MODIFIED BARGAINING PROTOCOLS FOR AUTOMATED NEGOTIATION IN OPEN MULTI-AGENT SYSTEMS

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ABSTRACT

Current research in multi-agent systems (MAS) has advanced to the development of open MAS, which are characterized by the heterogeneity of agents, free exit/entry and decentralized control. Conflicts of interest among agents are inevitable, and hence automated negotiation to resolve them is one of the promising solutions. This thesis studies three modifications on alternating-offer bargaining protocols for automated negotiation in open MAS. The long-term goal of this research is to design negotiation protocols which can be easily used by intelligent agents in accommodating their need in resolving their conflicts. In particular, we propose three modifications: allowing non-monotonic offers during the bargaining (non-monotonic-offers bargaining protocol), allowing strategic delay (delay-based bargaining protocol), and allowing strategic ignorance to augment argumentation when the bargaining comprises argumentation (ignorance-based argumentation-based negotiation protocol).

Utility theory and decision-theoretic approaches are used in the theoretical analysis part, with an aim to prove the benefit of these three modifications in negotiation among myopic agents under uncertainty. Empirical studies by means of computer simulation are conducted in analyzing the cost and benefit of these modifications. Social agents, who use common human bargaining strategies, are the subjects of the simulation.

In general, we assume that agents are bounded rational with various degrees of belief and trust toward their opponents. In particular in the study of the non-monotonic-offers bargaining protocol, we assume that our agents have diminishing surplus. We further assume that our agents have increasing surplus in the study of delay-based bargaining protocol. And in the study of ignorance-based argumentation-based

negotiation protocol, we assume that agents may have different knowledge and use different ontologies and reasoning engines.

Through theoretical analysis under various settings, we show the benefit of allowing these modifications in terms of agents' expected surplus. And through simulation, we show the benefit of allowing these modifications in terms of social welfare (total surplus). Several implementation issues are then discussed, and their potential solutions in terms of some additional policies are proposed. Finally, we also suggest some future work which can potentially improve the reliability of these modifications.

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Below are notations in general bargaining:

$\boldsymbol{X}$	*A set of all alternative solutions of the bargaining, or the set of all prices that can be offered by both buyer and seller in a single-attribute bargaining*
$\boldsymbol{x}$	*A point in* $\boldsymbol{X}$ *or a price in a single-attribute bargaining*
$\boldsymbol{Acc_i}$	*Acceptable set for agent-i*
$\boldsymbol{F}$	*Feasible set or all the feasible prices in a single-attribute bargaining*
$\boldsymbol{D}$	*Disagreement set, i.e.* $\boldsymbol{X} \setminus \boldsymbol{F}$, *where "* \ *" denotes the set difference*
$\succsim_i$	*Preference order of agent i over* $\boldsymbol{X}$
$\boldsymbol{V_i}$	*The agent's private valuation over the negotiated item*
$\boldsymbol{Sur_i(x)}$	*A surplus function of agent-i if* $\boldsymbol{x}$ *is the solution of the bargaining*

Below are notations used in a single-attribute (price) bargaining from buyer's perspective:

$\boldsymbol{T_d}$	*The buyer's time deadline*
$\boldsymbol{I_t}$	*The buyer's evaluation function*
$\boldsymbol{x}$	*A variable representing the price that the buyer will offer*
$\boldsymbol{x_t}$	*A price offered by the buyer at time* $\boldsymbol{t}$
$\boldsymbol{x_t^*}$	*A price offered by the buyer that maximizes the buyer's expected gain at time* $\boldsymbol{t}$
$\boldsymbol{y_t}$	*A price offered by the seller at time* $\boldsymbol{t}$
$\boldsymbol{B_t}$	*The buyer's valuation/reservation price at time* $\boldsymbol{t}$
$\boldsymbol{S_t}$	*The seller's valuation/reservation price at time* $\boldsymbol{t}$

$Sur_t(y_t)$	*A surplus made if* y_t *is accepted by the buyer*
$Sur_t^e(x_t)$	*A surplus estimated at time* t *if* x_t *is accepted by the seller at time* $t+1$
$Sur_t^e(y_j)$	*An estimation made at time* t *of the surplus from the seller's future offer at time* j
$Sur_{t+1}(x_t)$	*The surplus made by the buyer if* x_t *is accepted by the seller at time* $t+1$
$p_t(x_t)$	*Buyer's belief (subjective probability) that* x_t *will be accepted at time* $t+1$
$p_t(x)$	*Buyer's belief function that* x *will be accepted at time* $t+1$
$R_{t+1}(x_t)$	*The estimated future surplus (residue) if* x_t *is not accepted by the seller at time* $t+1$.
$\pi_t(x)$	*Buyer's belief function that* x *will be accepted by EvalF-II seller at time* $t+1$
$k_{t+1}(x)$	*Buyer's belief function that represents the likelihood of* x *being accepted by the seller because it is perceived as the best offer from the buyer at time* $t+1$
ϕ	*The time when the breakdown occurs*
q_t	*The buyer's belief of the likelihood of failure (breakdown) caused by the seller at time* t *which is independent of the buyer's offer* x_t
B_ϕ	*The buyer's valuation if the negotiation breaks down*
$EG_t(x)$	*The buyer's expected gain function from offering* x *at time* t
$EG'_{t+1}(x)$	*The buyer's expected gain function of the gain in the next period which is made on the current period* t
κ	*The buyer's belief that the seller is EvalF-I*
ω	*The ratio of* $\gamma(1 - q_t)(1 - p_t(x_t^*)) / [1 - \gamma(1 - q_t)(1 - p_t(x_t^*))]$

θ_t*	*The ratio of* $\gamma(1 - p_t(x_t^*))EG'_{t+1} / [p_t(x_t^*)(B_t - x_t^*)]$
α_S	*The seller's argument*
α_B	*The buyer's argument*
φ	*An imprecision factor that affects the judgment of* $p_t(x_t^*)$
$\psi(p_t(x_t^*), \varphi)$	*The weight function of imprecise* $p_t(x_t^*)$
χ	*The likelihood that verification by the buyer results in the negation of the seller's argument*
$p^{\alpha}_t(x)$	$p_t(x)$ *after the seller accepts* α_B
$\upsilon_t(\alpha_B)$	*The buyer's estimation of the likelihood that the seller will instantly accept* α_B *resulting in the increase of* $p_t(x)$ *to* $p^{\alpha}_t(x) = p_t(x\|\alpha_B)$
$\upsilon_{\phi}(\alpha_B)$	*The buyer's estimation of the likelihood that* α_B *will instantly trigger a breakdown*
$EG^{\alpha}_t(x_t)$	$EG_t(x_t)$ *after the seller accepts* α_B
$\mu_{f\,(p(x\|\alpha))}$	*The mean value of* $p^{\alpha}_t(x_t^*)$ *assigned by the buyer under uncertainty*
μ_f	*The same as* $\mu_{f\,(p(x\|\alpha))}$
$w_t(y)$	*The buyer's estimation of the seller's belief that the buyer will accept* y *at time* t
$g_t(\Delta t)$	*The buyer's estimation of the likelihood of the seller to sell the item to another buyer within interval* Δt *and there are no more items left*
ρ	*The likelihood of an overvaluation, i.e.* $B_t > B_r$
β	*The ratio of the buyer's estimated future surplus with respect to the current one*

ζ *The ratio of the buyer's estimated future non-breakdown rate to the current rate*

η *The ratio of the buyer's estimated belief of the acceptance of its future offer* $\boldsymbol{p_{t+1}(x_{t+1}^*)}$ *with respect to the belief of the acceptance of its current offer* $\boldsymbol{p_t(x_t^*)}$

λ *The ratio of the buyer's estimated future expected payoff after delay to that without delay*

Various agent types according to their myopia:

Myopic-0 agents *Agents who only consider the current state in generating their offers*

Myopic-1 agents *Agents who consider both the current and one-period-ahead state in generating their offers*

Myopic-N agents *Agents who consider the current state and all less-than-N-period-ahead states in generating their offers*

Various agent types according to their evaluation function:

EvalF-0 agents *Agents who accept their opponent's offer if it exceeds a fixed threshold, e.g. their valuation, thus generating a positive surplus; otherwise, counter offer or withdraw*

EvalF-I agents	*Agents who accept their opponent's offer if it generates a higher surplus than that from their about-to-submit counter-offers; otherwise, counter offer or withdraw*
EvalF-II agents	*Agents who accept their opponent's offer if it is perceived/predicted as the best offer that can be provided by their opponent (the 'best' in the sense of generating the highest surplus); otherwise, counter offer or withdraw*

CHAPTER 1

INTRODUCTION

A multi-agent system (MAS) is commonly defined as a system composed of multiple autonomous agents, characterized by incomplete capabilities of each agent, decentralized data, asynchronous computation, and no global system control [Jennings et al., 1998]. Even though there is no strict definition for an autonomous agent, it is commonly accepted that it could be one of the following three categories: biological agents (human or animal), and artificial agents, which can be robotic agents, and computational agents (software agents or artificial-life agents) [Franklin and Graesser, 1997]. Since the context of this thesis is automated negotiation, in the rest of this thesis the term 'agent' will be used to represent a software agent (i.e. an artificial computational agent) and the negotiation is between such an agent and human or another such agent.

To date, various kinds of agents have been created for various purposes; for example, personal-assistant agents, problem-solving agents, entertainment agents, trading agents, Internet-search agents, etc. During interaction among agents, conflict of interest happens naturally and negotiation is an indispensable solution. For example, several personal agents negotiate a meeting schedule for their users, trading agents negotiate the transaction of an item, etc. For an effective negotiation, all negotiators should follow an agreed negotiation protocol in finding a resolution. While the protocol is publicly known for regulating the type of messages and their passing method in the negotiation process, the strategy for generating the messages is privately owned by each negotiator.

In the early 1990s, the role of automated negotiation in a multi-agent system was to solve conflicts of interest among benevolent agents during task and resource allocation [Parsons and Wooldridge, 2002]. With the rapid growth of e-commerce in the mid 1990s, the study of automated negotiation became broader, especially the study of open electronic marketplaces where humans can delegate their software agents to negotiate with other agents, e.g. Kasbah [Chavez and Maes, 1996], Fishmarket [Rodriguez-Aguilar et al., 1998], Tete-a-Tete [Guttman et al., 1998], AuctionBot [Wurman et al., 1998], Shopbots and Pricebots [Greenwald and Kephart, 1999], eMediator [Sandholm, 2002b], AMELI [Esteva et al., 2004], iBundler [Giovannucci et al., 2004], etc.

When any agent can join a system at any time and negotiate with any agent, the MAS is called "open"; for example, an open electronic marketplace. In contrast, a closed MAS is one where there is a fixed set of agents. In an open MAS the degree of uncertainty is higher than that in a closed one, because agents can be more selfish (do not necessarily care about the interests of other agents), may not trust each other, may not know the negotiation strategy used by their opponents, and would not reveal their private information (e.g. their valuations of negotiated items, or reservation price) unless it can benefit them. Consequently, the goal and characteristics of the negotiation in open and closed MAS are different.

In a closed MAS, usually the negotiation protocol is designed so that the system can maintain a fair negotiation outcome that maximizes the total benefit of all agents. For example, if there are several users jointly using a number of printers within a department, then their personal-assistant agents will manage all printing jobs by means of negotiation with other agents so that the total waiting time is minimized and all urgent jobs can be finished on schedule. However, in an open MAS, a negotiation strategy is chosen by an agent for maximizing its own benefit regardless of the other agent's benefit; and the protocol is determined so that most (if not all) agents are willing to participate in the negotiation [Dastani et al., 2003]. In this case, the participation rate becomes the crucial indicator of the success of the system. The focus of my thesis is to study a particular negotiation protocol, i.e. alternating offers bargaining protocol, in open systems by proposing several modifications in order to gain a better negotiation outcome for all participating agents.

1.1 Some Economics Concepts and Terminology

Throughout this thesis we will use several terms from economics and game theory. These terms are briefly introduced here.

The first term is *dominant strategy* that represents a unique strategy played by an agent which always generates the best payoff in response to any moves by its opponents in a game [Mas-Colell et al., 1995]. Dominant strategy may not always exist. Frequently, the best response to two different moves by the agent's opponent in the same game may not be the same. Another term that is more commonly used in game theory is a *Nash equilibrium strategy* --- an agent's strategy that is the best response to all others' strategies assuming they are all playing a Nash equilibrium strategy. This assumption, then, will require rational behavior of all agents and that this be common knowledge.

When all players play a Nash equilibrium strategy, then the game will end up in a *Nash equilibrium* state.

The next term is *mechanism design* --- a process of designing a market (or other games) to achieve a specific outcome. Some examples of market mechanisms are auction, bargaining, etc. A mechanism consists of all agents' strategy sets (e.g. all allowed agent actions in a market) and rules governing the procedure for making the collective choice among agents (e.g. how to distribute the item(s) in the market for any given combination of agent actions). Some economists (and also some MAS researchers) are interested in designing a market mechanism that can achieve a fair allocation, maximize the total utility, and be immune from deceitful strategies [Ephrati and Rosenschein, 1991; Mas-Colell et al., 1995; Kfir-Dahav et al., 2000].

To achieve a fair allocation in a market, it is desirable to know the agents' preference or valuations to each item. Since the agents' valuation is privately known, they may have incentive to strategically misreport their valuation in order to manipulate the final outcomes in their favor. However, if truth-telling is a dominant strategy for all agents in a market mechanism, then we say this mechanism is *incentive compatible*. Sometimes, we say the mechanism is *strategy-proof*, because no strategy other than truth-telling should be used by the agents.

In some mechanisms, we need external funds in order to maintain the incentive-compatible property. In this case, the budget is not balanced, e.g. the amount received by a seller is more than the amount paid by the buyer. Usually, the discrepancy is paid by an authority (e.g. government). A mechanism is called *budget-balanced* if it does not need additional incentives in order to maintain an incentive-compatible property.

An allocation is called *Pareto efficient* or *Pareto optimal* when we cannot make any agent better off without making other(s) worse off. Nash equilibrium is not necessarily a Pareto optimal allocation. A *social optimum* is defined to be the situation where the total joint utilities of all agents are maximized. Generally, in the absence of economic externalities and when the utility of solutions is convex, then Pareto optimum is also social optimum.

Rationality and *bounded rationality* are used as usual to represent the agents' ability to choose the best/optimal choice without or with limited computation, respectively.

At last, we need to differentiate between a negotiation mechanism and a negotiation protocol. Broadly speaking, a mechanism consists of agents' strategy sets and rules governing the procedure for making the collective choice among those agents, which includes the negotiation protocol. Hence, a *negotiation protocol* deals with the implementation of a negotiation mechanism, specifically on the rules of agent interactions. However, since not all agents are rational or can communicate effectively in the real world, the protocol design problem needs to consider other issues beyond those considered in the mechanism design in game theory, such as the possibility of irrational agent behaviors, the standard of negotiation language, the presence of time delay or other communication costs, etc.

1.2 Research Background

Ideally, the negotiation protocol in open MAS should ensure that agents can earn fair profits and that the negotiation is efficient in order to increase the agents' participation rate. Here, efficiency can be measured by the cost of the negotiation (a time delay or other computational cost), the total surplus attained by both parties from the negotiation, and/or economic (allocation) efficiency, such as Pareto efficiency. Therefore the "ideal" protocol shall satisfy incentive compatibility. However, this is not considered a priority since such an ideal protocol is not always possible and for some users fairness of the outcome may not be the major factor of their participation. For instance, if the protocol is too complex, then the required negotiation strategy may become very complex too; thus, fewer agents will be able to participate in the system. Besides, if the protocol cannot help the agents to make transactions on a satisfactory level (e.g. at the price they are willing to sell/buy, and before their time deadline), then they may not participate in the market too. Finally, it is also not desirable to adopt a strict protocol that imposes several requirements before the negotiation takes place. Consequently, developing a negotiation protocol that can both accommodate the agent's need and be used easily in an open MAS is very important.

At present, a lot of literature in automated negotiation emphasizes theoretical analysis rather than empirical study, and game-theoretic analysis is the dominant approach. In general, game-theoretic research focuses on equilibrium analysis. If the equilibrium solution(s) exists and all agents satisfy the assumptions used in the analysis, then they will eventually solve the negotiation problem at that equilibrium. Hence, game-theoretic analysis can be used by a system designer to predict the outcome of a negotiation under a particular protocol.

However, there are serious limitations of the game-theoretic approach which come mainly from its strict assumptions [Jennings et al., 2001]. Hence, many MAS researchers have attempted to relax the assumptions and find solution(s) under these relaxed assumptions. Some of these alternative assumptions include computationally limited agents [Kfir-Dahav et al., 2000; Larson and Sandholm, 2001], the possibility of irrational behavior [Parkes and Ungar, 2000], asymmetric information [Fatima et al., 2001], etc. As a result, most negotiation problems become computational problems, such as finding optimal algorithms for bounded rational agents, designing negotiation protocols under computationally limited resources, etc. In more uncertain situations with dynamic, time-constrained, and unpredictable environments, normally artificial intelligence approaches are considered for the agent's negotiation strategy or finding the negotiation solution, for example using Bayesian learning [Zeng and Sycara, 1998], case-based reasoning [Gimenez-Funes et al., 1998], influence diagram [Mudgal and Vassileva, 2000], neural networks [Azoulay-Schwartz and Kraus, 2000], fuzzy logic [Faratin et al., 2000; Sim and Wang, 2004], heuristic search [Greenwald and Boyan, 2001; Fatima et al., 2001; Winoto and Tang, 2002; Byde et al., 2002], reinforcement learning [Tran and Cohen, 2002], etc.

Under uncertain conditions, the assurance of finding an optimal bargaining solution also decreases. As a result, modifications of existing bargaining protocols may be helpful to increase the chance of finding bargaining solution(s). This motivates the work described in this thesis.

1.3 Thesis Objective and Contributions

The objective of this thesis is to investigate the properties of extended alternating offers bargaining protocols. The domain is restricted to the bargaining between a buyer and a seller for a single attribute (e.g. price). However, the results can be applied in many open MAS, such as e-commerce systems, distributed systems (grid or peer-to-peer), wireless ad-hoc networks, etc.

As argued before, in the relevance to a higher uncertainty and the openness of MAS, we will not only focus on the study of protocol(s) that can generate highest benefit (low cost) or can maintain the fairness of a negotiation outcome; we will also focus on another criterion: participant preference to use the protocol, as postulated in the following statement.

Assumption 1-1. *An agent prefers one protocol over others if it believes that the protocol is more helpful in attaining the agent's goal, which could be a highest expected utility, a highest success (agreement) rate, or a combination of both.*

The first proposed modification is to allow arbitrary revisions (not just converging ones) of the negotiators' offers before agreement is reached, i.e. non-monotonic offers. The second proposed modification of the standard alternating offers protocol is to allow negotiators to use strategic delay --- a deliberative delay for achieving an agent's goal. The third modification is to allow negotiators to avoid argumentation (e.g. ignoring opponent's persuasion or avoiding certain argumentation). In particular, the contributions of each modification are as follows:

1. Allowing any revisions of the offer before agreement is reached (non-monotonic offers). In most literature, it is assumed that the sequence of offers of each negotiation converges monotonically to the agreement, for examples in [Rosenschein and Zlotkin, 1994; Chaves and Maes, 1996; Faratin et al., 2000; Fatima et al., 2001; Sim and

Wang, 2004]. Illustratively, the offers in a negotiation between a seller *S* and a buyer *B* may look like this:

S: $500 → *B*: $400
S: $450 → *B*: $425
S: $425 (agreement).

It is uncommon that sellers will revise their offer to be higher than the last offer. However, imposing a monotonic-offers protocol may not be always beneficial in an open MAS. The study of the cost and benefit of non-monotonic offers bargaining protocol becomes the first goal of this thesis.

2. Allowing negotiators to use strategic delay. Economics studies suggest that strategic delay is especially important at the beginning of the bargaining session. In fact, a great majority of agreements in human bargaining are concluded near the bargaining deadline (commonly known as the "deadline effect"), both in laboratory experiments [Roth et al., 1988] and in real bargaining [Craver, 2005]. In human bargaining, the purpose of delay is mainly for screening, signaling and attrition [Kennan and Wilson, 1990]. In addition, a strategic delay is also used to get more information, such as during holdouts in wage bargaining [Gu and Kuhn, 1998]. The second goal of this thesis is to propose an agent's decision structure when using a delaying strategy and demonstrate the costs and benefits of delay by naïve agents.

3. Allowing negotiators to avoid argumentation. Many real bargaining situations do not involve price, and in fact often implicitly or explicitly involve the change of attitudes. For example, a broker agent will persuade a customer to believe that the agent can help him/her to manage his/her money safely, or a seller may persuade a buyer that

his selling price is the lowest price in the city, etc. Some literature in MAS has addressed the issue of argumentation-based negotiation protocol [Sierra et al., 1998; McBurney et al., 2002], modeling of argumentation-based negotiation [Kraus et al., 1998; Amgoud et al., 2000; Ramchurn et al., 2003; Amgoud and Prade, 2005], applications of argumentation-based negotiation [Jung et al., 2001; Brito and Neves, 2002], and the benefit of argumentation in negotiation [Karunatillake and Jennings, 2005]. Argumentation can also be used for revealing information that speeds up the bargaining under time constraints, which is useful for both bargainers [Rahwan et al., 2004]. However, very few works analyze the disadvantages of allowing argumentation in bargaining, or under which condition the argumentation-based protocol is beneficial. A notable work is by Karunatillake and Jennings [2005]. Therefore, the final goal of this thesis is to analyze the condition(s) in which avoiding argumentation will be beneficial in bargaining model(s).

Finally, it is our expectation that this thesis will contribute not only to the fields of MAS, AI and computer science in general, but also to the fields of economics modeling, social science, management and other relevant fields of studies.

1.4 Thesis Organization

The organization of the thesis is as follows. In the next chapter, we outline some related work on automated negotiation based on various aspects: who is interested in automated negotiation (section 2.1), negotiation mechanisms (section 2.2 and 2.3), and the research methodology (section 2.4). The alternating offers bargaining protocol and the proposed model are provided in Chapter 3. In Chapter 4, we show the result of the theoretical analysis of the non-monotonic offers bargaining protocol (4.1), strategic delay (4.2), and strategic ignorance in argumentation-based negotiation (4.3). In Chapter 5, we show the experimental analysis of proposed protocols. In Chapter 6, we discuss the implication of our findings and the limitations of current work. Finally, Chapter 7 concludes this thesis and points out some future work.

CHAPTER 2

AUTOMATED NEGOTIATION: A REVIEW

2.1 Automated Negotiation: An Interdisciplinary Study

Automated negotiation is a complex interdisciplinary study [Jennings et al., 2001; Kraus, 1997, 2001]. Figure 2-1 shows some of the major contributors to the study of automated negotiation. They span from economics and political science to mathematics and computer science. Generally, they tackle three different problems: mechanism design, protocol design, and negotiation strategies.

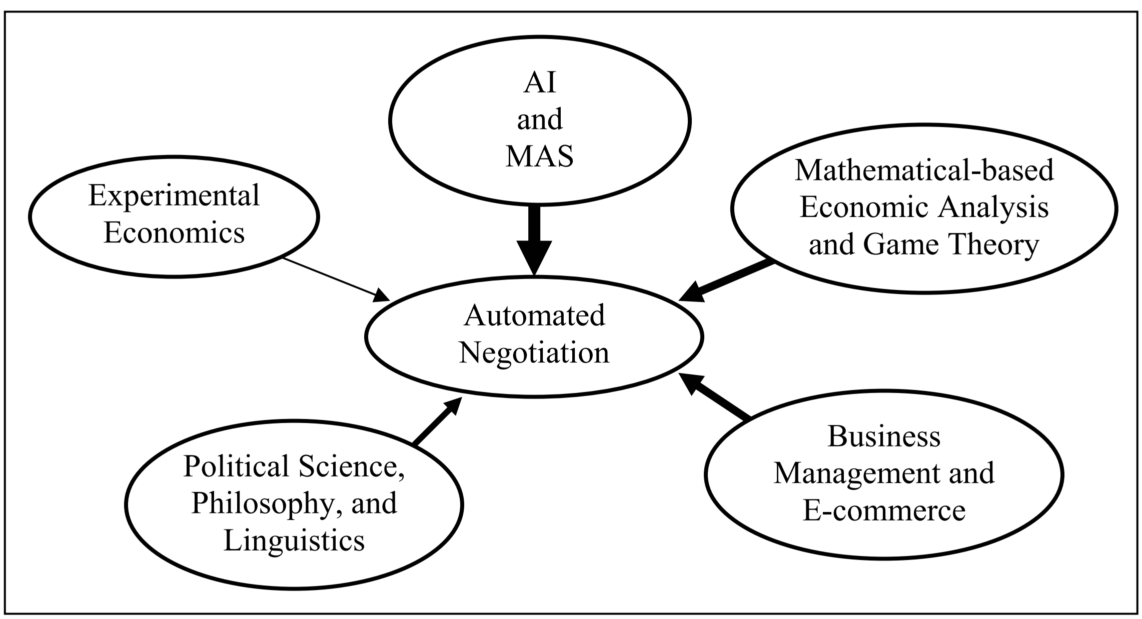

Figure 2-1. Automated negotiation and its major contributors

Broadly speaking, the study of mechanism design is to find a mechanism (an institution with rules governing the procedure for making collective choice) so that the objectives of the designer can be attained in the equilibrium state [Mas-Colell et al., 1995]. The common objectives are to maximize social welfare for all agents, to maximize revenue for specific agent(s) (e.g. auctioneer), to attain fair allocation of goods/services, etc. Usually, the approach assumes agents' rationality and unlimited computation, which may not always be available in MAS. Consequently, MAS researchers have initiated the design of negotiation protocols consisting of several negotiation rules, which can be followed by computationally limited agents and satisfies several negotiation criteria such as computationally tractability, fairness (symmetric treatment), etc. [Rosenschein and Zlotkin, 1995].

Given a negotiation protocol, implementing agents' negotiation strategies, including their reasoning engines, becomes another major problem in automated negotiation. The following sub-sections describe several fields that contribute to the development of automated negotiation.

2.1.1 Mathematical-based Economic Analysis and Game Theory

The major contribution of mathematical-based economic analysis and game theory is the development of several analytical tools, such as equilibrium analysis (e.g. Nash equilibrium, sequential Nash equilibrium, Bayesian Nash equilibrium), utility theory (e.g. von Neumann-Morgenstern expected utility), mechanism design (e.g. Vickrey auction), etc. In fact, much foundational work in negotiation theory was done by economists/mathematicians a long time ago, for example [Nash, 1950; Vickrey, 1961;

Harsanyi and Selten, 1972; Roth, 1979; Myerson, 1979, 1981; Milgrom and Weber, 1982; Rubinstein, 1982, 1985].

2.1.2 Experimental Economics

One of the major issues in the implementation of automated negotiation is user preference elicitation. For instance, how much a buyer is willing to pay if the seller offers an additional feature such as an extended warranty, or will the buyer take a risk by buying from a less reputable seller, or whether an earlier transaction matters for a buyer, etc. While many psychologists [Metzler et al., 1991; Yamagishi and Miyamoto, 1996; Roelofsma and van der Pligt, 2001] and AI researchers [Boutilier et al., 1997, 2006; Ha and Haddawy, 1998; Blum et al., 2004] have worked on this issue, experimental economists have also contributed a lot, e.g. [Karni and Safra, 1987; Johnson and Schkade, 1989; Camerer and Weber, 1992]. Moreover, experimental economists have led research on market design, which has become an important research topic in MAS recently; for example, research in double auctions [Smith, 1962, 1964] and combinatorial auction [Grether et al., 1981; Rassenti et al., 1982].

2.1.3 Business Management and E-commerce

The emergence of e-commerce provides a wider application area for automated negotiation, such as agent-human negotiation or agent-agent negotiation via the Internet. The main concerns of people from management and business are the impact of automated negotiation on the long-term development of e-commerce and how to deploy automated negotiation to facilitate e-commerce [Dworman et al., 1996; Segev and Beam, 1999; Goh et al., 2000].

2.1.4 Political Science, Philosophy and Linguistics

While game theorists and economists study numerical negotiation, such as determining the best bidding price in an auction, the study of qualitative negotiation is studied by political scientists, lawyers, linguists, etc., i.e., how to persuade others using argumentation. It is the most difficult issue because of the involvement of many aspects: natural language, cognitive style, belief, evidence, reputation, etc. Some of the contributions of political scientists, philosophers and linguists to automated negotiation are the study of argument structure [Toulmin, 1958], analysis and design of dialogue games [Mann, 1988; Carletta et al., 1997], formalization of argumentation [Pollock, 1992, 1994; Prakken and Sartor, 1997; Chesnevar et al., 2000], and application of automated negotiation in solving international crises [Wilkenfeld et al., 1995].

2.1.5 Artificial Intelligence and Multi-agent Systems

Currently, MAS researchers are heavily involved in research on automated negotiation. Some important contributions from AI and MAS are applying automated negotiation in various domains, extending traditional negotiation theory to fit computationally limited agents and finding better negotiation protocols, providing infrastructure such as standard MAS platforms, e.g. FIPA-OS [FIPA-OS, 2001], KQML [Finin et al., 1997], and designing artificial negotiators, equipped with various AI techniques, for example [Faratin et al., 1998; Mudgal and Vassileva, 2000; Greenwald and Boyan, 2001; Tran and Cohen, 2002; Keller et al., 2004].

2.2 Bargaining and Other Negotiation Types

A negotiation can be classified in many ways, for example, based on the items being negotiated, the character of the negotiators, the negotiation protocol, the characteristics of information (completeness and symmetry), the negotiation period (continuous, one-step, multiple stage), and many other factors (openness, with penalty, etc). Based on the negotiated items, negotiation can be differentiated into negotiation of single-attribute items or multiple-attribute items. An example of a multiple-attribute item is when negotiators consider price, quantity, quality, delivery time, and payment methods as a bundle. Moreover, negotiation can be categorized into one-to-one, one-to-many, or many-to-many negotiations. An English auction for antiques is a one-to-many negotiation (one auctioneer and many bidders), for example.

Depending on the character of the negotiators, a negotiation can be classified as cooperative or competitive. Cooperative negotiation is characterized by aiming for mutual social benefit (maximizing joint utility) for the negotiators. Competitive negotiation is characterized by seeking individual benefit for the negotiators (maximizing individual utility). Negotiation among agents in distributed problem solving usually falls into the former category, while negotiation in e-commerce falls into the latter.

Based on the protocol type, a negotiation can be categorized as an auction, a contract-net protocol, voting or bargaining. A brief description of these categories of negotiation is provided below.

2.2.1 Bargaining

Bargaining is among the oldest negotiation mechanisms in human history, even before the emergence of markets or money as a means of finding a resolution among interested parties in the presence of conflicts of interest (cooperative behavior in a competitive situation). We can divide bargaining theory into two main categories: axiomatic bargaining theory and strategic bargaining theory [Rubinstein, 1982; Kraus, 2001].

Axiomatic bargaining first sets several axioms (such as all bargainers are individually rational, the solution is invariant to independent changes of utility units, the solution is Pareto optimal, bargainers are symmetric and independent of irrelevant alternatives). It then finds unique bargaining solution(s) based on these axioms [Mas-Colell et al., 1995]. One of the applications of axiomatic bargaining is in labor arbitration, where union and company submit their proposals to an arbitrator (e.g. judge) who decides the final result. We may also have two salespersons who bargain on splitting their commission fee from their cooperative work in selling the same item (e.g. a used car or a house) where the final decision is made by their sales manager. The decision could be an egalitarian solution, e.g. split the commission fee such that both salespersons are equally happy in terms of their utility. Or, the decision could be a utilitarian solution, which splits their commission fee such that it maximizes the total utility of both; or, other solutions, such as Nash solution, Kalai-Smorodinsky solution, or Kalai-Rosenthal solution [Nash, 1950; Kalai and Rosenthal, 1978; Mas-Colell et al., 1995].

In contrast, strategic bargaining theory does not assume a centralized decision maker (arbitrator), but allows the bargainers to solve the dispute by offer and/or counter-offer proposals. A simple example of strategic bargaining is ultimatum (bargaining) game. In an ultimatum game, a bargainer A determines his/her demand (e.g. a proposal or offer on how to split a pie with his/her opponent B) which can either be accepted or rejected by

B. If B rejects A's offer, then the bargaining breakdowns and both get nothing. If B accepts it, then both will split the result according to A's offer. This bargaining method has been studied in both human domain [Davis and Holt, 1993] and agent domain [Todd and Borges, 1997; Katz and Kraus, 2006].

Alternating offers bargaining is another example. Here, a bargainer A starts the negotiation by sending a proposal to his/her opponent B, who chooses either to accept or reject the proposal. If B accepts it, then the negotiation terminates. If B rejects it, then he/she must send back a counter-proposal to specify his/her preferences to A. Now A will evaluate the proposal and choose either to accept or reject it. The process continues until agreement is reached or any party walks out resulting in a breakdown. Currently, there are many variants of alternating offers bargaining, such as a model with a time deadline [Sandholm and Vulkan, 1999], with various information levels (complete/incomplete, symmetric/ asymmetric) [Rubinstein, 1985; Kraus et al., 1995], with risk of breakdown (one party walks out before negotiation ends) [Debenham, 2004], with risk-averse agents [Harrington, 1990], etc. One of the seminal works in strategic bargaining theory is Rubinstein's dividing pie problem [Rubinstein, 1982, 1985], where two agents offer and counter-offer proposals about how to divide a pie in the presence of waiting cost.[1] Rubinstein uses backward induction to solve the problem and shows that the bargaining process only takes one step, i.e., an agent will send only one proposal that is accepted immediately by another agent. The outcome is based on some strict assumptions such as every agent is perfectly rational and has perfect foresight.

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[1] Some people use a melting (ice cream) cake rather than a pie to illustrate the waiting cost.

2.2.2 Other Negotiation Types

2.2.2.1 Auctions

English auction, Dutch auction and double auction are categories of negotiation characterized by sequential decision making and open-bidding. First-price sealed-bid auction and Vickrey auction are characterized by simultaneous decision-making and sealed-bidding [McAfee and McMillan, 1987; Davis and Holt, 1993].

In English auction, an auctioneer opens the auction, and all bidders bid openly (known by others) and sequentially until one active bidder remains (the winner). The winner pays the highest bid he/she submitted. The best strategy for bidders in an English auction is to increase the bid from zero until their private valuation. A bidder's valuation here means the minimum or maximum acceptable price depending on whether her/his position is as a buyer or a seller, respectively. A study by Roth and Ockenfels [2002] reports on the sniping strategy (bid near the end of auction) in English-type online auction (e.g. eBay.com).

Buyers in a Dutch auction do not propose a price, but a single seller will lower the price sequentially until a buyer (the winner) stops it. The strategy used in Dutch auction is fairly similar to that used in first-price sealed-bid auction, because bidders in a first-price sealed-bid auction submit their bids simultaneously. The winner is the bidder who submits the highest bid, and he/she pays the first highest bid (i.e. his/her own bid). First-price sealed-bid auctions in MASs have been studied in [David et al., 2002; Leyton-Brown et al., 2002; Zhu and Wurman, 2002].

Just like in a first-price sealed-bid auction, bidders in a Vickrey auction (or second-price sealed-bid auction) submit their bids simultaneously. The winner is the bidder who submits the highest bid, but he/she pays the second highest bid. The best strategy for bidders in a Vickrey auction is to bid their true valuation [Vickrey, 1961].

The double auction was introduced by Smith [1962, 1964]. Under double-auction rules, sellers/bidders announce their offers/bids sequentially, i.e. bids are raised and offers are lowered sequentially. In other words, sellers will compete to lower their offers and buyers will compete to raise their bids. Some papers investigating this auction are [Das et al., 2001], [Tesauro and Das, 2001], [Tesauro and Bredin, 2002], [Huang et al., 2002], [Grossklags and Schmidt, 2003], and [Lochner and Wellman, 2004].

In a combinatorial auction (CA) bidders can bid over bundles of items [Rassenti et al., 1982; Rothkopf et al., 1998]. For example, in an auction for an airport time slot, an airline company can submit the bid: <{Monday 8:00-9:30, Saturday 8:00-9:30}, $0.5 million> XOR <{Monday 10:00-11:30, Friday 8:00-9:30}, $0.4 million>, which means they are willing to pay either $0.5 million for time slot {Monday 8:00-9:30, Saturday 8:00-9:30} or $0.4 million for time slot {Monday 10:00-11:30, Friday 8:00-9:30} but not both. After all bidders submit their bids, the seller will determine the optimal allocation to the bidders so as to maximize his profit (optimal winner determination problem).

The optimal winner determination problem in combinatorial auction is NP-hard [Rothkopf et al., 1998], which makes it one of the most challenging problems in MAS [Sandholm, 2002a]. For example, Nisan [2000] and Tennenholtz [2000] attempt to find a class of combinatorial auctions with tractable (solvable by a polynomial time algorithm) optimal allocation. Gonen and Lehmann [2000] use branch and bound search, Sandholm and his colleagues [Sandholm et al., 2001] use heuristic search, Holland and O'Sullivan [2005] use *weighted super solutions* framework, etc.

2.2.2.2 Contract-net Protocol

Another approach, the contract-net protocol [Smith, 1980], on the other hand, provides a simple but powerful negotiation mechanism for solving a complex task by means of distributed problem solving. The common way to assign a task is to announce it to other agents (e.g. open an auction/bargaining) and assign the task to the winner. Moreover, every agent can sub-contract/re-contract its (previous) tasks to others who are willing to accept them. Theoretically, an agent will accept a contract if its marginal cost is less than its marginal benefit [Sandholm, 1993]. For example, if an agent already has many tasks to do, then any additional task will generate high marginal cost (e.g., cause slower computation). Theoretically, using this self-organizing mechanism, the system would perform task allocation, which is Pareto optimal when all agents are sincere in reporting their marginal cost and able to swap their tasks with others [Sandholm, 1999a].

Another important issue in the contract-net protocol is whether the agent can de-commit from a contract or not. Leveled commitment contract [Sandholm and Lesser, 1995; Sandholm et al., 1999], i.e. a contract where both parties can de-commit by paying a certain penalty, becomes a crucial mechanism in improving the social welfare of the contract-net. Some work with respect to the application of the contract-net protocol includes [Dellarocas and Klein, 2000; Tran and Cohen, 2002].

2.2.2.3 Voting

Another approach to resolving conflicts is through voting. Voting is a social choice mechanism in selecting social preferences over a set of alternatives, e.g. what is the society mostly prefer from available alternatives. One of the applications of voting in a MAS is resource allocation by means of majority voting. For example, in order to use a common resource (e.g. a supercomputer), an agent can broadcast a request to all other agents to collect access keys from these agents. If two agents compete to use the same resource, then the first who gets the majority votes (>50% of access keys) will be able to access the resource.

Some examples of the application of voting in MAS are [Ephrati and Rosenschein, 1991, 1993; Hunsberger and Zancanaro, 2000]. The voting mechanism is primarily used in making group decisions, in which the choice is simple such as ‘agree’ or ‘disagree’. However, in other approaches, voting can be very complex, because the voting result can be manipulated [Gibbard, 1973; Satterthwaite, 1975]. For example if the preference of an agent to three alternatives {A, B, C} is $A \succ B \succ C$,[2] and this agent knows that the chance of A to win is very small, but the chances of B and C are the same, then its best strategy is to vote B, not A. Some examples of current study in a voting protocol are [Guttmann and Zukerman, 2005] in finding better voting policies in the presence of unreliable agents (e.g. lazy, corrupt, selfish, or conservative agents), [Conitzer and Sandholm, 2005] in correcting noisy votes (e.g. irrational votes) by using maximum likelihood estimators, [Pitt et al., 2005] in formalizing the voting protocol using event calculus, and many others.

2.3 Alternating-offers Bargaining Protocol

This thesis focuses on alternating offer bargaining protocol, which is a kind of strategic bargaining. We do not focus on axiomatic bargaining because we want to explore inherently decentralized systems where negotiation can be conducted with a higher flexibility. Recall that the final decision in axiomatic bargaining is made by the arbitrator (third party) and enforced to both conflicting parties (bargainers). In this case, it may happen that the final decision by the arbitrator is actually a non-favorable one for either or both sides. Therefore, bargainers in axiomatic bargaining are less freely in obtaining what they want and they need an arbitrator which reduces the flexibility of the systems.

For similar reasons, we do not explore other negotiation mechanism. For instance, even when the auction mechanism is highly efficient, i.e., in terms of the trading surplus extracted [Davis and Holt, 1993; Gode and Sunder, 1993; Kagel, 1995; Gjerstad and Dickhaut, 1998; Das et al., 2001], there are still many limitations on auctions [Winoto et al., 2002]:

1. Auctions usually are scheduled in advance and with time restrictions, e.g. some online auctions range from 1 hour to 1 week and the schedule is announced a few days before it opens. Intrinsically, auctions need multiple buyers or sellers in order to work well, therefore needing some time for gathering participants. Some buyers/sellers may not want to wait until an auction opens or finalizes. Also, some may not like to attend auction with a lot of other bidders, because it will lessen the winning chance or end with "the winner's curse" (i.e. a bid higher than the real valuation, because everybody thought the item is really demanded, thus very valuable). Thorough discussions on the effects of the number of bidders on bidder's behavior can be found in [Kagel and Levin, 1993; Kagel, 1995; Kagel et al., 1995].

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[2] Throughout this thesis, the binary operator $\succsim$ refers to weak preference "at least as good as", and $\succ$ refers to strict preference "strictly preferred over", $\sim$ refers to the indifference relation "as preferred as".

2. In some circumstances, non-attribute factors are important, e.g., trusteeships, friendships, etc. Auctions may not accommodate these factors because it may affect the participation of other bidders due to unequal treatment. In fact, some regulations prevent auctioneers from discriminating bidders on these bases. For example, the Directive 2004/17/EC and 2004/18/EC of the European Parliament states that public contracts (e.g. in procurement auctions) are on the basis of transparency, non-discrimination and equal treatment, which guarantee that tenders are assessed based on "the lowest price" and "the most economically advantageous tender" only.

> *"In order to guarantee compliance with the principle of transparency, only the elements suitable for automatic evaluation by electronic means, without any intervention and/or appreciation by the contracting authority, may be the object of electronic auctions, that is, only the elements which are quantifiable so that they can be expressed in figures or percentages. On the other hand, those aspects of the tenders which imply an appreciation of nonquantifiable elements should not be the object of electronic auctions."*
>
> [Directive 2004/17/EC]

The contract-net protocol is useful for the transaction of compound items (e.g. a set of trading items/tasks). For a single item, the mechanism is equivalent to an auction, in which the announcer (initiator) agent acts as an auctioneer and all interested participants act as bidders. Again, the weakness of this approach lies on the time constraint and the number of participants. However, if we relax the negotiation in the contract-net protocol into a bilateral one, i.e. the negotiation is conducted between an initiator and a single participant, then the result is nothing but a bargaining mechanism. Therefore, our study here can also be applied in a contract-net protocol.

Finally, voting is more appropriate for resource allocation or social choice, not for trading items between two parties, which is our focus here. Considering these limitations, bargaining mechanisms still play an important role in automated negotiations. However, as stated before, in many actual bargaining situations, agents only have minimal or no information about their opponents and are only concerned about their own individual goal(s). This limitation may reduce the efficiency of strategic bargaining mechanisms in terms of a failure in finding a favorable solution for both parties. Thus, we should modify existing bargaining protocols to increase the chance of finding bargaining solution(s), which is the objective of this thesis.

Specifically, we concentrate on strategic bargaining in an alternating-offer setting in open system, which complies with the following assumption:

Assumption 2-1. *The system is open in the sense that*

1. *agents may join or leave the system anytime;*
2. *agents may be created by different designers and represent different owners; and*
3. *bargaining may be done simultaneously and asynchronously among agents.*

How negotiation factors are influenced by the degree of system openness is illustrated in Figure 2-2. The left column in Figure 2-2 represents an extreme situation where we have a highly closed MAS, while the right column represents another extreme for a highly open MAS. Certainly, we may have a relatively closed MAS where agents are created by multiple designers, hence with various predetermined (known) negotiation strategies, but all agents are benevolent and they cannot freely enter or exit the system.

Highly Closed MAS:		Highly Open MAS:
Single designer		Multiple designers
Predetermined strategy	→	Any strategy
Benevolent (honestly share all info.)		Malevolent (cheating may happen)
No entry and exit		Free entry and exit

Figure 2-2. The openness of MAS for automated negotiation

As MASs become more open, they become more flexible and less controllable. Several issues arise, such as how to design a safe negotiation protocol, how to preserve negotiators' privacy, how to prevent unfair ratings, etc. [Dellarocas, 2000; Parkes and Ungar, 2000; Jennings et al., 2001; Leyton-Brown et al., 2002; Sandholm and Wang, 2002].

Since we focus on an open system in our bargaining model, we assume that agents may be irrational and may enter or exit the system at their will. For example, they may use any bargaining strategy, including cheating, offering a random price, imitating their opponent's behaviors, etc. and they may open or leave a negotiation with other agents at any time. Moreover, we also want to study the bargaining protocol that allow persuasive argumentation, which is very common in human-human bargaining such as in used-car bargaining. We assume that our bilateral bargaining protocol allows persuasive argumentation among both parties. The next subsection will discuss the recent development of argumentation-based negotiation and its relation with other negotiation protocol.

2.3.1 Argumentation-based Negotiation (ABN)

Recently, one of the very active research areas in automated negotiation is argumentation-based negotiation. Indeed, argumentation-based negotiation has long been proposed for solving conflict of interest among artificial agents [Sycara, 1990; Kraus et al., 1998; Parsons et al., 1998; McBurney et al., 2003]. The common functions of argumentation are to inform, persuade, threaten, appeal to, or promise an opponent [Kraus et al., 1998; Rahwan et al., 2004]. For instance, an argument can be used to influence the opponent's stance, or, to justify the proponent's refusal [Rahwan et al., 2004]. The final purposes are to speed up the negotiation, attain a higher success rate, and increase the market efficiency.

Argumentation is mostly bidirectional in the form of a dialogue among conflicting agents. It can be blended into any negotiation protocol or executed separately. For example, [Sierra et al., 1998] propose an argumentation-based negotiation framework, which can be used in a contract net protocol, so that the proposed task allocation can be supported by persuasion. However, argumentation among bidders in an auction is not as useful as argumentation between a buyer and a seller in alternating-offer bargaining, because argumentation is naturally a useful tool in bargaining, i.e. to persuade other party to concede, while it is less effective for a bidder in an auction to persuade/threaten other bidders to quit.

Generally, research in argumentation-based negotiation follows from early research in dialogue games. Current topics in argumentation-based negotiation include:

- Argumentative reasoning, e.g. [Kraus et al., 1998; Parsons et al., 1998; Wooldridge et al., 2005]. This research is concerned with the evaluation, generation, and selection of arguments. Planning [Kraus et al., 1998; Parsons et al., 1998] and utility evaluation

[Sierra et al., 1998] are commonly used in the evaluation and selection of arguments. Rules and heuristics are commonly used for the selection. For example, "if *trust* is *low* and *utility* of the proposal is *high*, then send a *strong* argument" [Ramchurn et al., 2003]. In [Kraus et al., 1998], threat is the strongest argument, followed by a promise of future reward, and others.

- Communication language and negotiation protocols, e.g. [Sierra et al., 1998; McBurney et al., 2003; Amgoud and Prade, 2005]. So far, the majority of research in this direction is concerned with the design of (i) a better communication language to facilitate complex arguments; and (ii) a better protocol to govern the argumentation, such as maintaining rule consistency, avoiding disruption, assuring termination (success), etc.

Kraus et al. [1998] have proposed a BDI ("belief, desire, intention") framework that could be used as the foundation of an agent's reasoning in arguing and counter-arguing during a negotiation. In their setting, arguments can be in the form of appeal, threat, or promise, which are selected by the agent according to its goal, belief, desire, and intention. Argumentation is meant to be the only negotiation tool in finding a resolution, and all agents are benevolent and must respond to their opponent's argument. Implicitly, they assume that all agents can understand all arguments. McBurney et al. [2003] studied another ABN framework where agents could benefit from arguing in addition to offering a price in the context of purchasing a service or product. In their setting, a limited number of arguments are used by agents to interchange relevant information, such as their expectation as to the quality of the product/service. Again, they implicitly assume that all agents can understand all arguments. Most prior work, including [Sycara, 1990; Parsons et al., 1998] and many others mentioned in [Rahwan et al., 2004], assume that agents will automatically use arguments to solve the conflict. Even if the weight (importance) of each argument is not the same, which affects the selection of the argument, a frequent assumption is that agents will respond to their opponent's arguments. However, this may not be true in an open MAS because agents may not understand all arguments. For example, a naive buyer may join the system even if it is not able to understand or to generate most of the argument. This issue has not been studied yet, which becomes one of the motivations of this thesis.

2.3.2 The Application Areas

Figure 2-3 shows several application areas of general automated negotiation. In its early stages, automated negotiation was primarily used for task and resource allocation. Some examples of early work in automated negotiation for solving task/resource allocation are Smith's contract net protocol [Smith, 1980; Smith and Davis, 1983], Lesser's multistage negotiation [Conry, Meyer and Lesser, 1988; Kuwabara and Lesser, 1991], Durfee and Lesser's Partial Global Plan [Durfee and Lesser, 1987, 1989]. Generally, many early papers implicitly assumed that an agent is benevolent and is designed to achieve common goals (the designer's goals). However, with the emergence of e-commerce the primary goal of an automated negotiator has shifted to accomplishing individual goals and MASs have become more open; thus, the agent may become self-interested and may use nasty strategies to achieve its individual goals.

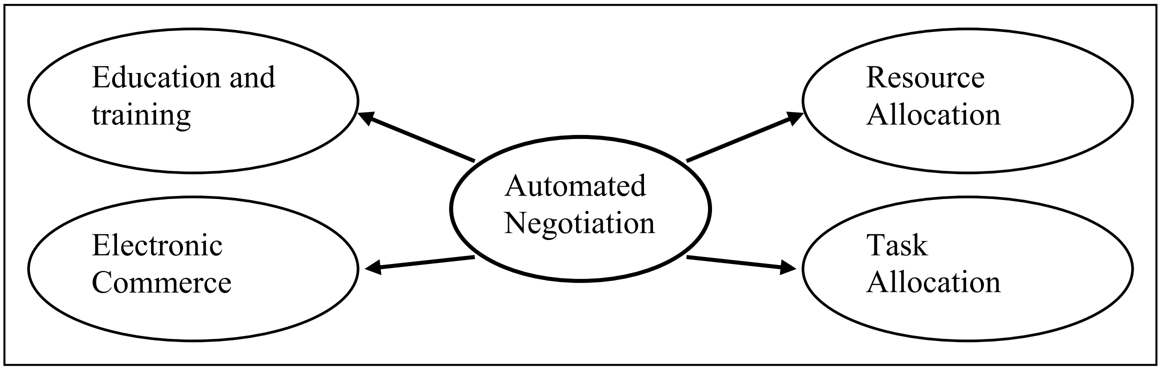

Figure 2-3. Applications of automated negotiation

Another interesting application area is to use automated negotiation in facilitating education or training, such as agent-agent negotiation in I-Help [Vassileva et al., 1999; Mudgal and Vassileva, 2000] and human-agent negotiation in Genie, a decision support system used for training U.S. foreign office personnel and German police [Wilkenfeld et al., 1995; Kraus, 2001]. TAC (Trading Agent Competition) is another facility used to help researchers in developing better trading agents [TAC, 2006]. TAC also has been used as a class project in many universities in the US and Europe [Wellman et al., 2002].

2.4 Research Methodology Used in the Study of Automated Negotiation

Figure 2-4 shows the main research methods used in the study of automated negotiation up to now. The theoretical analysis of negotiation falls into three main categories: game-theoretic analysis, computational analysis, and decision-theoretic analysis. The empirical study of negotiation can be divided into three main categories: simulation with artificial agents, experimentation with humans in the loop, and competition events. These categories are discussed in the next section.

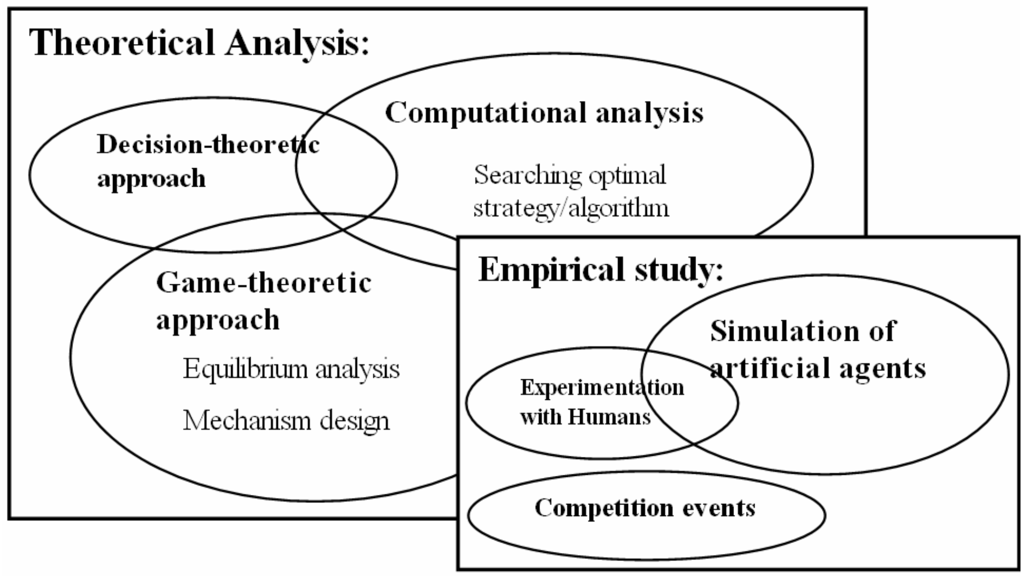

Figure 2-4. Research methodology used in the study of automated negotiation

2.4.1 Game-theoretic Approach

Some work in automated negotiation is based on game theoretic analysis [Parsons and Wooldridge, 2002], which is concerned with equilibrium analysis and finding optimal strategies. It is commonly used to prove the properties of a market mechanism. The main reasons are:

1. if all agents in a MAS follow the assumptions used in the analysis, then the MAS falls into an equilibrium state eventually;
2. the strategy that leads to the equilibrium becomes the candidate solution of the system;
3. if the equilibrium is not Pareto (or social) optimal, then it is important to find a way to approach the Pareto (or social) optimal frontier, either by changing the negotiation mechanism or by increasing central control (e.g., imposing a levy and redistributing it).

However, some limitations of the game-theoretic approach for direct application to MAS come from its strict assumptions [Jennings et al., 1998; Jennings et al., 2001]:

1. All agents are assumed to be perfectly rational;
2. Agents know the payoffs of each action and can predict their opponents' actions perfectly (perfect foresight);
3. Agents can search for the solution in exhaustive fashion and are able to consider all possible states of a game (computationally unlimited);
4. If there are game theoretic equilibrium strategies, such as the Nash equilibrium strategy or the dominant strategy, then all agents will eventually choose one of them;
5. Agents do not recall past experiences, and therefore no learning mechanism is involved.

Realizing that those assumptions are less applicable in the real world, many MAS researchers have attempted to relax them and find solution(s) under relaxed assumptions. Some alternative assumptions include computationally-limited agents, the possibility of irrational behavior, etc. However, some research, even under alternative assumptions, still uses game-theoretic analysis in finding solutions. For example, Monderer and Tennenholtz [1999] try to bridge the mechanism design problem with the protocol design problem using a game theoretic approach. Two main differences between game-theoretic results (mechanisms) and application-oriented designs (protocols) are considered, i.e. the issue of communication network and the structure of a message, which are ignored in game theory. Another example is the work by Larson and Sandholm [2001], which considers the limitation of agents' computational ability in the negotiation. They extend the study of bargaining so that all agents use anytime algorithms in their decision, and use deliberation control (performance profile) to stop the algorithm. However, they use a game-theoretic approach to perform equilibrium analysis (e.g. perfect Bayesian equilibrium), while they vary several factors, e.g. time constraints (who moves last), available information (complete/incomplete), character of performance profile (deterministic/stochastic), etc. The drawbacks of equilibrium analysis are on its applicability, because not many games are really played until reaching their equilibrium state, especially when the system is open. For instance, stock prices are fluctuated over time due to the dynamism of the stock values and the openness of the market (investment may flow in or out anytime from or to other market).

2.4.2 Beyond Classical Game Theory: What can Computational Analysis Do?

As more assumptions are relaxed, most negotiation problems become computational problems, such as finding optimal algorithms for bounded rational agents, finding optimal algorithms for winner determination in combinatorial auctions, designing negotiation protocols under computationally limited resources, etc. Two important topics that use computational analysis extensively are optimal winner determination in combinatorial auctions [Rothkopf et al., 1998; Hoos and Boutilier, 2000; Sandholm et al., 2001] and mechanism design [Lehmann et al., 1999; Kfir-Dahav et al., 2000; Mu'alem and Nisan, 2002]. As explained before, the mechanism design problem deals with the finding of efficient markets (to produce the highest social welfare), which satisfy incentive compatibility and budget balance. And the optimal winner determination problem deals with the finding of optimal allocation of goods (or services) by an auctioneer to all bidders in order to maximize the auctioneer's revenue or social welfare (depending on the aims of the auction).

In combinatorial auctions, a bidder can submit a bid for a combination of goods/services. After receiving all bids from bidders, an auctioneer allocates the goods/services to maximize total revenue, which is an NP-complete problem [Rothkopf et al., 1998]. Consequently, a tractable computational approach is needed to find the solution. For example, Gonen and Lehmann [2000] use the weighted multi-set packing problem as an analogy of the winner determination problem, and show that the problem is NP-hard and hard to approximate. Tennenholtz [2000] uses a b-matching technique to solve a specific class of combinatorial auction problems. Hoos and Boutilier [2000] introduce Casanova to solve optimal winner determination. Casanova is a stochastic local search algorithm that resembles Novelty+, which is used to solve SAT problems [Hoos, 1999]. Sandholm and his colleagues introduce CABOB, which uses decomposition techniques and heuristic search to solve the winner determination problem [Sandholm et al., 2001]. Holland and O'Sullivan [2005] use a *weighted super solutions* framework, which is borrowed from the field of constraint programming, to find a robust solution of combinatorial auctions.

On the mechanism design problem, Kfir-Dahav, Monderer and Tennenholtz [2000] introduce a heuristic Clarke's mechanism (a well-known efficient mechanism, after Clarke [1971]). It is based on a standard Clarke's mechanism, whose solution (i.e. finding optimal social welfare) is NP-hard, but agents are resource bounded. They prove that if agents follow specific heuristics, then the incentive compatibility and budget balance properties of Clarke's mechanism still hold. Lehmann et al. [1999] and Mu'alem and Nisan [2002] try to find a class of combinatorial auctions that satisfy both the property of VCG mechanism (after Vickrey [1961], Clarke [1971], and Groves [1973]), which ensure incentive compatibility and computational efficiency (tractability). Note that it is known from early studies that approximation methods for the winner determination problem in a combinatorial auction can destroy the incentive compatibility property, i.e. all agents will bid truthfully their valuation [Lehmann et al. 1999; Nisan and Ronen, 2000]. In other words, it is hard to find an efficient algorithm that maintains the incentive compatibility property at the same time, or, it is hard to find an incentive-compatible mechanism for which an efficient algorithm is sufficient. Lehmann et al. [1999] show that the combination of a greedy allocation and greedy payment scheme can maintain both incentive compatibility and computational efficiency. However, the method is correct if all agents are single-minded (only bid a set of items)[3] and if the method is applied to Generalized Vickrey Auctions (GVA). Later, Mu'alem and Nisan [2002] extended the method and applied it to a general combinatorial auction and multi unit combinatorial auction by adding linear programming and partial exhaustive search algorithms. Again, their method is correct if all agents are single-minded.

In the future, computational analysis will gain more popularity because:

1. pure game-theoretic analysis cannot solve most of the complex problems, especially combinatorial problems that inherently exist in some negotiation mechanisms;
2. in real application(s) many assumptions must be relaxed, which in turn makes pure game-theoretic analysis inappropriate;
3. more and more studies have adopted computational analysis, which will stimulate more research using computational analysis.

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[3] In a general combinatorial auction, a bidder can submit multiple bids simultaneously. For instance, s/he can bid <item X, $10>, <item X and item Y, $15> and <item Y and item Z, $12> at the same time. If a bidder is single-minded, then s/he only bids on one bundle, e.g. <item X and item Y, $15>.

2.4.3 When Everything Becomes Uncertain: The Decision-theoretic Approach

Most of the theoretical work in decision-theoretic approaches addresses the issue of design and complexity analysis of the agent's decision making algorithms, which are mostly supported by empirical study. This is not surprising since most of them assume an uncertain situation in which game-theoretic analysis cannot be used, and their main objective is to apply an existing AI technique or negotiation mechanism/protocol.

The decision-theoretic-based algorithms used in automated negotiation can be divided into three categories: i.e., static (off-line), dynamic (on-line), and long-term algorithms. Static algorithms are based on off-line analysis (*a priori*), i.e. to find the best strategy before the negotiation begins, e.g. using game-theoretic analysis. For example, Fatima et al. [2001] can determine the optimal bargaining strategies of an agent if the agent knows the probability value of its opponent's deadline and valuation. The strategies are based on a specific family of negotiation decision functions proposed in [Faratin et al., 1998], which satisfy strict monotonicity and are continuously differentiable. Therefore, after an agent knows the information, e.g. its opponent's type, it will pick a corresponding negotiation strategy and execute it until an agreement is reached or the negotiation ends. This strategy works efficiently if the information is known before the negotiation begins and the environment does not change during the negotiation. However, in many cases the agent may face a dynamic environment, e.g., where opponents switch their strategy. Thus, a dynamic strategy is needed to adapt to a new market condition. When the environment changes, the agent will utilize current and/or past information for its future decision. For example, Tesauro and Bredin [2002] use a dynamic programming (DP) method to update an agent's bidding strategy (GDX) when attending a sequential auction (i.e. Continuous Double Auction). First, their agent will estimate the future-clearing price and the number of remaining bidding opportunities. Then, the agent will decide how much it should bid in order to maximize its expected profit. They argue that the DP algorithm is quite feasible when the variation of commodities is very small, and the complexity increases exponentially when more types of commodities are introduced into the market.

An important factor in the DP algorithm and also other dynamic negotiation algorithms [e.g. Boutilier et al., 1999; Byde et al., 2002; Tesauro and Bredin, 2002; Greenwald and Boyan, 2004] is the availability of past information. For example, GDX [Tesauro and Bredin, 2002] uses information about 5 past periods to estimate the future price within a session. However, their agents do not retain past information for their future encounter, i.e. they will re-initialize everything on a new session. Indeed, in some situations past experiences may be useful for future negotiation (long-term memory), for example, by maintaining all information about its opponents [Zeng and Sycara, 1998; Mudgal and Vassileva, 2000; Tran and Cohen, 2002, 2004], or by memorizing the best strategy from its experiences [Dworman et al., 1996; Winoto and Tang, 2002]. Knowing that it encounters the same opponent, an agent may predict its opponent's behavior more accurately, which may lead to a better payoff.

2.4.4 Empirical Study: Problems and Promises

The three main purposes of conducting empirical studies in automated negotiation are:

1. Increasing the credibility of theoretical analysis or the proposed method [e.g., Andersson et al., 2000; Hoos and Boutilier, 2000; Tesauro and Bredin, 2002; Zhu and Wurman, 2002; Tran and Cohen, 2004];

2. Analyzing the characteristics of the proposed method [e.g., Faratin et al., 2000; Hunsberger and Grosz, 2000; Zhang and Lesser, 2002; Holland and O'Sullivan, 2005];
3. Searching for new phenomena [e.g., Dworman et al., 1996; Wurman et al., 1998; Wellman, 2002; TAC, 2006].

Empirical studies can also be categorized into several types, such as comparison of a proposed method with a benchmark (purpose 1), simulation by adjusting control variables (purposes 1 and 2), simulation by introducing uncontrolled variables/shocks (purposes 2 and 3), competition events (purpose 1 and 3), and experiments with human subject(s) (purposes 1, 2, and 3).

Comparing one method to other method(s) is the most common empirical study. For example, Andersson et al. [2000] compare their algorithm for optimal winner determination with those in Sandholm [1999b] and Fujishima et al. [1999]. Hoos and Boutilier [2000] compare their algorithm, i.e. Casanova, with CASS [Fujishima et al., 1999]. Tesauro and Bredin [2002] compare their algorithm for continuous double auction (i.e. GDX strategy) with the ZIP (zero intelligence plus) strategy [Cliff, 1997][4] and the Gjerstad and Dickhaut's strategy [Gjerstad and Dickhaut, 1998].

A simulation is sometimes used in the sensitivity analysis, either by adjusting control variables (e.g. Hunsberger and Grosz [2000], and Holland and O'Sullivan [2005]) or introducing shocks in order to understand the behavior of a MAS (e.g. Azoulay-Schwartz and Kraus [2002]). Simulation is very important, especially when we assume an uncertain environment. For instance, what happens if the opponent of an agent behaves

[4] ZIP is an extension of ZI (zero intelligence) strategy proposed by Gode and Sunder [1993].

irrationally in a negotiation? Will they destroy the equilibrium structure? In game-theoretic analysis, Bayesian Nash Equilibrium or Trembling Hand Perfect Nash Equilibrium analysis can be used to represent this situation, i.e., by assigning a probability of encountering an irrational agent (probability to deviate from equilibrium). However, the analysis is valid if all agents know that probability value, which is not likely to happen in the real world. Hence, a simulation, consisting of learning agents, is more desirable, because some emergent behaviors of agents may happen unpredictably.

Competition events and experimentation with human subjects are two other rarely studied issues in automated negotiation. TAC (Trading Agent Competition) and Santa Fe Double Auction Tournament [Rust et al., 1994] are two examples of competition events held to find better trading strategies. TACs have been held annually since July 2000 [TAC, 2006]. In the past, the task of agents in the TAC is to arrange a trip for a vacation, e.g., bid on air ticket, bid on hotel accommodation and buy/sell an entertainment ticket. Three different types of auction are used, i.e. continuous clearing auction, ascending multi-unit auction, and continuous double auction. Since 2003, TAC Supply-Chain Management is also held annually in conjunction with classical TAC [Sadeh et al., 2003]. To date, some research approaches have been reported with regard to the agents' capability in TAC, for example [Stone et al., 2001; Benisch et al., 2004; He and Jennings, 2004].

In order to build a smart negotiator we need to understand and solve many problems, such as how to build a (human) opponent model, how to collect information from the opponent, how to predict his/her behavior, how to argue using natural language, how to persuade people, etc. Unfortunately, only a small amount of literature reports experiments conducted with human player(s) in automated negotiation. One example is the study by Hoz-Weiss et al. [2002] who conducted experiments in which humans negotiate with an agent to solve an international crisis (modeled on a fishery dispute between Canada and Spain). They provide heuristics to help the agent in making its decision, such as when to send an offer, how to update a counter-offer, what argument/counter-argument is sent, etc. An agent uses backtracking to find the equilibrium strategy. They assume that the valuations/utilities of both parties are common knowledge. And the agent only maintains one parameter of the human model, i.e. risk attitude. Results show that agent-human negotiation outperforms human-human negotiation in terms of total utility gained.

Another example of human-agent experimentation is [Das et al., 2001], in which six humans and six agents are involved in a continuous double auction. Agents use the ZIP strategy [Cliff, 1997] and the Gjerstad and Dickhaut's strategy [Gjerstad and Dickhaut, 1998], in which no user model is needed. Results show that agents outperform humans by approximately 20% in terms of total trading surplus gained. The superiority of agents over humans decreases as the trading goes on, because humans are capable of learning from their experiences. However, agents still outperform humans by 5%-7% at the end of trading period (total 16 trading periods and 3 minutes each period).

2.5 Chapter Summary

In this chapter we have shown several contributions made toward automated negotiation, and also its classifications, applications, research problems, and some active research areas. Our research in this thesis is only concerned with a branch of automated negotiation: bargaining.

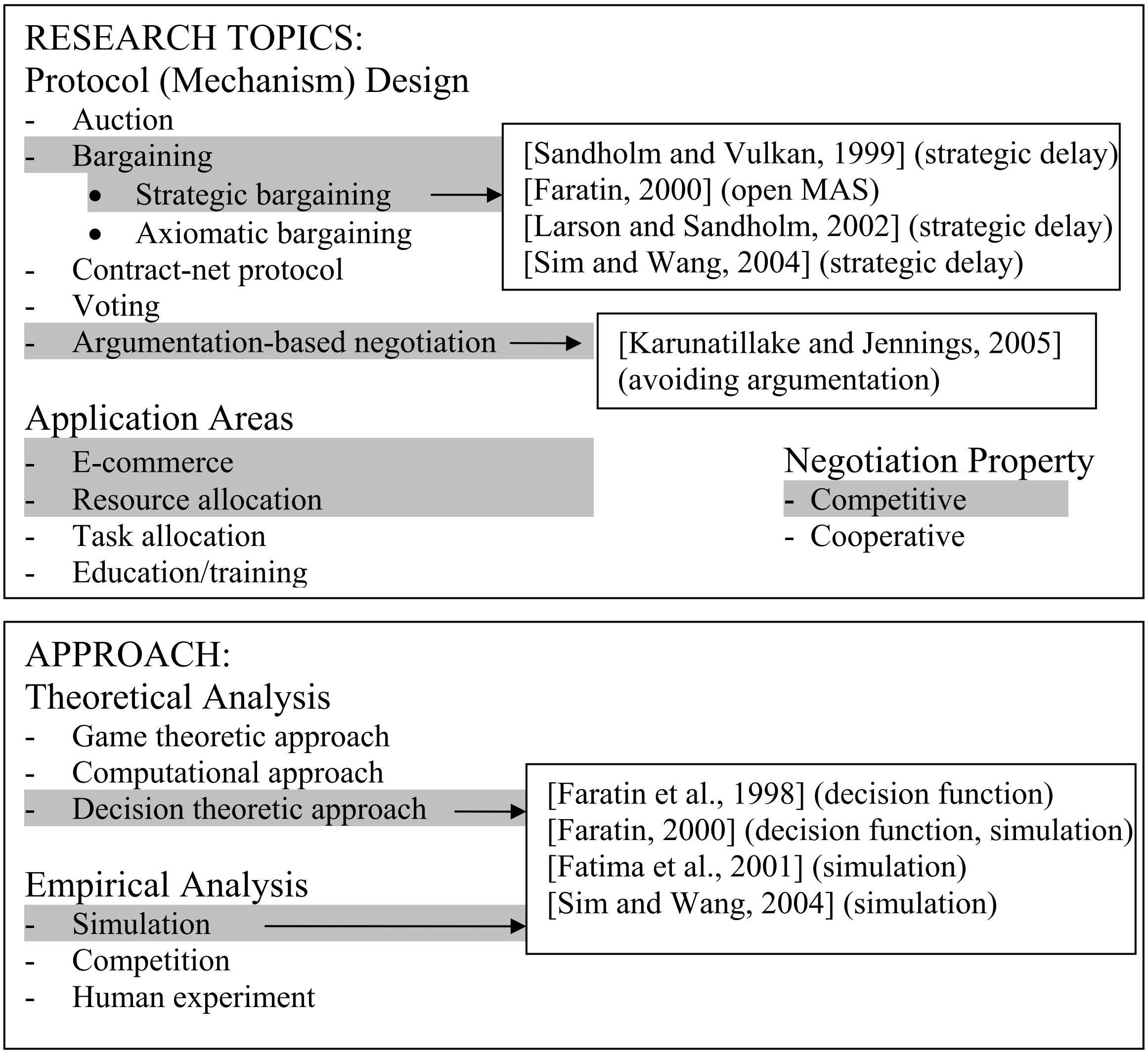


Figure 2-5. Context of this thesis in automated negotiation and some of its related work

Another purpose of this chapter is to show the context of this thesis among existing research in terms of three major dimensions (protocol design, application areas, problem solving approaches including theoretical and empirical analysis). As we will see later in the next chapters, it is concerned with bargaining protocol design supported by both theoretical and empirical analysis. Figure 2-5 shows the topics and approach used in this thesis and its relation to other work in automated negotiation.

The topics and approaches concerned are highlighted in Figure 2-5. The closest work to this thesis is Faratin's work on building an agent's decision function in an uncertain environment (assuming the agent does not know information about its opponents). Later, his approach is elaborated in [Fatima et al., 2001] and [Sim and Wang, 2004]. Our agent's decision function in the non-monotonic bargaining case is also derived from Faratin's work, with further extension to the strategic delay and strategic ignorance cases. The framework will be described in Chapter 3.

Sandholm and his colleagues [1999, 2002] use a game theoretic approach to analyze bargaining with a deadline. The results show the benefit of strategic delay, which is similar to our work. However, they did not verify the benefit empirically due to a restrictive domain of their study. Sim and Wang also propose a strategic delay under a specific condition, which is tested empirically. However, they did not provide a general agent decision framework using strategic delay. This thesis provides a general agent decision framework on deciding a delay and empirical results in terms of cost and benefit of using strategic delay in bargaining.

Karunatillake and Jennings [2005] briefly analyze the cost of argumentation by means of empirical study. They suggest a withdrawal by an agent from a negotiation when the argumentation in that negotiation is costly. Our work is different in the sense that we do not suggest a withdrawal, but rather to employ strategic ignorance on costly topics. In addition, we also propose a decision function for invoking strategic ignorance among agents and using simulator to show the benefit of it.

Finally, since this thesis discusses three different modifications to bargaining protocols, most related work for each modified protocol will be discussed separately in Chapter 4 when the modifications are introduced. Some other related work, which is relevant to the general framework of our agent's decision model, will be provided in Chapter 3.

CHAPTER 3

MODELLING AGENTS AND THEIR BARGAINING STRATEGIES

The strategy used by an agent in bargaining depends on the information and computational capacity of the agent. Without considering any information, the simplest strategy of a buyer is to submit consecutive offers based on a predefined sequence, e.g. <$100, $110, $125, …, $200>. Using this sequence, the buyer starts with an initial offer of $100, which then is revised to $110, $125, and so on until it is either accepted by the seller or reaches the ceiling $200. Since the sequence is set by the user and the agent only executes it regardless of the negotiation situation, we classify it as a static strategy. This method was commonly used in early systems, such as Kasbah [Chavez and Maes, 1996]. The advantage of this strategy lies in its simplicity and transparency for the user. However, it restricts the agents from taking advantage of relevant information including their opponent's behavior. Consequently, it tends to work well only when the bargaining information is known by the user, or when uncertain factors are not important in determining the success of the negotiation. For example, if the buyer knows that the seller will accept a $200 offer and it does not have a time deadline, then the aforementioned sequence can certainly be used by the buyer, even though it does not guarantee a maximum surplus.

In more sophisticated strategies, agents may have a higher autonomy to decide their offers, as long as they are not exceeding the limit set by the user. This is more likely to be used when the bargaining involves many uncertain factors. In these strategies, the offers may change according to the information sensed by the agents; thus, they are referred to as dynamic strategies. In this chapter, several dynamic strategies are discussed that may be implemented in bargaining under uncertainty. The structure of the discussion of these strategies presented in this chapter is shown in Figure 3-1.

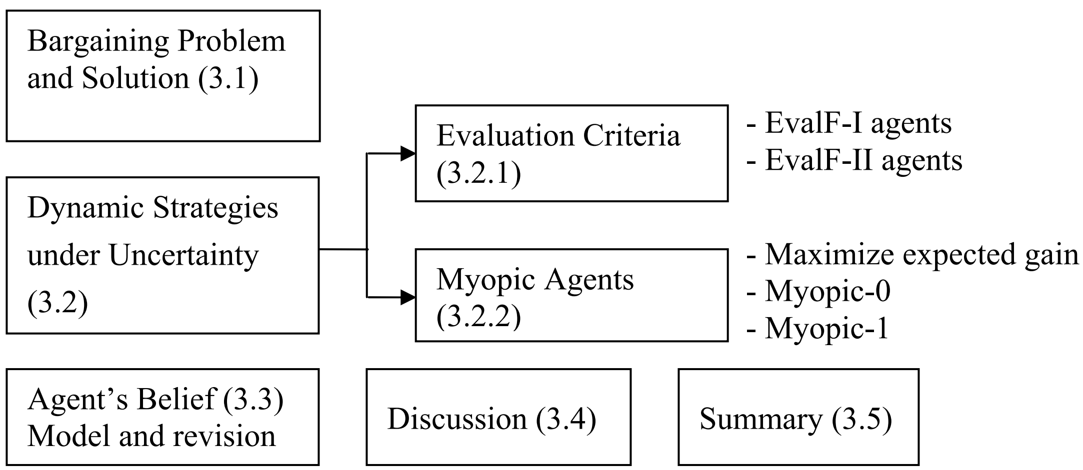


Figure 3-1. Topics covered in Chapter 3

First, we briefly discuss the characteristics of the bargaining problem and solution under uncertainty. Then we look at agents' behaviors, starting with their evaluation criteria in making a decision (section 3.2.1), followed by a description of agents according to their negotiation strategy (section 3.2.2). Then, we propose a model of agent belief and its revision mechanism, which is an important part of myopic agents, i.e. agents that only consider a few periods ahead in their decision making. More attention is given to myopic agents and their belief construct because they are used as the basis of the theoretical analysis, which makes this analysis different from traditional game-theoretic analysis that assumes a perfect forecast. After describing the model, we discuss the implication of the model in section 3.4. Finally, the summary of Chapter 3 is provided in section 3.5. All concepts described in this chapter will be used as the basis for the analysis of the modified bargaining protocols proposed in the next chapter.

3.1 Description of the Bargaining Problem and Solution

We consider the bargaining problem in which a buyer wants to buy an item from a seller where each party has some private bargaining attributes, e.g. price, quality, delivery time, warranty, etc. The following assumption is applied throughout this thesis.

Assumption 3-1. *Both the buyer and the seller have limited information and computational resources, and they are constrained by bargaining attributes and/or environmental factors, such as time deadlines, access to other buyers/sellers, etc., and all of those aspects are not necessarily symmetric for both of them.*

Assumption 3-1 assures the heterogeneity of our agents' characteristics and their environment. It makes our analysis more realistic for agents that represent contending users who enter an open market from a different time and/or place. Our analysis excludes situations where conflicts can be resolved analytically by game theory.

Assumption 3-2. *Both the buyer and the seller are bounded rational and selfish.*

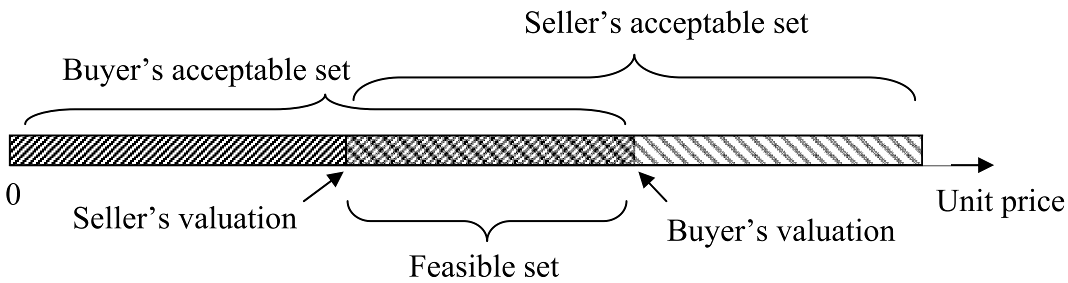
Assumption 3-2 further declares the limitation of our agents' reasoning and assures their self-interest which reflects their owner's interest. We assume bounded rationality because it is more realistic than perfect rationality [Russell, 1997]. However, a bounded rational property does not guarantee that agents can make an optimal decision in bargaining, especially if the decision is not computable. Intuitively, for finding an optimal decision, agents should be able to choose the best choice among all possible options. Thus, we further restrict our agents such that they have capability to order all available options in terms of computable utility value, and then derive the optimal decision by maximizing the utility value. We state this feature in Assumption 3-3.

Assumption 3-3. *(Zermelo's Axiom of Choice) Given limited information, limited computational resources, a time deadline, and other environmental constraints, both the buyer and the seller have totally ordered choices that can be expressed using utility functions, and both the buyer and the seller are utility maximizers.*

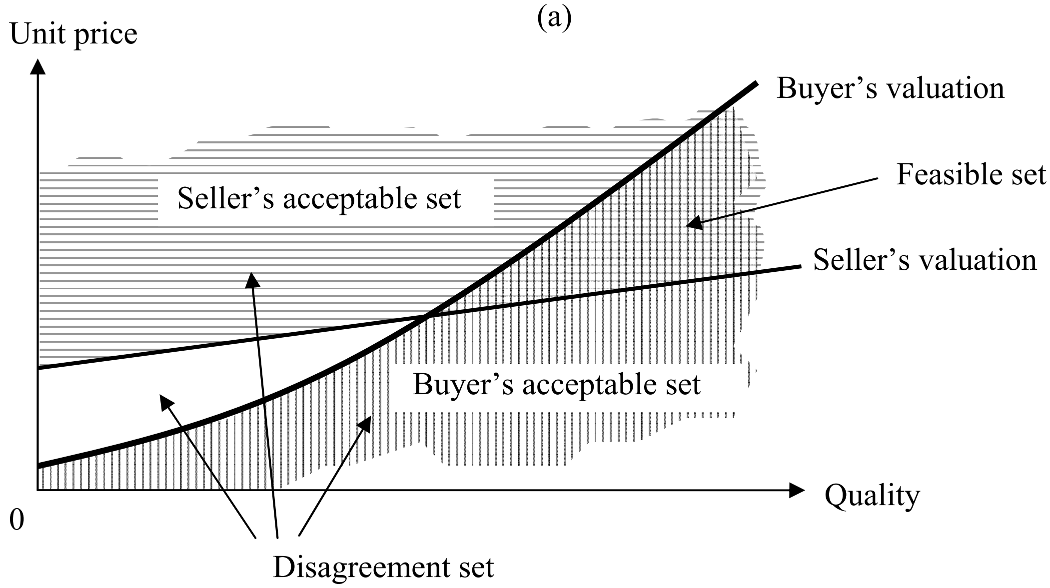
This assumption restricts our analysis to those agents with preference orders expressible in utility functions. In other words, they have the capability to choose the optimal choice by means of computation. Our approach here presumes an uncertain environment, in which deliberation and meta-reasoning is needed by the agent in maximizing their expected utility.

Without loss of generality, suppose that the negotiated issue is only for the item price, and both parties may persuade/threaten their opponent to accept their offer or to justify their refusal of their opponent's proposal. For the more general case of multiple-issues negotiation, we can combine those issues into a single utility function to represent agent choices as in multi-attribute utility theory/MAUT [Keeney and Raiffa, 1976].

In order to produce a bargaining solution, the valuations of seller and buyer should intercept or overlap (creating a feasible set). Figure 3-2 shows feasible sets in the bargaining of a single continuous attribute item (unit price) and an item with two continuous attributes (unit price and quality).



(a)



(b)

Figure 3-2. Bargaining solution in (a) one-dimensional attribute and (b) two-dimensional attributes

In Figure 3-2(a), the seller’s valuation is the minimum price at which it is willing to sell, and its acceptable set is the range of prices at which it is willing to sell; similarly for the buyer, the valuation is the maximum price it is willing to pay. The feasible set is the overlapping region of both acceptable prices. In Figure 3-2(b), the seller’s private valuation is a straight line representing the minimum selling price (reservation price) at different levels of quality. An upward sloping of that line means the minimum acceptable price increases as the quality increases. The seller's acceptable set is all acceptable bargaining points from the seller's view, in which the higher point (higher price for the same quality) is strictly preferred. Conversely, the buyer's private valuation is the buyer's maximum buying price (reservation price) at different qualities. The buyer's acceptable set is all acceptable bargaining points from the buyer's view, in which a lower point (lower price for the same quality) is strictly preferred. Conclusively, any point in the acceptable set farther down from the private valuation line is strictly preferred. The intersection between the two acceptable sets is a feasible set, where the result of bargaining will fall.

3.1.1 Formal Description of Bargaining Problem

Formally, suppose there are only two agents $\boldsymbol{i} \in \{\boldsymbol{b}, \boldsymbol{s}\}$ bargaining over N-attributes, where $\boldsymbol{b}$ denotes the buyer agent and $\boldsymbol{s}$ the seller agent. An alternative region $\boldsymbol{X}$ is defined as all possible points in the bargaining space of N-dimensional attributes. For instance, in a bargaining of a used car the dimensions and their range could be *price* (in dollars) = {0, 1, 2, …, 20000}, *dealer's warranty* (in months) = {0, 1, 2, …, 12}, *payment method* = {cash, installment}, and other bonus such as new tires, a new CD player, etc. In case of trade-in, the price of the buyer's car will be another attribute.

Further, both agents have their own preference order $\succsim_{\boldsymbol{i}}$ over $\boldsymbol{X}$ ($\succsim_{\boldsymbol{i}}$ is a total order in the bounded rationality sense) and private valuation set $\boldsymbol{V_i} \subseteq \boldsymbol{X}$, where $\boldsymbol{v_m}, \boldsymbol{v_n} \in \boldsymbol{V_i}$ $\Rightarrow \boldsymbol{v_m} \sim_i \boldsymbol{v_n}$ (agent $\boldsymbol{i}$ is indifferent in its preference over all its valuations). For instance, if we only consider price and dealer's warranty from previous example, then a buyer may strictly prefer ($12000, 3 months warranty) than ($12000, 0 month warranty). If the buyer is *not willing* to pay more than $12500 provided 3 months warranty, or more than $12000 without warranty, then we say it is *indifferent* between ($12500, 3 months warranty) and ($12000, 0 months warranty), which are its private valuations. Thus, the collection of all maximum prices that the buyer is willing to pay for various periods of extended warranty constitute its private valuation set $\boldsymbol{V_b}$. Then, for any buyer $\boldsymbol{b}$, if an alternative $\boldsymbol{x} \succ_{\boldsymbol{b}} \boldsymbol{V_b}$ ($\boldsymbol{x}$ is strictly preferred than $\boldsymbol{V_b}$), then $\boldsymbol{x}$ generates a positive surplus for it, denoted by $\boldsymbol{Sur^b(x)} > 0$; similarly for the seller we have $\boldsymbol{Sur^s(x)} > 0$ if $\boldsymbol{x} \succ_{\boldsymbol{s}} \boldsymbol{V_s}$. In the previous example, we could derive that ($12000, 3 months warranty) generates $500 surplus, because the buyer's valuation for the same warranty period is ($12500, 3 months warranty). Note here, sometimes we cannot measure surplus in term of money, especially in multi-attribute negotiation where the user's preference may not always be quantifiable or even unknown (preference elicitation problem). However, in this thesis, we assume all alternatives are comparable in term of price.

In some bargaining, agents' valuations dynamically change over time. In this thesis, we assume that the agents' valuations are dynamic. In one-dimensional bargaining in Figure 3-2(a), if we denote $\boldsymbol{V_b}$ with $\boldsymbol{B_t}$ and $\boldsymbol{V_s}$ with $\boldsymbol{S_t}$ at time $\boldsymbol{t}$ then the surplus functions are $\boldsymbol{Sur^b_t(x) = B_t - x}$ and $\boldsymbol{Sur^s_t(x) = x - S_t}$, where $\boldsymbol{x}$ is an alternative and $\boldsymbol{Sur^b_t(x)}$ and $\boldsymbol{Sur^s_t(x)}$ are time-dependent surpluses made by the buyer and the seller, respectively. In multidimensional bargaining, the surplus can be calculated using a weighted sum of the differences between vector $\boldsymbol{V}$ and $\boldsymbol{x}$. For example, in our previous example the buyer may value each 3 months extended warranty as $500. Given one of its valuations is ($12500, 3 months warranty), the surplus from $\boldsymbol{x}$ = ($12000, 6 months warranty) can be calculated as: $\boldsymbol{Sur^b(x)}$ = (\$12500 – \$12000) + (6 months – 3 months) × \$500/3 months = \$1000.

Definition 3-1. *An acceptable set* $\boldsymbol{Acc_i} \subseteq \boldsymbol{X}$ *of agent* $\boldsymbol{i}$ *is a set such that* $\forall \boldsymbol{x} \in \boldsymbol{Acc_i}$ *we have* $\boldsymbol{Sur^i(x)} > 0$*. In other words,* $\forall \boldsymbol{x} \in \boldsymbol{Acc_i} \Rightarrow \boldsymbol{x} \succsim_{\boldsymbol{i}} \boldsymbol{V_i}$*.*

Definition 3-2. *A feasible set* $\boldsymbol{F} \subseteq \boldsymbol{X}$ *is a compact set (closed and bounded) such that* $\forall \boldsymbol{x} \in \boldsymbol{F}$ *we have both* $\boldsymbol{Sur^b(x)} > 0$ *and* $\boldsymbol{Sur^s(x)} > 0$*.*

Definition 3-3. *A disagreement set is* $\boldsymbol{D} = \boldsymbol{X} \setminus \boldsymbol{F}$*.*

In Figure 3-2, the disagreement set is all points outside the feasible set. Now, it is the time to determine the basic protocol used in our bargaining analysis.

Assumption 3-4. *An alternating offer protocol is used as the basis of the bargaining, where the seller always starts by submitting a proposal/offer at time t = 0, and after receiving it the buyer will either accept the seller's proposal/offer or submit a counter proposal/offer at time t = 0 too. After that the virtual clock moves to t = 1 (next round) and it is then the seller's turn to evaluate the buyer's counteroffer. The process continues until a bargaining solution is found (all negotiated issues are solved) or a breakdown occurs (either or both parties left the negotiation without a solution).*

We assume here that the virtual clock represents the turn-taking of the bargaining; so it does not necessarily match the real time of the bargaining. However, in Chapter 4 we will slightly override this assumption and consider a real clock in the bargaining, especially in the study of strategic delay. Nevertheless we will use Assumption 3-4 in our analysis unless otherwise specified.

3.1.2 The Existence of Bargaining Solution

The existence of a feasible set does not guarantee the existence of bargaining solution, but it is a necessary condition for the existence of a solution. The existence of a bargaining solution also depends on the agents' strategies, i.e. whether their offers will converge to a solution or not. In real human negotiation the valuation is not necessarily fixed or known in advance, because new information is added over time, the market situation changes during the bargaining, or the utility of the bargained item also changes.

Example 3-1. *After a road test a human buyer may value a used car for $5,000, but finding an oil leak afterwards will change the buyer's valuation. Similarly, recent news of a 20% discount offered by another dealer will also affect the buyer's past valuation.*

In fact, most real human bargaining is conducted under uncertainty. Since agents are used to represent humans in automated negotiations, the agent's ability to perceive complex information is required in order to increase its effectiveness in the bargaining. For example, the agent may be able to check the market price from multiple fixed-price sellers, to assess the quality of the item and the seller's reputation, or to wait for other opportunities/sellers with better offers, etc. In this thesis, we assume that agent valuation is not always fixed, but may increase or decrease over time.

In the multiple-attribute bargaining between two bounded rational agents, the bargaining process can be illustrated as picking some initial pairs of alternative points and then repeatedly changing those points such that they become closer until one or more of them reaches a solution, as shown by the direction of the arrows in Figure 3-3.

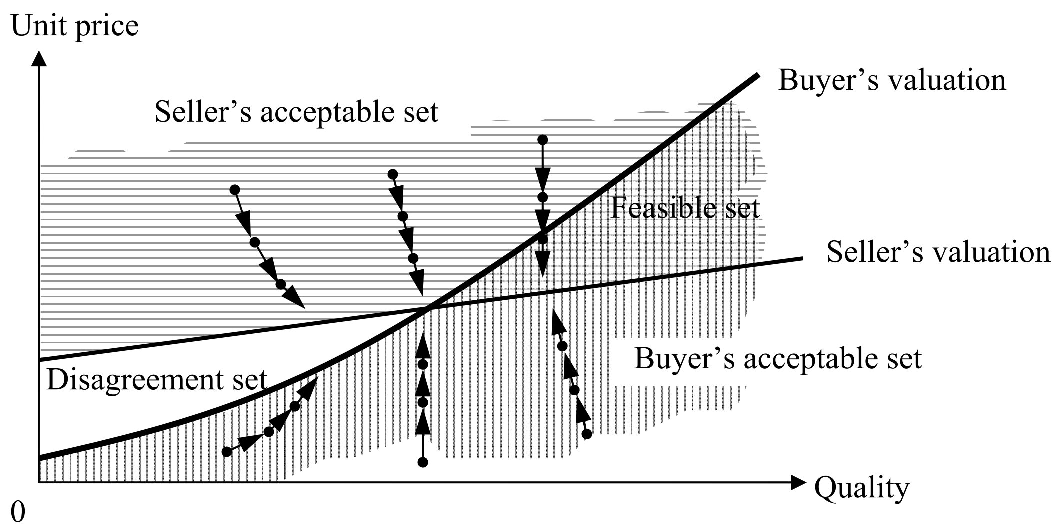


Figure 3-3. Multiple-attribute bargaining

Example 3-2. *During a bargaining over chemical goods a buyer might say "I am willing to increase my offer by \$1 if you could increase the purity to 99.5%", where the new offer is both increasing the price and the quality. Similarly, a store manager may also ask for a lower price for canned foods with closer expiry date.*

The action of lowering a score in one attribute and demanding a higher score in another attribute is known as making trade-offs [Faratin et al., 2000].

In a single-attribute negotiation, the process is less complicated; each agent only picks one point (e.g. price) and changes it over time (see Figure 3-4). In Figure 3-4, the seller's valuation (minimum price it is willing to sell) is fixed over time, but the buyer's valuation (maximum price it is willing to buy) is decreasing over time (horizontal axis). Since the agent only concentrates on a single attribute (issue) at any time, the computation is less complex compared to that in multiple-attribute cases. Following this argument, several researchers have reduced multiple-attribute negotiation into a sequential issue-by-issue negotiation [Bac, 2002; Fatima et al., 2004].

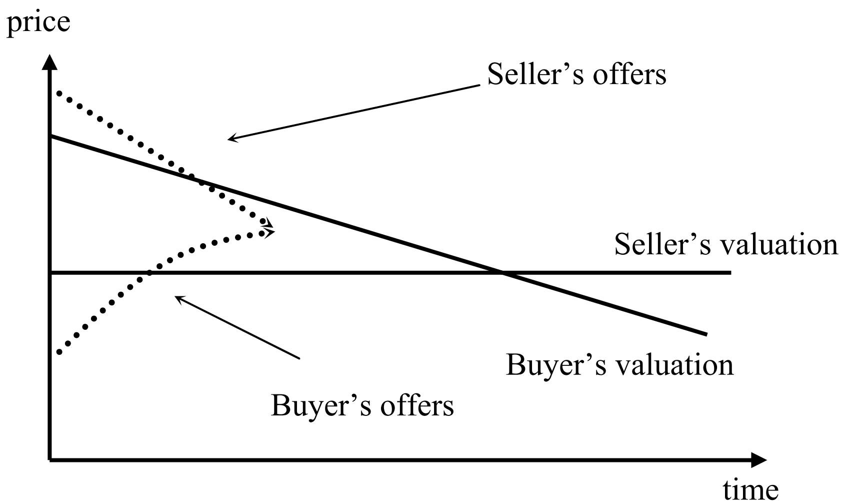


Figure 3-4. Single-attribute bargaining over time

Similarly, in this thesis, we will only consider *single-attribute bargaining*. Without loss of generality, we assume that the bargaining attribute is price. Therefore, $\boldsymbol{X}$ is finite and a subset of integer $\mathbb{Z}$, or could be converted to it depending on the precision and the unit currency being used. Conversely, if the range of $\boldsymbol{X}$ is large enough, we can also transform it into a finite set of 'pseudo' real number $\mathbb{R}^+$ (e.g. a single precision float number).

Example 3-3. *A set of prices with a precision 2 decimals* $\boldsymbol{X}$ *= {\$0.00, \$0.01, ..., \$10,000.00} can be transformed into integer set* $\boldsymbol{X}$ *= {¢0, ¢1, ..., ¢1,000,000}. Further we can scale-down* $\boldsymbol{X}$ *= {¢0, ¢1, ..., ¢1,000,000} into a finite set* $\boldsymbol{X}$ *= [0, 1] with a precision of 6 decimals.*

In this thesis we assume that the price can be approximated using real numbers (with continuous and differentiable properties). However, we will also analyze the case when it cannot be approximated, for example due to a small size of cardinality $|\boldsymbol{X}|$ or a large increment, e.g. \$100 for each increment. And since the bargainer's decisions are symmetric for the buyer and the seller, we will analyze them from the buyer's perspective only.

Assumption 3-5. *Unless otherwise specified, the theoretical analysis in this thesis is from the buyer's perspective and the offers and counteroffers are for the price only, which is in a continuous domain.*

Given Assumption 3-5, we can drop the superscript $\boldsymbol{b}$ in the notation of the buyer's surplus; for examples, the notations $\boldsymbol{Sur(x)}$ and $\boldsymbol{Sur_t(x)}$ are used to denote time-independent and time-dependent surplus functions, respectively. For the sake of consistency, we measure the buyer's surplus in the same unit as $\boldsymbol{B_t}$ and $\boldsymbol{x}$, e.g. in currency unit (cu.) or dollar (\$).

3.1.3 Surplus and Utility

As stated previously, if the transaction price of an item, e.g. $\boldsymbol{x}$, is lower than the buyer's valuation, then the buyer will make a positive surplus which can be calculated by the surplus function $\boldsymbol{Sur_t(x) = B_t - x}$. In economics, the ordering of human preference over a set of items is often represented by a continuous utility function, such that the preferred item will get higher utility value. Table 3-1 shows some common utility functions in economics, which some of them are also used in automated negotiation literature, e.g. quasi-linear function [MacKie-Mason and Wellman, 2006], time-dependent function [Kraus et al., 1995; Sandholm and Vulkan, 1999], and Von Neumann-Morgenstern expected utility function [Parson and Wooldridge, 2002; Sim and Wang, 2004; Cheng et al., 2005]. In addition, exponential utility function, which is part of constant absolute risk aversion family (not shown in Table 3-1) has also been used in designing risk-averse agents [Liu et al., 2003]. In the future, we believe that more complex utility functions will be adopted in designing agents, especially when the negotiation involves risk.

Table 3-1 Common utility functions [Mas-Colell et al., 1995]

Properties	**Sample Utility Function**	**Descriptions**
Perfect substitution	$U(v_1, v_2) = v_1 + v_2$	This is the simplest utility function. It is a special case of a quasi-linear utility function.
Quasi-linear	$U(v_1, v_2) = u(v_1) + v_2$ e.g. $U(v_1, v_2) = v_1^{0.5} + v_2$	This is a common consumer's utility function. The most representative interpretation: v_1 is the consumption good, and v_2 is the money paid for v_1.
Diminishing marginal rate of substitution	Cobb-Douglas: $U(v_1, v_2) = v_1^a v_2^{(1-a)}$ $a \in (0, 1)$ e.g. $U(v_1, v_2) = v_1^{0.3} v_2^{0.7}$	This function is commonly used to describe the "saturation" of consuming the same goods after a certain level.
Time dependent (discounting utility)	$U(v, t) = \delta^t u(v)$	This utility function is used to represent the future consumption in current value. This is commonly used in a sequential game (e.g. in bargaining game). $\delta \in (0, 1]$ is the discount factor and t is the time interval.
Von Neumann-Morgenstern expected utility	$EU = \sum p_i u(v_i)$	This is the most representative utility function of a risky asset with various outcomes. p_i is the probability of outcome v_i.
Constant Relative Risk Aversion (CRRA)	*Utility function such that* $-vU''(v) / U'(v) =$ constant	This family of utility functions is commonly used in a risky asset market (e.g. stock market). $U'(v)$ and $U''(v)$ are the first and second-order derivative, and v is a risky asset.

If the agent's utility function is irrelevant from the negotiation time (discount factor = 1), then we can consider the surplus as the utility value. Indeed, a time-independent surplus function $\boldsymbol{Sur}\ (\boldsymbol{x}) = \boldsymbol{B} - \boldsymbol{x}$ is a quasi-linear utility function, where $\boldsymbol{Sur}\ (\boldsymbol{x}) \rightarrow U(v_1, v_2)$, $\boldsymbol{B} \rightarrow u(v_1)$ and $-\boldsymbol{x} \rightarrow v_2$. However, if the utility function is time dependent, then we cannot use a time-independent surplus function.

Example 3-4. *Suppose the buyer's utility function is* $0.5^{\boldsymbol{t}} \cdot (\$200 - \boldsymbol{x})$*; then a transaction of \$150 at time* $\boldsymbol{t} = 0$ *and a transaction of \$100 at time* $\boldsymbol{t} = 1$ *will generate the same unit of utility* ($0.5^{\boldsymbol{0}} \cdot (\$200 - \$150) = 50$ *units and* $0.5^{\boldsymbol{1}} \cdot (\$200 - \$100) = 50$ *units, respectively), while their surplus values are different (\$200 – \$150 = \$50 and \$200 – \$100 = \$100, respectively).*

If various utility functions are used in the negotiation literature, why do we only use surplus in this thesis? The first reason is for the sake of clarity. Many prior analytical studies have used various types of time-dependent utility functions to represent different user models, e.g. constant discount rate, constant cost of delay, etc. [Kraus et al., 1995; Sandholm and Vulkan, 1999]. Adopting a complex function for analytical purposes is context-dependent. Therefore, rather than analyzing a list of complex utility functions, we choose a simpler one, i.e. surplus.

The second reason is due to the difficulty of preference elicitation and utility construction [Boutilier et al., 2006]. For instance, it is not easy to set a user's time-dependent utility function when it has high disparity/discontinuity. Depending on the negotiated item, our preference may change in hourly or daily basis rather than continuously in milliseconds. Also, our preference may not change a lot for a transaction made between 1 a.m. midnight and 6 a.m. morning at the same day, but it may change a lot for the transaction made between 8 a.m. and 1 p.m. at the same day; for instance, when the negotiated item is the analysis of the stock market.

Example 3-5. *Suppose an agent allows its owner to enter her valuation in discrete form, such as ((3, \$1000), (7, \$900), (10, \$850), (30, \$700)), which means their valuation is constant \$1000 for the first three days, \$900 for the next seven days, \$850 for the next ten days, and \$700 for the next 30 days. Then the agent will set its valuation as follows:* $\boldsymbol{B_1} = \boldsymbol{B_2} = \boldsymbol{B_3} = 1000$, $\boldsymbol{B_4} = \ldots = \boldsymbol{B_{10}} = 900$, *etc. From the user's perspective, using this method may be easier than using a discount rate function, where the system will ask the user to set two parameters* $\boldsymbol{B_0}$ *and* $\boldsymbol{\beta}$ *and set the agent's valuation in the form of* $\boldsymbol{B_t}=\boldsymbol{B_0}(1+\boldsymbol{\beta})^{-\boldsymbol{t}}$.

3.2 Decision under Uncertainty and Evaluation Functions

In this section we will discuss three typical bargainer actions in alternating-offers bargaining, i.e. withdraw, accept, and counter-offer. The evaluation criteria for each action depend on the bargainer's strategy. For instance, a buyer will accept an offer from the seller if the offer is better than the buyer's expectation; otherwise it will counter-offer or withdraw from the negotiation. Let's say a buyer expects to accept any price less than \$500 while the seller's offer happens to be \$450, thus, the buyer accepts it. Usually, the bargainer withdraws from a negotiation if the time deadline is passed, or further negotiation will generate negative surplus, or a better agreement has been made. Any withdrawal by any bargainer is considered as the failure (breakdown) of the negotiation.

We will first introduce two evaluation criteria (for withdrawing, accepting and counter-offering) used in this thesis (section 3.2.1). Then we will discuss the process to generate counteroffers by bounded rational agents (section 3.2.2). Under uncertainty, the agent's rationality is commonly defined as the behavior of an agent who only offers the price yielding the highest expected utility and accepts an opponent's offer if it maximizes the agent's utility.

3.2.1 Evaluation Criteria

In Assumption 3-1, we assume that the buyer is constrained by time deadline. We further assume here that when the deadline passes, the negotiation failed and the buyer must withdraw from the negotiation without penalty (zero surplus).

Three types of evaluation criteria (evaluation functions or EvalF for short) are discussed here, i.e. EvalF-0, EvalF-I, and EvalF-II. But only EvalF-I and EvalF-II are considered in our analysis. Under some conditions those two types are identical, but not under other conditions. Therefore, we will analyze them separately here. Let's start with EvalF-0 agent.

3.2.1.1 EvalF-0 agent

An EvalF-0 agent uses the simplest evaluation function in accepting an offer by its opponent, i.e., accept an offer if it generates positive surplus regardless of the amount of the surplus.

Example 3-6. *Suppose the buyer's deadline has not passed and its current valuation is \$100 and the seller's current offer is \$99; thus, the buyer will accept the seller's offer because it will generate \$100 – \$99 = \$1 surplus. However, if the seller's offer is \$105, then the buyer should not accept it but will counter offer, let say \$95. But if the deadline has passed while the seller's last offer is \$105, then the buyer can only withdraw from the negotiation.*

The evaluation criteria of EvalF-0 agents can be formulated in the following definition.

Definition 3-4. *Suppose it is the turn of an EvalF-0 buyer at time* $\boldsymbol{t}$*. Then it uses the following evaluation function in making its decision:*

$$\boldsymbol{I_t} = \begin{cases} \textit{Withdraw} & \textit{iff } \boldsymbol{t} > \boldsymbol{T_d} \\ \textit{Accept } \boldsymbol{y_t} & \textit{iff } \boldsymbol{Sur_t(y_t)} > 0 \textit{ and } \boldsymbol{t} \le \boldsymbol{T_d} \\ \textit{Counter offer } \boldsymbol{x_t} & \textit{otherwise} \end{cases} \tag{3-1}$$

where $\boldsymbol{I_t}$ *is the buyer's decision function at time* $\boldsymbol{t}$*,* $\boldsymbol{T_d}$ *is the buyer's time deadline,* $\boldsymbol{y_t}$ *is the offer by the seller at time* $\boldsymbol{t}$*,* $\boldsymbol{x_t}$ *is the buyer's offer that will be proposed,* $\boldsymbol{Sur_t(y_t)}$ *is the surplus if* $\boldsymbol{y_t}$ *is accepted by the buyer at time* $\boldsymbol{t}$*.*

The goal of EvalF-0 agents is to make a deal as soon as the resolution is acceptable regardless of the surplus, e.g. in [Tesauro, 2002]. It implicitly assumes that the user/owner does not care about the extent of the surplus.

3.2.1.2 EvalF-I agent

EvalF-I agents accept an offer if it generates a surplus that is greater than or equal to that of the counteroffer that will be sent by the agent in the next round as illustrated by Example 3-7.

Example 3-7. *Suppose the buyer's deadline has not passed and its current valuation is \$100 and the seller's current offer is \$80; thus, by accepting the seller's current offer the buyer will generate \$20 surplus. If the buyer's next valuation is the same \$100 and it believes by certainty that the seller can actually sell the item for \$75 in the next round, then it is better for the buyer to refuse seller's current offer but ask for \$75 in the next round, because the surplus will be \$25 which is higher than \$20 if it accepts the seller's current offer. But if the buyer's valuation is \$80 in the next round, then it is better* *for the buyer to accept seller's offer now, because it will only generate \$5 surplus in the next round.*

Under this rationale, the acceptance criterion for the buyer is to accept the seller's offer if it generates higher surplus than that by sending a counter offer in the next round. The evaluation criteria of EvalF-I agents can be formulated in the following definition.

Definition 3-5. *Suppose it is the turn of an EvalF-I buyer at time* $\boldsymbol{t}$. *Then it uses the following evaluation function in making its decision:*

$$\boldsymbol{I_t} = \begin{cases} \textit{Withdraw} & \textit{iff } \boldsymbol{t} > \boldsymbol{T_d} \textit{ or } (\boldsymbol{Sur_t^e(x_t)} \leq 0 \textit{ and } \boldsymbol{Sur_t(y_t)} \leq 0) \\ \textit{Accept } \boldsymbol{y_t} & \textit{iff } \boldsymbol{Sur_t(y_t)} > 0 \textit{ and } \boldsymbol{Sur_t(y_t)} \geq \boldsymbol{Sur_t^e(x_t)} \textit{ and } \boldsymbol{t} \leq \boldsymbol{T_d} \\ \textit{Counter offer } \boldsymbol{x_t} & \textit{otherwise} \end{cases} \tag{3-2}$$

where $\boldsymbol{x_t}$ *is the buyer's offer that will be proposed,* $\boldsymbol{Sur_t^e(x_t)}$ *is the estimated surplus at time* $\boldsymbol{t}$ *if* $\boldsymbol{x_t}$ *is accepted by the seller in the next bargaining round, i.e. at time* $\boldsymbol{t+1}$.

In the context of Example 3-7, we have $\boldsymbol{Sur_t(y_t)} = \boldsymbol{Sur_t}(\$80) = \$100 - \$80 = \$20$, and two possible estimated surpluses: $\boldsymbol{Sur_t^e(x_t)} = \boldsymbol{Sur_t^e}(\$75) = \$100 - \$75 = \$25$ when the buyer's valuation does not change, or $\boldsymbol{Sur_t^e}(\$75) = \$80 - \$75 = \$5$ if it decreases to \$80 at the next period. Here, the buyer estimated surplus is trivially calculated as the difference between the buyer's valuation and its offer, because the buyer is certain that $\boldsymbol{x_t}$ will be accepted by the seller. For instance, if both agents made similar deals at prices above $\boldsymbol{x_t}$ in the past encounters, then the buyer may believe that the seller will certainly accept $\boldsymbol{x_t}$. Similarly, if the buyer can predict the concession that will be made by the seller (e.g. the seller always concede at a constant rate), then at a certain point the buyer may also believe that the seller will certainly accept $\boldsymbol{x_t}$. For example, suppose the seller's and the buyer's last offers are < \$500, \$490, \$480, \$470, \$460> and < \$400, \$410, \$420,

$430, $440 >, respectively, where the seller moves first by offering $500. Since the seller always concedes at a rate of $10, then the buyer may believe that the seller will certainly accept $450. When the agent is certain that $\boldsymbol{x_t}$ will be accepted, then we have $\boldsymbol{Sur_t^e(x_t)}$ = $\boldsymbol{Sur_{t+1}(x_t)}$; for instance, $\boldsymbol{Sur_t^e(x_t)} = \boldsymbol{Sur_{t+1}(x_t)} = \boldsymbol{B_t} - \boldsymbol{x_t}$ for the buyer.

When the buyer is uncertain whether $\boldsymbol{x_t}$ will be accepted or not, then it may use $\boldsymbol{Sur_t^e(x_t) = p_t(x_t)\ Sur_{t+1}(x_t) + (1 - p_t(x_t))\ R_{t+1}(x_t)}$, where $\boldsymbol{p_t(x_t)}$ is its belief (subjective probability) that $\boldsymbol{x_t}$ will be accepted at time $\boldsymbol{t+1}$, $\boldsymbol{Sur_{t+1}(x_t)}$ is the surplus made if $\boldsymbol{x_t}$ is accepted by the seller at time $\boldsymbol{t+1}$, and $\boldsymbol{R_{t+1}(x_t)}$ is the estimated future surplus (residue) if $\boldsymbol{x_t}$ is not accepted. The discussion on $\boldsymbol{R_{t+1}(x_t)}$ and $\boldsymbol{p_t(x_t)}$ will be provided later in this chapter.

The evaluation function in equation (3-2) is also used in other automated negotiation literature [Faratin et al., 1998; Fatima et al., 2004; Li et al., 2006]. Sim and Wang [2004] have adopted fuzzy rules to modify the acceptance criteria of EvalF-I agents. Under their fuzzy criterion, an agent may accept an offer even if it is slightly worse than its next counteroffer.

3.2.1.3 EvalF-II agent

In addition to the evaluation criterion used by an EvalF-I agent, an EvalF-II agent uses an additional evaluation function in accepting an offer by its opponent, i.e., accept an offer if it is perceived to generate an optimal surplus.

Example 3-8. *Suppose a buyer's current valuation is $100 and the seller's current offer is $90, then the buyer will accept the offer if it predicts that the seller will not offer any better price until the end of the bargaining.*

The rationale is that some agents may be myopic and risk-averse towards breakdown in the future. Therefore, they may make a secure-yet-perceived-as-optimal offer even when a better one may exist in the future. While they also behave as EvalF-I agents, this additional evaluation criterion makes EvalF-II agents more 'vulnerable' to their opponent's strategy. For instance, a seller may stand on its offer because it believes that the buyer will be more likely to accept it. If the buyer thinks that the offer is acceptable and perceives it as the 'best' offer that it will ever get, then it will accept it, even when the seller may have offered a better one in the future. Later in sub-section 3.3.1 we will discuss how a buyer perceives a 'best' offer that it will ever get. Right now we will formalize the evaluation criterion of EvalF-II agents.

Definition 3-6. *Suppose it is the turn of an EvalF-II buyer at time* $\boldsymbol{t}$*. Then it uses the following evaluation function in making its decision:*

$$\boldsymbol{I_t} = \begin{cases} \textit{Withdraw} & \textit{iff } \boldsymbol{t} > \boldsymbol{T_d} \textit{ or } (\boldsymbol{Sur_t^e(x_t)} \leq 0 \textit{ and } \boldsymbol{Sur_t(y_t)} \leq 0) \\ \textit{Accept } \boldsymbol{y_t} & \textit{iff } [\boldsymbol{Sur_t(y_t)} \geq \boldsymbol{Sur_t^e(x_t)} \vee \wedge_j(\boldsymbol{Sur_t(y_t)} \geq \boldsymbol{Sur_t^e(y_j)})] \textit{ and} \\ & \qquad \boldsymbol{Sur_t(y_t)} > 0 \textit{ and } \boldsymbol{t} \leq \boldsymbol{T_d} \\ \textit{Counter offer } \boldsymbol{x_t} & \qquad \textit{otherwise} \end{cases} \tag{3-3}$$

where $\boldsymbol{Sur_t^e(y_j)}$ *is the estimated value of* $\boldsymbol{Sur_t(y_j)}$ *and* $\boldsymbol{j} \in \{\boldsymbol{t+1}, \boldsymbol{t+2}, \ldots, \boldsymbol{T_d}\}$ *are the turns of the buyer in the future until its time deadline* $\boldsymbol{T_d}$*.*

3.2.1.4 A brief discussion

EvalF-0 and EvalF-I agents are more common in the automated negotiation literature, e.g. in [Faratin et al., 1998; Tesauro, 2002; Fatima et al., 2004; Li et al., 2006], while EvalF-II agents have rarely been discussed. In this thesis, we introduce the EvalF-II agent to facilitate the analysis of non-monotonic-offers bargaining, because its acceptance criteria are more flexible than that of EvalF-I agents. Not all agents are surplus maximizers and able to counteroffer a price that maximizes their expected surplus. Intuitively, humans also use similar heuristics in negotiations, because we may accept an offer if we predict that it is already the best that our opponent can make and it generates positive surplus for us. Some experimental results in Chapter 5 indicate that EvalF-II agents may be better than EvalF-I agents in terms of success rate. Finally, Definitions 3-4, 3-5 and 3-6 are not only for the buyer but also apply for the seller's decision.

In addition to EvalF-I and EvalF-II agents, there are several other evaluation criteria which are not considered in this thesis. For example, an agent that will never accept any offer before its deadline but will accept any offer at its time deadline (deadline-driven), or an agent that will never accept their opponent's offer but will persuade their opponent to accept the proponent's offer (Boulwarism) [Raiffa, 1982]. We do not consider these acceptance criteria.

So far, we have defined the evaluation criteria of withdrawing, accepting and counter-offering by two different types of agents, i.e. EvalF-I and EvalF-II agents. But we have not described the generation of agents' offers. Unlike game-theoretic analysis, we cannot assume that agents have perfect foresight and are capable of generating offers that maximize their expected utility which directly generate the equilibrium solution (e.g. Rubinstein's bargaining solution [1982]). Instead, we will consider myopic agents that are able to generate a perceived optimal offer after considering partially observable information. These agents represent the decision-theoretic agents, who seek an optimal solution that maximizes their expected utility under uncertainty.

3.2.2 Myopic Agents that Maximize Expected Gain

Depending on the algorithm used, agents may consider different levels of future outcomes in their decisions to generate an offer. For instance, at time $\boldsymbol{t}$ the most myopic agent (namely myopic-0) will only consider all possible outcomes at time $\boldsymbol{t}$ regardless of whether or not the negotiation proceeds to the next round $\boldsymbol{t+1}$. On the other hand, myopic-1 agents will consider all possible outcomes at time $\boldsymbol{t}$ and $\boldsymbol{t+1}$ (one round in the future), which may happen if the negotiation at time t did not conclude with any result. Moreover, a myopic-K agent will take into account the situation when the negotiation does not yield any result until the K-th round. In open systems, agents are bounded rational and myopic to different degrees, and can never assume that their opponents will be the same type as them. For the sake of clarity and simplicity, in this thesis we will only consider myopic-0 and myopic-1 agents.

3.2.2.1 Myopic-0 agents

As a bounded rational agent, the buyer will never offer a price that generates negative surplus. Suppose that $\boldsymbol{Sur(x) = B_t - x}$, where $\boldsymbol{B_t}$ is the buyer valuation at time $\boldsymbol{t}$. Since the buyer only considers the immediate situation (myopic-0) and ignores the past and future situation, then its optimal decision depends only on the expected gain $\boldsymbol{EG_t}$ which is a function of the offer $\boldsymbol{x}$ defined as follows:

Definition 3-7. $\boldsymbol{EG_t(x) \equiv (1 - q_t)\, p_t(x)\, (B_t - x) + q_t\, B_\phi}$ (3-4)

where $\boldsymbol{q_t} \in [0, 1]$ *is the buyer's belief of the likelihood of negotiation breakdown caused by the seller at time* $\boldsymbol{t}$ *which is independent of* $\boldsymbol{x}$, $\boldsymbol{B_\phi}$ *is the buyer's valuation if the negotiation breaks down, and* $\boldsymbol{p_t(x)} \in [0, 1]$ *is the buyer's belief (subjective probability) function that price* $\boldsymbol{x}$ *will be accepted by the seller (acceptance rate) at time* $\boldsymbol{t}$.

We assume that q_t is independent of x, e.g. the seller has approached its deadline or has made a deal with another buyer, etc. And we use ϕ to denote the time of breakdown. If the negotiation does not break down, which the agent believes may happen with probability $(1 - q_t)$, then two possible states may happen. The first state is that the bargaining succeeds immediately (with subjective probability $p_t(x_t)$ where x_t is the price offered by the buyer), which gives the buyer positive surplus $(B_t - x_t)$. The other state is that the bargaining proceeds to the next round, which is ignored by the buyer since it is of myopic-0 type. Let us assume that no surplus is generated from a breakdown ($B_\phi = 0$), and due to the independence of x from q_t, then the optimization problem is to maximize $EG_t(x)$ in equation (3-4) which becomes

$$Max_x\, EG_t(x) = Max_x\, [\, p_t(x)(B_t - x)] \qquad (3\text{-}5)$$

Assumption 3-6. *A myopic-0 buyer will offer an optimum price* x_t^* *that yields the highest expected surplus at the present time* t*, i.e.,* $Max_x\, [\, p_t(x) \times Sur_t(x)]$.

For example, let the buyer's valuation B_t = \$100 and it believes that offering \$100 or above will be certainly accepted by the seller and offering \$$x$ will have the probability $p_t(x) = x/100$ for $0 \le x \le 100$. By equation (3-5) it is easily calculated that the optimum price x_t^* = \$50, i.e. with 0.5 chance of being accepted.

3.2.2.2 Myopic-1 agents

In our model, we define a myopic-1 buyer as an agent that will offer a price that yields the highest weighted sum of expected surplus at the present time t and the next round $t+1$. The buyer's problem can be stated as $Max_x\, EG_t(x)$ where

Definition 3-8. $EG_t(x) \equiv (1 - q_t)\, [p_t(x)(B_t - x) + \gamma(1 - p_t(x))EG'_{t+1}(x)] + q_t\, B_\phi$

(3-6)

where $\gamma \in [0, 1]$ is the weight of the expected gain in the next round ***t+1***, $\boldsymbol{EG'_{t+1}(x)}$ *is the estimation of the expected gain in the next round which is made by the buyer at the current round* ***t***, *and the rest of parameters are as defined in Definition 3-7.*

We assume that $\gamma \leq 1$, because the agent is always averse toward uncertainty in the future and $\gamma \boldsymbol{EG'_{t+1}(x)}$ is the estimated future surplus or residue $\boldsymbol{R_{t+1}(x_t)}$. It is also important here to differentiate between $\boldsymbol{EG'_{t+1}(x)}$ and the function $\boldsymbol{EG_{t+1}(x)}$, because both of them may not be the same function given that the buyer may revise its beliefs at time ***t+1*** (a more detailed discussion is provided later in sub-section 3.2.2.3 and 3.3.6). Similar to the analysis of myopic-0 buyers, if we assume that no surplus is generated from a breakdown and $\boldsymbol{x}$ is independent of $\boldsymbol{q_t}$, then equation (3-6) becomes

$$\boldsymbol{Max_x\, EG_t(x) = Max_x\, [p_t(x)(B_t - x) + \gamma(1 - p_t(x))EG'_{t+1}(x)]} \qquad (3\text{-}7)$$

Assumption 3-7. *A myopic-1 buyer will offer a price* $\boldsymbol{x_t^*}$ *that yields the highest expected gain of the combination of positive surplus at time* ***t*** *and expected gain at time* ***t+1***, *i.e.* $\boldsymbol{Max_x\, [p_t(x) \times Sur_t(x) + \gamma(1 - p_t(x))EG'_{t+1}(x)]}$ *subject to* $\boldsymbol{Sur_t(x)} > 0$.

For example, let the buyer's valuation $\boldsymbol{B_t}$ = \$100 and it believes that offering \$100 or above will be certainly accepted by the seller and offering \$$\boldsymbol{x}$ will have the probability $\boldsymbol{p_t(x)} = \boldsymbol{x}/100$ for $0 \leq \boldsymbol{x} \leq 100$. By equation (3-5) it is easily calculated that the optimum price $\boldsymbol{x_t^*}$ = \$50, i.e. with 0.5 chance of being accepted.

3.2.2.3 Remarks

Depending on the agent's computation, $\boldsymbol{EG'_{t+1}(x)}$ may or may not depend on $\boldsymbol{x}$. In human negotiation, if a high offer $\boldsymbol{x_h}$ is rejected, then the expected gain in the future may be lower than it would be if a low offer $\boldsymbol{x_l}$ is rejected.

Example 3-9. *Suppose the buyer's valuation is constant over time, say \$100. If a buyer offered \$90 and was rejected, then his/her expected gain in the future may not be more than \$10, because the buyer now believes that the seller may only accept a price above \$90. On the other hand, if the buyer offered \$80 and was rejected, then his/her expected gain in the future may be more than \$10, because there is no reason for him/her to believe that the seller will not accept a price below \$90. Thus, in this case* $\boldsymbol{EG'_{t+1}(x)}$ *is a decreasing function with respect to* $\boldsymbol{x}$*, i.e.* $\boldsymbol{EG'_{t+1}}(\$80) > \boldsymbol{EG'_{t+1}}(\$90)$*. However, this is correct only if the buyer believes that the seller will react identically in both cases, for example by counter-offering the same price \$100, which may not be true in some cases. For example, if the buyer conceded more by offering \$90, then it is more likely that a cooperative (benevolent) seller will also concede more by offering a closer price, say \$95, which may increase the expected gain of the buyer due to a higher belief function* $\boldsymbol{p_{t+1}(x)}$ *in estimating* $\boldsymbol{EG'_{t+1}(x)}$*.*

Conclusively, in human negotiation, our estimation of our future gain varies according to the interaction and observation of our opponent's behavior. However, in artificial agents the situation may not be exactly the same as in human negotiation, because the agent's behavior depends on the algorithm used. We will discuss this issue in more detail in subsection 3.3.6 after describing the belief mechanism.

Another issue is both equations (3-5) and (3-7) may not be solved analytically if $\boldsymbol{p_t(x)}$ is a discrete, discontinuos or piecewise function. However, we assume that they can be solved numerically by the agents. If there are two or more values of $\boldsymbol{x}$ (offers) maximizing the equations, i.e. more than one $\boldsymbol{x_t^*}$, then we assume that the buyer will choose the one with higher $p_t(x_t{}^*)$. Another constraint that must be considered in solving the equation is $Sur_t(x)$, which must be positive.

In both equations, the function $p_t(x)$ is crucial in determining the buyer's correct decision that maximizes its surplus and results in a high success rate of negotiation. If $p_t(x)$ is generated incorrectly in the sense of mistaken belief, then it may affect the buyer's offer and may lead to a lower success rate or surplus. Since $p_t(x)$ is very important in determining the agent's offer, we will discuss it separately in section 3.3. For the purpose of analysis, we will use a simplified model of $p_t(x)$ and $EG'_{t+1}(x)$ in order to derive our propositions.

3.3 Agent's Belief

As stated before, the agent's belief has an important role in deciding the negotiation outcomes, especially in myopic agents. Fundamentally, the agent's belief has been considered as one of the characteristics of intelligent agents, commonly denoted as BDI agents (agents with belief, desire, and intention) [Bratman, 1987; Wooldridge, 2000]. In much early work, the BDI architecture has been used for various purposes, including cooperation among agents and their interaction with the environment; for instances, Procedural Reasoning System (PRS) [Georgeff and Lansky, 1987], dMARS [d'Inverno et al., 1998], JADEX [Pokahr et al., 2005], etc. In this section, we will describe various belief functions and their revision mechanism that will be used in both the analytical and empirical study of our proposed bargaining protocols.

Specifically, there are several factors in agents' decision-making that involve belief. For example, EvalF-II agents must believe that $Sur_t(y_t) \geq Sur_t^e(y_j)$ for all $j \in \{t+1, t+2, \ldots, T_d\}$ in order to accept y_t. In this case, agents must have the capability to estimate all future outcomes and believe that they are correctly estimated and will not generate higher surplus than the current one. Another example is that myopic agents must assign belief (in term of subjective probability) over the acceptance rate of their offer, i.e. $\boldsymbol{p_t(x)}$. Moreover, myopic agents must also have beliefs regarding the breakdown rate $\boldsymbol{q_t}$. Finally, in a more complex interaction that involves argumentation, which will be explained later, agents must also assess the truthfulness of their opponent's arguments. In this case, trust and argument verification are two important factors. In this section, we will describe those beliefs which will be used in our analysis. Certainly, this is not an exhaustive list of all possible agents' beliefs, but a list representing some commonsense beliefs that may be used by bargaining agents and are adopted in our study.

3.3.1 Belief toward Maximum Surplus in EvalF-II Agents

Suppose the seller's offers in the first six rounds are as follows < \$125, \$100, \$95, \$90, \$90, \$90>. Then as a human buyer, we may predict that it is likely the seller will not sell it at a price lower than \$90. So, if our valuation is a constant \$100 and we are not willing to wait for a longer time, then we will accept this price, which generates \$10 surplus. Our decision here has reflected the characteristics of EvalF-II agents, who must decide whether or not the current offer from their opponent has generated the highest possible surplus.

Since agents do not have perfect foresight, then they must rely on limited information to estimate their future surplus, which may not be correct. Suppose that the past and future surplus is defined consistently as $\boldsymbol{Sur(y_t) = B_t - y_t}$, $\boldsymbol{t} \in \{\boldsymbol{0}, \boldsymbol{1}, \ldots, \boldsymbol{T_d}\}$. If $\boldsymbol{B_t}$ is constant, say equal to $\boldsymbol{B}$, then agents may observe the past surplus values and use the first order derivative condition to decide whether the current surplus is maximum or not, i.e. $\partial(\boldsymbol{B} - \boldsymbol{y_t})/\partial \boldsymbol{t} = 0$ and $\partial^2(\boldsymbol{B} - \boldsymbol{y_t})/\partial \boldsymbol{t}^2 < 0$. In a discrete domain it can be accomplished by first deriving the continuous function of $\boldsymbol{y(t)}$, e.g. by using regression analysis. However, this task is not easy, especially if the data is inconsistent (no regularity in their opponent's offers) and agents do not know the strategy used by their opponent. If the opponent's offer is asymptotic (e.g. follows a logarithmic function), then the first order condition would never be met.

Adopting a more flexible condition, such as $\partial(\boldsymbol{B} - \boldsymbol{y_t})/\partial \boldsymbol{t} \approx 0$ and $\partial^2(\boldsymbol{B} - \boldsymbol{y_t})/\partial \boldsymbol{t}^2 < 0$, may help, but not always. For example, if the seller uses a random strategy, then the standard error of the regression will be high and the confidence level will be low, which in turn cannot convince the buyer to believe the regression function. Thus, in the absence of consistent data and with low certainty of other factors (e.g. agents' valuation is not constant or unknown), then the accuracy of this method will be low.

Alternatively, the agents can adopt another statistical method; for example by comparing the current surplus with the surplus they have made in the past and predicting the likelihood that the current surplus is the highest one. Or, the agents may use a more complex method by predicting the characteristics of their opponent, a research area known as agent modeling [Carmel and Markovitch, 1996; Riley and Veloso, 2000; Denzinger and Hamdan, 2004]. For example, the agents may predict whether its oppoinent is a deceiver or not, or predicting its valuation by looking at its history if any historic data is available. However, these methods may not be accurate if there are not many experiences or information gathered by the agent.

Since there are several methods that can be adopted by the agents, we will not use all of them in our analysis; rather, we will use a more general one based on our commonsense, which tells us that the likelihood of maximum surplus is low if our opponent's offers are steeply changed. For example, consider the following two sequences of offer by two sellers: <$91, $90, $90> and <$99, $99, $90>. In the absence of any other information about sellers' valuation, it is more convincing that the first seller has reached its limit (e.g. its valuation), while it is less convincing that the second seller has. The reason is that the first seller has only made a small amount of concession during the three offers, while the second seller made a relatively large amount of concession; thus, it is very likely that the second seller may still be willing to concede more. Thus, we can derive a conclusion that if an agent's consecutive offers are close, then it is more likely that it has reach a limit; thus, the surplus is maximized. Using this heuristic we will analyze EvalF-II agents in this thesis.

3.3.2 Belief Function $p_t(x)$ if the Opponent is EvalF-I Agent

3.3.2.1 Belief function

Suppose that the buyer knows that the seller is an EvalF-I agent. Then the buyer knows that the seller will accept its offer *iff* its offer is greater than or the same as the seller's next offer. Under uncertainty, the seller's next offer is unknown by the buyer, but it could be predicted to follow a subjective probabilistic distribution function (subjective because it depends on the buyer's algorithm). Then if the buyer offers a price, the likelihood that it will be accepted by the seller is the cumulative distribution value of the seller's next offer. Then offering a higher price will have a higher chance of exceeding the seller's next offer. Thus, $\boldsymbol{p_t(x)}$ is an increasing function of $\boldsymbol{x}$ (i.e., an accumulative distribution function of the seller's next offer).

Proposition 3-1. *Suppose the buyer knows that the seller is EvalF-I. Then the buyer's belief function that its offer at time* $\boldsymbol{t}$ *will be accepted by a seller,* $\boldsymbol{p_t(x)}$ *is an increasing function of* $\boldsymbol{x}$*, where for any* $\boldsymbol{x}$ *we have* $\boldsymbol{p_t(x)} \in [0, 1]$*.*

Another characteristic is $\boldsymbol{p_t(x)} = 1$ for $\boldsymbol{x} \geq \boldsymbol{y_t}$, where $\boldsymbol{y_t}$ is the seller's current offer; and $\boldsymbol{p_t(x)} = 0$ for $\boldsymbol{x} < \boldsymbol{z}$, where $\boldsymbol{z}$ is the buyer's estimation of the seller's valuation. Thus, the shape of $\boldsymbol{p_t(x)}$ will be from the family of step functions, linear functions, logarithmic functions or logistic functions, depending on the initial program, the value of $\boldsymbol{y_t}$ and $\boldsymbol{z}$, and the updating method (belief revision).

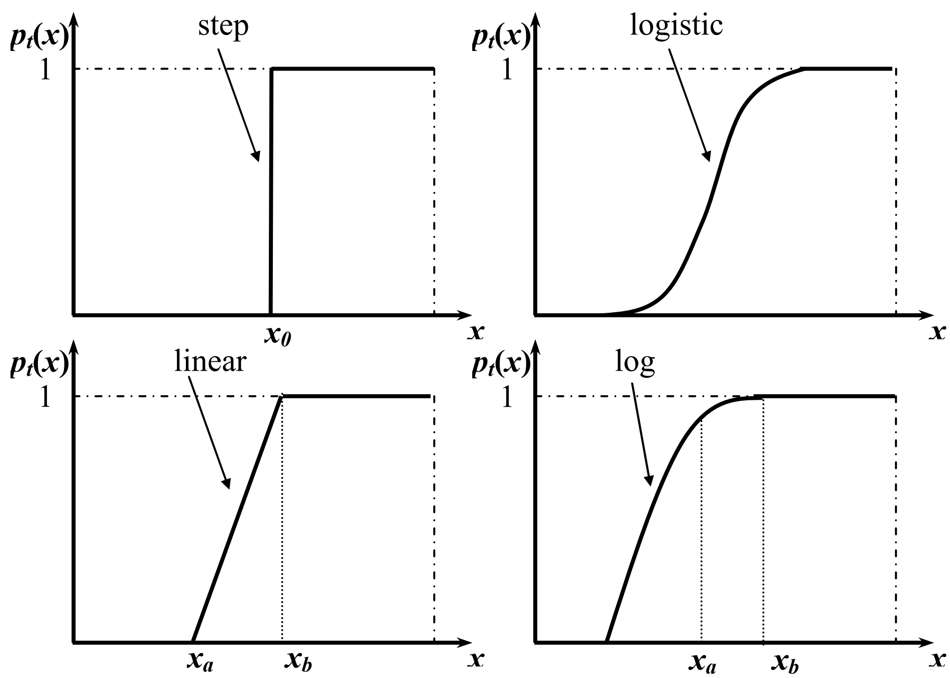


Figure 3-5. Various agents' belief functions $\boldsymbol{p_t(x)}$

Figure 3-5 shows four different belief functions that satisfy Proposition 3-1. A step function represents a two-state belief, i.e. the buyer believes that the seller will only accept a price higher than or equal to $\boldsymbol{x_0}$ ($\boldsymbol{x_0}$ is the seller's next offer with certainty). If the agent is more risk averse, then it may choose a higher value of x_0. A linear function represents a uniform probability that the seller's next offer will appear between x_a and x_b. A logarithm function represents an agent characteristics with a high indecisiveness on a wide range of offers (a range between x_a and x_b in Figure 3-5), which means the seller's next offer is not likely greater than x_a. Which belief function and what parameters should be chosen by an agent depend on the designer and the user characteristics (if the users can set their agent's parameter explicitly). However, in this thesis our goal is not seeking the best belief function or their characteristics, but finding a better protocol that can help those agents to find a better resolution.

3.3.2.2 Belief revision

When an agent receives new information, then it may update its belief function. Intuitively, a buyer will reduce $p_t(x_t^*)$ if its optimal offer, x_t^* has not been accepted by the seller, or increase $p_t(x_t^*)$ if the seller's counteroffer, y_t is very close to x_t^*. Figure 3-6(a) shows an example of a buyer's belief function at time t and $t+1$, i.e., after the seller drops its offer from \$200 to \$190.

Example 3-10. *Suppose the buyer's valuation is \$180 and its previous offer* $x_t^* =$ *\$160, with* $p_t(\$160) = 0.5$. *If the seller reduces its offer from \$200 to \$190, then the buyer may change its belief function from* $p_t(x)$ *to* $p_{t+1}(x)$. *The new beliefs are higher for prices above \$175, and lower for prices below \$175 (the intersection of* $p_t(x)$ *and* $p_{t+1}(x)$*). Thus, the buyer now believes that the acceptance rate of offering \$160 is 0.3, which may force the buyer to raise its offer. However, if the seller insists on its current price, the buyer's belief will shift to the right, because by now, the buyer may think that the seller will not be able to concede (Figure 3-6(b)). In this case,* $p_t(x)$ *is first order stochastic dominant over* $p_{t+1}(x)$ *in Figure 3-6(b). More seriously, if the seller increases the price rather than* *conceding, then the buyer may think that the seller cannot even insist on its previous offer; thus, the buyer's belief will reduce more and becomes steeper (Figure 3-6(c)). In this case,* $\boldsymbol{p_{t+1}(x)}$ *in Figure 3-6(c) is first order stochastic dominant over* $\boldsymbol{p_{t+1}(x)}$ *in Figure 3-6(b).*

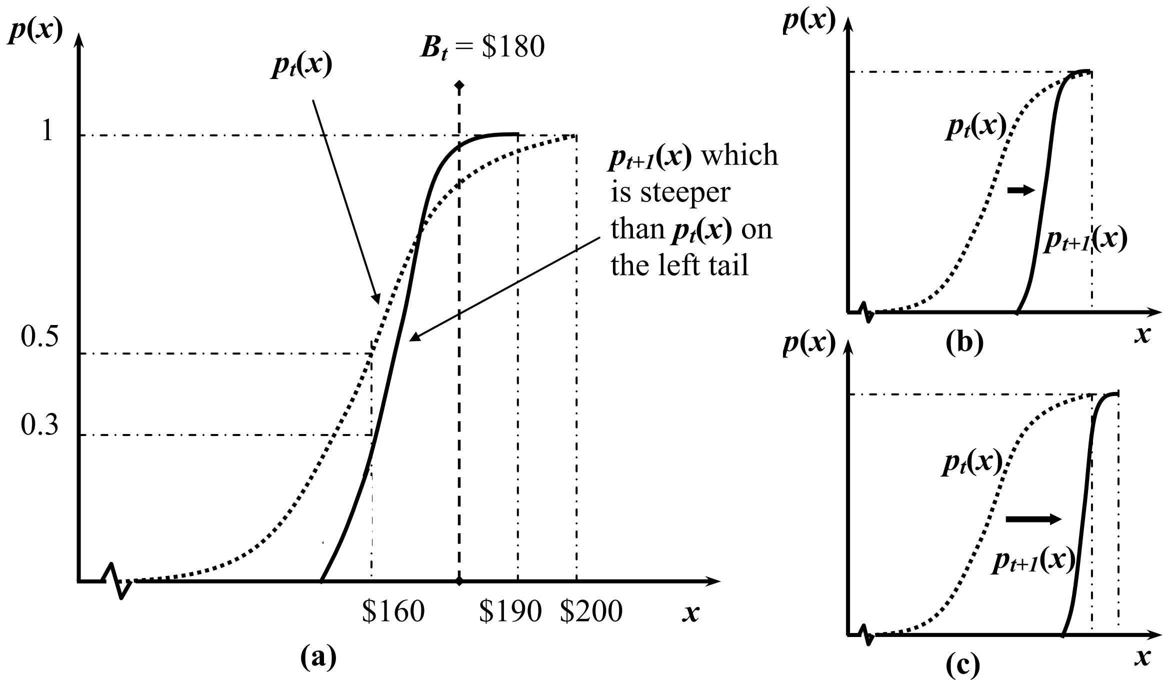


Figure 3-6. An example of the shifting of a buyer's belief towards the seller's acceptance rate after (a) seller concedes, (b) seller insists, and (c) seller increases its price

Surely, the magnitude and frequency of update depends on the buyer's characteristics. For example, a skeptical buyer may not update its belief frequently even after the seller rejected its offer several times, and if an update occurs the magnitude may not be large. In contrast, a believing buyer may update its belief more frequently with higher magnitude. Regardless of the agent's characteristics, we may come up with the following assumption that is applied in our model for the updating mechanism of the buyer's belief when facing a EvalF-I seller.

Assumption 3-8. *Suppose that the buyer knows that the seller is EvalF-I. (i) If the buyer's optimal offer* $\boldsymbol{x_t^*}$ *is rejected, then the buyer will reduce* $\boldsymbol{p_t(x)}$ *for all* $\boldsymbol{x \in Acc}$ *and* $\boldsymbol{x \leq x_t^*}$*, where a price smaller than* $\boldsymbol{x_t^*}$ *is reduced faster; thus yielding a steeper function* $\boldsymbol{p_{t+1}(x)}$ *at* $\boldsymbol{x \leq x_t^*}$*. (ii) If the seller concedes such that* $\boldsymbol{y_{t+1} < y_t}$*, then the buyer will increase its belief such that* $\boldsymbol{p_{t+1}(x)} = 1$ *for all* $\boldsymbol{x \geq y_{t+1}}$*, and may also increase* $\boldsymbol{p_t(x)}$ *for* $\boldsymbol{x}$ *that is slightly lower than* $\boldsymbol{y_{t+1}}$*. (iii) If the seller's offer is unchanged or raised to* $\boldsymbol{y_{t+1} > y_t}$*, then the buyer will decrease all belief of* $\boldsymbol{x < y_{t+1}}$*, i.e.* $\boldsymbol{p_{t+1}(x)}$ *becomes a steeper function.*

In Assumption 3-8(ii) an imprecise concept "slightly lower than $\boldsymbol{y_{t+1}}$" is used to represent the increase of belief between \$175 and \$190 in Figure 3-6(a), i.e. $\boldsymbol{p_{t+1}(x)} > \boldsymbol{p_t(x)}$ for $\boldsymbol{x} \in$ [\$175, \$190], where \$175 is the intersection point between $\boldsymbol{p_{t+1}(x)}$ and $\boldsymbol{p_t(x)}$, and \$190 is $\boldsymbol{y_{t+1}}$. In this case the phrase "slightly lower" represents prices lower than \$190 until \$175. Depending on the updating algorithm and the belief functions, the intersection point in Figure 3-6(a) may be closer to $\boldsymbol{y_{t+1}}$ or further from it. Combining Assumption 3-8(i) and 3-8(ii), $\boldsymbol{p_{t+1}(x_t^*)}$ will decrease if $\boldsymbol{y_{t+1}}$ is significantly higher than $\boldsymbol{x_t^*}$, and may increase if $\boldsymbol{y_{t+1}}$ is only slightly higher than $\boldsymbol{x_t^*}$.

Example 3-11. *Suppose the buyer's offer at time* $\boldsymbol{t}$ *is \$100 with prior belief* $\boldsymbol{p_t}$*(\$100) = 0.8. If the seller refuses to accept the offer but counter-offers \$200, then the buyer's posterior belief* $\boldsymbol{p_{t+1}}$*(\$100) may drop to 0.7. However, if the seller's counteroffer is \$101, then the buyer's posterior belief* $\boldsymbol{p_{t+1}}$*(\$100) may increase to 0.95.*

Proposition 3-2. *If an optimal offer* $\boldsymbol{x_t^*}$ *is rejected by an EvalF-I seller, who does not concede significantly from its previous offer* $\boldsymbol{y_t}$*, then the buyer's posterior belief will be steeper at* $\boldsymbol{x \leq x_t^*}$*.*

Again, in Proposition 3-2 an imprecise concept "concede significantly" should be interpreted at an algorithm level which may vary for different type of buyers.

3.3.3 Belief Function $p_t(x)$ if the Opponent is EvalF-II Agent

3.3.3.1 Belief function

Suppose the seller is EvalF-II and the buyer knows it. To avoid confusion, in this section we will use $p_t(x)$ to represent the buyer's belief function if the seller is EvalF-I, and use $\pi_t(x)$ if the seller is EvalF-II. When the seller's type is unknown or unimportant, we will use $p_t(x)$ for the sake of consistency. Suppose at time t the buyer has a monotonically increasing belief function $\pi_t(x)$ as shown in Figure 3-7(a) and offers a price x_t^*. Also, let x_t^* be rejected so the bargaining goes to the next round $t+1$.

Knowing that the seller is EvalF-II, the buyer realizes that the seller may accept any price if the price falls in the feasible set and is perceived by the seller as the best offer from the buyer; thus, $\pi_{t+1}(x)$ may not be an increasing function. Instead, $\pi_{t+1}(x)$ is the sum of the revised $\pi_t(x)$ according to Assumption 3-8, namely $p_{t+1}(x)$ (dashed curve in Figure 3-7(b)), and a belief function $k_{t+1}(x)$ that represents the likelihood of x being accepted by the seller because it is perceived as the best offer from the buyer at time $t+1$ (top curve in Figure 3-7(b)). Or,

Definition 3-9. $\pi_{t+1}(x) = min(p_{t+1}(x) + k_{t+1}(x), 1)$. (3-8)

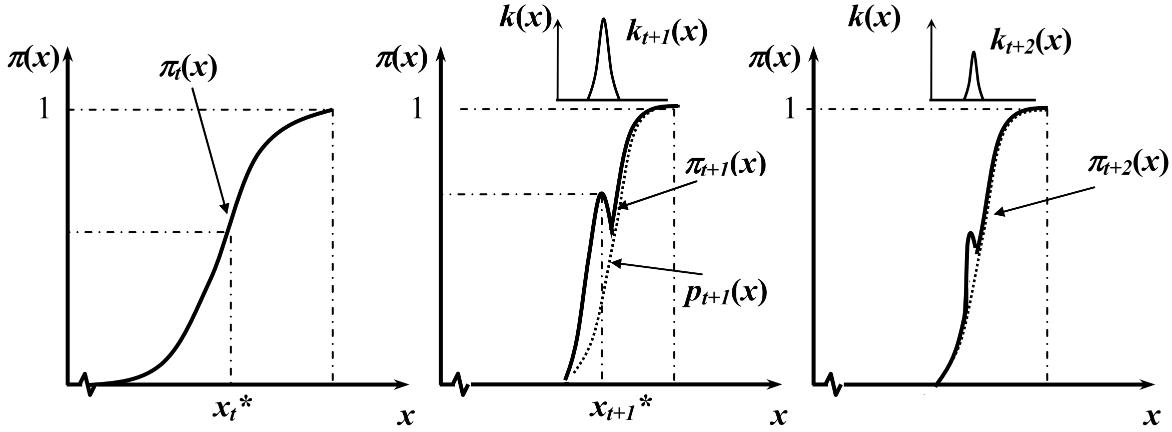


Figure 3-7. Belief revision of EvalF-II agents when (a) before rejection, (b) after first rejection, (c) after second rejection

The belief function $\boldsymbol{k(x)}$ can be regarded as the tendency of the buyer to deceive the seller by insisting on its previous offer. Using the rationale of the buyer's belief toward the maximum surplus that it can get from the seller as explained in section 3.3.1, we can derive that the highest value of $\boldsymbol{k_{t+1}(x)}$ is around $\boldsymbol{x_t^*}$ for the following reasons.

1. If the buyer offers a price $\boldsymbol{x_{t+1}^*}$ that is much higher than $\boldsymbol{x_t^*}$, then the seller will learn that the buyer may concede more in the future; thus, for the seller $\boldsymbol{x_{t+1}^*}$ is not the best offer from the buyer, or $\boldsymbol{k_{t+1}(x_{t+1}^*)} = 0$ for $\boldsymbol{x_{t+1}^*} >> \boldsymbol{x_t^*}$. This is one of the reasons why human bargainers rarely concede quickly especially at the beginning of the bargaining, i.e. to avoid sending a signal that s/he will easily concede. Again, the degree of "much higher" depends on the algorithm used by the buyers.
2. If the buyer offers a price $\boldsymbol{x_{t+1}^*}$ slightly higher or lower than $\boldsymbol{x_t^*}$ (say a price within a range $[\boldsymbol{x_t^*}-\boldsymbol{\delta}, \boldsymbol{x_t^*}+\boldsymbol{\delta}]$), then the seller will perceive that the buyer has reached its limit of conceding; thus, $\boldsymbol{x_{t+1}^*}$ may be perceived as the best offer from the buyer.

3. If the buyer offers a price $x_{t+1}*$ much lower than x_t*, then the seller realizes that the buyer may not offer a higher price in the future, or $x_{t+1}*$ is the best offer, but $x_{t+1}*$ may not be in the feasible set that can be accepted by the seller, or $k_{t+1}(x_{t+1}*) = 0$ for $x_{t+1}* << x_t*$.

3.3.3.2 Belief revision

As stated in Definition 3-9, $\pi_{t+1}(x) = min(p_{t+1}(x) + k_{t+1}(x), 1)$. If the peak of $k_{t+1}(x)$ is high enough, the buyer has incentive to offer $x_{t+1} \in [x_t*-\delta, x_t*+\delta]$ (stay close to its previous offer) to deceive the seller to accept it (as if it is the best offer that could be made by the buyer). In response, the seller may accept it (because the seller believes that x_{t+1} is the best) or reject it (because the seller is skeptical or x_{t+1} is lower than its valuation), and counter offer y_{t+1}. Receiving counteroffer y_{t+1}, the buyer will update $k_{t+1}(x)$ and $p_{t+1}(x)$ separately.

First, the buyer will reduce $k_{t+1}(x)$ to $k_{t+2}(x)$ (the top of Figure 3-7(c)), because it realizes that x_{t+1} is less likely within the feasible set. The reduction rate reflects the buyer's belief assigned to the reason for the seller's refusal, i.e. whether the seller is skeptical or has valuation higher than x_{t+1}. If the buyer believes that the refusal is very likely due to the seller's skepticism, then the reduction of $k_{t+1}(x)$ is small. Conversely, if the buyer believes that the refusal is due to the seller's high valuation, then the reduction is bigger.

Second, the buyer will update $p_{t+1}(x)$ to $p_{t+2}(x)$ following Assumption 3-8 (as if the seller is an EvalF-I agent). After updating $k_{t+2}(x)$ and $p_{t+2}(x)$, the buyer can compose a new $\pi_{t+2}(x) = min(p_{t+2}(x) + k_{t+2}(x), 1)$. If under new belief $\pi_{t+2}(x)$ the buyer's optimal offer $x_{t+2}*$, which is less than $x_t*+\delta$, is still rejected by the seller, then the buyer will use the same updating method as described before. Thus, if within $\boldsymbol{n}$ rounds the buyer submits offers $\boldsymbol{x_{t+1}}$*,…, $\boldsymbol{x_{t+n}}$* which are less than $\boldsymbol{x_t}$*+$\boldsymbol{\delta}$ and all are rejected by the seller, then eventually $\boldsymbol{k_{t+n}(x)}$ will equal to zero, which leaves $\boldsymbol{\pi_{t+n}(x)} = \boldsymbol{p_{t+n}(x)}$; thus, $\boldsymbol{\pi_{t+n}(x)}$ is a monotonically increasing function again. However, if at any round $\boldsymbol{t+j} < \boldsymbol{t+n}$ the buyer's offer $\boldsymbol{x_{t+j}}$ is higher than $\boldsymbol{x_t}$*+$\boldsymbol{\delta}$ and it is rejected by the seller, then the buyer will only update $\boldsymbol{p_{t+j}(x)}$ to $\boldsymbol{p_{t+j+1}(x)}$ and generate a new $\boldsymbol{k_{t+j+1}(x)}$ with a peak at around $\boldsymbol{x_{t+j}}$, because the rejection of $\boldsymbol{x_{t+j}}$ is not relevant to the update of $\boldsymbol{k_{t+j}(x)}$. In other words, $\boldsymbol{k_{t+j}(x)}$ will be updated only if $\boldsymbol{x_{t+j}}$ is lower than $\boldsymbol{x_t}$*+$\boldsymbol{\delta}$.

Assumption 3-9. *Suppose the buyer knows that the seller is an EvalF-II agent and the buyer's offer* $\mathbf{x_t}$* *is rejected by the seller. Then the buyer will update* $\boldsymbol{\pi_t(x)}$ *by decomposing it into* $\boldsymbol{p_t(x)}$ *and* $\mathbf{k_t}(\boldsymbol{x})$*, and update them separately.* $\boldsymbol{p_t(x)}$ *will be updated according to Assumption 3-8, and* $\mathbf{k_t}(\boldsymbol{x})$ *will be updated according to the following rules:*

(i) If $\mathbf{x_t}$* *is less than or equal to* $\mathbf{x_{t-1}}$*+$\boldsymbol{\delta}$*, then* $\mathbf{k_t}(\boldsymbol{x})$ *will be reduced to* $\mathbf{k_{t+1}}(\boldsymbol{x})$*, where the reduction rate depends on the buyer's belief about the reason for the seller's refusal, either due to the seller's skepticism or the seller's high valuation.*

(ii) If $\mathbf{x_t}$* *is greater than* $\mathbf{x_{t-1}}$*+$\boldsymbol{\delta}$*, then* $\mathbf{k_t}(\boldsymbol{x})$ *will not be updated. Instead, a new* $\mathbf{k_{t+1}}(\boldsymbol{x})$ *will be generated whose a peak is around* $\mathbf{x_t}$**.*

Assumption 3-10. *There exists* $\mathbf{n}$ *and* $\boldsymbol{\delta}$ *such that if the buyer's consecutive offers* $\mathbf{x_{t+1}}$**,…,* $\mathbf{x_{t+n}}$* *are less than* $\mathbf{x_t}$*+$\boldsymbol{\delta}$ *and all are rejected by the seller, then* $\mathbf{k_{t+n}}(\boldsymbol{x}) = \boldsymbol{0}$ *and* $\boldsymbol{\pi_{t+n}(x)} = \boldsymbol{p_{t+n}(x)}$*.*

Depending on the characteristics of $\boldsymbol{k(x)}$ and $\boldsymbol{n}$ (the speed of reducing $\boldsymbol{k(x)}$ to zero), we can characterize a buyer as a skeptical/believing and deceitful/benevolent one. If both the peak of $\boldsymbol{k(x)}$ is high (e.g. causing $\boldsymbol{\pi_{t+1}(x_t^*)} > \boldsymbol{\pi_t\ (x_t^*)}$) and $\boldsymbol{n}$ is large (e.g.

greater than 5), then the buyer is a deceitful and skeptical one because it insists on its offer for several rounds (e.g. 5 or more rounds) just to mislead the seller to accept it. If the peak of $\boldsymbol{k(x)}$ is high but $\boldsymbol{n = 1}$, then the buyer is deceitful but believing because it will try to insist on its previous offer only in order to mislead the seller to accept its offer. If $\boldsymbol{k(x) = 0}$ for any $\boldsymbol{x}$, then the buyer is a benevolent one because it never misleads the seller, even if it knows that the seller is EvalF-II.

Proposition 3-3. *Suppose that the buyer knows that the seller is EvalF-II. If the buyer's consecutive offers which are lower than* $\boldsymbol{x_t^*+\delta}$ *were rejected for* $\boldsymbol{n}$ *rounds and the seller concedes insignificantly during these rounds, then the buyer's posterior belief* $\boldsymbol{\pi_{t+n}(x)}$ *will be steeper than that of* $\boldsymbol{\pi_t(x)}$ *for all* $\boldsymbol{x \leq x_t^*}$.

In addition to the above characteristic, $\boldsymbol{\pi_t(x)} = 1$ for $\boldsymbol{x} \geq \boldsymbol{y_t}$, where $\boldsymbol{y_t}$ is the seller's current offer; and $\boldsymbol{\pi_t(x)} = 0$ for $\boldsymbol{x} < \boldsymbol{z}$, where $\boldsymbol{z}$ is the buyer's estimation of the seller's valuation. Here, $\boldsymbol{z}$ may be assigned using a simple heuristic, e.g. as a fraction of the buyer's valuation $\boldsymbol{z} = 0.7\ \boldsymbol{B_0}$, or predicted according to the seller's type and its initial offer $\boldsymbol{y_0}$, e.g. $\boldsymbol{z} = 0.3\ \boldsymbol{y_0}$, if the seller is a total stranger, or $\boldsymbol{z} = 0.7\ \boldsymbol{y_0}$, if the seller is a well-known one. Facing an EvalF-II seller, the buyer's belief $\boldsymbol{\pi_t(x)}$ may not be an increasing function. However, the buyer does not need to consider all prices in order to find an optimal offer. Intuitively, if there are several prices with the same probability of being accepted by the opponent (see Figure 3-8(a)), then the lowest price will be selected by the buyer because it will generate higher surplus than the higher price, unless the expected gain is indirectly related to the price (e.g. choosing a higher price will invoke the seller to concede more in the future). Thus, the search space of the buyer (effective belief function) is always an increasing function $\boldsymbol{\pi_{t+1}(x)}$, as shown in Figure 3-8(b).

Proposition 3-4. *Suppose the buyer knows that the seller is EvalF-II. Then the effective buyer's belief that its offer at time* $\boldsymbol{t}$ *be accepted by a seller,* $\boldsymbol{\pi_t(x)}$*, is an increasing function with respect to its offer* $\boldsymbol{x}$*, where* $\boldsymbol{\pi_t(x)} \in [0, 1]$ *for all* $\boldsymbol{x}$*. However,* $\boldsymbol{\pi_t(x)}$ *may not be a continuous function.*

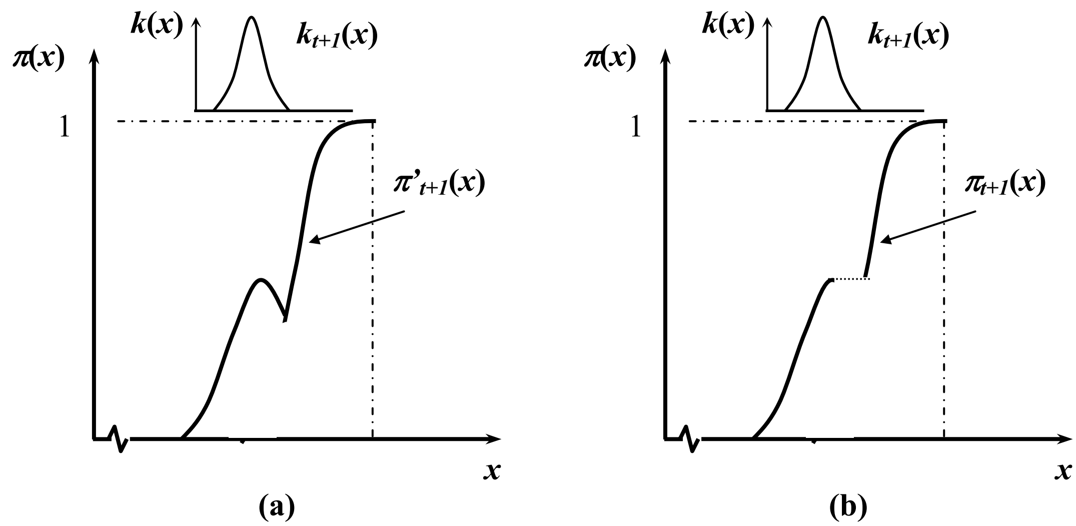


Figure 3-8. Belief of EvalF-II agents that is (a) stored and (b) effectively considered

3.3.3.3 Remarks

If the buyer does not know whether or not the seller is EvalF-II, then $\boldsymbol{k_{t+1}(x)}$ depends on both $\boldsymbol{x}$ and the estimated ratio of sellers who perceived it as the signal of the best offer from the buyer. Thus, the buyer may consider the seller as an EvalF-II seller and Assumption 3-9 and 3-10 apply, but with a lower value of $\boldsymbol{k_{+1}(x)}$ as stated in the following assumption.

Assumption 3-11. *If the buyer agent does not know the type of the seller, then it will assign a probability that the seller is EvalF-I and EvalF-II of* $\boldsymbol{\kappa}$ *and* $\boldsymbol{1 - \kappa}$*, respectively; and consider the seller as an EvalF-II seller with* $\boldsymbol{k_{+1}(x) = (1 - \kappa)\, k_{+1}(x)'}$*, where* $\boldsymbol{k_{+1}(x)'}$ *is the value if the buyer certainly believes that the seller is EvalF-II.*

As mentioned before, some buyers may be skeptical while others are not. The reason for a buyer to be skeptical is because the seller may be skeptical as illustrated in the following example.

Example 3-12. *Suppose the buyer's current offer is \$100 and the buyer knows that the seller may accept it if it is above the seller's valuation and the buyer also knows that the seller is skeptical. Consequently, the buyer must insist on that price in order to convince the seller. However, insisting on the price in one or two rounds may not be so convincing, because the seller is skeptical. Suppose the seller refuses to sell at that price, then the buyer cannot differentiate whether the refusal is due to the skepticism of the seller or the non-feasibility of the price (\$100 is lower than seller's valuation). If it is due to the non-feasibility of the price, then there is no reason for the buyer to stand on its offer. However, if it is due to the skepticism of the seller, then it is better for the buyer to stand on its offer. Thus, the buyer will insist on its offer for several rounds before believing that the rejection is due to the non-feasibility of the price.*

If both parties are highly skeptical, then it will cause a deadlock (both insist on their offer). If one party is more skeptical than the other, then the more skeptical one may get a higher surplus (this is analogous to the agent's patience in Rubinstein's bargaining solution [1982], in which the more patient agent gets more surplus). However, it is unwise to always use a skeptical agent, because it may prolong the negotiation and thus increase the breakdown rate as will be explained in the next section. Again, we are not seeking the best belief construct and are not explicitly studying skepticism in this thesis. Rather, we include them in our analysis for representing a wider range of BDI agents that may use the bargaining protocols that will be proposed in the thesis.

3.3.4 Belief toward Probability of Breakdown

When no information regarding the opponents' time deadline is revealed, then agents can only predict the probability of breakdown (e.g. q_t in Definition 3-7 and 3-8). The time deadline could come from two sources --- statically set by the user or system or dynamically influenced by the market.

Example 3-13. *(Static deadline) A user may instruct her/his agent to buy an item before 18:30. In another case, the server may set a rule such that any negotiation session should be done within 3 hours or it will be terminated by the server.*

(Dynamic deadline) A seller may terminate its negotiation with a buyer if the item has been sold to another buyer. In other cases, the buyer may terminate a negotiation session, because it found another seller with a lower price than its current offer.

Usually, the buyer's static deadline is known by the agent itself, but the opponent's static deadline is unknown. To predict this, the agent can generate a cumulative distribution function of the likelihood of the deadline: as the negotiation proceeds, the likelihood of the deadline gets higher. The dynamic deadline, on the other hand, depends on market conditions. If there are many buyers looking for the same item at the same time (high demand), then the buyer will estimate that the dynamic time deadline is shorter, because the likelihood of the seller to sell the item to another buyer is higher.

Any breakdown caused by the time deadline is denoted as a time-dependent breakdown. Intuitively, as time goes by, the likelihood that agents are approaching their deadline is higher; thus, the probability of breakdown is higher too, or q_t is an increasing function of t. For example, it may be a logistic function, as in Figure 3-9(a).

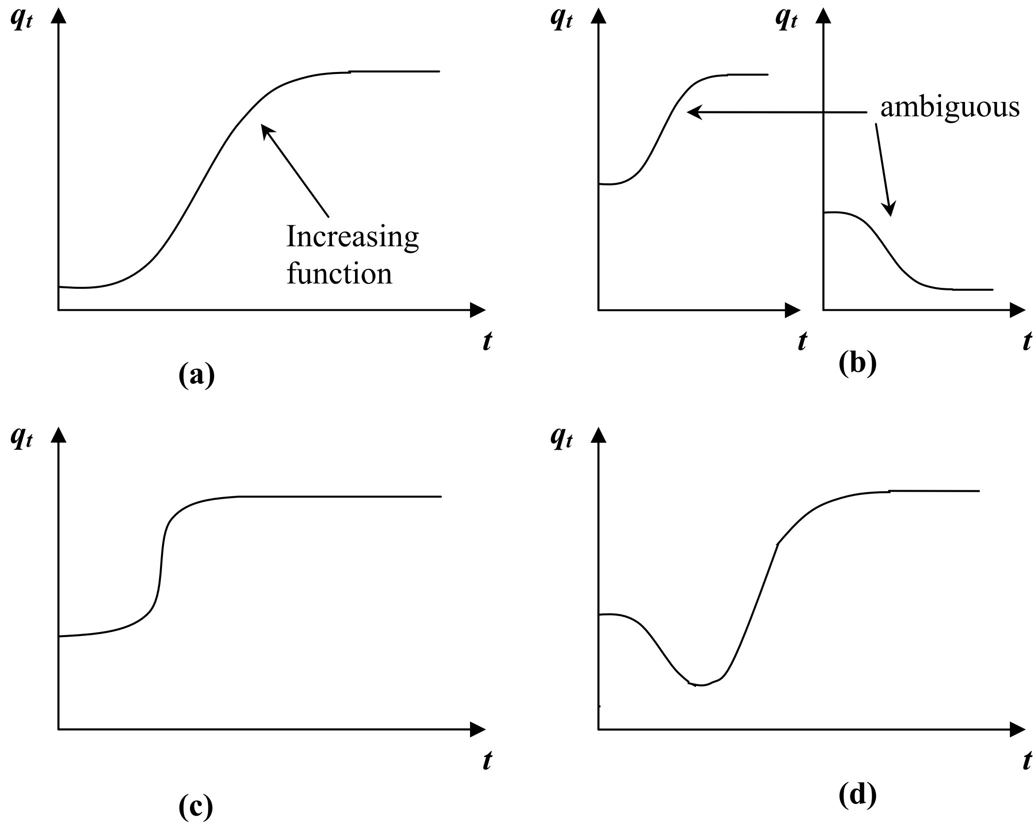


Figure 3-9. An agent's belief toward the probability of breakdown from (a) time-deadline, (b) loss-of-interest, (c) amplification of time-deadline and loss-of-interest, and (d) reduction of time-deadline and loss-of-interest

Another factor that commonly causes a breakdown is the loss of interest toward the negotiation (e.g. believing that the negotiation outcome may not generate positive surplus, or distrusting the credibility of the seller in delivering the contract). If not much information can be gathered such as in the early stage of the negotiation, then agents will not trust each other and will assign a moderate probability of breakdown. Depending on the information collected, agents may update their trust toward their opponent (e.g. the seller is very likely a deceitful one and the negotiation should be terminated promptly), moving lower or higher, which increases or decreases the probability of breakdown.

Similarly, as the negotiation proceeds, agents may predict more confidently the surplus they may get from the negotiation (as their belief function $\boldsymbol{p_t(x)}$ becomes steeper, see Figure 3-6(a)), which may either be a positive surplus (decrease the probability of breakdown) or negative surplus (increase the probability of breakdown). Thus, from this factor the trend of the probability of breakdown is ambiguous. However, the effect of this factor toward probability of breakdown may not be longer than the effect of the time deadline (compare Figure 3-9(a) and (b)), because after several rounds both parties may eventually know the credibility of their opponents or conclude whether it is worthy to continue the negotiation or not. Thus, in the long run, $\boldsymbol{q_t}$ is an increasing function of time $\boldsymbol{t}$, as shown in Figure 3-9(c) and (d).

Proposition 3-5. *In the long run* $\boldsymbol{q_t}$ *is an increasing function of time* $\boldsymbol{t}$.

3.3.5 Belief toward Opponent's Argument

In human bargaining, argumentation (persuasion, threat, or appeal) is commonly used. Recent studies in MAS have also paid more attention to dialog-based or argumentation-based negotiation (ABN) [Rahwan et al., 2004]. However, little attention has been directed to study the disadvantages of the protocol. As stated in Chapter 1, this thesis also includes the study of the ABN protocol, especially the need to use ignorance in ABN. In our setting we would assume that the environment is uncertain and agents have limited capability to assess the credibility of their opponent's arguments.

Similar to the offer/counter-offer setting, agents may also believe or not believe the arguments provided by their opponent. The framework of agent belief toward argumentation actually is almost the same as that toward prices. For example, suppose the seller promises "I will deliver the item on schedule." Then the buyer must assign a belief (or trust) value toward the credibility of the seller in executing its promise. If no information is available, then the buyer may use a default value (base) and look for more resources such as the reputation of the seller, or the past experience of other buyers, etc. in order to update its trust. After receiving two or more arguments, the buyer may build its trust toward the seller by verifying those arguments. If all arguments are checked and verified to be true, then the buyer can assign a higher trust toward the seller. If the seller is trustworthy, then the buyer may assign a higher belief value to the arguments from the seller in the future, even without verification.

Example 3-14. *Suppose a seller claims "My price is the lowest price in this market," and the buyer verifies it by checking the transaction price(s) in the market. Suppose the buyer finds that all the past transaction prices for that good from other sellers are higher than the seller's offer, so the buyer asserts a high belief toward the seller's statement and updates* $\boldsymbol{p_t(x)}$ *and its trust towards the seller, accordingly. Since the seller is deemed trustworthy now (e.g. with a higher reputation), then the seller's threat such as "There is another buyer who is willing to pay $100, so your offer must be higher than that or I will terminate the negotiation" is credible, even without further verification.*

The order of arguments matters here. For the seller, it is better to provide arguments that can be verified by the buyer in order to convince the buyer to trust the seller. Only after the seller believes that the buyer trusts it can the seller send arguments that are non-verifiable.

Practically, agents can use machine learning techniques to update their trust toward their opponent, e.g. reinforcement learning or Bayesian updating [Tran and

Cohen, 2002]. In such a case, skeptical agents may use a lower learning rate while believing agents use a higher one. In order to understand the arguments used in the negotiation, all agents must agree to use the same ontology (i.e. a vocabulary used to represent concepts in the arguments) or otherwise some arguments cannot be interpreted correctly. However, in this thesis, the verification techniques and the ontology of the arguments are not our main concern. The focus will be on the assessment of the cost of using the ABN protocol by assuming that it will take some efforts for the agents to understand/generate an argument.

Persuasive arguments and threats are the most common arguments used in negotiation. We discuss several factors that may be influenced by argumentation: the belief toward the opponent's acceptable offer, the private valuation, and the likelihood of breakdown.

Example 3-15. *A buyer of a used car may say "I must replace those tires, and it will cost at least $500" in order to convince the seller that the buyer's valuation is not as high as the seller's expectation. In reply, the seller may say "They are still in a good condition, at least for half a year." If the buyer successfully convinces the seller that the tires must be replaced, then the seller may reduce its valuation by a value up to $500 and the buyer will update* $\boldsymbol{p_t(x)}$ *to a higher value. In another situation the seller may threaten the buyer by using the statement "Take my offer now, or I will sell it to another buyer." If the buyer believes the statement, then it will assign the probability of breakdown to one; thus there will be no expected gain from a counter-offer.*

The other parameters that may be affected by argumentation are the expected gain in the future, trust toward the opponent, and also information about the negotiation strategy used by agents, which will not be studied in this thesis.

3.3.6 $EG'_{t+1}(x)$ and Belief Update: Revisited

When a myopic-1 buyer considers the expected gain from its offer x_t, it will consider the future expected gain $EG'_{t+1}(x)$ if x_t is rejected by the seller. In section 3.2.2.3 we have argued by an example that $EG'_{t+1}(x)$ may increase or decrease over the x submitted at time t.

Proposition 3-6. *(Ambiguity of $EG'_{t+1}(x)$ with respect to x) $EG'_{t+1}(x)$ may increase or decrease with respect to x submitted at time t.*

Indeed, the situation in artificial agents may not be exactly the same as that in human negotiation, because agents' behavior depends on the algorithm used. For example, the buyer may calculate EG'_{t+1} either independently of x, or as a function of x, or as a fraction of $EG_t(x)$, or recursively as if x_t has been rejected, etc. Certainly, the easiest way is to choose a constant value that is independent of x. The buyer can also use an estimated surplus from accepting the seller's offer, e.g. $EG'_{t+1} = B_{t+1} - y_t$. A more complex way is to predict it recursively using the following formula.

Definition 3-10. *The estimation of $EG'_{t+1}(x)$ by recursive method is calculated by*

$$EG'_{t+1}(x) \equiv \underset{x}{Max}[(1 - q'_{t+1})[p'_{t+1}(x)(B'_{t+1} - x) + \gamma(1 - p'_{t+1}(x))EG'_{t+2}(x)] + q'_{t+1} B_{\phi}] \tag{3-9}$$

where all parameters except γ and B_ϕ must be predicted or calculated iteratively by assuming that x_t has been rejected, and $p'_{t+1}(x)$ is the buyer's estimation of its revised *belief if $\boldsymbol{x_t}$ has been rejected. And for a myopic-1 buyer with no payoff from breakdown ($\boldsymbol{B_\phi}$ =0) the estimation becomes*

$$\boldsymbol{EG'_{t+1}(x) \equiv \underset{x}{Max}[(1 - q'_{t+1})p'_{t+1}(x)(B'_{t+1} - x)]} \tag{3-10}$$

If $\boldsymbol{x'_{t+1}*}$ solves the above equation, then we have

$$\boldsymbol{EG'_{t+1}(x'_{t+1}*) = (1 - q'_{t+1})\, p'_{t+1}(x'_{t+1}*)(B'_{t+1} - x'_{t+1}*)} \tag{3-11}$$

Note here that the actual revised belief function at time $\boldsymbol{t+1}$, i.e. $\boldsymbol{p_{t+1}(x)}$, may not be the same as its estimated value $\boldsymbol{p'_{t+1}(x)}$. In addition, if the buyer is facing a time pressure in the long run, e.g. a strictly increasing probability of breakdown, or $\boldsymbol{q'_{t+1} - q_t} > 0$, and a decreasing valuation over time, or $\boldsymbol{B'_{t+1} - B_t} \leq 0$, then $\boldsymbol{EG'_{t+1}(x)}$ may be a decreasing function over time.

Proposition 3-7. *If a myopic-1 buyer is facing time pressure in the long run such that its belief at time* $\boldsymbol{t+1}$ *satisfies* $\boldsymbol{B'_{t+1} - B_t} \leq 0$ *and* $\boldsymbol{(q'_{t+1} - q_t) \geq (1 - (1 + \omega)p_t(x_t*))(1 - q_t)}$*,where*

$$\omega \equiv \frac{\gamma(\mathbf{1} - \boldsymbol{q_t})(\mathbf{1} - \boldsymbol{p_t}(\boldsymbol{x_t^*}))}{[\mathbf{1} - \gamma(\mathbf{1} - \boldsymbol{q_t})(\mathbf{1} - \boldsymbol{p_t}(\boldsymbol{x_t^*}))]}\ , \tag{3-12}$$

and its estimated optimal offer $\boldsymbol{x'_{t+1}*} > \boldsymbol{x_t*} + \boldsymbol{B'_{t+1} - B_t}$*, then its expected future gain will be lower than its current expected gain, i.e.* $\boldsymbol{EG'_{t+1}(x'_{t+1}*) < EG_t(x_t*)}$.

Proof. *(a detail proof is provided in the appendix)* First, we use $\boldsymbol{EG'_{t+1}(x'_{t+1}*) < EG_t(x_t*)}$ to find the conditions under which the inequality holds. The condition should be

$\boldsymbol{(1-q_t)p_t(x_t*)(\Delta_B-\Delta_x)(1+\omega)+[(1-q_t)(\Delta_p-p_t(x_t*)\omega) - \Delta_q\, p'_{t+1}(x'_{t+1}*)](B'_{t+1}-x'_{t+1}*) < 0}$ (3-13)

Where $\Delta_B = \boldsymbol{B'_{t+1} - B_t}$, $\Delta_x = \boldsymbol{x'_{t+1}* - x_t*}$, $\Delta_q = \boldsymbol{q'_{t+1} - q_t}$ and $\Delta_p = \boldsymbol{p'_{t+1}(x'_{t+1}*) - p_t(x_t*)}$. Then we use the characteristics of those parameters such as $\boldsymbol{q_t} \in [0, 1]$, $\boldsymbol{\gamma} \in [0, 1]$, $\boldsymbol{p_t(x_t*)} > 0$, $\Delta_q > 0$ and $\Delta_B \leq 0$, etc. to derive a simplified condition:

$$\Delta_x > \Delta_B \text{ and } \Delta_q \geq \mathbf{[1 - \mathit{p_t}(\mathit{x_t}^*)(1 + \omega)](1 - \mathit{q_t})} \tag{3-14}$$

This condition (3-14) is exactly the premise in the proposition that

$$\boldsymbol{x'_{t+1}}^* > \boldsymbol{x_t}^* + \boldsymbol{B'_{t+1}} - \boldsymbol{B_t} \text{ and } (\boldsymbol{q'_{t+1}} - \boldsymbol{q_t}) \geq [1 - \boldsymbol{p_t}(\boldsymbol{x_t}^*)(1 + \omega)](1 - \boldsymbol{q_t}). \qquad \blacksquare$$

Intuitively, the sufficient condition in inequality (3-14) holds when the time pressure is intense and either $\boldsymbol{p_t(x_t^*)(1 + \omega)}$ and $\boldsymbol{q_t}$, or both, are high (close to 1).

Example 3-16. *Suppose* $\boldsymbol{p_t(x_t^*)} = 0.9$*,* $\boldsymbol{q_t} = 0.5$*, and* $\gamma = 0.5$*, then* $\boldsymbol{p_t(x_t^*)(1 + \omega)} = \boldsymbol{p_t(x_t^*)} / \boldsymbol{(1 - \gamma(1 - q_t)(1 - p_t(x_t^*)))} = .9/(1-.5(.5)(.1)) = .9/(1-.025) = .9/.975$*, or* $\boldsymbol{[1 - p_t(x_t^*)(1 + \omega)](1 - q_t)} = (1 - .9/.975)(1 - .5) = .0385$*; hence,* $\Delta_q \geq .0385$ *is sufficient. However, if* $\boldsymbol{p_t(x_t^*)} = 0.5$ *with other values are the same, then* $\boldsymbol{p_t(x_t^*)} / \boldsymbol{(1 - \gamma(1 - q_t)(1 - p_t(x_t^*)))} = .5/(1-.5(.5)(.5)) = .5/(1-.125) = .5/.875$*, and* $\boldsymbol{[1 - p_t(x_t^*)(1 + \omega)](1 - q_t)} = (1-.5/.875)(1-.5) = .2143$*, or* $\Delta_q < .2143$ *is not sufficient.*

Bear in mind that this condition is only a sufficient one, which is not a necessary condition for $\boldsymbol{EG'_{t+1}(x'_{t+1}{}^*) < EG_t(x_t^*)}$. In this thesis we will consider three possibilities: $\boldsymbol{EG'_{t+1}(x'_{t+1}{}^*)}$ is a constant value over $\boldsymbol{x_t}$, a decreasing function over $\boldsymbol{t}$, or iteratively calculated according to equation (3-10).

3.4 Discussion

3.4.1 Commonsense Reasoning

As shown in section 3.1, 3.2 and 3.3, our model is based on commonsense reasoning --- the agent's decision is not designed to follow directly the rules of inference or any of the well known solutions of mathematical logic or game theory, but are rather a set of conjectures to find the best available solutions. The agent's decisions depend heavily on its observation of its ill-defined world and do not necessarily follow the monotonicity property often used in knowledge and reasoning. For instance, the buyer's belief function $\boldsymbol{p_t(x)}$ can be updated abruptly after receiving new information as long as it preserves commonsense properties, such as $\boldsymbol{p_t(x)} = 1$ for all $\boldsymbol{x} > \boldsymbol{y_t}$. However, a refusal of the buyer's offer $\boldsymbol{x_t}$* by the seller does not necessarily make the buyer infer that $\boldsymbol{p_t(x_t{*})} = 0$. Instead, depending on the buyer's belief about the seller's skepticism (or bluffing behavior) the buyer may increase/decrease $\boldsymbol{p_t(x_t{*})}$. In the next chapter we will show that it is even plausible for the buyer to decrease its next offer, or $\boldsymbol{x_{t+1}{*}} < \boldsymbol{x_t{*}}$. We believe that the agent's ambivalent/ contradictory decision does not violate agent rationality; rather, it represents our commonsense reasoning which often conforms to nonmonotonic reasoning.

Our model may also serve as an abstraction of symbolic reasoning. For example, the buyer may assume (by default) that the seller is an EvalF-I agent and then implement a circumscription ("jumping to a certain conclusion in commonsense reasoning" [McCarthy, 1986]) that the seller may be an EvalF-II agent. Then, the buyer may infer that a price $\boldsymbol{x}$ should be accepted by the seller after asserting other conditions; thus, all prices below it will be rejected. When a binary choice (T/F or Accept/Reject) is the only available option, we may represent the output using a step function (cf. Figure 3-5). Alternatively, we can use a preferential system [Kraus et al., 1990] or modal logic to represent our agent's knowledge and decision procedures. The reason for not using a symbolic approach in our analysis is to avoid a complex representation, especially in the belief revision and deliberation to find an optimal offer. Beside, Dubois et al. [2004] have shown the compatibility of probabilistic and logical representation of accepted belief.

Therefore, in this thesis we will not investigate symbolic representation unless quantitative representation is inappropriate.

Earlier we have assumed bounded rationality (optimality) of agents. This suggests a prescriptive model of our agents, where a utility maximizer in terms of myopic-0 and myopic-1 are used. Certainly, it creates some deviations between our model and human behavior. However, the objective of this thesis is not to analyze human behavior or to build a normative standard for an agent's reasoning in bargaining. In fact, our model is aimed at solving real world problems that arise in some artificial intelligent systems. But we cannot ignore human intervention with their agents. Thus, several commonly observed strategies used in human society (sub-optimal from the perspective of a prescriptive standard) should also be considered. For example, tit-for-tat and random strategy, which are common strategies in human society, will be used in our experimental study to analyze the cost and benefit of our proposed bargaining protocols in Chapter 5. In conclusion, most of the properties of the agent's negotiation model in this thesis are based on our commonsense knowledge, and those that cannot be analyzed theoretically will be tested by using experimentation.

3.4.2 Iterative Belief

Other important phenomena in agents' interactions are iterative belief and common knowledge. For example, suppose the buyer knows its own type as EvalF-II (represented as a level 0 belief, i.e. knowing the fact ***P***). If the seller knows that the buyer is EvalF-II (or it has level-1 belief: ***bel*(*s*, *P*)**), then the seller will stand on its offer in order to mislead the buyer to accept it. If however, the buyer believes that the seller knows that it is an EvalF-II agent (it has level-2 belief: ***bel*(*b*, *bel*(*s*, *P*))**), then the buyer will be more skeptical when the seller stands on its offer. Now, if the seller believes that the buyer believes the above fact and becomes skeptical (it has level-3 belief: ***bel*(*s*, *bel*(*b*, *bel*(*s*, *P*)))**), then there is no reason for the seller to mislead the buyer because it will only waste its time.

This iteration of belief may continue to higher levels, thus requiring more complex iterative reasoning. In this thesis we do not address complex iterative reasoning, going up to two levels only. The reason is that the higher levels of iterative belief cannot be known easily due to asymmetric and private information. In addition, there are increasing costs and decreasing returns on considering a high level of reciprocal modeling and bias toward the desired solution [Durfee et al., 1993]. Thus, no agent is certain that it knows more than its opponent, unless the fact is declared explicitly such as “I am an EvalF-II agent” which makes it common knowledge – this is unlikely to happen.

3.4.3 Unpredictable Bargaining Rounds

Usually agents are facing a time deadline, e.g. 10 minutes from now or tomorrow afternoon at 17:30. Given this, the opportunity to (counter) offer may depend on the communication speed between agents. For instance, if a buyer believes that the likelihood of breakdown of a session is 0.9 after 10 minutes, and the communication speed is 3 seconds per offer, then with probability 0.1 the buyer can make up to 100 counter offers before a breakdown occurs. However, if due to some reason the communication speed slows down to 30 seconds per offer then with the same probability the buyer can make up to 10 counter offers only.

Assuming that the speed is constant and known by the agents, they can easily predict the number of opportunities to counter-offer before hitting the deadline.

Sometimes a deliberate delay is needed while collecting or assessing opponent's information or arguments, or, agents may change the speed of response and bargaining rounds. In addition, unpredictable network bottlenecks may also affect the communication speed. Therefore, planning in advance may not be suitable for negotiating agents in our model. If planning is involved, then it can be only for a few rounds ahead, such as building trust using few consecutive arguments or standing on its offer for few rounds. However, in most situations agents would react according to the partially observed information.

3.4.4 Preference over a Longer Bargaining Opportunity

Given the expected gain in equation (3-5) and (3-7), we can derive the next proposition regarding the agent's preference to make an additional offer during a series of consecutive offers.

Proposition 3-8. *A series of consecutive offers* $<\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_{t-1}}, \boldsymbol{x_t}, \phi>$ *is preferred to* $<\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_{t-1}}, \phi>$ *for* $0 \le \boldsymbol{x_t} < \boldsymbol{B_t}$.

Proposition 3-8 basically states that an additional opportunity to submit an offer is always preferred to ending up with a breakdown, no matter what the value of the last offer is. In fact, this proposition applies not only for myopic-0 and myopic-1 agents, but also for myopic-K agents. By the transitivity of preference and iteration that $<\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_{t+i}}, \phi>$ is preferred to $<\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_{t+i-1}}, \phi>$, we can derive that a longer bargaining opportunity is preferred to a shorter one, or $<\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_{t+i}}, \phi>$ is preferred to $<\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_t}, \phi>$, for any $\boldsymbol{i} > 0$.

Certainly, this proposition is only valid if a longer negotiation incurs the same cost as the shorter one, which means the buyer can negotiate with multiple sellers simultaneously and extending its offers will not affect the computation on other negotiation threads (negotiation with other sellers). Later, Proposition 3-8 will be used to prove Theorem 4-1 and 4-2.

3.4.5 Examples of Agents' Decision Model

Integrating an agent's evaluation function and an agent's strategy in generating offers will give us a decision model for the agent. For instance, if a buyer is an EvalF-II agent, then its decision activity after receiving the seller's counteroffer can be illustrated in Figure 3-10. If the buyer is myopic-0, then generating its counteroffer $\boldsymbol{x_t}$* can be decomposed into several processes as in Figure 3-11(a). If the buyer uses tit-for-tat, then the process is illustrated in Figure 3-11(b).

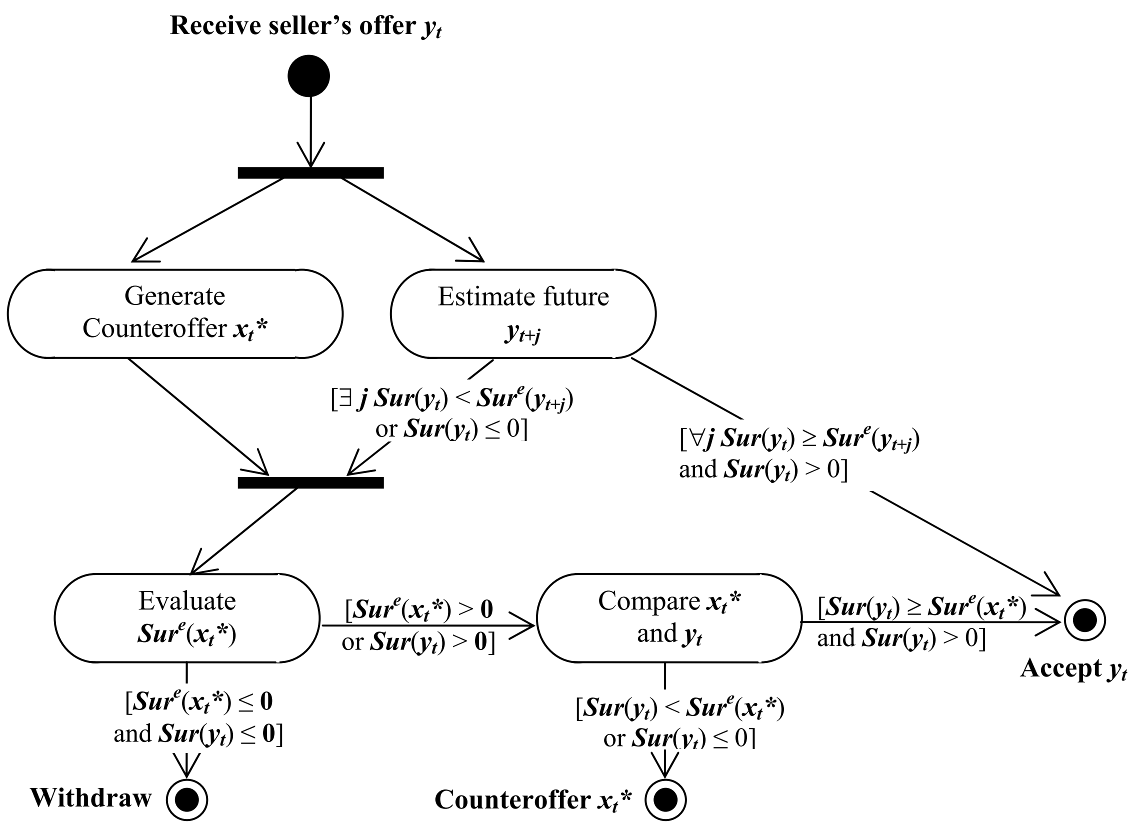


Figure 3-10. Activity of an EvalF-II buyer after receiving a seller's offer

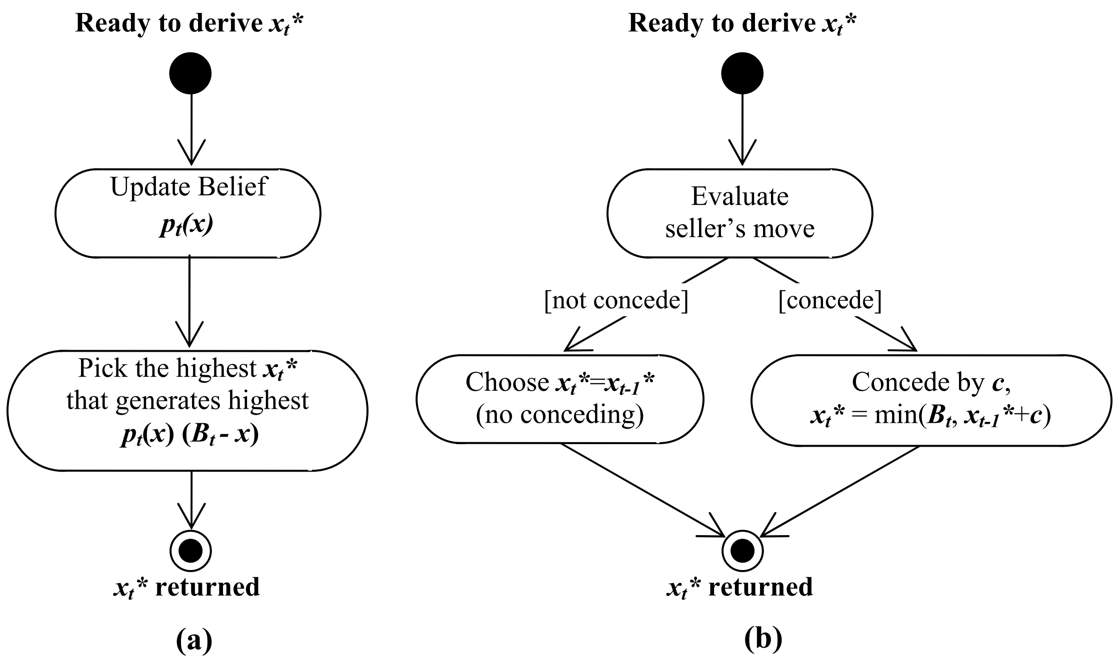


Figure 3-11. Process of generating a counteroffer for (a) a myopic-0 buyer and (b) a buyer using tit-for-tat strategy

Combining Figure 3-10 and 3-11 will give the decision model of an EvalF-II-myopic-0 buyer and an EvalF-II-tit-for-tat buyer. In the example, the buyer can use its expected gain to evaluate $\boldsymbol{Sur^e(x_t^*)}$, or $\boldsymbol{Sur^e(x_t^*) = p_t(x_t^*)\ (B_t - x_t^*)}$.

3.4.6 Convergence of Offers

When two agents use two reactive (irrational) strategies, it may happen that their offers never converge because of deadlock. However, if both of them are utility maximizer (myopic) agents, then their offers will converge to a resolution. For converging to a resolution, the offers of both myopic agents must be eventually in the feasible set and at least one of the agents concedes to the resolution. Therefore, we will discuss two conditions here. First, the feasible set must exist at the time when the resolution will be made. Second, agents must eventually concede due to a higher expected gain to be achieved by conceding than not conceding.

3.4.6.1 Facilitating feasible set

If the feasible set does not exist, then no resolution can be made regardless what bargaining protocol is used. Therefore, we will not consider this case. If the feasible set exists in certain periods of the negotiation, then the protocol must accommodate the agents to make a resolution within those periods. Moreover, if the existence of the feasible set depends on the agent's belief, e.g. the existence can be altered by altering the agent's belief through persuasive arguments, then the protocol must facilitate the negotiation so that the agent can alter its own and the opponent's belief. In the next chapter we will show how protocols that allow strategic delay and argumentation can be used to accommodate the existence of a feasible set, especially by increasing the buyer's valuation over time.

3.4.6.2 Higher expected gain from conceding

Three factors affect the agent's expected gain by conceding. First, a lower surplus compared to without conceding; second, higher $\boldsymbol{p_t(x_t*)}$ due to the increasing function of effective belief (cf. Figure 3-8(b)) as stated in Proposition 3-1 and Proposition 3-4; and third, ambiguous change of expected future gain depending on the predicted future condition (applied to myopic-1 agents). For a myopic buyer, if it knows that the seller is EvalF-I, then Assumption 3-8 applies. If it knows that the seller is EvalF-II or does not know the type of seller, then Assumption 3-9 applies. Nevertheless, by Proposition 3-2 and Proposition 3-3 that follow Assumption 3-8 and 3-9 respectively, if an offer $\boldsymbol{x_t*}$ is not accepted after one or $\boldsymbol{n}$ rounds by the seller who only concedes insignificantly, then $\boldsymbol{p_{t+n}(x)}$ and $\boldsymbol{\pi_{t+n}(x)}$ become steeper for $\boldsymbol{x \leq x_t*}$. If we ignore the process when $\boldsymbol{k_t(x)}$ is updated, then Proposition 3-2 becomes the special case of Proposition 3-3 with $\boldsymbol{n} = 1$. If the bargaining round is large enough, then we may use Proposition 3-2 in our analysis by ignoring the updating of $\boldsymbol{k_t(x)}$.

These propositions have implications for bargaining in the long run. For example, if the valuations of both agents are temporarily constant and the seller concedes insignificantly, then the buyer's offers will eventually converge to the seller's offer if the agents' expected future gain satisfies specific criteria such as it is a non-increasing value over time and non-decreasing over $\boldsymbol{x}$ as stated in the following proposition.

Proposition 3-9. *Suppose both bargainers are myopic agents and their valuations are constant within a long period, and* $\boldsymbol{x}$, $\boldsymbol{p_t(x)}$, *and* $\boldsymbol{EG'_{t+1}(x)}$ *are continuous and differentiable. If the seller does not concede significantly within that period so that the buyer will update its belief* $\boldsymbol{p_t(x)}$ *to a steeper one* for all $\boldsymbol{x \le x_t^*}$, *then the buyer concedes provided*

(a) the buyer is myopic-0, or

(b) the buyer is myopic-1 with convex belief $\boldsymbol{p_t(x)}$ *at* $\boldsymbol{x \le x_t^*}$, *i.e.* $\partial \boldsymbol{p_t(x)}/\partial \boldsymbol{x} > 0$ *and* $\partial^2 \boldsymbol{p_t(x)}/\partial \boldsymbol{x}^2 \le 0$, *and with expected future gain* $\boldsymbol{EG'_{t+2}(x)}$ *that satisfies:*

$$(\forall x' \le x_t^*)\ \left[\left(\frac{\partial EG'_{t+1}(x)}{\partial x}\right)_{x=x_t^*} \le \left(\frac{\partial EG'_{t+2}(x)}{\partial x}\right)_{x=x'} \wedge\ EG'_{t+1}(x_t^*) \ge EG'_{t+2}(x')\right]$$

Proof. (A detailed proof is provided in the appendix) We first establish $\boldsymbol{\pi_{t+n}(x)} = \boldsymbol{p_{t+n}(x)}$. If before time $\boldsymbol{t+n}$ the buyer concedes, then the proposition is proven. If the buyer does not concede, then by Assumption 3-9 eventually $\boldsymbol{k_{t+n}(x)} = 0$, and $\boldsymbol{\pi_{t+n}(x)} = \boldsymbol{p_{t+n}(x)}$. So, we can ignore the intermediate transition state between $\boldsymbol{t}$ and $\boldsymbol{t+n}$ and only consider

$\pi_t(x) = p_t(x)$ and $\pi_{t+n}(x) = p_{t+n}(x)$, where $p_t(x)$ and $p_{t+n}(x)$ are continuous and differentiable.

(a) Since in this case the buyer is a myopic-0 agent, then it will offer optimal price x_t^* that maximizes its expected gain, i.e. $Max_x\,[p_t(x)(B_t - x)]$, which can be illustrated by the largest area of the rectangle bounded by belief $p_t(x)$, x-axis and buyer's valuation B_t in Figure 3-12.

Since the seller rejects x_t^* and the counter offer is insignificantly different, then by Proposition 3-2 the slope $\partial p_t(x)/\partial x$ becomes steeper for $x \le x_t^*$, or $\partial(p_t(x) - p_{t+1}(x))/\partial x \le 0$ for $x \le x_t^*$ and $p_{t+1}(x) > 0$. Proof by contradiction: suppose the buyer's optimal offer is not a concession but still generates a positive expected gain, or $x_{t+1}^* \le x_t^*$ and $p_{t+1}(x_{t+1}^*) > 0$. Then by integrating $\partial(p_t(x) - p_{t+1}(x))/\partial x \le 0$ from x_{t+1}^* to x_t^*, we have

$$[p_t(x) - p_{t+1}(x)]_{xt^*} - [p_t(x) - p_{t+1}(x)]_{xt+1^*} \le 0$$

$$\Leftrightarrow p_t(x_t^*)(B - x_{t+1}^*) - p_{t+1}(x_t)(B - x_{t+1}^*) \le p_t(x_{t+1}^*)(B - x_{t+1}^*) - p_{t+1}(x_{t+1}^*)(B - x_{t+1}^*) \tag{3-15}$$

Since $x_{t+1}^* \le x_t^*$, then $(B - x_t^*) \le (B - x_{t+1}^*)$, or

$$\Leftrightarrow p_t(x_t^*)(B - x_t^*) - p_{t+1}(x_t^*)(B - x_t^*) \le p_t(x_{t+1}^*)(B - x_{t+1}^*) - p_{t+1}(x_{t+1}^*)(B - x_{t+1}^*) \tag{3-16}$$

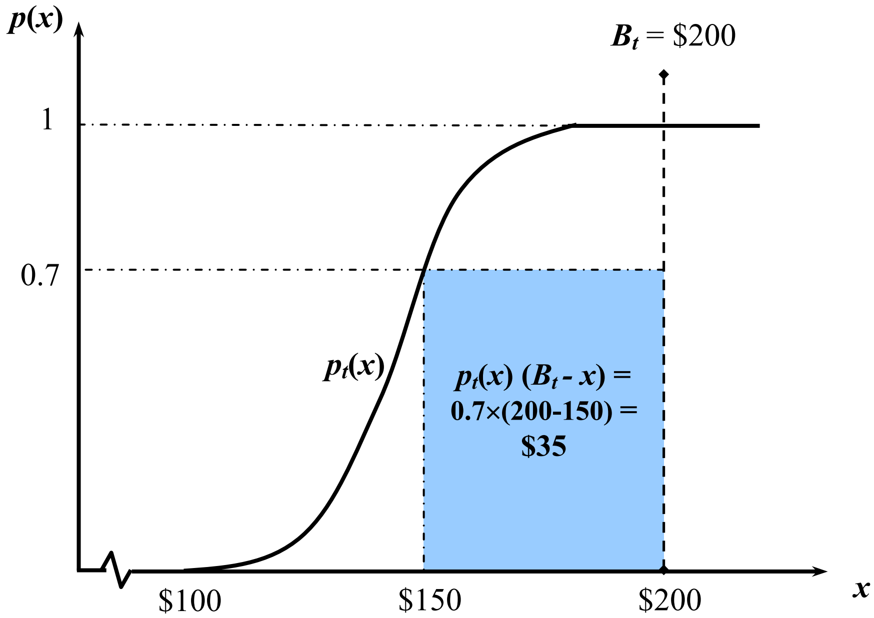


Figure 3-12. The expected surplus of myopic-0 agents

However, $p_t(x_t^*)(B - x_t^*)$ is the maximum expected gain at time t, or $p_t(x_t^*)(B - x_t^*) > p_t(x_{t+1}^*)\ (B - x_{t+1}^*)$; or, $p_t(x_t^*)(B - x_t^*) = p_t(x_{t+1}^*)\ (B - x_{t+1}^*) + d^+$, where d^+ is some positive value. Hence, by substituting $p_t(x_t^*)(B - x_t^*) = p_t(x_{t+1}^*)\ (B - x_{t+1}^*) + d^+$ into (3-16) we get

$$p_t(x_{t+1}^*)\ (B - x_{t+1}^*) + d^+ - p_{t+1}(x_t^*)(B - x_t^*) \leq p_t(x_{t+1}^*)(B - x_{t+1}^*) - p_{t+1}(x_{t+1}^*)(B - x_{t+1}^*)$$

$$\Leftrightarrow \qquad p_{t+1}(x_t^*)(B - x_t^*) \geq p_{t+1}(x_{t+1}^*)(B - x_{t+1}^*) + d^+ \qquad (3\text{-}17)$$

But at time $t+1$, $p_{t+1}(x_{t+1}^*)(B - x_{t+1}^*)$ is the maximum gain, or $p_{t+1}(x_{t+1}^*)(B - x_{t+1}^*) > p_{t+1}(x_t^*)(B - x_t^*)$; thus $p_{t+1}(x_t^*)(B - x_t^*) > p_{t+1}(x_t^*)(B - x_t^*) + d^+$, which is a contradiction. Thus, $x_{t+1}^* > x_t^*$ or a myopic-0 buyer concedes.

(b) Since the buyer in this case is a myopic-1 agent, then it will offer the optimal price x_t^* that maximizes its expected gain, i.e. $Max_x\ [p_t(x)(B_t - x) + \gamma(1 - p_t(x))\mathrm{E}G'_{t+1}(x)]$. If x_t^* is an optimal offer, then the necessary condition is that x_t^* be a critical point which satisfies

$\partial[p_t(x)(B - x) + \gamma(1 - p_t(x))EG'_{t+1}(x)] /\partial x = 0$ at x_t^* (first order derivative test), or

$$(B-x)\frac{\partial p_t(x)}{\partial x} - p_t(x) + (1 - p_t(x))\gamma\frac{\partial EG'_{t+1}(x)}{\partial x} - \gamma EG'_{t+1}(x)\frac{\partial p_t(x)}{\partial x} = 0 \text{ at } x_t^*$$

$$\Leftrightarrow \quad \left.\frac{\partial p_t(x)}{\partial x}\right|_{x=x_t^*} = \left.\frac{p_t(x) - (1 - p_t(x))\gamma\frac{\partial EG'_{t+1}(x)}{\partial x}}{(B - x - \gamma EG'_{t+1}(x))}\right|_{x=x_t^*} \tag{3-18}$$

Similarly, for the optimal offer x_{t+1}^* at time $t+1$, we also have:[1]

$$\left.\frac{\partial p_{t+1}(x)}{\partial x}\right|_{x_{t+1}^*} = \left.\frac{p_{t+1}(x) - (1 - p_{t+1}(x))\gamma\frac{\partial EG'_{t+2}(x)}{\partial x}}{(B - x - \gamma EG'_{t+2}(x))}\right|_{x_{t+1}^*} \tag{3-19}$$

Since the seller rejects x_t^* and concedes insignificantly in its counter-offer, then by Proposition 3-2 the slope $\partial p_t(x)/\partial x$ becomes steeper for all $x \leq x_t^*$, or $\partial p_t(x)/\partial x \leq \partial p_{t+1}(x)/\partial x$ for $x \leq x_t^*$. If the buyer does not concede, or $x_{t+1}^* \leq x_t^*$, then we have $\left.\frac{\partial p_t(x)}{\partial x}\right|_{x_{t+1}^*} \leq \left.\frac{\partial p_{t+1}(x)}{\partial x}\right|_{x_{t+1}^*}$. And since $p_t(x)$ is convex for all $x \leq x_t^*$, then we have $\left.\frac{\partial p_t(x)}{\partial x}\right|_{x_t^*} \leq \left.\frac{\partial p_t(x)}{\partial x}\right|_{x_{t+1}^*}$, therefore, $\left.\frac{\partial p_t(x)}{\partial x}\right|_{x_t^*} \leq \left.\frac{\partial p_{t+1}(x)}{\partial x}\right|_{x_{t+1}^*}$. By substituting equation (3-18) and (3-19) into the inequality above we get

$$\left.\frac{p_t(x) - (1 - p_t(x))\gamma\frac{\partial EG'_{t+1}(x)}{\partial x}}{(B - x - \gamma EG'_{t+1}(x))}\right|_{x_t^*} \leq \left.\frac{p_{t+1}(x) - (1 - p_{t+1}(x))\gamma\frac{\partial EG'_{t+2}(x)}{\partial x}}{(B - x - \gamma EG'_{t+2}(x))}\right|_{x_{t+1}^*}$$

[1] For the sake of brevity, we will use " $|_{xt}$ " instead of " $|_{x=xt}$ " to represent the condition in which the formula hold.

The inequality condition above tells us that if $x_{t+1}*$ is an optimal offer at time $t+1$ that satisfies $x_{t+1}* \leq x_t*$, then the inequality holds. If the inequality does not hold, then by modus tollens, we can conclude that $x_{t+1}*$ is not an optimal offer, or a contradiction. Thus, we come out with a sufficient condition for $x_{t+1}* > x_t*$, which is the negation of the inequality above, i.e.

$(\forall\ x' \leq x_t*)$

$$\left.\frac{p_t(x) - (1 - p_t(x))\gamma \dfrac{\partial EG'_{t+1}(x)}{\partial x}}{(B - x - \gamma EG'_{t+1}(x))}\right|_{x_t*} > \left.\frac{p_{t+1}(x) - (1 - p_{t+1}(x))\gamma \dfrac{\partial EG'_{t+2}(x)}{\partial x}}{(B - x - \gamma EG'_{t+2}(x))}\right|_{x'}$$

which leads to two joint conditions:

$(\forall\ x' \leq x_t*)$

$$p_t(x_t*) - (1 - p_t(x_t*))\gamma \left[\frac{\partial EG'_{t+1}(x)}{\partial x}\right]_{x_t*} > p_{t+1}(x') - (1 - p_{t+1}(x'))\gamma \left[\frac{\partial EG'_{t+2}(x)}{\partial x}\right]_{x'}$$

$$\wedge \qquad (B - x_t* - \gamma EG'_{t+1}(x_t*)) \leq (B - x' - \gamma EG'_{t+2}(x'))$$

Or,

$(\forall\ x' \leq x_t*)$

$$p_t(x_t*) - p_{t+1}(x') + (1 - p_{t+1}(x'))\gamma \left[\frac{\partial EG'_{t+2}(x)}{\partial x}\right]_{x'} > (1 - p_t(x_t*))\gamma \left[\frac{\partial EG'_{t+1}(x)}{\partial x}\right]_{x_t*}$$

$$\wedge \quad \gamma EG'_{t+2}(x') \leq \gamma EG'_{t+1}(x_t*) + (B - x') - (B - x_t*)$$

Since $x' \leq x_t*$, then $(B - x_t*) \leq (B - x')$ and $p_t(x_t*) \geq p_t(x') > p_{t+1}(x')$, or $p_t(x_t*) > p_{t+1}(x')$. Given $p_t(x_t*) - p_{t+1}(x') > 0$ and $(B - x') - (B - x_t*) \geq 0$, then the joint condition above holds if

$$(\forall\ x' \leq x_t*) \quad (1 - p_t(x_t*))\gamma \left[\frac{\partial EG'_{t+1}(x)}{\partial x}\right]_{x_t*} \leq (1 - p_{t+1}(x'))\gamma \left[\frac{\partial EG'_{t+2}(x)}{\partial x}\right]_{x'}$$

$$\wedge \qquad \boldsymbol{EG'_{t+1}(x_t^*) \geq EG'_{t+2}(x')}$$

And since $\mathbf{(1 - \boldsymbol{p_t}(\boldsymbol{x_t}^*)) < (1 - \boldsymbol{p_{t+1}}(\boldsymbol{x'}))}$, then the sufficient condition becomes

$$(\forall x' \leq x_t^*) \left[\left(\frac{\partial EG'_{t+1}(x)}{\partial x} \right)_{x_t^*} \leq \left(\frac{\partial EG'_{t+2}(x)}{\partial x} \right)_{x'} \wedge \quad EG'_{t+1}(x_t^*) \geq EG'_{t+2}(x') \right]$$

(3-20)

Which is the premise in the proposition, the buyer's optimal offer $\boldsymbol{x_{t+1}^* > x_t^*}$. ■

The condition of Proposition 3-9(a) is straightforward, but not for Proposition 3-9(b), which will be explained here. First, the convexity of belief function $\boldsymbol{p_t(x)}$ in Proposition 3-9(b) represents the diminishing of the increasing rate of belief value over $\boldsymbol{x}$. Or, the left tail of the belief function in Figure 3-12 cannot exist. Proposition 3-9(b) applies for linear, logarithmic, and step belief functions, but not for logistic functions. Second, the sufficient condition in (3-20) **is** related to the buyer's expected gain in the future after an offer $\boldsymbol{x' \leq x_t^*}$ is rejected by the seller. If we interpret them separately, then the first condition restricts our expected future gain function at time $\boldsymbol{t+1}$ so that the slope of all $\boldsymbol{x' \leq x_t^*}$ must be greater than the slope of the expected future gain of $\boldsymbol{x_t^*}$. If $\boldsymbol{EG'_{t+1}(x)}$ and $\boldsymbol{EG'_{t+2}(x)}$ are decreasing functions over $\boldsymbol{x}$, then all $\boldsymbol{x' \leq x_t^*}$ $\boldsymbol{EG'_{t+2}(x')}$ must decrease slower than $\boldsymbol{EG'_{t+1}(x_t^*)}$. If both are constant values, then the second condition alone is enough. The second condition states that if any offer $\boldsymbol{x' \leq x_t^*}$ is rejected by the seller, then the buyer's expected future gain must be lower than when $\boldsymbol{x_t^*}$ is rejected. Intuitively, if the buyer's expected future gain is much higher than what it had expected previously, then there is an incentive for the buyer to delay the bargaining by submitting a lower offer than its previous offer (a strategic delay that will be discussed in the next chapter).

However, this condition is only a sufficient condition, which means the buyer may also concede even without satisfying this condition.

3.5 Chapter Summary

The decision model described in this chapter is the general framework that will be used in our theoretical analysis. Based on agent bargaining strategy we have classified our model into two large groups: myopic-0 and myopic-1 agents that maximize expected gain. According to their evaluation criteria, we have EvalF-I and EvalF-II agents which may affect their belief updating mechanism as shown in sections 3.3.2.2 and 3.3.3.2 respectively.

Certainly, the model described here is not a general model that represents all possible agent behaviors. As explained before, the scope of our current bargaining model is bounded to single attribute item (i.e. price), and the decision model only represents a specific set of agents. For instance, the decision model excludes myopic-N agents, where N > 1 and it does not consider other evaluation criteria, as mentioned in section 3.2.1.4. However, as argued in section 3.4.1, the model is based on commonsense reasoning with various belief updating mechanisms. Hence, the model has represented a partial model of bargaining strategies under uncertainty. As our purpose is not to analyze the bargaining strategy but the bargaining protocol, we believe that our model is sufficiently representative of bounded-rational agents as discussed in current literature.

In the next chapter, we will analyze three different protocols, which have specific advantages/disadvantages within our model. First, we will use a modified model with time pressure, such as decreasing valuation for the buyer, increasing likelihood of breakdown over time, and decreasing expected future gain for myopic agents. In fact, this modified model is the most common one in alternating offers bargaining, e.g. when the bargaining incurs some cost over time [Rubinstein, 1982; Kraus et al., 1995; Sandholm and Vulkan, 1999]. Then, we will analyze the effect of unconstrained agent valuation of their expected future gain, but still with some pressure from the likelihood of breakdown. The reason is to represent the uncertainty of agent valuation, which is also very common in an ill-informed bargaining. For instance, when the buyer is not certain about the quality of the item, then its valuation and expected future gain may increase or decrease over time. Therefore, agents may need some time to observe their environment and update their beliefs. Finally, instead of using a standard belief updating mechanism of $\boldsymbol{p_t(x)}$, $\boldsymbol{q_t}$, etc., we will allow agents to control their updating mechanism by generating trust toward their opponents. Intuitively, this approach needs more sophisticated agent reasoning than what we have described here. For example, we will allow a skeptical agent to switch to a believing one and vice versa, or allow it to switch its belief that its opponent is EvalF-I instead of EvalF-II, etc. The dynamism comes from the agent's richer interaction --- by argumentation. Hence, our theoretical analysis will focus on the implication of communication/argumentation and its cost in belief-updating speed. We will prove that in some situations agents may ignore argumentation, thus suggesting a more flexible argumentation-based negotiation protocol.

CHAPTER 4

AGENT DECISIONS AND NEGOTIATION PROTOCOLS

In this chapter we analyze separately three modifications made to the traditional alternating-offers (with argumentation) bargaining protocol. For each modification, we first illustrate its motivation and necessity. Then, we analyze buyer behavior under the modifications, including any deviated behavior that may occur due to selfishness and rationality. We start the analysis with the non-monotonic-offers bargaining protocol.

4.1 Non-monotonic-offers Bargaining Protocol

4.1.1 Motivation

In human bargaining, one of the agreed upon properties in alternating-offer bargaining is the (weak) monotonic (counter-) offer by bargainers, i.e. human buyers/sellers may only insist on their previous offers or concede monotonically until an agreement is reached. In other words, changing their offers arbitrarily is normally avoided.

The reasons for the monotonic-offers property could be either human rationality or social norms, or both. For instance, our mental models may satisfy the monotonicity of $\boldsymbol{p_t(x)}$, the belief updating mechanism stated in Proposition 3-2 and 3-3, and also the buyer's conceding mechanism in Proposition 3-9. Hence, if we are convinced that our current offer would not be accepted by our opponent, then very likely we also believe that other non-conceding offers would not be accepted either, which pushes us to concede. Yet, even if a human buyer realizes that a non-conceding action is often better, e.g. after receiving new information, reducing his/her offers may be interpreted as an absurdity or insincere behavior; thus, it is not acceptable.

Unlike humans, agents are not constrained by norms or emotions. As long as the protocol is announced and understandable, their designer will provide them with a relevant bargaining strategy. [Rosenschein and Zlotkin, 1994] were the first to propose a monotonic concession protocol in multi-agent systems, in which agents monotonically increase or decrease their offers. Their most important assumption is that agents are selfish with private information and negotiation is the only option to reach a solution. The work of many others also uses the same assumption, e.g. [Faratin et al., 1998; Fatima et al., 2004; Sim and Wang, 2004]. In this section we will show that imposing a non-monotonic-offers protocol (N-protocol, for short) in agents' bargaining may be better than imposing a monotonic-offers protocol (M-protocol). Let us first look at an illustrative example in Example 4-1.

Example 4-1. *Suppose a buyer wants to buy a service (e.g. predicting the future price of stocks), which it would like within a specific time (e.g., before the stock market opens). However, if it cannot get the service during that period, its preference towards the service reduces over time, and becomes zero if it got the service after a time deadline (e.g., after the market closes). Thus, its valuation towards the service will be high when the market opens, and will decrease until the market closes. Suppose the seller's valuation is constant over time. Since both parties may hold their valuations privately,* *the initial spread will be relatively big. The initial spread will decrease as the bargaining proceeds (see Figure 4-1(a)). However, under M-protocol, the bargaining may be stuck even if both parties repeat it several times after a short delay, as illustrated in Figure 4-1(a). Here, the failures are caused by the buyer who cannot resume the negotiation after its offer approaches its valuation because its next offer will be higher than its next valuation (under M-protocol). However, under a non-monotonic offers protocol (N-protocol for short), the failure can be remedied and the delay from re-opening a bargaining session disappears as illustrated in Figure 4-1(b).* ■

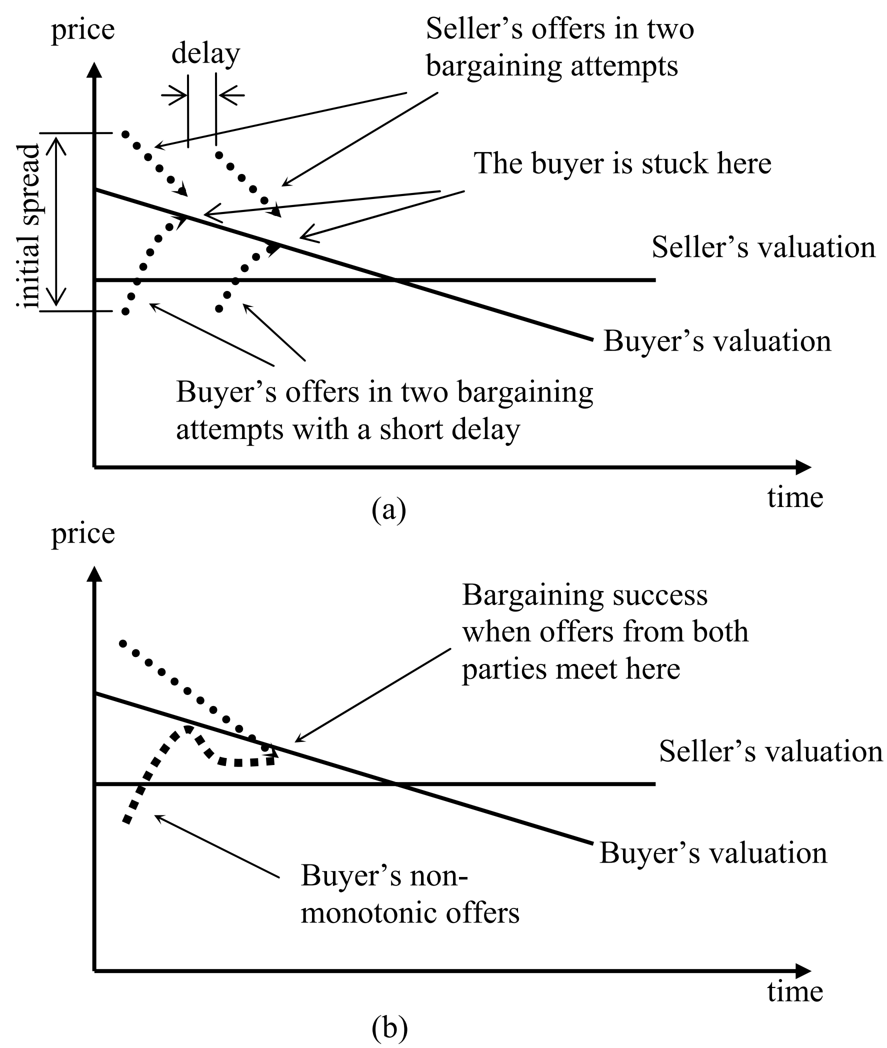


Figure 4-1 An example of the bargaining under (a) M- protocol and (b) N-protocol

Example 4-1 above only shows a partial scenario, which favors the N-protocol. The goal of the N-protocol is to allow agents to bargain as flexibly as possible so that they can always maximize their (expected) utility or increase the success rate, or both, depending on their goals. Indeed, some bargainers' decision processes under the N-protocol may suppress the convergence of spread in reaching a concession, which will be explained later. However, assuming that both bargainers are rational, slower convergence rates due to non-monotonic offers will not significantly reduce the overall performance of the bargaining in terms of agents' expected surplus.

4.1.2 Model Description

In our formal model described in Chapter 3, we have discussed myopic-0 and myopic-1 agents, whose expected gains are defined in equations (3-4) and (3-6) respectively, and whose decisions follow equations (3-5) and (3-7). If a zero surplus is generated from a breakdown, then the buyers' problems are as follows:

(myopic-0) $$EG_t(x) = (1 - q_t)\, p_t(x)\, (B_t - x) \quad (3\text{-}4)$$

$$Max_x\, EG_t(x) = Max_x\, [p_t(x)(B_t - x)] \quad (3\text{-}5)$$

(myopic-1) $$EG_t(x) = (1 - q_t)\, [p_t(x)(B_t - x) + \gamma(1 - p_t(x))EG'_{t+1}(x)] \quad (3\text{-}6)$$

$$Max_x\, EG_t(x) = Max_x\, [p_t(x)(B_t - x) + \gamma(1 - p_t(x))EG'_{t+1}(x)] \quad (3\text{-}7)$$

Along with the above formulae, our model here also includes Assumptions 3-8 and 3-9 and also Propositions 3-1, 3-2, 3-3 and 3-4 regarding their belief functions and updating mechanisms (as described in section 3.3.2 and 3.3.3). The aforementioned properties hold for a buyer who knows the type of the seller which remains unchanged. If the buyer does not know or is unsure about the type of the seller, which is more reasonable, then we assume that the buyer will follow Assumption 3-11, which assumes that the buyer will treat the seller as an EvalF-II seller but with lower value of $\boldsymbol{k_t(x)}$. Finally, we also use Proposition 3-8 regarding the buyer's preference over additional opportunities.

In addition to those assumptions and properties, we will add some assumptions to make our analysis tractable. These assumptions will assure the convergence of the bargaining. The first two assumptions are:

Assumption 4-1. $\boldsymbol{q_t}$ *is an increasing function over time* $\boldsymbol{t}$.

Assumption 4-2. $\boldsymbol{B_t}$ *is a decreasing function over time* $\boldsymbol{t}$.

These two assumptions state that the buyer is facing pressure from the bargaining and believes that the likelihood of breakdown is increasing over time. In fact, these two assumptions are very common in both human and agent bargaining models. In much negotiation literature, a discounted utility function and time deadlines are used to represent the pressure of time [Kraus et al., 1995]. In contrast, we use surplus instead of a general utility function to represent an agent's payoff from the bargaining; thus, we use a decreasing buyer's valuation (Assumption 4-2) to represent a discounted surplus, as shown in Example 4-1. Recall that we have discussed this issue in section 3.1.3, Surplus and Utility.

Given assumptions 4-1 and 4-2, the buyer will have less expected future gain due to the increasing probability of breakdown in the future and lower valuation which causes lower expected surplus in the future. Thus, we will assume that the buyer's expected future gain $\boldsymbol{EG'_{t+1}(x)}$ is decreasing over time, i.e. satisfies Proposition 3-7. Moreover, in Proposition 3-6 we have shown that $\boldsymbol{EG'_{t+1}(x)}$ is ambiguous with respect to $\boldsymbol{x}$. For the sake of simplicity we assume it is constant with respect to $\boldsymbol{x}$. Rather than using Proposition 3-7, we postulate the properties of $\boldsymbol{EG'_{t+1}(x)}$ using Assumption 4-3:

Assumption 4-3. $\boldsymbol{EG'_{t+1}(x)}$ *is a decreasing function over* $\mathbf{t}$ *and a constant function over* $\mathbf{x}$*, denoted* $\mathbf{EG'_{t+1}}$ *for convenience.*

In the next two sub-sections, we will analyze the buyer's characteristics when facing EvalF-I and EvalF-II sellers.

4.1.3 A Society of EvalF-I Agents

Given the characteristics of $\boldsymbol{p_t(x)}$ and expected gain, a myopic buyer will be able to maximize its expected gain by offering $\boldsymbol{x_t*}$, either in a continuous or discrete domain. In a continuous domain when $\boldsymbol{p_t(x)}$ and $\boldsymbol{EG_t(x)}$ are continuous and differentiable, we can derive the necessary condition of optimal offer $\boldsymbol{x_t*}$ by taking the first order derivative condition of $max_x\ \boldsymbol{EG_t(x)}$, i.e. $\boldsymbol{dEG_t/dx} = 0$ at $\boldsymbol{x_t*}$. For a myopic-0 buyer, we use equation (3-4) for $\boldsymbol{EG_t(x) = (1 - q_t)\ p_t(x)\ (B_t - x)}$; or the first order derivative condition can be written as

$$\left.\frac{dEG_t(x)}{dx}\right|_{x_t*} = (1 - q_t)\left.\left(\frac{\partial p_t(x)}{\partial x}(B_t - x) - p_t(x)\right)\right|_{x_t*} = 0$$

$$\Leftrightarrow \qquad \left.\frac{\partial p_t(x)}{\partial x}\right|_{x_t*}(B_t - x_t*) - p_t(x_t*) = 0$$

$$\Leftrightarrow \qquad \left.\frac{\partial p_t(x)}{\partial x}\right|_{x_t*}(B_t - x_t*) = p_t(x_t*)$$

$$\Leftrightarrow \qquad \left.\frac{dp_t(x)}{dx}\right|_{x_t*} = \frac{p_t(x_t*)}{B_t - x_t*} \qquad (4\text{-}1)$$

For a myopic-1 buyer, we use equation (3-6) for $\boldsymbol{EG_t(x) = (1 - q_t)[\, p_t(x)(B_t - x)} + \boldsymbol{\gamma(1 - p_t(x))EG'_{t+1}(x)]}$, where the first order derivative condition has been derived previously in the proof of Proposition 3-9:

$$\left.\frac{\partial p_t(x)}{\partial x}\right|_{x_t^*} = \left.\frac{p_t(x) - (1 - p_t(x))\gamma \dfrac{\partial EG'_{t+1}(x)}{\partial x}}{(B - x - \gamma EG'_{t+1}(x))}\right|_{x_t^*} \qquad (3\text{-}18)$$

Since $\boldsymbol{EG'_{t+1}(x)}$ is a constant function with respect to $\boldsymbol{x}$ but decreasing over time (by Assumption 4-3), then $\frac{\partial EG'_{t+1}(x)}{\partial x} = 0$ and we can simplify equation (3-18) into:

$$\left.\frac{dp_t(x)}{dx}\right|_{x_t^*} = \frac{p_t(x_t^*)}{B_t - x_t^* - \gamma EG'_{t+1}} \qquad (4\text{-}2)$$

However, in real bargaining sometimes the bargaining attributes (e.g. price) are continuous (as stated in Assumption 3-5); hence, the conditions are slightly different, so we use finite difference instead of derivative. In the derivation of the necessary condition, we use the fact that an optimal offer $\boldsymbol{x_t^*}$ satisfies $\boldsymbol{EG_t(x_t^*)} \geq \boldsymbol{EG_t(x_t^* \pm \Delta x)}$ where $\boldsymbol{\Delta x}$ is the smallest interval (increment) of $\boldsymbol{x}$ (e.g. \$1). If $\boldsymbol{EG_t(x)}$ has only a single peak value, then this condition is sufficient. To derive the necessary condition, let's divide it into backward difference $\boldsymbol{EG_t(x_t^*)} \geq \boldsymbol{EG_t(x_t^* - \Delta x)}$ (or, $\nabla \boldsymbol{EG_t(x_t^*)} \geq 0$) and forward difference $\boldsymbol{EG_t(x_t^*)} \geq \boldsymbol{EG_t(x_t^* + \Delta x)}$ (or, $\Delta \boldsymbol{EG_t(x_t^*)} \geq 0$). Using equation (3-4), then

$\boldsymbol{EG_t(x_t^*) = (1 - q_t)\, p_t(x_t^*)\, (B_t - x_t^*)}$ and

$\boldsymbol{EG_t(x_t^* - \Delta x) = (1 - q_t)\, p_t(x_t^* - \Delta x)\, (B_t - (x_t^* - \Delta x))}$

From the condition $\boldsymbol{EG_t(x_t^*)} \geq \boldsymbol{EG_t(x_t^* - \Delta x)}$, then

$\Leftrightarrow \qquad \boldsymbol{(1 - q_t)\, p_t(x_t^*)\, (B_t - x_t^*) \geq (1 - q_t)\, p_t(x_t^* - \Delta x)\, (B_t - (x_t^* - \Delta x))}$

$\Leftrightarrow \qquad \boldsymbol{p_t(x_t^*)\, (B_t - x_t^*) \geq p_t(x_t^* - \Delta x)\, (B_t - (x_t^* - \Delta x))}$

$$\Leftrightarrow \quad p_t(x_t^*)\,(B_t - x_t^*) \geq p_t(x_t^* - \Delta x)\,(B_t - x_t^*) + p_t(x_t^* - \Delta x)\,\Delta x \quad (4\text{-}3)$$

If we assume the backward difference $[\nabla p_t]_{xt^*} \equiv p_t(x_t^*) - p_t(x_t^* - \Delta x)$, then $p_t(x_t^* - \Delta x) = p_t(x_t^*) - [\nabla p_t]_{xt^*}$. Inequality (4-3) becomes

$$\Leftrightarrow \quad p_t(x_t^*)\,(B_t - x_t^*) \geq (p_t(x_t^*) - [\nabla p_t]_{xt^*})\,(B_t - x_t^*) + p_t(x_t^* - \Delta x)\,\Delta x$$

$$\Leftrightarrow \quad p_t(x_t^*)\,(B_t - x_t^*) \geq p_t(x_t^*)\,(B_t - x_t^*) - [\nabla p_t]_{xt^*}\,(B_t - x_t^*) + p_t(x_t^* - \Delta x)\,\Delta x$$

$$\Leftrightarrow \quad 0 \geq - [\nabla p_t]_{xt^*}\,(B_t - x_t^*) + p_t(x_t^* - \Delta x)\,\Delta x$$

$$\Leftrightarrow \quad [\nabla p_t]_{xt^*}\,(B_t - x_t^*) \geq p_t(x_t^* - \Delta x)\,\Delta x$$

$$\Leftrightarrow \quad [\nabla p_t]_{xt^*} / \Delta x \;\geq p_t(x_t^* - \Delta x) / (B_t - x_t^*)$$

$$\Leftrightarrow \quad [\nabla p_t / \Delta x \;\geq p_t(x - \Delta x) / (B_t - x)]_{xt^*} \quad (4\text{-}4)$$

In addition, from the forward difference we have

$EG_t(x_t^* + \Delta x) = (1 - q_t)\,p_t(x_t^* + \Delta x)\,(B_t - (x_t^* + \Delta x))$ and

$[\Delta p_t]_{xt^*} \equiv p_t(x_t^*) - p_t(x_t^* + \Delta x)$ or $p_t(x_t^* + \Delta x) = p_t(x_t^*) - [\Delta p_t]_{xt^*}$. Hence,

$EG_t(x_t^*) \geq EG_t(x_t^* + \Delta x)$

$$\Leftrightarrow \quad (1 - q_t)\,p_t(x_t^*)\,(B_t - x_t^*) \geq (1 - q_t)\,p_t(x_t^* + \Delta x)\,(B_t - (x_t^* + \Delta x))$$

$$\Leftrightarrow \quad p_t(x_t^*)\,(B_t - x_t^*) \geq p_t(x_t^* + \Delta x)\,(B_t - x_t^*) - p_t(x_t^* + \Delta x)\,\Delta x$$

$$\Leftrightarrow \quad p_t(x_t^*)\,(B_t - x_t^*) \geq (p_t(x_t^*) - [\Delta p_t]_{xt^*})\,(B_t - x_t^*) - (p_t(x_t^*) - [\Delta p_t]_{xt^*})\,\Delta x$$

$$\Leftrightarrow \quad p_t(x_t^*)(B_t - x_t^*) \geq p_t(x_t^*)(B_t - x_t^*) - [\Delta p_t]_{xt^*}(B_t - x_t^*) - (p_t(x_t^*) - [\Delta p_t]_{xt^*})\Delta x$$

$$\Leftrightarrow \quad 0 \geq - [\Delta p_t]_{xt^*}\,(B_t - x_t^*) - p_t(x_t^*)\Delta x + [\Delta p_t]_{xt^*}\,\Delta x$$

$$\Leftrightarrow \quad p_t(x_t^*)\Delta x \geq - [\Delta p_t]_{xt^*}\,(B_t - x_t^* - \Delta x)$$

$$\Leftrightarrow \quad p_t(x_t^*) / (B_t - x_t^* - \Delta x) \geq - [\Delta p_t]_{xt^*} / \Delta x$$

$$\Leftrightarrow \quad - [\Delta p_t]_{xt^*} / \Delta x \leq p_t(x_t^*) / (B_t - (x_t^* + \Delta x))$$

$$\Leftrightarrow \quad - [\Delta p_t]_{xt^*} / \Delta x \leq p_t((x_t^* + \Delta x) - \Delta x) / (B_t - (x_t^* + \Delta x))$$

but $-[\Delta \boldsymbol{p_t}]_{xt^*} = -(\boldsymbol{p_t}(\boldsymbol{x_t}^*) - \boldsymbol{p_t}(\boldsymbol{x_t}^* + \Delta \boldsymbol{x})) = \boldsymbol{p_t}(\boldsymbol{x_t}^* + \Delta \boldsymbol{x}) - \boldsymbol{p_t}(\boldsymbol{x_t}^*) \equiv [\nabla \boldsymbol{p_t}]_{xt^*+\Delta x}$, thus

$$\Leftrightarrow \qquad [\nabla \boldsymbol{p_t} / \Delta \boldsymbol{x} \leq \boldsymbol{p_t}(\boldsymbol{x} - \Delta \boldsymbol{x}) / (\boldsymbol{B_t} - \boldsymbol{x})]_{xt^*+\Delta x} \qquad (4\text{-}5)$$

Thus, combining inequality (4-4) and (4-5), we will have a compound condition:

$[\nabla \boldsymbol{p_t} / \Delta \boldsymbol{x} \geq \boldsymbol{p_t}(\boldsymbol{x} - \Delta \boldsymbol{x}) / (\boldsymbol{B_t} - \boldsymbol{x})]_{xt^*}$ and $[\nabla \boldsymbol{p_t} / \Delta \boldsymbol{x} \leq \boldsymbol{p_t}(\boldsymbol{x} - \Delta \boldsymbol{x}) / (\boldsymbol{B_t} - \boldsymbol{x})]_{xt^*+\Delta x}$.

Or, we can express the condition as

$$\left.\frac{\nabla p_t}{\Delta x}\right|_{x_t^*} \approx \frac{p_t(x_t^b)}{B_t - x_t^*} \qquad (4\text{-}6)$$

Here $\boldsymbol{x_t^b} \equiv \boldsymbol{x_t}^* - \Delta \boldsymbol{x}$ is the price just lower than $\boldsymbol{x_t}^*$, and $\nabla \boldsymbol{p_t} \equiv \boldsymbol{p_t}(\boldsymbol{x}) - \boldsymbol{p_t}(\boldsymbol{x} - \Delta \boldsymbol{x})$. The equality sign "≈" means that at $\boldsymbol{x_t}^*$ the difference of LHS and RHS is minimized but LHS ≥ RHS, and at $\boldsymbol{x_t}^* + \Delta \boldsymbol{x}$ we have LHS ≤ RHS. To differentiate it with an ordinary equality, we denote it as ε-equality.

Using a similar method, we can derive the following condition for a myopic-1 buyer.

$$\left.\frac{\nabla p_t}{\Delta x}\right|_{x_t^*} \approx \frac{p_t(x_t^b)}{B_t - x_t^* - \gamma EG'_{t+1}} \qquad (4\text{-}7)$$

Given the optimal-offer conditions above, we can prove that a buyer needs to adjust its offer in either an increasing or a decreasing way; which can only be realized under the N-protocol.

Proposition 4-1. *Suppose all agents under the N-protocol are EvalF-I agents and this is common knowledge. If* $\boldsymbol{x_t}^*$ *is an optimal offer at time* $\boldsymbol{t}$, *then in order to maximize its expected gain at time* $\boldsymbol{t}+1$:

(a) a myopic-0 buyer will monotonically increase its offer if $\boldsymbol{B_{t+1}} - \boldsymbol{x_t}^* >> \boldsymbol{0}$, *and decrease its offer if* $\boldsymbol{B_{t+1}} - \boldsymbol{x_t}^* \rightarrow \boldsymbol{0}$ *or* $\boldsymbol{B_{t+1}} - \boldsymbol{x_t}^* < \boldsymbol{0}$;

(b) a myopic-1 buyer will monotonically increase its offer if $\boldsymbol{B_{t+1} - x_t^* >> \gamma EG'_{t+2}}$, *and decrease its offer if* $\boldsymbol{B_{t+1} - x_t^* \rightarrow \gamma EG'_{t+2}}$ *or* $\boldsymbol{B_{t+1} - x_t^* < \gamma EG'_{t+2}}$.

Proposition 4-1 describes the self-adjustment of the buyer's optimal offer in order to maximize its expected gain. When the projected surplus by sticking on its previous offer (= $\boldsymbol{B_{t+1} - x_t^*}$) is large, the buyer tends to increase its offer. But when the projected surplus is small or negative, then it will reduce its offer. The convergence of a buyer's offers to its valuation is guaranteed as shown in Proposition 4-2.

Proposition 4-2. *Under the N-protocol, if all agents are EvalF-I agents and this is common knowledge, then* $\boldsymbol{x^*}$ *converges to* $\boldsymbol{B}$ *over time.*

From Proposition 4-1 and 4-2, we can derive several properties of EvalF-I buyers:

- The likelihood of breakdown does not affect myopic-0 buyers but does affect myopic-1 buyers. If the likelihood increases ($\gamma \boldsymbol{EG'_{t+1}}$ decreases), then a myopic-1 buyer's optimal offers will converge quickly to its valuation.
- If the buyer's valuation decreases sharply, then its optimal offer may decrease too.

In our previous analysis (Proposition 4-1 and 4-2), the agents are less concerned about failure or breakdown. However, in a real situation, humans may be more concerned about this than about the surplus. In such a situation, agents would be required to find a concession as soon as possible, even with a zero surplus. Intuitively, agents in the N-protocol will outperform agents in the M-protocol in performing such tasks, as stated in proposition 4-3 below.

Proposition 4-3. *If EvalF-I agents are only concerned about the success rate, then the N-protocol is preferred over the M-protocol.*

Combining Propositions 4-1 through 4-3, we get the following theorem:

Theorem 4-1. *The N-protocol is at least as good as the M-protocol for EvalF-I agents.*

4.1.4 A Society of EvalF-II Agents

Recall that an EvalF-II agent uses an additional evaluation function in accepting an offer by its opponent, i.e., accept an offer if it is *perceived* to generate an *optimal* surplus. While they also behave as EvalF-I agents, this additional evaluation criterion makes EvalF-II agents more 'vulnerable' to their opponent's strategy in the N-protocol. For instance, a seller may increase its price (which can only happen under N-protocol) because it maximizes the seller's expected utility. If the buyer thinks that the offer is acceptable and perceives it as the 'best' offer, then it will accept it, even though the seller may make a better one in the future.

Recall also from section 3.3.3 that the buyer's belief $\pi_t(x)$ may consist of an ordinary belief $p_t(x)$ as if it is an EvalF-I buyer and $k_t(x)$ represents the likelihood of x being accepted by the seller because it is perceived as the best offer. However, if a sequence of the buyer's offers for n+1 consecutive rounds $< x_t^*, x_{t+1}^*, x_{t+2}^*, \ldots, x_{t+n}^* >$ that are within $[x_t^*-\delta, x_t^*+\delta]$ is rejected, then $\pi_{t+n}(x) = p_{t+n}(x)$ (Assumption 3-10). Given that in reality $\pi_t(x)$ may not be a continuous function (cf. Proposition 3-4), we modify equation (3-5) and (3-7) into

$$argmax_x\ EG_t = argmax_x\ [\pi_t(x)\ (B_t - x)] \quad (4\text{-}8)$$

$$argmax_x\ EG_t = argmax_x\ [\pi_t(x)(B_t - x) + \gamma(1 - \pi_t(x))EG'_{t+1}] \quad (4\text{-}9)$$

In some situations, equation (4-8) and (4-9) can only be solved numerically. However, under certain conditions, the optimal offer may increase or decrease. For instance, x_t^* will increase at a moment $\pi_t(x) = p_t(x)$ (by Assumption 3-10) and $B_{t+1} - x_t^*$

>> 0 (myopic-0) or $B_{t+1} - x_t^* >> \gamma EG'_{t+2}$ (myopic-1). And x_t^* will decrease if $B_{t+1} - x_t^* <$ 0 (myopic-0) or $B_{t+1} - x_t^* < \gamma EG'_{t+2}$ (myopic-1). Thus, similar to the situation in EvalF-I agents, we may conclude that EvalF-II buyers will benefit from the N-protocol, because they can maximize their expected gain either by increasing or decreasing their offers. But, the benefit for EvalF-II agents may not be as high as for EvalF-I agents due to the possibility of abuses of the N-protocol by greedy or irrational agents, which reduces the convergence speed of the bargaining spread and may reduce the success rate. However, we can prove that the buyers' offers will eventually approach their valuations, and if they are only concerned about the success rate, then the N-protocol is better than the M-protocol.

Proposition 4-4. *Under the N-protocol, if all agents are EvalF-II agents and this is common knowledge, then* $x_\infty^* \to B_\infty$.

Proposition 4-5. *If EvalF-II agents are only concerned about the success rate, then the N-protocol is preferred to the M-protocol.*

As for EvalF-I agents, the N-protocol is better than the M-protocol for EvalF-II agents if they are only concerned about the success rate (Proposition 4-5). Also, the N-protocol is better than the M-protocol for EvalF-II agents because it gives them flexibility to offer an optimal price. However, if both agents are not concerned about the success rate, then the success rate of the N-protocol may not be as high as the success rate of the M-protocol, especially when most agents are relatively skeptical, i.e. n is relatively big compared to time deadline T_d (recall that n represents the speed of reducing $k(x)$ to zero in the agents' belief update). The reason is that non-monotonic offers by skeptical agents may reduce the convergence speed in reaching an agreement. This becomes the serious weakness of the N-protocol.

Proposition 4-6. *Suppose the deadline for EvalF-I agents is the same as the deadline for EvalF-II agents. Then the success rate of the N-protocol for EvalF-II agents is as low as (1/*$\boldsymbol{n}^{\sim}$*)-th of the success rate for EvalF-I agents, where,* $\boldsymbol{T_d}^{\sim}$ *and* $\boldsymbol{n}^{\sim}$ *are the average value of* $\boldsymbol{T_d}$ *and* $\mathbf{n}$ *for EvalF-II agents.*

If $\boldsymbol{n}^{\sim}$ is small, then the success rate of both types will not differ much. In other words, if the agents are benevolent then Theorem 4-1 can be applied for EvalF-II agents. This leads to Theorem 4-2.

Theorem 4-2. *The N-protocol is at least as good as the M-protocol for EvalF-II agents if the agents are benevolent.*

4.1.5 Summary

According to Theorem 4-1 and 4-2, we conclude that the N-protocol is better than the M-protocol in terms of success rate and expected gain for buyers if the agents are benevolent. However, this conclusion can be drawn only if strict assumptions about agents' rationality as utility maximizers and their other behaviors are satisfied. These assumptions may become a serious constraint in a real situation. For example, a designer may use belief revision that violates Assumption 3-8 or 3-9, or use production rules that may not really maximize the expected utility or unintentionally cause nasty behaviors that violate Theorem 4-2.

Indeed, exhaustive analyses are not realistic considering the infinite possible belief functions and revision methods and also infinite possible seller's behaviors. For this reason, we cannot (and do not) prescribe optimal decision or belief revision procedures for the buyers in our analysis. Rather, the protocol design is intended to help agents to gain higher utility in a more general case under various assumptions stated previously. However, to a certain degree, we can test the usefulness of a protocol by alternative analysis through experimentation, as will be explained in the next chapter.

4.2 Dynamic Delay Bargaining Protocol

In the previous section we have shown the benefit of allowing non-monotonic offers in avoiding the cost from the delay generated from establishing a new bargaining session, in which we assume a decreasing valuation of the buyer (see Figure 4-1). In this section we analyze other bargaining situations. Under these situations, we will show the benefit of allowing a buyer to delay their offer without assuming a decreasing valuation over time.

4.2.1 Motivation

One of the advantages of artificial agents is their fast response, resulting in a short negotiation session. For example, a bargaining session between two agents may conclude within seconds or even milliseconds. Generally, when a negotiation needs no complex information processing, efficiency can be gained by increasing the speed of agent responses. However, in some cases, faster resolution may not be the ultimate goal of the negotiation. Consider the following example.

Example 4-2. *Suppose a buying agent is delegated to buy a product within three days. Then a deal made on the first day is no different from a deal on the third day; but finding a lower price and/or better quality item is a priority. If the buyer does not have enough information about the market conditions, such as where to find a good product or* *a cheaper price, then it may act cautiously on the first day, i.e. assigning a relatively low* $\boldsymbol{q_t}$ *and/or* $\boldsymbol{B_t}$*, and/or a relatively high* $\boldsymbol{p_t(x)}$ *and/or* $\boldsymbol{EG'_{t+1}}$*, which causes a very slow concession rate by the buyer. As time goes by, the buyer will act more aggressively, i.e. assigning a higher* $\boldsymbol{q_t}$ *and/or* $\boldsymbol{B_t}$*, and/or a relatively high* $\boldsymbol{p_t(x)}$ *and/or* $\boldsymbol{EG'_{t+1}}$*, which speeds up its concession rate.*

In Example 4-2 above, the ideal strategy of the buyer is to open negotiation with all possible sellers within three days and accept the *global best offer* from the sellers (e.g. the lowest price that those sellers are willing to sell for before the buyer's deadline passes). Unfortunately, this strategy may not work because the seller who offers the global lowest price may leave the negotiation whilst the buyer is still bargaining with other seller(s), or a seller may accept a buyer's counter-offer which is higher than the global best offer. This situation may happen because the buyer does not know whether or not a *local best offer* (i.e. the best offer in a given time interval) is globally best, and it cannot predict what will be the global best offer. Thus, the best strategy of the buyer is to concede slowly and keep its negotiation open with preferred seller(s), who provide local best offers, until it is convinced that that a local best offer is globally best. This option of keeping a negotiation open is known as a *strategic delay*. In general, any attempt to prolong a negotiation by a deliberate slow concession, or by taking no action within an allowed time, or, by submitting a message such as "please wait…" or irrelevant/meaningless messages, is considered as a strategic delay.

People use strategic delay in both complete and incomplete information games [Roth et al., 1988]. Delay is also observed in both finite and infinite horizon games. Game-theoretic work on strategic delay has focused on finite horizon games (bargaining with a deadline), including those with complete yet imperfect information [Dekel, 1990], complete information with inadvertent random delay [Ma and Manove, 1993], complete information with increasing surplus [Larson and Sandholm, 2002], incomplete information of agents' valuation [Cho, 1990; Gu and Kuhn, 1998], stochastic agents' deadline [Sandholm and Vulkan, 1999; Yildiz, 2004], etc. In the infinite horizon game context, most studies concern incomplete information cases with discount rate or fixed cost, such as two-sided uncertain valuations [Cramton, 1992], etc. Nevertheless, most of the work above is from a game-theoretic perspective. Even when an agent decision function is prescribed, it is only valid under very restrictive assumptions and bargaining settings, such as both bargainers are rational with unlimited computational power, with perfect foresight to solve the game, without risk of breakdown from a strategic delay, etc. These assumptions are less realistic in open multi-agent systems. In contrast to game-theoretic work, our goals here are to show the importance of allowing delay in automated negotiation in open systems, to propose an agent's decision structure when using a delaying strategy, and to demonstrate the costs and benefits of delay by naïve (myopic) agents.

4.2.2 Strategic Delay in Automated Negotiation

As in our previous analysis, we assume that agents can depict all negotiated issues into a single real number (e.g. utility value) and are able to make choices and their offers according to this value. However, in some cases, agents may not have exact properties but only estimates, e.g. estimated reservation price or a fuzzy time deadline. Also, we assume that an agent can bilaterally negotiate with multiple opponents simultaneously. Now, we will look at several potential applications that may fit our model.

Example 4-3. *Consider an e-market in which buying and selling agents can bargain over a product. Since both agents represent different users with different valuations, they will not reveal their private information. Besides, identical/similar items may be dynamically available from multiple sellers; thus, a buyer may simultaneously bargain with several sellers and prolong its negotiation with a preferred seller until it is convinced that no more sellers can provide a better offer.*

Example 4-4. *Similar to Example 4-3 except that the price of the negotiated item is decreasing over time, e.g. as in electronics. Suppose a potential buyer is not in a hurry to buy the item. Then, his/her agent should prolong the negotiation until the deadline is approaching because by that time the market price may be lower and, thus, it can ask for a lower price.*

Example 4-5. *Similar to Example 4-3 except that users have a vague valuation toward the item, for example, in a P2P system in which virtual currency is used to buy/sell digital items. When similar items are observed from many sources, users may change their preferences very frequently, especially at the beginning of the negotiation when the users have not made a definite decision. Thus, they may change the valuation fed to their agents and their agents may use strategic delay until the valuation is stable. The first reason for delay is that overpricing may occur if the valuation is reduced after the transaction is made. For example, let the user's initial valuation be $100 and a minute later becomes $80. If within few seconds his/her buying agent has bought the item for $90, then an overpricing occurs. The second reason for delay is that a missed-opportunity may happen if the valuation is raised after the agent leaves a bargaining session due to its initial low valuation.*

Example 4-6. *Consider an e-service in which a client may rent a scarce resource from a server within a specific time, e.g. doing a data analysis after all data have been collected. Since the complexity of the service varies for different cases, the renting price could be bargained and could depend on the service/renting duration. If the client does not have an exact schedule to do the analysis (e.g. the data collection can only be stochastically determined) but it wants to analyze the data soon after they are available, then it may delay the transaction until the data are ready for a shorter renting time (thus, a lower cost).*

Example 4-7. *An argumentation-based negotiation in which agents need time to verify arguments from their opponent. Suppose a seller claims that "My price is very cheap compared to others' prices." For its rebuttal, the buyer may verify the seller's claim by checking any available past-transaction information, which may need a certain time. If the buyer does not have any other counter-argument but does not want to accept the seller's offer immediately, then a strategic delay can be used until the verification is completed.*

We can see from Examples 4-3 to 4-7 that the benefit of delay can be in the form of direct utility (e.g. paying a lower price in Example 4-3), and/or indirect utility (e.g. a more certain market price in Example 4-7). Generally, agents will use strategic delay if

- they have increasing valuation/surplus over time;
- they are averse to ambiguity/vagueness (imprecise probability/outcomes) that is reducing over time;[1] and
- they are willing to wait for a better opportunity.

[1] In decision science, ambiguity aversion has received tremendous attention since the 1960's and has been proven to be a separate human behavior from their risk aversion [Camerer and Weber, 1992].

In conclusion, there are many applications in automated negotiation in which agents may use strategic delay. Intuitively, a delay may increase the utility of agents, but it may deteriorate joint utility. In the next sections we will formally analyze the benefit from delay.

4.2.3 Basic Model

The following assumptions are used in our model.

Assumption 4-4. *(Concurrent bilateral negotiation) A buyer may bargain with a set of sellers {$\boldsymbol{S_1}$, $\boldsymbol{S_2}$, …, $\boldsymbol{S_n}$}, but all bargaining sessions are independent and asynchronous in the sense that the decision by two sellers are independently made (no collusion among sellers).*

Assumption 4-5. *(Persuasive negotiation) An agent may persuade their opponent to accept their offer or to justify their refusal.*

Assumption 4-6. *(Uncertain valuation) The buyer's estimated valuation $\boldsymbol{B_t}$ may increase over time, and the real value $\boldsymbol{B_r}$ may only be known by the buyer after the item is used or received.*

These assumptions are common in human negotiation. For instance, a salesperson may persuade buyers such that they increase their valuation, which may cause the buyers to overpay for the item. Similarly, underestimation may also happen if the buyer finds that the item is more useful than what s/he previously thought. And the buyer may negotiate with several suppliers to seek the item, as shown in our previous examples.

After receiving the seller's offer and argument, the buyer must decide whether to accept the offer or to counter-offer. We assume that the buyer is myopic-1 and also $\boldsymbol{EG'_{t+1}}$ is independent of $\boldsymbol{x}$. Thus, the expected gain from its counter offer is:

$$\boldsymbol{EG_t(x) = (1 - q_t)\,[p_t(x)(B_t - x) + \gamma(1 - p_t(x))EG'_{t+1}]} \qquad (3\text{-}6)$$

To simplify the notation, we may use $\boldsymbol{\theta_t}$ to represent the ratio of future expected value to current expected surplus, or,

$$\theta_t = \frac{\gamma(1 - p_t(x))EG'_{t+1}}{p_t(x)(B_t - x)} \qquad (4\text{-}10)$$

Thus, equation (3-6) becomes

$$\boldsymbol{EG_t(x) = (1 - q_t)\,p_t(x)(B_t - x)(1 + \theta_t)} \qquad (4\text{-}11)$$

If the buyer chooses a delay, then the expected gain after the delay is $\boldsymbol{[EG'_{t+1}]_{delay}}$. In its decision to accept the seller's offer, or to counter offer, or to delay the bargaining, the buyer must choose an optimal offer that maximizes its expected payoff in equation (3-6). If the buyer's optimal offer and the seller's offer are $\boldsymbol{x_t^*}$ and $\boldsymbol{y}$ respectively, then we can formalize the buyer's decision making as follows:

Definition 4-1. *A buyer's evaluation function is:*

$$I_t = \begin{cases} \textit{Delay} & \textit{iff } \gamma\,[EG'_{t+1}]_{delay} > EG_t(x_t^*) \textit{ and } \gamma\,[EG'_{t+1}]_{delay} > EG_t(y) \\ \textit{Accept} & \textit{iff } EG_t(y) \geq EG_t(x_t^*) \textit{ and } EG_t(y) \geq \gamma\,[EG'_{t+1}]_{delay} \\ \textit{Counter offer} & \textit{otherwise} \end{cases} \qquad (4\text{-}12)$$

Intuitively, after a buyer has received a seller's offer and/or arguments $\boldsymbol{\alpha_S}$, it will evaluate $\boldsymbol{\alpha_S}$ and update its belief function $\boldsymbol{p_t(x)}$ and its valuation $\boldsymbol{B_t}$ accordingly. In addition, it can also select its best counter-argument $\boldsymbol{\alpha_B}$ that can influence $\boldsymbol{p_t(x)}$ which may increase $\boldsymbol{EG_t(x_t^*)}$. The selection of those arguments can be realized by evaluating the strength of each argument [Kraus et. al. 1998], the acceptability of the arguments [Parsons et al., 1998], etc.[2] Only after the buyer has estimated $\boldsymbol{EG_t(x_t^*)}$, $\boldsymbol{EG'_{t+1}}$ and

[2] A more complete study of argumentation-based negotiation can be found in Rahwan et al. [2004].

$[EG'_{t+1}]_{delay}$ at time t, will it decide whether to accept the seller's offer or to send a counter-offer x_t* (plus a counter-argument if necessary) or to delay the bargaining. The value of $[EG'_{t+1}]_{delay}$ may be lower than the value of EG'_{t+1}, because a deliberate delay may increase the risk of breakdown (the seller may walkout if the buyer delays the negotiation).

4.2.4 Delay as a Result of an Increasing Surplus

When a buyer's valuation is increasing over time, it is more willing to wait for a later transaction, because a later transaction with the same price generates higher surplus than an earlier one. Example 4-6 falls in this category because a later transaction means less renting cost or higher valuation. Similarly, if the valuation is constant, but the price is decreasing, then the buyer may also wait as shown in Example 4-4. This will only be possible, of course, if the buyer can foresee or surely know that there will be an increasing valuation/surplus in the next bargaining round; otherwise, this fact will not affect the buyer's decision.

Given Definition 4-1 and $\gamma\, [EG'_{t+1}]_{delay} > EG_t(y)$, we can derive the condition of strategic delay by a buyer who is facing an increasing surplus.

Proposition 4-7. *A delay will be used by a buyer if the ratio of its future surplus with respect to the current one, denoted by* β, *satisfies*

$$\beta > \frac{1}{\varsigma\eta\gamma(1+\theta_{t+1}*)[\lambda-(1-q_t)(1-p_t(x_t*))]} \tag{4-13}$$

Where

$$\beta = \frac{B_{t+1} - x_{t+1}*}{B_t - x_t*}, \varsigma = \frac{1-q_{t+1}}{1-q_t},\ \eta = \frac{p_{t+1}(x_{t+1}*)}{p_t(x_t*)}, \text{ and}$$

$$\lambda = \frac{[EG'_{t+1}]_{delay}}{EG'_{t+1}} > (1-q_t)(1-p_t(x_t{*})) \tag{4-14}$$

Here, ζ is the ratio of the estimated future non-breakdown rate to the current rate; η is the ratio of the buyer's estimated future belief that its offer will be accepted to its current belief that its offer will be accepted; and λ is the ratio of the buyer's estimated future expected payoff after delay to that without delay. When β is large (the buyer has much higher future surplus compared to the current surplus), then it is more likely to satisfy the condition in inequality (4-13), or a delay will be used. Similarly, a rise of ζ, η, or λ, will reduce the RHS of inequality (4-13) which promotes a delay.

It follows that buyers must assess ζ, η, β, λ, and $\theta_{t+1}{*}$, which depends on the implementation. For instance, an optimistic buyer may consider $\zeta = 1$ and $\eta = 1$; and a myopic buyer may consider $EG'_{t+2} = 0$ which implies $\theta_{t+1}{*} = 0$. Thus, a delay may be used when $\beta > [\gamma(\lambda - (1-q_t)(1-p_t(x_t{*})))]^{-1}$. A more realistic implementation of myopic buyers is by assumption $EG'_{t+2} = EG'_{t+1}$, resulting in the following corollary.

Corollary 4-1. *If a buyer uses* $EG'_{t+2} = EG'_{t+1}$*, then a delay will be used if*

$$\beta > \frac{1-\gamma(1-q_{t+1})(1-p_{t+1}(x_{t+1}{*}))}{\zeta\eta\gamma[\lambda-(1-q_t)(1-p_t(x_t{*}))]} \tag{4-15}$$

The delay is self-adjusting. For example, a longer delay may cause a higher risk of breakdown or a lower $[EG'_{t+1}]_{delay}$, or $\lambda_{\text{long}} < \lambda_{\text{short}}$. Thus, even if the surplus is constantly increasing (β is constant over time), at one point a delay will not be used, e.g. when $\lambda \to (1-q_t)(1-p_t(x_t{*}))$. Practically, the estimation of λ must include the length of the delay that has been used. And it is not only affected by the change of the risk of breakdown, but also the change of $p_{t+1}(x_{t+1}{*})$ and EG'_{t+2} after a delay, if any. Intuitively,

$p_{t+1}(x_{t+1}*)$ may be slightly different when a delay is used at time t, because it may have been revised after x_t* is rejected. However, this difference is minor compared to the risk of breakdown. Therefore, we may ignore it in the real implementation.

4.2.5 Delay as a Result of Ambiguity

Example 4-5 is a case when a buyer is averse to ambiguity, because overpricing and missed-opportunity may occur due to uncertain valuation. Generally speaking, buyers may use strategic delay when they are averse toward ambiguity (either imprecise probability or outcome) that is reducing over time. The ambiguity may be in the form of the vagueness of the buyer's valuation, belief, and other bargaining factors.

One of the most influential theories in explaining ambiguity aversion in human behavior is Kahneman & Tversky's prospect theory [1979]. Under this theory, the weight of a precise prospect is greater than the weight of an imprecise prospect. For example, if there is an imprecision factor φ that influences the judgment of $p_t(x_t*)$, then the weight of current and future payoffs, i.e. $(p_t(x_t*)$ and $(1 - p_t(x_t*))$, will be mapped to $\psi(p_t(x_t*), \varphi)$ and $\psi(1 - p_t(x_t*) , \varphi)$, respectively; thus

$$EG_t(x_t*) = (1–q_t)[\ \psi(p_t(x_t*), \varphi)(B_t – x_t*) + \gamma\ \psi(1–p_t(x_t*), \varphi)\ EG'_{t+1}] \quad (4\text{-}16)$$

where the mapping satisfies $\psi(p_t(x_t*), \varphi) + \psi(1 - p_t(x_t*), \varphi) \leq 1$ (subcertainty property). This property explains why people prefer a precise prospect φ_1 rather than an imprecise one φ_2, because we can have both $\psi(p_t(x_t*), \varphi_1) > \psi(p_t(x_t*), \varphi_2)$ and $\psi(1 - p_t(x_t*), \varphi_1) > \psi(1 - p_t(x_t*), \varphi_2)$, which implies $[EG_t]_{\varphi 1} > [EG_t]_{\varphi 2}$. Since imprecision reduces over time, we could have $\gamma\,\lambda\ [EG'_{t+1}]_{\varphi 1} > [EG_t]_{\varphi 2}$, which causes a delay from the buyer.

Proposition 4-8. *A delay will be used by a buyer if the ratio of the weight of its future precise belief with respect to the current imprecise belief satisfies*

$$\frac{\psi(p_{t+1}(x_{t+1}*),\varphi_1)}{\psi(p_t(x_t*),\varphi_2)} > \frac{1}{\zeta\beta\gamma(1+\theta_{t+1}*)[\lambda-(1-q_t)\psi(1-p_t(x_t*),\varphi_2)]} \qquad (4\text{-}17)$$

$$Where\ \theta_{t+1}* = \frac{\gamma\psi(1-p_{t+1}(x_{t+1}*),\varphi_1)EG'_{t+2}}{\psi(p_{t+1}(x_{t+1}*),\varphi_1)(B_{t+1}-x_{t+1}*)} \qquad (4\text{-}18)$$

From this proposition, it is possible that $\boldsymbol{p_{t+1}(x_{t+1}*) < p_t(x_t*)}$ but $\boldsymbol{\psi(p_{t+1}(x_{t+1}*), \varphi_1)} > \boldsymbol{\psi(p_t(x_t*), \varphi_2)}$, because the influence of the increasing of precision may be stronger than that of the decreasing of belief. Figure 4-2 illustrates the effect, in which the imprecision causes $\boldsymbol{\psi(p_t(x_t*), \varphi_2) < p_t(x_t*)}$.

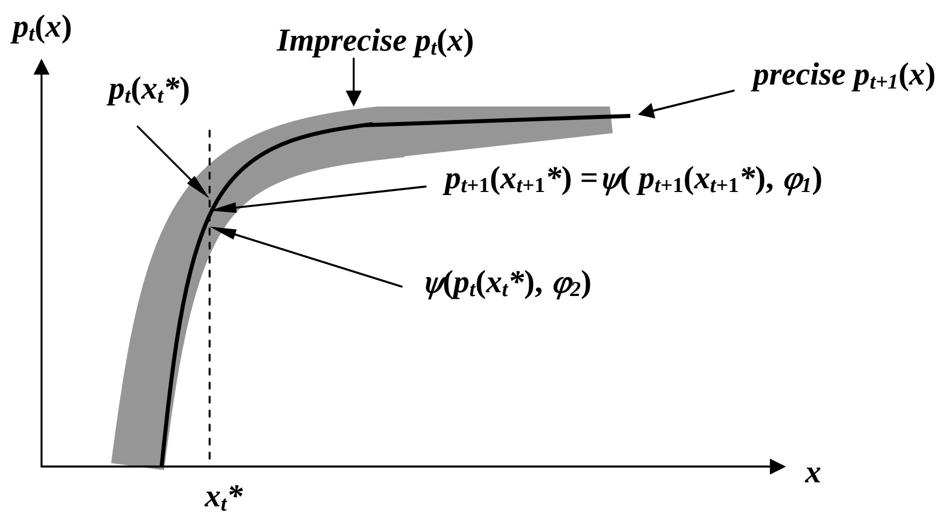


Figure 4-2 Imprecise belief versus precise belief functions

Similarly, most people also prefer a precise outcome over an imprecise outcome [Camerer and Weber, 1992]. Suppose the buyer is uncertain of its valuation at time $\boldsymbol{t}$, but can estimate it within a range, or $\boldsymbol{B^e_t} \in [\boldsymbol{B_L}, \boldsymbol{B_H}]$. Since the ambiguity reduces over time (after more information is collected), the buyer expects that the range is converging over time, or

$$(\boldsymbol{B_H} - \boldsymbol{B_L})_{t+n} < \ldots < (\boldsymbol{B_H} - \boldsymbol{B_L})_{t+1} < (\boldsymbol{B_H} - \boldsymbol{B_L})_{t} \qquad (4\text{-}19)$$

According to common observation on human ambiguity aversion, the buyer will prefer a narrower range over a wider one even if the wider range has a higher mean value [Camerer and Weber, 1992]. Thus, the buyer's estimated valuation satisfies

$$\boldsymbol{B^e}_{t+n}(\varphi_1) > \boldsymbol{B^e}_{t+n-1}(\varphi_2) > \ldots > \boldsymbol{B^e}_{t+1}(\varphi_{n-2}) > \boldsymbol{B^e}_{t}(\varphi_{n-1}). \qquad (4\text{-}20)$$

Given this, Proposition 4-7 can be applied by replacing β.

Proposition 4-9. *A delay will be used by a buyer if the ratio of its estimated future surplus with respect to the current estimated surplus satisfies*

$$\beta = \frac{B_{t+1}^{e}(\varphi_1) - x_{t+1}{*}}{B_t^e(\varphi_2) - x_t{*}} > \frac{1}{\varsigma\eta\gamma(1+\theta_{t+1}{*})[\lambda - (1-q_t)(1-p_t(x_t{*}))]} \qquad (4\text{-}21)$$

Using similar analysis, we can analyze the strategic delay from imprecise risk of breakdown. If the imprecision of the risk of breakdown reduces such that the risk of breakdown is estimated to be decreasing, then a delay may be used under similar conditions.

Proposition 4-10. *A delay will be used by a buyer if the ratio of its estimated future probability of breakdown with respect to the current estimate satisfies*

$$\varsigma = \frac{1 - q_{t+1}^{e}(\varphi_1)}{1 - q_t^e(\varphi_2)} > \frac{1}{\eta\beta\gamma(1+\theta_{t+1}{*})[\lambda - (1-q_t^e(\varphi_2))(1-p_t(x_t{*}))]} \qquad (4\text{-}22)$$

The effects of ambiguity may be more than just those described above. However, we cannot analyze imprecision that arises from all possible bargaining factors, but only provide the general idea of a delay from ambiguity.

4.2.6 Delay as a Result of Verifying Arguments

When the seller tries to persuade the buyer who does not believe the seller's argument, then the buyer needs some time to verify it, as shown in Example 4-7. Since the buyer is rational, it must have an incentive for verifying the argument, such as reducing the ambiguity and/or finding a good reason for a rebuttal. Generally speaking, verification is useful if the buyer's expected utility after verifying the seller's argument to be false exceeds its expected utility without verifying the argument, which is helpful for its rebuttal. The following proposition describes this.

Proposition 4-11. *Let the true value of the seller's argument (*$\boldsymbol{\alpha_S}$*) induce an increasing of the buyer's valuation from* $\mathbf{B_{t-1}}$ *to* $\mathbf{B_t}$ *but reduce* $\mathbf{p_{t-1}(x)}$ *to* $\mathbf{p_t(x)}$*, while the negation of it (*$\neg\boldsymbol{\alpha_S}$*) does not change the buyer's valuation (*$\mathbf{B_{t+1}} = \mathbf{B_t} = \mathbf{B_{t-1}}$*) but increases* $\mathbf{p_{t-1}(x)}$ *to* $\mathbf{p_{t+1}(x)}$*. Then, the seller's argument will be verified, which causes a delay, if*

$$\beta\eta > \frac{1}{\varsigma\gamma(1+\theta_{t+1}^*)[\lambda^{\#} - (1-q_t)(1-p_t(x_t^*))]} \qquad (4\text{-}23)$$

where $\beta = \dfrac{B_{t+1} - x_{t+1}^*}{B_t - x_t^*}$, $\eta = \dfrac{p_{t+1}(x_{t+1}^*)}{p_t(x_t^*)}$, $\varsigma = \dfrac{1-q_{t+1}}{1-q_t}$, *and*

$\lambda^{\#} = \chi \dfrac{[EG'_{t+1}]_{verify-false}}{EG'_{t+1}} > (1-q_t)(1-p_t(x_t^*))$ *and* χ *is the likelihood that the verification results in the negation of the seller's argument.*

The value χ can be obtained by calculating the trust toward the seller. If the seller is trustworthy, χ will be low and the buyer may not verify the seller's argument. The condition above is similar to that in Proposition 4-7 except that the delay comes from verifying the argument after considering the benefit of the negation of the seller's argument. A seller's argument "This car is in a very good condition" is an example of Proposition 4-11 above. If the buyer accepts it to be true, then it may raise the buyer's valuation from $\boldsymbol{B_{t-1}}$ to $\boldsymbol{B_t}$ and reduce the buyer's belief from $\boldsymbol{p_{t-1}(x)}$ to $\boldsymbol{p_t(x)}$. If the argument is verified to be false, then it justifies the buyer's rebuttal and raises its bargaining power; or the buyer will have a reason to persuade the seller to reduce its valuation which in turn will increase the buyer's belief from $\boldsymbol{p_{t-1}(x)}$ to $\boldsymbol{p_{t+1}(x)}$. Another example of Proposition 4-11 is the argument "Your offer is not acceptable, because it is lower than market price." If the buyer accepts it, then it reduces the buyer's estimated belief such that $\boldsymbol{p_t(x_t*)} < \boldsymbol{p_{t-1}(x_t*)}$ but $\boldsymbol{B_t} = \boldsymbol{B_{t-1}}$. If it is proven to be false, i.e. the buyer's offer is acceptable or higher than market price, then the seller should accept the buyer's offer, or $\boldsymbol{p_{t+1}(x_t*)} > \boldsymbol{p_t(x_t*)}$.

Intuitively, if the negation is useless (e.g. $\boldsymbol{p_{t+1}(x)}$ is slightly higher than $\boldsymbol{p_t(x)}$) or very unlikely be proven (e.g. the seller has supporting evidence such that $\chi \to 0$), then the buyer will not verify the seller's argument. Also, if the buyer trusts the seller ($\chi \to 0$), then the buyer will not verify the seller's argument. However, this may not be true in some situations. For example, the previous argument "This car is in a *very good* condition" is sometimes interpreted as a vague argument, thus needing further verification before the buyer can estimate $\boldsymbol{B^e_t}$. Or, a delay will still be used even if the buyer trusts the seller. Thus, verification is also useful when verifying the argument will generate higher expected utility than would happen without verifying it, the purpose of which is mainly for reducing imprecision/ambiguity, e.g. estimating $\boldsymbol{B^e_t}$. The analysis, then, is similar to that in section 4.2.5.

4.2.7 Delay as a Result of Waiting for Better Opportunities

When a buyer can negotiate with more than one seller concurrently, then it will have incentive to wait for better opportunities as in Example 4-3. Sim and Wang [2004] have recognized and proposed the use of delay (slow concession) in this kind of situation. They suggest setting the concession speed according to the probability of finding a better offer from other (potential) sellers, which is a function of the number of sellers in the market, e.g. a slow concession will be used if more sellers join the market. In contrast to their probabilistic approach, our approach relies more on the calculation in Definition 4-1.

Suppose an offer by a seller $\boldsymbol{S_k}$ satisfies $\boldsymbol{EG_t(y) \geq EG_t(x_t^*)}$. Then we must check whether or not $\boldsymbol{EG_t(y) \geq \gamma\, [EG'_{t+1}]_{delay}}$ from all possible sellers, i.e. those in current and future encounters. If $\boldsymbol{EG_t(y) < \gamma\, [EG'_{t+1}]_{delay}}$, then a delay will be used. Certainly, when a larger number of sellers are involved in the negotiation, the likelihood of $\boldsymbol{EG_t(y) < \gamma\, [EG'_{t+1}]_{delay}}$ is higher, which may support Sim and Wang's approach. The calculation of $\boldsymbol{[EG'_{t+1}]_{delay}}$ from a seller which is already bargaining is not difficult, but estimating this for an incoming seller is more complicated. A probabilistic approach, such as that used by Sim and Wang, could be used for the estimation. Alternatively, statistical confidence can be built if there are enough bargaining sellers for whom it is estimated a delay would be useful.

Proposition 4-12. *Let an offer by a seller* $\boldsymbol{k}$ *satisfy* $\boldsymbol{EG_t(y)_k \geq EG_t(x_t^*)}$. *A delay will be used by the buyer if any of the following conditions applies:*

(i) $\boldsymbol{EG_t(y)_k < \gamma\, [EG'_{t+1}]_{delay}}$

(ii) there exists a seller $\boldsymbol{j \neq k}$ *s.t.* $\boldsymbol{EG_t(y)_k < \gamma\, [EG'_{t+1}]_j}$

(iii) there exists in the future a seller $\boldsymbol{v \neq k}$ *s.t.* $\boldsymbol{EG_t(y)_k < \gamma [EG'_{t+1}]_v}$

When the buyer's bargaining space is narrow (e.g. it can only increase its offer a few times), then the buyer may offer the same price repeatedly, which may result in a bargaining breakdown. The Boulware strategy is an example that uses repetitive same offers at the beginning of bargaining [Faratin et al., 1998]. This strategy has been known to cause a high breakdown rate [Raiffa, 1976].

Suppose the buyer is expecting an increase of its valuation, e.g. an adjustment by its owner, or a decrease of the seller's valuation, e.g. the negotiation approaches the seller's time deadline. Then the buyer will delay its counter-offer, which can help it to avoid a breakdown. In this case $\boldsymbol{\gamma [EG'_{t+1}]_{delay} > EG_t(x_t^*) \geq EG_t(y)}$ where $\boldsymbol{EG_t(x_t^*)} \to 0$ due to a high possibility of breakdown, and $\boldsymbol{EG_t(y) = B_t - y_t} < 0$ (negative surplus). Thus, the condition of delay is similar to that in formula (4-13) in proposition 4-7, except that we have $\boldsymbol{\zeta}$ instead of $\boldsymbol{\beta}$ on the LHS.

4.2.8 Summary

Proposition 4-7 to 4-12 have demonstrated the benefit of strategic delay for the buyer. Since the analysis is symmetric for the seller, we may expect that under similar circumstances the seller will also use strategic delay. From Proposition 4-7 to 4-12 we can derive Theorem 4-3:

Theorem 4-3. *(The existence of strategic delay) A myopic-1 agent may benefit from strategic delay.*

In existing MAS platforms, a long waiting time for a response may be considered as a network congestion or a connection failure, thus possibly resulting in a breakdown which is usually enforced by the platform manager. To overcome this, we may add a waiting signal in the agent communication language (ACL) such that a communication will still be maintained during the waiting period. Thus, all agents must have common understanding of the waiting/delaying signal sent by their opponent. We denote this negotiation platform as a delay-enabled negotiation protocol, and the following theorem is applied.

Theorem 4-4. *If the primary goal of agents is to maximize their expected utility, then myopic-1 agents prefer a delay-enabled bargaining protocol.*

Proof. See Appendix

4.3 Strategic Ignorance in Argumentation-based Negotiation

4.3.1 Motivation

To date, argument-based negotiation (ABN) among artificial agents has of necessity been less complicated than among humans, because humans have the creativity to use arguments which are beyond/broader than the context of the negotiated issues, and to seek/request the exact meaning of their opponent's argument when it is ambiguous; none of which can be done by artificial agents due to limitations in their ontology, knowledge and reasoning.

When the negotiated issues or protocol are complicated such as in an ABN, the cost incurred from the negotiation process, in terms of time and computational complexity, cannot be ignored. Intuitively, this cost increases with an increasing number of possible arguments in the ABN. And it will also increase if agents do not trust each other, because it will take some time to assess the validity of each argument, as described in section 4.2.6. Even worse, if the ABN protocol allows agents to use their own ontology in generating/interpreting an argument, then those agents may need more time to respond and/or understand the meaning of their opponent's argument. Excessive costs occurring from these factors may diminish the agent's gains from using an ABN, thus discouraging the agent from using argumentation, especially when there are alternative solutions such as negotiating with other sellers/buyers who will not use argumentation [Karunatillake and Jennings, 2005].

In this section, we assume that the context of the negotiation is the same, i.e. the purchasing of a product or service with its price as the primary issue, but based on ABN protocol. Our focus here is on an ABN between a buyer and a seller agent who do not necessarily trust each other, and both agents may have different ontologies, knowledge and/or reasoning engines. Note that these cannot be completely different, because for the negotiation to work properly there needs to be at least some common ground such as the description of the service/item and the unit price. Here, argumentation could be used either to persuade an opponent to accept an offer or to change its stance (e.g. trust) toward the proponent, as shown in Figure 4-3.

Realizing that an agent's argument may not be understood by its opponent, agents should agree at least to inform their opponent that they do not understand the argument being received or they are unable to reply to the argument. This is important because misunderstanding or ignorance may cause an indeterminate process/stall in the negotiation. However, allowing agents to do so means allowing them to ignore their opponent's arguments when they do not want to respond to them. Thus, a rational agent may use ignorance as a strategy to avoid a dialog which may lead it into a weaker position (e.g. lower expected payoff) during the negotiation. In addition, an agent may also deliberately choose not to use specific arguments during negotiation. For example, if a property agent (seller) knows that a buyer drives a car, then persuading the buyer by argument "this apartment is less than a minute walking distance from the bus station" will be useless or even cause a negative impact because the buyer may not like a crowded area. We call this agent's strategy of avoiding argumentation, either by pretending that it cannot understand the argument or avoiding the use of specific arguments, as *strategic ignorance*.

Negotiation over a used car in natural language

Buyer: *How much does it cost?*
Seller: *$5000, plus a new tire and a stereo.*
Buyer: *How about $4000?*
Seller: *$5000 is very cheap. It only has 120,000 Km. It's rarely been used.*
Buyer: *Yeah… But it's manual. I can have an automatic one for $5000.*
Seller: *Believe me, you can't find a better one for this price.*
.….

Negotiation in agent language

Buyer: ask(price)
Seller: offer(price(this, 5000) $\wedge$ add(tire, 1) $\wedge$ add(stereo, 1))
Buyer: offer(price(this, 4000))
Seller: inform(quality(kms(this, 120000) $\wedge$ usage(this, rare)) $\rightarrow$ offer_price(5000, cheap))
Buyer: inform(quality(transmission(this, manual)) $\rightarrow \neg$ offer_price(5000, cheap))
Seller: inform($\neg\exists$ X (price(X, 5000) $\wedge$ better_quality(X, this)))
.….

Figure 4-3 An illustrative example of ABN in natural and agent language

To the best of our knowledge, this strategy has not been broadly explored in the automated negotiation literature, especially in the context of ABN. Perhaps the work that comes closest to exploring strategic ignorance is by Karunatillake and Jennings [2005] who have shown the benefit of avoiding argumentation (evading conflict) when the ABN is costly and there are many non-arguing options, such as buying a similar product from other sellers (or what they called ‘resources’ in their paper). Using empirical simulation, they found that when the availability of ‘resources’ are high, then arguing is less favorable, because the cost from arguing may be higher than the cost of seeking other ‘resources’. Our work is similar to theirs in that argumentation may incur some costs, but different in the sense that we do not assume the existence of any other non-arguing ‘resources’. So, our agent can avoid argumentation by sending a statement “refusal to argue” for specific issues, such as “I do not understand your argument” or “No comments” without breaking the negotiation. For this supposition to work, we assume that all agents have universal understanding toward this statement and will avoid reiterating similar argumentation upon receiving it.

Finally, it is worth mentioning that ignorance in argumentation is not the same as argument from ignorance (*argumentum ad ignorantiam*) [Walton, 1995]. The latter concept is used to describe arguments without solid knowledge/evidence that are subject to rebuttal, an issue commonly studied in the area of non-monotonic logic and reasoning.

4.3.2 Basic Model

Suppose that a buyer ***B*** wants to buy an item from a seller ***S***, where other buyers/sellers may be available but each negotiation session is exclusively between a buyer and a seller only (concurrent bilateral negotiation in Assumption 4-4). In our ABN protocol, either party opens the negotiation by sending a message (an offer and/or an argument) to the other party, who is obliged to reply by counter-offer and/or counter-argument. As usual, our approach here presumes an uncertain environment, in which deliberation and meta-reasoning is needed by the agent in maximizing their expected utility. Moreover, arguments are used by both parties to persuade their opponent to accept their offer or to justify their refusal of their opponent's proposal (Assumption 4-5).

Since both parties represent different owners, they may have different (or identical) ontologies, knowledge, and reasoning engines for generating and understanding the arguments. Both parties know that their opponent may not necessarily understand all their arguments, and they can inform them if any argument is not understood. Suppose also that the buyer does not necessarily know the real value of the negotiated item (this assumption is slightly different from Assumption 4-6).

Assumption 4-7. *(Uncertain valuation) The buyer's estimated valuation* $\boldsymbol{B_t}$ *is not fixed over time, and the real value* $\boldsymbol{B_r}$ *may only be known by the buyer after the item is used or received.*

Thus, the buyer must assess/predict the risk of believing in the salesperson's arguments; and also assign a belief value as to the likelihood that the seller may believe the buyer's counter-argument, which leads us to the next assumption.

Assumption 4-8. *Suppose a seller uses argument* $\boldsymbol{\alpha_S}$ *at time* $\boldsymbol{t}$*. Then the buyer's belief over* $\boldsymbol{\alpha_S}$ *at time* $\boldsymbol{t}$*, denoted by* $\boldsymbol{\upsilon_t(\alpha_S)}$*, depends on the seller's reputation and other information, such as the truth of the seller's prior arguments and the truth value of* $\boldsymbol{\alpha_S}$ *itself. Similarly, the buyer's belief of its own argument* $\boldsymbol{\alpha_B}$ *being accepted by the seller at time* $\boldsymbol{t}$*, denoted by* $\boldsymbol{\upsilon_t(\alpha_B)}$*, depends on the buyer's reputation and other information that reflects the seller's belief toward the buyer and the truth value of* $\boldsymbol{\alpha_B}$ *itself.*

In practice, agents may not have autonomy to change their valuation, because it is determined exogenously. For instance, the owner may set the reservation price using a time-dependent parameter, e.g. as in Kasbah [Chavez and Maes, 1996]. However, as autonomy increases, agents may set the valuation endogenously, for instance by monitoring the fluctuation of the market price. We thus assume that the valuation is dynamic and agents may change their valuation according to the environment (including the influence of the seller's arguments). To keep our model simple, we also assume that the buyer is risk neutral and is myopic-0 with $\boldsymbol{q_t} = 0$; therefore the expected payoff of the buyer is:

Definition 4-2. *(i)* $\boldsymbol{EG_t(x) \equiv p_t(x)\,(B_t - x)}$ (4-24)

(ii) $\boldsymbol{EG_t(y) \equiv B_t - y}$ (4-25)

Usually, after the buyer receives a seller's offer and/or arguments $\boldsymbol{\alpha_S}$, it will evaluate $\boldsymbol{\alpha_S}$ and may update $\boldsymbol{B_t}$ and $\boldsymbol{p_t(x)}$ accordingly. The updating is necessary in order to avoid stalling the negotiation and to allow both agents to deliberate and learn to maximize their utility. The updating mechanism may use the heuristics stated in the following assumption.

Assumption 4-9. *Upon receiving a seller's argument and/or offer* $\boldsymbol{y}$*, a buyer may update its valuation and belief according to the following heuristics:*

(a) $\mathbf{p_t(x)}$ *will be reduced for all prices less than the buyer's last offer (inclusive) if the seller convinces the buyer that the seller will not accept the buyer's last offer, either by insisting on its previous offer or using arguments.*

(b) $\mathbf{p_t(x)}$ *will be raised for prices close to the seller's new offer if the seller decreases its previous offer.*

(c) In both heuristic (a) and (b) above, the updating of $\mathbf{p_t(x)}$ *will not affect its monotonic property.*

(d) $\boldsymbol{B_t}$ *will be increased if new credible information has been received, either directly from the seller or from other sources.*

We assume EvalF-I agents use a slightly different evaluation function as follows:

Definition 4-3. *A buyer's evaluation function is:*

$$\boldsymbol{I_t} = \begin{cases} \textit{Withdraw} & \textit{iff } \boldsymbol{t} > \boldsymbol{T_d} \textit{ or } \boldsymbol{maxEG_t(x)} \leq 0 \\ \textit{Accept} & \textit{iff } \boldsymbol{EG_t(y)} \geq \boldsymbol{maxEG_t(x)} \textit{ and } \boldsymbol{t} \leq \boldsymbol{T_d} \\ \textit{Counter offer and/or argument} & \textit{otherwise} \end{cases} \tag{4-26}$$

Only after the buyer has estimated $\boldsymbol{maxEG_t(x)}$, will it compare it to $\boldsymbol{EG_t(y)}$ and decide whether to accept the seller's offer or to send a counter-offer (plus a counter-argument if necessary). If a counter-offer must be sent, then it will counter offer $\boldsymbol{x_t^*}$ which generates the maximum expected payoff. Depending on the bargaining protocol, the counter-offer $\boldsymbol{x_t^*}$ could be monotonically increasing or not. To maintain the generality of our analysis, we do not assume the monotonicity property of the offers.

Assumption 4-10. *A buyer will always choose the best counter-argument* $\boldsymbol{\alpha_B}$ *that can affect the value of* $\boldsymbol{p_t(x)}$ *which may increase* $\boldsymbol{EG_t(x)}$.

The mechanism of choosing the best arguments could be based on the buyer's expectation of those arguments being accepted, $\boldsymbol{\upsilon_t(\alpha_B)}$, and the influences (strength) of those arguments in changing the seller's internal states [Rahwan et al., 2004]. In our model, we assume that the buyer can always generate the best argument which may cost it some time; thus leaving our analysis as a choice between using counter-argument and not using it.

The purpose of our model is to analyze general bounded-rational agents in ABN and thus it does not explicitly restrict the reasoning mechanism used by agents. As long as the agent behavior follows or approximates our assumptions, then our analysis in the next two sections can be applied. Indeed, some of our analyses have extended our assumptions into more specific cases, such as expected utility for future outcome or time delay in processing arguments. Nevertheless, we will demonstrate the existence of strategic ignorance in our model.

4.3.3 Analysis I: Proactive Ignorance

As stated before, to maximize its expected payoff the buyer may decide to offer $\boldsymbol{x_t}$***** which maximizes $\boldsymbol{EG_t(x)}$, and/or to persuade the seller using argument $\boldsymbol{\alpha_B}$ that increases $\boldsymbol{p_t(x)}$ for some $\boldsymbol{x}$ (including $\boldsymbol{x_t}$*****). Intuitively, if the buyer believes that the expected marginal benefit of sending argument $\boldsymbol{\alpha_B}$ exceeds the cost incurred from the argumentation, then it is worth doing. Suppose that the buyer believes that with a positive probability $\boldsymbol{\upsilon_t(\alpha_B)}$ the seller will *instantly* accept argument $\boldsymbol{\alpha_B}$ resulting in the increase of $\boldsymbol{p_t(x)}$ to $\boldsymbol{p^{\alpha}_t(x) = p_t(x|\alpha_B)}$, a situation denoted by $\boldsymbol{\upsilon_t(\alpha_B) = P(\ p_t(x) \uparrow p^{\alpha}_t(x) \mid \alpha_B\ ) > 0}$, where "↑" represents an "increase to" binary relation. Then, we could easily show that it is always worthwhile for the buyer to argue.

Proposition 4-13. *If the buyer believes that the probability* $\boldsymbol{\upsilon_t(\alpha_B) = P(\ p_t(x) \uparrow p^{\alpha}_t(x) \mid \alpha_B\ )}$ *is positive, then* $\boldsymbol{\alpha_B}$ *will always be sent.*

Proof. First, we can re-write equation (4-24) such that by probability $\boldsymbol{\upsilon_t(\alpha_B)}$ the expected payoff is $\boldsymbol{p^{\alpha}_t(x)\ (B_t - x)}$ and by probability $\boldsymbol{(1 - \upsilon_t(\alpha_B))}$ the expected payoff is $\boldsymbol{p_t(x)\ (B_t - x)}$, or

$$\boldsymbol{EG^{\alpha}_t(x) = \upsilon_t(\alpha_B)\, p^{\alpha}_t(x)\, (B_t - x) + (1 - \upsilon_t(\alpha_B))\, p_t(x)\, (B_t - x)} \qquad (4\text{-}27)$$

Since $\boldsymbol{p^{\alpha}_t(x) > p_t(x)}$, then we have

$$\upsilon_t(\alpha_B) p^{\alpha}_t(x)\,(B_t - x) + (1 - \upsilon_t(\alpha_B))\, p_t(x)\,(B_t - x) >$$

$$\upsilon_t(\alpha_B) p_t(x)(B_t - x) + (1 - \upsilon_t(\alpha_B)) p_t(x)(B_t - x) \qquad (4\text{-}28)$$

But the RHS of inequality (4-28) equals to $p_t(x)(B_t - x) = EG_t(x)$; or, $EG^{\alpha}_t(x) > EG_t(x)$ for any $\upsilon_t(\alpha_B) \in (0, 1]$. Thus, α_B will always be sent. ■

In Proposition 4-13, we assume that the cost of argumentation is zero or neglected by the buyer, or the argument will be instantly accepted or not at all. However, this is not true in many situations, because it may take some time to create a convincing argument and then to persuade an opponent to accept an argument (due to rebuttal, verification of the argument, building trust, etc.). In such a situation, we need to assign a cost to the negotiation delay and analyze the buyer's meta-reasoning about whether or not to avoid the argumentation.

4.3.3.1 Ignorance due to costly negotiation time

Consider that it will take some time for the seller to process or be convinced by the buyer's argument so that an argument α_B may not be instantly accepted by the seller, but instead be subject to rebuttal for some finite rounds of interaction. Then we can modify equation (4-27) by distributing the probability $\upsilon_t(\alpha_B)$ to several bargaining rounds from t to $t+n$, where n represents the processing time until the time when the buyer believes that it has failed to convince the seller using argument α_B. For the sake of simplicity, assume the estimation of the belief function $p^{\alpha}_t(x)$ is the same over time, or $p^{\alpha}_t(x) = p^{\alpha}_{t+1}(x) = p^{\alpha}_{t+2}(x) = \ldots = p^{\alpha}_{t+n}(x)$. Suppose also that if the buyer can estimate the time n, then its expected payoff can be expressed in a time series as follows:

$$EG^{\alpha}_t(x) = \upsilon_t(\alpha_B)\, p^{\alpha}_t(x)\,(B_t - x) + \upsilon_{t+1}(\alpha_B)\, p^{\alpha}_t(x)\,(B_{t+1} - x) +$$

$$\upsilon_{t+2}(\alpha_B)\, p^{\alpha}_t(x)\, (B_{t+2} - x) + \quad \ldots + \upsilon_{t+n}(\alpha_B)\, p^{\alpha}_t(x)\, (B_{t+n} - x) +$$

$$(1 - \Sigma_{i=0}^{n} \upsilon_{t+i}(\alpha_B))\, p_t(x)\, (B_{t+n} - x) \qquad (4\text{-}29)$$

where $\upsilon_t(\alpha_B)$ represents the probability that argument α_B will be instantly accepted by the seller, and $\upsilon_{t+1}(\alpha_B)$ represents the probability that argument α_B will be accepted by the seller at time $t+1$ after it is not accepted at time t, and $\upsilon_{t+2}(\alpha_B)$ represents the probability that argument α_B will be accepted by the seller at time $t+2$ after it is not accepted at time t and $t+1$, and so on.

The distribution of $\upsilon_t(\alpha_B)$ … $\upsilon_{t+n}(\alpha_B)$ depends on the buyer's beliefs about the processing time needed by the seller and the estimated length of the argumentation; thus, it depends on the complexity of the arguments, the possibility of attack/rebuttal, and the trust between them. Practically, those values can be set externally by the owner or adjusted internally by the agent after several encounters using a specific learning mechanism.

Example 4-8. *Suppose* $\mathbf{n} = 2$ *and* $\upsilon_t(\alpha_B) = \upsilon_{t+1}(\alpha_B) = 0$ *and* $\upsilon_{t+2}(\alpha_B) = 1$*, which means it will take at least two rounds of argument before the buyer can convince the seller that the seller will change* $p_t(x_t)$ *to* $p^{\alpha}_t(x_t)$*. Let* $p_t(x_t) = 0.7$*,* $p^{\alpha}_t(x_t) = 1.0$*,* $x_t = \$100$*,* $B_t = \$150$*, and* $B_{t+2} = \$120$*. If we plug these values into equation (4-29) and (4-24) then* $EG^{\alpha}_t(x_t) = \$20$ *and* $EG_t(x_t) = \$35$*, that is* $EG^{\alpha}_t(x_t) < EG_t(x_t)$*. Thus, the buyer will not use argument* α_B*.*

From Example 4-8, we can derive Proposition 4-14 about the existence of a non-arguing strategy.

Proposition 4-14. *(Strategic ignorance due to costly negotiation time) If there is a cost incurred from sending argument* $\boldsymbol{\alpha_B}$ *as shown in equation (4-29), and the buyer's valuation is decreasing over time, then* $\boldsymbol{\alpha_B}$ *may not always be sent.*

If we compare equations (4-27) and (4-29), the cost incurred from sending $\boldsymbol{\alpha_B}$ in equation (4-29) is mainly from the decreasing of the valuation $\boldsymbol{B_t}$. This situation is similar to that used by Karunatillake and Jennings [2005] in their analysis. The difference is that their agents use overall argumentation cost which may lead to a withdrawal, while our agents consider a subset of argument topics which may lead to ignorance. However, is there any situation when a buyer with a non-decreasing valuation will also avoid argumentation? The answer is "yes", which will be explained in the following two sub-sections, section 4.3.3.2 and 4.3.3.3.

4.3.3.2 Ignorance due to recoiling arguments

If the buyer is aware of the negative impact of the argumentation, i.e. the possibility of $\boldsymbol{p^{\alpha}_t(x_t*) < p_t(x_t*)}$, then using argumentation may incur unpredicted costs to the buyer. This could happen, for example, if the buyer unintentionally reveals some information after sending $\boldsymbol{\alpha_B}$ or ignites an assessment that makes the seller refuse $\boldsymbol{x_t*}$ rather than accepting it. For instance, sending an argument "$\boldsymbol{x_t*}$ is a fair market price" will prompt the seller to verify it, which may lead to a reverse conclusion by the seller such as "$\boldsymbol{x_t*}$ is not a fair market price" or "reject $\boldsymbol{x_t*}$ with certainty". Facing the possibility of its arguments recoiling to its disadvantage, the buyer should first assess the likelihood of this happening. Certainly, a deliberation is needed here and depending on the level of consideration we may have various types of buyer, exemplarily a sophisticated buyer versus a naïve one.

Suppose that the negotiation time is not as costly as in section 4.3.3.1, so we can neglect deliberation cost in our analysis here. Suppose also that the buyer is unsure that $p^{\alpha}_{t}(x) > p_{t}(x)$ for x_t* after sending α_B (ambiguity), but it can assign a probability density function to $p^{\alpha}_{t}(x_t*)$, denoted by $f(p^{\alpha}_{t}(x_t*))$ with mean value $\mu_{f\,(p(x|\alpha))}$ or μ_f for short. For the sake of simplicity, let agents be neutral toward both risk and uncertainty (ambiguity), such that their decision depends on the mean value μ_f only. Given the above assumptions, then we can derive the following proposition.

Proposition 4-15. *(Strategic ignorance due to recoiling arguments) Let the buyer be neutral to uncertainty and risk. If* $\mu_f < p_t(x_t*)$ *and* $\upsilon_t(\alpha_B)$ *is strictly positive, then* α_B *will not be sent.*

Intuitively, Proposition 4-15 states that a buyer will avoid an argument if the buyer believes that the argument will reduce the expected probability of the seller's acceptance of its offer. Relaxing the assumption of neutrality, Proposition 4-15 can be extended to an uncertainty-averse buyer. If the buyer is uncertainty-averse, then it may still avoid argumentation despite μ_f being slightly higher than $p_t(x_t*)$. In other words, an uncertainty averse buyer will still avoid argumentation even if it may slightly increase the *expected* probability of its offer being accepted.

Another similar situation that may cause ignorance is $p^{\alpha}_{t}(x) = p_t(x)$, i.e. when the argument is not valuable or does not have persuasive power. In this case, a neutral buyer is indifferent between sending α_B and not sending it. If sending it incurs a small processing time (cost), then α_B will not be sent. However, if $\mu_f = p_t(x)$, then it is more likely for the buyer not to send α_B, even if the buyer is neutral toward uncertainty.

Nevertheless, the real implementation depends on the algorithm and user model --- whether the owners allow their agent to accept uncertainty or not, and this is beyond the scope of this thesis.

In contrast to the previous section, this subsection has considered the value of the arguments in changing the opponent's perception regardless of the importance of time. In the next sub section, we will show another reason for strategic ignorance due to the risk of breakdown from sending an argument, also without considering the cost of negotiation time.

4.3.3.3 Ignorance due to risk of breakdown

In the previous two subsections, we considered ignorance from the perspective of the buyer's expected utility. However, sometimes an argument by the buyer may trigger the seller to terminate the negotiation prematurely (breakdown). We assume that the breakdown comes from the argument rather than exogenous factors in $\boldsymbol{q_t}$. There are at least three possible causes here, which are explained below.

1. The buyer's argument, $\boldsymbol{\alpha_B}$, decreases the buyer's estimation of the seller's belief, $\boldsymbol{w_t(S_t)}$,[3] that the buyer will accept the seller's valuation $\boldsymbol{S_t}$. From the buyer's perspective, if the seller's belief approaches zero, then there is no reason for the seller to continue the negotiation. Therefore, the buyer has to have a belief about its argument possibly affecting the seller's belief in causing a breakdown, which can be expressed by $\boldsymbol{\upsilon_{\phi}(\alpha_B) = P(\, w_t(S_t) = 0 \mid \alpha_B\,)}$. An example of such a buyer argument is "I

[3] We use $\boldsymbol{w_t(x)}$ for the buyer's estimation of the seller's belief that the seller's offer will be accepted by the buyer.

cannot pay any price higher than $\boldsymbol{x}$." when the buyer knows that $\boldsymbol{x} < \boldsymbol{S_t}$. This argument will only make the seller walkout, especially at the beginning of the negotiation.

2. The buyer's argument delays the negotiation by Δt, and during the course of the delay the seller has sold the item to another buyer, with probability $\boldsymbol{P}(\ \boldsymbol{g_t}(\Delta t) = 1 \mid \boldsymbol{\alpha_B}\ )$, where $\boldsymbol{g_t}(\Delta t)$ is the likelihood of the seller selling the item to another buyer within interval Δt and there are no more items left; thus, the negotiation will be terminated immediately by the seller. We may consider this condition as the opportunity cost of negotiation delay, which is similar to our previous discussion in subsection 4.3.3.1. However, since the effect is breakdown instead of the decreasing of the buyer valuation, the buyer may consider it separately.
3. The buyer's argument cannot be processed by the seller, which eventually causes a breakdown of the negotiation. This condition relates to the robustness of the agent reasoning; for instance, in handling contradictive arguments, recursive computation, exceptional error, etc.

Since the seller's mental state is not transparent to the buyer, it is not possible for the buyer to know exactly when such breakdowns will happen. Therefore, the only way to handle breakdown situations is by assigning a probability value to each argument, denoted by $\boldsymbol{\upsilon_\phi(\alpha_B)}$, and use a meta-reasoning engine to assess the likelihood of that argument triggering a breakdown. Intuitively, if the buyer uses too many arguments from ignorance or threats, it will reduce the seller's belief $\boldsymbol{w_t(S_t)}$. And if the buyer uses third-party information that may take a longer verification time (therefore longer Δt), it will increase $\boldsymbol{g_t}(\Delta t)$. And repeated or illogical arguments may trigger a failure in the seller's reasoning.

If a breakdown costs nothing but a zero expected payoff, then we may consider its cost by integrating $\upsilon_{\phi}(\alpha_B)$ into $p_t(x)$ in equation (4-24) such that the equation becomes

$$EG^{\alpha}_t(x) = (1 - \upsilon_{\phi}(\alpha_B))\, p^{\alpha}_t(x)\, (B_t - x) \tag{4-30}$$

where $\upsilon_{\phi}(\alpha_B) \in [0, 1]$.

Proposition 4-16. *(Strategic ignorance due to risk of breakdown) If* $\upsilon_{\phi}(\alpha_B)$ *is greater than the increasing rate of* $p_t(x_t^*)$ *from* α_B*, i.e.* $\upsilon_{\phi}(\alpha_B) > (p^{\alpha}_t(x_t^*) - p_t(x_t^*)) / p^{\alpha}_t(x_t^*)$*, then* α_B *will not be sent.*

This proposition states that an argument will be used only if the benefit of using it in terms of the increasing rate of $p_t(x_t^*)$ exceeds the cost of using it, in terms of the probability of breakdown caused.

Corollary 4-2. *If* $\upsilon_{\phi}(\alpha_B) > 1 - p_t(x_t^*)$*, then* α_B *will not be sent.*

4.3.3.4 Summary

So far, we have shown that the buyer may avoid argumentation when it is estimated as a costly and/or risky action. Thus, we can be sure that proactive ignorance may be used by a rational agent, that is it may actively choose not to make an argument as a rational choice.

Theorem 4-5. *(The existence of proactive ignorance) A rational agent may not use argument* α_B *if it*

- *incurs a high cost;*
- *reduces the expected probability of the seller's acceptance of its offer; and/or*
- *raises the risk of breakdown more than the increasing rate of* $p_t(x_t^*)$.

Proof. By Proposition 4-14, 4-15, and 4-16. ■

Depending on the design of the agent and the particular argument, a buyer may take into account all or some conditions of ignorance described in the previous section. Certainly, it may also consider other conditions of ignorance which are beyond the scope of our analysis here. When an argument about a certain topic is costly or risky, then the buyer may choose to pick it or not. However, it is not the buyer's sole right to open an argumentation, because the seller may have the same right to touch upon those topics that may threaten the buyer's bargaining position. In this case, if the buyer has a reason not to counter argue, strategic ignorance will be used too. For instance, if the seller argues about a topic that is predicted to be a time consuming argumentation by the buyer, then the best strategy of the buyer is to avoid it. Thus, all conditions described in subsections 4.3.3.1, 4.3.3.2, and 4.3.3.3 may also apply in response to the seller arguments. That is, they can be used in responsive ignorance as well as proactively. However, there are additional conditions beyond these that may cause responsive ignorance. In the next section we will show these other conditions, where the buyer may also avoid arguing in response to the seller's argument.

4.3.4. Analysis II: Responsive Ignorance

In this section we will analyze several reasons for responsive ignorance in addition to the proactive conditions described in section 4.3.3. We will describe them according to the intention of the seller using an argument. We may classify the seller's argument into the following categories:

- It persuades the buyer to increase $\boldsymbol{B_t}$ (that the item offered by the seller is worth more than what the buyer has estimated), denoted by $\boldsymbol{\alpha_{S1}}$.

- It justifies the seller's offer such that $(\forall x < y)\ p_t(x) \rightarrow 0$ (that it is almost impossible for the buyer to buy the item with a price below the seller's offer, or the seller will not sell the item for a price lower than y), denoted by α_{S2}.

Undoubtedly, arguments can also be used to demonstrate the seller's reputation, which increases the value of $\upsilon_t(\alpha_S)$, or to rebut the buyer's argument. However, in this section we will only analyze the two categories. We will first discuss several types of response a buyer may have.

Suppose during a negotiation of buying/selling an apartment, the seller uses the following argument "The government has planned to build a sport-center close to this apartment next year, so the price of the nearby properties may rise over 30%." Under uncertain conditions and depending on the ABN protocol, upon receiving the seller's argument the buyer may reply with one of the following arguments:

- Affirmation --- inform the seller that the buyer understands and agrees to the statement. (e.g. "I see.")
- Rebuttal --- inform the seller that the buyer understands the seller's statement but is skeptical as to the truthfulness of the statement. (e.g. "I don't believe that the government will really build it; they changed a similar plan before.")
- Query --- inform the seller that the buyer does not understand some (or all) of the statement and ask for a clarification. (e.g. "What is a "sport-center"?"); or inform the seller that the buyer understands the seller's statement and ask for further information. (e.g. "Is the government plan approved?")

- Responsive Ignorance --- inform the seller that its previous statement cannot be processed and has been therefore ignored. (e.g. "Let's not discuss this further; please continue with the negotiation.")

If the buyer replies affirmatively, it will inform the seller that the buyer may update its valuation, which implies that the seller's offer is more reasonable than that of the buyer. Consequently, the buyer has an obligation to revise its offer by increasing it. A buyer may reply using this argument if it knows that both the seller and the government are trustworthy, and thus raises its offer accordingly.

However, if the buyer believes that the seller's argument is not true, then the buyer will rebut. It may lead to a degradation of the buyer's trust toward the seller's arguments, unless the seller could provide more data/facts to convince the buyer. In human negotiation, rebuttal may come from the buyer's skepticism toward the seller's trustworthiness regardless of the truth of the argument. When data that contradicts the seller's argument exists, then the buyer should deny the seller's argument, which may lead to a rebuttal.

When the buyer, who trusts the seller, cannot assess the truthfulness of the seller's argument or cannot understand the argument, then it may use a query as its counter-argument. Such counter-arguments (R3) imply that the buyer has not changed its valuation and is still in the process of interpreting the seller's argument. To convince the buyer, the seller may explain its argument, and the argumentation may continue for several rounds until the buyer understands the argument, which eventually leads to an affirmation or rebuttal, or a failure to fully clarify the argument, which leads to ignorance.

Certainly, a rational buyer will choose ignorance if it is better than all other responses, that is when the expected utility of ignorance is higher than the expected utility of any other responses (R1 to R3). In the next two sub sections we will demonstrate responsive ignorance for two different intentions of the seller's argument: (i) to persuade the buyer to increase its valuation and (ii) to accept the seller's offer.

4.3.4.1 Ignoring α_{S1}

Suppose the seller uses argument α_{S1} to persuade the buyer to increase B_t to B_t'. Given equation (4-24) and (4-25), a buyer will have a higher expected payoff after increasing its estimated valuation from B_t to B_t'. But at the same time, it will also increase the chance that the real valuation B_r is lower than B_t' (cf. Assumption 4-7). Since B_t is chosen by the buyer without the seller's interference, then any cost generated from it ($\Delta B = B_t - B_r$) should not be blamed on the seller's argument. Intuitively, from the buyer's perspective, it will choose B_t as the estimated value of B_r. However, any cost generated in term of $\Delta B' = B_t' - B_t$ (when $B_t \geq B_r$) or $\Delta B' = B_t' - B_r$ (when $B_t < B_r$), should be charged to the seller's argument (buyer's overvaluation due to seller's persuasion). Since the buyer does not know the value of B_r, it can only assign a probabilistic value to the occurrence of overvaluation using the known value $\Delta B' = B_t' - B_t$. Let's denote the expected cost incurred from this risk as $\rho\Delta B'$, where ρ is the likelihood that the overvaluation occurs. If the seller's reputation is high and its information is accurate, i.e. $\upsilon_t(\alpha_{S1})$ is high, then the risk is low; conversely, if the seller's reputation is low or the information is less accurate, then the risk is high. Under this rationale, we can infer that ρ

is proportional to $\mathbf{(1 - \upsilon_t(\alpha_{S1}))}$, or $\boldsymbol{\rho} \propto \mathbf{(1 - \upsilon_t(\alpha_{S1}))}$. Given this estimation, the new valuation that should be used by the buyer is $\boldsymbol{B_t^* = B_t + (1-\rho)\,(B_t'-B_t)}$.

Example 4-9. *Suppose a buyer wants to buy a property from a renowned agent, and the buyer's initial valuation was $150K. Suppose that the seller told the buyer that the government has planned to build a sport-center so the price of the nearby property may rise over 30%. Given this new information, the buyer now is willing to pay up to $200K if the government will really build the facility. However, since the buyer is also aware of the possibility that the government plan will not be approved, then the buyer may still have a risk of overpaying $50K for the property. If the buyer believes that the government can realize its plan with probability 0.6, then the buyer's new valuation will be $150K + 0.6($200K-$150K) = $180K.*

From Example 4-9 we can see that the buyer is still facing a risk despite the high credibility of the seller. In this example the buyer can calculate its new valuation because it knows the credibility of the government. But what if the buyer does not know the reputation of the seller and has no ability to verify the truthfulness of the seller's argument either from the third party or by itself? One possibility is to stick to its previous valuation and inform the seller, and another possibility is to try to assess the seller's argument, which may be very costly or time consuming. Thus, the buyer may choose ignorance as its reply. If after several rounds of negotiation or from other argumentation the buyer has been convinced that the seller's reputation is high (a truth-teller), then it may re-consider to accept the entire seller's previous argument(s) that could not be verified. Thus, responsive ignorance buys time for the buyer.

Another reason for using strategic ignorance could be the consequence of the buyer's self interest of pretending it is a low-valuation buyer, regardless of seller reputation or the truth of the argument. By doing so, it may reduce the seller's expectation of selling the item at a high price, which in turn forces the seller to make greater concessions. This strategy may work if the buyer knows that the seller valuation is low and it is impatient or cannot find another buyer easily. In other words, the buyer must believe that its offer is very likely to be accepted, or $\boldsymbol{p_t(x_t^*)}$ is high. However, if the buyer does not know this, then it will be very risky for the buyer to ignore the seller's argument(s), because the seller may walk out due to its updated belief that the buyer will never buy at a reasonable price ($\boldsymbol{w(S_t)} = 0$).

Proposition 4-17. *(Valuation hiding) Let the buyer believe that both the seller valuation and the risk of breakdown are low so that the seller is willing to accept the buyer's offer and also* $\boldsymbol{\upsilon_t(\alpha_{S1})}$ *is high so that the seller's argument is trusted and* $\boldsymbol{\rho}$ *is low so that the buyer will increase its valuation; then*

(i) $\boldsymbol{\alpha_{S1}}$ *will be affirmed if* $\boldsymbol{p_t(x_t^*)}$ *is steep; and*

(ii) $\boldsymbol{\alpha_{S1}}$ *will be ignored if* $\boldsymbol{p_t(x_t^*)}$ *is flat near certainty.*

4.3.4.2 Ignoring α_{S2}

Suppose the seller uses argument $\boldsymbol{\alpha_{S2}}$ to justify that the seller will not reduce its offer anymore, and the buyer can fully understand $\boldsymbol{\alpha_{S2}}$. Normally, if the buyer believes that $\boldsymbol{\alpha_{S2}}$ is true and the seller's offer is acceptable, then the buyer will just accept the seller's offer. Conversely, if the buyer does not believe that $\boldsymbol{\alpha_{S2}}$ is true and it has a low-cost counter-argument(s), then rebuttal will be used, because it can force the seller to reduce its offer. But if counter-arguing is very costly, then the buyer will use strategic ignorance as described in section 4.3.3.1. Therefore, ignorance will be used when the buyer does not believe that α_{S2} is true and counter-arguments are very costly, as stated in the following proposition.

Proposition 4-18. *Let* $\upsilon_t(\alpha_{S2})$ *be low so that the seller's argument is trusted and assume all possible counter-arguments are very costly, then* α_{S2} *will be ignored.*

However, there is a special situation in which ignorance will be used even when the buyer believes that α_{S2} is true, i.e. when the buyer benefits from the delay in the negotiation; for instance, when it has a temporary increasing valuation $\boldsymbol{B_t}$ over time or when it is waiting for a response by other seller(s). In this situation, the buyer basically agrees that the seller will not reduce its offer further and the seller's offer is acceptable from the buyer's perspective. Thus, the reason for not promptly accepting the seller's offer is not because it is not a reasonable price, but expressly to incur a delay. Consequently, the buyer will not affirm α_{S2} because it wants to accept the seller's offer at a later time. Instead, ignorance may be used, especially when there is no reasonable rebuttal or the rebuttal may increase the risk of breakdown. Alternatively, the buyer can also use strategic delay when it is applicable, which can be realized by sending a sentence like "Please wait; I am currently considering your offer but it is taking me time to factor in everything …" Indeed, the buyer's action of delaying the negotiation by pretending to not fully understand the seller's argument can be regarded as a 'soft' ignorance, in contrast with a 'hard' ignorance which informs the seller that the buyer does not understand the argument at all. The following proposition states the condition when responsive ignorance is used.

Proposition 4-19. *Let* $\upsilon_t(\alpha_{S2})$ *be high so that the seller's argument is not trusted and assume* B_t *is increasing over time. If rebuttal and query are more costly and/or may increase the risk of breakdown more than ignorance, then* α_{S2} *will be ignored as long as its incurred cost is lower than the marginal gain of* B_t.

4.3.4.3 Summary

In the previous two sub sections, we have shown that the buyer may use responsive ignorance toward a weak seller argument; especially when rebuttal is costly or risky. In addition, the buyer may also use ignorance for hiding its valuation and as a replacement for strategic delay, despite a strong seller argument. These choices are based on buyer rationality in wanting to increase its expected revenue. Therefore, we can conclude our analysis with the following theorem.

Theorem 4-6. *(The existence of responsive ignorance) A rational agent may benefit from ignoring argument* α_{S1} *and* α_{S2}.

Proof. By Proposition 4-17, 4-18 and 4-19. ■

4.4 Chapter Summary

The following conclusions regarding bargaining protocols can be derived from our analysis in this chapter.

First, a bargaining protocol should not restrict agents to submit monotonic offers only, because they may expect a higher expected utility from non-monotonic offers. However, as we stated previously in subsection 4.1.5, this protocol may be misused by malevolent agents, which eventually reduces the success rate of the negotiation. To avoid this, we may restrict the frequency of non-monotonic offers, e.g. each agent cannot switch to submit non-monotonic offers more than N times. However, we cannot prove the effectiveness of this restriction now and it is left for our future work.

Second, the protocol should allow agents to delay their offer, because it can be used to wait a better opportunity, increasing the accuracy of its estimation, or other purposes. Since the platform designer may restrict a communication delay which may result from a connection error or failure, the negotiation protocol should include a message (in Agent Communication Language) to inform explicitly that an agent is delaying its offer/decision.

Third, the protocol, if incorporating argumentation, should allow agents to ignore their opponents' argument. This ignorance-based protocol can improve the system's robustness and also increase the benefit of agents. Again, the argumentation-based negotiation protocol should include a message that an agent cannot understand the argument of others.

The above results are true based on the agents' individual interest. However, bounded rationality is the central assumption in our analysis here. If either or both parties are not utility maximizers, it is unclear that the proposed modifications are still useful in increasing the social welfare. Intuitively, both parties may be benefit from the modified protocol under specific circumstances. For example, if both parties need time to clarify their information, then both may use strategic delay concurrently. In the next chapter, we will analyze empirically the impact of the modified protocol to the social welfare empirically.

CHAPTER 5

SIMULATION AND EXPERIMENTAL RESULTS

In this chapter we present experimental results of our study of the non-monotonic-offers bargaining protocol, the delay-based bargaining protocol, and the ignorance-based argumentation-based negotiation. These experiments will be used to shed light on what happens when we impose more realistic constraints on the bargaining events than in the theoretical analysis in Chapter 4. In the real world, agents may not be able to maximize their expected utility due to computational limitations and incomplete information. Most of these agents cannot be analyzed formally. Hence, we need to analyze our protocols empirically. After describing these agents in section 5.1, we present an experimental study of the three bargaining protocols that are the focus of this research.

5.1 Social Agents that Use Random or Reactive Strategy

Many scholars have challenged the "utility maximization" principle of bargainers. For instance, several experiments with human subjects have shown that not all people are utility-maximizers, even under complete information. Some people seek fairness in bargaining [Forsythe et al., 1994], while others fail to find the optimal (equilibrium) solution because they fail to use backward induction [Neelin et al., 1988; Johnson et al., 2002]. In terms of MAS, many people also question the possibility of building agents that can have correct prior beliefs and an updating mechanism that can really help them to maximize their utility [Faratin, 2000].

To overcome these constraints, a user may use social agents that have more straightforward strategies. Random and reactive strategies are commonly used in human society due to their simplicity. Static strategies (time-dependent tactics) and reactive strategies (behavioral tactics) are also commonly used in human negotiation and also common in the negotiation literature and will also be considered in some of our experiments. Hence, social agents will use social clues derived from human-human negotiation. It is not only important in attaining a satisfactory, or maybe an optimal, transaction, but also may improve the agents' reputation in the electronic marketplaces where they are residing.

5.1.1 Random Strategy

In the random strategy, if the buyer valuation is 100 cu. (currency units) and the seller asks for 90 cu., then the buyer can randomly offer a price between 0 and 90 cu. without considering any factor. If the seller refuses to sell it at the randomly selected price, say 45 cu., then the buyer may try a new random price between 45 and 90 cu. In the negotiation literature, those agents using pure random strategies are known as zero intelligence (ZI) agents [Gode and Sunder, 1993; Duffy, 2006].

In a more complex algorithm the agents may consider their characteristics in choosing a random value. For instance, a patient buyer may generate a random price using a distribution function with a lower mean value compared to a less patient buyer. Figure 5-1 shows a uniform distribution function (symmetric), a normal distribution function (symmetric), and a lognormal distribution function (asymmetric) that may be used by agents in generating their offers.

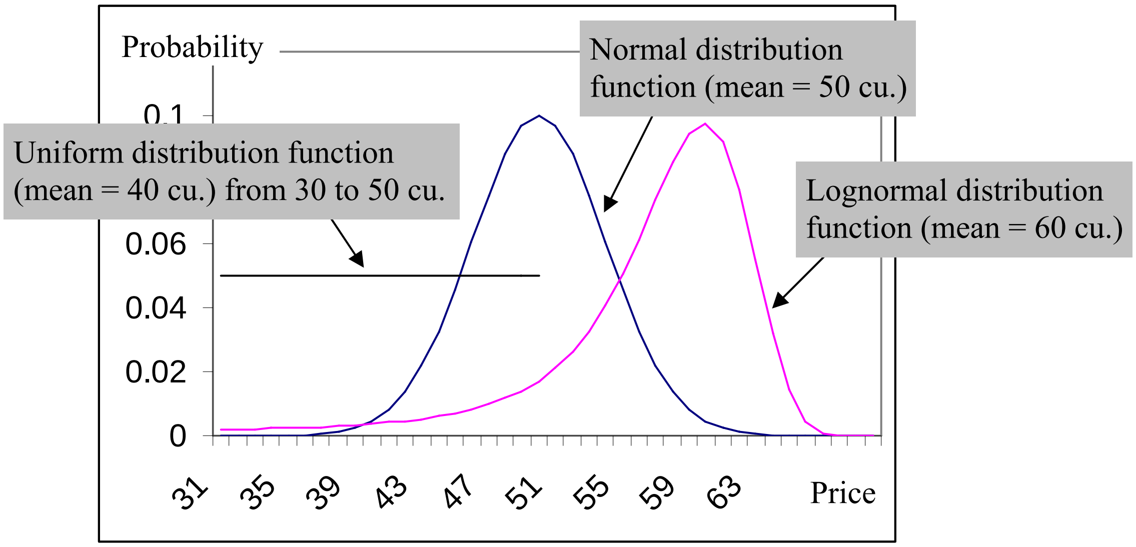


Figure 5-1. Various distribution functions for generating agent's offers

In addition, the buyer may generate a random price using a specific distribution function it has learned from its past experience as the most effective one (e.g. generating highest average surplus). Or, it may also use different distribution functions for different groups of sellers at different times. In this case, the buyer's decision is governed by social clues. For example, in the market of used cars, a buyer may use a normal distribution function, while in the market of used books it will use a lognormal distribution function, because the demand of used books is higher which leads to a smaller chance for the seller to reduce its price. Besides, it is more likely for the buyer to encounter the same book seller in the future (it is unlikely to buy several cars within a year); thus, the buyer's reputation can be built by a higher successful negotiation (as a result of a fast concession from the lognormal distribution). These social clues, then, will be transformed into heuristics and ported to agents by their owner or designer.

5.1.2 Reactive Strategy

In contrast to a random strategy, a reactive strategy generates an offer according to the behavior of the agent's opponent. Tit-for-tat is the most common reactive strategy and has been observed in many biological interactions. It is known as the best strategy in solving the repeated Prisoner's Dilemma game [Axelrod, 1981]. For example, if the seller reduces its offer by 50 cu., then the buyer reacts by increasing its offer by 50 cu. too. Similarly, if the seller stands on its previous offer, then the buyer will follow it by standing on its previous one. Knowing that the buyer will follow its move, the seller can speed up the negotiation by reducing its offer faster. Thus, the advantage of tit-for-tat is its simplicity; yet, it is also powerful enough that it can guide the negotiation to a resolution. However, the disadvantage of this strategy is the possibility of deadlock. For example, if both parties use tit-for-tat, and one of them stands on its previous offer, then they will stand on their offers forever. To avoid the deadlock, at least one party must adopt a tie-breaker, e.g. never stand on its offer for more than j rounds. However, depending on the proponent's characteristics and other constraints (e.g. time deadline and valuation), the proponent's move may not be exactly the same as the opponent's move. For example, after the seller reduces its offer by 50 cu., the buyer may increase its offer by 40 cu. only, because increasing more will generate negative surplus. Or, if the buyer is impatient, then it will reduce its offer by 60 cu. in order to speed up the negotiation. Again, in this thesis we will not study the strategy used by the agent because it could be varied according to designer/user characteristics, but rather, we include them in order to facilitate studying negotiation protocols.

Other reactive strategies considered in this chapter are tat-for-tit, tit-for-two-tat, and spread-driven strategy. In short, those four reactive strategies can be explained as follows.

- Tit-for-tat: the proponent's move is the same as the opponent's previous move. For example, if the buyer concedes by increasing its offer, then the seller also concedes by decreasing its offer. Note here, their concession rate is not necessarily at the same amount, e.g. the seller may decrease its offer by 5 cu. as the response to an increasing of 10 cu. by the buyer.
- Tit-for-2tat: if the opponent's previous two consecutive moves are in the same direction but different from the proponent's previous move, then the proponent switches its move to resemble the opponent's previous move; otherwise the proponent does not switch. For example, suppose at time *t–2* and *t–1* the buyer did not concede, i.e. stood firm or decreased its offers, then the seller will respond by not conceding at time *t*, e.g. it also stands firm or increases its offer.
- Tat-for-tit: the proponent's move is the reverse of the opponent's previous move. For example, if the buyer conceded, then the seller will not concede.
- Spread-driven: the proponent's move tries to reduce the spread of negotiation (the difference between the buyer's and the seller's offer) by a constant fraction.

Even though tat-for-tit is not commonly used due to its odd behavior, we include it in some of our analysis as noise.

5.1.3 Time-dependent Strategy

Time-dependent strategy is a static strategy which only depends on the time factor. Using this strategy, a buyer will increase its offer monotonically until the value of the offer reaches the buyer's expected valuation or a specific predetermined price. Figure 5-2 shows some time-dependent strategies. The increment of a buyer's offer represents its characteristics. For example, a less patient buyer will concede faster (Conceder) than a more patient one (Boulware) [Faratin et al., 1998]. If a steady increment rate is chosen, it is regarded as a linear strategy. And if a mixture of two or more time-dependent strategies is chosen, we have a compound strategy (for example, combining Conceder and Boulware as in Figure 5-2). Most time-dependent strategies have been studied widely in the automated negotiation literature [Faratin et al., 1998; Fatima et al., 2004]. In our experiments, we will use linear and conceder strategies with various increment rates.

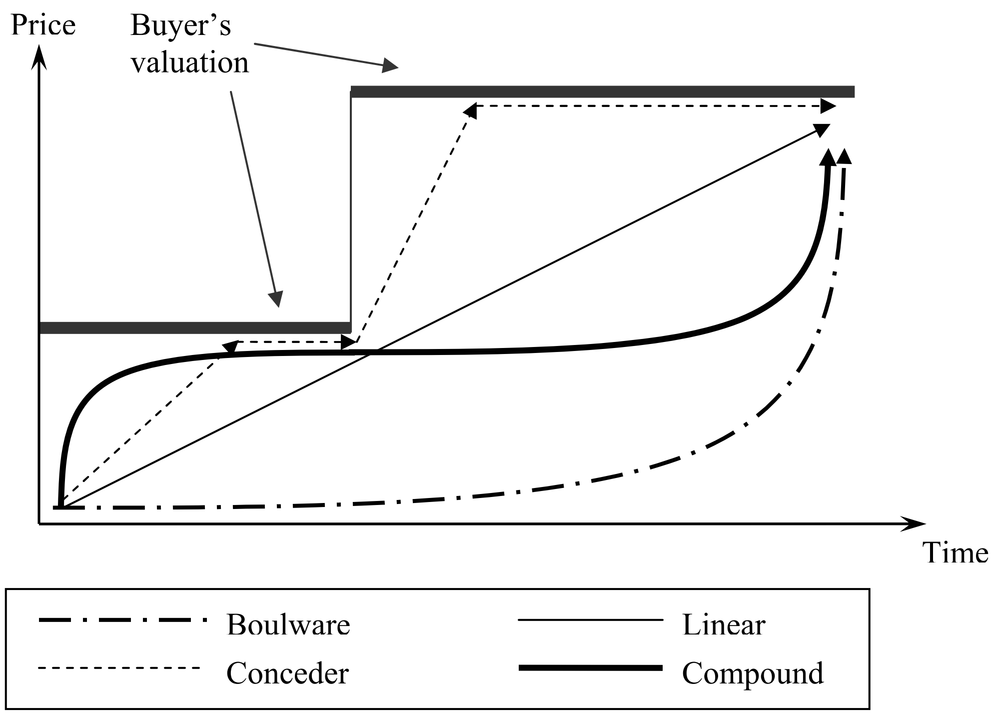


Figure 5-2. Some examples of time-dependent strategies

5.2 Empirical Study of the Non-monotonic-offers Bargaining Protocol

The purpose of this empirical study is to analyze the cost and benefit of the non-monotonic-offers bargaining protocol among irrational agents. Since exhaustive experiments are unrealistic, our experiment is restricted to providing insight as to the effect of agents' irrational behaviors in both monotonic and non-monotonic-offers bargaining protocols.

These irrational behaviors are hard to be analyzed theoretically, because there is a large amount of possible combinations between two irrational strategies in a negotiation. For instance, we can have tens or even hundreds of different random strategies depending on the size of the random outcomes.

5.2.1 Agents' Valuation

First, 100 pairs of non-decreasing valuations (both buyers and sellers have non-decreasing valuations over time) are generated randomly under a pre-specified range. Figure 5-3 shows four representative pairs of agents' valuations (out of 100 different ones generated), which are generated randomly. The vertical axis represents the price, and the horizontal axis represents the time line (in rounds). The thick line represents the buyer's valuation in each round, and the thin line represents the seller's valuation. The transaction may be made within the area where the thick line is on the top of the thin line, or when both of them are on the same horizontal line. Near the end of the bargaining period, both lines always overlap at 100 cu. This gives us a higher assurance of a success in the bargaining if the bargaining prolongs to the deadline. In all pairs of generated valuations, there is always a non empty feasible set (a period when the buyer's valuation is higher than the seller's valuation; or, a period when a resolution can be reached). However, the length of these periods varies. For instance, the right upper diagram of Figure 5-3 provides shorter periods than the right bottom.

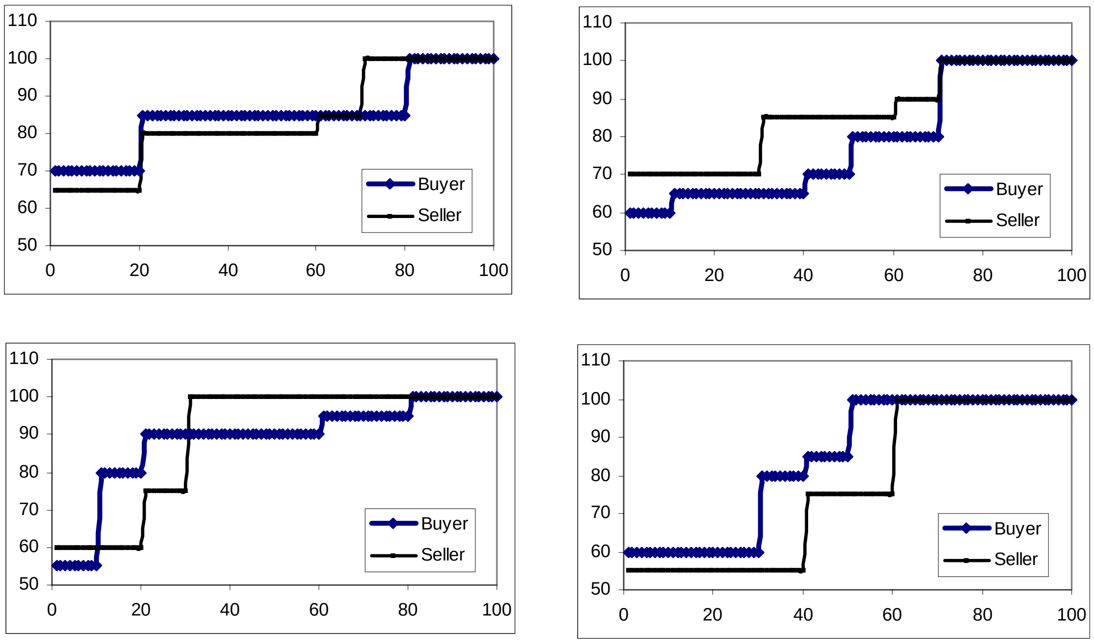


Figure 5-3. Four pairs (out of 100) of upward valuation used in the experiments

Under this setting, the buyer's offer will never get stuck in its valuation, because its valuation is non-decreasing. However, the seller's offer may get stuck in its valuation because its next valuation may be higher than current one. This is especially true when the seller is risk-seeking and quickly offers a price lower than 100 cu. at the beginning of the bargaining.

5.2.2 Experimental Design

Two main parts of the experiment have been designed, based on the protocol and the strategies used by agents:

- Part 1: Agents use random strategies in bargaining
- Part 2: Agents use reactive (behavior-dependent) strategies in bargaining

Furthermore, we divide the experiment in Part 1 into four smaller groups according to the following strategies used by the sellers:

- Risk-averse seller (R-averse): a seller who offers a monotonically decreasing price but will not offer any price below its maximum valuation (in this case 100 cu.);
- Risk-seeking seller (R-seeking): a seller who offers a monotonically decreasing price but may offer any price above its present valuation (in this case it may offer a price less than 100 cu.);
- Nonmonotonic-offer seller (N-seller): a seller who offers any price above its present valuation and may increase it as its valuation increases, but only when it is stuck on its valuation;
- Nonmonotonic with random change (NR-seller): a seller who is similar to N-seller, except that it may increase its price randomly (with probability equal to 0.1)[1] in order to attract EvalF-II buyers.

For each group in the experiment above, we subdivide it into four treatments based on the strategies used by sellers and buyers in accepting an offer by their opponents. These four treatments are shown in Table 5-1, labeled as S1B1, S1B2, S2B1, and S2B2. S1B1 means that all sellers are EvalF-I agents and all buyers are EvalF-I agents. S2B1 means that all sellers are EvalF-II agents and all buyers are EvalF-I agents. Similar explanations are used for S1B2 and S2B2.

Given these treatments, a total of 16 groups of experiments are conducted in part 1. Each group is repeated 300 times for each pair of valuations, resulting in 16 groups x 300 repetitions x 100 pairs of valuation = 480000 trials in part 1. In this experiment,

[1] This probability 0.1 is particularly chosen for the current simulation only. The small value is chosen since we believe that agents will not change their offers arbitrarily and frequently.

agents only follow random strategies, i.e. sellers (buyers) randomly increase (decrease) their offers until the valuations are reached. Both the increment and decrement vary from 0 to 5 cu. Depending on agents' characteristics, some may raise/drop their offers faster than others, but they never take their opponents' offers/behavior into consideration.

Table 5-1. Four treatments based on agents' evaluation (non-monotonic-offers protocols)

		Seller	
		EvalF-I	EvalF-II
Buyer	EvalF-I	S1B1	S2B1
	EvalF-II	S1B2	S2B2

The realization of an EvalF-I agent is by using the following method:

- if my opponent's current offer generates higher positive surplus than my offer which will be sent in the next round, then accept my opponent's current offer;

The realization of an EvalF-II agent is by adding the following method in addition to the criterion in an EvalF-I agent:

- if my opponent's current offer generates higher positive surplus than the previous one, do nothing;
- if my opponent's current offer generates lower positive surplus than the previous one, accept it.

The experiment in part 2 is almost the same as that in part 1, except that most agents use reactive strategies. Four reactive strategies are considered: tit-for-tat, tit-for-2tat, tat-for-tit, and spread-driven. These reactive strategies are slightly different from those used in [Faratin et al., 1998], but with some overlap, e.g. tit-for-tat. The spread-driven strategy in our experiment is also slightly different from the market-driven strategy used in [Sim, 2002].

Each experiment in part 2 consists of agents with reactive strategies plus agents with random strategies. Eight different treatments are conducted in part 2: the 4 different treatments given in Table 5-1, and 2 different protocols for each of them (N-protocol and M-protocol). For statistical analysis purposes, each possible combination is repeated more than 30 times; thus 900 trials are conducted for each treatment, because there are 5 different agent strategies which results in 25 possible pair combinations. Therefore, on average 900/25 = 36 repetitions are done for each possible combination, which is enough for statistical analysis. In total, we have conducted 8 treatments x 900 trials x 100 pairs of valuation = 720000 trials in part 2.

Since two reactive agents may stand on their offers (e.g. two tit-for-tat agents will always use the same strategies if they meet), then we use a "tie-breaker" mechanism such that an agent will not stand on their current offer for more than 3 rounds. Moreover, we assume a high cost in repeating a bargaining session. Thus, if an agent is stuck in their valuation, then we will consider it as a breakdown. Table 5-2 shows general parameters used in both part 1 and part 2 of our experiments.

5.2.3 Evaluation Criteria

In both part 1 and part 2, two main variables are recorded for evaluation purposes, i.e. total surplus generated from each group experiment, in terms of the sum of surplus for both buyers and sellers, and, the number of breakdowns/successes.

Based on those two values, three pieces of information are computed: average surplus, average surplus per successful transaction, and success rate (percentage of successful negotiation).

Table 5-2. Parameters used in both parts of experiments (non-monotonic-offers protocols)

Parameters	Values
Maximum bargaining rounds	99
Sellers' and buyers' initial valuation	50 – 85 cu.
Sellers' and buyers' final valuation	100 cu.
Increment of valuation	5 cu.
Range of sellers' initial offers	100 – 120 cu.
Range of buyers' initial offers	30 – 50 cu.
Min. increment/decrement of offers	1 cu.
Max. increment/decrement of offer	5 cu.

Note: cu. is the currency unit of the offer

5.2.4 Results

5.2.4.1 Results of Part 1: Agents use random strategies in bargaining

It is shown in Table 5-3 that agents in N-protocol (the two bottom rows) generate higher surplus compared to agents in M-protocol (the two upper rows). The success rates of N-protocol are also higher compared with the setting where sellers are risk-seeking in an M-protocol (2nd row).

Table 5-3. Result of experiment in part 1 (non-monotonic-offers protocols)

	Ave. surplus (cu.)*	Ave.surplus (cu.)/ transaction	Success rate (%)
R-averse (M-protocol)	6.690 (*10.543*)	7.047	94.937
R-seeking (M-protocol)	7.069 (*10.767*)	8.817	80.183
N-seller (N-protocol)	9.902 (*11.619*)	10.083	98.200
NR-seller (N-protocol)	10.091 (*11.736*)	10.815	93.307

*The corresponding standard deviation is inside the parentheses

Table 5-4. The p-values of the pair-wise t-test on "Average surplus" and "Success rate"

Average surplus			
	R-seeking (M-protocol)	N-seller (N-protocol)	NR-seller (N-protocol)
R-averse (M-protocol)	2.80242E-18	0	0
R-seeking (M-protocol)		0	0
N-seller (N-protocol)			7.17231E-05
Success rate			
	R-seeking (M-protocol)	N-seller (N-protocol)	NR-seller (N-protocol)
R-averse (M-protocol)	0	0	1.17069E-64
R-seeking (M-protocol)		0	0
N-seller (N-protocol)			0

A relatively large standard deviation (approx. 11 cu.) of the average surplus in Table 5-3 reveals that the results are distributed in a quite large range. However, very low *p*-values from the t-test (assuming two tails and unequal variances) shown in Table 5-4 justify that the differences of the mean values reported in Table 5-3 are statistically significant. These results justify our theoretical analysis that N-protocol favors negotiation under sellers' upward valuations. We can also show that similar results may appear for bargaining under downward valuation (both sellers and buyers decrease their valuation over time), since it is a dual (symmetric) of this experiment.

If we compare the results of each group of strategies used by buyers and sellers (four possible combinations of buyer/seller acceptance strategies), we find out that the effect of various strategies used by sellers and buyers are not significant in N-protocol (Figure 5-4). Smaller average surpluses are only observed in the case when sellers are EvalF-II and buyers are EvalF-I (shown as S2B1). When we check the effect in the M-protocol, much smaller average surpluses are only observed when sellers are EvalF-I (see Figure 5-5, in case S1B1 and S1B2). The result suggests that EvalF-I sellers may reduce their overall surplus under M-protocol, in other words, EvalF-II sellers outperform EvalF-I sellers in generating surplus.

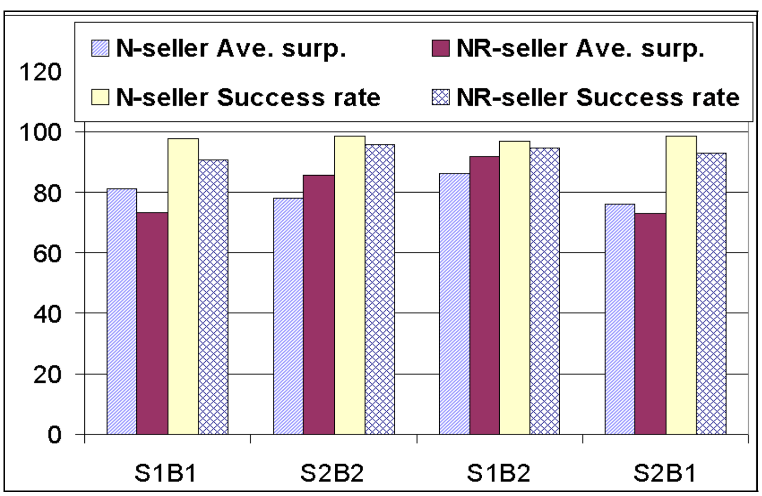


Figure 5-4. A comparison between N-seller and NR-seller in four possible combinations of EvalF-I and EvalF-II agents as shown in Table 5-1 (average surplus is normalized)

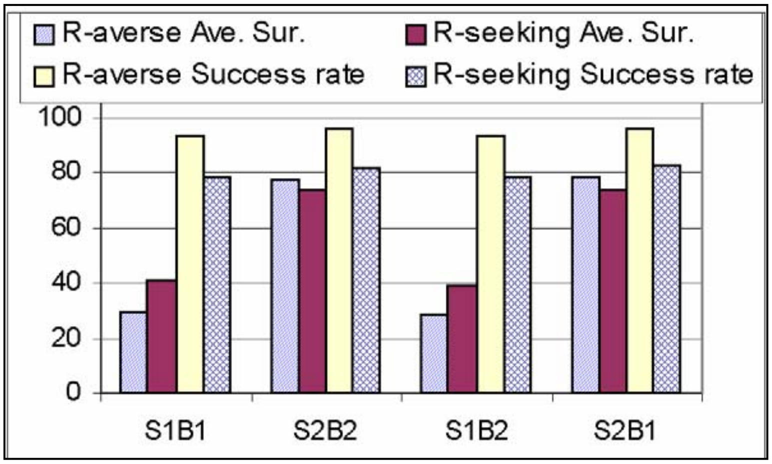
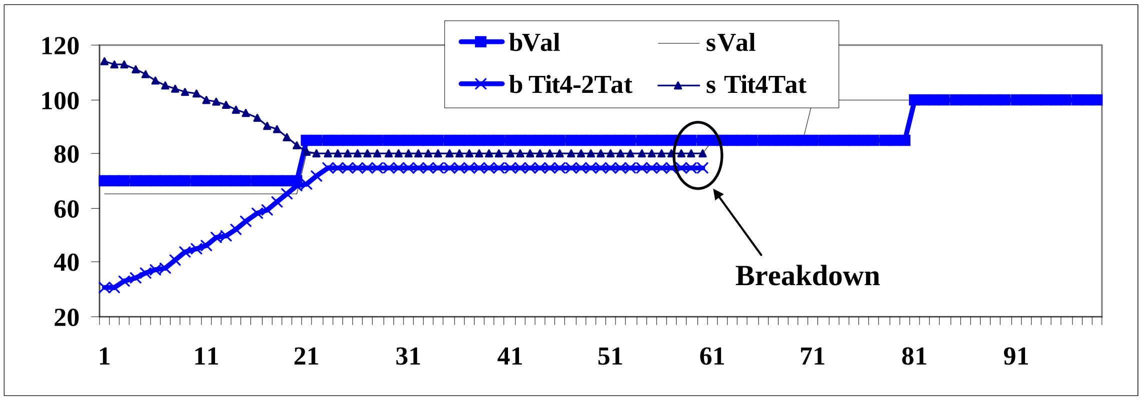


Figure 5-5. A comparison between R-averse and R-seeking in four possible combinations of EvalF-I and EvalF-II agents as shown in Table 5-1 (average surplus is normalized)

Moreover, introducing NR-sellers in the N-protocol reduces the success rate as predicted. The effort to increase the price in order to convince the buyer to make concessions earlier will prolong the bargaining, thus increasing the risk of breakdown. However, as expected, it generates higher total surplus; since more concessions are made, surplus is generated for both parties.

5.2.4.2 Results of Part 2: Agents use reactive strategies in bargaining

Figure 5-6 shows some examples of encounters among agents with various reactive strategies. Unlike the results in part 1, more failures are detected due to the characteristics of reactive agents who are more risk-seeking and "stubborn" to insist on their offers. However, the success rate and the average surplus generated are higher in the N-protocol, as shown in Table 5-5.



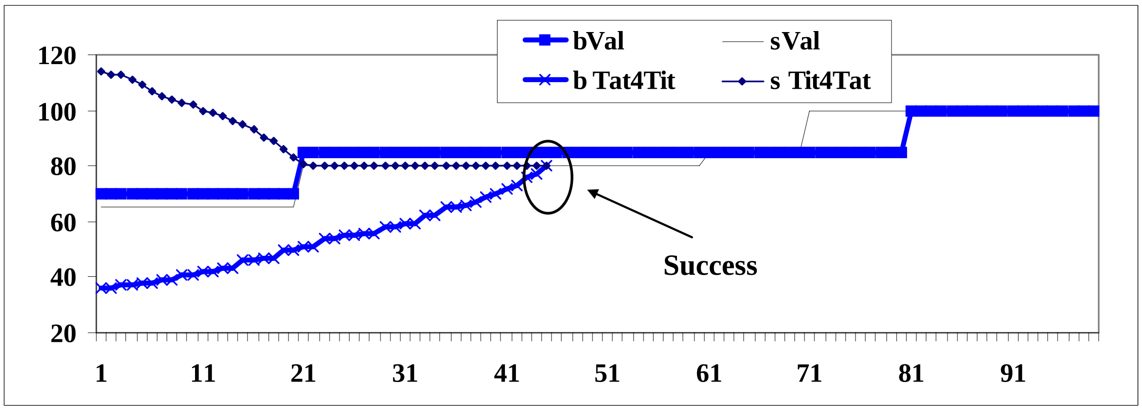


Figure 5-6 Examples of encounters between agents with reactive strategies

Table 5-5. Result of experiment in part 2 (non-monotonic-offers protocols)

	M-protocol				N-protocol			
	S1B1	S1B2	S2B1	S2B2	S1B1	S1B2	S2B1	S2B2
Ave. Surplus (cu.)*	7.50 *(11.86)*	7.47 *(11.81)*	7.61 *(10.09)*	7.60 *(10.09)*	8.46 *(11.87)*	8.55 *(11.73)*	8.38 *(10.11)*	8.48 *(10.04)*
Success rate (%)	57.6	57.7	65.9	65.5	100.0	100.0	100.0	100.0

*The corresponding standard deviation is inside the parentheses

Within the same group (either within M-protocol or N-protocol) the results are similar. Some of them are not statistically different. For instance, the *p*-values from the comparison of the average surplus between “S1B1” and “S1B2” groups under M-protocol is approximately 0.564. The corresponding *p*-value of “S1B1” and “S2B2” groups under N-protocol is approximately 0.736, which means they are unlikely to represent different groups. However, the *p*-value of the average surplus made under M-protocol and N-protocol is close to zero (approximately = $3.8842\ 10^{-277}$), which means they are statistically different. Hence, the effect of allowing non-monotonic offers among reactive-strategy and random-strategy agents, in terms of the generated surplus, is significant.

Finally, analyzing success rates, 100% success rate is gained under our experiment in N-protocol, which is much higher than 57% - 66% in M-protocol. This result justifies that N-protocol is better than M-protocol in terms of success rate, when both parties have increasing valuations.

5.2.5 Summary

In general, agents in N-protocol generate higher surplus compared to agents in M-protocol. The success rates of negotiation under N-protocol are also higher in most cases. Thus, our experimental results support our theoretical analysis in Chapter 4 regarding the benefit of allowing non-monotonic offers, even under the more relaxed agent constraints described in this chapter.

5.3 Empirical Study of Delay-based Bargaining Protocol

The purpose of the empirical study in this section is to analyze the cost and benefit of a delay-based bargaining protocol under more forgiving constraints allowing for irrational agents as with the other studies in this chapter. Specifically, we compare agents’ surplus and success rate under delay- and non-delay-based bargaining protocols.

We assume that buyers under the non-delay-based protocol will always make a concession unless they are stuck on their valuation. And buyers under the delay-based protocol may stick on the same offer for several rounds if they decide to use strategic delay. Buyers will use strategic delay only after observing their future valuation, i.e. delay will be used if their future valuation is higher than their current one. For that purpose, we only simulate cases of buyers' increasing valuation, while the sellers' valuation is kept constant. The seller will never delay the bargaining and both will concede to offer their valuation at their time deadline.

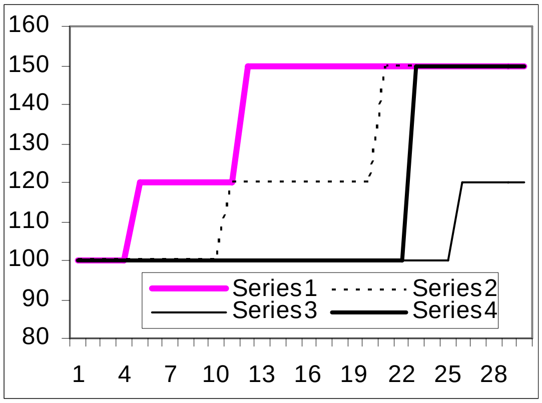
5.3.1 Experimental Design

10 different sellers' valuations and 3 groups of 10 different buyers' valuations are used in this experiment. The buyers' valuations are increasing over time from 100 cu. to as high as 150 cu. with time deadline at the 30th round. In contrast, the sellers' valuations are constant over time (= 100 cu.), but they have various time deadlines (from the 21st to 30th round). The reason for varying the sellers' time deadline is to simulate the cost of strategic delay. For example, a long delay by the buyer may cause a breakdown when the seller's time deadline is short. If the seller's time deadline is very long, then no cost will occur.

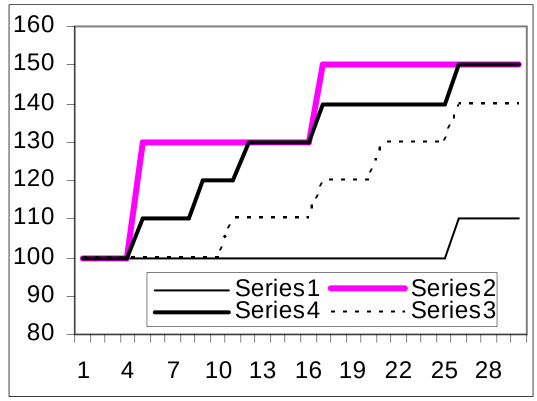
Since we could have various increments of the buyer's valuation, we study three groups here: high-increment valuations (HI), medium-increment valuations (MI), and low-increment valuations (LI). The reasons of studying these groups are:

(i) a larger increment of valuation within a short period of time may be more likely to overcome the cost of delay;

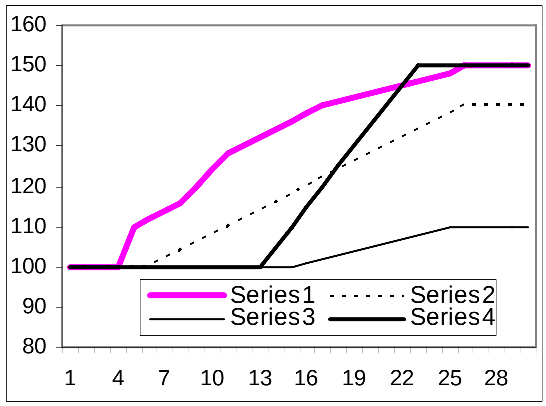
(ii) a higher frequency of increment may induce a more frequent delay.



(a) HI



(b) MI



(c) LI

Figure 5-7. Three groups of buyer valuations used in the experiments

Figure 5-7 (a), (b), and (c) show some examples of each of these valuations (respectively HI, MI, and LI). The horizontal line represents the time and the vertical line represents the valuation. For the sake of clarity, only four different valuations (out of ten) are shown in each diagram.

With regard to the negotiation strategy, we consider two groups of sellers, namely:

- *Time-dependent group*: sellers who monotonically reduce their offers with a constant rate;
- *Tit-for-tat group*: sellers who follow an opponent's concession, but with various initial offers.

The buyer will always use a time-dependent strategy by increasing its offer at a predefined rate, with or without a delay. The delaying strategy uses the following rule:

Buyer's delaying rule: If the buyer observes an increment of its valuation within the d-th round ahead (compared to that in the current round), then a delay will be used at the current round.

Here, d represents the buyer's myopic horizon. When $d = 0$ the buyer will never delay its offer, so it resembles non-delaying bargaining. Since the buyer's valuation is increasing over time, a higher d implies a higher likelihood of a longer delay. We use $d = \{0, 1, 2, .., 12\}$, and this becomes the control variable. Table 5-6 shows other parameters used in our experiment. Given these parameters, for each group of buyers' valuations we have (10 x 10) pairs of agents' valuations, (3 x 3) pairs of their initial offers, (4 x 4) pairs of their increment/decrement of offers, 2 different seller strategies, and 12 different buyers' delaying strategies, totaling to 345600 different trials. Therefore, for 3 different groups of buyers' valuations we run 3 x 345600 = 1036800 trials.

Table 5-6. Parameters used in both parts of experiment (delay-based protocols)

Parameters	*Values*
Maximum bargaining rounds	≤ 30
Sellers' initial offer	{120, 130, 140} cu.
Buyers' initial offer	{70, 80, 90} cu.
Buyers'/sellers' increment/decrement of offers	{1, 2, 3, 4} cu.
Acceptance criteria of both buyers and sellers	EvalF-I

5.3.2 Evaluation Criteria

In all trials, two main variables are recorded for evaluation purposes: surplus generated, in terms of the sum of surplus for both buyers and sellers, and breakdown/success of the negotiation. Then, three pieces of information are computed: total surplus, average surplus per successful transaction, and success rate.

5.3.3 Results

Figure 5-8 shows the results of our experiment for various values of $d = \{0, 1, 2, \ldots, 12\}$ with respect to the success rate (in %), surplus rate (in %), and total surplus (in cu.).

The success rate is calculated by dividing the number of successes by the total number of trials (= 28800 trials). The surplus rate is calculated by dividing the average surplus of successful negotiations with the estimate of the maximum surplus. And the estimate is calculated as the average surplus when the negotiation succeeds between round 21 and 30, i.e. $\frac{1}{100}\sum_{j=1}^{10}\sum_{t=21}^{30}(B_{jt}-100)$, where j is the index of 10 different buyers' valuations and t is their valuation from round 21 to 30. Since this estimate is for the average of maximum surplus, it is possible that the real surplus is higher than this estimate, as shown in Figure 5-8 (e).

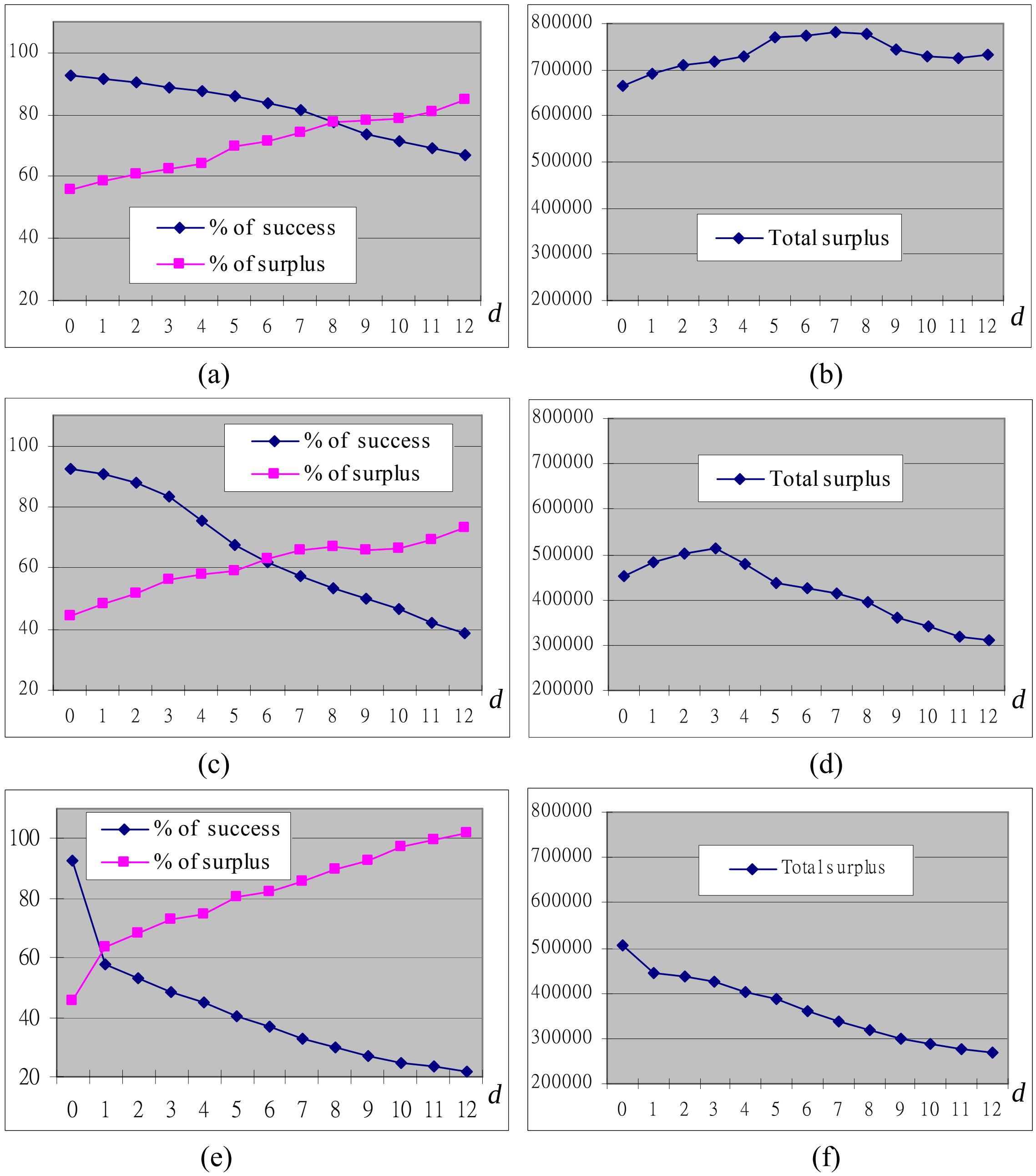
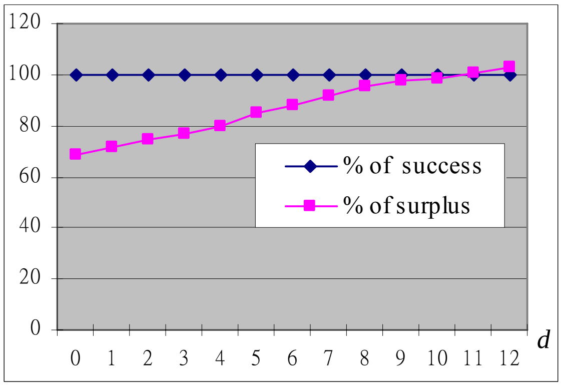


Figure 5-8. The experimental results of HI (a and b), MI (c and d), and LI (e and f) for various *d* (x-axis)

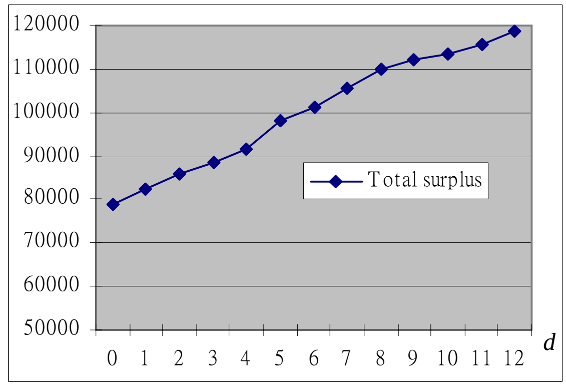
It is shown from Figure 5-8 (a), (c), and (e) that a longer delay from a higher horizon *d* may increase the surplus rate but reduce the success rate. This is intuitive, because a longer delay may defer the transaction to the time where the buyers' valuation is higher, but at the same time causes a higher rate of breakdown due to the sellers' earlier deadline. However, the results on total surplus are not as intuitive as shown in Figure 5-8 (b), (d) and (f). Several more important results are as follows:

- The decreasing of success rate is slower in the HI group compared to that in the MI and LI groups, and in the LI group's it is the fastest. We observe that the frequency of the increment of the buyers' valuation in those groups has affected the success rate. When the buyers' valuation increases more frequently over time, such as in the LI group, delay will be invoked more frequently and continuously until a stable valuation is reached; thus prolonging the delaying time. However, when the buyers' valuation increases less frequently, such as in the HI group, only very few delays will be invoked. Conclusively, HI is more stable in terms of the success rate; thus a moderate horizon ($d \leq 6$) could be applied without increasing the risk of breakdown (less than 10% reduction of success rate).
- A fast reduction of success rate appears in the LI group when a delay is adopted, i.e. from 92.5% for d = 0, to 57.9% for d = 1, as shown in Figure 5-8(e). This phenomenon is accompanied by a drop of the total surplus (a decrease from approximately 507000 to 444000 cu. in the left-most data in Figure 5-8(f)). However, the average surplus is steadily increasing until d = 12, where it exceeds the estimation of maximum surplus.
- The total surplus is increasing when d is less than or equal to 7 and 3 in HI and MI respectively; but it is decreasing over d in the LI group (Figure 5-8(f)). The increases of total surplus in HI and MI groups justify our hypothesis that the society may benefit from adopting strategic delay when the buyers' valuations are increasing and

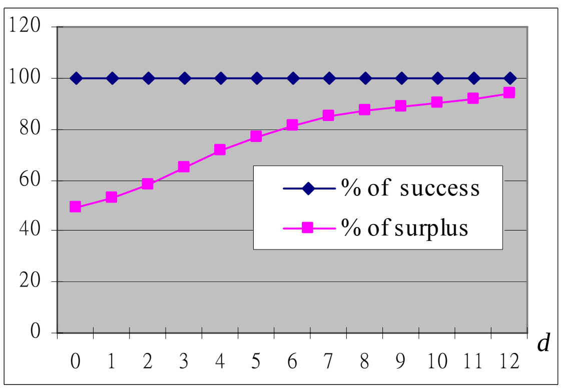
the sellers' valuations are constant. This result provides counter evidence to the common belief that a delay in negotiation deteriorates social welfare in terms of total surplus (joint utility) in the presence of a time deadline (shorter sellers' time deadline).



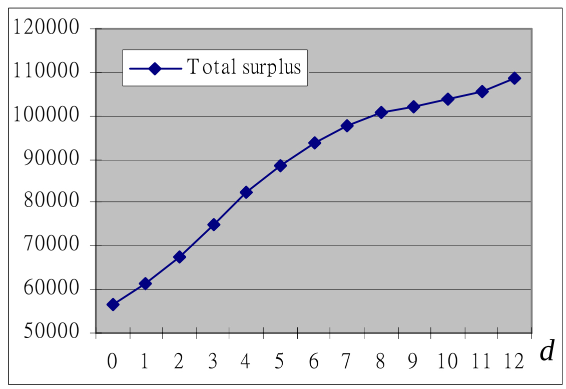
(a)



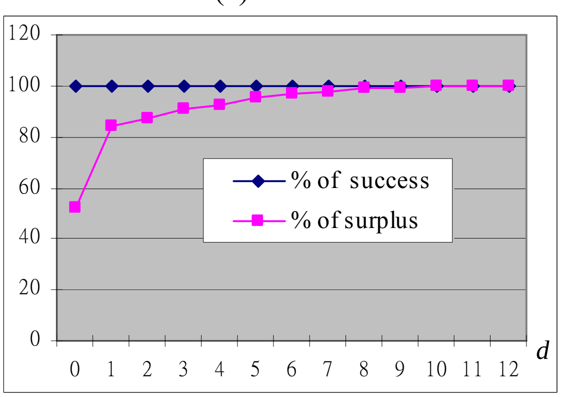
(b)



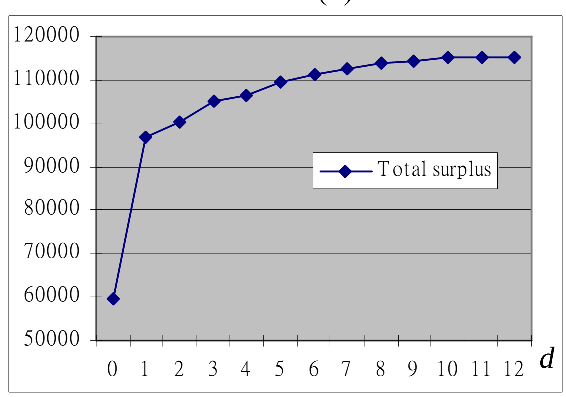
(c)



(d)



(e)



(f)

Figure 5-9. The experimental results of HI (a and b), MI (c and d), and LI (e and f) when the sellers' time deadline is the same as that of the buyer (the 30th round)

From our experimental results, delay deteriorates social welfare only when *d* is high or when the buyer's valuation is increasing more frequently (LI group). Note here that lower social welfare occurs because the sellers have a shorter time deadline. If the sellers' time deadlines are equal to those of the buyers', then the cost can be avoided. Figure 5-9 shows partial experimental results for the case when the sellers' time deadline is equal to the buyers' (at the 30th round). It is shown that the total surplus is always increasing and the success rate is always 100%, because by the end of the negotiation the buyers' valuation is always exceeding the sellers', thus always resulting in a transaction. From this partial result, the LI group benefits most from adopting delay, a jump of percentage and total surplus when *d* increases from 0 to 1. However, from the general results (Figure 5-8), we conclude that a longer delay could be applied when the frequency of increment is low (HI group).

Another interesting setting is to study the worst-case scenario in which the sellers are facing the shortest time deadline. Figure 5-10 shows the experimental results.

An interesting anomaly is observed in the HI group, where the total surplus is not monotonically decreasing after a delay is adopted (see Figure 5-10 (b) when *d* = 0 to 5). This anomaly originates from the increasing speed of the breakdown rate that is systematically caused by the buyers' valuation. Figure 5-11 shows (a) the absolute success rate and (b) three buyer's valuations that cause the drop of success rate at *d* = 1, 2, and 3. From Figure 5-11(a) we observe that a fast drop of success rate occurs when *d* increases from 0 to 1 and from 2 to 3. These drops explain the decreasing of total surplus when *d* is 1 and 3 (Figure 5-10(b)).

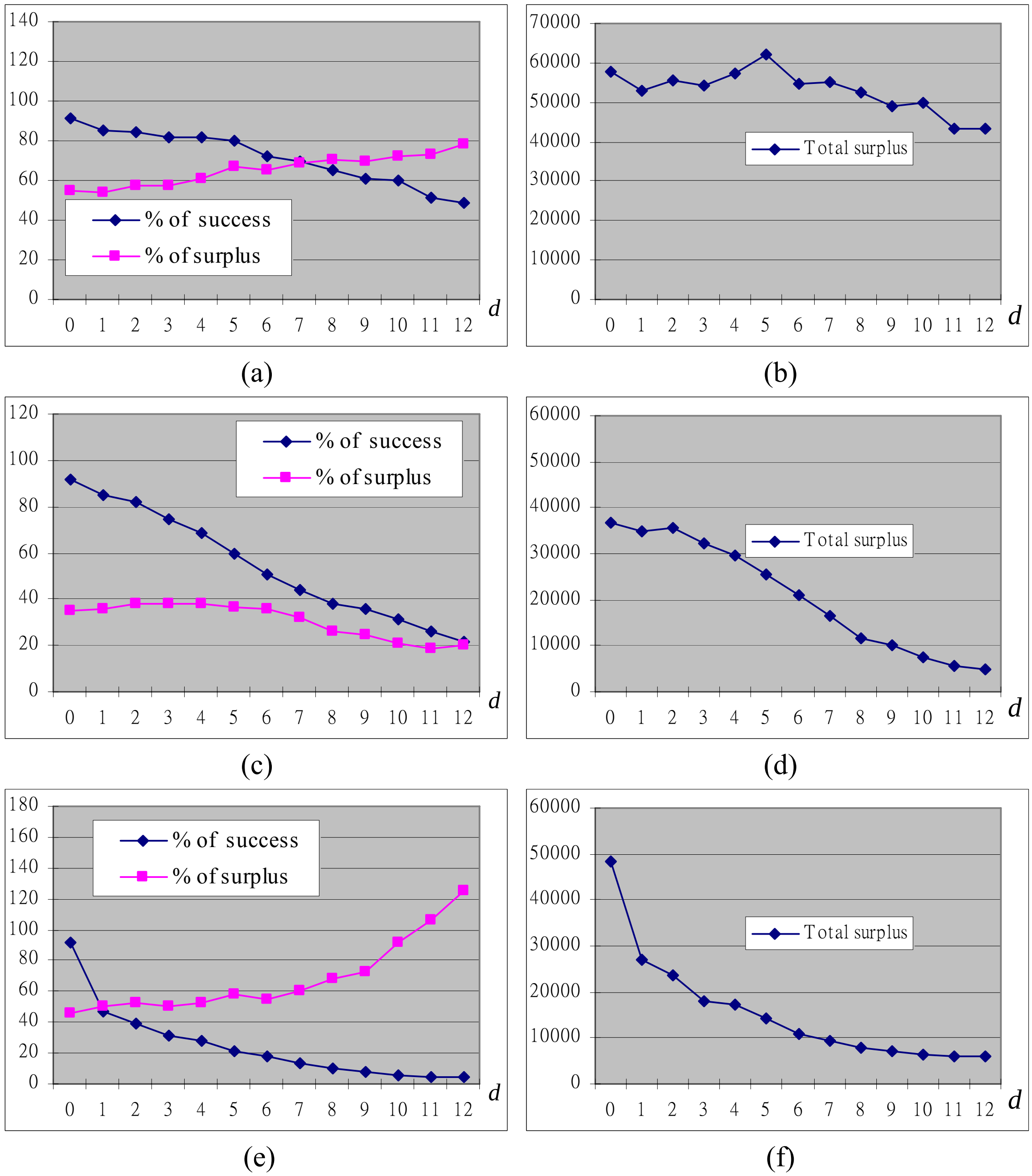


Figure 5-10. The experimental results of HI (a and b), MI (c and d), and LI (e and f) when the sellers' time deadline is the shortest one (the 21th round)

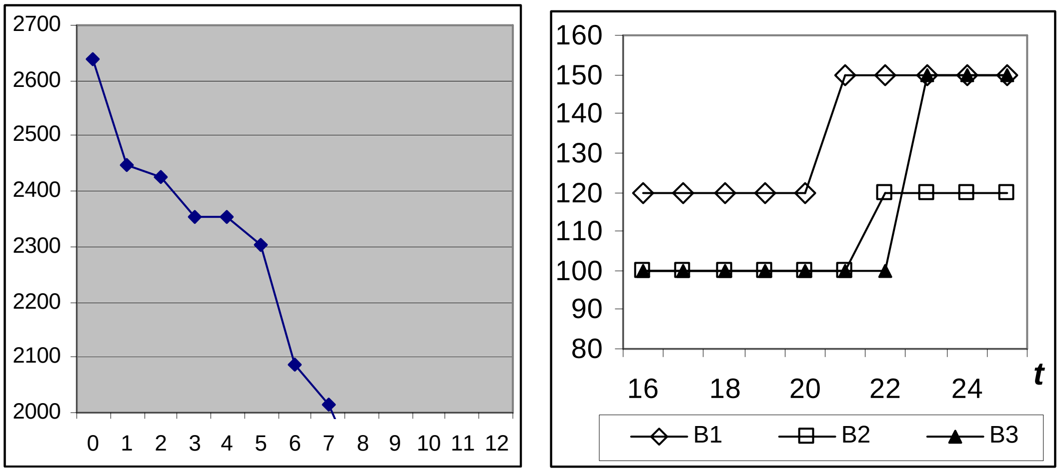


Figure 5-11. (a) The absolute success rate in HI group for the shortest sellers' deadline and (b) the buyers' valuations that caused the decreasing of success rate at $d = 1$, 2, and 3.

But where does the drop of success rate come from? Recall here that the seller's deadline is at $t = 21$, or it will accept any buyer's offer at $t = 20$ as long as it is greater than or equal to 100 cu. In other words, a delay by the buyer at $t = 20$ may cause a breakdown because the delay also delays its offer reaching 100 cu. Thus, the buyers' valuation around $t = 20$ will determine whether a breakdown occurs or not. From Figure 5-11 (b), the buyers with valuation B1 will delay their offer at $t = 20$ when $d = 1$, 2, or 3; similarly, buyers with B2 will delay their offer at $t = 20$ when $d = 2$ or 3, and buyers with B3 will delay their offer at $t = 20$ when $d = 3$. Or, B1, B2, and B3 will cause some breakdowns for $d = 1$, 2, and 3, respectively. Since the magnitude of B2 is only 20 cu. (see Figure 5-11(b)), the loss caused by it, in terms of total surplus, is not as severe as those from B1 and B3; therefore, the decreasing of total surplus happened when d is 1 and 3 only.

Nevertheless, there is only one case where the total surplus exceeds the surplus without delay ($d = 0$), that is at $d = 5$ (62000 cu.) (in Figure 5-11 (b)) which is not much higher than that in $d = 0$ (57620 cu.). And this is the only case for all conditions shown in Figure 5-10 (b), (d), and (f). Therefore, we may conclude that delay is generally not helpful, in terms of increasing the total surplus, when the seller's deadline is much shorter than that of the buyer's.

Since sellers may use either the time-dependent strategy or tit-for-tat, we also would like to know the impacts of those different responses toward buyers' delay. When a buyer delays its decision, it is reasonable for the seller to wait until the buyer has made its decision. So, tit-for-tat can be seen either as a retaliation by the seller or a norm in bilateral negotiation. Sometimes, it may also happen that the seller steadily changes its offers, even when the buyer has delayed its decision: especially when the seller is eager to sell the item. Thus, both the time-dependent strategy and tit-for-tat become our focus here. Figure 5-12 and 5-13 show the total and average surplus, respectively, when the sellers use either time-dependent or tit-for-tat, respectively.

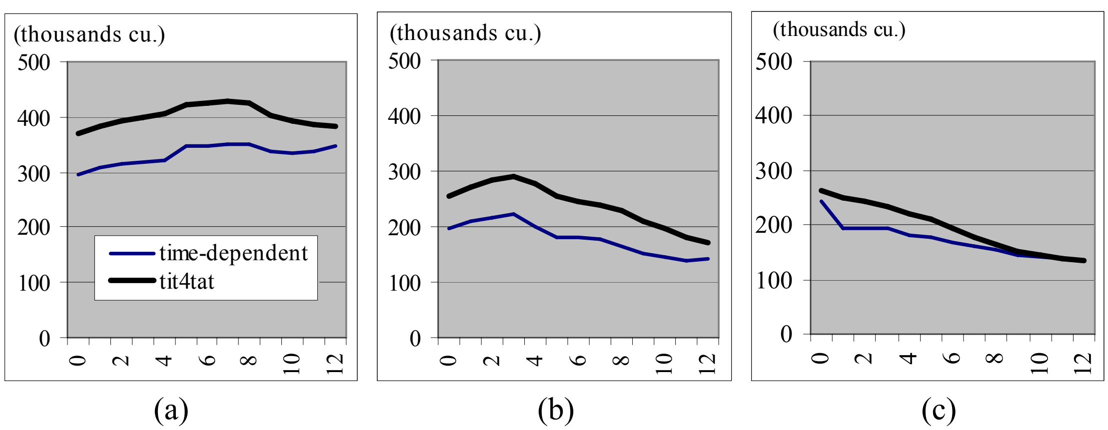


Figure 5-12. The total surplus for various d (x-axis) in (a) HI, (b) MI, and (c) LI groups for both time-dependent and tit-for-tat groups

It is shown here that higher total and average surpluses are observed in the tit-for-tat group. This phenomenon is not surprising because a longer delay will be generated in the tit-for-tat group due to the delay by the sellers when the buyers delayed the negotiation. The peaks of social welfare in the HI group are at $d = 8$ (time-dependent) and $d = 7$ (tit-for-tat), and are the same at $d = 3$ in MI the group (Figure 5-12 (a) and (b)). This result shows that the pattern of social welfare depends on the horizon of delay, not on the sellers' strategy. However, this conclusion and also all previous conclusions are only applied in our experimental setting. They may not be generalized into other situations, such as when the sellers use tat-for-tit or other time-dependent strategies (e.g. Conceder or Boulware strategies), etc.

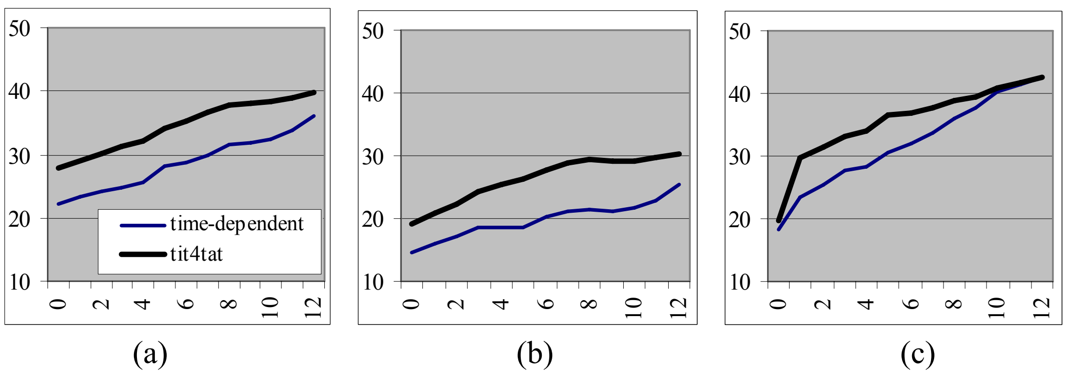


Figure 5-13. The average surplus in (a) HI, (b) MI, and (c) LI groups

5.3.4 Summary

So far, we have shown through simulations the benefit of strategic delay, especially in terms of total and percentage surplus. This result complements our theoretical analysis in Chapter 4 that strategic delay may be beneficial from an agent's individual perspective.

5.4 Empirical Study of Ignorance-based ABN

The purpose of this empirical study is to analyze the benefit of the ignorance- and argumentation-based negotiation protocol. In Chapter 4 we have shown the benefit of strategic ignorance from an individual agent's perspective. In this section we will show the benefit of allowing strategic ignorance from the agent society's perspective. Specifically, we compare social welfare under ignorance-based and non-ignorance-based ABN protocols. We assume that agents under a non-ignorance-based protocol will always argue on each topic/issue, while agents under an ignorance-based protocol may ignore a topic if they believe that arguing is costly. Since the agents' beliefs may deviate from reality, i.e. a deemed beneficial topic may be costly and a deemed costly topic may actually be beneficial, we will also simulate the deviation of the agents' beliefs. In addition, ignorance may also cause distrust among agents or generate a side-cost, which will also be simulated by cost-bearing ignorance.

5.4.1 Experimental Design

We assume agents have 20 different topics and each agent has their own estimated surplus (benefit/cost) from discussing each topic. We randomly generate the estimated surplus between -1 cu. and 1 cu., where the value within [-1, 0] cu. represents the cost of argumentation and within (0, 1] cu. the benefit of argumentation. We use either uniform or normal probability distribution functions to generate those values. The actual values are generated by randomly deviating the estimated values so that each value can deviate by up to $\pm b$ cu., where b (bias) is a control variable with discrete value = {0 cu., 0.01 cu., 0.02 cu., …, 2.00 cu.}. The bias b is also generated randomly by using either a uniform or a normal distribution function, but is always consistent with the distribution function in generating the estimated surplus. In both cases the uniform distribution function is within the range [-1, 1] cu., while the normal distribution function's mean value equals to 0 cu. and its standard deviation equals to 0.5 cu respectively. However, we restrict agents' surplus within a range [-1, 1] cu.; thus, if an out-of-range value was generated from a normal-distribution random generator, we will randomly generate it once again. If the second generated value is still out-of-range, then we replace it with the boundary value -1 cu. (if the secondly generated value is less than -1) or 1 cu. (if the second generated value is greater than 1).

Under the non-ignorance protocol, we assume both agents will argue on all topics; thus, the social welfare is calculated by adding actual surplus after discussing all topics. We divide the ignorance-based protocol into two groups: proactive-ignorance and full-ignorance (both proactive and reactive). In the proactive-ignorance case we assume argumentation over a topic will be performed if the topic is beneficial for either side; thus, we exclude topics that generate negative surplus for both sides. Conversely, in the full-ignorance case, only mutually beneficial topics will be discussed by the agents; thus, we sum the surplus of topics that are estimated as beneficial topics by both.

The cost of ignorance is deducted from the agents' surplus. It is intended to simulate the side-cost from ignorance, such as distrust, penalty, etc. We varied the cost from 0 cu. (cost-free) to 2 cu. to study its effect. This factor becomes the second major control variable in our experiment. Finally, we also varied the mean value of the uniform distribution function to study its effect toward the social welfare; thus, the ranges of uniform distribution varied from [-1, 1] cu. to [0, 1] cu. The latter is to simulate the situation where agents are very optimistic or they believe that no cost will be generated from the argumentation. The variation of this mean value is the last control variable in our experiment.

In total, we use 2 different random generators, 200 different costs of ignorance, 200 different agents' bias *b*, and 200 different ranges of uniform distribution functions. And we repeat each setting with 10 different random values for statistical analysis purposes. But we do not test all combinations of those factors. Instead, we sub-divide them into 20 groups of experiment and run 2000 trials for each group. So this leads to 40000 trials, overall. These groups can be categorized into the following four parts:

- Part 1: study the effect of bias *b* in each of the protocol (non-ignorance, proactive-ignorance and full-ignorance), where both the estimated surplus and bias are generated from uniform-distribution random generator. We run 3 groups of experiments in this part, i.e. {non-ignorance, proactive-ignorance, and full-ignorance}.
- Part 2: the same as those in Part 1, except that both the estimated surplus and bias are generated form normal-distribution random generator. We also run 3 groups of experiments in this part.
- Part 3: study the effect of the cost of ignorance under a full-ignorance protocol for both uniform- and normal-distribution random generator. Here, the bias *b* is restricted to {0, 2} only. Thus, we run 4 groups of experiments in this part, i.e. {(uniform-distribution; b = 0), (normal-distribution; b = 0), (uniform-distribution; b = 2), (normal-distribution; b = 2)}.
- Part 4: study various mean values (μ) in normal-distribution random generator. It includes all three protocols and four different biases (*b* = 0, 0.5, 1, and 2). We assume zero cost and run 10 groups of experiments in this part: all combination of three

protocols for various biases except for the proactive-ignorance protocol for $b = \{0, 2\}$ only.

5.4.2 Evaluation Criteria

Since we are only concerned with the overall social welfare, the total actual surplus is our important measure. We assume that the ignorance does not affect the success rate, but incurs some costs.

5.4.3 Results

Figure 5-14 shows the results of our experiment for various values of bias b (x-axis) with respect to the total surplus (y-axis) when we use the uniform distribution function to generate the surplus (Part 1 of our experiment).

It is shown in Figure 5-14 (a) that the average of total surplus in non-ignorance-based ABN is approximately zero regardless of the bias of agents' estimation, as shown by a very small slope parameter of the linear regression function (= 0.0772). This result is not surprising because all agents are forced to discuss all topics no matter whether they incur cost or benefit, which, in aggregate, will cancel out each other (the average surplus is zero). Conversely, Figure 5-14 (b) and (c) show that the average of total surplus is decreasing over b, as shown by a negative slope parameter (-2.9994 and -1.3348, respectively) of the linear-regression function. A positive average total surplus is observed at $b = 0$, because strategic ignorance is used to filter out those topics that generate negative surplus, thus leaving a positive surplus in the summation of the total surplus. The negative slope can be explained as follows: an increasing of bias causes a higher estimation error, which more severely deteriorates the filtering by ignorance; thus, more topics picked by agents are actually generating costs which reduce the average total surplus.

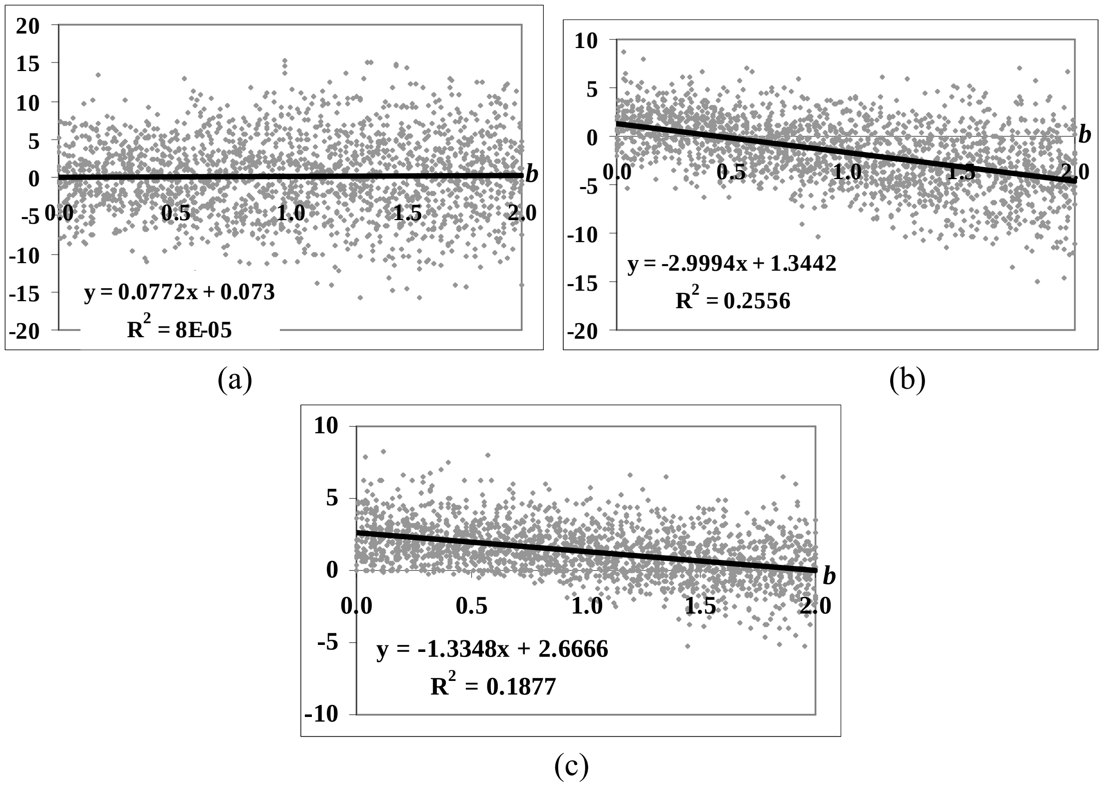
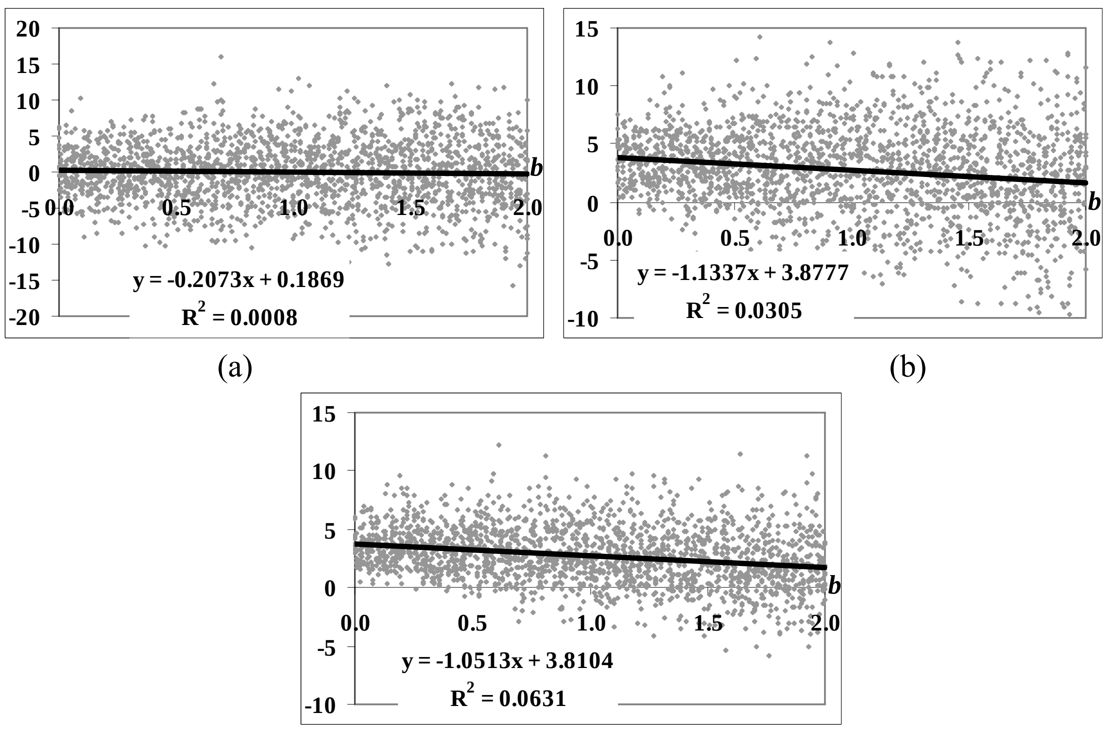


Figure 5-14. The total surplus (y-axis) in (a) non-ignorance, (b) proactive-ignorance, and (c) full-ignorance-based ABN (cost-free) against various biases in predicting the real surplus (x-axis) under the uniform-distribution random generator

If we compare Figure 5-14(b) and (c), then higher total surplus is observed in Figure 5-14(c), which means that allowing full-ignorance is better than only allowing proactive-ignorance. However, in all cases the R-square values are low, indicating that those linear-regression functions cannot predict the stochastic outcomes (total surplus) given a bias *b*. The results from normal-distribution random generators are different from those from the uniform-distribution random generator, as no significant difference is observed from proactive and full-ignorance cases in the normal-distribution-random-generator group (Part 2 of our experiments), as shown in Figure 5-15 (b) and (c).

Since the normal-distribution random generator uses a lower standard deviation (= 0.5) compared with the uniform-distribution random generator (= 0.58), we expect less estimation error from the former. This explains the flatter negative slopes, i.e. -1.1337 and -1.0513 in Figure 5-15(b) and (c), compared with the slopes in the uniform-distribution generator, i.e. -2.9994 and -1.3348 in Figure 5-14 (b) and (c), respectively. In conclusion, a higher uncertainty or error in estimating the surplus from the argumentative topics causes a lower expected social welfare (total surplus).

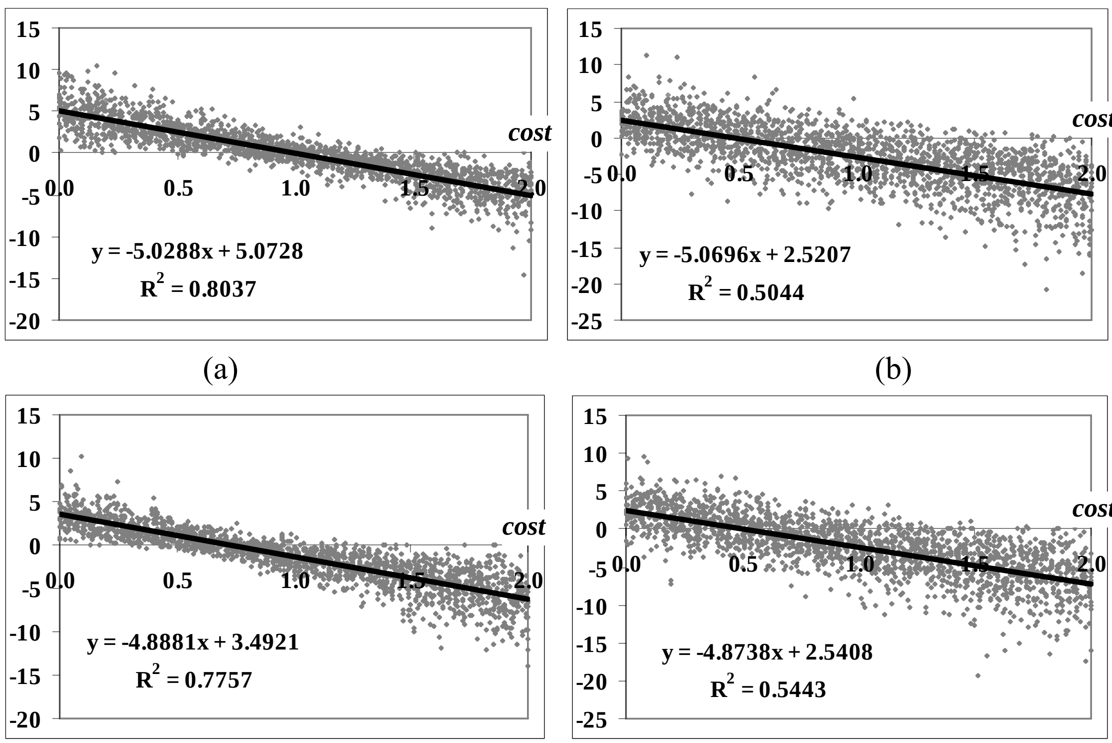


(a) (b) (c)

Figure 5-15. The total surplus (y-axis) in (a) non-ignorance, (b) proactive-ignorance, and (c) full-ignorance-based ABN (cost-free) against various biases in predicting the real surplus (x-axis) under the normal-distribution random generator

Next, we want to see the effect of various costs on full-ignorance cases (Part 3 of our experiments). Figure 5-16 shows the simulation results with various costs (x-axis) for both random generators and for b = 0 and 2.

Since the cost of ignorance reduces the total surplus, we expect a lower social welfare as the cost increases. Figure 5-16 conforms to this rationale as shown by decreasing linear-regression trends in all cases. At a certain cut-off, a negative social welfare will be generated, where the cut-off is lower in the right-side diagrams (Figure 5-16 (b) and (d)) compared with their left-side counterparts, and also lower in bottom-side diagrams compared with their upper-side counterparts. These results come from the uncertainty factor or bias in estimating surplus (higher bias in the right-side diagrams, and higher standard deviation in the bottom two diagrams).

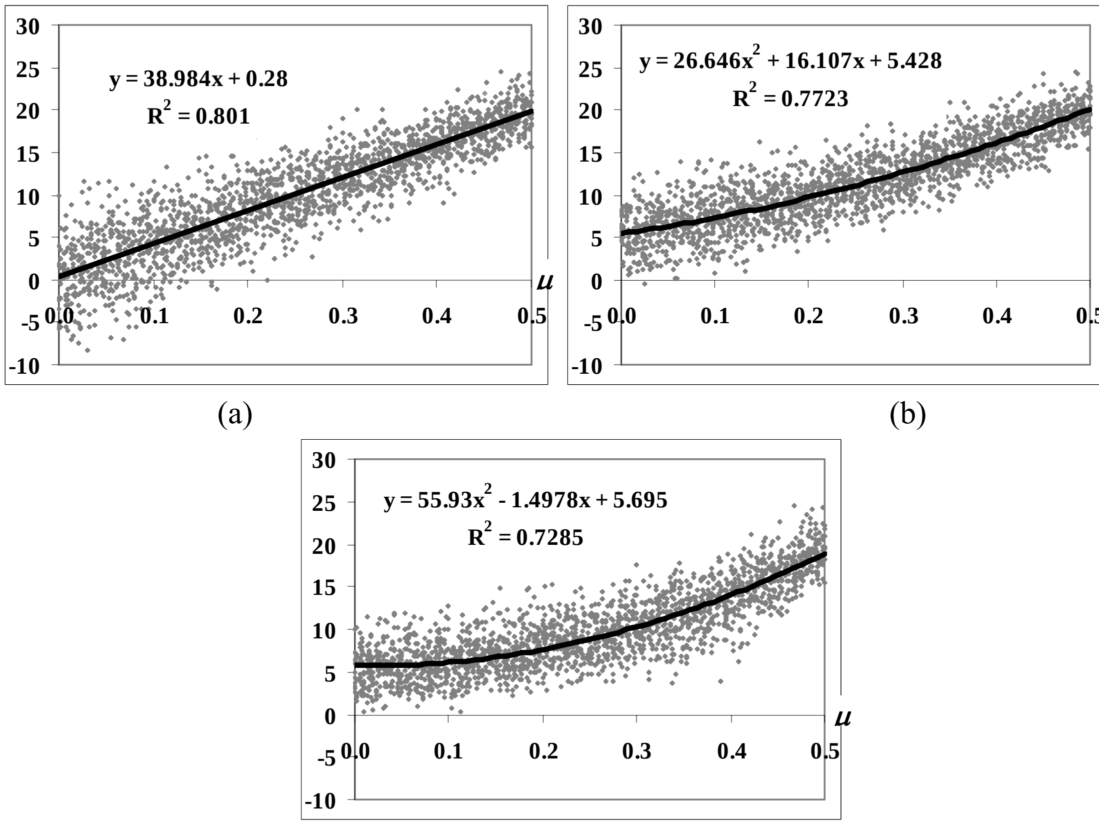


(c) (d)

Figure 5-16. The total surplus (y-axis) under the uniform-distribution random generator (a and b) and normal-distribution random generator (c and d) against various costs of ignorance (x-axis), where the bias is 0 (a and c) or 2 (b and d)

The above results support the usage of responsive-ignorance, especially when both the cost of ignorance and the uncertainty of the estimated surplus are reasonably low.

However, that conclusion is applied when the mean value of surplus is zero. What if the mean value is not zero? Figure 5-17 shows the result after we alter the uniform-distribution random generator so that the distribution range of surplus varies from [-1, 1] to [0, 1], with mean values changing from 0 to 0.5, respectively (x-axis of Figure 5-17), and with zero cost (Part 4 of our experiments).



(a) (b)

(c)

Figure 5-17. The total surplus (y-axis) against various mean values (μ) of the uniform-distribution random generator (x-axis), for bias $b = 0$ and for (a) non-ignorance, (b) proactive-ignorance, and (c) full-ignorance-based protocols

It is shown in Figure 5-17 (b) and (c) that both proactive and full ignorance help agents attain higher social welfare compared to non-ignorance when the mean values are low (< 0.15). The superiority of the ignorance-based protocol increases when the bias increases (see Figure 5-18). The intersections of quadratic equations (full-ignorance) and linear equations (non-ignorance) shift further to 0.2 as the bias increases (orderly from (a) to (d)). It is conclusive, therefore, that the full-ignorance-based protocol is more likely to outperform the non-ignorance-based one when the bias is larger.

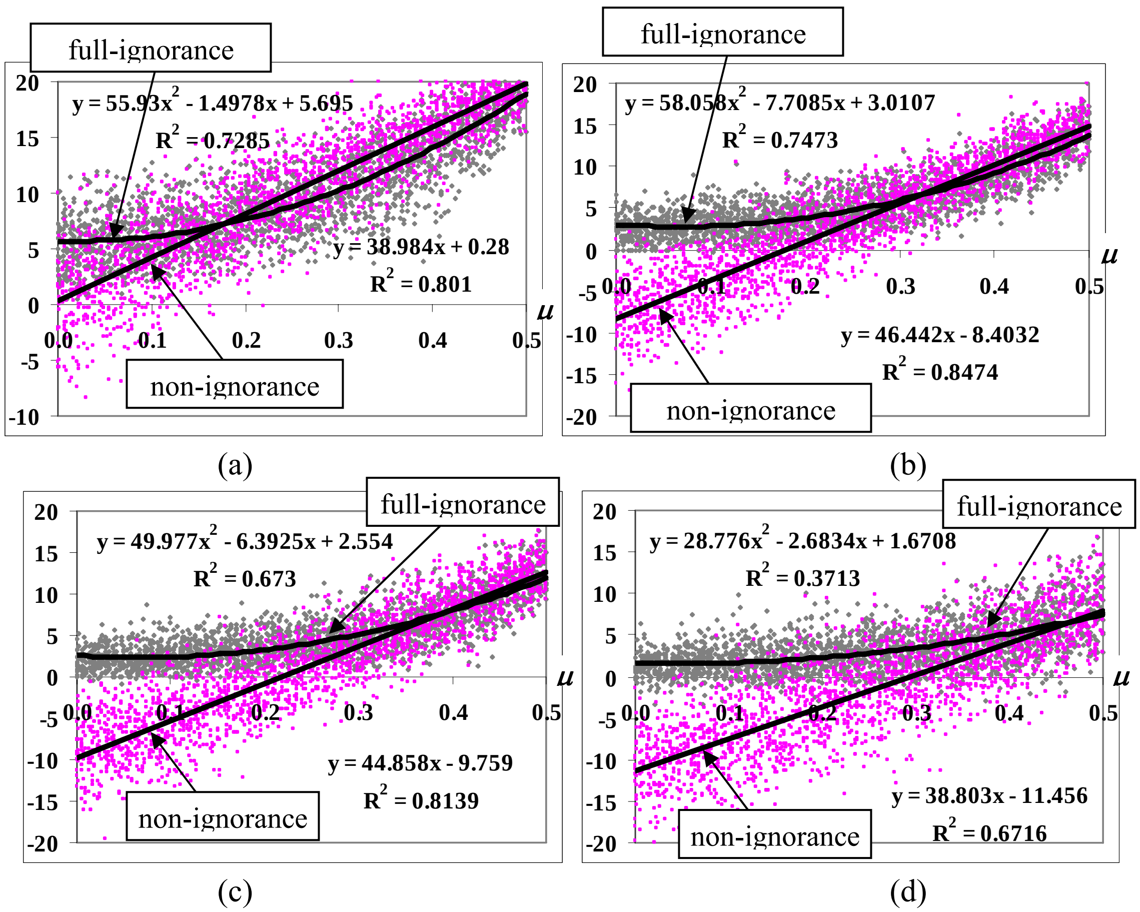
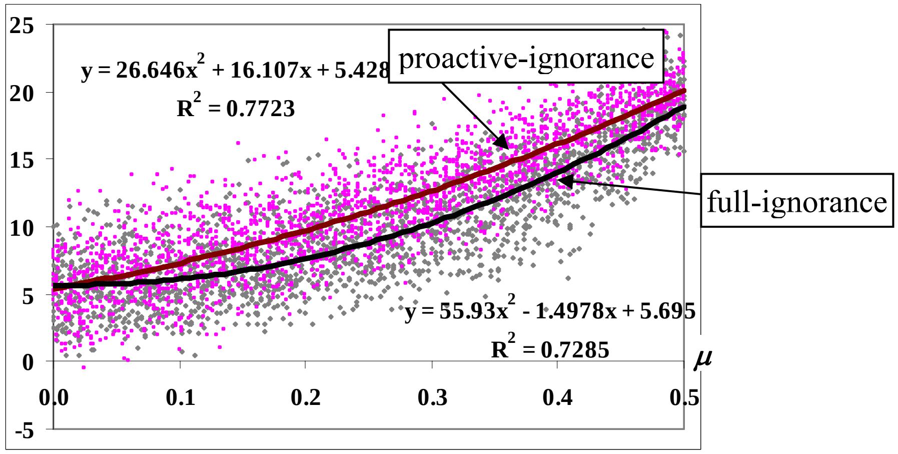
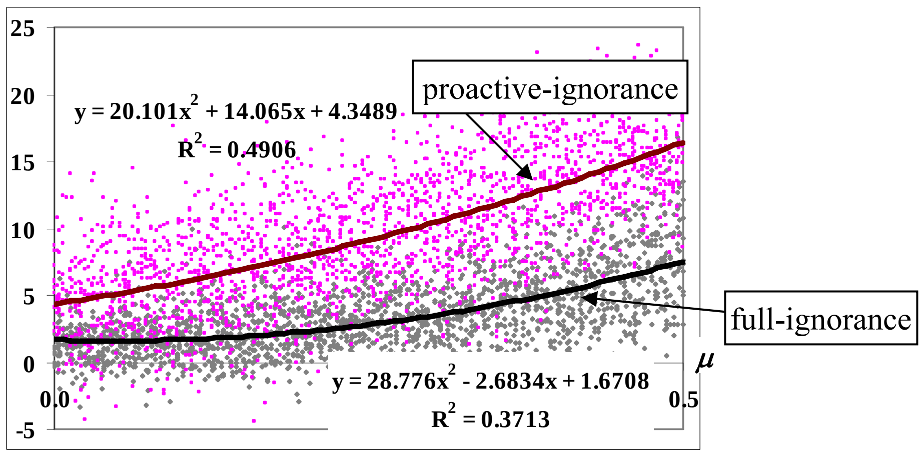


Figure 5-18. The total surplus (y-axis) against various mean values (μ) of the uniform-distribution random generator (x-axis), for bias equal to (a) 0, (b) 0.5, (c) 1.0 and (d) 2.0, and under both non-ignorance and full-ignorance protocols



(a)



(b)

Figure 5-19. The total surplus (y-axis) against various mean values (μ) of the uniform-distribution random generator (x-axis), for bias equal to (a) 0 and (b) 2.0, under both proactive-ignorance and full-ignorance protocols

A more interesting result is shown in Figure 5-19 when we compare proactive and full ignorance for various mean values. From both Figure 5-19 (a) and (b) we conclude that proactive ignorance (shown by upper quadratic equations) outperforms full ignorance (shown by lower quadratic equations) when we change the range of the random generator. This result can be explained from the calculation of total surplus. Under the full-ignorance case, we only add surpluses that are considered positively by both agents (both-win), while in proactive-ignorance we also add surpluses that are considered positively by a single agent only (single-win). Therefore, in addition to both-win topics as in the full-ignorance case, some single-win topics will also be argued in the proactive-ignorance case. Since the mean values are increasing, the benefit gained by the agent in single-win topics may supersede the cost incurred from its opponent. Thus, in aggregate, a positive marginal surplus will be generated, which causes the total surplus of the proactive-ignorance case to be higher than that of the full-ignorance case.

5.4.4 Summary

We have shown through simulation the benefit of full and proactive ignorance in terms of total surplus. Specifically, they will be beneficial if the cost of ignorance is relatively low, the bias is also reasonably low, and the cost of argumentation is relatively high (low mean value).

This result complements our theoretical analysis in Chapter 4 that strategic ignorance may be beneficial from an agent's perspective.

5.5 Chapter Summary

This chapter is concerned with empirical studies of modified protocols, i.e. allowing non-monotonic offers, strategic delay, and strategic ignorance in argumentation-based negotiation. The results have proven the benefit of these modified protocols under various settings. In Chapter 4 we have shown the benefit of the modifications by means of theoretical analysis from agents' perspective. In this chapter we demonstrate their benefits and drawbacks in term of social welfare and success rate, which complement the results drawn in Chapter 4.

For example, it is difficult to conclude from the theoretical analysis whether or not the society will really benefit from allowing non-monotonic offers, delaying the negotiation or ignoring arguments from others. The primary reason is due to the large combinations of (irrational) strategies in a negotiation and many uncertain factors affecting agent decisions.

From theoretical analysis we have proved that agents tend to prefer a non-monotonic-offers bargaining protocol to monotonic-offers bargaining one; however, we do not know if allowing non monotonic offers bargaining protocols will be able to increase the social welfare of both agents. In order to understand it, experimental studies were carried out. We find out that allowing non-monotonic offers can increase both the success rate and total surplus of the negotiation. Similarly, allowing a relatively short delay can increase the total surplus and slightly reduce the success rate. Furthermore, allowing ignorance in argumentation-based negotiation can be useful for both parties, especially when they can accurately predict the benefit from the ignorance. All of these are hard to be derived from theoretical analysis.

CHAPTER 6

DISCUSSION

In this concluding chapter we further discuss the implications of our findings, some possible weaknesses of our approach, and some suggested solutions or policies to improve the protocols. First, we will discuss each modified protocol separately, including some problems that may arise in the implementation of those protocols and possible solutions to those problems. Then, we will discuss the limitations and boundaries of our approach.

6.1 Non-monotonic-offers Protocol

As mentioned earlier, alternative solutions for an agent who has diminishing valuation include re-opening negotiation with the same opponent or starting a new one every time the agent gets stuck in its valuation. This situation is not analyzed and simulated in our experiment for the following reasons:

1. Intuitively, if two agents have met before, then they may restart their initial offers on the same item closer than if they do not know each other in the first place, which may speed up the convergence to the concession, as shown in Figure 6-1 (where the seller always restarts a bargaining by offering its last offer in the previous session). However, there are several weaknesses with the restarting mechanism, among them:

- There are some extra costs to restarting the bargaining, in terms of delay (illustrated by several gaps during the bargaining in Figure 6-1). A long delay may cause a failure. Or even if an agreement is reached, the total surplus may be lower than that from an early agreement.
- If the market server is involved in the process of restarting the bargaining, then restarting will incur an overhead to the server, which may charge a fee for each match-making between a buyer and a seller.

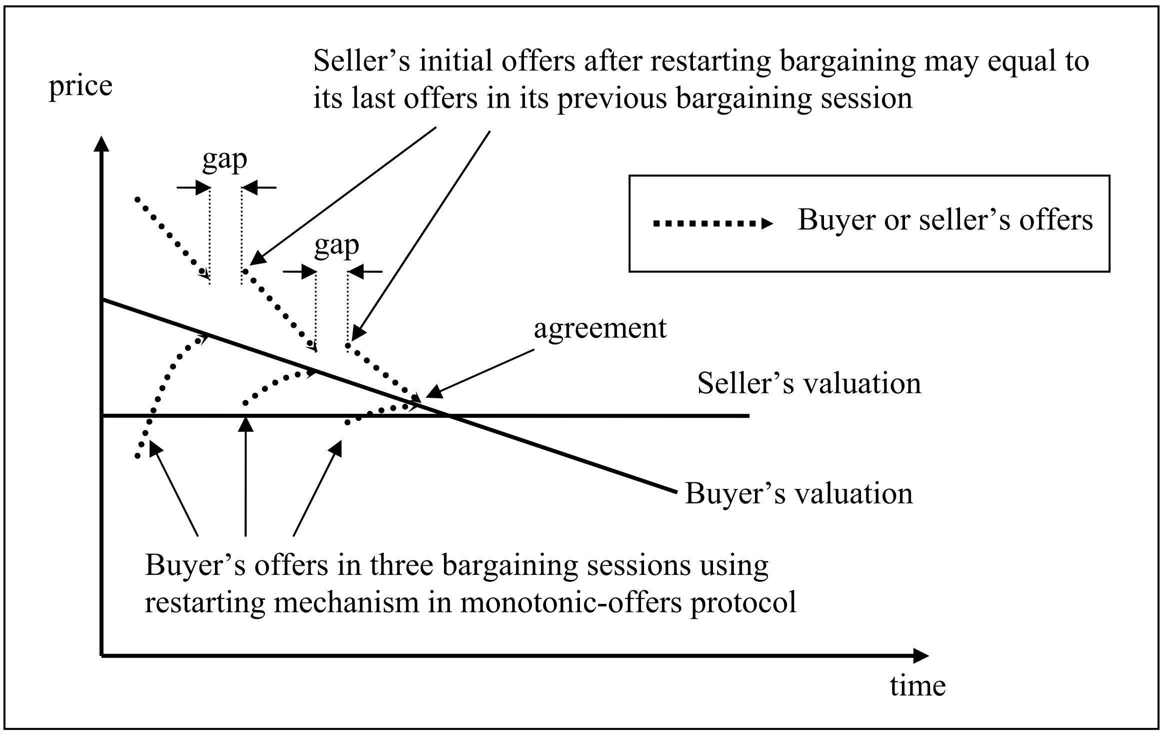


Figure 6-1 An illustration of the cost from restarting a bargaining session under the monotonic-offers bargaining protocol

2. If two agents can repeat their bargaining without friction in the monotonic-offers protocol, then the efficiency (in terms of expected gain and success rate) gained by them is at most as high as the efficiency gained in the non-monotonic-offers protocol, which becomes the upper bound of the efficiency of the monotonic-offers

protocol. Thus, allowing a restarting mechanism in the monotonic-offers protocol cannot be better than allowing non-monotonic offers in the non-monotonic-offers protocol.

Another issue in the empirical analysis is the coverage of the study. From our simulation in non-monotonic-offers protocol, agents who use a simple bargaining strategy, such as random strategy or tit-for-tat, can gain higher efficiency (in terms of success rate and surplus) compared to agents in the monotonic-offers protocol. This suggests that the non-monotonic-offers protocol may work better than the monotonic-offers protocol for many classes of agents, including those able to maximize expected gain, as shown in Chapter 4. However, it does not mean that non-monotonic-offers protocols are resilient to any manipulative strategies. Indeed, this is the weakness of the simulation approach in general. For example, our simulation results may not be true if we have the following agents in the bargaining.

- Misinformed agents, who cannot accept a non-monotonic offer, and therefore retreat from the bargaining immediately;
- Nasty agents, who use non-monotonic offers to threaten their opponents, arbitrarily increase or decrease their offers, or mislead their opponents.

Nasty agents' behaviors can be avoided by imposing a restriction on the number of monotonic-offers sequences in a negotiation session. For example, the protocol may only allow three monotonic-offers sequences. Suppose the buyer starts the negotiation by increasing its offers sequentially until a certain price level, and then reduces its offers sequentially until another price level. By this restriction, now the buyer can only have an opportunity to increase its offers sequentially until an agreement or breakdown is reached. Imposing this restriction will restrict fluctuated offers by nasty agents.

In the case of misinformed agents, these agents may perceive a non-monotonic offer as a sign of the lack of seriousness of their opponent, or an indication of prolonged bargaining, or a higher likelihood of breakdown. Thus, they will retreat from the bargaining, and try to find a new opponent. This situation cannot be avoided unless they can self-identify themselves as agents who cannot accept a non-monotonic offer. Early announcement about the possibility of non-monotonic offers by their opponents may help to avoid misinformation. However, nothing can be done if the agents are ported with a program to deny any non-monotonic offers.

In conclusion, allowing a non-monotonic-offers bargaining strategy is useful when agents have a decreasing surplus over time --- either a decreasing valuation for the buyer or an increasing one for the seller, or both. Empirical results have shown the benefit of that strategy in the absence of the agents' ability to maximize their expected utility. Therefore, an alternating-offer bargaining protocol should not restrict agents from non-monotonically changing their offers when their valuation is discounted over time.

6.2 Delay-based Bargaining Protocol

Allowing strategic delay during the negotiation is useful when agents gain positive utility during the delay, either in terms of direct utility (e.g. a higher surplus) or indirect utility (e.g. more certain bargaining information). The empirical study of the former setting (i.e. increasing of the buyer's valuation) has proven the benefit of this strategy in terms of total surpluses.

According to several examples that we have pointed out in section 4.2.2, we believe that allowing strategic delay is very important in attracting agents' participation in a bargaining-based market, especially in e-commerce or service-oriented markets. A delayed-based protocol is also important when agents are not deployed in a fully autonomous mode, e.g. agents need to report the possible outcomes to their owner for final approval (e.g. the last acceptable offer by its opponent) or seek help in the middle of the negotiation (e.g. tuning some parameters or changing their strategy). In such cases, a delay-based protocol is a must in maintaining the continuity of the negotiation.

Given this benefit, we suggest a modification in the standard bargaining protocol, such that agents are allowed to delay their negotiation, either by sticking to their previous offer or by sending a waiting signal. Note here that in the current standard negotiation protocol, a long waiting time for a response in a multi-agent system platform may be considered as network congestion or a connection failure, thus resulting in a breakdown usually enforced by the system manager rather than the seller.

To differentiate a strategic delay and a connection failure, we suggest that a message of delay be added to the agent communication language. Given this modification, the only shortcoming of allowing delay is the communication cost during the delay. This may not be a problem when the delay is within minutes, or when the communication cost is low. Intuitively, we may let the agent, who initiates a strategic delay, to disconnect the bargaining session and reconnect it later by recalling its opponent. However, agents may suffer from a commitment problem --- an opponent may not be willing to go back to the negotiation with its previous proposal after a recall. To remedy the problem, the following actions can be taken by agents who initiate a delay.

1. Enforcing an intermediate contract before the delay --- This action is used to ensure that their opponent will come back to the negotiation according to the intermediate contract (e.g. start from the last proposal).
2. Contacting the opponent repetitively during the delay --- This is used to ensure the existence/readiness of their opponent during the delay, and to update any relevant information (e.g. the item is still unsold).
3. Rewarding a commitment by the opponent --- Agents can provide an incentive to their opponent to come back to the negotiation (e.g. by a higher rating in reputation system or monetary reward).

Certainly, the above actions may incur cost; thus, they are useful only if the incurred cost is less than the cost of maintaining the connection during the delay.

Other policies may be imposed by the protocol in order to maintain the robustness of the system from malicious agents and to increase the participation of those who do not like to wait, for example, restricting the bargaining session within a limited time to avoid unnecessary long delays, disallowing a frequent delay in order to avoid frustration of those who do not want to wait, forbidding agents who have used excessive delay in the past, etc.

In conclusion, allowing delay in bargaining may be useful when agents have an increasing expected surplus over time and the deadline is long enough so that the delay will not jeopardize the negotiation (causing a breakdown). Empirical results have also shown the benefit of strategic delay in the absence of the agents' ability to maximize their expected utility. Therefore, the bargaining protocol should incorporate a waiting message in order to facilitate strategic delay.

6.3 Ignorance-based ABN Protocol

Allowing strategic ignorance in argumentation-based negotiations (ABNs), could increase the aggregate surplus when the cost of ignoring expensive arguments is low. As stated previously in section 4.3.1, allowing ignorance in ABNs is also part of the effort in designing a robust open system, especially when agents do not have a common ontology and reasoning. But from the agents' perspective, this can be strategically used to avoid costly argumentation, which could be beneficial for both agents.

If the direct cost of argumentation is high, such as incurring a high waiting cost or risk of breakdown, then agents may prefer an ignorance-based ABN protocol that allows them to strategically ignore argumentation. If there is no direct cost of argumentation, then they may also prefer the ignorance-based ABN protocol for allowing them to avoid the opportunity cost of argumentation. However, if there is no opportunity cost of argumentation and the direct cost of argumentation is low, then they will prefer an ABN protocol that forbids the use of strategic ignorance. Therefore, a conflict of interest will appear when two parties have different attitudes toward strategic ignorance (low-cost versus high-cost agents). This conflict may reduce the efficiency of ABN in terms of the rate of success and waiting cost in making a transaction, because the risk of breakdown will increase due to the unacceptability of responsive ignorance by the low cost agent. We propose several methods to resolve this issue.

First, we can use a matchmaker to select agents such that both parties have the same attitude toward ignorance (both are low-cost or high-cost agents). Therefore, we can avoid situations where only one prefers to use ignorance in ABN. Since agents can assess the cost of argumentation, there is no incentive for them to lie about their preference whether allowing ignorance or not. Suppose a buyer is low-cost but lies to the matchmaker that it is a high-cost buyer, so the matchmaker assigns it with a high-cost seller which will use the ignorance-based ABN protocol. But the buyer will not utilize strategic ignorance (because argumentation does not cost it much) and will be worse off from the strategic ignorance used by the seller (because the ignorance deteriorates the persuasion of the buyer). Thus, a low-cost agent prefers a non-ignorance-based ABN protocol and will not lie about its preference.

On the other hand, if the high-cost buyer pretends to be a low-cost one, it will meet a low-cost seller and they will use a non-ignorance-based ABN protocol. Then the buyer cannot use strategic ignorance because it is restricted by the protocol or alternatively, can displease the seller so that it causes a breakdown, and thus is worse off from pretending as a low-cost buyer. In other words, agents will tell their type truthfully. This method can work well if there are many similar agents who want to negotiate at the same time. Otherwise, agents may miss opportunities while waiting for a similar type of opponent.

Second, we may allow both parties to set the number of ignorance arguments that they may use during the ABN. If the number of ignorance arguments used by either party is over this limit, then the negotiation will be terminated automatically. Since both parties are rational, they will use responsive ignorance more carefully for a necessary situation only. However, the overhead incurred by the setting of this limit may reduce the efficiency of the negotiation itself, because agents may need to negotiate this limit (first stage) before the real negotiation over the item (second stage). Therefore, this method may only work if we can have an efficient (fast) first-stage negotiation and agents can incorporate/reason according to the limit in using responsive ignorance.

Third, we could set a limit on the total number of responsive ignorance arguments by both parties during the negotiation. If the total number of responsive ignorance arguments used by both parties is over this limit, then the negotiation will be terminated. This method will also restrict the abuse of responsive ignorance, and it does not have the overhead of the first stage negotiation. However, it may cause unnecessary termination, especially when there is a big discrepancy between agent ontologies or knowledge bases. For example, when one party cannot understand most arguments used by the other, repetitive non-strategic ignorance will cause a premature termination of the negotiation. Thus, the limit must be chosen very carefully, because it will be used for all agents in the system.

Fourth, we may impose a penalty for both parties when either of them uses responsive ignorance; for example, by paying a certain amount of 'money'. Since the penalty is per a responsive ignorance argument, it will suppress both parties in using complex or unnecessary arguments which may prolong the argumentation. However, this method may not be supported by the agent owner, unless the penalty is reasonable and both agents have common and synchronized ontologies, knowledge, and reasoning engines.

Fifth, we may adopt a reputation mechanism to record the amount of responsive ignorance arguments used by an agent and also the trust given by others toward the agent. Therefore, agents can assess and predict the argumentation capability and also the reputation of others. For an effective result, this mechanism needs agents to have the capability to evaluate each other and report truthfully their trust toward other agents. Moreover, the reliability of the system can only be gained after many participants have used the system.

In a simple-yet-crowded ABN system, such as when the amount of possible arguments is very limited and there are plenty of agents with various attitudes toward ignorance, the system may not need any method to restrict the usage of ignorance argumentation and could leave agents to adjust their beliefs and trust by themselves, because the conflict of agent attitude toward ignorance will not seriously affect the efficiency of the negotiation. For example, if a buyer finds that the seller is the opposite of its type, then it may walkout and find another seller. However, when the argumentation is very complex and the number of participants is very few, then it is more likely that agents will use responsive ignorance more frequently which will reduce the overall efficiency of the negotiation; therefore, we need to adopt any combination of the aforementioned methods according to the nature of the negotiation and users. The analysis of the effectiveness of those methods is beyond the scope of this thesis, and should be studied more thoroughly in the future.

6.4 Theoretical Analysis: What We Have Learned?

The theoretical analysis in Chapter 4 is based on a decision-theoretic approach, in the sense of neglecting the interaction among agents in deciding a move by an agent. The moves of other agents are seen as probability values not as the result of motivated action by these agents. Therefore, a proponent has no incentive to influence its opponent's move. In fact, we assume that the proponent may not know the type of its opponent, who might be an irrational agent. The proponent's moves, then, are guided by its belief about the states of the world ($\boldsymbol{p_t}$, $\boldsymbol{q_t}$, $\boldsymbol{EG'_{t+1}}$, etc.). This approach is more realistic in open MAS, where very limited information can be sensed by an agent prior to its interaction.

One of the critical issues in the real implementation is the assignment of those prior (initial) beliefs. However, it is not our focus, because our theoretical framework can be seen as an abstraction beyond other particular reasoning model. For instance, an agent may use a Bayesian net to generate $\boldsymbol{p_t}$, $\boldsymbol{q_t}$, or even $\boldsymbol{EG_t}$. Reinforcement learning or Bayesian update or something else may be used to update those beliefs. It is at the discretion of the agent's owner or designer to adjust or assign its prior beliefs. The crucial thing in our analysis is that we can use our simple framework to analyze the benefits of using the proposed protocols. As long as the agent gains benefit from the protocol, it will use the system, which eventually increases the participation rate. However increasing the participation rate is also not our terminal goal. Our main interest is to evaluate the benefit of the proposed protocols in terms of social welfare, as described in Chapter 5.

Nevertheless, the decision-theoretic approach in this thesis can be seen as an alternative to complement the traditional game-theoretic approach, especially in an ill-informed environment or when the bargaining cannot be iterated too many times (that is, no effective learning mechanism can be used to infer the opponent's type).

6.5 Limitations of the Current Work

Both our theoretical and empirical studies are restricted to the models described in Chapter 3, 4 and 5. Certainly, the situations and conditions considered in these chapters are not exhaustive. We believe there are many more situations in which agents may submit non-monotonic offers, use strategic delay, and deploy ignorance-based argumentation. Definitely, there are many research issues about further uses of those strategies that remain unexplored here. Specifically, the following high-level problems have limited our analyses (the responses in italics are given below each):

1. Many other behavior-based strategies can be adopted by agents, including irrational strategies, and these have not been tested empirically in our model.

   Response: Exhaustive testing is not possible. Our empirical studies have adopted common human strategies, which is the first step in understanding agents' bargaining behavior under the proposed protocols.

2. Our current model and experiment does not include qualitative negotiation.

   Response: Indeed, our model does not include qualitative negotiation. Many methods can be used to quantify the qualitative properties of an item, e.g. fuzzy quantification [Barro et al., 2003]. If by quantification we can map the qualitative properties into agents' valuation, then our model can still be applied. However, if this cannot be done, our model may serve as the abstraction of the agent's meta-reasoning.

3. Our current model and experiment does not include multiple-issues negotiation, which is very common in complex negotiation.

   Response: Study in multiple-issues negotiation is relatively new. Some suggest issue-by-issue negotiation in solving this problem. If this is the case, then our model can be applied. If it requires a simultaneous negotiation, then we may combine the bundle into a single value. If this cannot be done, then our model needs a further extension.

4. The empirical studies are limited to simulation, not a real implementation; hence, the actual results are not predictable.

Response: This is a common objection to a theory or simulation approach. The non-monotonic offer protocol is rarely used in human negotiation, but we believe it will be used and accepted if both bargainers are informed of the possibility of using it prior to the negotiation. Strategic delay is common in human negotiation; thus, we believe it has a real application value. In fact, snipping agents in an online auction wait until the last minute before submitting their bid, which reflects the value of delay. Strategic ignorance is also rarely used in human negotiation. But it is one of the requirements of designing a robust open ABN. Therefore, we believe it has a real application value too. However, these protocols have not been explored in MAS and have not been implemented in the real world (except for a delay-based one), so their benefits are still uncertain and need more experimental studies. In this thesis, we analyze them by simulation only. However, we have used the most common strategies in our simulation.

At a lower level, limitations of our analysis come from our assumptions and the experimental settings. Several of them are discussed here.

First, the temporal factors (e.g. deadline, delay duration, etc.) may not be proportional to the speed of the negotiation. For example, a strategic delay in an e-commerce negotiation could be as long as hours or days, while the duration between an offer and a counter-offer can be as short as several milliseconds. In such a case, disconnection between bargainers is very likely to happen; thus, intermediate contracts may be used to enforce commitment to a delay. A similar situation can happen for some cases in a non-monotonic-offers protocol, i.e. the agents' valuation could be flat relative to the negotiation session, because the valuation may change in the order of hours. Thus, agents are unlikely to meet their weak deadline. Unfortunately, this issue is context-dependent and can only be fully known in the real application. Certainly, if all valuations are flat relative to the negotiation session, then there is no reason to use the non-monotonic-offers protocol.

Second, our simulation in the ignorance-based ABN protocol is restricted to only two random generators, and the other settings explained in section 5.3.1; thus, the conclusions derived from our simulation may not be generalized to a broader setting. For example, if agents use a reactive strategy, such as tit-for-tat, then strategic ignorance may be used improperly, thus, reducing the benefit of argumentation. So far, we cannot make any conjecture on this issue, but plan to address it in the future.

Third, another drawback of strategic ignorance in argumentation-based negotiations may happen if most topics are excessively important for a party, but not at all for another party. In this case, allowing strategic ignorance may suppress the argumentation on those topics which causes a lower social welfare than would result from disallowing the ignorance. This situation is not addressed in our analysis and, again, is context-dependent. Nevertheless, we suggest the explicit inclusion of an ignorance message in ABN for the sake of the robustness of open ABN and for the benefit generated under the settings discussed in our previous chapters.

Given those limitations, we still need to study many things before a real deployment of our proposed protocols. For example, we may add some policies into these protocols to avoid irrational behavior, such as restricting the frequency of switching in the non-monotonic-offers protocol, limiting the number of responsive ignorance replies in the ignorance-based ABN protocol, or other policies that we have suggested previously.

CHAPTER 7

CONCLUSIONS AND FUTURE WORK

7.1 Conclusions

This thesis has shown three important modifications of the traditional alternating-offers bargaining protocol for automated negotiation in an open system. Both theoretical and empirical analyses have shown the benefit of these modifications either from an agent's or society's perspective under various settings, such as a discounting surplus in non-monotonic-offers protocol or an increasing surplus in the delay-based protocol.

We assume that the system is open, the agents are selfish, and various negotiation factors are uncertain. In the theoretical analysis, we assume our agents are bounded rational and not aware of their opponent's type. In the empirical analysis, our agents replicate common human strategies. These assumptions are weak in the sense they cover a wide variety of agents. Relaxing these assumptions may be one of our future goals, as shown in section 7.2.

The proposed modifications may be combined. For example, a bargaining protocol may allow both delay and ignorance, or delay and non-monotonic offers, or all of them. However, they may not be resilient to all malicious strategies. To remedy this, we have also pointed out some potential problems and proposed several policies to refine the protocols, so that they can prevent malicious strategies from destroying the advantage of the protocols.

In addition to proposing modified protocols, our study has also used decision-theoretic analysis to prove the benefit of these protocols. The analytical method is a complement of the existing game-theoretic method. This, to the best of our knowledge, is a method that has not been used in MAS community so far.

Within research in automated negotiation, this thesis has specifically contributed to the mechanism and protocol design to be used in negotiation and also to the development of open MAS. Other contributions include a novel decision-theoretic analysis of bargaining (Chapter 3 and 4) and an agent simulation (Chapter 5) to provide insight on the range of situations for which the modified protocols will be useful.

7.2 Future Work

The proposed future work can be derived from the limitations of our analysis. Basically, we can classify this future work into (i) more empirical studies, (ii) extension to other bargaining models and (iii) implementation in real applications. Several important future directions are described below.

First, we may perform more empirical studies including of various irrational strategies. Specifically, we may test various reactive strategies in the simulation of the delay-based protocol, and may explore effective policies in filtering out irrational agents in the non-monotonic-offers protocol. Testing various policies (described in Chapter 6), that deter agents' irrational behaviors is another reasonable extension. In addition, we may also incorporate knowledge about some risks of breakdown in the agents' reasoning, which can be used to adjust the delay or the concession rate in order to maximize their gain from delay. This feedback may tune the agents' concession rate or delay and thus will need to incorporate repetitive bargaining and learning mechanisms. Finally, we may extend the negotiation session (longer deadlines) and increase the fluctuation of the buyer's and seller's valuations in order to study the sensitivity of the proposed protocols, especially the non-monotonic-offers and delay-based ones. The goal is to analyze situations where the negotiation session is relatively longer than the negotiation time and/or the dynamism of the market reflected by highly volatile valuations.

A second main direction for future work is to extend the theoretical model in Chapter 3 and 4. A natural extension is to develop a model that includes qualitative and multiple-issues negotiation, for example, by adopting a multi-attribute utility function. We may divide this work into (i) incorporating a simple combination of multi-attribute utility functions in our proposed protocols, and (ii) studying a qualitative negotiation under the theoretical model in Chapter 3 (especially section 3.2 and 3.3). Another possibility is to extend our proposed protocols into other negotiation settings, for example, by allowing agents in a contract-net protocol to use strategic delay or non-monotonic offers in solving task allocation. The effect of those protocols on the overall social welfare needs a further study.

A third direction is to implement the protocols proposed in this thesis and to test them against human players. This is crucial before we can have a real system using the proposed protocols. The robustness of the protocols in avoiding malicious strategies is one of the major concerns in our design. As mentioned previously, human players are very creative in finding the weaknesses of a protocol and manipulating the outcomes. The testing of these proposed protocols in a real world domain with human players is one of the most important stages before the final deployment. Some possible real world applications of these protocols are e-commerce, service-oriented computing, mobile ad-hoc networks, and game-oriented interactive learning environments.

It is our hope that the proposed protocols can open up a new direction in automated negotiation, and shed light on how they can be applied or integrated to current applications. For example, we may use it in a contract-net protocol such that a task will be negotiated bilaterally among agents in order to achieve a better allocation. Or, we may develop a new e-commerce site which can handle multi-attribute negotiation while preserving the high flexibility characteristic for open systems to allow agents to join the system at any time. We also believe that our work will stimulate more research in exploring better negotiation protocols other than alternating offers bargaining and other kinds of traditional human negotiation.

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APPENDIX

FORMAL PROOFS

List of Assumptions

Assumption 1-1. *An agent prefers one protocol over others if it believes that the protocol is more helpful in attaining the agent's goal, which could be a highest expected utility, a highest success (agreement) rate, or a combination of both.*

Assumption 2-1. *The system is open in the sense that*

1. *agents may join or leave the system anytime;*
2. *agents may be created by different designers and represent different owners; and*
3. *bargaining may be done simultaneously and asynchronously among agents.*

Assumption 3-1. *Both the buyer and the seller have limited information and computational resources, and they are constrained by bargaining attributes and/or environmental factors, such as time deadlines, access to other buyers/sellers, etc., and all of those aspects are not necessarily symmetric for both of them.*

Assumption 3-2. *Both the buyer and the seller are bounded rational and selfish.*

Assumption 3-3. *(Zermelo's Axiom of Choice) Given limited information, limited computational resources, a time deadline, and other environmental constraints, both the buyer and the seller have totally ordered choices that can be expressed using utility functions, and both the buyer and the seller are utility maximizers.*

Assumption 3-4. *An alternating offer protocol is used as the basis of the bargaining, where the seller always starts by submitting a proposal/offer at time $t = 0$, and after receiving it the buyer will either accept the seller's proposal/offer or submit a counter proposal/offer at time $t = 0$ too. After that the virtual clock moves to $t = 1$ (next round) and it is then the seller's turn to evaluate the buyer's counteroffer. The process continues until a bargaining solution is found (all negotiated issues are solved) or a breakdown occurs (either or both parties left the negotiation without a solution).*

Assumption 3-5. *Unless otherwise specified, the theoretical analysis in this thesis is from the buyer's perspective and the offers and counteroffers are for the price only, which is in a continuous domain.*

Assumption 3-6. *A myopic-0 buyer will offer an optimum price $\boldsymbol{x_t^*}$ that yields the highest expected surplus at the present time $\boldsymbol{t}$, i.e., $\boldsymbol{Max_x\ [\ p_t(x) \times Sur_t(x)]}$.*

Assumption 3-7. *A myopic-1 buyer will offer a price $\boldsymbol{x_t^*}$ that yields the highest expected gain of the combination of positive surplus at time $\boldsymbol{t}$ and expected gain at time $\boldsymbol{t+1}$, i.e. $\boldsymbol{Max_x\ [p_t(x) \times Sur_t(x) + \gamma(1 - p_t(x))EG'_{t+1}(x)]}$ subject to $\boldsymbol{Sur_t(x)} > 0$.*

Assumption 3-8. *Suppose that the buyer knows that the seller is EvalF-I. (i) If the buyer's offer $\boldsymbol{x_t}$ is rejected, then the buyer will reduce $\boldsymbol{p_t(x)}$ for all $\boldsymbol{x \in Acc}$ and $\boldsymbol{x \leq x_t}$, where a price smaller than $\boldsymbol{x_t}$ is reduced faster; thus yielding a steeper function $\boldsymbol{p_{t+1}(x)}$ at $\boldsymbol{x \leq x_t}$. (ii) If the seller concedes such that $\boldsymbol{y_{t+1} < y_t}$, then the buyer will increase its belief such that $\boldsymbol{p_{t+1}(x)} = 1$ for all $\boldsymbol{x \geq y_{t+1}}$, and may also increase $\boldsymbol{p_t(x)}$ for $\boldsymbol{x}$ that is slightly lower than $\boldsymbol{y_{t+1}}$. (iii) If the seller's offer is unchanged or raised to $\boldsymbol{y_{t+1} > y_t}$, then the buyer will decrease all belief of $\boldsymbol{x < y_{t+1}}$, i.e. $\boldsymbol{p_{t+1}(x)}$ becomes a steeper function.*

Assumption 3-9. *Suppose the buyer knows that the seller is an EvalF-II agent and the buyer's offer* $\boldsymbol{x_t}$** *is rejected by the seller. Then the buyer will update* $\boldsymbol{\pi_t(x)}$ *by decomposing it into* $\boldsymbol{p_t(x)}$ *and* $\boldsymbol{k_t(x)}$*, and update them separately.* $\boldsymbol{p_t(x)}$ *will be updated according to Assumption 3-8, and* $\boldsymbol{k_t(x)}$ *will be updated according to the following rules:*

(i) If $\boldsymbol{x_t^*}$ *is less than or equal to* $\boldsymbol{x_{t-1}^*+\delta}$*, then* $\boldsymbol{k_t(x)}$ *will be reduced to* $\boldsymbol{k_{t+1}(x)}$*, where the reduction rate depends on the buyer's belief about the reason for the seller's refusal, either due to the seller's skepticism or the seller's high valuation.*

(ii) If $\boldsymbol{x_t^*}$ *is greater than* $\boldsymbol{x_{t-1}^*+\delta}$*, then* $\boldsymbol{k_t(x)}$ *will not be updated. Instead, a new* $\boldsymbol{k_{t+1}(x)}$ *will be generated whose a peak is around* $\boldsymbol{x_t^*}$*.*

Assumption 3-10. *There exists* $\boldsymbol{n}$ *and* $\boldsymbol{\delta}$ *such that if the buyer's consecutive offers* $\boldsymbol{x_{t+1}^*, \ldots, x_{t+n}^*}$ *are less than* $\boldsymbol{x_t^*+\delta}$ *and all are rejected by the seller, then* $\boldsymbol{k_{t+n}(x) = 0}$ *and* $\boldsymbol{\pi_{t+n}(x) = p_{t+n}(x)}$*.*

Assumption 3-11. *If the buyer agent does not know the type of the seller, then it will assign a probability that the seller is EvalF-I and EvalF-II of* $\boldsymbol{\kappa}$ *and* $\boldsymbol{1-\kappa}$*, respectively; and consider the seller as an EvalF-II seller with* $\boldsymbol{k_{+1}(x) = (1-\kappa)\, k_{+1}(x)'}$*, where* $\boldsymbol{k_{+1}(x)'}$ *is the value if the buyer certainly believes that the seller is EvalF-II.*

Assumption 4-1. $\boldsymbol{q_t}$ *is an increasing function over time* $\boldsymbol{t}$*.*

Assumption 4-2. $\boldsymbol{B_t}$ *is a decreasing function over time* $\boldsymbol{t}$*.*

Assumption 4-3. $\boldsymbol{EG'_{t+1}(x)}$ *is a decreasing function over* $\boldsymbol{t}$ *and a constant function over* $\boldsymbol{x}$*, denoted* $\boldsymbol{EG'_{t+1}}$ *for convenience.*

Assumption 4-4. *(Concurrent bilateral negotiation) A buyer may bargain with a set of sellers* $\boldsymbol{\{S_1, S_2, \ldots, S_n\}}$*, but all bargaining sessions are independent and* *asynchronous in the sense that the decision by two sellers are independently made (no collusion among sellers).*

Assumption 4-5. *(Persuasive negotiation) An agent may persuade their opponent to accept their offer or to justify their refusal.*

Assumption 4-6. *(Uncertain valuation) The buyer's estimated valuation* $\boldsymbol{B_t}$ *may increase over time, and the real value* $\boldsymbol{B_r}$ *may only be known by the buyer after the item is used or received.*

Assumption 4-7. *(Uncertain valuation) The buyer's estimated valuation* $\boldsymbol{B_t}$ *is not fixed over time, and the real value* $\boldsymbol{B_r}$ *may only be known by the buyer after the item is used or received.*

Assumption 4-8. *Suppose a seller uses argument* $\boldsymbol{\alpha_S}$ *at time* $\boldsymbol{t}$*. Then the buyer's belief over* $\boldsymbol{\alpha_S}$ *at time* $\boldsymbol{t}$*, denoted by* $\boldsymbol{\upsilon_t(\alpha_S)}$*, depends on the seller's reputation and other information, such as the truth of the seller's prior arguments and the truth value of* $\boldsymbol{\alpha_S}$ *itself. Similarly, the buyer's belief of its own argument* $\boldsymbol{\alpha_B}$ *being accepted by the seller at time* $\boldsymbol{t}$*, denoted by* $\boldsymbol{\upsilon_t(\alpha_B)}$*, depends on the buyer's reputation and other information that reflects the seller's belief toward the buyer and the truth value of* $\boldsymbol{\alpha_B}$ *itself.*

Assumption 4-9. *Upon receiving a seller's argument and/or offer* $\boldsymbol{y}$*, a buyer may update its valuation and belief according to the following heuristics:*

(a) $\boldsymbol{p_t(x)}$ *will be reduced for all prices less than the buyer's last offer (inclusive) if the seller convinces the buyer that the seller will not accept the buyer's last offer, either by insisting on its previous offer or using arguments.*

(b) $\boldsymbol{p_t(x)}$ *will be raised for prices close to the seller's new offer if the seller decreases its previous offer.*

(c) In both heuristic (a) and (b) above, the updating of $\boldsymbol{p_t(x)}$ *will not affect its monotonic property.*

(d) $\boldsymbol{B_t}$ *will be increased if new credible information has been received, either directly from the seller or from other sources.*

Assumption 4-10. *A buyer will always choose the best counter-argument* $\boldsymbol{\alpha_B}$ *that can affect the value of* $\boldsymbol{p_t(x)}$ *which may increase* $\boldsymbol{EG_t(x)}$.

List of Definitions

Definition 3-1. *An acceptable set* $\boldsymbol{Acc_i} \subseteq \boldsymbol{X}$ *of agent* $\boldsymbol{i}$ *is a set such that* $\forall \boldsymbol{x} \in \boldsymbol{Acc_i}$ *we have* $\boldsymbol{Sur^i(x)} > 0$*. In other words,* $\forall \boldsymbol{x} \in \boldsymbol{Acc_i} \Rightarrow \boldsymbol{x} \succsim_{\boldsymbol{i}} \boldsymbol{V_i}$.

Definition 3-2. *A feasible set* $\boldsymbol{F} \subseteq \boldsymbol{X}$ *is a compact set (closed and bounded) such that* $\forall \boldsymbol{x} \in \boldsymbol{F}$ *we have both* $\boldsymbol{Sur^b(x)} > 0$ *and* $\boldsymbol{Sur^s(x)} > 0$.

Definition 3-3. *A disagreement set is* $\boldsymbol{D} = \boldsymbol{X} \setminus \boldsymbol{F}$.

Definition 3-4. *Suppose it is the turn of an EvalF-0 buyer at time* $\boldsymbol{t}$*. Then it uses the following evaluation function in making its decision:*

$$\boldsymbol{I_t} = \begin{cases} \textit{Withdraw} & \textit{iff } \boldsymbol{t} > \boldsymbol{T_d} \\ \textit{Accept } \boldsymbol{y_t} & \textit{iff } \boldsymbol{Sur_t(y_t)} > 0 \textit{ and } \boldsymbol{t} \le \boldsymbol{T_d} \\ \textit{Counter offer } \boldsymbol{x_t} & \textit{otherwise} \end{cases} \tag{3-1}$$

where $\boldsymbol{I_t}$ is the buyer's decision function at time $\boldsymbol{t}$, $\boldsymbol{T_d}$ is the buyer's time deadline, $\boldsymbol{y_t}$ is the offer by the seller at time $\boldsymbol{t}$, $\boldsymbol{x_t}$ is the buyer's offer that will be proposed, $\boldsymbol{Sur_t(y_t)}$ is the surplus if $\boldsymbol{y_t}$ is accepted by the buyer at time $\boldsymbol{t}$.

Definition 3-5. *Suppose it is the turn of a EvalF-I buyer at time $\boldsymbol{t}$. Then it uses the following evaluation function in making its decision:*

$$\boldsymbol{I_t} = \begin{cases} \textit{Withdraw} & \textit{iff } \boldsymbol{t} > \boldsymbol{T_d} \textit{ or } (\boldsymbol{Sur_t^e(x_t)} \le 0 \textit{ and } \boldsymbol{Sur_t(y_t)} \le 0) \\ \textit{Accept } \boldsymbol{y_t} & \textit{iff } \boldsymbol{Sur_t(y_t)} > 0 \textit{ and } \boldsymbol{Sur_t(y_t)} \ge \boldsymbol{Sur_t^e(x_t)} \textit{ and } \boldsymbol{t} \le \boldsymbol{T_d} \\ \textit{Counter offer } \boldsymbol{x_t} & \textit{otherwise} \end{cases} \quad (3\text{-}2)$$

where $\boldsymbol{x_t}$ is the buyer's offer that will be proposed, $\boldsymbol{Sur_t^e(x_t)}$ is the estimated surplus at time $\boldsymbol{t}$ if $\boldsymbol{x_t}$ is accepted by the seller in the next bargaining period, i.e. at time $\boldsymbol{t+1}$.

Definition 3-6. *Suppose it is the turn of an EvalF-II buyer at time $\boldsymbol{t}$. Then it uses the following evaluation function in making its decision:*

$$\boldsymbol{I_t} = \begin{cases} \textit{Withdraw} & \textit{iff } \boldsymbol{t} > \boldsymbol{T_d} \textit{ or } (\boldsymbol{Sur_t^e(x_t)} \le 0 \textit{ and } \boldsymbol{Sur_t(y_t)} \le 0) \\ \textit{Accept } \boldsymbol{y_t} & \textit{iff } [\boldsymbol{Sur_t(y_t)} \ge \boldsymbol{Sur_t^e(x_t)} \vee \wedge_j(\boldsymbol{Sur_t(y_t)} \ge \boldsymbol{Sur_t^e(y_j)})] \textit{ and} \\ & \quad \boldsymbol{Sur_t(y_t)} > 0 \textit{ and } \boldsymbol{t} \le \boldsymbol{T_d} \\ \textit{Counter offer } \boldsymbol{x_t} & \textit{otherwise} \end{cases} \quad (3\text{-}3)$$

where $\boldsymbol{Sur_t^e(y_j)}$ is the estimated value of $\boldsymbol{Sur_t(y_j)}$ and $\boldsymbol{j} \in \{\boldsymbol{t+1}, \boldsymbol{t+2}, \ldots, \boldsymbol{T_d}\}$ are the turns of the buyer in the future until its time deadline $\boldsymbol{T_d}$.

Definition 3-7. $\boldsymbol{EG_t(x) \equiv (1 - q_t)\, p_t(x)\, (B_t - x) + q_t\, B_\phi}$ (3-4)

Where $\boldsymbol{q_t} \in [0, 1]$ is the buyer's belief of the likelihood of negotiation breakdown caused by the seller at time $\boldsymbol{t}$ which is independent of $\boldsymbol{x}$, $\boldsymbol{B_\phi}$ is the buyer's valuation if the *negotiation breaks down, and* $\boldsymbol{p_t(x)} \in [0, 1]$ *is the buyer's belief (subjective probability) function that price* $\boldsymbol{x}$ *will be accepted by the seller (acceptance rate) at time* $\boldsymbol{t}$.

$$\boldsymbol{Max_x\, EG_t(x) = Max_x\, [\, p_t(x)\, (B_t - x)]} \tag{3-5}$$

Definition 3-8. $\boldsymbol{EG_t(x) \equiv (1-q_t)\, [p_t(x)(B_t-x) + \gamma(1-p_t(x))EG'_{t+1}(x)] + q_t B_\phi}$ (3-6)

where $\boldsymbol{\gamma} \in [0, 1]$ *is the weight of the expected gain in the next round* $\boldsymbol{t+1}$, $\boldsymbol{EG'_{t+1}(x)}$ *is the estimation of the expected gain in the next round which is made by the buyer at the current round* $\boldsymbol{t}$, *and the rest of parameters are as defined in Definition 3-7.*

$$\boldsymbol{Max_x\, EG_t(x) = Max_x\, [p_t(x)(B_t - x) + \gamma(1 - p_t(x))EG'_{t+1}(x)]} \tag{3-7}$$

Definition 3-9. $\boldsymbol{\pi_{t+1}(x) = min(p_{t+1}(x) + k_{t+1}(x)}, 1)$. (3-8)

Definition 3-10. *The estimation of* $\boldsymbol{EG'_{t+1}(x)}$ *by recursive method is calculated by*

$$\boldsymbol{EG'_{t+1}(x) \equiv \underset{x}{Max}[(1 - q'_{t+1})[p'_{t+1}(x)(B'_{t+1} - x) + \gamma(1 - p'_{t+1}(x))EG'_{t+2}(x)] + q'_{t+1}\, B_\phi]} \tag{3-9}$$

where all parameters except $\boldsymbol{\gamma}$ *and* $\boldsymbol{B_\phi}$ *must be predicted or calculated iteratively by assuming that* $\boldsymbol{x_t}$ *has been rejected, and* $\boldsymbol{p'_{t+1}(x)}$ *is the buyer's estimation of its revised belief if* $\boldsymbol{x_t}$ *has been rejected. And for a myopic-1 buyer with no payoff from breakdown* ($\boldsymbol{B_\phi} = 0$) *the estimation becomes*

$$\boldsymbol{EG'_{t+1}(x) \equiv \underset{x}{Max}[(1 - q'_{t+1})p'_{t+1}(x)(B'_{t+1} - x)]} \tag{3-10}$$

Definition 4-1. *A buyer's evaluation function is:*

$$\boldsymbol{I_t} = \begin{cases} \textit{Delay} & \textit{iff } \boldsymbol{\gamma\, [EG'_{t+1}]_{delay} > EG_t(x_t^*)} \textit{ and } \boldsymbol{\gamma\, [EG'_{t+1}]_{delay} > EG_t(y)} \\ \textit{Accept} & \textit{iff } \boldsymbol{EG_t(y) \geq EG_t(x_t^*)} \textit{ and } \boldsymbol{EG_t(y) \geq \gamma\, [EG'_{t+1}]_{delay}} \\ \textit{Counter offer} & \textit{otherwise} \end{cases} \tag{4-12}$$

Definition 4-2. (i) $\boldsymbol{EG_t(x) \equiv p_t(x)\,(B_t - x)}$ (4-24)

(ii) $\boldsymbol{EG_t(y) \equiv B_t - y}$ (4-25)

Definition 4-3. *A buyer's evaluation function is:*

$$I_t = \begin{cases} \textit{Withdraw} & \textit{iff } t > T_d \textit{ or } maxEG_t(x) \le 0 \\ \textit{Accept} & \textit{iff } EG_t(y) \ge maxEG_t(x) \textit{ and } t \le T_d \\ \textit{Counter offer and/or argument} & \textit{otherwise} \end{cases} \tag{4-26}$$

List of Propositions and Their Proofs

Proposition 3-1. *Suppose the buyer knows that the seller is EvalF-I. Then the buyer's belief function that its offer at time* $\boldsymbol{t}$ *will be accepted by a seller,* $\boldsymbol{p_t(x)}$ *is an increasing function of* $\boldsymbol{x}$*, where for any* $\boldsymbol{x}$ *we have* $\boldsymbol{p_t(x)} \in [0, 1]$.

Proposition 3-1 follows from the property of an EvalF-I seller: accepting any buyer's offer when it is higher than the seller's (about-to-submit) next offer. Since offering a higher price will have a higher chance of exceeding the seller's about-to-offer price, it implies an increasing function of $\boldsymbol{p_t(x)}$ in terms of $\boldsymbol{x}$.

Proposition 3-2. *If an optimal offer* $\boldsymbol{x_t^*}$ *is rejected by an EvalF-I seller, who does not concede significantly from its previous offer* $\boldsymbol{y_t}$*, then the buyer's posterior belief will be steeper at* $\boldsymbol{x \le x_t^*}$.

Proof. The proof can be trivially derived from Assumption 3-8(i) and instantiation of 3-8(iii) if the seller does not concede at all, where 3-8(ii) is not satisfied. ■

Proposition 3-3. *Suppose that the buyer knows that the seller is EvalF-II. If the buyer's consecutive offers which are lower than* $\boldsymbol{x_t^*+\delta}$ *were rejected for* $\boldsymbol{n}$ *periods and the seller concedes insignificantly during these periods, then the buyer's posterior belief* $\boldsymbol{\pi_{t+n}(x)}$ *will be steeper than that of* $\boldsymbol{\pi_t(x)}$ *for all* $\boldsymbol{x \le x_t^*}$.

Proof. When an offer $\boldsymbol{x_t^*}$ is rejected, the buyer will update its belief according to Assumption 3-9, where $\boldsymbol{p_{t+1}(x)}$ is steeper than $\boldsymbol{\pi_t(x)}$ for all $\boldsymbol{x \le x_t^*}$. If the buyer's offers lower than $\boldsymbol{x_t^*+\delta}$ are still rejected for $\boldsymbol{n}$ rounds, then by Assumption 3-10 we have $\boldsymbol{\pi_{t+n}(x)} = \boldsymbol{p_{t+n}(x)}$, where $\boldsymbol{p_{t+n}(x)}$ is derived using Assumption 3-8 from $\boldsymbol{p_{t+n-1}(x)}$, and $\boldsymbol{p_{t+n-1}(x)}$ is derived from $\boldsymbol{p_{t+n-2}(x)}$, and so on until $\boldsymbol{p_{t+2}(x)}$ is derived from $\boldsymbol{p_{t+1}(x)}$. Therefore, for all $\boldsymbol{x \le x_t^*}$ we have $\boldsymbol{\pi_{t+n}(x)} = \boldsymbol{p_{t+n}(x)}$ is steeper than $\boldsymbol{p_{t+n-1}(x)}$ which is steeper than $\boldsymbol{p_{t+n-2}(x)}$ and so on which is steeper than $\boldsymbol{p_{t+1}(x)}$ which is steeper than $\boldsymbol{\pi_t(x)}$; or by transitivity, $\boldsymbol{\pi_{t+n}(x)}$ is steeper than $\boldsymbol{\pi_t(x)}$ for all $\boldsymbol{x \le x_t^*}$. ■

Proposition 3-4. *Suppose the buyer knows that the seller is EvalF-II. Then the effective buyer's belief that its offer at time* $\boldsymbol{t}$ *be accepted by a seller,* $\boldsymbol{\pi_t(x)}$*, is an increasing function with respect to its offer* $\boldsymbol{x}$*, where* $\boldsymbol{\pi_t(x)} \in [0, 1]$ *for all* $\boldsymbol{x}$. *However,* $\boldsymbol{\pi_t(x)}$ *may not be a continuous function.*

Proposition 3-4 follows from the rationality of the buyer. Suppose $\boldsymbol{x_1} > \boldsymbol{x_2}$ but $\boldsymbol{x_1}$ is less likely be accepted by the seller than $\boldsymbol{x_2}$. Then there is no reason for the buyer to offer $\boldsymbol{x_1}$. Thus, the search space of the buyer (effective belief function) is always an increasing function. Since $\boldsymbol{x_1}$ is not considered, then $\boldsymbol{\pi_t(x_1)}$ is undefined, or $\boldsymbol{\pi_t(x)}$ is a discontinuous function.

Proposition 3-5. *In the long run* $\boldsymbol{q_t}$ *is an increasing function of time* $\boldsymbol{t}$.

Proposition 3-5 follows from the explanation described in section 3.3.4. Intuitively, the likelihood that the seller's deadline is met is higher as time goes by; thus, $\boldsymbol{q_t}$ is an increasing function of time t.

Proposition 3-6. *(Ambiguity of* $\boldsymbol{EG'_{t+1}(x)}$ *with respect to* $\boldsymbol{x}$*)* $\boldsymbol{EG'_{t+1}(x)}$ *may increase or decrease with respect to* $\boldsymbol{x}$ *submitted at time* $\boldsymbol{t}$.

Proof. Suppose there are two different offers that can be submitted by a buyer at time $\boldsymbol{t}$, i.e. a higher and a lower one, denoted by $\boldsymbol{x_t^h}$ and $\boldsymbol{x_t^l}$, where $\boldsymbol{x_t^h} > \boldsymbol{x_t^l}$. Let the buyer's belief at time $\boldsymbol{t}$ be $\boldsymbol{p_t(x)}$. From the buyer's perspective, if it offered $\boldsymbol{x_t^h}$ and it was rejected, then it will update $\boldsymbol{p_t(x)}$ to $\boldsymbol{p^h_{t+1}(x)}$. Similarly, if it offered $\boldsymbol{x_t^l}$ and this was rejected, then it will update $\boldsymbol{p_t(x)}$ to $\boldsymbol{p^l_{t+1}(x)}$. Now, suppose the buyer knows that the seller is EvalF-I, then by Assumption 3-8(i) a new belief $\boldsymbol{p^l_{t+1}(x)}$ will be steeper for all $\boldsymbol{x} \leq \boldsymbol{x_t^l}$ and also $\boldsymbol{p^h_{t+1}(x)}$ will be steeper for all $\boldsymbol{x} \leq \boldsymbol{x_t^h}$. Since $\boldsymbol{x_t^h} > \boldsymbol{x_t^l}$, then $\boldsymbol{p^h_{t+1}(x)}$ must be steeper than $\boldsymbol{p^l_{t+1}(x)}$ for all $\boldsymbol{x} \leq \boldsymbol{x_t^h}$, or $\boldsymbol{p^h_{t+1}(x)} < \boldsymbol{p^l_{t+1}(x)}$ for all $\boldsymbol{x} \leq \boldsymbol{x_t^h}$. Given this, the expected surplus at time $\boldsymbol{t+1}$ will be lower if $\boldsymbol{x_{t+1}} \leq \boldsymbol{x_t^h}$. Thus, in this case $\boldsymbol{EG'_{t+1}(x)}$ may be a decreasing function of $\boldsymbol{x}$. However, this can only be guaranteed if the buyer believes that the seller will react identically in both cases, i.e. $\boldsymbol{p^h_{t+1}(x)} = \boldsymbol{p^l_{t+1}(x)}$ for all $\boldsymbol{x} > \boldsymbol{x_t^h}$. If this is not true, e.g. due to the seller's different counter-offers after receiving two different buyer's offers, then by Assumption 3-8(ii), $\boldsymbol{p^h_{t+1}(x)}$ may not be equal to $\boldsymbol{p^l_{t+1}(x)}$ for all $\boldsymbol{x} > \boldsymbol{x_t^h}$. Note here, $\boldsymbol{EG'_{t+1}(x)}$ depends on the buyer's offer $\boldsymbol{x_{t+1}}$, which may be higher than $\boldsymbol{x_t^h}$. Therefore, it may happen that $\boldsymbol{p^h_{t+1}(x)} > \boldsymbol{p^l_{t+1}(x)}$ for all $\boldsymbol{x} > \boldsymbol{x_t^h}$, or $\boldsymbol{EG'_{t+1}(x)}$ may be an increasing function of $\boldsymbol{x}$ at time $\boldsymbol{t}$. In conclusion, without knowing the seller's reaction, the characteristic of $\boldsymbol{EG'_{t+1}(x)}$ over $\boldsymbol{x}$ at time $\boldsymbol{t}$ is ambiguous. ■

Proposition 3-7. *If a myopic-1 buyer is facing time pressure in the long run such that its belief at time* $\boldsymbol{t+1}$ *satisfies* $\boldsymbol{B'_{t+1} - B_t} \le 0$ *and* $\boldsymbol{(q'_{t+1} - q_t) \ge (1 - (1 + \omega)p_t(x_t^*))(1 - q_t)}$*, where*

$$\omega \equiv \frac{\gamma(\mathbf{1} - \boldsymbol{q_t})(\mathbf{1} - \boldsymbol{p_t}(\boldsymbol{x_t^*}))}{[\mathbf{1} - \gamma(\mathbf{1} - \boldsymbol{q_t})(\mathbf{1} - \boldsymbol{p_t}(\boldsymbol{x_t^*}))]}\ , \tag{3-12}$$

and its estimated optimal offer $\boldsymbol{x'_{t+1}{}^*} > \boldsymbol{x_t^*} + \boldsymbol{B'_{t+1} - B_t}$*, then its expected future gain will be lower than its current expected gain, i.e.* $\boldsymbol{EG'_{t+1}(x'_{t+1}{}^*) < EG_t(x_t^*)}$.

<u>Proof</u>. First, we will see what kind of conditions should be met for the buyer's expected future gain to be lower than its current one.

$\boldsymbol{EG'_{t+1}(x'_{t+1}{}^*) < EG_t(x_t^*)}$

But $\boldsymbol{EG_t(x_t^*) = (1 - q_t)\ [p_t(x_t^*)(B_t - x_t^*) + \gamma(1 - p_t(x_t^*))EG'_{t+1}(x'_{t+1}{}^*)]}$

$\Leftrightarrow \boldsymbol{EG'_{t+1}(x'_{t+1}{}^*) < (1 - q_t)p_t(x_t^*)(B_t - x_t^*) + \gamma(1 - q_t)(1 - p_t(x_t^*))EG'_{t+1}(x'_{t+1}{}^*)}$

$\Leftrightarrow \boldsymbol{(1 - \gamma(1 - q_t)(1 - p_t(x_t^*)))EG'_{t+1}(x'_{t+1}{}^*) < (1 - q_t)p_t(x_t^*)(B_t - x_t^*)}$

$\Leftrightarrow \boldsymbol{EG'_{t+1}(x'_{t+1}{}^*) < (1 - q_t)p_t(x_t^*)(B_t - x_t^*) \,/\, (1 - \gamma(1 - q_t)(1 - p_t(x_t^*)))}$

And from $\boldsymbol{EG'_{t+1}(x'_{t+1}{}^*) = (1 - q'_{t+1})\ p'_{t+1}(x'_{t+1}{}^*)(B'_{t+1} - x'_{t+1}{}^*)}$ the LHS becomes

$\Leftrightarrow \boldsymbol{(1 - q'_{t+1})\ p'_{t+1}(x'_{t+1}{}^*)(B'_{t+1} - x'_{t+1}{}^*)} <$

$\boldsymbol{(1 - q_t)p_t(x_t^*)(B_t - x_t^*) \,/\, (1 - \gamma(1 - q_t)(1 - p_t(x_t^*)))}$

Now, suppose the estimated value $\boldsymbol{q'_{t+1}} = \boldsymbol{q_t} + \Delta_q$ and $\boldsymbol{p'_{t+1}(x'_{t+1}{}^*)} = \boldsymbol{p_t(x_t^*)} + \Delta_p$, Substituting these into the inequality yields:

$\Leftrightarrow \boldsymbol{(1 - q_t - \Delta_q)(p_t(x_t^*) + \Delta_p)(B'_{t+1} - x'_{t+1}{}^*)} <$

$\boldsymbol{(1 - q_t)p_t(x_t^*)(B_t - x_t^*) \,/\, (1 - \gamma(1 - q_t)(1 - p_t(x_t^*)))}$

$\Leftrightarrow \boldsymbol{(1 - q_t)(p_t(x_t^*) + \Delta_p)(B'_{t+1} - x'_{t+1}{}^*) - \Delta_q\ (p_t(x_t^*) + \Delta_p)(B'_{t+1} - x'_{t+1}{}^*)} <$

$\boldsymbol{(1 - q_t)p_t(x_t^*)(B_t - x_t^*) \,/\, (1 - \gamma(1 - q_t)(1 - p_t(x_t^*)))}$

Suppose also that the estimated value $B'_{t+1} = B_t + \Delta_B$, and $x'_{t+1}{}^* = x_t{}^* + \Delta_x$. Substituting these into the first term yields:

$$\Leftrightarrow (1 - q_t)(p_t(x_t^*) + \Delta_p)(B_t + \Delta_B - x_t^* - \Delta_x) - \Delta_q\,(p_t(x_t^*) + \Delta_p)(B'_{t+1} - x'_{t+1}{}^*) <$$

$$(1 - q_t)p_t(x_t^*)(B_t - x_t^*) \,/\, (1 - \gamma(1 - q_t)(1 - p_t(x_t^*)))$$

$$\Leftrightarrow (1 - q_t)p_t(x_t^*)(B_t + \Delta_B - x_t^* - \Delta_x) + (1 - q_t)\Delta_p(B'_{t+1} - x'_{t+1}{}^*) -$$

$$\Delta_q(p_t(x_t^*) + \Delta_p)(B'_{t+1} - x'_{t+1}{}^*) <$$

$$(1 - q_t)p_t(x_t^*)(B_t - x_t^*) \,/\, (1 - \gamma(1 - q_t)(1 - p_t(x_t^*)))$$

And by expanding $(B_t + \Delta_B - x_t^* - \Delta_x)$ into $(B_t - x_t^*) + (\Delta_B - \Delta_x)$, we get

$$\Leftrightarrow (1 - q_t)p_t(x_t^*)(B_t - x_t^*) + (1 - q_t)p_t(x_t^*)(\Delta_B - \Delta_x) + (1 - q_t)\Delta_p(B'_{t+1} - x'_{t+1}{}^*) -$$

$$\Delta_q(p_t(x_t^*) + \Delta_p)(B'_{t+1} - x'_{t+1}{}^*) <$$

$$(1 - q_t)p_t(x_t^*)(B_t - x_t^*) \,/\, (1 - \gamma(1 - q_t)(1 - p_t(x_t^*)))$$

And by regrouping $(1 - q_t)p_t(x_t^*)(B_t - x_t^*)$ to the RHS of inequality, we get

$$\Leftrightarrow (1 - q_t)p_t(x_t^*)(\Delta_B - \Delta_x) + (1 - q_t)\Delta_p(B'_{t+1} - x'_{t+1}{}^*) -$$

$$\Delta_q(p_t(x_t^*) + \Delta_p)(B'_{t+1} - x'_{t+1}{}^*) <$$

$$[(1 - q_t)p_t(x_t^*)(B_t - x_t^*) \,/\, (1 - \gamma(1 - q_t)(1 - p_t(x_t^*)))] - (1 - q_t)p_t(x_t^*)(B_t - x_t^*)$$

From $p'_{t+1}(x'_{t+1}{}^*) = p_t(x_t^*) + \Delta_p$, we get

$$\Leftrightarrow (1 - q_t)p_t(x_t^*)(\Delta_B - \Delta_x) + [(1 - q_t)\Delta_p - \Delta_q\, p'_{t+1}(x'_{t+1}{}^*)](B'_{t+1} - x'_{t+1}{}^*) <$$

$$(1 - q_t)p_t(x_t^*)(B_t - x_t^*)[1 - (1 - \gamma(1 - q_t)(1 - p_t(x_t^*)))] \,/\, [1 - \gamma(1 - q_t)(1 - p_t(x_t^*))]$$

$$\Leftrightarrow (1 - q_t)p_t(x_t^*)(\Delta_B - \Delta_x) + [(1 - q_t)\Delta_p - \Delta_q\, p'_{t+1}(x'_{t+1}{}^*)](B'_{t+1} - x'_{t+1}{}^*) <$$

$$(1 - q_t)p_t(x_t^*)(B_t - x_t^*)\gamma(1 - q_t)(1 - p_t(x_t^*)) \,/\, [1 - \gamma(1 - q_t)(1 - p_t(x_t^*))]$$

From $\boldsymbol{B'_{t+1}} = \boldsymbol{B_t} + \Delta_B$ and $\boldsymbol{x'_{t+1}}* = \boldsymbol{x_t}* + \Delta_x$, we have $\boldsymbol{B_t - x_t* = (B'_{t+1} - x'_{t+1}*) - (\Delta_B - \Delta_x)}$;

substituting $\omega \equiv \dfrac{\gamma(1-q_t)(1-p_t(x_t^*))}{[1-\gamma(1-q_t)(1-p_t(x_t^*))]}$ and $\boldsymbol{B_t - x_t* = (B'_{t+1} - x'_{t+1}*) - (\Delta_B - \Delta_x)}$

$$\Leftrightarrow \boldsymbol{(1-q_t)p_t(x_t*)(\Delta_B - \Delta_x) + [(1-q_t)\Delta_p - \Delta_q\, p'_{t+1}(x'_{t+1}*)](B'_{t+1} - x'_{t+1}*)} <$$

$$\boldsymbol{(1-q_t)p_t(x_t*)((B'_{t+1} - x'_{t+1}*) - (\Delta_B - \Delta_x))\omega}$$

$$\Leftrightarrow \boldsymbol{(1-q_t)p_t(x_t*)(\Delta_B - \Delta_x)(1+\omega)} +$$

$$\boldsymbol{[(1-q_t)(\Delta_p - p_t(x_t*)\omega) - \Delta_q\, p'_{t+1}(x'_{t+1}*)](B'_{t+1} - x'_{t+1}*)} < \mathbf{0} \qquad (3\text{-}13)$$

Since $\boldsymbol{q_t} \in [0, 1]$, $\boldsymbol{\gamma} \in [0, 1]$ and $\boldsymbol{p_t(x_t*)} > 0$, then $0 \le \gamma\boldsymbol{(1-q_t)(1-p_t(x_t*))} < 1$, or $\boldsymbol{\omega} > 0$. Moreover, due to the time pressure in the long run, $\boldsymbol{q_t}$ is strictly increasing and $\boldsymbol{B_t}$ is decreasing over time, or $\Delta_q > 0$ and $\Delta_B \le 0$. And since $\boldsymbol{(B'_{t+1} - x'_{t+1}*)}$ is an expected surplus, then it must be positive. However, $\boldsymbol{p'_{t+1}(x'_{t+1}*)}$ may be greater or less than $\boldsymbol{p_t(x_t*)}$, and $\boldsymbol{x'_{t+1}*}$ may be greater or less than $\boldsymbol{x_t*}$; hence, the sign of Δ_p and Δ_x are ambiguous. Consequently, the conditions for inequality (3-13) to hold are

$$(\Delta_B - \Delta_x) < 0 \text{ and } \boldsymbol{(1-q_t)(\Delta_p - p_t(x_t*)\omega) - \Delta_q\, p'_{t+1}(x'_{t+1}*)} \le 0$$

$$\Leftrightarrow \qquad \Delta_x > \Delta_B \text{ and } \boldsymbol{(\Delta_p - p_t(x_t*)\omega) / p'_{t+1}(x'_{t+1}*) \le \Delta_q / (1-q_t)}$$

Since $\Delta_B \le 0$, then $\Delta_x > 0$ will guarantee the first condition. The second condition can be modified as follows:

$$\boldsymbol{(\Delta_p - p_t(x_t*)\omega) / p'_{t+1}(x'_{t+1}*) \le \Delta_q / (1-q_t)}$$

$$\Leftrightarrow \qquad \boldsymbol{(p'_{t+1}(x'_{t+1}*) - p_t(x_t*) - p_t(x_t*)\omega) / p'_{t+1}(x'_{t+1}*) \le \Delta_q / (1-q_t)}$$

$$\Leftrightarrow \qquad \boldsymbol{1 - p_t(x_t*)(1+\omega) / p'_{t+1}(x'_{t+1}*) \le \Delta_q / (1-q_t)}$$

Since $0 < \boldsymbol{p'_{t+1}(x'_{t+1}*)} \le 1$, then $\boldsymbol{1 - p_t(x_t*)(1+\omega) \le \Delta_q / (1-q_t)}$ is a sufficient condition for the above inequality. Therefore, the condition can be further simplified as follows:

$$\boldsymbol{1 - p_t(x_t*)(1+\omega) \le \Delta_q / (1-q_t)}$$

$$\Leftrightarrow \qquad [1 - p_t(x_t^*)(1 + \omega)](1 - q_t) \leq \Delta_q$$

Thus, the sufficient condition that satisfies inequality (3-13) can be rewritten as follows:

$$\Delta_x > \Delta_B \text{ and } \Delta_q \geq [1 - p_t(x_t^*)(1 + \omega)](1 - q_t) \qquad (3\text{-}14)$$

The conditions 3-14 are in fact the premises in the proposition that

$$x'_{t+1}{}^* > x_t^* + B'_{t+1} - B_t \text{ and } (q'_{t+1} - q_t) \geq [1 - p_t(x_t^*)(1 + \omega)](1 - q_t). \qquad ■$$

Proposition 3-8. *A series of consecutive offers* $<x_1, x_2, \ldots, x_{t-1}, x_t, \phi>$ *is preferred to* $<x_1, x_2, \ldots, x_{t-1}, \phi>$ *for* $0 \leq x_t < B_t$.

Proof. Given equation (3-4) and (3-6), the probability of bargaining success in each round can be calculated as follows. The probability that the bargaining will succeed in the first round is $(1 - q_1)\, p_1(x_1)$, and the probability that the bargaining proceeds to the second round is $(1 - q_1)(1 - p_1(x_1))$. The probability that the bargaining will succeed in the second round is equal to the probability that the bargaining proceeds to the second round times the probability that the bargaining succeeds in the second round, i.e. $(1 - q_1)(1 - p_1(x_1))(1 - q_2)p_2(x_2)$, and the probability that the bargaining proceeds to the third round is $(1 - q_1)(1 - p_1(x_1))(1 - q_2)(1 - p_2(x_2))$. Therefore, the probability that the bargaining will succeed in the i-th round is

$$(1 - q_i)p_i(x_i)\prod_{j=1}^{i-1}(1 - q_j)(1 - p_j(x_j))$$

Suppose the total probability that any offer in a sequence $X1 = <x_1, x_2, \ldots, x_{t-1}, \phi>$ is accepted by the seller is f_{t-1}. Or,

$$f_{t-1} = (1 - q_1)p_1(x_1) + \sum_{i=2}^{t-1}\left((1 - q_i)p_i(x_i)\prod_{j=1}^{i-1}(1 - q_j)(1 - p_j(x_j))\right)$$

Then by adding $\boldsymbol{x_t} > 0$, the total probability that any offer in $\boldsymbol{X2} = <\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_t}, \phi>$ is accepted by the seller becomes $f_t = f_{t-1} + (1-q_t)p_t(x_t)\prod_{j=1}^{t}(1-q_j)(1-p_j(x_j))$, which results in $\boldsymbol{f_t} > \boldsymbol{f_{t-1}}$. Since the last round is a breakdown, then the expected gain from ***X1*** or ***X2*** is the same for both myopic-I and myopic-II agents, which can be expressed consecutively as follows:

$$EG_{X1} = (1-q_1)p_1(x_1)Sur(x_1) + \sum_{i=2}^{t-1}\left((1-q_i)p_i(x_i)Sur(x_i)^{+}\prod_{j=1}^{i-1}(1-q_j)(1-p_j(x_j))\right)$$

$$EG_{X2} = EG_{X1} + (1-q_t)p_t(x_t)Sur(x_t)\prod_{j=1}^{t-1}(1-q_j)(1-p_j(x_j))$$

where $\boldsymbol{Sur(x_i) = (B_i - x_i)}$ is the positive surplus if the bargaining succeeds at round *i*. From both equations above we have $\boldsymbol{EG_{X2}} \geq \boldsymbol{EG_{X1}}$. Since the probability of finding a concession in ***X2*** is strictly greater than in ***X1*** and the expected surplus of ***X2*** is weakly greater than of ***X1***, then we can conclude that ***X2*** is preferred to ***X1***. ■

Proposition 3-9. *Suppose both bargainers are myopic agents and their valuations are constant within a long period, and* $\boldsymbol{x}$, $\boldsymbol{p_t(x)}$, *and* $\boldsymbol{EG'_{t+1}(x)}$ *are continuous and differentiable. If the seller does not concede significantly within that period so that the buyer will update its belief* $\boldsymbol{p_t(x)}$ *to a steeper one* for all $\boldsymbol{x \leq x_t^*}$, *then the buyer concedes provided*

(a) the buyer is myopic-0, or

(b) the buyer is myopic-1 with convex belief $\boldsymbol{p_t(x)}$ *at* $\boldsymbol{x \leq x_t^*}$, *i.e.* $\partial \boldsymbol{p_t(x)}/\partial \boldsymbol{x} > 0$ *and* $\partial^2 \boldsymbol{p_t(x)}/\partial \boldsymbol{x}^2 \leq 0$, *and with expected future gain* $\boldsymbol{EG'_{t+2}(x)}$ *that satisfies:*

$$(\forall \boldsymbol{x}' \leq \boldsymbol{x_t}^*)\ \left[\left(\frac{\partial \boldsymbol{EG'_{t+1}(x)}}{\partial \boldsymbol{x}}\right)_{\boldsymbol{x}=\boldsymbol{x_t}^*} \leq \left(\frac{\partial \boldsymbol{EG'_{t+2}(x)}}{\partial \boldsymbol{x}}\right)_{\boldsymbol{x}=\boldsymbol{x}'} \wedge\ \boldsymbol{EG'_{t+1}(x_t^*)} \geq \boldsymbol{EG'_{t+2}(x')}\right]$$

Proof. Suppose at time $\boldsymbol{t}$ the buyer with $\boldsymbol{\pi_t(x)}$ offered $\boldsymbol{x_t^*}$ and it was rejected. Since the bargaining period is long enough without significant concession from the seller, then there is enough time for the buyer to update $\boldsymbol{\pi_t(x)}$ to $\boldsymbol{\pi_{t+n}(x) = p_{t+n}(x)}$. If before time $\boldsymbol{t+n}$ the buyer concedes, then the proposition is proven. If the buyer does not concede, then by Assumption 3-9 eventually $\boldsymbol{k_{t+n}(x)} = 0$, and $\boldsymbol{\pi_{t+n}(x)} = \boldsymbol{p_{t+n}(x)}$. So, let us ignore the intermediate transition state between $\boldsymbol{t}$ and $\boldsymbol{t+n}$ and only consider $\boldsymbol{\pi_t(x)} = \boldsymbol{p_t(x)}$ and $\boldsymbol{\pi_{t+n}(x)} = \boldsymbol{p_{t+n}(x)}$, where $\boldsymbol{p_t(x)}$ and $\boldsymbol{p_{t+n}(x)}$ are continuous and differentiable.

(a) Since in this case the buyer is a myopic-0 agent, then it will offer optimal price $\boldsymbol{x_t^*}$ that maximizes its expected gain, i.e. $\boldsymbol{Max_x\, [p_t(x)(B_t - x)]}$, which can be illustrated by the largest area of the rectangle bounded by belief $\boldsymbol{p_t(x)}$, $\boldsymbol{x}$-axis and buyer's valuation $\boldsymbol{B_t}$ in Figure 3-12. Since the seller rejects $\boldsymbol{x_t^*}$ and the counter-offer is insignificant, then by Proposition 3-2 the slope $\partial \boldsymbol{p_t(x)}/\partial \boldsymbol{x}$ becomes steeper for $\boldsymbol{x \le x_t^*}$, or $\partial\boldsymbol{(p_t(x) - p_{t+1}(x))}/\partial \boldsymbol{x} \le 0$ for $\boldsymbol{x \le x_t^*}$ and $\boldsymbol{p_{t+1}(x)} > 0$. Also, from by Assumption 3-8 we have $\boldsymbol{p_{t+1}(x) < p_t(x)}$ for $\boldsymbol{x \le x_t^*}$. Given these conditions, we can prove that a myopic-0 buyer will concede as shown by the following Lemma.

Lemma 3-1. Let $\partial\boldsymbol{(p_t(x) - p_{t+1}(x))}/\partial \boldsymbol{x} \le 0$ and $\boldsymbol{p_t(x) > p_{t+1}(x)} > 0$ for all $\boldsymbol{x \le x_t^*}$, then a myopic-0 buyer will concede, or $\boldsymbol{x_{t+1}^* > x_t^*}$.

Proof by contradiction: suppose the buyer does not concede but still generates a positive expected gain, or $\boldsymbol{x_{t+1}^* \le x_t^*}$ and $\boldsymbol{p_{t+1}(x_{t+1}^*)} > 0$, then by integrating $\partial\boldsymbol{(p_t(x) - p_{t+1}(x))}/\partial \boldsymbol{x} \le 0$ from $\boldsymbol{x_{t+1}^*}$ to $\boldsymbol{x_t^*}$, we have

$$\boldsymbol{[p_t(x) - p_{t+1}(x)]_{\,xt^*} - [p_t(x) - p_{t+1}(x)]_{\,xt+1^*}} \le 0$$

$$\Leftrightarrow \boldsymbol{[p_t(x_t^*) - p_{t+1}(x_t^*)] - [p_t(x_{t+1}^*) - p_{t+1}(x_{t+1}^*)]} \le 0$$

$$\Leftrightarrow \boldsymbol{p_t(x_t^*) - p_{t+1}(x_t^*) \le p_t(x_{t+1}^*) - p_{t+1}(x_{t+1}^*)}$$

$$\Leftrightarrow p_t(x_t*)(B - x_{t+1}*) - p_{t+1}(x_t*)(B - x_{t+1}*) \leq p_t(x_{t+1}*)(B - x_{t+1}*) - p_{t+1}(x_{t+1}*)(B - x_{t+1}*)$$

(3-15)

Since $x_{t+1}* \leq x_t*$, then $(B - x_t*) \leq (B - x_{t+1}*)$; and given $p_t(x_t*) > p_{t+1}(x_t*)$ we have:

$(p_t(x_t*) - p_{t+1}(x_t*))(B - x_t*) \leq (p_t(x_t*) - p_{t+1}(x_t*))(B - x_{t+1}*)$, or

$$p_t(x_t*)(B - x_t*) - p_{t+1}(x_t*)(B - x_t*) \leq p_t(x_t*)(B - x_{t+1}*) - p_{t+1}(x_t*)(B - x_{t+1}*)$$

Then by transitivity we can replace the LHS of (3-15), and it becomes:

$$p_t(x_t*)(B - x_t*) - p_{t+1}(x_t*)(B - x_t*) \leq p_t(x_t*)(B - x_{t+1}*) - p_{t+1}(x_t*)(B - x_{t+1}*) \leq$$
$$p_t(x_{t+1}*)(B - x_{t+1}*) - p_{t+1}(x_{t+1}*)(B - x_{t+1}*)$$

$$\Leftrightarrow p_t(x_t*)(B - x_t*) - p_{t+1}(x_t*)(B - x_t*) \leq p_t(x_{t+1}*)(B - x_{t+1}*) - p_{t+1}(x_{t+1}*)(B - x_{t+1}*) \quad (3\text{-}16)$$

However, $p_t(x_t*)(B - x_t*)$ is the maximum expected gain at time t, or $p_t(x_t*)(B - x_t*) > p_t(x_{t+1}*)\ (B - x_{t+1}*)$; or, $p_t(x_t*)(B - x_t*) = p_t(x_{t+1}*)\ (B - x_{t+1}*) + d^+$, where d^+ is some positive value. Hence, by substituting $p_t(x_t*)(B - x_t*) = p_t(x_{t+1}*)\ (B - x_{t+1}*) + d^+$ into (3-16) we get

$$p_t(x_{t+1}*)\ (B - x_{t+1}*) + d^+ - p_{t+1}(x_t*)(B - x_t*) \leq p_t(x_{t+1}*)(B - x_{t+1}*) - p_{t+1}(x_{t+1}*)(B - x_{t+1}*)$$

$$\Leftrightarrow \quad d^+ - p_{t+1}(x_t*)(B - x_t*) \leq - p_{t+1}(x_{t+1}*)(B - x_{t+1}*)$$

$$\Leftrightarrow \quad d^+ - p_{t+1}(x_t*)(B - x_t*) \leq - p_{t+1}(x_{t+1}*)(B - x_{t+1}*)$$

$$\Leftrightarrow \quad p_{t+1}(x_t*)(B - x_t*) \geq p_{t+1}(x_{t+1}*)(B - x_{t+1}*) + d^+ \qquad (3\text{-}17)$$

But at time $t+1$, $p_{t+1}(x_{t+1}*)(B - x_{t+1}*)$ is the maximum gain, or $p_{t+1}(x_{t+1}*)(B - x_{t+1}*) > p_{t+1}(x_t*)(B - x_t*)$; thus,

$$\Leftrightarrow \quad p_{t+1}(x_t*)(B - x_t*) > p_{t+1}(x_t*)(B - x_t*) + d^+$$

which is a contradiction. Thus, $x_{t+1}* > x_t*$ or a myopic-0 buyer concedes (End of the proof of Lemma 3-1).

(b) Since the buyer is a myopic-1 agent, then it will offer optimal price x_t^* that maximizes its expected gain, i.e. $\boldsymbol{Max_x\,[p_t(x)(B_t - x) + \gamma(1 - p_t(x))EG'_{t+1}(x)]}$. If x_t^* is an optimal offer, then the necessary condition is that x_t^* be a critical point which satisfies $\partial[p_t(x)(B - x) + \gamma(1 - p_t(x))EG'_{t+1}(x)]\ /\partial x = 0$ at x_t^* (first order derivative test), or

$$\left(B-x\right)\frac{\partial p_t(x)}{\partial x}-p_t(x)+\left(1-p_t(x)\right)\gamma\frac{\partial EG'_{t+1}(x)}{\partial x}-\gamma EG'_{t+1}(x)\frac{\partial p_t(x)}{\partial x}=0 \quad \text{at } x_t^*$$

$$\Leftrightarrow \left(B-x-\gamma EG'_{t+1}(x)\right)\frac{\partial p_t(x)}{\partial x}-p_t(x)+\left(1-p_t(x)\right)\gamma\frac{\partial EG'_{t+1}(x)}{\partial x}=0 \ \text{ at } x_t^*$$

$$\Leftrightarrow \left.\frac{\partial p_t(x)}{\partial x}\right|_{x=x_t^*} = \left.\frac{p_t(x)-\left(1-p_t(x)\right)\gamma\dfrac{\partial EG'_{t+1}(x)}{\partial x}}{\left(B-x-\gamma EG'_{t+1}(x)\right)}\right|_{x=x_t^*} \qquad (3\text{-}18)$$

Similarly, for the optimal offer x_{t+1}^* at time $t+1$, we also have:

$$\left.\frac{\partial p_{t+1}(x)}{\partial x}\right|_{x_{t+1}^*} = \left.\frac{p_{t+1}(x)-\left(1-p_{t+1}(x)\right)\gamma\dfrac{\partial EG'_{t+2}(x)}{\partial x}}{\left(B-x-\gamma EG'_{t+2}(x)\right)}\right|_{x_{t+1}^*} \qquad (3\text{-}19)$$

Since the seller rejects x_t^* and counter offers insignificantly, then by Proposition 3-2 the slope $\partial p_t(x)/\partial x$ becomes steeper for all $x \le x_t^*$, or $\partial p_t(x)/\partial x \le \partial p_{t+1}(x)/\partial x$ for $x \le x_t^*$. If the buyer does not concede, or $x_{t+1}^* \le x_t^*$, then we have $\left.\frac{\partial p_t(x)}{\partial x}\right|_{x_{t+1}^*} \le \left.\frac{\partial p_{t+1}(x)}{\partial x}\right|_{x_{t+1}^*}$

And since $p_t(x)$ is convex for all $x \le x_t^*$, then we have $\left.\frac{\partial p_t(x)}{\partial x}\right|_{x_t^*} \le \left.\frac{\partial p_t(x)}{\partial x}\right|_{x_{t+1}^*}$. Or, by transitivity we have $\left.\frac{\partial p_t(x)}{\partial x}\right|_{x_t^*} \le \left.\frac{\partial p_{t+1}(x)}{\partial x}\right|_{x_{t+1}^*}$

At this point, we can substitute equation (3-18) and (3-19) into the inequality above:

$$\left.\frac{p_t(x)-(1-p_t(x))\gamma\dfrac{\partial EG'_{t+1}(x)}{\partial x}}{(B-x-\gamma EG'_{t+1}(x))}\right|_{x_t*} \le \left.\frac{p_{t+1}(x)-(1-p_{t+1}(x))\gamma\dfrac{\partial EG'_{t+2}(x)}{\partial x}}{(B-x-\gamma EG'_{t+2}(x))}\right|_{x_{t+1}*}$$

The inequality condition above tells us that if $\boldsymbol{x_{t+1}*}$ is an optimal offer at time $\boldsymbol{t+1}$ that satisfies $\boldsymbol{x_{t+1}* \le x_t*}$, then the inequality holds. If the inequality does not hold, then by modus tollens, we can conclude that $\boldsymbol{x_{t+1}*}$ is not an optimal offer, or a contradiction. Thus, we come out with a sufficient condition for $\boldsymbol{x_{t+1}* > x_t*}$, which is the negation of the inequality above, i.e.

$$(\forall\ \boldsymbol{x' \le x_t*})\ \left.\frac{p_t(x)-(1-p_t(x))\gamma\dfrac{\partial EG'_{t+1}(x)}{\partial x}}{(B-x-\gamma EG'_{t+1}(x))}\right|_{x_t*} > \left.\frac{p_{t+1}(x)-(1-p_{t+1}(x))\gamma\dfrac{\partial EG'_{t+2}(x)}{\partial x}}{(B-x-\gamma EG'_{t+2}(x))}\right|_{x'}$$

which can be elaborated into

$(\forall\ \boldsymbol{x' \le x_t*})$

$$\frac{p_t(x_t*)-(1-p_t(x_t*))\gamma\left[\dfrac{\partial EG'_{t+1}(x)}{\partial x}\right]_{x_t*}}{(B-x_t*-\gamma EG'_{t+1}(x_t*))} > \frac{p_{t+1}(x')-(1-p_{t+1}(x'))\gamma\left[\dfrac{\partial EG'_{t+2}(x)}{\partial x}\right]_{x'}}{(B-x'-\gamma EG'_{t+2}(x'))}$$

and more specifically into two joint conditions:

$(\forall\ \boldsymbol{x' \le x_t*})$

$$p_t(x_t*)-(1-p_t(x_t*))\gamma\left[\frac{\partial EG'_{t+1}(x)}{\partial x}\right]_{x_t*} > p_{t+1}(x')-(1-p_{t+1}(x'))\gamma\left[\frac{\partial EG'_{t+2}(x)}{\partial x}\right]_{x'}$$

$$\wedge \qquad (B-x_t*-\gamma EG'_{t+1}(x_t*)) \le (B-x'-\gamma EG'_{t+2}(x'))$$

Or,

$(\forall\, x' \le x_t^*)$

$$p_t(x_t^*) - p_{t+1}(x') + \left(1 - p_{t+1}(x')\right)\gamma\left[\frac{\partial EG'_{t+2}(x)}{\partial x}\right]_{x'} > \left(1 - p_t(x_t^*)\right)\gamma\left[\frac{\partial EG'_{t+1}(x)}{\partial x}\right]_{x_t^*}$$

$$\wedge \quad \gamma EG'_{t+2}(x') \le \gamma EG'_{t+1}(x_t^*) + (B - x') - (B - x_t^*)$$

Since $x' \le x_t^*$, then $(B - x_t^*) \le (B - x')$ and $p_t(x_t^*) \ge p_t(x') > p_{t+1}(x')$, or $p_t(x_t^*) > p_{t+1}(x')$. Given $p_t(x_t^*) - p_{t+1}(x') > 0$ and $(B - x') - (B - x_t^*) \ge 0$, then the joint condition above holds if

$$(\forall\, x' \le x_t^*) \quad \left(1 - p_t(x_t^*)\right)\gamma\left[\frac{\partial EG'_{t+1}(x)}{\partial x}\right]_{x_t^*} \le \left(1 - p_{t+1}(x')\right)\gamma\left[\frac{\partial EG'_{t+2}(x)}{\partial x}\right]_{x'}$$

$$\wedge \qquad EG'_{t+1}(x_t^*) \ge EG'_{t+2}(x')$$

And since $(1 - p_t(x_t^*)) < (1 - p_{t+1}(x'))$, then the sufficient condition becomes

$$(\forall x' \le x_t^*) \quad \left[\left(\frac{\partial EG'_{t+1}(x)}{\partial x}\right)_{x_t^*} \le \left(\frac{\partial EG'_{t+2}(x)}{\partial x}\right)_{x'} \wedge \quad EG'_{t+1}(x_t^*) \ge EG'_{t+2}(x')\right] \tag{3-20}$$

Thus, given the condition in (3-20), the buyer's optimal offer $x_{t+1}^* > x_t^*$. ■

The following propositions and theorems are for the non-monotonic-offers protocol

Proposition 4-1. *Suppose all agents under the N-protocol are EvalF-I agents and this is common knowledge. If x_t^* is an optimal offer at time t, then in order to maximize its expected gain at time t+1:*

(a) a myopic-0 buyer will monotonically increase its offer if $B_{t+1} - x_t^* >> 0$, and decrease its offer if $B_{t+1} - x_t^* \to 0$ or $B_{t+1} - x_t^* < 0$;

(b) a myopic-1 buyer will monotonically increase its offer if $B_{t+1} - x_t^* >> \gamma EG'_{t+2}$, *and decrease its offer if* $B_{t+1} - x_t^* \rightarrow \gamma EG'_{t+2}$ *or* $B_{t+1} - x_t^* < \gamma EG'_{t+2}$.

<u>*Proof.*</u> (a) Suppose the buyer has already offered an optimal price at time t, i.e., x_t^* such that

$$[\nabla p_t]_{xt^*} / \Delta x \approx p_t(x_t^b)/(B_t - x_t^*) \tag{4-8}$$

If this offer is rejected by the seller, then by Assumption 3-8(i) the buyer will reduce both $p_t(x_t^*)$ and $p_t(x_t^b)$ to $p_{t+1}(x_t^*)$ and $p_{t+1}(x_t^b)$ respectively at time t +1, such that $p_t(x_t^b)$ is reduced more than $p_t(x_t^*)$, or $[\nabla p_{t+1}]_{xt^*} = p_{t+1}(x_t^*) - p_{t+1}(x_t^b)$ is greater than $[\nabla p_t]_{xt^*} = p_t(x_t^*) - p_t(x_t^b)$. Thus, the LHS of ε-equation (4-8) increases from $[\nabla p_t]_{xt^*}/\Delta x$ to $[\nabla p_{t+1}]_{xt^*} /\Delta x$, and the RHS decreases from $p_t(x_t^b)/(B_t - x_t^*)$ to $p_{t+1}(x_t^b)/(B_t - x_t^*)$, which changes the ε-equality such that there exists a value μ that satisfies

$$[\nabla p_{t+1}]_{xt^*} / \Delta x > \mu \approx p_{t+1}(x_t^b) / (B_t - x_t^*) \tag{4-9}$$

Since the buyer's valuation decreases over time from B_t to B_{t+1}, say by ΔB, then the denominator of the RHS of (4-8) decreases, or $p_{t+1}(x_t^b)/(B_{t+1} - x_t^*) > p_{t+1}(x_t^b)/(B_t - x_t^*)$.

Let's consider three cases. First, if ΔB is relatively small compared to $(B_t - x_t^*)$, i.e. when $(B_t - x_t^*) >> \Delta B$ or $(B_{t+1} - x_t^*) >> 0$, then the drop of B_t to B_{t+1} is not enough to increase the RHS of (4-9) to be ε-equal to the LHS, or there still exists a μ such that

$$[\nabla p_{t+1}]_{xt^*} / \Delta x > \mu \approx p_{t+1}(x_t^b) / (B_{t+1} - x_t^*) \tag{4-10}$$

Thus, the buyer needs to increase x_t^* to a new optimal offer x_{t+1}^*, which in turn increases $p_{t+1}(x_t^b)$ to $p_{t+1}(x_{t+1}^b)$ and decreases $(B_{t+1} - x_t^*)$ to $(B_{t+1} - x_{t+1}^*)$ (therefore, the RHS of (4-10) increases to $p_{t+1}(x_{t+1}^b)/(B_{t+1} - x_{t+1}^*)$), and may also decrease $[\nabla p_{t+1}]_{xt^*}$ to

$[\nabla \boldsymbol{p}_{t+1}]_{xt+1^*}$ (therefore, the LHS of (4-10) decreases to $[\nabla \boldsymbol{p}_{t+1}]_{xt+1^*} / \Delta \boldsymbol{x}$) such that the $\boldsymbol{\varepsilon}$-equality of (4-8) holds again at time $\boldsymbol{t+1}$, or $[\nabla \boldsymbol{p}_{t+1}]_{xt+1^*}/\Delta \boldsymbol{x} \approx \boldsymbol{p}_{t+1}(\boldsymbol{x}_{t+1}{}^{\boldsymbol{b}})/(\boldsymbol{B}_{t+1} - \boldsymbol{x}_{t+1}{}^*)$.

Second, if $\Delta \boldsymbol{B}$ is relatively large compared to $(\boldsymbol{B}_t - \boldsymbol{x}_t{}^*)$, i.e. when $(\boldsymbol{B}_t - \boldsymbol{x}_t{}^*)$ is almost equal to $\Delta \boldsymbol{B}$ or $\boldsymbol{B}_{t+1} - \boldsymbol{x}_t{}^* \to \mathbf{0}$, then the drop of $\boldsymbol{B}_t$ to $\boldsymbol{B}_{t+1}$ may increase the RHS of (4-9) even more than the LHS of (4-9), or

$$[\nabla \boldsymbol{p}_{t+1}]_{xt^*} / \Delta \boldsymbol{x} < \boldsymbol{p}_{t+1}(\boldsymbol{x}_t{}^{\boldsymbol{b}}) / (\boldsymbol{B}_{t+1} - \boldsymbol{x}_t{}^*) \qquad (4\text{-}11)$$

Consequently, the buyer needs to decrease $\boldsymbol{x}_t{}^*$ to a new optimal offer $\boldsymbol{x}_{t+1}{}^*$, which in turn decreases $\boldsymbol{p}_{t+1}(\boldsymbol{x}_t{}^{\boldsymbol{b}})$ to $\boldsymbol{p}_{t+1}(\boldsymbol{x}_{t+1}{}^{\boldsymbol{b}})$ and increases $(\boldsymbol{B}_{t+1} - \boldsymbol{x}_t{}^*)$ to $(\boldsymbol{B}_{t+1} - \boldsymbol{x}_{t+1}{}^*)$ (thus, the RHS of (4-11) decreases), and may increase $[\nabla \boldsymbol{p}_{t+1}]_{xt^*}$ to $[\nabla \boldsymbol{p}_{t+1}]_{xt+1^*}$ (thus, the LHS of (4-11) may increase) such that the ε-equality of (4-8) holds again.

In the third case, if $\boldsymbol{B}_{t+1} - \boldsymbol{x}_t{}^* < \mathbf{0}$, then a negative surplus is generated at $\boldsymbol{x}_t{}^*$; thus, the buyer must offer a lower price $\boldsymbol{B}_{t+1} - \boldsymbol{x}_{t+1}{}^* > \mathbf{0}$.

(b) The proof for a myopic-1 buyer is similar to the proof for a myopic-0 buyer (a), except with an additional term $\gamma \boldsymbol{EG'}_{t+1}$ that decreases over time. The decrease of this term, say by $\Delta \boldsymbol{EG}$, will increase the denominator of the RHS of (4-7). However, if $\Delta \boldsymbol{B} - \Delta \boldsymbol{EG}$ is relatively small compared to $(\boldsymbol{B}_t - \boldsymbol{x}_t{}^* - \gamma \boldsymbol{EG'}_{t+1})$, i.e. when $(\boldsymbol{B}_t - \boldsymbol{x}_t{}^* - \gamma \boldsymbol{EG'}_{t+1}) >> \Delta \boldsymbol{B} - \Delta \boldsymbol{EG}$, or $(\boldsymbol{B}_{t+1} - \boldsymbol{x}_t{}^*) >> \gamma \boldsymbol{EG'}_{t+2}$, then the buyer needs to increase $\boldsymbol{x}_t{}^*$ to a new optimal offer $\boldsymbol{x}_{t+1}{}^*$ as in the proof of (a). The proofs of the case of $\boldsymbol{B}_{t+1} - \boldsymbol{x}_t{}^* \to \gamma \boldsymbol{EG'}_{t+2}$ and $\boldsymbol{B}_{t+1} - \boldsymbol{x}_t{}^* < \gamma \boldsymbol{EG'}_{t+2}$ are also similar to the steps used in the proof of (a). ■

Proposition 4-2. *Under the N-protocol, if all agents are EvalF-I agents and this is common knowledge, then* $\boldsymbol{x}^*$ *converges to* $\boldsymbol{B}$ *over time.*

Proof. To show the convergence of $\boldsymbol{x}^*$ to $\boldsymbol{B}$ we only need to show that $\boldsymbol{B}_{t+1} - \boldsymbol{x}_{t+1}{}^* < \boldsymbol{B}_t - \boldsymbol{x}_t{}^*$ at any $\boldsymbol{t}$, either when $\boldsymbol{B}_{t+1} - \boldsymbol{x}_t{}^* >> \mathbf{0}$, $\boldsymbol{B}_{t+1} - \boldsymbol{x}_t{}^* \to \mathbf{0}$, or $\boldsymbol{B}_{t+1} - \boldsymbol{x}_t{}^* < \mathbf{0}$.

First, we can prove the case for myopic-0 buyers using Proposition 4-1(a). If $\boldsymbol{B_{t+1} - x_t^* >> 0}$, then the buyer will increase its offer to $\boldsymbol{x_{t+1}^*} > \boldsymbol{x_t^*}$. Since the buyer's valuation decreases over time, then $\boldsymbol{B_{t+1} < B_t}$. Or, $\boldsymbol{B_{t+1} - x_{t+1}^* < B_t - x_{t+1}^* < B_t - x_t^*}$. Thus, $\boldsymbol{B_{t+1} - x_{t+1}^* < B_t - x_t^*}$. If $\boldsymbol{B_{t+1} - x_t^*} \to \boldsymbol{0}$ or $\boldsymbol{B_{t+1} - x_t^*} < \boldsymbol{0}$, then the buyer will decrease its optimal offer to $\boldsymbol{x_{t+1}^*}$, resulting in $\boldsymbol{x_{t+1}^b < x_t^b}$. Given this, by Assumption 3-8 we have $\boldsymbol{p_t(x_t^b) > p_{t+1}(x_t^b) > p_{t+1}(x_{t+1}^b)}$ and by Proposition 3-2 we have $[\nabla \boldsymbol{p_t}]_{xt^*} \leq [\nabla \boldsymbol{p_{t+1}}]_{xt+1^*}$. Rewriting equation (4-6), we have

$$[\nabla \boldsymbol{p_t}]_{xt^*} / \boldsymbol{\Delta x \approx p_t(x_t^b) / (B_t - x_t^*)}$$

$$\Leftrightarrow \qquad \boldsymbol{(B_t - x_t^*)}\, [\nabla \boldsymbol{p_t}]_{xt^*} / \boldsymbol{\Delta x \approx p_t(x_t^b)}$$

$$\Leftrightarrow \qquad \boldsymbol{B_t - x_t^* \approx p_t(x_t^b)\, \Delta x} / [\nabla \boldsymbol{p_t}]_{xt^*} \text{ and } \boldsymbol{B_{t+1} - x_{t+1}^*} \approx \boldsymbol{p_{t+1}(x_{t+1}^b)\, \Delta x} / [\nabla \boldsymbol{p_{t+1}}]_{xt+1^*}.$$

Since, $\boldsymbol{p_t(x_t^b)}$ decreases to $\boldsymbol{p_{t+1}(x_{t+1}^b)}$ and $[\nabla \boldsymbol{p_t}]_{xt^*}$ increases to $[\nabla \boldsymbol{p_{t+1}}]_{xt+1^*}$, then $\boldsymbol{B_t - x_t^*}$ should also decrease to $\boldsymbol{B_{t+1} - x_{t+1}^*}$ for the ε-equality to hold; thus, $\boldsymbol{B_{t+1} - x_{t+1}^* < B_t - x_t^*}$, or $\boldsymbol{x^*}$ converges to $\boldsymbol{B}$ over time.

Similarly, to prove the case for myopic-1 buyers we use proposition 4-1(b) and an alteration of ε-equation (4-7), $\boldsymbol{B_t - x_t^*} \approx \boldsymbol{p_t(x_t^b)}\ \Delta\boldsymbol{x}/\ [\nabla \boldsymbol{p_t}]_{xt^*} + \boldsymbol{\gamma EG'_{t+1}}$. When the buyer increases its offer, then $\boldsymbol{B_{t+1} - x_{t+1}^* < B_t - x_t^*}$, or $\boldsymbol{x}$ converges to $\boldsymbol{B}$. When the buyer decreases its offer, by the fact that $\boldsymbol{p_t(x_t^b)}$ decreases to $\boldsymbol{p_{t+1}(x_{t+1}^b)}$, $[\nabla \boldsymbol{p_t}]_{xt^*}$ may increase, and $\boldsymbol{\gamma EG'_{t+1}}$ decreases over time, then $\boldsymbol{B_t - x_t^*}$ should also decrease to $\boldsymbol{B_{t+1} - x_{t+1}^*}$ for the ε-equality to hold; or, $\boldsymbol{x^*} \to \boldsymbol{B}$. ■

Proposition 4-3. *If EvalF-I agents are only concerned about the success rate, then the N-protocol is preferred over the M-protocol.*

Proof. Suppose both the buyer and the seller are only concerned about the success rate. Then, under the N-protocol the best strategy of the buyer is to offer its valuation, which decreases over time, because $\boldsymbol{p_t(B_t)}$ will be the highest (cf. Proposition 3-1). The seller also always offers its valuation. But under the M-protocol the buyer cannot do this. Thus, both the buyer's and seller's offers will be more likely to fall in the feasible set under the N-protocol which increases the success rate. Now, suppose that only the buyer is concerned about the success rate. Then, under the N-protocol the buyer will also offer its valuation, which decreases over time. If the seller's initial offer falls in the feasible set, then the bargaining succeeds. If it does not fall in the feasible set, then the buyer will decrease its offer along with its valuation; thus, by applying the inverse of Assumption 3-8(iii) to the seller, the seller will decrease its belief faster than the buyer increases its offer. Or, the seller will concede faster when the buyer offers its valuation. Therefore, both offers will be more likely to fall in the feasible set under the N-protocol. ■

Theorem 4-1. *The N-protocol is at least as good as the M-protocol for EvalF-I agents.*

Proof. A protocol is preferred by agents if it can help the agents to achieve their goals better than another protocol. First, if the goal of agents is to maximize their expected gain, then it is shown by Proposition 4-1 that the N-protocol guarantees a way for the agents to offer a price that maximizes their expected gain; while agents in the M-protocol cannot because they are restricted to increase their offers only. This is true not only for a decreasing valuation, but also for an increasing one, because it is possible that $\boldsymbol{B_{t+1} - \gamma EG'_{t+2}} > \boldsymbol{B_t - \gamma EG'_{t+1}}$ (cf. equation (4-7) for myopic-1 buyers), or the buyer has incentive to decrease its offer at time $\boldsymbol{t+1}$ to maximize its expected gain. Second, if the goal of agents is to find a resolution as soon as possible, then it is shown by Proposition 4-3 that the N-protocol is preferred to the M-protocol. Finally, if the agents cannot optimize their offers but submit them according to a pre-defined strategy (a pre-defined sequence of offers), then buyers in the M-protocol may get stuck in their valuation (in case of decreasing valuation), which incurs some cost in order to repeat the bargaining. Buyers under the N-protocol will never get stuck, and by Proposition 3-8, this is preferred over getting stuck. While this does not apply in the case of increasing valuation, still the N-protocol incurs no extra cost. ■

Proposition 4-4. *Under the N-protocol, if all agents are EvalF-II agents and this is common knowledge, then* $x_\infty^* \to B_\infty$.

<u>*Proof.*</u> If x_{t+1}^* is always beyond the region $[x_t^* - \delta, x_t^* + \delta]$ for any t, then equation (4-8) and (4-9) become equation (3-5) and (3-7) respectively. Thus, we can apply the maximization analysis done for EvalF-I agents; therefore, Proposition 4-2 holds and x_t^* converges to B. Consequently, a divergence appears only if x_{t+1}^* is within the region $[x_t^* - \delta, x_t^* + \delta]$ (we have a discontinuous function $\pi_{t+1}(x)$). Let the buyer consecutively offer prices within the region. However, under Assumption 3-10, within interval n the buyer will have $\pi_{t+n}(x) = p_{t+n}(x)$. And n rounds after that, the buyer will have $\pi_{t+2n}(x) = p_{t+2n}(x)$, where $p_{t+2n}(x)$ is the updated version of $p_{t+n}(x)$. If x_{t+2n}^* and x_{t+n}^* are the optimal offers at time $t+2n$ and $t+n$, respectively, and we ignore the situation between time $t+2n$ and $t+n$, then from Proposition 4-2 we get $x_{t+2n}^* \to B_{t+2n}$, or x_{t+2n}^* relatively converges to B_{t+2n} compared to x_{t+n}^* from B_{t+n}. Iteratively, x_{t+3n}^* also relatively converges to B_{t+3n} compared to x_{t+2n}^* from B_{t+2n}. Therefore, $x_\infty^* \to B_\infty$. ■

Proposition 4-5. *If EvalF-II agents are only concerned about the success rate, then the N-protocol is preferred to the M-protocol.*

The proof of Proposition 4-5 is similar to the proof of Proposition 4-3, with an additional condition that the seller may concede faster because a decreasing of the buyer's offers along its valuation may be perceived by the seller as the 'best' offer from the buyer. And a faster concession by the seller will result in higher probability of finding a resolution; thus, the N-protocol is preferred to the M-protocol.

Proposition 4-6. *Suppose the deadline for EvalF-I agents is the same as the deadline for EvalF-II agents. Then the success rate of the N-protocol for EvalF-II agents is as low as* $(1/\boldsymbol{n}^{\sim})$*-th of the success rate for EvalF-I agents, where,* $\boldsymbol{T_d}^{\sim}$ *and* $\boldsymbol{n}^{\sim}$ *are the average value of* $\boldsymbol{T_d}$ *and* $\boldsymbol{n}$ *for EvalF-II agents.*

Proof. The worst case is that all EvalF-II agents always choose an offer from the region $[\boldsymbol{x_t}^*-\delta, \boldsymbol{x_t}^*+\delta]$ which never yields a success. Since the effect of $\boldsymbol{k}(\boldsymbol{x})$ disappears after on average $\boldsymbol{n}^{\sim}$ rounds, then on average, we can find $\boldsymbol{T_d}^{\sim}/\boldsymbol{n}^{\sim}$ rounds where EvalF-II agents are under the same condition as EvalF-I agents. If the success rate of EvalF-I agents under the N-protocol is $\boldsymbol{m}$, then the success rate of EvalF-II agents will be as low as $\boldsymbol{m}/\boldsymbol{n}^{\sim}$. ■

Theorem 4-2. *The N-protocol is at least as good as the M-protocol for EvalF-II agents if the agents are benevolent.*

Proof. The same as the proof of Theorem 4-1, except that agents are benevolent.■

The following propositions and theorems are for strategic-delay-based protocols

Proposition 4-7. *A delay will be used by a buyer if the ratio of its future surplus with respect to the current one, denoted by* β*, satisfies*

$$\beta > \frac{1}{\varsigma\eta\gamma(1+\theta_{t+1}*)[\lambda-(1-q_t)(1-p_t(x_t*))]} \tag{4-13}$$

Where

$$\beta = \frac{B_{t+1}-x_{t+1}*}{B_t - x_t *}, \varsigma = \frac{1-q_{t+1}}{1-q_t},\ \eta = \frac{p_{t+1}(x_{t+1}*)}{p_t(x_t*)}, \text{ and}$$

$$\lambda = \frac{[EG'_{t+1}]_{delay}}{EG'_{t+1}} > (1-q_t)(1-p_t(x_t*)) \tag{4-14}$$

<u>*Proof.*</u>

$$\gamma\left[EG'_{t+1}\right]_{delay} > EG_t(x_t*)$$
$$\Leftrightarrow \gamma\lambda EG'_{t+1} > (1-q_t)\left[p_t(x_t*)(B_t - x_t*) + \gamma(1-p_t(x_t*))EG'_{t+1}\right]$$
$$\Leftrightarrow \gamma EG'_{t+1}\left(\lambda - (1-q_t)(1-p_t(x_t*))\right) > (1-q_t)p_t(x_t*)(B_t - x_t*)$$
$$\Leftrightarrow \gamma EG'_{t+1} > \frac{(1-q_t)p_t(x_t*)(B_t - x_t*)}{\lambda-(1-q_t)(1-p_t(x_t*))}$$
but $EG'_{t+1} = (1-q_{t+1})p_{t+1}(x_{t+1}*)(B_{t+1}-x_{t+1}*)(1+\theta_{t+1}*)$
$$\Leftrightarrow \gamma(1-q_{t+1})p_{t+1}(x_{t+1}*)(B_{t+1}-x_{t+1}*)(1+\theta_{t+1}*) > \frac{(1-q_t)p_t(x_t*)(B_t - x_t*)}{\lambda-(1-q_t)(1-p_t(x_t*))}$$
$$\Leftrightarrow \frac{\gamma(1-q_{t+1})p_{t+1}(x_{t+1}*)(B_{t+1}-x_{t+1}*)}{(1-q_t)p_t(x_t*)(B_t - x_t*)}(1+\theta_{t+1}*) > \frac{1}{\lambda-(1-q_t)(1-p_t(x_t*))}$$
since $\zeta = \frac{1-q_{t+1}}{1-q_t}, \eta = \frac{p_{t+1}(x_{t+1}*)}{p_t(x_t*)}, \beta = \frac{B_{t+1}-x_{t+1}*}{B_t - x_t*}$
$$\Leftrightarrow \zeta\eta\gamma\beta(1+\theta_{t+1}*) > \frac{1}{\lambda-(1-q_t)(1-p_t(x_t*))}$$
$$\Leftrightarrow \beta > \frac{1}{\zeta\eta\gamma(1+\theta_{t+1}*)\left[\lambda-(1-q_t)(1-p_t(x_t*))\right]}$$

■

Corollary 4-1. *If a buyer uses* ***EG'$_{t+2}$*** = ***EG'$_{t+1}$****, then a delay will be used if*

$$\beta > \frac{1-\gamma(1-q_{t+1})(1-p_{t+1}(x_{t+1}*))}{\zeta\eta\gamma[\lambda-(1-q_t)(1-p_t(x_t*))]} \tag{4-15}$$

<u>*Proof*</u>. From the definition of $\boldsymbol{\theta_t}$*

$$\theta_{t+1}* = \frac{\gamma(1-p_{t+1}(x_{t+1}*))EG'_{t+2}}{p_{t+1}(x_{t+1}*)(B_{t+1}-x_{t+1}*)}$$

Since ***EG'$_{t+2}$*** = ***EG'$_{t+1}$***, we have

$$\theta_{t+1}* = \frac{\gamma(1-p_{t+1}(x_{t+1}*))EG'_{t+1}}{p_{t+1}(x_{t+1}*)(B_{t+1}-x_{t+1}*)}$$
$$\theta_{t+1}* = \frac{\gamma(1-p_{t+1}(x_{t+1}*))}{p_{t+1}(x_{t+1}*)(B_{t+1}-x_{t+1}*)}(1-q_{t+1})p_{t+1}(x_{t+1}*)(B_{t+1}-x_{t+1}*)(1+\theta_{t+1}*)$$
$$\theta_{t+1}* = \gamma(1-q_{t+1})(1-p_{t+1}(x_{t+1}*))(1+\theta_{t+1}*)$$
$$\theta_{t+1}* = \gamma(1-q_{t+1})(1-p_{t+1}(x_{t+1}*)) + \gamma(1-q_{t+1})(1-p_{t+1}(x_{t+1}*))\theta_{t+1}*$$
$$\theta_{t+1}*(1-\gamma(1-q_{t+1})(1-p_{t+1}(x_{t+1}*))) = \gamma(1-q_{t+1})(1-p_{t+1}(x_{t+1}*))$$
$$\theta_{t+1}* = \frac{\gamma(1-q_{t+1})(1-p_{t+1}(x_{t+1}*))}{1-\gamma(1-q_{t+1})(1-p_{t+1}(x_{t+1}*))}$$

the inequality condition in Proposition 4-17 becomes

$$\beta > \frac{1}{\zeta\eta\gamma\left(1+\dfrac{\gamma(1-q_{t+1})(1-p_{t+1}(x_{t+1}*))}{1-\gamma(1-q_{t+1})(1-p_{t+1}(x_{t+1}*))}\right)[\lambda-(1-q_t)(1-p_t(x_t*))]}$$
$$\beta > \frac{1}{\zeta\eta\gamma\left(\dfrac{1}{1-\gamma(1-q_{t+1})(1-p_{t+1}(x_{t+1}*))}\right)[\lambda-(1-q_t)(1-p_t(x_t*))]}$$
$$\beta > \frac{1-\gamma(1-q_{t+1})(1-p_{t+1}(x_{t+1}*))}{\zeta\eta\gamma[\lambda-(1-q_t)(1-p_t(x_t*))]}$$

■

Proposition 4-8. *A delay will be used by a buyer if the ratio of the weight of its future precise belief with respect to the current imprecise belief satisfies*

$$\frac{\psi(p_{t+1}(x_{t+1}*),\varphi_1)}{\psi(p_t(x_t*),\varphi_2)} > \frac{1}{\zeta\beta\gamma(1+\theta_{t+1}*)[\lambda-(1-q_t)\psi(1-p_t(x_t*),\varphi_2)]} \qquad (4\text{-}17)$$

$$\textit{where } \theta_{t+1}* = \frac{\gamma\psi(1-p_{t+1}(x_{t+1}*),\varphi_1)EG'_{t+2}}{\psi(p_{t+1}(x_{t+1}*),\varphi_1)(B_{t+1}-x_{t+1}*)} \qquad (4\text{-}18)$$

Proof. The proof is similar to the proof of Proposition 4-7 except we use $\boldsymbol{\psi(p, \varphi)}$ instead of $\boldsymbol{p(\,)}$. ■

Proposition 4-9. *A delay will be used by a buyer if the ratio of its estimated future surplus with respect to the current estimated surplus satisfies*

$$\beta = \frac{B^e_{t+1}(\varphi_1) - x_{t+1}*}{B^e_t(\varphi_2) - x_t*} > \frac{1}{\varsigma\eta\gamma(1+\theta_{t+1}*)[\lambda-(1-q_t)(1-p_t(x_t*))]} \qquad (4\text{-}21)$$

Proof. The proof is similar to the proof of proposition 4-7 except we use the estimated value of ***B***. ■

Proposition 4-10. *A delay will be used by a buyer if the ratio of its estimated future probability of breakdown with respect to the current estimate satisfies*

$$\varsigma = \frac{1-q^e_{t+1}(\varphi_1)}{1-q^e_t(\varphi_2)} > \frac{1}{\eta\beta\gamma(1+\theta_{t+1}*)[\lambda-(1-q^e_t(\varphi_2))(1-p_t(x_t*))]} \qquad (4\text{-}22)$$

Proof. The proof is similar to the proof of Proposition 4-7 except we use estimated value of ***q***. ■

Proposition 4-11. *Let the true value of the seller's argument ($\boldsymbol{\alpha_S}$) induce an increasing of the buyer's valuation from $\boldsymbol{B_{t-1}}$ to $\boldsymbol{B_t}$ but reduce $\boldsymbol{p_{t-1}(x)}$ to $\boldsymbol{p_t(x)}$, while the negation of it ($\neg\boldsymbol{\alpha_S}$) does not change the buyer's valuation ($\boldsymbol{B_{t+1} = B_t = B_{t-1}}$) but increases $\boldsymbol{p_{t-1}(x)}$ to $\boldsymbol{p_{t+1}(x)}$. Then, the seller's argument will be verified, which causes a delay, if*

$$\beta\eta > \frac{1}{\varsigma\gamma(1+\theta_{t+1}*)[\lambda^{\#} - (1-q_t)(1-p_t(x_t*))]} \tag{4-23}$$

where $\beta = \frac{B_{t+1} - x_{t+1}*}{B_t - x_t*}$, $\eta = \frac{p_{t+1}(x_{t+1}*)}{p_t(x_t*)}$, $\varsigma = \frac{1-q_{t+1}}{1-q_t}$, *and*

$\lambda^{\#} = \chi \frac{[EG'_{t+1}]_{verify-false}}{EG'_{t+1}} > (1-q_t)(1-p_t(x_t*))$ *and* χ *is the likelihood that the verification results in the negation of the seller's argument.*

<u>*Proof.*</u> For the verification taken place, we must have

$\gamma[EG'_{t+1}]_{verify-false} > EG_t(x_t*)$, or

$$\Leftrightarrow \gamma\lambda^{\#} EG'_{t+1} > (1-q_t)[p_t(x_t*)(B_t - x_t*) + \gamma(1-p_t(x_t*))EG'_{t+1}]$$

Using the same method as in the proof of Proposition 4-7 we get

$$\beta\eta > \frac{1}{\varsigma\gamma(1+\theta_{t+1}*)[\lambda^{\#} - (1-q_t)(1-p_t(x_t*))]}$$

■

Proposition 4-12. *Let an offer by a seller* $\boldsymbol{k}$ *satisfy* $\boldsymbol{EG_t(y)_k \geq EG_t(x_t*)}$. *A delay will be used by the buyer if any of the following conditions applies:*

(i) $\boldsymbol{EG_t(y)_k < \gamma\,[EG'_{t+1}]_{delay}}$

(ii) there exists a seller $\boldsymbol{j \neq k}$ *s.t.* $\boldsymbol{EG_t(y)_k < \gamma\,[EG'_{t+1}]_j}$

(iii) there exists in the future a seller $\boldsymbol{v \neq k}$ *s.t.* $\boldsymbol{EG_t(y)_k < \gamma\,[EG'_{t+1}]_v}$

<u>*Proof.*</u> The proof is straightforward from Definition 4-1**.** ■

Theorem 4-3. *(The existence of strategic delay) A myopic-1 agent may benefit from strategic delay.*

<u>*Proof.*</u> By Proposition 4-7 to 4-12. ■

Theorem 4-4. *If the primary goal of agents is to maximize their expected utility, then myopic-1 agents prefer a delay-enabled bargaining protocol.*

Proof. A protocol is preferred by agents if it can help the agents to achieve their goals better than another protocol. Since the goal of agents is to maximize their expected gain, then it is shown by Proposition 4-7 to 4-12 that strategic delay can maximize their expected gain; while agents in the non-delay-enabled protocol cannot because they cannot delay their responses. ■

The following propositions and theorems are for ignorance-based ABN protocols

Proposition 4-13. *If the buyer believes that the probability* $\upsilon_t(\alpha_B) = P(\, p_t(x) \uparrow p^{\alpha}_t(x) \mid \alpha_B\,)$ *is positive, then* α_B *will always be sent.*

Proof. First, we can re-write equation (4-24) $EG_t(x) \equiv p_t(x)\,(B_t - x)$ such that by probability $\upsilon_t(\alpha_B)$ the expected payoff is $p^{\alpha}_t(x)\,(B_t - x)$ and by probability $(1 - \upsilon_t(\alpha_B))$ the expected payoff is $p_t(x)\,(B_t - x)$, or

$$EG^{\alpha}_t(x) = \upsilon_t(\alpha_B)\, p^{\alpha}_t(x)\,(B_t - x) + (1 - \upsilon_t(\alpha_B))\, p_t(x)\,(B_t - x) \qquad (4\text{-}27)$$

Since $p^{\alpha}_t(x) > p_t(x)$, then we have

$$\upsilon_t(\alpha_B) p^{\alpha}_t(x)\,(B_t - x) + (1 - \upsilon_t(\alpha_B))\, p_t(x)\,(B_t - x) >$$
$$\upsilon_t(\alpha_B) p_t(x)(B_t - x) + (1 - \upsilon_t(\alpha_B)) p_t(x)(B_t - x) \qquad (4\text{-}28)$$

But the RHS of inequality (4-28) equals to $p_t(x)(B_t - x) = EG_t(x)$; or, $EG^{\alpha}_t(x) > EG_t(x)$ for any $\upsilon_t(\alpha_B) \in (0, 1]$. Thus, α_B will always be sent. ■

Proposition 4-14. *(Strategic ignorance due to costly negotiation time) If there is a cost incurred from sending argument* $\boldsymbol{\alpha_B}$ *as shown in equation (4-29), and the buyer's valuation is decreasing over time, then* $\boldsymbol{\alpha_B}$ *may not always be sent.*

Proposition 4-14 follows from situations where the negotiation time is costly. Example 4-8 shows one of the possible case.

Proposition 4-15. *(Strategic ignorance due to recoiling arguments) Let the buyer be neutral to uncertainty and risk. If* $\boldsymbol{\mu_f < p_t(x_t^*)}$ *and* $\boldsymbol{\upsilon_t(\alpha_B)}$ *is strictly positive, then* $\boldsymbol{\alpha_B}$ *will not be sent.*

<u>Proof</u>. Since the agents are neutral toward uncertainty, then we can substitute $\boldsymbol{p^{\alpha}_t(x_t^*)}$ with its mean value $\boldsymbol{\mu_f < p_t(x_t^*)}$. By altering the inequality sign of equation (4-28) we have

$$\boldsymbol{EG^{\alpha}_t(x_t^*) = \upsilon_t(\alpha_B)p^{\alpha}_t(x_t^*)\,(B_t - x_t^*) + (1 - \upsilon_t(\alpha_B))\,p_t(x_t^*)\,(B_t - x_t^*) <}$$

$$\boldsymbol{\upsilon_t(\alpha_B)p_t(x_t^*)(B_t - x_t^*) + (1 - \upsilon_t(\alpha_B))p_t(x_t^*)(B_t - x_t^*) = EG_t(x_t^*)}$$

or, $\boldsymbol{EG^{\alpha}_t(x_t^*) < EG_t(x_t^*)}$ for any $\boldsymbol{\upsilon_t(\alpha_B)} \in (0, 1]$. Therefore, $\boldsymbol{\alpha_B}$ will not be sent. ■

Proposition 4-16. *(Strategic ignorance due to risk of breakdown) If* $\boldsymbol{\upsilon_\phi(\alpha_B)}$ *is greater than the increasing rate of* $\boldsymbol{p_t(x_t^*)}$ *from* $\boldsymbol{\alpha_B}$*, i.e.* $\boldsymbol{\upsilon_\phi(\alpha_B)} > \boldsymbol{(p^{\alpha}_t(x_t^*) - p_t(x_t^*))\,/\,p^{\alpha}_t(x_t^*)}$*, then* $\boldsymbol{\alpha_B}$ *will not be sent.*

<u>Proof</u>. Suppose $\boldsymbol{\upsilon_\phi(\alpha_B) > (p^{\alpha}_t(x_t^*) - p_t(x_t^*))\,/\,p^{\alpha}_t(x_t^*) = 1 - (p_t(x_t^*)\,/\,p^{\alpha}_t(x_t^*))}$. By equation (4-30)

$$\boldsymbol{EG^{\alpha}_t(x_t^*) = (1 - \upsilon_\phi(\alpha_B))\,p^{\alpha}_t(x_t^*)\,(B_t - x_t^*)}$$

$$\Rightarrow \quad \boldsymbol{EG^{\alpha}_t(x_t^*) < (1 - 1 + (p_t(x_t^*)\,/\,p^{\alpha}_t(x_t^*)))\,p^{\alpha}_t(x_t^*)\,(B_t - x_t^*)}$$

$$\Leftrightarrow \quad \boldsymbol{EG^{\alpha}_t(x_t^*) < p_t(x_t^*)(B_t - x_t^*)}$$

$$\Leftrightarrow \quad \boldsymbol{EG^{\alpha}_t(x_t^*) < EG_t(x_t^*)}.$$

Since $EG^{\alpha}_t(x_t^*)$ less than $EG_t(x_t^*)$, α_B will not be sent. ■

Corollary 4-2. *If* $\upsilon_\phi(\alpha_B) > 1 - p_t(x_t^*)$*, then* α_B *will not be sent.*

<u>Proof</u>. Suppose $\upsilon_\phi(\alpha_B) > 1 - p_t(x_t^*)$. From the fact that $p_t(x_t^*) \in [0, 1]$ then

$$EG^{\alpha}_t(x_t^*) = (1 - \upsilon_\phi(\alpha_B))\, p^{\alpha}_t(x_t^*)\, (B_t - x_t^*)$$

$$\Rightarrow \quad EG^{\alpha}_t(x_t^*) < (1 - 1 + p_t(x_t^*))\, p^{\alpha}_t(x_t^*)\, (B_t - x_t^*)$$

$$\Leftrightarrow \quad EG^{\alpha}_t(x_t^*) < p_t(x_t^*)\, p^{\alpha}_t(x_t^*)\, (B_t - x_t^*)$$

$$\Leftrightarrow \quad EG^{\alpha}_t(x_t^*) < p^{\alpha}_t(x_t^*)\, EG_t(x_t^*)$$

Since $p^{\alpha}_t(x_t^*) \leq 1$

$$\Rightarrow \quad EG^{\alpha}_t(x_t^*) < EG_t(x_t^*)$$

Thus, α_B will not be sent regardless of the value of $p^{\alpha}_t(x_t^*)$. ■

Theorem 4-5. *(The existence of proactive ignorance) A rational agent may not use argument* α_B *if it*

- *incurs a high cost;*
- *reduces the expected probability of the seller's acceptance of its offer; and/or*
- *raises the risk of breakdown more than the increasing rate of* $p_t(x_t^*)$.

<u>Proof</u>. By Proposition 4-14, 4-15, and 4-16. ■

Proposition 4-17. *(Valuation hiding) Let the buyer believe that both the seller valuation and the risk of breakdown are low so that the seller is willing to accept the buyer's offer and also* $\upsilon_t(\alpha_{S1})$ *is high so that the seller's argument is trusted and* ρ *is low so that the buyer will increase its valuation; then*

(i) α_{S1} *will be affirmed if* $p_t(x_t^*)$ *is steep; and*

(ii) α_{S1} *will be ignored if* $p_t(x_t^*)$ *is flat near certainty.*

Proof. Since $\boldsymbol{\upsilon_t(\alpha_{S1})}$ is high, then the buyer has only two choices in its reply: affirmation or ignorance. For ignoring $\boldsymbol{\alpha_{S1}}$ to be an appropriate response it is sufficient to show that the utility of ignorance is higher than that of affirmation. From the fact that $\boldsymbol{\rho}$ is low, we have a new valuation $\boldsymbol{B_t* = B_t + (1-\rho)(B_t'-B_t)} > \boldsymbol{B_t}$. Since the buyer is a utility maximizer, a shift of $\boldsymbol{B_t}$ may change the buyer's offer. First, we can derive the sensitivity of optimal offer $\boldsymbol{x_t*}$ to $\boldsymbol{B_t}$. Suppose $\boldsymbol{p_t(x_t*)}$ is continuous and differentiable, then we can use the first-order differential condition of equation (4-24) to obtain its optimal offer, i.e.

$$\left.\frac{dEG}{dx}\right|_{x_t*} = p_t'(x_t*)(B_t - x_t*) - p_t(x_t*) = 0, \text{ or } x_t* = B_t - \frac{p_t(x_t*)}{p_t'(x_t*)}$$

By differentiation on both sides we get

$$\begin{aligned} dx_t* &= dB_t - d\left(\frac{p_t(x_t*)}{p_t'(x_t*)}\right) \\ &= dB_t - \frac{p_t'(x_t*)^2\,dx_t* - p_t(x_t*)p_t''(x_t*)dx_t*}{p_t'(x_t*)^2} \\ &= dB_t - \left(1 - \frac{p_t(x_t*)p_t''(x_t*)}{p_t'(x_t*)^2}\right)dx_t* \end{aligned}$$

or $$\frac{dx_t*}{dB_t} = \frac{p_t'(x_t*)^2}{2p_t'(x_t*)^2 - p_t(x_t*)p_t''(x_t*)} \qquad \text{(A-31)}$$

Now, consider case (i) when $\boldsymbol{p_t(x_t*)}$ is steep, or $\boldsymbol{p_t'(x_t*)} >> 0$. Since $\boldsymbol{p_t(x_t*)} > 0$ and $\boldsymbol{p_t''(x_t*)} \leq 0$ (except in the left part of logistic function which is unlikely to be the buyer's optimal offer), then we can conclude that the denominator is positive; thus $\boldsymbol{dx_t*/dB_t} > 0$, or the buyer's optimal offer $\boldsymbol{x_t*}$ will increase when its valuation increases. Since the buyer will increase its optimal offer $\boldsymbol{x_t*}$, then it has no incentive to ignore $\boldsymbol{\alpha_{S1}}$; thus, an affirmation will be sent.

Now, consider case (ii) when $\boldsymbol{p_t(x_t^*)}$ is flat near certainty, or $\boldsymbol{p_t'(x_t^*)} \rightarrow 0$ and $\boldsymbol{p_t(x_t^*)} \approx 1$. Since $\boldsymbol{p_t''(x_t^*)} < 0$, by equation (A-31) we have $\boldsymbol{dx_t^*/dB_t} \rightarrow 0$. Or, the change of the buyer valuation will not affect its optimal offer. This is also true in non-differentiable cases, such as in the step and linear functions, where $\boldsymbol{x_t^*}$ is the upper breaking point (cf. $\boldsymbol{x_o}$ and $\boldsymbol{x_b}$ in Fig. 3-4). Because in this case $\boldsymbol{p_t(x_t^*)} = 1$, therefore the buyer does not have incentive to increase $\boldsymbol{x_t^*}$ more. Since the buyer does not have incentive to increase its offer, then the buyer will ignore the seller's argument. ■

Proposition 4-18. *Let $\boldsymbol{\upsilon_t(\alpha_{S2})}$ be low so that the seller's argument is trusted and assume all possible counter-arguments are very costly*, *then $\boldsymbol{\alpha_{S2}}$ will be ignored.*

<u>*Proof.*</u> This can be derived straightforwardly from Proposition 4-14 by substituting $\boldsymbol{\alpha_B}$ with all the buyer's counter-arguments. ■

Proposition 4-19. *Let $\boldsymbol{\upsilon_t(\alpha_{S2})}$ be high so that the seller's argument is not trusted and assume $\boldsymbol{B_t}$ is temporarily increasing over time. If rebuttal and query are more costly and/or may increase the risk of breakdown more than ignorance, then $\boldsymbol{\alpha_{S2}}$ will be ignored as long as its incurred cost is lower than the marginal gain of $\boldsymbol{B_t}$.*

<u>*Proof.*</u> Suppose by affirmation the buyer's expected payoff is $\boldsymbol{EG_t(y)}$, and if the deal is made at time $\boldsymbol{t+n}$, then the buyer's expected payoff is $\boldsymbol{EG_{t+n}(y)} = \boldsymbol{EG_t(y)} + \boldsymbol{g}$, where $\boldsymbol{g}$ is a positive marginal gain. Since the buyer is rational and the expected cost of ignorance is the lowest compared to rebuttal and query, and it is lower than $\boldsymbol{g}$, then it will be chosen. ■

Theorem 4-6. *(The existence of responsive ignorance) A rational agent may benefit from ignoring argument $\boldsymbol{\alpha_{S1}}$ and $\boldsymbol{\alpha_{S2}}$.*

<u>*Proof.*</u> By Proposition 4-17, 4-18 and 4-19. ■