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An Application of Soft Sets in A Decision Making Problem

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Abstract—In this paper, we apply the theory of soft sets to solve a decision making problem using rough mathematics.

Keywords—Soft set, Reduct-soft-set, Choice value, Weighted choice value, Fuzzy set, Rough set.

1. INTRODUCTION

Most of our traditional tools for formal modeling, reasoning, and computing are crisp, deterministic, and precise in character. But many complicated problems in economics, engineering, environment, social science, medical science, etc., involve data which are not always all crisp. We cannot always use the classical methods because of various types of uncertainties present in these problems. The important existing theories *viz.* theory of probability, theory of fuzzy sets [1], theory of intuitionistic fuzzy sets [2,3], theory of vague sets [4], theory of interval mathematics [3,5], theory of rough sets [6] can be considered as mathematical tools for dealing with uncertainties. But all these theories have their own difficulties as pointed out in [7]. The reason for these difficulties is, possibly, the inadequacy of the parameterization tool of the theories; and consequently, Molodtsov [7] initiated the concept of soft theory as a new mathematical tool for dealing with uncertainties which is free from the above difficulties. Soft set theory has a rich potential for applications in several directions, few of which had been shown by Molodtsov in his pioneer work [7]. Soft sets are called (binary, basic, elementary) neighbourhood systems [8] and are a special case of context dependent fuzzy sets, as defined by Thielle [9]. In [10], we have made a theoretical study of the "soft set theory". In this paper, we present an application of soft sets in a decision making problem with the help of rough mathematics of Pawlak [6,11]. Earlier, a

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rough set representation, and hence, application done by Lin [12] and Yao [8]. We have used here an almost analogous representation of the soft sets in the form of a binary information table.

2. PRELIMINARIES

In this section, we present the notion of soft sets introduced by Molodtsov in [7], and some useful definitions from [11] on rough mathematics.

2.1. Definition of Soft Set

Let U be an initial universe set and let E be a set of parameters.

DEFINITION 2.1. *(See [7].) A pair (F, E) is called a soft set over U if and only if F is a mapping of E into the set of all subsets of the set U, i.e., $F : E \to P(U)$, where $P(U)$ is the power set of U.*

In other words, the soft set is a parameterized family of subsets of the set U. Every set $F(\epsilon)$, for $\epsilon \in E$, from this family may be considered as the set of ϵ-elements of the soft set (F, E), or as the set of ϵ-approximate elements of the soft set.

As an illustration, let us consider the following examples (quoted from [7]).

(1) A soft set (F, E) describes the attractiveness of the houses which Mr. X is going to buy.
U = the set of houses under consideration.
E = the set of parameters. Each parameter is a word or a sentence.
E = {expensive; beautiful; wooden; cheap; in the green surroundings; modern; in good repair; in bad repair}.
In this case, to define a soft set means to point out expensive houses, beautiful houses, and so on. It is worth noting that the sets $F(\epsilon)$ may be empty for some $\epsilon \in E$.

(2) Zadeh's fuzzy set may be considered as a special case of the soft set. Let A be a fuzzy set of U with membership μ_A, i.e., μ_A is a mapping of U into $[0, 1]$.
Let us consider the family of α-level sets for the function μ_A given by
$$F(\alpha) = \{x \in U : \mu_A(x) \geq \alpha\}, \qquad \alpha \in [0, 1].$$
If we know the family F, we can find the functions $\mu_A(x)$ by means of the following formulae:
$$\mu_A(x) = \sup_{\alpha \in [0,1],\, x \in F(\alpha)} (\alpha).$$
Thus, every Zadeh's fuzzy set A may be considered as the soft set $(F, [0, 1])$.

(3) Let (X, τ) be a topological space, that is, X is a set and τ is a topology, in other words, τ is a family of subsets of X, called the open sets of X.
Then, the family of open neighbourhoods $T(x)$ of point x, where $T(x) = \{V \in \tau : x \in V\}$, may be considered as the soft set $(T(x), \tau)$.

The way of setting (or describing) any object in the soft set theory principally differs from the way in which we use classical mathematics. In classical mathematics, we construct a mathematical model of an object and define the notion of the exact solution of this model. Usually the mathematical model is too complicated and we cannot find the exact solution. So, in the second step, we introduce the notion of approximate solution and calculate that solution.

In the soft set theory, we have the opposite approach to this problem. The initial description of the object has an approximate nature, and we do not need to introduce the notion of exact solution.

The absence of any restrictions on the approximate description in soft set theory makes this theory very convenient and easily applicable in practice. We can use any parameterization we prefer: with the help of words and sentences, real numbers, functions, mappings, and so on. It means that the problem of setting the membership function or any similar problem does not arise in the soft set theory.

DEFINITION 2.2. *(See [11].) A knowledge representation system can be formulated as follows: knowledge representation system is a pair $S = (U, A)$, where*

U = *a nonempty, finite set called the universe.*

A = *a nonempty, finite set of primitive attributes.*

Every primitive attribute $a \in A$ is a total function $a : U \rightarrow V_a$, where V_a is the set of values of a, called the domain of a.

DEFINITION 2.3. *(See [11].) With every subset of attributes $B \subseteq A$, we associate a binary relation* $\mathrm{IND}(B)$, *called an indiscernibility relation, defined by*

$$\mathrm{IND}(B) = \left\{(x,y) \in U^2 : \text{for every } a \in B,\ a(x) = a(y)\right\}.$$

Obviously, $\mathrm{IND}(B)$ *is an equivalence relation and*

$$\mathrm{IND}(B) = \bigcap_{a \in B} \mathrm{IND}(a).$$

Every subset $B \subseteq A$ will be called an *attribute*. If B is a single element set, then B is called *primitive*, otherwise the attribute is said to be *compound*. Attribute B may be considered as a name of the relation $\mathrm{IND}(B)$, or in other words—as a name of knowledge represented by an equivalence relation $\mathrm{IND}(B)$.

DEFINITION 2.4. *(See [11].) Let R be a family of equivalence relations and let $A \in R$. We will say that A is dispensable in R if* $\mathrm{IND}(R) = \mathrm{IND}(R - \{A\})$; *otherwise A is indispensable in R. The family R is independent if each $A \in R$ is indispensable in R; otherwise R is dependent.*

If R is independent and $P \subseteq R$, then P is also independent.

$Q \subset P$ is a reduct of P if Q is independent and $\mathrm{IND}(Q) = \mathrm{IND}(P)$.

Intuitively, a reduct of knowledge is its essential part, which suffices to define all basic concepts occurring in the considered knowledge, whereas the core is in a certain sense its most important part. The set of all indispensable relations in P will be called the core of P, and will be denoted $\mathrm{CORE}(P)$. *Clearly,* $\mathrm{CORE}(P) = \cap\mathrm{RED}(P)$, *where* $\mathrm{RED}(P)$ *is the family of all reducts of P.*

The use of the concept of the core is twofold. First, it can be used as a basis for computation of all reducts, for the core is included in every reduct, and its computation is straightforward. Secondly, the core can be interpreted as the set of the most characteristic part of knowledge, which cannot be eliminated when reducing the knowledge.

DEFINITION 2.5. DEPENDENCY OF KNOWLEDGE. *(See [11].) Formally, the dependency can be defined as shown below: Let $K = (U, R)$ be a knowledge base and let $P, Q \subseteq R$.*

1. *Knowledge Q depends on knowledge P iff* $\mathrm{IND}(P) \subseteq \mathrm{IND}(Q)$.
2. *Knowledge P and Q are equivalent, denoted as $P \equiv Q$, iff $P \Rightarrow Q$ and $Q \Rightarrow P$.*
3. *Knowledge P and Q are independent, denoted as $P \not\equiv Q$, iff*

$$\text{neither } P \Rightarrow Q \text{ nor } Q \Rightarrow P \text{ hold.}$$

Obviously, $P \equiv Q$, iff $\mathrm{IND}(P) = \mathrm{IND}(Q)$.

DEFINITION 2.6. *(See [10].) For two soft sets (F, A) and (G, B) over a common universe U, we say that (F, A) is a soft subset of (G, B) if*

(i) *$A \subset B$, and*

(ii) *$\forall \epsilon \in A$, $F(\epsilon)$, and $G(\epsilon)$ are identical approximations.*

We write $(F, A)\tilde{\subset}(G, B)$.

(F, A) is said to be a soft super set of (G, B), if (G, B) is a soft subset of (F, A). We denote it by $(F, A)\tilde{\supset}(G, B)$.

3. AN APPLICATION OF SOFT SET THEORY

Molodtsov [7] presented some applications of the soft set theory in several directions *viz.* study of smoothness of functions, game theory, operations research, Riemann-integration, Perron integration, probability, theory of measurement, etc. In this section, we present an application of soft set theory in a decision making problem with the help of rough approach [11]. The problem we consider is as below.

Let $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$, be a set of six houses, $E =$ {expensive; beautiful; wooden; cheap; in the green surroundings; modern; in good repair; in bad repair}, be a set of parameters.

Consider the soft set (F, E) which describes the 'attractiveness of the houses', given by $(F, E) =$ {expensive houses $= \phi$, beautiful houses $= \{h_1, h_2, h_3, h_4, h_5, h_6\}$, wooden houses $= \{h_1, h_2, h_6\}$, modern houses $= \{h_1, h_2, h_6\}$, in bad repair houses $= \{h_2, h_4, h_5\}$, cheap houses $= \{h_1, h_2, h_3, h_4, h_5, h_6\}$, in good repair houses $= \{h_1, h_3, h_6\}$, in the green surroundings houses $= \{h_1, h_2, h_3, h_4, h_6\}$}.

Suppose that, Mr. X is interested to buy a house on the basis of his choice parameters 'beautiful', 'wooden', 'cheap', 'in the green surroundings', 'in good repair', etc., which constitute the subset $P =$ {beautiful; wooden; cheap; in the green surroundings; in good repair} of the set E. That means, out of available houses in U, he is to select that house which qualifies with all (or with maximum number of) parameters of the soft set P.

Suppose that, another customer Mr. Y wants to buy a house on the basis of the sets of choice parameters $Q \subset E$, where, $Q =$ {expensive; beautiful; in the green surroundings; in good repair}, and Mr. Z wants to buy a house on the basis of another set of parameters $R \subset E$.

The problem is to select the house which is most suitable with the choice parameters of Mr. X. The house which is most suitable for Mr. X, need not be most suitable for Mr. Y or Mr. Z as the selection is dependent upon the set of choice parameters of each buyer.

To solve the problem, we do some theoretical characterizations of the soft set theory of Molodtsov, which we present below.

3.1. Tabular Representation of a Soft Set

Tabular representation of soft sets were done by Lin [12] and Yao [8] earlier. We present an almost analogous representation in the form of a binary table. For this consider the soft set (F, P) above on the basis of the set P of choice parameters of Mr. X. We can represent this soft set in a tabular form as shown below. This style of representation will be useful for storing a soft set in a computer memory. If $h_i \in F(\epsilon)$ then $h_{ij} = 1$, otherwise $h_{ij} = 0$, where h_{ij} are the entries in Table 1.

Table 1.

U	e_1	e_2	e_3	e_4	e_5
h_1	1	1	1	1	1
h_2	1	1	1	1	0
h_3	1	0	1	1	1
h_4	1	0	1	1	0
h_5	1	0	1	0	0
h_6	1	1	1	1	1

Thus, a soft set now can be viewed as a knowledge representation system (Definition 2.2) where the set of attributes is to be replaced by a set of parameters.

3.2. Reduct-Table of a Soft Set

Consider the soft set (F, E). Clearly, for any $P \subset E$, (F, P) is a soft subset of (F, E). We will now define a reduct-soft-set of the soft set (F, P).

Consider the tabular representation of the soft set (F, P). If Q is a reduct of P, then the soft set (F, Q) is called the reduct-soft-set of the soft set (F, P).

Intuitively, a reduct-soft-set (F, Q) of the soft set (F, P) is the essential part, which suffices to describe all basic approximate descriptions of the soft set (F, P).

The core soft set of (F, P) is the soft set (F, C), where C is the CORE(P).

3.3. Choice Value of an Object h_i

The choice value of an object $h_i \in U$ is c_i, given by

$$c_i = \sum_j h_{ij},$$

where h_{ij} are the entries in the table of the reduct-soft-set.

3.4. Algorithm for Selection of the House

The following algorithm may be followed by Mr. X to select the house he wishes to buy

1. input the soft set (F, E),
2. input the set P of choice parameters of Mr. X which is a subset of E,
3. find all reduct-soft-sets of (F, P),
4. choose one reduct-soft-set say (F, Q) of (F, P),
5. find k, for which $c_k = \max c_i$.
   Then h_k is the optimal choice object. If k has more than one value, then any one of them could be chosen by Mr. X by using his option.

Now we use the algorithm to solve our original problem.

Clearly, from the table we see that $\{e_1, e_2, e_4, e_5\}$, $\{e_2, e_3, e_4, e_5\}$ are the two reducts of $P = \{e_1, e_2, e_3, e_4, e_5\}$. Choose any one say, $Q = \{e_1, e_2, e_4, e_5\}$.

Incorporating the choice values, the reduct-soft-set can be represented in Table 2 below.

Table 2.

U	e_1	e_2	e_4	e_5	choice value
h_1	1	1	1	1	$c_1 = 4$
h_2	1	1	1	0	$c_2 = 3$
h_3	1	0	1	1	$c_3 = 3$
h_4	1	0	1	0	$c_4 = 2$
h_5	1	0	0	0	$c_5 = 1$
h_6	1	1	1	1	$c_6 = 4$

Here $\max c_i = c_1$ or c_6.

Decision: Mr. X can buy either the house h_1 or the house h_6.

It may happen that for buying a house, all the parameters belonging to P are not of equal importance to Mr. X. He likes to impose weights on his choice parameters, i.e., corresponding to each element $p_i \in P$, there is a weight $w_i \in (0, 1]$.

3.5. Weighted Table of a Soft Set

Lin [13] asked a very fundamental question: should a membership function be regarded as the only characteristic function of a fuzzy set? While answering this question by himself in [13], he defined a new theory of mathematical analysis which is "theory of W-softsets". 'W-softsets' means weighted soft sets. Following Lin's style, we define the weighted table of the reduct-soft-set (F, Q) will have entries $d_{ij} = w_j \times h_{ij}$, instead of 0 and 1 only, where h_{ij} are the entries in the table of the reduct-soft-set (F, Q).

3.6. Weighted Choice Value of an Object h_i

The weighted choice value of an object $h_i \in U$ is c_i, given by

$$c_i = \sum_j d_{ij},$$

where $d_{ij} = w_j \times h_{ij}$. Imposing weights on his choice parameters, Mr. X now could use the following revised algorithm for arriving at his final decision.

3.7. Revised Algorithm for Selection of the House

1. Input the soft set (F, E),
2. input the set P of choice parameters of Mr. X which is a subset of E,
3. find all reduct-soft-sets of (F, P),
4. choose one reduct-soft-set say (F, Q) of (F, P),
5. find weighted table of the soft set (F, Q) according to the weights decided by Mr. X,
6. find k, for which $c_k = \max c_i$.
   Then h_k is the optimal choice object. If k has more than one value, then any one of them could be chosen by Mr. X, by using his option.

Let us solve now the original problem using the revised algorithm.

Suppose that Mr. X sets the following weights for the parameters of Q: for the parameter "beautiful", $w_1 = 0.8$, for the parameter "wooden", $w_2 = 0.3$, for the parameter "in the green surroundings", $w_4 = 0.9$, for the parameter "in good repair", $w_5 = 0.8$.

From Table 3 it is seen that Mr. X will select the house h_1 or h_6 for buying according to his choice parameters in P.

Table 3.

U	$e_1, w_1 = .8$	$e_2, w_2 = .3$	$e_4, w_4 = .9$	$e_5, w_5 = .8$	Choice Value
h_1	1	1	1	1	$c_1 = 2.8$
h_2	1	1	1	0	$c_2 = 2.0$
h_3	1	0	1	1	$c_3 = 2.5$
h_4	1	0	1	0	$c_4 = 1.7$
h_5	1	0	0	0	$c_5 = 0.8$
h_6	1	1	1	1	$c_6 = 2.8$

4. CONCLUSION

The soft set theory of Molodtsov [7] offers a general mathematical tool for dealing with uncertain, fuzzy, or vague objects. Molodtsov in [7] has given several possible applications of soft set theory. In the present paper, we give an application of soft set theory in a decision making problem by the rough technique of Pawlak [11].

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