $F_\mu \tilde{\cup} G_\delta = G_\delta \tilde{\cup} F_\mu$
(ii) $F_\mu \tilde{\cap} G_\delta = G_\delta \tilde{\cap} F_\mu$
(iii) $F_\mu \tilde{\cup} (G_\delta \tilde{\cup} H_\lambda) = (F_\mu \tilde{\cup} G_\delta) \tilde{\cup} H_\lambda$
(iv) $F_\mu \tilde{\cap} (G_\delta \tilde{\cap} H_\lambda) = (F_\mu \tilde{\cap} G_\delta) \tilde{\cap} H_\lambda$

Proof. The properties follow from definition. □

Note 3.18. The following does not hold here:

(i) $F_\mu \tilde{\cap} (G_\delta \tilde{\cup} H_\lambda) = (F_\mu \tilde{\cap} G_\delta) \tilde{\cup} (F_\mu \tilde{\cap} H_\lambda)$
(ii) $F_\mu \tilde{\cup} (G_\delta \tilde{\cap} H_\lambda) = (F_\mu \tilde{\cup} G_\delta) \tilde{\cap} (F_\mu \tilde{\cup} H_\lambda)$

But if we take standard fuzzy operations then distributive property holds.

Proposition 3.19. *Let F_μ and G_δ are two GFSS over (U, E), then the following holds:*

(i) $(F_\mu \tilde{\cap} G_\delta)^c = (F_\mu^c \tilde{\cup} G_\delta^c)$
(ii) $(F_\mu \tilde{\cup} G_\delta)^c = (F_\mu^c \tilde{\cap} G_\delta^c)$.

Proof. The proof follows from definition. □

4. Relation on generalised fuzzy soft sets

The notion of relation on two fuzzy soft sets and fuzzy soft relation was introduced by Som (Definition 2.14). Later the concept of relations on intuitionistic fuzzy soft sets was introduced by Mukherjee and Chakraborty [12]. In this section we have defined fuzzy soft relation and generalised fuzzy soft relation in GFSS settings.

Definition 4.1. Let F_μ and G_δ be two GFSS over the parametrized universe (U, E) and $C \subseteq E^2$. Then a fuzzy soft relation R from F_μ to G_δ is a function $R : C \to I^U \times I$, defined as follows:

$$R(e, f) = F_\mu(e) \tilde{\cap} G_\delta(f) \quad \text{for all } (e, f) \in C.$$

A Generalisation of this may be:

Definition 4.2. Let $F = \{F^i_{\mu_i}, i \in \Delta\}$, where Δ is the index set, be any collection of GFSS over (U, E) and $C \subseteq E^n$. Then an n-ary generalised fuzzy soft relation R on F is the mapping $R : C \to I^U \times I$, defined by $R(e_{i_1}, e_{i_2}, \ldots .e_{i_n}) = \bigcap_{j=1}^n F^{i_j}_{\mu_j}(e_{i_j})$, where $(e_{i_1}, e_{i_2}, \ldots, e_{i_n}) \in C$.

An application of this Generalised fuzzy soft relation in a decision making problem is shown below.

Suppose the universe consists of four machines, x_1, x_2, x_3, x_4, i.e. $U = \{x_1, x_2, x_3, x_4\}$ and there are three parameters $e_i, i = 1, 2, 3$ which describe their performances according certain specific task. Hence $E = \{e_1, e_2, e_3\}$. Suppose a firm wants to buy one such machine depending on any two of the parameters only. Let there be two observations F_μ and G_δ by two experts A and B respectively.

Let their corresponding membership matrices be as follows:

$$F_\mu = (F, \mu) = \begin{pmatrix} 0.4 & 0.2 & 0.1 & 0.6 & 0.5 \\ 0.7 & 0.8 & 0.5 & 0.4 & 0.6 \\ 0.6 & 0.4 & 0.5 & 0.6 & 0.8 \end{pmatrix} \quad \text{and} \quad G_\delta = (G, \delta) = \begin{pmatrix} 0.4 & 0.6 & 0.5 & 0.3 & 0.5 \\ 0.8 & 0.4 & 0.9 & 0.6 & 0.7 \\ 0.1 & 0.2 & 0.1 & 0.4 & 0.3 \end{pmatrix}.$$

Let $R : C \to I^U \times I$, be the generalised fuzzy soft relation between F_μ and G_δ, defined as follows:

R	x_1	x_2	x_3	x_4	λ
(e_1, e_1)	(0.4)	0.2	0.1	0.3	0.5
(e_1, e_2)	0.1	0.2	0.1	(0.6)	0.5
(e_1, e_3)	0.1	0.2	0.1	(0.4)	0.3
(e_2, e_1)	0.4	(0.6)	0.5	0.3	0.5
(e_2, e_2)	(0.7)	0.4	0.5	0.4	0.6
(e_2, e_3)	0.1	0.2	0.1	(0.4)	0.3
(e_3, e_1)	0.4	0.4	(0.5)	0.3	0.5
(e_3, e_2)	0.6	0.4	(0.5)	0.6	0.7
(e_3, e_3)	0.1	0.2	0.1	(0.4)	0.3

Now to determine the best machine we first mark the highest numerical grade (indicated in parenthesis) in each row excluding the last column which is the grade of such belongingness of a machine against each pair of parameters. Now the score of each of such machines is calculated by taking the sum of the products of these numerical grades with the corresponding values of λ. The machine with the highest score is the desired machine. We do not consider the numerical grades of the machines against the pairs (e_i, e_i), $i = 1, 2, 3$, as both the parameters are same.

Grade table

R	(e_1, e_1)	(e_1, e_2)	(e_1, e_3)	(e_2, e_1)	(e_2, e_2)	(e_2, e_3)	(e_3, e_1)	(e_3, e_2)	(e_3, e_3)
x_i	x_1	x_4	x_4	x_2	x_1	x_4	x_3	x_3	x_4
highest numerical grade	×	0.6	0.4	0.6	×	0.4	0.5	0.5	×
λ		0.5	0.3	0.5		0.3	0.5	0.7	

$Score(x_1) = 0$, $\quad Score(x_2) = 0.6 \times 0.5 = 0.30$, $\quad Score(x_3) = 0.5 \times 0.5 + 0.5 \times 0.7 = 0.60$ and
$Score(x_4) = 0.6 \times 0.5 + 0.4 \times 0.3 + 0.4 \times 0.3 = 0.54$.

Then the firm will select the machine with highest score. Hence they will buy machine x_3.

5. Similarity between two generalised fuzzy soft sets

In several problems it is often required to compare two sets. The sets may be fuzzy, may be vague etc. We often interested to know whether two patterns or images are identical or approximately identical or at least to what degree they are identical. Several researchers have studied the problem of similarity measurement between fuzzy sets, fuzzy numbers and vague sets. Recently Majumdar and Samanta [7,8] have studied the similarity measure of soft sets and fuzzy soft sets. Similarity measures have extensive application in several areas such as pattern recognition, image processing, region extraction, coding theory etc.

In this section a measure of similarity between two GFSS has been given. The set theoretic approach has been taken in this regard because it is easier for calculation and is a very popular method too.

Let $U = \{x_1, x_2, \ldots, x_n\}$ be the universal set of elements and $E = \{e_1, e_2, \ldots, e_m\}$ be the universal set of parameters. Let F_ρ and G_δ be two GFSS over the parametrized universe (U, E). Hence $F_\rho = \{(F(e_i), \rho(e_i)),\ i = 1, 2, \ldots, m\}$ and $G_\delta = \{(G(e_i), \delta(e_i)),\ i = 1, 2, \ldots, m\}$.

Thus $\hat{F} = \{F(e_i),\ i = 1, 2, \ldots, m\}$ and $\hat{G} = \{G(e_i),\ i = 1, 2, \ldots, m\}$ are two families of fuzzy soft sets.

Now the similarity between $\hat{F}$ and $\hat{G}$ is found first and it is denoted by $M(\hat{F}, \hat{G})$. Next the similarity between the two fuzzy sets ρ and δ is found and is denoted by $m(\rho, \delta)$. Then the similarity between the two GFSS F_ρ and G_δ is denoted as $S(F_\rho, G_\delta) = M(\hat{F}, \hat{G}) \cdot m(\rho, \delta)$.

Here

$$M(\hat{F}, \hat{G}) = \max_i M_i(\hat{F}, \hat{G}), \quad \text{where } M_i(\hat{F}, \hat{G}) = 1 - \frac{\sum_{j=1}^{n} \left|\hat{F}_{ij} - \hat{G}_{ij}\right|}{\sum_{j=1}^{n} (\hat{F}_{ij} + \hat{G}_{ij})}, \quad \hat{F}_{ij} = \mu_{\hat{F}(e_i)}(x_j) \text{ and } \hat{G}_{ij} = \mu_{\hat{G}(e_i)}(x_j).$$

Also

$$m(\rho, \delta) = 1 - \frac{\sum |\rho_i - \delta_i|}{\sum (\rho_i + \delta_i)}, \quad \text{where } \rho_i = \rho(e_i) \text{ and } \delta_i = \delta(e_i).$$

Example 5.1. Consider the following two GFSS where $U = \{x_1, x_2, x_3, x_4\}$ and $E = \{e_1, e_2, e_3\}$:

$$F_\rho = \begin{pmatrix} 0.2 & 0.5 & 0.9 & 1.0 & 0.6 \\ 0.1 & 0.2 & 0.6 & 0.5 & 0.8 \\ 0.2 & 0.4 & 0.7 & 0.9 & 0.4 \end{pmatrix} \quad \text{and} \quad G_\delta = \begin{pmatrix} 0.4 & 0.3 & 0.2 & 0.9 & 0.5 \\ 0.6 & 0.5 & 0.2 & 0.1 & 0.7 \\ 0.4 & 0.3 & 0.2 & 0.1 & 0.9 \end{pmatrix}.$$

Here

$$m(\rho, \delta) = 1 - \frac{\sum |\rho_i - \delta_i|}{\sum (\rho_i + \delta_i)} = 1 - \frac{0.1 + 0.1 + 0.5}{1.1 + 1.5 + 1.3} = 0.82.$$

And

$$M_1(\hat{F}, \hat{G}) \cong 0.73, \qquad M_2(\hat{F}, \hat{G}) \cong 0.43, \qquad M_3(\hat{F}, \hat{G}) = 0.50.$$
$$\therefore M(\hat{F}, \hat{G}) = 0.73.$$

Hence the similarity between the two GFSS F_ρ and G_δ will be

$$S(F_\rho, G_\delta) = M(\hat{F}, \hat{G}) \cdot m(\rho, \delta) = 0.73 \times 0.82 \cong 0.60.$$

Proposition 5.2. *Let F_μ and G_δ be two GFSS over (U, E). Then the following holds:*

(i) $S(F_\mu, G_\delta) = S(G_\delta, F_\mu)$,
(ii) $0 \le S(F_\mu, G_\delta) \le 1$,
(iii) $F_\mu = G_\delta \Rightarrow S(F_\mu, G_\delta) = 1$,
(iv) $F_\mu \subseteq G_\delta \subseteq H_\tau \Rightarrow S(F_\mu, H_\tau) \le S(G_\delta, H_\tau)$,
(v) $F_\mu \tilde{\cap} G_\delta = \tilde{\Phi} \Leftrightarrow S(F_\mu, G_\delta) = 0$

where minimum operation has been taken as generalised fuzzy intersection.

Proof. The proofs are straightforward and follow from definition. □

6. An application of this similarity measure in medical diagnosis

This technique of similarity measure between two GFSS can be applied to detect whether an ill person is suffering from a certain disease or not.

We first give the following definition:

Definition 6.1. Let F_μ and G_δ be two generalised fuzzy soft sets over the same soft universe (U, E). We call the two generalised fuzzy soft sets to be significantly similar if $S(F_\mu, G_\delta) > \frac{1}{2}$.

Table 1
Model GFSS for pneumonia.

M_μ	e_1	e_2	e_3	e_4	e_5	e_6	e_7
y	1	1	0	1	0	0	1
n	0	0	1	0	1	1	0
μ	1	1	1	1	1	1	1

Table 2
GFSS for the first ill person.

G_δ	e_1	e_2	e_3	e_4	e_5	e_6	e_7
y	0.7	0	0.6	0.6	0.5	0.1	0.1
n	0.1	0.3	0.2	0.1	0.3	0.7	0.6
δ	0.2	0.8	0.7	0.4	0.3	0.8	0.9

Table 3
GFSS for the second ill person.

H_ν	e_1	e_2	e_3	e_4	e_5	e_6	e_7
y	0.8	0.9	0.2	0.6	0.5	0.1	0.8
n	0.1	0.1	0.2	0.1	0.3	0.7	0.1
ν	0.9	0.8	0.7	0.8	0.7	0.8	0.9

Table 4
GFSS for the third ill person.

P_λ	e_1	e_2	e_3	e_4	e_5	e_6	e_7
y	0.8	0.9	0.2	0.6	0.5	0.1	0.8
n	0.1	0.1	0.2	0.1	0.3	0.7	0.1
λ	0.2	0.3	0.2	0.6	0.3	0.2	0.4

In the following example we will try to estimate the possibility that an ill person having certain visible symptoms is suffering from pneumonia. For this we first construct a model generalised fuzzy soft set for pneumonia and the generalised fuzzy soft set of symptoms for the ill person. Next we find the similarity measure of these two sets. If they are significantly similar then we conclude that the person is possibly suffering from pneumonia.

Let our universal set contain only two elements 'yes' and 'no', i.e. $U = \{y, n\}$. Here the set of parameters E is the set of certain visible symptoms. Let $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$, where e_1 = body temperature, e_2 = cough with chest congestion, e_3 = cough with no chest congestion, e_4 = body ache, e_5 = headache, e_6 = loose motion, e_7 = breathing trouble.

Our model generalised fuzzy soft set for pneumonia M_μ is given in Table 1 and this can be prepared with the help of a physician.

Now the ill person is having fever, cough and headache. After talking to him we can construct his GFSS G_δ as in Table 2.

Now here $S(M_\mu, G_\delta) \cong 0.23 < \frac{1}{2}$. Hence the two GFSS are not significantly similar. Therefore we conclude that the person is not suffering from pneumonia.

Again consider another ill person with some symptoms whose corresponding GFSS is given in Table 3.

Here $S(M_\mu, H_\nu) \cong 0.8 > \frac{1}{2}$. Hence the two GFSS are significantly similar. So we conclude that this person is suffering from pneumonia.

One should note that unlike [2] here the result depends not only on $H(e_i)$ but also on $\nu(e_i)$, i.e. on the reliability of the data also. For example consider the ill person with data as in Table 4.

Here

$$\mu_{P(e_i)}(x_j) = \mu_{H(e_i)}(x_j) \quad \forall\, i, j.$$

But

$$S(M_\mu, P_\lambda) \cong 0.43 < \frac{1}{2}.$$

This is because the values of $\nu(e_i)$ and $\lambda(e_i)$ are different.

This is only a simple example to show the possibility of using this method for diagnosis of disease which could be improved by incorporating clinical results and other competing diagnosis.

7. Conclusion

In this paper we have introduced the concept of generalised fuzzy soft set and studied some of its properties. An application of this theory has been applied to solve a decision making problem. Similarity measure of two generalised fuzzy soft sets is discussed and an application of this to medical diagnosis has been shown. The authors are hopeful that this modified concept will be helpful in dealing with several problems related to uncertainty and will yield more natural results.

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