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Boundary layer flow of nanofluid over an exponentially stretching surface

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Abstract

The steady boundary layer flow of nanofluid over an exponential stretching surface is investigated analytically. The transport equations include the effects of Brownian motion parameter and thermophoresis parameter. The highly nonlinear coupled partial differential equations are simplified with the help of suitable similarity transformations. The reduced equations are then solved analytically with the help of homotopy analysis method (HAM). The convergence of HAM solutions are obtained by plotting *h*-curve. The expressions for velocity, temperature and nanoparticle volume fraction are computed for some values of the parameters namely, suction injection parameter α, Lewis number *Le*, the Brownian motion parameter *Nb* and thermophoresis parameter *Nt*.

Keywords: nanofluid, porous stretching surface, boundary layer flow, series solutions, exponential stretching

1 Introduction

During the last many years, the study of boundary layer flow and heat transfer over a stretching surface has achieved a lot of success because of its large number of applications in industry and technology. Few of these applications are materials manufactured by polymer extrusion, drawing of copper wires, continuous stretching of plastic films, artificial fibers, hot rolling, wire drawing, glass fiber, metal extrusion and metal spinning etc. After the pioneering work by Sakiadis [1], a large amount of literature is available on boundary layer flow of Newtonian and non-Newtonian fluids over linear and nonlinear stretching surfaces [2-10]. However, only a limited attention has been paid to the study of exponential stretching surface. Mention may be made to the works of Magyari and Keller [11], Sanjayanand and Khan [12], Khan and Sanjayanand [13], Bidin and Nazar [14] and Nadeem et al. [15,16].

More recently, the study of convective heat transfer in nanofluids has achieved great success in various industrial processes. A large number of experimental and theoretical studies have been carried out by numerous researchers on thermal conductivity of nanofluids [17-22]. The theory of nanofluids has presented several fundamental properties with the large enhancement in thermal conductivity as compared to the base fluid [23].

In this study, we have discussed the boundary layer flow of nanofluid over an exponentially stretching surface with suction and injection. To the best of our knowledge, the nanofluid over an exponentially stretching surface has not been discussed so far. However, the present paper is only a theoretical idea, which is not checked experimentally. The governing highly nonlinear partial differential equation of motion, energy and nanoparticle volume fraction has been simplified by using suitable similarity transformations and then solved analytically with the help of HAM [24-39]. The convergence of HAM solution has been discussed by plotting h-curve. The effects of pertinent parameters of nanofluid have been discussed through graphs.

2 Formulation of the problem

Consider the steady two-dimensional flow of an incompressible nanofluid over an exponentially stretching surface. We are considering Cartesian coordinate system in such a way that x-axis is taken along the stretching surface in the direction of the motion and y-axis is normal to it. The plate is stretched in the x-direction with a velocity $U_w = U_0 \exp(x/l)$. defined at $y = 0$. The flow and heat transfer characteristics under the boundary layer approximations are governed by the following equations

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$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=v\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{(\rho c)_p}{(\rho c)_f}\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right], \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial y^2}\right), \tag{4}$$

where (u, v) are the velocity components in (x, y) directions, ρ_f is the fluid density of base fluid, v is the kinematic viscosity, T is the temperature, C is the nanoparticle volume fraction, $(\rho c)_p$ is the effective heat capacity of nanoparticles, $(\rho c)_f$ is the heat capacity of the fluid, $\alpha = k/(\rho c)_f$ is the thermal diffusivity of the fluid, D_B is the Brownian diffusion coefficient and D_T is the thermophoretic diffusion coefficient.

The corresponding boundary conditions for the flow problem are

$$\begin{aligned}&u=U_w(x)=U_0 exp\,(x/l)\,,\quad v=-\beta\,(x)\,,\quad T=T_w,\quad C=C_w\quad at\quad y=0,\\&u=0,\quad T=T_\infty\quad C=C_\infty\quad as\ y\rightarrow\infty,\end{aligned} \tag{5}$$

in which U_0 is the reference velocity, $\beta(x)$ is the suction and injection velocity when $\beta(x) > 0$ and $\beta(x) < 0$, respectively, T_w and T_∞ are the temperatures of the sheet and the ambient fluid, C_w, C_∞ are the nanoparticles volume fraction of the plate and the fluid, respectively.

We are interested in similarity solution of the above boundary value problem; therefore, we introduce the following similarity transformations

$$\begin{aligned}&u=U_0 exp\left(\frac{x}{l}\right)f'(\eta)\,,\quad v=-\sqrt{\frac{vU_0}{2l}}exp\left(\frac{x}{2l}\right)\{f(\eta)+\eta f'(\eta)\}\,,\\&\eta=y\sqrt{\frac{U_0}{2vl}}exp\left(\frac{x}{2l}\right),\quad\theta=\frac{T-T_\infty}{T_w-T_\infty},\quad g=\frac{C-C_\infty}{C_w-C_\infty}.\end{aligned} \tag{6}$$

Making use of transformations (6), Eq. (1) is identically satisfied and Equations (2)-(4) take the form

$$f_{\eta\eta\eta}+ff_{\eta\eta}-2f_\eta^2=0, \tag{7}$$

$$\theta_{\eta\eta}+Pr\left(f\theta_\eta-f_\eta\theta+Nb\theta_\eta g_\eta+Nt\theta_\eta^2\right)=0 \tag{8}$$

$$g_{\eta\eta}+Le\left(fg_\eta-f_\eta g\right)+\frac{Nt}{Nb}\theta_{\eta\eta}=0, \tag{9}$$

$$\begin{aligned}&f=-v_w,\quad f_\eta=1,\quad\theta=1,\quad g=1\ at\ \eta=0,\\&f_\eta\rightarrow 0,\quad\theta\rightarrow 0,\quad g\rightarrow 0\quad as\ \eta\rightarrow\infty,\end{aligned} \tag{10}$$

where

$$Nt=D_B\frac{(\rho c)_p}{(\rho c)_f}(C_w-C_\infty)\,,\quad Nb=\frac{D_T}{T_\infty}\frac{(\rho c)_p}{(\rho c)_f}\frac{(T_w-T_\infty)}{\upsilon},\quad Le=\frac{\upsilon}{D_B},\quad Pr=\frac{\upsilon}{\alpha}.$$

The physical quantities of interest in this problem are the local skin-friction coefficient C_f, Nusselt number Nu_x and the local Sherwood number Sh_x, which are defined as

$$\begin{aligned}&C_{f_x}=\frac{\tau_w|_{y=0}}{\rho U_0^2 e^{\frac{2x}{l}}},\quad Nu_x=-\frac{x}{(T_w-T_\infty)}\frac{\partial T}{\partial y}\bigg|_{y=0},\quad Sh_x=-\frac{x}{(C_w-C_\infty)}\frac{\partial C}{\partial y}\bigg|_{y=0},\\&\sqrt{2\mathrm{Re}}C_{f_x}=f''(0)\,,\quad Nu_x/\sqrt{2\mathrm{Re}_x}=-\sqrt{\frac{x}{2l}}\theta'(0)\,,\quad Sh_x/\sqrt{2\mathrm{Re}_x}=-\sqrt{\frac{x}{2l}}g'(0),\end{aligned} \tag{11}$$

where $\mathrm{Re}_x = U_w x/v$ is the local Renolds number.

3 Solution by homotopy analysis method

For HAM solutions, the initial guesses and the linear operators L_i (i = 1 - 3) are

$$f_0(\eta)=1-v_w-e^{-\eta},\quad\theta_0(\eta)=e^{-\eta},\quad g_0(\eta)=e^{-\eta}, \tag{12}$$

$$\mathcal{L}_1(f)=f'''-f',\quad\mathcal{L}_2(\theta)=\theta''-\theta,\quad\mathcal{L}_3(g)=g''-g. \tag{13}$$

The operators satisfy the following properties

$$\mathcal{L}_1\left[c_1e^{-\eta}+c_2e^{\eta}+c_3\right]=0, \tag{14}$$

$$\mathcal{L}_2\left[c_4e^{-\eta}+c_5e^{\eta}\right]=0, \tag{15}$$

$$\mathcal{L}_3\left[c_6e^{-\eta}+c_7e^{\eta}\right]=0, \tag{16}$$

in which C_1 to C_7 are constants. From Equations (7) *to* (9), we can define the following zeroth-order deformation problems

$$(1-p)\,\mathcal{L}_1\left[\hat f(\eta,p)-f_0(\eta)\right]=p\hbar_1 H_1\tilde N_1\left[\hat f(\eta,p)\right], \tag{17}$$

$$(1-p)\,\mathcal{L}_2\left[\hat\theta(\eta,p)-\theta_0(\eta)\right]=p\hbar_2 H_2\tilde N_2\left[\hat\theta(\eta,p)\right], \tag{18}$$

$$(1-p)\,\mathcal{L}_3\left[\hat g(\eta,p)-g_0(\eta)\right]=p\hbar_3 H_3\tilde N_3\left[\hat g(\eta,p)\right], \tag{19}$$

$$\hat f(0,p)=-v_w,\quad\hat f'(0,p)=1,\quad\hat f'(\infty,p)=0, \tag{20}$$

$$\hat\theta(0,p)=1,\quad\hat\theta(\infty,p)=0, \tag{21}$$

$$\hat g'(0,p)=1,\quad\hat g(\infty,p)=0. \tag{22}$$

In Equations (17)-(22), $\hbar_1$, $\hbar_2$, and $\hbar_3$ denote the nonzero auxiliary parameters, H_1, H_2 and H_3 are the nonzero auxiliary function ($H_1 = H_2 = H_3 = 1$) and

$$\tilde{N}_1\left[\hat{f}\left(\eta,p\right)\right]=\frac{\partial^3 f}{\partial\eta^3}-2\left(\frac{\partial f}{\partial\eta}\right)^2+f\frac{\partial^2 f}{\partial\eta^2}, \quad (23)$$

$$\tilde{N}_2\left[\hat{\theta}\left(\eta,p\right)\right]=\frac{\partial^2\theta}{\partial\eta^2}+Pr\left(f\frac{\partial\theta}{\partial\eta}-\frac{\partial f}{\partial\eta}\theta+Nb\frac{\partial\theta}{\partial\eta}\frac{\partial g}{\partial\eta}+Nt\left(\frac{\partial\theta}{\partial\eta}\right)^2\right), \quad (24)$$

$$\tilde{N}_3\left[\hat{g}\left(\eta,p\right)\right]=\frac{\partial^2 g}{\partial\eta^2}+Le\left(f\frac{\partial g}{\partial\eta}-\frac{\partial f}{\partial\eta}g+\frac{Nt}{Nb}\theta_{\eta\eta}\right)+\frac{Nt}{Nb}\frac{\partial^2\theta}{\partial\eta^2}. \quad (25)$$

Obviously

$$\hat{f}\left(\eta,0\right)=f_0\left(\eta\right), \quad \hat{f}\left(\eta,1\right)=f\left(\eta\right), \quad (26)$$

$$\hat{\theta}\left(\eta,0\right)=\theta_0\left(\eta\right), \quad \hat{\theta}\left(\eta,1\right)=\theta\left(\eta\right), \quad (27)$$

$$\hat{g}\left(\eta,0\right)=g_0\left(\eta\right), \quad \hat{g}\left(\eta,1\right)=g\left(\eta\right). \quad (28)$$

When p varies from 0 to 1, then $\hat{f}\left(\eta,p\right)$, $\hat{\theta}\left(\eta,p\right)$, $\hat{g}\left(\eta,p\right)$ vary from initial guesses $f_0\left(\eta\right)$, $\theta_0\left(\eta\right)$ and $g_0\left(\eta\right)$ to the final solutions $f\left(\eta\right)$, $\theta\left(\eta\right)$ and $g\left(\eta\right)$, respectively. Considering that the auxiliary parameters $\hbar_1$, $\hbar_2$ and $\hbar_3$ are so properly chosen that the Taylor series of $\hat{f}\left(\eta,p\right)$, $\hat{\theta}\left(\eta,p\right)$ and $\hat{g}\left(\eta,p\right)$ expanded with respect to an embedding parameter converge at $p=1$, hence Equations (17)-(19) become

$$\hat{f}\left(\eta,p\right)=f_0\left(\eta\right)+\sum_{m=1}^{\infty}f_m\left(\eta\right)p^m, \quad (29)$$

$$\hat{\theta}\left(\eta,p\right)=\theta_0\left(\eta\right)+\sum_{m=1}^{\infty}\theta_m\left(\eta\right)p^m, \quad (30)$$

$$\hat{g}\left(\eta,p\right)=g_0\left(\eta\right)+\sum_{m=1}^{\infty}g_m\left(\eta\right)p^m, \quad (31)$$

$$f_m\left(\eta\right)=\frac{1}{m!}\left.\frac{\partial^m\hat{f}\left(\eta,p\right)}{\partial p^m}\right|_{p=0}, \quad (32)$$

$$\theta_m\left(\eta\right)=\frac{1}{m!}\left.\frac{\partial^m\hat{\theta}\left(\eta,p\right)}{\partial p^m}\right|_{p=0}, \quad (33)$$

$$g_m\left(\eta\right)=\frac{1}{m!}\left.\frac{\partial^m\hat{g}\left(\eta,p\right)}{\partial p^m}\right|_{p=0}. \quad (34)$$

The mth-order problems are defined as follow

$$\mathcal{L}_1\left[f_m\left(\eta\right)-\chi_m f_{m-1}\left(\eta\right)\right]=\hbar_1\check{R}_m^1\left(\eta\right), \quad (35)$$

$$\mathcal{L}_2\left[\theta_m\left(\eta\right)-\chi_m\theta_{m-1}\left(\eta\right)\right]=\hbar_2\check{R}_m^2\left(\eta\right), \quad (36)$$

$$\mathcal{L}_3\left[g_m\left(\eta\right)-\chi_m g_{m-1}\left(\eta\right)\right]=\hbar_3\check{R}_m^3\left(\eta\right), \quad (37)$$

$$f_m\left(0\right)=f_m'\left(0\right)=f_m'\left(\infty\right)=0, \quad (38)$$

$$\theta_m\left(0\right)=\theta_m\left(\infty\right)=0, \quad (39)$$

$$g_m'\left(0\right)=g_m\left(\infty\right)=0, \quad (40)$$

where

$$\chi_m=\begin{cases}0, & m\leq 1,\\ 1, & m>1.\end{cases} \quad (41)$$

$$\check{R}_m^1\left(\eta\right)=f_{m-1}'''\left(\eta\right)+\sum_{k=0}^{m-1}f_{m-1-k}f_k''-2\sum_{k=0}^{m-1}f_{m-1-k}'f_k', \quad (42)$$

$$\check{R}_m^2\left(\eta\right)=\theta_{m-1}''+Pr\sum_{k=0}^{m-1}\left\{f_{m-1-k}\theta_k'-f_{m-1-k}'\theta_k+Nb\theta_{m-1-k}'g_k'+Nt\theta_{m-1-k}'\theta_k'\right\}, \quad (43)$$

$$\check{R}_m^3\left(\eta\right)=g_{m-1}''+Le\sum_{k=0}^{m-1}\left\{f_{m-1-k}g_k'-f_{m-1-k}'g_k\right\}+\frac{Nt}{Nb}\theta_{m-1}''. \quad (44)$$

Employing MATHEMATICA, Equations (35)-(40) have the following solutions

$$f\left(\eta\right)=\sum_{m=0}^{\infty}f_m\left(\eta\right)=\lim_{M\to\infty}\left[\sum_{m=0}^{M}a_{m,0}^0+\sum_{n=1}^{M+1}\mathrm{e}^{-n\eta}\left(\sum_{m=n-1}^{M}\sum_{k=0}^{m+1-n}a_{m,n}^k\eta^k\right)\right], \quad (45)$$

$$\theta\left(\eta\right)=\sum_{m=0}^{\infty}\theta_m\left(\eta\right)=\lim_{M\to\infty}\left[\sum_{n=1}^{M+2}\mathrm{e}^{-n\eta}\left(\sum_{m=n-1}^{M+1}\sum_{k=0}^{m+1-n}A_{m,n}^k\eta^k\right)\right], \quad (46)$$

$$g\left(\eta\right)=\sum_{m=0}^{\infty}g_m\left(\eta\right)=\lim_{M\to\infty}\left[\sum_{n=1}^{M+2}\mathrm{e}^{-n\eta}\left(\sum_{m=n-1}^{M+1}\sum_{k=0}^{m+1-n}F_{m,n}^k\eta^k\right)\right], \quad (47)$$

in which $a_{m,0}^0$, $a_{m,n}^k$, $A_{m,n}^k$, $F_{m,n}^k$ are the constants and the numerical data of above solutions are shown through graphs in the following section.

4 Results and discussion

The numerical data of the solutions (45)-(47), which is obtained with the help of Mathematica, have been discussed through graphs. The convergence of the series solutions strongly depends on the values of non-zero auxiliary parameters $\hbar_i$ ($i=1, 2, 3$, $h_1=h_2=h_3$), which can adjust and control the convergence of the solutions. Therefore, for the convergence of the solution, the $\hbar$-curves is plotted for velocity field in Figure 1. We have found the convergence region of velocity for different values of suction injection parameter ν_w. It is seen that

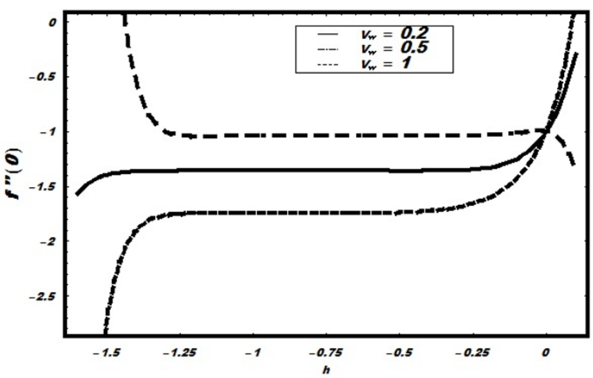

Figure 1 *h*-Curve for velocity.

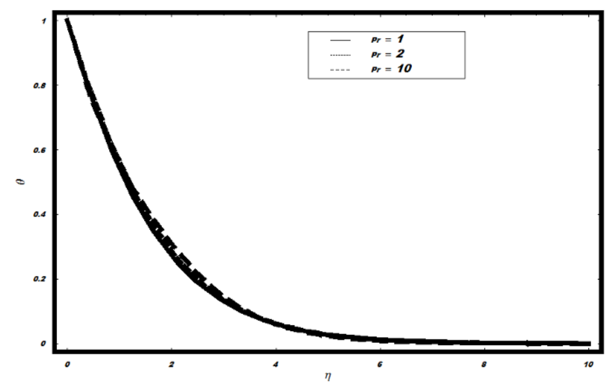

Figure 3 Variation of temperature for different values of Pr when *Le* = 2, *h* = -0.1, *Nt* = *Nb* = 0.5, v_w= 1.

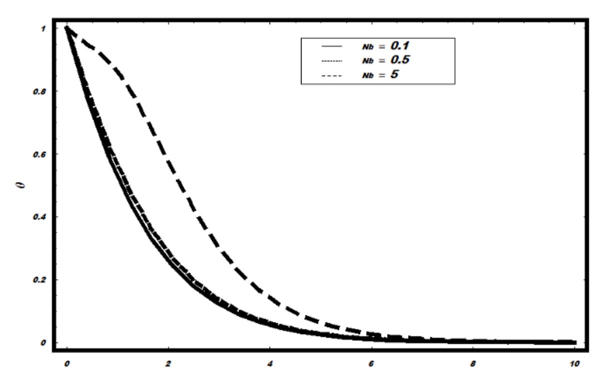

Figure 4 Variation of temperature for different values of *Nb* when *Le* = 2, *h* = -0.1, *Nt* = 0.5, v_w = 1, Pr = 2.

with the increase in suction parameter v_w, the convergence region become smaller and smaller. Almost similar kind of convergence regions appear for temperature and nanoparticle volume fraction, which are not shown here. The non-dimensional velocity f' against η for various values of suction injection parameter is shown in Figure 2. It is observed that velocity field increases with the increase in v_w. Moreover, the suction causes the reduction of the boundary layer. The temperature field θ for different values of Prandtle number Pr, Brownian parameter *Nb*, Lewis number *Le* and thermophoresis parameter *Nt* is shown in Figures 3, 4, 5 and 6. In Figure 3, the temperature is plotted for different values of Pr. It is observed that with the increase in Pr, there is a very slight change in temperature; however, for very large Pr, the solutions seem to be unstable, which are not shown here. The variation of *Nb* on θ is shown in Figure 4. It is depicted that with the increase in *Nb*, the temperature profile increases. There is a minimal change in θ with the increase in *Le* (see Figure 5). The results remain unchanged for very large values of *Le*. The effects of *Nt* on θ are seen in Figure 6. It is seen that temperature profile increases with the increase in *Nt*; however, the thermal boundary layer thickness reduces. The nanoparticle volume fraction g for different values of Pr, *Nb*, *Nt* and *Le* is plotted in Figures 7, 8, 9 and 10. It is observed from Figure 7 that with the increase in

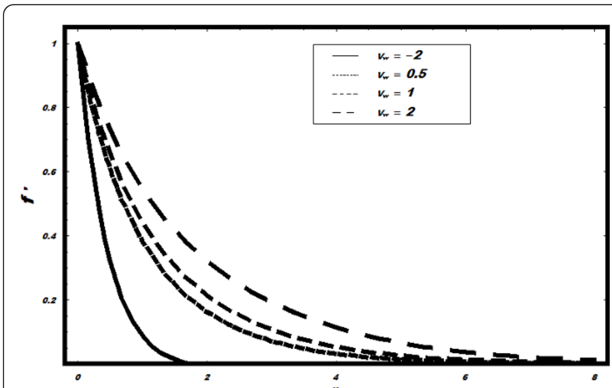

Figure 2 Velocity for different values of suction and injection parameter.

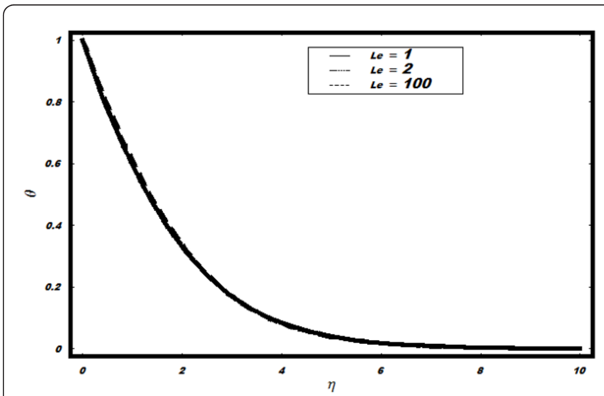

Figure 5 Variation of temperature for different values of *Le* when *h* = -0.1, *Nt* = *Nb* = 0.5, v_w = 1, Pr = 2.

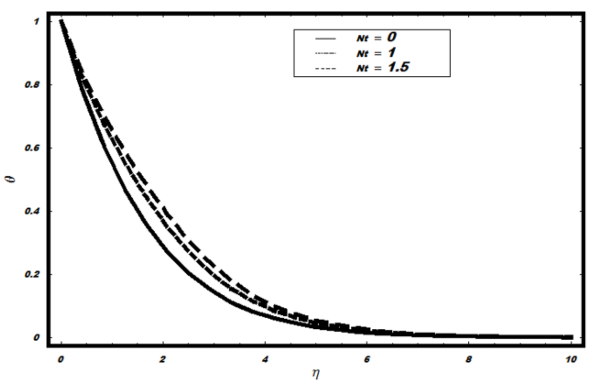

Figure 6 Variation of temperature for different values of *Nt* when *Le* = 2, *h* = -0.1, *Nb* = 0.5, v_w = 1, Pr = 2.

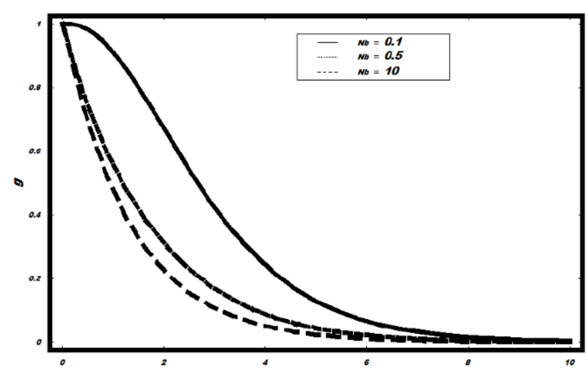

Figure 7 Variation of nanoparticle fraction *g* for different values of *Nb* when *Le* = 2, *h* = -0.1, *Nt* = 0.5, v_w = 1, Pr = 2.

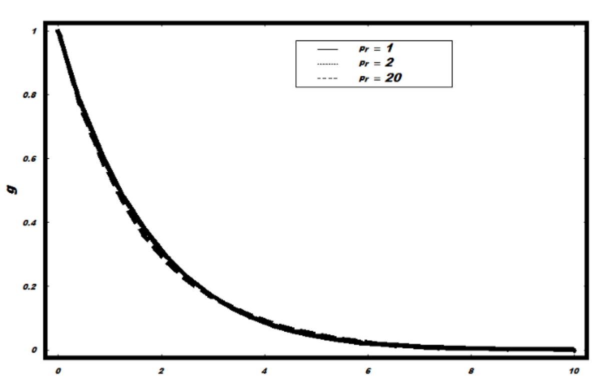

Figure 8 Variation of nanoparticle fraction *g* for different values of Pr when *Le* = 2, *h* = -0.1, *Nt* = 0.5, v_w = 1, *Nb* = 0.5.

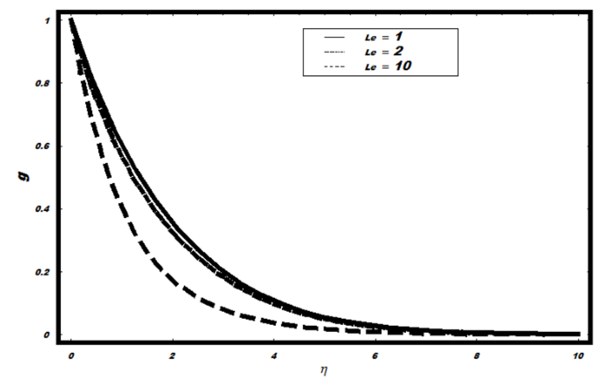

Figure 9 Variation of nanoparticle fraction *g* for different values of *Le* when Pr = 2, *h* = -0.1, *Nt* = 0.5, v_w = 1, *Nb* = 0.5.

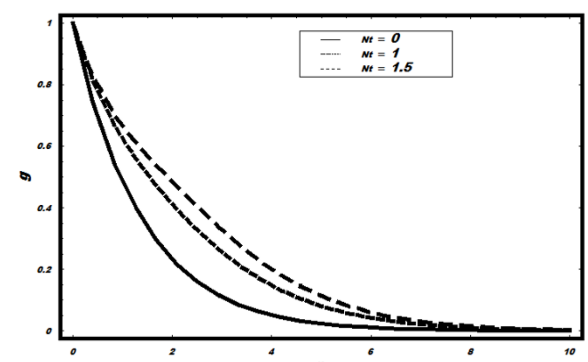

Figure 10 Variation of nanoparticle fraction *g* for different values of *Nt* when *Le* = 2, *h* = -0.1, *Nt* = 0.5, v_w = 1, Pr = 2.

Nb, g decreases and boundary layer for *g* also decreases. The effects of Pr on *g* are minimal. (See Figure 8). The effects of *Le* on *g* are shown in Figure 9. It is observed that *g* decreases as well as layer thickness reduces with the increase in *Le*. However, with the increase in *Nt*, *g* increases and layer thickness reduces (See Figure 10).

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Authors' contributions

SN done the major part of the article; however, the funding and computational suggestions and proof reading has been done by CL. All authors read and approved the final manuscript.

Competing interests

This is just the theoretical study, every experimentalist can check it experimentally with our consent.

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