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SCAKD 2016 – The Second International Workshop on Soft Computing Applications and Knowledge Discovery

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Preface

This volume contains the papers presented at the Second International Workshop on Soft Computing Applications and Knowledge Discovery (SCAKD 2016) held on July 18, 2016 at the National Research University Higher School of Economics, Moscow, Russia.

Soft computing is a collection of methodologies, which aim to exploit tolerance for imprecision, uncertainty and partial truth to achieve tractability, robustness and low solution cost in real life tasks. The workshop proposes to present high quality scientific results and promising research in the area of soft computing and data mining, particularly by young researchers, with an objective of bringing them to the focus while promoting collaborative research activities. By holding the workshop in conjunction with CLA 2016, we hope to provide the participants exposure and interaction with eminent scientists, engineers, and researchers in the related fields.

Each submission has been reviewed by at least two Program Committee members. Six regular papers have been accepted for publication as well as four research proposals. The program also includes one invited industry talk by the representatives of ExactPro company on Using intelligent systems and structural analysis to ensure orderly operations of the modern trading and exchange platforms.

We would like to thank all the authors of submitted papers and the Program Committee members for their commitment. We are grateful to our invited speaker and our sponsors: National Research University Higher School of Economics (Moscow, Russia), Russian Foundation for Basic Research, and ExactPro. Finally, we would like to acknowledge the EasyChair system which helped us to manage the reviewing process.

July 18 2016
Moscow

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Using Intelligent Systems and Structural Analysis to Assure Orderly Operations of the Modern Trading and Exchange Platforms

Olga Moskaleva and Anna Gromova

Exactpro company

Abstract. Maintaining orderly operations of its markets is the London Stock Exchange Group's (LSEG) utmost priority. As a regulated entity, LSEG has a legal responsibility to ensure correct and stable behavior of its platforms and monitor its markets.
This represents two major dimensions of Exactpro work. Technical stability and search for possible software defects. Fraud detection to prevent market manipulation, money laundering and other illegal activities.
Market fraud detection can be carried out by monitoring and analyzing all market events. Market surveillance systems ensure this monitoring.
Defect management is an essential part of improving the technical stability of software by using test tools. Identifying and correcting defects saves software costs. Additionally, the prediction of testing metrics should give project managers a better picture of risks associated with a particular software defect.
We demonstrate how intelligent systems and structural analysis could solve such tasks.

Keywords: fraud detection, surveillance systems, technical stability, defect management, intelligent systems, Data Mining

ACL-Scale as a Tool for Preprocessing of Many-Valued Contexts

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Abstract. One of the formal technique in Data mining is Formal Concept Analysis (FCA). During preprocessing of a many-valued context many applications of FCA require the partitioning of numerical data attributes into some smaller intervals. Designation of such numerical intervals with linguistic terms without domain experts will help researchers to understand attributes and their dependencies better. To solve this task we propose the notion of a special ACL-scale, which can be considered as a linguistic variable with ordered linguistic terms, modeled by fuzzy sets. The notion of ACL-scale, algorithms of its creation and application are presented. The example how many-valued context can be transformed into formal context using ACL-scale is shown in the paper. The main contribution is a new uniform tool for preprocessing of numerical attributes of given tables which simplify their transformation into a formal context with linguistic attributes.

Keywords: data mining, data preprocessing, ACL-scale, formal context, linguistic values

1 Introduction

One of the formal techniques in Data Mining and Knowledge Discovery in Databases (DM&KDD) process for extraction and representation of useful information, of objects (attributes) and of data dependencies is the Formal Concept Analysis (FCA) [1,2]. The first steps in applying of FCA is data preprocessing, where a many-valued context has to be transformed into a formal context to represent a data table with values of suitable granularity. When the input values are numerical, they have to be partitioned into numerical intervals. There are three main approaches to do this transformation, based on scaling theory. The conceptual scaling approach is well established and it uses conceptual scales [3,4] to derive a formal context. Logical scaling was introduced in [5] as a method using some expert knowledge to transform given data into the data from which conceptual hierarches can be explored. The fuzzy scaling approach beeing considered for example in [6,7,8] applies the notion of a linguistic variable [9]. The latter adds information to the structure of a formal context and can give linguistic description of numerical values of attributes and their dependences. The comparison of conceptual and fuzzy scaling theories for FCA was considered in [6]. The different approaches to embed fuzzy logic into FCA and application in KDD are given in [10]. The authors described the most important theories connected with fuzzy attributes, fuzzy concepts and fuzzy concept lattice.

The main problems in applying fuzzy scaling theory to FCA were discussed in [11] and some solutions were presented. One of the problems the author mentioned was the problem of using and interpreting the membership functions in FCA, so the short alternative conceptual description of fuzziness without using membership functions was given in [11].

In this paper we propose the approach for transforming numerical attributes of a many-valued context into linguistic variables. This transformation is considered as preprocessing based on the fuzzy scaling theory, where the membership functions are used to derive linguistic values of the partitions of numerical attributes only. The advantage of this approach is a linguistic granulation of numerical attributes in a many-valued context. This linguistic granulation can be useful in segmentation of objects with similar features. Mining the dependencies among several objects expressed in linguistic terms is another application of that linguistic granulation. To solve this task we propose the notion of a special Absolute & Comparative scale (ACL-scale). Using ACL-scale the partitions of numerical data and their linguistic descriptions can be derived. Therefore, the formal context can be presented in a traditional form, and well-known algorithms for FCA can be applied without computing of membership functions.

2 Problem Definition

Here we recall the definition of many-valued context [12] in respect to attributes *m* having numerical values *w*.

Definition 1. A many-valued context $K = (G, M, W, J)$ is a set of objects G, a set of attributes M, a set of possible values W, and a ternary relation $J \subseteq G \times M \times W$, with

$$(g, m, w) \in J, (g, m, v) \in J \Rightarrow w = v,$$

where $(g, m, w) \in J$ indicates that object g has the attribute *m* with value *w*. In this case, we also write $m(g) = w$, regarding the attribute *m* as a partial function from G to W.

Definition 2. A formal context is a triple $C = \langle G, Y, I \rangle$ where G is a set of objects, Y is a set of attributes and $I \subseteq G \times Y$ is a binary relation between G and Y. For $\langle g, y \rangle \in I$ it is said “The object g has the attribute y”.

The task is to transform given many-valued context into a formal context. We denote this transformation as $K \Rightarrow C$.

Each value $y \in Y$ is a linguistic value (some linguistic description of a numerical value w), derived by scaling. This means that for each attribute $m \in M$ on the set of its possible numerical values W a special scale has to be defined and then applied to transform a given numerical value w into a linguistic value y. Therefore we consider a task of a scale construction for each attribute $m \in M$ on the set of its possible numerical values W. The main demands for this scale construction are simple adaptation to a set of numerical values W and minimizing of an expert participation. To solve this task the scale must be formed in automatic way using uniform quantity of parameters and of operations. Beside that the scale must be considered as a linguistic variable to associate its linguistic terms to the scaling values.

So, the problem is to denote the notion of a special scale, which satisfies the mentioned above demands, and algorithms of its construction and its application. Application of this special scale will allow to decrease preprocessing time of a transformation of a given many-valued context into a formal context using uniform formal tool.

3 Notion of an ACL-scale

In this section we propose a special scale, named an ACL-scale (Absolute & Comparative scale) to do the transformation of given many-valued context into a formal context.

Let $\{x_i \in W, W \subseteq \mathbb{R}, i = 1,2, \dots, n\}$ be the set of possible ordered values of a numerical attribute *m* in respect to definition 1.

We assume that the binary relation $x \leq y$ is defined possessing the following properties:

- reflexivity: $x \leq x, \forall x \in W$.
- transitivity: if $x \leq y$ and $y \leq z$, then $x \leq z, \forall x, y, z \in W$.
- anti-symmetry: if $x \leq y$ and $y \leq x$, then $x = y, \forall x, y \in W$.

Let suppose several partially ordered intervals of equal length cover a set W and they are used for building a linguistic variable $\tilde{X}$ with fuzzy terms $\tilde{x}_k = \{x_i, \mu_{\tilde{x}_k}(x_i)\}$, $x_i \in W$, $\tilde{x}_k \in \tilde{X}$, $i = 1,2, \dots, n, k = 1,2, \dots, r, r < n$. Here $\mu_{\tilde{x}_k}(x_i), i = 1,2, \dots, n$ denotes the membership function of a fuzzy term with a linguistic value $\tilde{x}_k$. Therefore it can be said that a set of linguistic values covers a set W. Each linguistic value $\tilde{x}_k \in \tilde{X}$ can be considered as an ordered gradation of a scale and as linguistic estimation of every numerical value with some truth value.

Definition 3. ACL-scale for an attribute *m* with possible numerical values from the set *W* is an algebraic system

$$ACL = \{H, \Psi, \Omega\},$$

where the set $H = \{W, \tilde{X}\}$ denotes possible numerical values and possible fuzzy terms for an attribute *m*; $\Psi = \{nmin, nmax, r, MF\}$ is a set of parameters of an ACL-scale; $\Omega = \{Fuzzy, DeFuzzy\}$ is a set of operations, defined on a set H.

Below the components Ψ and Ω of an ACL-scale will be considered in details.

3.1 Parameters of an ACL-scale

Parameterization of an ACL-scale is useful as a tool for domain specific adaptation.
To adopt an ACL-scale to real values of a set W we consider two alternatives. The first one corresponds to the case when experts evaluate quantaty, parameters and shape of membership functions of linguistic variables $\tilde{X}$. Unfortunately this case is difficult to realize in practice. In the second alternative the goal is to minimize the work of expert and some algorithm is used to adopt an ACL-scale to real values of a set W. We apply the second alternative and consider four parameters of an ACL-scale adaptation:

$$\Psi \;= \{nmin, nmax, r, MF\}, \tag{1}$$

where $nmin \;=\; inf(W), nmax \;=\; sup(W)$; MF is the uniform shape of the membership functions of fuzzy terms (for example in a triangular form) [13]; r is the quantity of fuzzy terms, r+1 is the quantity of numerical intervals of equal length d, used for membership functions construction:

$$[x-d, x] \subset\; W, \qquad d = \frac{nmax - nmin}{r+1}. \tag{2}$$

Notice, that these intervals are the result of partitioning of the set W and any numerical value $w \;\in\; [x-d, x]$ is considered according to an ACL-scale as identical, with the same linguistic value, but having different truth degree. According to (2) the length of numerical intervals d depends on quantity of fuzzy terms.
In this case researcher must define the shape and the quantity of fuzzy terms r . Parameter r determines a quantity of numerical intervals and their length d. It means that parameter r determines a level of linguistic granulation: smaller value of parameter r corresponds to larger linguistic granulation and vice versa. Therefore the quantity of fuzzy terms r depends on research goals and required level of granulation. Taking into account human perception the recommendation for choosing the value of parameter r are: $3 < r < 10$.
The example of ACL-scale for a numerical attribute m with possible values defined in $W = [-26, 66]$ is shown on the Figure 1. Here partitioning into six ordered intervals was done, on which five triangular fuzzy terms (r=5) were constructed with linguistic values $\tilde{X} = \{A_{0-1}, A_0, A_1, A_2, A_{2+1}\}$.

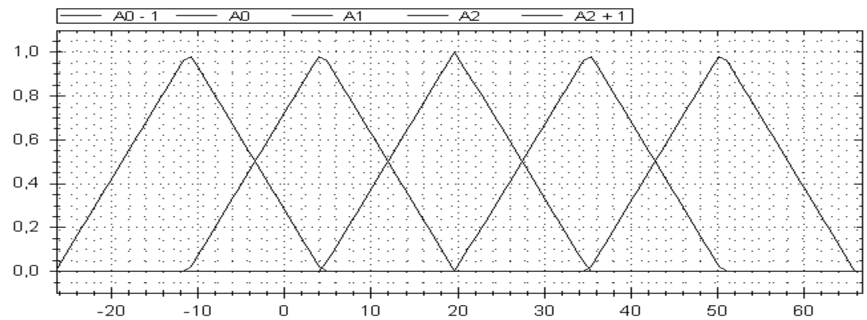

Fig. 1. Example of an ACL-scale

We assume that the following is fulfilled for an ACL-scale:

1. The numerical values *w* of attributes *m* corresponding to real or ideal objects are estimated.
2. Numerical and linguistic estimates are various, but they are equally essential aspects at the different levels of granularity.
3. Linguistic values of numerical attributes can be estimated by expert or a modeling estimation procedure.

The usage of parameters of an ACL-scale for linguistic description of numerical attributes allows to determine linguistic values practically in an automatic way, better understood by researchers.

3.2 The operations of an ACL-scale

The set of operations, defined on a set *H*, can be based on fuzzified/defuzzified functions. The operation $Fuzzy$ for linguistic description of each numerical value is defined as the following function:

$$\tilde{x}_i = \tilde{x}_s, if\ \tilde{x}_s(x_i) \geq \tilde{x}_j(x_i), s \in \{1,2,\ldots,r\}, \forall j = 1,2,\ldots,\mathrm{r}. \quad (3)$$

In respect to (3) for every $x_i \in W$ there will be only one linguistic value $\tilde{x}_s \in \tilde{X}$ with the maximum value among all of membership functions, *s* – is a number of that membership function.

We denote the operation $deFuzzy$ for numerical estimation of linguistic value as function $x_i' = DeFuzzy(\tilde{x}_i), x_i \in W, \tilde{x}_i \in \tilde{X}$, for example, as centroid of area:

$$x_t' = \frac{\int_{nmin}^{nmax} x\cdot\tilde{x}(x)dx}{\int_{nmin}^{nmax} \tilde{x}(x)dx}.$$

It is obvious, that $DeFuzzy$ function calculates approximate value with some error of estimation, and the latter can be computed in different ways, for example in a form:

$$Er_{x_i} = |x_i' - x_i|,$$

where the approximate value is $x_i' = DeFuzzy(\tilde{x}_i)$; x_i is the actual numerical value of some attribute.
The usage of uniform scaling by an ACL-scale will allow to transform given many-valued context into a formal context in automatic way and to explore the concepts having linguistic values which are better understood by researchers.

4 Transformation of numerical values into linguistic ones using an ACL-scale

The transformation of a numerical value $x_i \in W, i = 1,2,\dots,n$ into a linguistic value $\tilde{x}_i \in \tilde{X}$ with an ACL-scale means, that it is possible to define several fuzzy terms $\tilde{x}_j(x_i), j = 1,2,\dots,\mathrm{r}$ with different truth degree for $\forall x_i$.
Let $W \subseteq \mathbb{R}$ be a set of possible numerical values of an attribute.
First of all, it is required to construct an ACL-scale on the set W, containing the ordered fuzzy terms with linguistic values $\tilde{x}_k \in \tilde{X}$, $k = 1,2,\dots,r$.
Below we propose the Algorithm 1 for an ACL-scale creation by the determining its parameters on the set of possible numerical values *W* of a many-valued context.
Algorithm 1.
Step 1. Define the parameter *r* (the number of fuzzy terms) of ACL-scale.
Step 2. Compute the parameter *nmin* as the minimum value on a set of *W*.
Step 3. Compute the parameter *nmax* as the maximum value on a set of *W*.
Step 4. Order the possible values on W. Partition the ordered set of possible values $W \subseteq \mathbb{R}$, into *r*+1 intervals in respect to (2).
Step 5. Define the shape of the membership functions *MF* of fuzzy terms. Determine the linguistic values of fuzzy terms $\tilde{x}_k \in \tilde{X}$, $k = 1,2,\dots,r$.

To output the linguistic values for the numerical values of the set *W*, using an ACL-scale, Algorithm 2 is proposed.
Algorithm 2.
For each numerical value $x_i \in W, i = 1,2,\dots,n$ do the following:
Step 1. Using operation $Fuzzy$ (3) and well-known notion of fuzzy terms of chosen shape (for details you can see [13]) compute the values of their membership functions $\tilde{x}_k = \{x_i, \mu_{\tilde{x}_k}(x_i)\}, \tilde{x}_k \in \tilde{X}$, $k = 1,2,\dots,r$.
Step 2. Determine the fuzzy term $\tilde{x}_s(x_i)$ with the maximum value of membership function according to (3).
Step 3. Assign the output linguistic value as $\tilde{x}_i = \tilde{x}_s$ for input x_i Here *s* is the number of linguistic value on the set $\tilde{X}$, corresponding to an ACL-scale for the set of numerical values W.

5 Example

To illustrate how the ACL-scale can be applied to transform a many-valued context into a formal context we use the input data, which characterize hardware by two at-

tributes x_{cpu} ="Load of the central processor - CPU" and x_{ram} ="Load of the memory - RAM" (see Table 1).
We created one ACL-scale using the Algorithm 1 for both attributes, as their numerical values are contained in the same set of possible numerical values [0,100] presented in percentage. For this domain we defined $nmin = 0\%, nmax = 100\%$. Then seven fuzzy terms ($r = 7$) with linguistic values "very low", "low", "below an average", "average", "above an average", "high", "very high" were defined.

Table 1. Input many-valued data

id_obiect	x_{cpu}, %	x_{ram}, %
1	84,31	82,94
2	50,67	58,93
3	66,89	68,18
4	97,06	77,56
5	92,04	33,58
6	97,33	93,42
7	97,44	94,78
8	88,30	80,05
9	66,64	48,49

The shape of membership function was chosen as triangular with parameters shown in Table 2 (a - left, c - right, b - middle of numerical interval on which membership function is build).

Table 2. The parameters of membership functions of fuzzy terms in the form of triangular fuzzy number for attributes of hardware

	Linguistic values	The parameters of membership functions		
		a	***b***	***c***
x_{ram} x_{cpu}	very low	0	0	16,5
	low	0	16,5	33
	below an average	16,5	33	50
	average	33	50	66,5
	above an average	50	66,5	83
	high	66,5	83	100
	very high	83	100	100

After an ACL-scale has been created, it was used to output the linguistic value for every numerical value of the hardware attributes, applying the Algorithm 2. Table 3 illustrates the results of transformation of input data (see Table 1) into linguistic values.

Table 3. The results of linguistic estimation of the numerical values of the hardware attributes, using ACL-scale

id_obiect	**linguistic values x_{cpu}**	**linguistic values x_{ram}**
1	high	high
2	average	above an average
3	above an average	above an average
4	very high	high
5	high	below an average
6	very high	very high
7	very high	very high
8	high	high
9	above an average	average

Table 4 presents the formal context with linguistic values of hardware numerical attributes (here vl ="very low", lo= "low", ba = "below an average", av = "average", aa="above an average", hi="high", vh ="very high" for short).

Table 4. The formal context for a many-valued data derived by ACL-scale

id_obiect	x_{cpu}							x_{ram}						
	vl	lo	ba	av	aa	hi	vh	vl	lo	ba	av	aa	hi	vh
1						**x**							**x**	
2				**x**								**x**		
3					**x**							**x**		
4							**x**						**x**	
5						**x**				**x**				
6							**x**							**x**
7							**x**							**x**
8						**x**							**x**	
9					**x**						**x**			

The results in Table 4 show the transformation of the numerical attributes of a many-valued context (see Table 1) into linguistic variables for more understandable description of these attributes, which can be used for mining dependencies or for clustering. For further analysis the additional characteristics of a linguistic value of attributes are useful: the truth degree and the membership function.

6 Conclusion

During the past years preprocessing became an important step of data mining. For better understanding and analyzing numerical data, it is useful to have their linguistic description. To derive the latter description the transformation tecniques based on scaling are used usually.

In this paper the notion of an ACL-scale as the tool for transformation a many-valued context with a numerical attributes into a formal context with linguistic attributes is proposed. The algorithm of an ACL-scale creation by adaptation of its parameters on a set of numerical values is described. Application of an ACL-scale provides the linguistic granulation which can be useful in segmentation and investigation of objects with similar features. Mining the dependencies among attributes and among several objects expressed in linguistic terms is another application of that linguistic granulation. In these tasks time reduction on preprocessing stage will be obtained due to usage of the proposed uniform scaling algorithm for different numerical attributes.

The given example shows applicability and suitability of an ACL-scale for the preprocessing of a many-valued context with numerical attributes and deriving formal context with linguistic values.

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Coherence Analysis of Financial Analysts' Recommendations in the Framework of Evidence Theory*

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Abstract. This article is devoted to the analysis of coherence of financial recommendations with respect to securities of the Russian companies. The study is based on the analysis of approximately 4000 recommendations and forecasts of 23 investment banks with respect to around forty securities of Russian stock market over the period of 2012-2014 years. The predictive history of each of the investment bank was considered as evidence in the framework of evidence theory. The coherence of recommendations was evaluated with the help of the so-called conflict measure between the evidence, which determined on the subsets of the set of all evidence. Then the study of coherence was reduced to analysis of values of the conflict measure. This analysis was performed with the help of game-theoretic methods (Shapley index, interaction index), network analysis methods (centralities), fuzzy relation methods, hierarchical clustering methods.

Keywords: analysts' recommendations, conflict measure, interaction index, network analysis, hierarchical clustering.

1 Introduction

The forecasts and recommendations of financial analysts' (of investment banks) are the important sources of information in decision making by the participants of the financial market. The different aspects connected to the recommendations of financial analysts' are reflected in the research literature. The influence of forecasts of financial analysts' on the investors and the reaction of market on these forecasts is estimated in [13]. The relationship between analysts' fame and the reaction of investors on the corrected forecast is investigated in [2]. The "asymmetry" of analysts' forecasts and the manipulability of the recommendations is analyzed in [10].

The analysis of the coherence of forecasts and recommendations is one of the important directions of research. The coherence of recommendations is determined as a

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rule as similarity of recommendations that is given by different analysts with respect to the same securities. The level of coherence of the recommendations is evaluated more often as an average of all recommendations for a particular security. For example, the dependence of coherence of the forecasts from a number of the shares characteristics was investigated in [7].

In this study, analysis of the coherence of financial analysts' recommendations about the value of the shares of Russian companies in 2012-2014 will be performed in the framework of evidence theory (Dempster-Shafer theory, [4, 14]). Namely, the recommendation of the analyst (the recommendation of the investment bank) is described as evidence. The evidence determined by the set of focal elements and the mass function. The set of focal elements is a set of intervals of the relative value of the shares corresponding to the recommendations (buy/hold/sell). The mass function is equal to the relative frequency of recommendations in each interval (focal element). In [3] the conflict measure $K \in [0,1]$ was introduced on the set of all evidence of this type. This measure characterizes the inconsistency between the evidence. Then the value $C = 1 - K$ has the sense of the degree of the coherence of recommendations. The study, which started in [3], will continue in present article. Namely, the coherence of the recommendations will be evaluated and the set of the investment banks will be structured with respect to this coherence. The analysis of coherence will be performed with the help of game-theoretic methods (Shapley index, interaction index), network analysis methods (centralities), fuzzy relation methods, hierarchical clustering methods. In addition, the expressions for some of computational characteristics (Shapley index, interaction index) will be obtained in this study in the terms of the evidence of the type under consideration.

The work is structured as follows. The main notions of the evidence theory, the notion of the conflict measure are given in Section 2. The axiomatic of the conflict measure is discussed in this section too. The research database is described in Section 3. Section 4 is devoted to the description of evidence corresponding to database and the used conflict measure in the term of evidence. Section 5 is the main part of the work, in which study the coherence by the different methods. Finally, some conclusions from research are presented in Section 6.

2 The Evidence Functions Theory and Conflict Measures

Let X be a finite set and 2^X be a powerset of X. The mass function and the focal element are the fundamental notions in evidence theory. The mass function is a set function $m: 2^X \to [0,1]$ that satisfy the following conditions

$$m(\varnothing) = 0, \ \sum\nolimits_{A \subseteq X} m(A) = 1. \tag{1}$$

The value $m(A)$ characterizes the degree that true alternative from X belongs to the set $A \in 2^X$. The subset $A \in 2^X$ is called a focal element if $m(A) > 0$. Let $\mathcal{A} = \{A\}$ be a set of all focal elements. Then the pair $F = (\mathcal{A}, m)$ is called a body of evidence. Let $F\ (X)$ be a set of all bodies of evidence on X. Note that the body of evidence can be considered for an arbitrary nonempty set X, if the set function $m: L \to [0,1]$ is defined on the some nonempty set L of subsets from X that satisfy the conditions (1).

Let we have two bodies of evidence $F_1 = (A_1, m_1)$ and $F_2 = (A_2, m_2)$. For example, these evidences can be obtained from two sources of information. Then we have a question about the conflict between the two evidences. Historically the function $K_0(F_1, F_2)$ connected with Dempster's combining rule [4, 14] was the first conflict measure: $K_0 = K_0(F_1, F_2) = \sum_{B \cap C = \varnothing,\, B \in A_1,\, C \in A_2} m_1(B) m_2(C)$.

The axioms of the conflict measure are considered in [5]. There are few approaches to the estimation of the conflict of evidence. The analyses of these approaches can be found in [3]. It can be allocated conditionally the metric approach [8], the structural approach [11], the algebraic approach [9].

The notion of a conflict measure (and corresponding axioms) was generalized in [3] for arbitrary finite set of evidence. Suppose that we have some finite set of evidence $M = \{F_1, ..., F_l\}$, $F_i \in F\ (X)$, $i = 1, ..., l$. Let 2^M be a powerset of M. We shall put by definition that $K(B) = 0$, if $|B| = 1$, $B \in 2^M$ and $K(\varnothing) = 0$. Note that the conflict measure K_0 that considered on 2^M in the form

$$K_0(\{F_{i_1}, ..., F_{i_k}\}) = \sum_{A_{i_1} \cap ... \cap A_{i_k} = \varnothing} m_{i_1}(A_{i_1}) ... m_{i_k}(A_{i_k}),\ F_{i_s} = (\{A_{i_s}\}, m_{i_s}),\ s = 1, ..., k, \qquad (2)$$

satisfies the monotonicity condition: $K(B') \le K(B'')$, if $B' \subseteq B''$ and $B', B'' \in 2^M$. This means that the adding of new evidence to the set of evidence does not reduce the conflict measure.

3 The Description of the Database

The conflictness (and coherence as the dual concept of) of the evidences about analysts' forecasts (investment banks) is investigated in this article. The conflictness characterizes in this case the degree of non coherence of forecasts for some set of experts.

The study is based on the analysis of approximately 4000 recommendations and forecasts of 23 investment banks with respect to around forty securities of Russian stock market over the period of 2012-2014 years. The databases of the agencies Bloomberg and RBC are the sources of information. The forecasts are presented by experts of the world's largest investment banks including such renowned companies as Goldman Sachs, Credit Suisse, UBS, Deutsche Bank and others.

Each investment bank makes recommendations of three types to sell/hold/buy with forecast of target price of the security. The target prices of forecasts are recalculated into the so-called relative values of target prices. The relative value of a target price is a ratio of the predicted price to the quotation of the security on the date of the forecast.

The boundaries of relative prices between the recommendations of various types were determined by maximizing number of recommendations that fall into the "corresponding" intervals: [0, 0.97), [0.97, 1.22), [1.22, +∞). Thus, we have nine sets, each of which represents the interval and a label of recommendation type: $A_1^{(t)} = [0, 0.97)$, $A_2^{(t)} = [0.97, 1.2)$, $A_3^{(t)} = [1.2, +\infty)$, $t = 1, 2, 3$, where $t = 1$ – to sell, $t = 2$ – to hold, $t = 3$ – to buy.

4 The Description of Evidence and the Used Conflict Measures

The belonging of the relative price of the forecast of a certain type (to buy/hold/sell) to one of the three intervals can be considered as an evidence of the investment bank. Then we can found the body of evidence for given investment bank. Let we fixed the i-th investment bank, $i = 1,...,l$ (l is a number of investment banks), $c_{ik}^{(t)}$ is a number of belonging of relative price to interval $A_k^{(t)}$, N_i is a general number of forecasts for i-th investment bank. Then $m_i(A_k^{(t)}) = c_{ik}^{(t)} / N_i$ is a frequency of belonging of relative price to interval $A_k^{(t)}$. The mass function m_i satisfies the normalization condition: $\sum_t \sum_k m_i(A_k^{(t)}) = 1$ for all $i = 1,...,l$. Then $F_i = \left(A_k^{(t)}, m_i(A_k^{(t)})\right)_{k,t}$ is a body of evidence of i-th investment bank, $i = 1,...,l$. We can consider that all evidences have the same set of focal elements (even if $m_i(A_k^{(t)}) = 0$ for certain indexes) and all different focal elements $A_k^{(t)}$ are pairwise disjoint. Thus, the vector $\mathbf{m} = (m^{(s)})_{s=1}^{9}$, $m^{(k+3(t-1))} = m(A_k^{(t)})$, $k = 1, 2, 3$, $t = 1, 2, 3$ corresponds bijectively to the body of evidence $F = \left(A_k^{(t)}, m(A_k^{(t)})\right)_{k,t}$. The set of all such evidence forms a simplex $S = \{m^{(s)} : m^{(s)} \geq 0\ \forall s, \quad \sum_{s=1}^{9} m^{(s)} = 1\}$.

The formula (2) for calculation of conflict measure $K_0(F_1,...,F_l)$ can be simplified.

Proposition 1 [3]. *If a bodies of evidence* $F_i = \left(\{A_k\}, m_i(A_k)\right)$, $i = 1,...,l$ *satisfy the conditions* $A_s \cap A_k = \varnothing$ *for* $s \neq k$, *then the conflict measure* $K_0(B)$, $B \subseteq M$ *is equal to* $K_0(B) = 1 - \sum_k \prod_{i:F_i \in B} m_i(A_k)$.

The following measure

$$K(B) = 1 - \sum_k \min_{i:F_i \in B} m_i(A_k), \tag{3}$$

satisfies also the monotonicity condition and will be considered as a conflict measure below instead of measure K_0 in this paper.

Let $m_i^{(k)} = m_i(A_k)$ $\forall k$, $i = 1,...,l$ and we denote $F_i \in B \subseteq M$ for shot $i \in B$. We denote the measure $K(F_{i_1},...,F_{i_s})$ as $K(\mathbf{m}_{i_1},...,\mathbf{m}_{i_s})$, if $\mathbf{m}_{i_p} \leftrightarrow F_{i_p}$, $p = 1,...,s$ with consideration of the vector representation of evidence.

We will consider a coherence measure $C = 1 - K$ which is defined on 2^M together with a conflict measure K . This measure characterized the degree of coherence of financial analysts' recommendations.

Below, we are interested in estimation of increments of the individual analysts' contribution in the total conflict: $\Delta_i K(B) = K(B \cup \{i\}) - K(B)$, $B \subseteq M \setminus \{i\}$, $\Delta_{ij} K(B) = K(B \cup \{i,j\}) - \quad K(B \cup \{i\}) - K(B \cup \{j\}) + K(B)$, $B \subseteq M \setminus \{i,j\}$. Let $(t)_+ = \begin{cases} t, t \geq 0, \\ 0, t < 0. \end{cases}$ The following proposition is true for measure (3) and the increments $\Delta_i K(B)$ and $\Delta_{ij} K(B)$.

Proposition 2. *The following equalities are true for any* $\mathbf{m}_i, \mathbf{m}_j \in S$ *and* $B \subseteq S$:

1) $K_{ij} = K(\mathbf{m}_i, \mathbf{m}_j) = \sum_k \max\{m_i^{(k)}, m_j^{(k)}\} - 1 = \frac{1}{2} \sum_k \left| m_i^{(k)} - m_j^{(k)} \right|$;

2) $\Delta_i K(\varnothing) = 0$ *and*

$$\Delta_i K(B) = \sum_k \max\left\{\min_{s \in B} m_s^{(k)}, m_i^{(k)}\right\} - 1 = \sum_k \left(\min_{s \in B} m_s^{(k)} - m_i^{(k)}\right)_+ , \textit{ if } \varnothing \neq B \subseteq M \setminus \{i\};$$

3) $\Delta_{ij} K(\varnothing) = K_{ij}$ *and*

$$\Delta_{ij} K(B) = -\sum_k \left(\min_{s \in B} m_s^{(k)} - \max\left\{m_i^{(k)}, m_j^{(k)}\right\}\right)_+ , \textit{ if } \varnothing \neq B \subseteq M \setminus \{i,j\}.$$

Remark 1. The equality 1) in Proposition 2 shows us that the measure of pair conflict $K(\cdot,\cdot)$ is a metric on the simplex S .

Remark 2. All pair increments of the conflict measure with non empty coalitions are not positive as follows from 3): $\Delta_{ij} K(B) \leq 0$ $\forall B \neq \varnothing$, $B \subseteq M \setminus \{i,j\}$. This means that the inclusion of any analyst in the greater coalition increases the conflict measure by a smaller amount than the inclusion of the analyst in the smaller coalition.

5 An Analysis of Evidence Coherence

5.1 The Finding of the Most Conflict Analysts Using the Shapley Vector

If the monotone measure K is defined on the set of all subsets of M then we can determine the contribution of *i*-th analyst in general conflict $K(M)$ of the set of all analysts M as the difference $K(M) - K(M \setminus \{i\})$. More accurately the contribution of *i*-th analyst in general conflict can be determined as a average contribution in the conflict of the group (coalition) of analysts B : $\Delta_i K(B) = K(B \cup \{i\}) - K(B)$, where the average is computed for all groups (coalitions) of analysts B , $B \subseteq M \setminus \{i\}$. In this case we will get so called Shapley value [15], which is widely used in the coalition (cooperative) game theory: $v_i = \sum_{B \subseteq M \setminus \{i\}} \alpha_l\left(|B|,1\right)\Delta_i K(B)$, $i = 1,...,l$, $\alpha_l\left(s,r\right) = \frac{(l-s-r)!s!}{(l-r+1)!}$, $s+r = 1,...,l$. The vector $\mathbf{v} = (v_i)_{i=1}^l$ is called by Shapley vector and it satisfies the condition $\sum_{i=1}^l v_i = K(M)$. We will find an expression for the Shapley values of conflict measure (3) in terms of evidence $F_i \leftrightarrow \mathbf{m}_i$, $i = 1,...,l$.

Proposition 3. *The following formula is true for Shapley values of conflict measure (3):* $v_i = \sum_{\varnothing \neq B \subseteq M \setminus \{i\}} \alpha_l\left(|B|,1\right)\sum_k \max\left\{m_k^{(i)}, \min_{s \in B} m_k^{(s)}\right\} - \frac{l-1}{l}$, $i = 1,...,l$.

The contributions of all investment banks in the general conflictness of recommendations in period 2012-2014 are shown in the Fig. 1. These contributions were estimated with the help of Shapley values. The general conflictness for all 23 investment banks is equal 0.625.

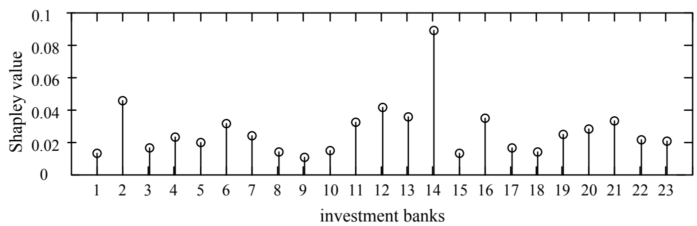

Fig. 1. The Shapley values of investment banks

Remark 3. The following denotations of investment banks are used on Fig. 1–4, in Tables 1–2: 1 – Alfa-Bank, 2 – Aton Bank, 3 – BCS, 4 – Veles Capital, 5 – VTB Capital, 6 – Gazprombank, 7 – Metropol Bank, 8 – Discovery Bank, 9 – Renaissance Capital, 10 – Uralsib Capital, 11 – Finam, 12 – Barclays, 13 – Citi group, 14 – Credit suisse, 15 – Deutsche Bank, 16 – Goldman Sachs, 17 – HSBC, 18 – J.P. Morgan, 19 – Morgan Stanley, 20 – Raiffeisen, 21 – Rye. Man&GorSecurities, 22 – Sberbank CIB, 23 – UBS.

The interrelation between the Shapley values of investment banks and the profitability of forecasts was analyzed in [3].

5.2 An Analysis of the Mutual Coherence of the Recommendations of Analysts with the help of Interaction Index

In addition to the detection of key analysts (investment banks) with the help of Shapley values that have the greatest influence on the coherence of forecasts, it is important to analyze the mutual influence of investment banks on the coherence of forecasts. This can be done with the help of the so-called interaction index [6], which is equal $I(T) = \sum_{B \subseteq M \setminus T} \alpha_l\left(|B|,|T|\right)\sum_{C \subseteq T}(-1)^{|T \setminus C|}K\left(C \cup B\right)$ for arbitrary coalition T and monotone measure K, defined on the finite set M, $|M| = l$. The interaction index $I(T)$ of the set of analysts T characterizes in our case the value of added contribution (synergistic effect) of this set in general conflict as compared with the summary contribution of separate analysts and improper subsets of T in the conflict. In particular, $I(\{i\}) = v_i$ is a Shapley value, $i = 1,...,l$. The interaction index for coalitions from two elements $I(\{i, j\}) = I_{ij}$ has an important value. This index was introduced earlier in [12]: $I_{ij} = \sum_{B\subseteq M\setminus\{i,j\}} \alpha_l \left(|B|,2\right) \Delta_{ij} K(B)$. The interaction index has value in the interval $[-1,1]$. If I_{ij} is close to 1, then this means that these analysts in pair increase the conflict in combination with the other coalitions to a larger value than each of them individually. If I_{ij} is close to -1, then the union of two analysts in the pair will not cause the synergistic effect in calculation of conflict. We will find an expression for the pair interaction index of the conflict measure (3) in terms of evidence $F_i \leftrightarrow \mathbf{m}_i$, $i = 1,...,l$.

Proposition 4. *The following formula is true for pair interaction index of the conflict measure (3)* $I_{ij} = \frac{1}{l-1} K_{ij} - \sum_{\emptyset\neq B\subseteq M\setminus\{i,j\}} \alpha_l \left(|B|,2\right) \sum_k \left(\min_{s\in B} m_s^{(k)} - \max\left\{m_i^{(k)}, m_j^{(k)}\right\}\right)_+$.

The values of the interaction index I_{ij} that characterized the contributions of pairs of investment banks in the general conflict of forecasts about the value of shares of Russian companies in period 2012-2014 are shown in Table 1. The values for which $|I_{ij}| \geq 0.013$ are indicated only in the table.

Table 1. The values of the interaction index I_{ij} , $|I_{ij}| \geq 0.013$

	11	**14**	**19**	**20**	**21**	**22**	**23**
6	-0,017			0,013			
7		0,013			-0,02		
11					-0,015		
12		-0,023	-0,013	-0,015	0,013		
13							-0,015
14				-0,014	0,014	0,015	
16						-0,014	

Since we are interested in the coherence measure of recommendations $C = 1 - K$ then and $I_{ij}(C) = -I_{ij}(K)$, then the pair with negative and large absolute values are interesting for us in Table 1. It is the pairs (in decreasing order of $|I_{ij}|$): (12,14), (7,21), (6,11), (11,21), (12,20), (13,23).

5.3 A Network Analysis of the Coherence of Analysts' Recommendations

We consider the coherence graph of recommendations $G = (N, C)$ on the set of all investment banks, where $N = \{n_i\}$ be a set of all nodes (investment banks), $C = \{C_{ij}\}$ be a set of edges with weights $C_{ij} = 1 - K_{ij}$ and K_{ij} be a value of conflict measure between the i-th and j-th investment banks, which calculated by formula (3). We can consider the "roughenned" coherence graph instead of the graph G for a better visualization with weights $C_{ij} = \begin{cases} 1, & K_{ij} < h, \\ 0, & K_{ij} \geq h, \end{cases}$ where h is a threshold value. The such graph, which constructed by the data of value of shares of Russian companies in period 2012-2014, is shown in the Fig. 2 for $h = 0.15$.

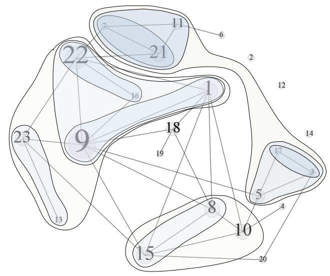

Fig. 2. The coherence graph of recommendations of investment banks

The matrix of pair coherence of recommendations $C = \{C_{ij}\}$ is a symmetric and non-negative. We investigate the problem of finding such investment banks, which have a most influence on coherence of recommendations. We will consider the so-called eigenvector centrality [1]. This centrality takes into account not only neighbor links but also distant links of nodes. The calculation of the measure of centrality for each node associated with the solution of the eigenvector problem with respect to the adjacency matrix A of the network graph. The vector of the relative centralities x is an eigenvector of the adjacency matrix corresponding to the largest eigenvalue $\lambda_{\max}$, i.e. $A\mathbf{x} = \lambda_{\max}\mathbf{x}$.

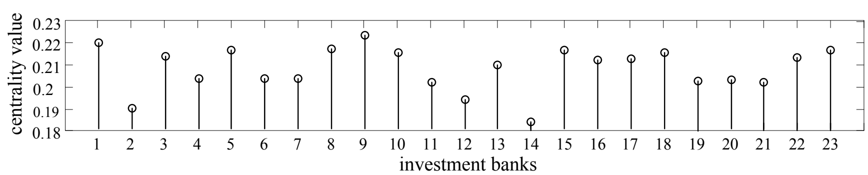

Fig. 3. The values of coordinates of centrality vector for the coherence graph of recommendations of investment banks

We have $\lambda_{\max} = 17.9$ for the data of value of shares of Russian companies in period 2012-2014. The values of coordinates of corresponding eigenvector (centrality vector) are shown in the Fig. 3. As can be seen from this figure, the greatest influence on the coherence of the recommendations in accordance with the values coordinates of the centrality vector have the banks (in descending order of influence) 9,1,18,15,23, etc.

The centrality vector correlated greatly and negatively with the Shapley vector. The corresponding correlation coefficient is equal to -0.86.

However, pairwise coherencies of recommendations do not give a complete picture of the more complex (not pairwise) interactions. This kind of interaction can be revealed with the help of analysis of the cluster structures of relations on the set of evidence, which is given by a conflict measure.

5.4 An Analysis of Fuzzy Relations on the Set of Evidence

Let $M = \{F_1,...,F_l\}$ be a set of evidence. Then the pair conflict measure $K_{ij} = K(F_i,F_j)$ and the corresponding coherence measure $C_{ij} = 1 - K_{ij}$ can be considered as binary fuzzy relations, which are given on the Cartesian square M^2. The relation $C = (C_{ij})$ is a similarity relation (i.e. reflexive and symmetric fuzzy relation) [18]. It is easy to verify that the relation $C = (C_{ij})$ is not a max-min transitive relation [18]. But we can construct the relation $\hat{C} = (\hat{C}_{ij})$ with the help of a transitive closure operator $\hat{C} = \bigcup_{n=1}^{\infty} C^n$. This relation will be a max-min transitive relation and, consequently, will be a fuzzy equivalence relation. Then the relation $\hat{K} = 1 - \hat{C}$ will be dissimilitude relation. The dissimilitude relation $\hat{K}$ defines the ultrametric on M^2 (i.e. $\hat{K}$ satisfies the axioms: 1) $\hat{K}(F,G) = 0 \Leftrightarrow F = G$; 2) $\hat{K}(F,G) = \hat{K}(G,F)$; 3) $\hat{K}(F,G) \le \max\{\hat{K}(F,J),\hat{K}(J,G)\}$ for all $F,G,J \in M$).

Thus, the matrix $(\hat{K}_{ij})$ can considered as a matrix of distances between the analysts. The corresponding matrix of coherence $\hat{C} = (\hat{C}_{ij})$ can considered as a similarity matrix between the investment banks.

The structure of coherence of investment bank recommendations can be analyzed with the help of the α-cut $\hat{C}_\alpha = \{(F,G) : \hat{C}(F,G) \ge \alpha\}$, $\alpha \in (0,1]$ of the fuzzy similarity relation $\hat{C}$. For every fixed $\alpha \in (0,1]$ the set $\hat{C}_\alpha$ defines the equivalence relation, which induces a partition of evidence M on the equivalence classes.

The equivalence classes of coherence indicated in Table 2 (only not singletons) for some values of $\alpha \in (0,1]$ for the data of value of shares of Russian companies in period 2012-2014. Each of these classes represents set of analysts, whose recommendations have a large degree of coherence. This degree is defined by threshold α.

Table 2. The equivalence classes of coherence of investment bank recommendations

α	equivalence classes
0.95	(3,17), (7,21)
0.9	(3,5,17), (1,9,16,22), (8,15), (7,21)
0,85	(1,…,11,13,15,…23)

5.5 A Cluster Analysis of the Coherence of Analysts' Recommendations

We consider the matrix $\hat{K} = 1 - \hat{C}$, where $\hat{C}$ is a transitive closure of similarity relation $C = 1 - K$, $K = (K_{ij})$ and K_{ij} is a value of conflict measure between the i-th and j-th investment banks, which calculated by formula (4). A conflict measure considered on the set of evidence $M = \{F_1, ..., F_l\}$.

The cluster analysis of coherence of analysts' recommendations will be performed using one of the methods of hierarchical clustering. For example, we will use the Unweighted Pair-Group Method Using Arithmetic Averages (UPGMA) [16], which is the most simple and popular from the agglomerative methods of clustering. In this method a union of closest clusters is performed on each iteration step beginning with the singletons (clusters with the unit cardinality). The binary tree of decision (or dendrogram) is constructed as a result of the algorithm. The ultrametricity of data guarantees the uniqueness of construction of such tree [17]. The dendrogram of coherence of investment bank recommendations for the data of value of shares of Russian companies in period 2012-2014 is shown in the Fig. 4[1]. The dendrogram presents the full picture of the cluster structures. In particular, we can indicate the following basic cluster structures of investment banks with respect to the coherence of recommendations (these clusters highlighted in various shades of gray in the Fig. 2): (((7,21), 11), ((3,17), 5)), (((1,9), (16,22)), (13,23)), ((8,15), 10). We can see that the result of hierarchical clustering agrees well with the partition of similarity relation $\hat{C}$ on the equivalence classes.

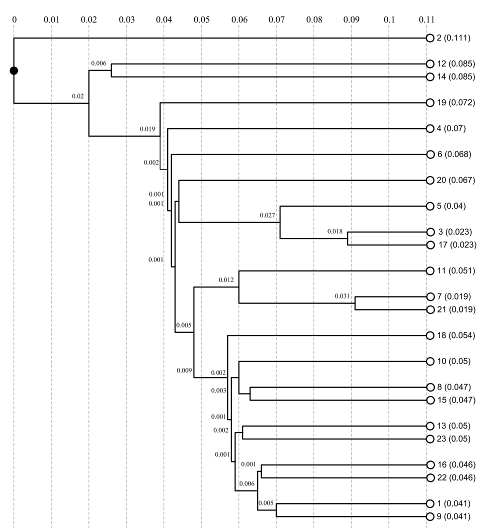

Fig. 4. The dendrogram of cluster structure of coherence of investment bank recommendations

[1] The dendrogram was obtained with the help of the utility http://genomes.urv.cat/UPGMA/

6 Conclusion

In this paper, the coherence of investment bank recommendations was studied for the data of value of shares of Russian companies in period 2012-2014. The specific of the study consists in using the conflict measure defined in the framework of the belief function theory for determination of the coherence of recommendations. The analysis of coherence was reduced to analysis of values of the conflict measure. This analysis was performed with the help of game-theoretic methods (Shapley index, interaction index), network analysis methods (centralities), fuzzy relation methods, hierarchical clustering methods.

The following results were obtained:

- the ranking of investment banks with respect to their contribution to the overall coherence of the recommendations using the Shapley value was obtained;
- the contributions of the separate pairs of investment banks in the total conflict of recommendations of the Russian companies with the help of the interaction index were evaluated;
- the investment banks rendering the greatest influence on the coherence of the recommendations were detected with the help of the analysis of the centrality;
- the sets of analysts whose recommendations have a greater degree of coherence were identified with the help of analysis of fuzzy similarity relations generated by the coherence measure;
- the main cluster structures of investment banks with respect to coherence of the recommendations were identified by the method of hierarchical clustering;
- the expressions for some of the calculated parameters (Shapley values, interaction index) were obtained in the terms of evidence.

In addition, we have shown that the set of the key investment banks, have made the greatest contribution to the overall coherence of the recommendations obtained with the help of Shapley values and the methods of analysis of the centrality, are close together. Similarly, the cluster structures of analysts, whose recommendations have a greater degree of coherence, obtained by the methods of analysis of the fuzzy similarity relations and methods of the hierarchical clustering, are close to each other. Indirectly, this confirms the importance of the results.

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Recovering Noisy Contexts with Probabilistic Formal Concepts*

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Abstract. The uncertainty in the environment typically generates noisy concept alternatives and leads to an overpopulated concept lattice. From a computational point of view, a straightforward filtering of the noisy concept lattice will suffer from an exponential-size computational overkill, and from a semantical one – will face numerous ambiguities due to an overfitting. We managed to bypass the filtering problem by applying a sort of probabilistic approach. We developed a probabilistic generalization of formal concepts which seems to avoid a monstrous combinatorial complexity of a complete context lattice construction. The theoretical base for this method is described, as well as a ready-to-work noise resistant algorithm. The algorithm has been tested and showed a moderate precision and recall rate on various datasets, including a toy one presented with the presence of a 2, 3 or 5% random noise.

Keywords: formal concept analysis, concept lattice, inductive learning, data mining, association rules, classification task

1 Introduction

Formal concepts may be successfully used as classification units [1, 2]. However, reviewing the concept lattice as a plain graph with the Formal Concept Analysis (FCA) works well until data become uncertain, when lattices can become prohibitively huge even on small-sized datasets.

There are some attempts to get rid of noise in data by concepts selection or filtering. E.g. measures of the concept stability has been shown to pick out the most reliable formal concepts [4, 5]. It was demonstrated that the stability index is relevant to data mining tasks and possesses several attractive properties [9].

Nevertheless, this is still not enough for uncertain environments [4]. Noisy clones overloading makes the calculation intractable even on small datasets. Formally speaking, a zero-populated concept context superimposed with a random Bernoulli noise is expected to produce exponential-size lattices [8].

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Approaches based on the hypothesis-making has been analyzed: performance of a model still suffers in the practical tasks environment [7]. The closest research domains are probably connected with the fuzzy concepts analysis, like [18].

In the paper we reconsider the problem of handling a possible noise in data by means of probability and logic. The origin of the probabilistic pattern for formal concepts lies in cognitive science, where they are closely related to the "natural classes" [16]. We will focus on developing a context recovery method keeping eye on the next key capabilities:

1. The stability of a reproduced context concept lattice with respect to a possible minor noise;
2. Computational tractability, avoiding filtering the whole concept lattice;
3. Handling the prediction ambiguity problem;
4. Relationship with the theory of category formation [17, 16].

The first step has been made in [12] – a probabilistic generalization of formal concepts goes here. The next step is to equip concepts with possible attribute negations and develop a logical language of a context probabilistic reasoning. We will also prove some technical facts about a consistency of probabilistic reasoning.

2 Formal concept analysis foundations

This section suggests a brief overview for a formal concept analysis framework [1, 2, 13] exploited in the paper.

A dataset is represented by an attribute-value cross-table. Formally speaking,

Definition 1. *A formal context is a triple (G, M, I) where G and M are the sets of an arbitrary nature and $I \subseteq G \times M$ is a binary relation.*

On the formal context (or simply context) a derivation operator $'$ is defined:

Definition 2. *$A \subseteq G$, $B \subseteq M$. Then*

1. $A' = \{m \in M \mid \forall g \in A, (g, m) \in I\}$
2. $B' = \{g \in G \mid \forall m \in B, (g, m) \in I\}$
3. *The pair (A, B) is called a formal concept if $A' = B$ and $B' = A$.*

Generally speaking, formal concepts analysis concentrates on a concept lattice arising on concept extents from the natural subset order. However, we aim to avoid considering a concept lattice and exploit the intrinsic properties of the data. The implication is a core notion.

Definition 3. *The implication is a pair (B, C), $B, C \subseteq M$, which we write as $B \to C$. The implication $B \to C$ is true on $K = (G, M, I)$, if $\forall g \in G(B \nsubseteq g'$ or $C \subseteq g')$. We denote the set of all true implications as $Imp(K)$.*

Implications are not only the forms a conceptual bridge from FCA to logic structures, but are an essential way of reasoning within a machine logic and prediction task particularly.

Definition 4. *For any set of implications L we construct an operator of a direct inference f_L that adds all conclusions of applicable implications:*

$$f_L(X) = X \cup \{C \mid B \subseteq X,\ B \to C \in L\}$$

The following theorem characterizes concepts by means of fixed points.

Theorem 1 (see [2]). *For any set $B \subseteq M$, $f_{Imp(K)}(B) = B \Leftrightarrow B'' = B$.*

The theorem application may be illustrated on a simple formal context.

m_1	m_2	m_3
1	1	0
1	1	1
0	1	1
0	0	0

Table 1: A very simple formal context K_0

We can reformulate concept lattice construction task by means of implications and an inference operator. It can be easily found out from Table 1 that attributes m_1 and m_3 determine the class of an object. In fact, m_1 implies m_2 and so does m_3. This is written as $m_1 \to m_2$ and $m_3 \to m_2$. The sets $\{m_1, m_2\}$, $\{m_2, m_3\}$ and $\{m_1, m_2, m_3\}$ are the formal concepts, so do fixed points of the direct inference operator. For example,

$$\{m_1\} \xrightarrow{f_{Imp(K)}} \{m_1, m_2\} \xrightarrow{f_{Imp(K)}} \{m_1, m_2\}$$

Thus indeed, $\{m_1, m_2\}$ is a fixed point and a formal concept simultaneously.

3 Probabilistic logic on a formal context

Let us add some noise on K_0. We also extend K_0, by adding redundant objects duplicates. It will help to keep the noise level rather low in order to make a concept recovery practically possible.

Every single altering will change a concept lattice a lot. The first context is equivalent to K_0 above and has the same concept lattice. However, the second one generates a lot of side concepts, provoked by noise: the sets $\{m_2\}$ and $\{m_1, m_3\}$ also become formal concepts. The amount of side concepts is increasing as more noise is incoming – the dependency tends to be asymptotically exponential [8].

The stability may be obtained in various ways. The most obvious way is computing some stability index in order to evaluate, does the concept from noisy context is good enough, either does it is produced by noise [4, 5].

The other one may be based on a different subject: instead of measuring a stability of concepts, a stability of implications is measured. An essential way to do this is to exploit a likelihood of attribute implications, but we will also generalize them up to logical formulas.

m_1	m_2	m_3
1	1	0
1	1	1
0	1	1
1	1	0
1	1	1
0	1	1
1	1	0
1	1	1
0	1	1
0	0	0

Table 2: K_0 populated with duplicates

m_1	m_2	m_3
0	1	0
1	1	1
0	1	1
1	1	0
1	0	1
0	1	1
1	1	0
1	1	1
0	1	1
0	0	0

Table 3: K_{noise}, with a little bit noise

Definition 5. *For a formal context $K = (G, M, I)$ we introduce classical logical definitions:*

- L_K *is a letters set and includes any $m \in M$ as well as their negations $\neg m$;*
- Φ_K *is a formulas set and is defined inductively: a letter is a formula and for any $\phi, \psi \in \Phi_K$ products of $\phi \wedge \psi, \phi \vee \psi, \phi \to \psi, \neg\phi$ are formulas, too;*

Remark 1. For brevity, we assume $\bigwedge L = \underset{P \in L}{\wedge} P$ (or $\bigwedge L = 1$ if $L = \varnothing$). Similarly, $\neg L = \{\neg P \mid P \in L\}$.

For every object $\{g\}$, a logic model of the object K_g is defined. We say that the object g respects the formula $\phi \in \Phi_K$, if the formula is true for the model K_g. We will write this fact as $g \vDash \phi \Leftrightarrow K_g \vDash \phi$. The set $G_\phi = \{g \in G \mid g \vDash \phi\}$ is called the support of ϕ.

Definition 6. *Let us consider an arbitrary probability measure μ, i.e. μ is a finite countably additive measure on the set G. Then the contextual probability measure is defined by the following:*

$$\nu : \Phi_K \to [0, 1],\ \ \nu(\phi) = \mu(\{g \mid g \vDash \phi\}).$$

.

The most common understanding of formula probability may be linked with a well-known confidence index for context implications: $\mathrm{conf}(X \to Y) = \frac{|\mathrm{supp}(X \cup Y)|}{|\mathrm{supp}(X)|}$. The formula probability will express exactly the same, if we keep things simple and assume μ to be a counting measure: $\mu(\{g\}) = \frac{1}{|G|}$.

For practical applications, here and further we will suppose that G is finite and does not contain any objects of a zero measure, i.e. $\forall g \in G,\ \mu(\{g\}) \neq 0$.

Definition 7. *The set of attributes M is compatible, if $M' \neq \varnothing$.*

The same may be expressed as $\nu(\bigwedge M) > 0$.

Now let us consider the set $L = \{m_i, m\}_{i=1...k} \subseteq \mathrm{L}_K$. The formula $m_1 \wedge m_2 ... \wedge m_k \to m$ will look like the classical context implication $(\{m_i\}, \{m\})$, except when it is possible to include negations of attributes, like in this one: $m_1 \wedge \neg m_2 ... \wedge m_k \to \neg m$. The concept of the implication as a formula is reified in definition of the rule:

Definition 8. *Let $C, H_i \in \mathrm{L}_K$, $C \notin \{H_1, H_2, ...H_k\}, k \geq 0$. Then:*

1. *The **rule** $R = (H_1, H_2 ..., H_k \to C)$ is an implication $(H_1 \wedge H_2 ... \wedge H_k \to C)$;*
2. *The **premise** $R^{\leftarrow}$ of a rule R is a set of letters $\{H_1, H_2 ..., H_k\}$;*
3. *The **conclusion** is $R^{\rightarrow} = C$;*
4. *If $R_1^{\leftarrow} = R_2^{\leftarrow}$ and $R_1^{\rightarrow} = R_2^{\rightarrow}$, then $R_1 = R_2$.*

Definition 9. *The probability of the rule R is a conditional probability*

$$\eta(R) = \nu(R^{\rightarrow} \mid R^{\leftarrow}) = \frac{\nu(R^{\leftarrow} \wedge R^{\rightarrow})}{\nu(R^{\leftarrow})}$$

If $\nu(R^{\leftarrow})$ is zero, the probability of the rule remains undefined.

Keeping eye on K_0, let us try to watch what is happening on K_{noise} with the implications $m_1 \to m_3$ and $m_1 \to m_3$. They stopped to be contextual tautologies, but we still can stick to the corresponding rules with reasonable likelihoods: $\eta(m_1 \to m_2) = \frac{4}{5}$ and $\eta(m_3 \to m_2) = \frac{5}{6}$.

The core idea of the approach is to exploit Theorem 1. An operator of a direct inference could be easily adapted to employing probabilistic rules in contrast to formal context implications.

Definition 10. *The prediction operator Π on the set of the rules $\mathcal{R}$ works as follows:*

$$\Pi_{\mathcal{R}}(L) = L \cup \{C \mid \exists R \in \mathcal{R} \colon R^{\leftarrow} \subseteq L, R^{\rightarrow} = C\}.$$

Definition 11. *A closure $\overline{L}$ of the set of the letters L is the smallest fixed point of the prediction operator: $\overline{L} = \Pi^{\infty}(L)$.*

4 Rule classes

Note, that the definition 10 accepts any set of rules. To produce a relevant and consistent set of generalized concepts, additional restrictions for this set are needed. Following the [6], we will prove the compatibility theorem and ensure the correctness property for the prediction closure operator.

Definition 12 (subrule). $R_1 \sqsubset R_2$*, if* $R_1^{\leftarrow} \subset R_2^{\leftarrow}$ *and* $R_1^{\rightarrow} = R_2^{\rightarrow}$*.*

Definition 13 (refinement). $R_1 > R_2$*, if* $R_2 \sqsubset R_1$ *and* $\eta(R_1) > \eta(R_2)$*.*

For example, the rule $m_1 \to m_2$ from K_{noise} has the only unconditional subrule: $(\varnothing \to m_2) \sqsubset (m_1 \to m_2)$. This could not be considered as a refinement relation: $\eta(\varnothing \to m_2) = \frac{8}{10} = \frac{4}{5} = \eta(m_1 \to m_2)$. However, $(m_3 \to m_2) > (\varnothing \to m_2)$ because $\eta(\varnothing \to m_2) = \frac{8}{10} < \frac{5}{6} = \eta(m_3 \to m_2)$.

The class M_1 requires that the rules have a greater conditional probability than an unconditional probability of C, i.e. the rule is guaranteed to be useful in reasoning:

Definition 14. $R \in M_1(C) \Leftrightarrow \eta(R) > \nu(R^{\rightarrow}), R^{\rightarrow} = C$.

The class M_2 requires a rule to be specific – we cannot improve probability by refining the rule:

Definition 15. $R \in M_2(C) \Leftrightarrow R \in M_1(C)$ *and* $[R \sqsubset \tilde{R} \Rightarrow \eta(\tilde{R}) \leq \eta(R)]$

The rule $(m_3 \to m_2)$ could be refined up to the $(\neg m_1 \wedge m_3 \to m_2)$ due to the inequality: $\eta(m_3 \to m_2) = \frac{5}{6} < 1 = \frac{3}{3} = \eta(\neg m_1 \wedge m_3 \to m_2)$. The last rule satisfies all M_2 conditions, and thus $(\neg m_1 \wedge m_3 \to m_2) \in M_2(m_2)$.

The class Imp contains all exact implications. So does any contextual tautology:

Definition 16. $R \in Imp(C) \Leftrightarrow R^{\rightarrow}(R) = C$ *and* $\eta(R) = 1$

We also consider compound classes for entire set of letters:

Definition 17. $M_1 = \bigcup_{C \in \mathrm{L}_K} M_1(C)$

Remark 2. M_2 and Imp are defined similarly.

All exact implications are indeed necessary to ensure a completeness property for the prediction operator. In turn, a set of rules must consist only from the M_2 rules in order to obtain a consistency property. The set of the letters L is called consistent, if it does not contain an atom C and its negation $\neg C$.

Definition 18. *If $Imp \subset \mathcal{R}$, then the set of rules $\mathcal{R}$ is called complete.*

Definition 19. *By a system of the rules, we will call any $\mathcal{R} \subseteq M_2$.*

5 Prediction consistency

Definition 20. *The set of attributes M is consistent, if $L \in M \Rightarrow \neg L \notin M$.*

$\Pi_{\mathcal{R}}$ must avoid inconsistent inferences [3]. The following theorem is the main theoretical result of the paper. It proves predictions to be consistent and compatible (see def. 7). For the proof and technical details, see [6].

Theorem 2 (Compatibility). *If L is compatible, then $\Pi_{\mathcal{R}}(L)$ is also compatible and consistent for any system of the rules $\mathcal{R}$.*

Somewhat more difficult, but still solvable, is the question of the inconsistency of prediction closures. Let us assume $\mathcal{R}$ to be a complete set of rules and $\Pi_{\mathcal{R}}$ to be the corresponding prediction operator. It is important to note that the rule systems containing M_2 are always complete.

Theorem 3. *If L is incompatible, then $\Pi_{\mathcal{R}}(L)$ is inconsistent and incompatible.*

6 Probabilistic formal concepts

The fixed points of a prediction operator are clear to be the candidates for concept intents. What about concept extents? The principles proposed in [5, 9] give us a cue. The idea is to take all possible closure preimages attribute sets, i.e. all $M : \Pi(M) = B$, and compose their derivative sets together into a derived concept extent A. This will allow restoring the actual concept reference by applying the prediction operator and include all the objects of the same class into a conjoined extent.

For example, let $K_{squares}$ be a context depicted as two disjoint squares (which are two independent formal concepts). To bring extra complexity, we also alter some entries:

G	m_1	m_2	m_3	m_4	m_5	m_6	m_7	m_8
g_1	1	1	1	1	1	0	0	0
g_2	1	1	1	1	0	0	0	0
g_3	1	1	1	1	0	0	0	0
g_4	1	1	1	1	0	0	0	0
g_5	0	0	0	0	1	1	1	1
g_6	0	0	0	0	1	1	0	1
g_7	0	0	0	0	1	1	1	1
g_8	0	0	0	0	1	1	1	1

Table 4: A two-concept context $K_{squares}$ with a minor noise

Note that the most specific rules referring to M_2 are $(m_{i=1...4} \to \neg m_{j=5...8})$, however rule $m_5 \to \neg m_{i=1...4}$ is not. There is a more specific rule for the last one: $\eta(m_5 \wedge m_6 \to \neg m_1) = 1 > \frac{4}{5} = \eta(m_5 \wedge \to \neg m_1)$. This is how noise is handled being encapsulated in probability and refinement.

To pick up the first object from $K_{squares}$, firstly, the prediction operator computes a closure: $\Pi(g_1') = \{m_1, m_2, m_3, m_4, \neg m_5, \neg m_6, \neg m_7, \neg m_8\}$. And secondly, all objects with the same closure are composed into a concept with the extent $\{g_1, g_2, g_3, g_4\}$.

Definition 21. *By a probabilistic formal concept on $K = (G, M, I)$ we mean any pair (A, B) which satisfies*

$$\Pi(B) = B, \; A = \bigcup_{C \subset B, \; \Pi(C) = B} G_C$$

Our selection is also justified by the following statement, relating probabilistic and ordinary formal concepts on the same context.

Theorem 4 (Ordinary concepts inclusion [12]). *Let K be a formal context.*

1. *If (A, B) is an ordinary concept on K, then there is a probabilistic concept (N, M) such that $A \subseteq N$, and $B \subseteq M$.*

2. *If (N, M) is a probabilistic concept on K, then there is a set of ordinary concepts $\mathcal{C}$, such that*

$$\forall (A, B) \in \mathcal{C} \ (\Pi(B) = M),$$
$$N = \bigcup_{(A,B)\in\mathcal{C}} A.$$

7 Probabilistic concepts discovery

For practical applications, a computational problem should be solved. It is still exponentially hard if we require a full M_2 set enumeration.

A semantic probabilistic inference as an enumeration procedure has been described in details in [15]. The idea is to perform a kind of a greedy search combined with a branches and boundaries search on the inference tree. The last aims to obtain an M_2 subset, which will be enough for the most practical tasks.

Definition 22. *R is a probabilistic law, if for any $\tilde{R}$, $(\tilde{R} \sqsubset R) \Rightarrow (\tilde{R} < R)$.*

Definition 23. *The rule $\tilde{R}$ is semantically probabilistic inferred from the rule R. We write $R \triangleright \tilde{R}$, if R, $\tilde{R}$ are the probabilistic laws, and $\tilde{R} > R$.*

Definition 24. *The probabilistic law R is the strongest, or $R \in \mathrm{SPL}$, if there is no other probabilistic law $\tilde{R}$ such that $(\tilde{R} > R)$.*

Proposition 1. *All strongest probabilistic laws are in M_2.*

The rules extraction routine is based on exploiting a Semantic Probabilistic Inference (SPI) approach. It requires each path in the inference graph to be a sequence of semantic inferences:

Definition 25. *SPI is a sequence of the rules $R_0 \triangleright R_1 \triangleright R_2 ... \triangleright R_m$, such that $R_0^{\leftarrow} = \varnothing$ and R_m is the strongest probabilistic law.*

Now let us assume that some system of the rules $\mathcal{R}$ on a context K has already been discovered by semantic probabilistic inference. The probabilistic concept definition implies the following **closure-search procedure**.

1. Set the step counter $k = 1$ and generate the set $C^{(0)} = \{\Pi_{\mathcal{R}}(R^{\leftarrow}) \mid R \in \mathcal{R}\}$. In fact, this may be an arbitrary family of letter sets to be extended up to their probabilistic concepts closures. The set $C^{(0)}$ is almost always redundant, but it should be enough to cover all statistically significant attribute sets;
2. On the step $k > 1$ in case $C^{(k)} = \varnothing$ the algorithm finishes the execution and outputs a list of detected probability concepts;
3. On the step $k > 1$ the set $A = \{g \in G \mid \Pi_{\mathcal{R}}(g' \cap B) = B\}$ is computed for each $B \in C^{(k)}$. If $A \neq \varnothing$, the pair (A, B) is added to the list of the found concepts. It corresponds to a join operation on the concept lattice and subsequently climbs to superordinate levels of the lattice;

4. The set $C^{(k+1)} = \{\Pi_{\mathcal{R}}(B \cup C) \mid B, C \in C^{(k)}\} \setminus C^{(k)}$ is generated. In fact, actual prediction closures are computed on this step;
5. Let $k := k + 1$ and go to the step 2.

The algorithm could be applied to a context recovery task as well as to a wide variety of data mining problems, such as classification and clusterisation tasks. In the final section we will focus on handling noise in a toy, a rather hard context recovery task.

8 An example

m0	m1	m2	m3	m4	m5	m6	m7
0	0	0	0	0	1	1	0
0	0	0	0	0	1	1	0
0	0	0	0	0	1	1	0
0	0	0	0	0	1	1	0
0	0	0	0	0	1	1	0
1	1	1	1	0	1	1	0
1	1	1	1	0	1	1	0
1	1	1	1	0	1	1	0
1	1	1	1	0	1	1	0
1	1	1	1	0	1	1	0
1	1	1	1	0	0	0	0
1	1	1	1	0	0	0	0
1	1	1	1	0	0	0	0
1	1	1	1	0	0	0	0
1	1	1	1	0	0	0	0
1	1	1	1	0	0	1	1
1	1	1	1	0	0	1	1
1	1	1	1	0	0	1	1
1	1	1	1	0	0	1	1
1	1	1	1	0	0	1	1
0	0	0	0	0	0	1	1
0	0	0	0	0	0	1	1
0	0	0	0	0	0	1	1
0	0	0	0	0	0	1	1
0	0	0	0	0	0	1	1
0	0	0	1	1	0	1	1
0	0	0	1	1	0	1	1
0	0	0	1	1	0	1	1
0	0	0	1	1	0	1	1
0	0	0	1	1	0	1	1
0	0	0	1	1	0	1	1

m0	m1	m2	m3	m4	m5	m6	m7
0	0	0	1	1	0	0	0
0	0	0	1	1	0	0	0
0	0	0	1	1	0	0	0
0	0	0	1	1	0	0	0
0	0	0	1	1	0	0	0
0	0	1	1	0	0	0	0
0	0	1	1	0	0	0	0
0	0	1	1	0	0	0	0
0	0	1	1	0	0	0	0
0	0	1	1	0	0	0	0
0	0	1	1	0	0	0	0
0	0	1	1	0	0	0	0
0	0	1	1	0	0	0	0
0	0	1	1	0	1	1	0
0	0	1	1	0	1	1	0
0	0	1	1	0	1	1	0
0	0	1	1	0	1	1	0
0	0	1	1	0	1	1	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	0
0	0	0	0	0	1	1	0
0	0	0	0	0	1	1	0
0	0	0	0	0	1	1	0
0	0	0	0	0	1	1	0

Fig. 1: Initial context

Earlier we considered the $K_{squares}$ context very simple but illustrative. A more sophisticated example should contain more interactions between concepts, both in extent and intent components. Also more noise should be produced.

To measure some performance issues, we will set up several modifications of a single context. Modifications differ at levels of a noise and there may be a number of data duplicates, when producing more data is necessary. An initial context has been composed from rectangle blocks, easy to be recognized as formal concepts (let them be denoted as "solid" concepts).

A set of experiments was based on:

1. K_{exp} – the initial context, depicted on Fig. 1.
2. K_{x3} – similar to K_{exp}, except it contains 3 duplicates of each K_{exp} object;
3. $K_{x3.n05} = K_{x3}$ + randomly inflicted binary noise, Bernoulli distributed with $p = 0.05$
4. $K_{x3.n04} = K_{x3}$ + noise, $p = 0.04$
5. $K_{x3.n03} = K_{x3}$ + noise, $p = 0.03$

The primary characteristics of the datasets are presented in Table 5.

Context	$\|G\|$	$\|M\|$	# Concepts	# Solid	# Logical	Noise
K_{exp}	61	8	5 + 4 + 2	5	6 + 4	0
K_{x3}	183	8	5 + 4 + 2	5	6 + 4	0
$K_{x3.n05}$	183	8	5 + 4 + 2	5	6 + 4	0.05
$K_{x3.n04}$	183	8	5 + 4 + 2	5	6 + 4	0.04
$K_{x3.n03}$	183	8	5 + 4 + 2	5	6 + 4	0.03

Table 5: Experimental data summary: $|G|$ and $|M|$ stay for the amount of objects and attributes in a context. The number of concepts is a sum of three different types of concepts: solid concepts, which are indicated on Fig. 1 as solid sequences of ones; join-concepts, which are made of two sequences; and meet-concepts, which are produced by an intersection of two solid concepts. Note that a logical approach is a little bit specific: the model excludes meet-concepts from the consideration and accepts attributes negations, while respecting the empty concept. Thus, the column logical sums solid and join-concepts add an empty one to solid.

The first stage in executing a **closure-search procedure** is a rules extraction routine. According to the method discussed in Section 7, a computer program was implemented to perform a semantical probabilistic inference. For each context a set of rules has been obtained and has eventually been used in a **closure-search procedure**.

Context	# Rules	# Rules ($p > 0.5$)	# Rules ($p > 0.9$)
$K_{x3.n05}$	1712	1358	682
$K_{x3.n04}$	1193	967	497
$K_{x3.n03}$	1103	916	532

Table 6: Rules extraction routine summary: # Rules are a total number of discovered rules via semantic probabilistic inference. The next two values are the numbers of the rules with conditional probability thresholds of 0.5 and 0.9.

While increasing a noise level, a context becomes less and less clear and requires more and more rules for describing attribute associations. A minor noise produces an insignificant effect and affords to solve the problem almost exactly.

Following [10], we will compare an original concept lattice O with a predicted one E and measure the method performance by calculating two ratios:

$$Precision = \frac{|O \cap E|}{|E|},\ Recall = \frac{|O \cap E|}{|O|}$$

The experiment results are presented in Table 7. In addition to the performance indexes, the data are presented separately for the solid concepts and the join-concepts.

Context	$\|O\|$	$\|O \cap E\|$	$\|O \setminus E\|$	$\|E \setminus O\|$	Precision	Recall
K_{x3}	6 + 4	6 + 4	0 + 0	0 + 0	1.0 + 1.0	1.0 + 1.0
$K_{x3.n03}$	6 + 4	6 + 4	0 + 0	0 + 0	1.0 + 1.0	1.0 + 1.0
$K_{x3.n04}$	6 + 4	6 + 1	0 + 3	0 + 0	1.0 + 1.0	1.0 + 0.25
$K_{x3.n05}$	6 + 4	6 + 3	0 + 1	0 + 1	1.0 + 0.75	1.0 + 0.75

Table 7: Concepts recovery summary on the same contexts: $|O|$ is the total number of expected concepts; $|O \cap E|$ is the number of correctly predicted concepts; $|O \setminus E|$ is the number of lost concepts; and $|E \setminus O|$ is the number of incorrectly predicted concepts.

It was rather easy for a **closure-search procedure** to determine all formal concepts without noise.

However, even on noisy contexts the algorithm has been able to restore the original set of concepts with a moderate accuracy. All probabilistic concepts encountered by the algorithm may be essentially associated with the original images in ordinary concepts, while some non-primal concepts have been leaked. Nevertheless, it seems that probabilistic formal concepts perform more accurately, in comparison with a stability approach [4].

Indeed, the main advantage may not even be the method accuracy: probabilistic formal concepts are able to discover concepts on big data frames. The estimated computational complexity for SPI is $|M|^{d+2} * |G|$, and one for a closure-search seems to be $|\Pi| * |M|^c * |G|$, where $3 < c < 4$ (the estimation is empirical and still needs to be checked). Noise induces extra complexity but using a Pentium 4 2-core 2.4GHz computer is enough to solve a 321x26 context with 10% noise in about 10 minutes, while it takes 5 minutes to complete a 3% noise task.

9 Conclusion

The introduced method has been experimentally and theoretically proven to be correct and accurate. Some extra experiments have been proposed in earlier works [12, 20]. Probabilistic formal concepts are also very profitable as they may serve to construct exact concept lattices from real, noisy raw data immediately instead of performing a filtering on a overpopulated concept lattices, possibly exponentially sized. The further work includes theoretical evaluation for computational complexity as well as more sophisticated experiments on a big dataset. We are also planning to compare our results with some famous classification methods in terms of prediction accuracy and speed.

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Logics for Representation of Propositions with Fuzzy Modalities

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Abstract. In the paper we introduce logical calculi for representation of propositions with modal operators indexed by fuzzy values. There calculi are called Heyting-valued modal logics. We introduce the concept of a Heyting-valued Kripke model and consider a semantics of Heyting-valued modal logics at the class of Heyting-valued Kripke models.

1 Introduction

The formalism of propositional modal logic and its proof technique is one of the most powerful approaches for knowledge representation and reasoning about dynamic systems, databases, etc.

In the present paper we introduce more general modal propositional formalism, which allows to express propositions with modalities indexed by elements of a complete Heyting algebra. In this formalism any proposition A can be augmented with a modal operator of the form $\Box_a$, which can be interpreted, for example, as a value of necessity of A (or a value of confidence of A, or a value of plausibility of A, or a probability of A, or something else). This formalism can be considered as a logical foundation for

- reasoning about objects that are incomplete and inconsistent, such as databases with incomplete and unclear information,
- model checking for discrete models which are rough approximations of analyzed systems.

Mathematical approaches to representation of knowledges with taking into account an uncertainty and incompleteness of knowledges were considered in several papers, in particular, in [3]–[13]. The most of them are related to quantitative evaluation of uncertainty.

Uncertainty of information can appear by several causes.

1. An information under processing can be unclear, approximate, and not verified, and for correct processing of such information it is necessary to have a formalism for taking into account a value of reliability of information under processing.
2. If we investigate a complex system, such that its detail and exact representation is impossible, then we construct a rough model of this system, which has small complexity, and instead of this system we investigate its rough model.

> But because the original system and its model are essentially not identical, then their properties can differ. Thus, for correct investigation of the system on the base of such model it is necessary to have an approach to evaluation the difference between properties of a rough model and properties of original system. Values of the difference can be not only quantitative, but also qualitative. For example, the set of such values can be a boolean algebra of subsets of some set of situations (i.e. states of an environment), in which the analyzed system does work. A value of equivalence between the system and its model (with respect to the properties under checking) can be defined for example as a set of situations in which these properties are equivalent for the original system and for its model. A value of truth of the properties under checking can be defined as a subset of this set, which consists of situations, in which the analysed properties does hold. These situations can be augmented by quantitative parameters (their weights, probabilities, etc.), and the set of such values can be more complex (if the sets of the parameters are totally ordered sets, then the set of values of truth is a Heyting algebra).

The main goal of the present paper is to construct a logical framework, which can serve as a logical foundation for representation of such uncertain information. The proposed formalism can be used also for design of specification languages of a behavior of dynamic systems with uncertain information about their structure and behavior, by analogy with the specification languages based on temporal logic for description of properties of program systems and electronic circuits ([2]). Some recent approaches to logic representation of propositions with fuzziness can be found in [16], [17].

The paper is organized as follows. In section 2 we introduce the syntax of Heyting-valued modal logics and define a minimal Heyting-valued modal logic HVK. In section 3 we introduce the concept of a Heyting-valued Kripke model and define the semantics of Heyting-valued modal formulas at the class of Heyting-valued Kripke models. We also consider an example of a Heyting-valued Kripke model related to description logics. In section 4 we introduce a concept of a canonical model of a Heyting-valued modal logic, and in section 5 we use the concept of a canonical model for the proof of completeness for minimal Heyting-valued modal logic HVK at the class of Heyting-valued Kripke models. In the conclusion we summarize the results of the paper and describe problems for future research.

2 Heyting-valued modal logics

2.1 Complete Heyting algebras

We shall assume that a set of fuzzy values which can occur in formulas of Heyting-valued modal logics has some algebraic properties, namely, it is a complete Heyting algebra. In this section we remind a definition of this concept.

A **complete lattice** is a partially ordered set $\mathcal{H}$, such that for every subset $Q \subseteq \mathcal{H}$ there are elements $\inf(Q)$ and $\sup(Q)$ of $\mathcal{H}$ such that for every $b \in \mathcal{H}$

$$\begin{array}{lcl} (\forall q \in Q \quad b \leq q) & \Leftrightarrow & b \leq \inf(Q), \\ (\forall q \in Q \quad q \leq b) & \Leftrightarrow & \sup(Q) \leq b. \end{array}$$

The elements $\inf(\mathcal{H})$ and $\sup(\mathcal{H})$ will be denoted by the symbols 0 and 1 respectively.

For every finite subset

$$Q = \{a_1, \ldots, a_n\} \subseteq \mathcal{H}$$

the elements $\inf(Q)$ and $\sup(Q)$ will be denoted by the symbols

$$a_1 \wedge \ldots \wedge a_n \quad \text{and} \quad a_1 \vee \ldots \vee a_n$$

respectively.

These elements will be denoted also by the symbols

$$\left\{ \begin{array}{c} a_1 \\ \ldots \\ a_n \end{array} \right\} \quad \text{and} \quad \left[\begin{array}{c} a_1 \\ \ldots \\ a_n \end{array} \right]$$

respectively.

A **complete Heyting algebra** can be defined as a complete lattice $\mathcal{H}$, with a binary operation

$$\to: \ \mathcal{H} \times \mathcal{H} \to \mathcal{H},$$

such that for every $a, b, c \in \mathcal{H}$

$$a \wedge b \leq c \quad \Leftrightarrow \quad a \leq b \to c \tag{1}$$

Below the symbol $\mathcal{H}$ denotes some fixed complete Heyting algebra.

For every $a, b \in \mathcal{H}$ the symbol $a \leftrightarrow b$ denotes the element $\left\{ \begin{array}{c} a \to b \\ b \to a \end{array} \right\}$.

One of the most important examples of a complete Heyting algebra is a set of n–tuples

$$\{(a_1, \ldots, a_n) \mid a_1 \in M_1, \ldots, a_n \in M_n\}$$

where $M_1, \ldots, M_n$ are complete totally ordered sets (for example, every M_i is a segment $[0, 1]$), and $(a_1, \ldots, a_n) \leq (b_1, \ldots, b_n)$ iff for every $i = 1, \ldots, n \quad a_i \leq b_i$. For every pair $a = (a_1, \ldots, a_n), b = (b_1, \ldots, b_n)$

$$a \to b = (c_1, \ldots, c_n), \quad \text{where } c_i = \begin{cases} 1, & \text{if } a_i \leq b_i \\ b_i, & \text{otherwise} \end{cases}$$

2.2 Heyting-valued modal formulas

Let PV be a countable set, elements of which will be called **propositional variables**.

The set Fm of **Heyting-valued modal formulas (HVMFs)** is defined inductively as follows.

- Every $p \in PV$ is a HVMF.
- Every $a \in \mathcal{H}$ is a HVMF.
- If A and B are HVMFs, then the strings $A \wedge B$, $A \vee B$, and $A \to B$ are HVMFs.
- If A is a HVMF, and $a \in \mathcal{H}$, then $\Box_a A$ if a HVMF.

The symbols $\Box_a$ are called **Heyting-valued modal operators**.

A HVMF $\Box_a A$ can be interpreted as the proposition

"the plausibility value of A is equal to a".

For every list $A_1, \ldots, A_n$ of HVMFs the strings

$$A_1 \wedge A_2 \wedge \ldots \wedge A_n \text{ and } A_1 \vee A_2 \vee \ldots \vee A_n$$

are the restricted notations of the HVMFs

$$A_1 \wedge (A_2 \wedge (\ldots \wedge A_n)\ldots) \text{ and } A_1 \vee (A_2 \vee (\ldots \vee A_n)\ldots)$$

respectively.

These HVMFs will be denoted also by the symbols

$$\left\{ \begin{array}{c} A_1 \\ \ldots \\ A_n \end{array} \right\} \quad \text{and} \quad \left[\begin{array}{c} A_1 \\ \ldots \\ A_n \end{array} \right]$$

respectively.

For every pair A, B of HVMFs the string $A \leftrightarrow B$ is a restricted notation of the HVMF $\left\{ \begin{array}{c} A \to B \\ B \to A \end{array} \right\}$.

2.3 Substitutions

A **substitution** is a pair

$$\theta = ((p_1, \ldots, p_n),\ (A_1, \ldots, A_n)) \tag{2}$$

where $p_1, \ldots, p_n$ are distinct variables, and $A_1, \ldots, A_n$ are HVMFs.

For every substitution (2) and every HVMF A the symbol $\theta(A)$ denotes a result of substitution for every $i = 1, \ldots, n$ the HVMF A_i instead of all occurrences of p_i in A.

2.4 Tautologies

Let A and B be HVMFs. We shall say that B is obtained from A by an equivalent transformation, if

- there is a subformula of A of the form $a \wedge b, a \vee b$, or $a \to b$, where $a, b \in \mathcal{H}$,
- B is a result of a substitution in A the corresponded element of $\mathcal{H}$ instead of this subformula.

We shall consider HVMFs A and B as equal (and write $A = B$) iff the pair (A, B) belongs to the least equivalency relation generated by pairs of the form (C, D), where D can be obtained from C by an equivalent transformation.

Let A be a HVMF without modal operators, and the list of variables of A has the form $(p_1, \ldots, p_n)$. A is said to be a **tautology**, if $\theta(A) = 1$ for every substitution (2), such that $\forall i \in \{1, \ldots, n\} \quad A_i = a_i \in \mathcal{H}$.

2.5 Heyting-valued modal logics

A **Heyting-valued modal logic (HVML)** is a set L of HVMFs such that

- every tautology belongs to L,
- for every A, B of HVMFs and every $a \in \mathcal{H}$

$$\Box_a \left\{ \begin{array}{c} A \\ B \end{array} \right\} \leftrightarrow \left\{ \begin{array}{c} \Box_a A \\ \Box_a B \end{array} \right\} \quad \in L, \tag{3}$$

- for every $a \in \mathcal{H}$

$$a \to \Box_a 1 \quad \in L, \tag{4}$$

- for every HVMF A and every $a \in \mathcal{H}$

$$\Box_a A \to a \quad \in L, \tag{5}$$

- for every HVMFs A, B

$$\begin{array}{l} \textbf{if} \quad A \in L \textbf{ and } A \to B \in L \\ \textbf{then } B \in L \end{array} \tag{6}$$

- for every HVMF A and every substitution θ

$$\begin{array}{l} \textbf{if} \quad A \in L \\ \textbf{then } \theta(A) \in L \end{array} \tag{7}$$

- for every HVMFs A, B and every $a, b \in \mathcal{H}$

$$\begin{array}{l} \textbf{if} \quad a \to (A \to B) \in L \\ \textbf{then } a \to (\Box_b A \to \Box_b B) \in L \end{array} \tag{8}$$

– for every HVMF A and every subset $\{a_i \mid i \in \Im\} \subseteq \mathcal{H}$

$$\begin{array}{ll} \textbf{if} & \forall i \in \Im \quad a_i \to A \ \in L \\ \textbf{then} & (\sup_{i \in \Im} a_i) \to A \ \in L. \end{array} \tag{9}$$

This definition implies that there is a minimal (with respect to the inclusion) HVML, which we shall denote by the symbol HVK.

It is not so difficult that the inference rule

$$\begin{array}{ll} \textbf{if} & a_1 \to A_1 \ \in L, \\ & \ldots \\ & a_n \to A_n \ \in L \\ & \left(\begin{array}{l} \text{where } a_1, \ldots, a_n \in \mathcal{H}, \quad \text{and} \\ \qquad A_1, \ldots, A_n \quad \text{are HVMFs} \end{array} \right) \\ \textbf{then} & \left\{ \begin{array}{c} a_1 \\ \ldots \\ a_n \end{array} \right\} \to \left\{ \begin{array}{c} A_1 \\ \ldots \\ A_n \end{array} \right\} \ \in L \end{array} \tag{10}$$

is admissible for every HVML.

For every HVMF A and every HVML L the symbol

$$[\![A]\!]_L$$

denotes a supremum of the set

$$\{a \in \mathcal{H} \mid a \to A \ \in L\}. \tag{11}$$

This definition and (9) imply

$$\forall a \in \mathcal{H} \qquad a \to A \ \in L \quad \Leftrightarrow \quad a \leq [\![A]\!]_L.$$

3 Heyting-valued Kripke models

3.1 Heyting–valued sets

Remind ([1]) that a **Heyting–valued set (HS)** (over a complete Heyting algebra $\mathcal{H}$) is a pair

$$W = (X, \mu) \tag{12}$$

where

- X is a set (which is called a **support** of W), and
- μ is a mapping of the form

$$\mu : X \times X \to \mathcal{H}$$

such that

$$\forall x, y \in X \qquad \mu(x,y) = \mu(y,x) \tag{13}$$

$$\forall x, y, z \in X \qquad \left\{ \begin{array}{c} \mu(x,y) \\ \mu(y,z) \end{array} \right\} \leq \mu(x,z) \tag{14}$$

For every pair $x, y \in X$ the element $\mu(x, y)$ is called a **similarity value between** x **and** y.

For example, let

- X be a set of humans,
- $\{a_1, \ldots, a_n\}$ be a list of some their characteristics (age, sex, salary, reputation, health, etc.),
- $M_1, \ldots, M_n$ are complete totally ordered sets of similarity values related to the characteristics $a_1, \ldots, a_n$ respectively,
- a Heyting algebra $\mathcal{H}$ has the form

$$M_1 \times \ldots \times M_n \tag{15}$$

We can consider X as a Heyting–valued set over (15), where for every pair $x, y \in X$ their similarity $\mu(x, y)$ is a n–tuple $c_1, \ldots, c_n$, such that for every $i \in \{1, \ldots n\}$ if x and y are similar with respect to the characteristics a_i, then c_i is in proximity to the maximal element of M_i.

For every $x \in X$ the element $\mu(x, x)$ is called a **membership value** of x at the HS (12).

Let $W = (X, \mu)$ be a HS. A **Heyting–valued binary relation (HR)** on W is a mapping R of the form $R : X \times X \to \mathcal{H}$, such that

$$\forall x, y, x', y' \in X \qquad \left\{\begin{matrix} R(x, y) \\ \mu(x, x') \\ \mu(y, y') \end{matrix}\right\} \leq R(x', y'), \tag{16}$$

$$\forall x, y \in X \qquad R(x, y) \leq \left\{\begin{matrix} \mu(x, x) \\ \mu(y, y) \end{matrix}\right\}. \tag{17}$$

For every pair $(x, y) \in X \times X$ the element $R(x, y)$ can be interpreted as a **belonging value** of this pair to the HR R.

A **Heyting–valued subset (HSS)** of a HS (12) is a mapping s of the form

$$s : X \to \mathcal{H} \tag{18}$$

such that

$$\forall x, x' \in X \qquad \left\{\begin{matrix} s(x) \\ \mu(x, x') \end{matrix}\right\} \leq s(x'), \tag{19}$$

$$\forall x \in X \qquad s(x) \leq \mu(x, x). \tag{20}$$

For every $x \in X$ the element $s(x)$ can be interpreted as a **membership value** of x at the HSS (18).

The set of all HSSs of a HS (12) will be denoted by the symbol $Sub(W)$.

Below

- for every HS W its support will be denoted by the same symbol W,
- for every pair of elements of the support the similarity value between x and y will be denoted by the symbol $W(x, y)$, and
- for every $x \in W$ the membership value of x at the HS W will be denoted by the symbol $W(x)$.

3.2 Definition of a Heyting-valued Kripke model

A **Heyting-valued Kripke model (HVKM)** is a triple M of the form

$$M = (W, \{R_a \mid a \in \mathcal{H}\}, \xi) \tag{21}$$

where

- W is a HS, elements of which are called **objects** (or **worlds**),
- $\{R_a \mid a \in \mathcal{H}\}$ is a $\mathcal{H}$–tuple of HRs on W, which are called **transition relations**,
- ξ is a mapping of the form

  $$\xi : PV \to Sub(W) \tag{22}$$

  which is called an **evaluation of variables**.

3.3 Evaluation of HVMFs at HVKMs

For every HVMF A and every HVKM (21) an **evaluation of** A **at** M is the mapping

$$[\![A]\!]_M : W \to \mathcal{H},$$

which maps every $x \in W$ to the element $[\![A]\!]_x \in \mathcal{H}$, which is defined as follows:

- if $A = p \in PV$, then $[\![A]\!]_x \stackrel{\text{def}}{=} \xi(p)(x)$,
- if $A = a \in \mathcal{H}$, then $[\![A]\!]_x \stackrel{\text{def}}{=} \left\{ \begin{array}{l} a \\ W(x) \end{array} \right\}$,
- if $A = B \wedge C$, then $[\![A]\!]_x \stackrel{\text{def}}{=} [\![B]\!]_x \wedge [\![C]\!]_x$,
- if $A = B \vee C$, then $[\![A]\!]_x \stackrel{\text{def}}{=} [\![B]\!]_x \vee [\![C]\!]_x$,
- if $A = B \to C$, then $[\![A]\!]_x \stackrel{\text{def}}{=} \left\{ \begin{array}{l} [\![B]\!]_x \to [\![C]\!]_x \\ W(x) \end{array} \right\}$,
- if $A = \Box_a B$, then $[\![A]\!]_x \stackrel{\text{def}}{=} \left\{ \begin{array}{l} a \\ \inf\limits_{y \in W} (R_a(x,y) \to [\![B]\!]_y) \\ W(x) \end{array} \right\}$.

It is not so difficult to prove that $[\![A]\!]_M$ is a HSS of the HS W.

3.4 An example of a HVKM

In this section we give an example of a HVKM related to description logic ([14]).

Description Logic is a language for formal description of complex concepts on the base of *atomic concepts* and binary relations, called *atomic roles.* Assume that there are given

- a set $\mathcal{I}$ of **individuals**,
- a set $\mathcal{C}$ of **atomic concepts**, and every atomic concept $c \in \mathcal{C}$ represents a subset $[\![c]\!] \subset \mathcal{I}$

– a set $\mathcal{R}$ of **atomic roles**, and every atomic role $r \in \mathcal{R}$ represents a binary relation $[\![r]\!] \subseteq \mathcal{I} \times \mathcal{I}$.

Description Logic allows to represent complex notions by **concept terms**, i.e. expressions that are built from atomic concepts and atomic roles with use of the concept constructors:

– boolean operations (conjunction ($\sqcap$), etc.), and
– quantifier operations of the form $\forall r$, where $r \in \mathcal{R}$.

Every concept term represents a subset $[\![t]\!] \subseteq \mathcal{I}$, which is defined by induction as follows:

– $[\![t_1 \sqcap t_2]\!] \stackrel{\rm def}{=} [\![t_1]\!] \cap [\![t_2]\!]$,
– $[\![\forall r.t]\!] \stackrel{\rm def}{=} \{a \in \mathcal{I} \mid \text{for every } \ b \in \mathcal{I} \quad (a,b) \in [\![r]\!] \Rightarrow b \in [\![t]\!]\}$.

For example (the example is borrowed from [15]), if

– $\mathcal{I}$ consists of all humans,
– the atomic concept `Woman` is interpreted as the set of all women, and
– the atomic role `child` is interpreted as the set of all pairs (a,b) of humans, such that b is a child of a

then the concept of all women having only daughters can be represented by the concept term

$$\texttt{Woman} \sqcap \forall \texttt{child}.\texttt{Woman}$$

Let $\mathcal{R}^*$ be the set of all finite sequences of elements of $\mathcal{R}$.
Every sequence $r = (r_1, \ldots, r_n) \in \mathcal{R}^*$ represents a binary relation

$$[\![r]\!] \stackrel{\rm def}{=} [\![r_1]\!] \circ \ldots \circ [\![r_n]\!] \quad \subseteq \mathcal{I} \times \mathcal{I}$$

Elements of $\mathcal{R}^*$ can be interpreted as *derivative roles*, and will be referred briefly as *roles*.

Let $\mathcal{H}$ be the set $\mathcal{P}(\mathcal{R}^*)$ of all subsets of the set $\mathcal{R}^*$. $\mathcal{H}$ is a complete Heyting algebra, because it is a complete boolean algebra.

We can consider the set $\mathcal{I}$ of humans as a Heyting-valued set (over $\mathcal{H} = \mathcal{P}(\mathcal{R}^*)$), where for every pair $(x,y) \in \mathcal{I} \times \mathcal{I}$ the similarity value $\mathcal{I}(x,y)$ consists of all roles $r \in \mathcal{R}^*$ such that x and y are equal with respect to r (we do not clarify the concepts of equality of humans with respect to a role, because it seems to be intuitively clear, but the precise definition of this notion requires a strong linguistic foundation).

A HVKM related to this example has the following components.

– Objects of this HVKM are humans, and similarity value between them was described above.
– For every $\rho \in \mathcal{H}$ and every pair x, y of humans the set $R_\rho(x,y)$ consists of all roles $r \in \rho$ such that $(x,y) \in [\![r]\!]$.

– The set PV is equal to the set $\mathcal{C}$ of atomic concepts, and for every $c \in \mathcal{C}$ the evaluation
$$\xi(c) : \mathcal{I} \to \mathcal{P}(\mathcal{R}^*)$$
is defined as follows: for every human $x \in \mathcal{I}$
$$\xi(c)(x) \overset{\text{def}}{=} \begin{cases} \mathcal{I}(x), & \text{if } x \in [\![c]\!] \\ \emptyset, & \text{otherwise.} \end{cases}$$

3.5 Truth of HVMFs at HVKMs

A HVMF A is said to be **true at an object** x of a HVKM (21), if
$$[\![A]\!]_x = W(x). \tag{23}$$

A HVMF A is said to be **true at a HVKM** (21), if A is true at every object of (21).

It is not so difficult to prove that every HVMF $A \in HVK$ is true at every HVKM, because

– every tautology is true at every HVKM,
– HVMFs from (3), (4) and (5) are true at every HVKM, and
– inference rules (6), (7), (8) and (9) preserve the truth property at every HVKM.

Below we prove the inverse statement: if a HVMF A is true at every HVKM, then $A \in HVK$.

4 Canonical models of HVMLs

4.1 Consistent HVMLs

A HVML L is **consistent**, if for every $a \in \mathcal{H}$ $\quad a \in L \quad \Rightarrow \quad a = 1.$

It is not so difficult to prove that HVK is consistent.

Below every HVML under consideration is assumed to be consistent.

4.2 *L*–consistent sets of HVMFs

Let

– L be a consistent HVML, and
– u be a set of HVMFs.

The set u is said to be L–**consistent**, if for

– every finite subset of the set u, which has the form
$$\{a_1 \to A_1, \ldots, a_n \to A_n\} \tag{24}$$
(where $a_1, \ldots, a_n \in \mathcal{H}, \quad A_1, \ldots, A_n$ are HVMFs, and

– every $b \in \mathcal{H}$

the statement
$$\left\{\begin{array}{c} A_1 \\ \dots \\ A_n \end{array}\right\} \to b \quad \in L \tag{25}$$
implies the inequality
$$\left\{\begin{array}{c} a_1 \\ \dots \\ a_n \end{array}\right\} \leq b. \tag{26}$$

4.3 Properties of L–consistent sets

For every pair u_1, u_2 of sets of HVMFs the inequality
$$u_1 \leq u_2 \tag{27}$$
means that
$$\begin{array}{l} \text{for every HVMF of the form } \ a \to A \ \in u_1 \\ a = 0 \ \text{ or } \ \exists\, b \geq a : \ b \to A \ \in u_2. \end{array}$$

Theorem 1. For every pair u_1, u_2 of sets of HVMFs the inequality (27) implies that
$$u_2 \text{ is } L\text{–consistent} \quad \Rightarrow \quad u_1 \text{ is } L\text{–consistent}.$$

Theorem 2. Every consistent HVML is a L–consistent set.

Below the symbol L denotes some fixed consistent HVML.

Theorem 3. Let

– u be a L–consistent set,
– A be a HVMF, and
– Q be the set of all elements $a \in \mathcal{H}$ such that
$$u \cup \{a \to A\} \quad \text{is } L\text{–consistent}. \tag{28}$$

Then for every $a \in \mathcal{H}$
$$a \leq \sup(Q) \quad \Leftrightarrow \quad a \in Q.$$

The element $\sup(Q)$, which corresponds to A and u, will be denoted by the symbol
$$[\![A]\!]_u \tag{29}$$

The definition of the element $[\![A]\!]_u$ implies that for every set u of HVMFs the following implication holds:

$$\begin{array}{l} u \quad \text{is } L\text{–consistent} \quad \Rightarrow \quad \forall\, A \in Fm \\ u \cup \{[\![A]\!]_u \to A\} \quad \text{is } L\text{–consistent} \end{array} \tag{30}$$

Theorem 4. Let u_1 and u_2 be L–consistent sets, such that

$$u_1 \leq u_2.$$

Then for every HVMF A

$$[\![A]\!]_{u_2} \leq [\![A]\!]_{u_1}. \tag{31}$$

Theorem 5. Let

- u be a L–consistent set of HVMFs, and
- A, B be a pair of HVMFs, such that

$$A \to B \quad \in L \tag{32}$$

Then

$$[\![A]\!]_u \leq [\![B]\!]_u. \tag{33}$$

Theorem 6. For

- every L–consistent set u, and
- every HVMF A

the following inequality holds:

$$[\![A]\!]_L \leq [\![A]\!]_u. \tag{34}$$

4.4 L–complete sets of HVMFs

Let x be a set of HVMFs.

The set x is said to be L–**complete**, if

- x is L–consistent, and
- for every HVMF A

$$[\![A]\!]_x \to A \quad \in x. \tag{35}$$

4.5 Completion of L–consistent sets

Let

- u be a L–consistent set, and
- x be a L–complete set.

x is said to be a **completion of** u, if

$$u \leq x \tag{36}$$

Theorem 7. For every L–consistent set u there is its completion x.

Below we shall assume that $\mathcal{H}$ satisfies the additional condition:

$$\forall\, a \in \mathcal{H} \quad (a \to 0) \to 0 \;=\; a. \tag{37}$$

This condition is equivalent to the condition that $\mathcal{H}$ is a boolean algebra with respect to the operations $\wedge, \vee, \neg$, where $\forall a \in \mathcal{H} \quad \neg a \overset{\text{def}}{=} a \to 0$.

4.6 Canonical models of HVMLs

A **canonical model** of a HVML L is a HVKM

$$M_L \stackrel{\text{def}}{=} (W_L, \{R_{L,a} \mid a \in \mathcal{H}\}, \xi_L)$$

the components of which are defined as follows.

- W_L consists of all L–complete sets.
  For every pair $x, y \in W_L$

$$W_L(x, y) \stackrel{\text{def}}{=} \inf_{A \in Fm} ([\![A]\!]_x \leftrightarrow [\![A]\!]_y) \tag{38}$$

  Note that this definition implies that

$$\forall x \in W_L \qquad W_L(x) = 1. \tag{39}$$

- For every $a \in \mathcal{H}$ $\quad R_{L,a}$ is a HR on W_L, $R_{L,a} : W_L \times W_L \to \mathcal{H}$, where

$$\begin{gathered} \forall x, y \in W_L \\ R_{L,a}(x, y) \stackrel{\text{def}}{=} \inf_{A \in Fm} ([\![\Box_a A]\!]_x \to [\![A]\!]_y) \end{gathered} \tag{40}$$

- ξ_L is a mapping of the form $\xi_L : PV \to Sub(W_L)$, where for every $p \in PV$ the HSS $\xi_L(p) : W_L \to \mathcal{H}$ is defined as follows:

$$\forall x \in W_L \qquad \xi_L(p)(x) \stackrel{\text{def}}{=} [\![p]\!]_x. \tag{41}$$

It is not so difficult to prove that

- W_L satisfies (13) and (14),
- $R_{L,a}$ satisfies (16) and (17), and
- $\xi_L(p)$ satisfies (19) and (20).

4.7 Main property of canonical models

Theorem 8. For every HVMF A and every $x \in W_L$

$$[\![A]\!](x) = [\![A]\!]_x. \tag{42}$$

5 Completeness of *HVK*

Theorem 9. If a HVMF A is true at every HVKM, then $A \in HVK$.

Proof.

Assume that $A \notin HVK$. Prove that A is not true at a certain object of the canonical model of HVK.

Note that the set

$$\{([\![A]\!]_{HVK} \to 0) \to (A \to 0)\} \tag{43}$$

is HVK–consistent, because for every $b \in \mathcal{H}$ the statement

$$(A \to 0) \to b \quad \in HVK \tag{44}$$

implies the inequality

$$[\![A]\!]_{HVK} \to 0 \leq b \tag{45}$$

Indeed, (44) implies that

$$\begin{array}{lll} (b \to 0) \to A \quad \in HVK & \Rightarrow & \\ b \to 0 \ \leq \ [\![A]\!]_{HVK} & \Rightarrow & (45) \end{array}$$

Theorem 7 implies that HVK–consistency of the set (43) implies that

$$\exists x \in W_{HVK}: \quad [\![A]\!]_{HVK} \to 0 \ \leq \ [\![A \to 0]\!]_x \tag{46}$$

Since the set x is HVK–complete, then (46) implies that

$$[\![A \to 0]\!]_x = [\![A]\!]_x \to [\![0]\!]_x = [\![A]\!]_x \to 0 \tag{47}$$

(42), (46) and (47) imply the inequality

$$[\![A]\!]_{HVK} \to 0 \ \leq \ [\![A]\!](x) \to 0 \tag{48}$$

which is equivalent to the inequality

$$[\![A]\!](x) \ \leq \ [\![A]\!]_{HVK} \tag{49}$$

Prove that A is not true at the object x.
If A is true at x, then (23) and (39) imply that

$$[\![A]\!](x) = 1 \tag{50}$$

(49) and (50) imply the equality $[\![A]\!]_{HVK} = 1$, which implies $A \in HVK$.
This contradicts to the assumption that $A \notin HVK$.

6 Conclusion

In the paper we have introduced a new framework for representation of propositions which can contain fuzzy modalities. We have defined the concept of a Heyting-valued modal logic and have proved the completeness theorem for the minimal Heyting-valued modal logic. The directions of further research related to the introduces concepts and results can be the following.

1. Prove the completeness theorem without the condition $(a \to 0) \to 0 = a$ for every $a \in \mathcal{H}$.
2. Investigate the problems of finite model property and decidability of minimal HVML.
3. Define the concept of a Heyting-valued proof for first-order logics, and introduce a Heyting-valued provability logics related to the concept of a Heyting-valued proof, investigate properties of Heyting-valued provability logics.
4. Design a specification language and model checking algorithms for Heyting-valued dynamic systems based on the proposed framework.

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Modification of Good Tests in Dynamic Contexts: Application to Modeling Intellectual Development of Cadets

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Abstract. An approach to incremental learning of Good Maximally Redundant Diagnostic Tests (GMRTs) is considered. GMRT is a special formal concept in Formal Concept Analysis. Mining GMRTs from data is based on Galois lattice construction. Four situations of learning GMRTs are considered: inserting an object (value) and deleting an object (value). An application to modeling intellectual development of cadets is proposed. We explore two datasets of female medical cadets. First dataset is formed at the moment of admission to academy, and another is formed at the end of second year of learning. Classification attribute (dynamics of cadets' intellectual development) is based on analysis of psychological questionnaire invented by M.M. Reshetnikov and B.V. Kulagin. Structural model attributes are based on MMPI questionnaire adopted by L.N. Sobchik.

Keywords: good classification test, formal concept, concept lattice, incremental learning, dynamic formal context, educational data mining, intellectual development, medical cadets

1 Introduction

Good Maximally Redundant Diagnostic Tests (GMRTs) [11] can be considered as formal concepts with minimal by the inclusion relation intents, see, please also minimal hypotheses in [5]. Mining of GMRTs from data is based on constructing the Galois Lattice. The main motivation of incremental learning GMRTs is to provide an expert a way of step-by-step changing of the prediction model. This can be useful, for example, to evaluate an impact of attribute (object) to a prediction model, as well as to improve efficiency of inferring GMRTs (only a part of GMRTs should be recalculated instead of whole set in a batch inferring case).

Incremental learning to construct formal concepts (FCs) require incremental algorithms for Galois lattice generation. In this process, it is generally assumed that the data (objects, itemsets, or transactions) are added gradually but not deleted. Much attention has been paid in recent years to the problem of concept lattice incremental construction [6],[9],[16],[20]. An algorithm of incremental generating GMRTs has been considered in [14].

On the other hand, there is a practical demand to modify the concept lattice already constructed under dynamic data changes. In this case, it is necessary to consider the possibility of both adding and deleting the data (objects, attributes). This problem is not yet investigated sufficiently. Deleting objects (and only objects) is considered in [2] and [21]. Algorithms RemoveObject and DeleteObject are proposed in first and second papers, respectively. These algorithms have been essentially improved with respect to their computational complexity in [22]: in newly proposed algorithm FastDeletion, it is necessary to compare a modified concept only with one of its lower neighbours by the order relation in the concept lattice, whereas, in previous two algorithms, this comparison is performed with all of its lower neighbours.

However modifying the data or the formal classification contexts with the use of which a concept lattice has been constructed can be realized not only by adding or deleting objects but also by adding or deleting attributes. Such a modification of the concept lattice is even less explored than the problem of deleting objects. Of interest in this regard, the paper [8], in which the authors solve the problem of removing an incidence from a formal context.

All the four variants of changing formal contexts, i.e. adding (deleting) an object and adding (deleting) an attribute are considered in [2]. Recent publication on this topic [16] provides an efficient algorithm of modifying the formal contexts by adding objects, which may include, in their descriptions, some new attributes. Modification of the order relation in the concept lattice is also determined. A peculiarity of the proposed algorithm is that it defines new and modified concepts without using previously built lattice but the only available data. New formal concept is a concept in new data with some of added objects to its object set (extent) and attribute set (intent) not equal to intent of any concept in the data before updating. Modified formal concept is a concept in new data with the same intent as some existing concept in the data before updating; and its extent is enlarged by some introduced objects. The algorithm proposed is based on algorithm Close-by-One (CbO) of generating formal concepts [10] and its next refinements FCbO [16], PFCbO [17].

As for removing objects, this process is reduced to adding objects. If description of an object is changed, then this object is removed from the formal context and after that it is treated as an introduced object with new description. Adding and removing attributes is seen as similar to adding and removing objects, but objects and attributes in the algorithm and formal context simply swap places. Updating GMRTs proposed in the present paper covers, likewise in [16], four cases adding/removing objects and adding/removing attributes (values of attributes). New and modified GMRTs (formal concepts) are determined. Modification algorithms are based on decomposition of formal classification context into attributive and objects sub-contexts and using a previously developed incremental algorithm for inferring GMRTs given in [14].

The paper is organized as follows: Sec. 2 gives the main definitions of GMRTs. A decomposition of the formal classification contexts in two kinds of subcontexts is considered. Two kinds of corresponding sub-tasks are required for updating GMRTs in subcontexts. A dataset is described in Sec. 3. An application of four updating GMRTs cases is considered in Sec. 4. The application is supplied by illustrative examples using cadet dataset.

2 Basics of Good Test Analysis

Let G be the set of objects (object indices for short). Assume that objects are described by a set U of symbolic (numeric) attributes, and $\mathrm{dom}(attr_i) \cap \mathrm{dom}(attr_j) = \emptyset, \forall attr_i, attr_j \in U, i \neq j$, where $\mathrm{dom}(attr_i)$ is the set of values of $attr_i$.

Let $M = \{\cup \mathrm{dom}(attr), attr \in U\}$; then one can construct $\delta : G \to D$, where $D = 2^M$ is a set of all possible object descriptions. We denote a description of $g \in G$ by $\delta(g)$, and the sets of positive and negative object descriptions by $D_+ = \{\delta(g) |\ g \in G_+\}$ and $D_- = \{\delta(g) |\ g \in G_-\}$, respectively. The Galois connection [15] between the ordered sets $(2^G, \subseteq)$ and $(2^M, \subseteq)$, i.e. $2^G \to 2^M$ and $2^M \to 2^G$, is defined by the following mappings called derivation operators: for $A \subseteq G$ and $B \subseteq M$, $A' = \mathrm{val}(A) = \{$intersection of all $\delta(g) |\ g \in A\}$ and $B' = \mathrm{obj}(B) = \{g |\ g \in G, B \subseteq \delta(g)\}$. The notation $(\cdot)'$ is from [3], see also similar notation $(\cdot)^\diamond$ in [4].

There are two closure operators: generalization_of$(B) = \mathrm{val}(\mathrm{obj}(B))$ and generalization_of$(A) = \mathrm{obj}(\mathrm{val}(A))$. A set A is closed if $A = \mathrm{obj}(\mathrm{val}(A))$ and a set B is closed if $B = \mathrm{val}(\mathrm{obj}(B))$. If $(A' = B)$ & $(B' = A)$, then a pair (A, B) is called a formal concept [3], subsets A and B are called concept extent and intent, respectively. All formal concepts form a Galois (concept) lattice. A triplet (G, M, I), where I is a binary relation between G and M, is a formal context $\mathbb{K}$.

According to the goal attribute Cl we get some possible forms of the formal contexts: $\mathbb{K}_\epsilon := (G_\epsilon, M, I_\epsilon)$ and $I_\epsilon := I \cap (G_\epsilon \times M)$, where $\epsilon \in \{+, -\}$ (if necessary the value τ can be added to provide the undefined objects) [5]. These contexts form a classification context $\mathbb{K}_\pm = \mathbb{K}_+ \cup \mathbb{K}_-$.

Definition 1. *A Diagnostic Test (DT) for* G_+ *is a pair* (A, B) *such that* $B \subseteq M$, $A = \mathrm{obj}(B) \neq \emptyset$, $A \subseteq G_+$, *and* $\mathrm{obj}(B) \cap G_- = \emptyset$ *[12].*

Definition 2. *A diagnostic test* (A, B) *for* G_+ *is maximally redundant if* $\mathrm{obj}(B \cup m) \subset A$ *for all* $m \in M \setminus B$ *[12].*

Definition 3. *A diagnostic test* (A, B) *for* G_+ *is good iff any extension* $A_* = A \cup i$, $i \in G_+ \setminus A$, *implies that* $(A_*, \mathrm{val}(A_*))$ *is not a test for* G_+ *[12].*

In the paper, we deal with Diagnostic Tests, which are good and maximally redundant simultaneously (GMRTs). If a good test (A, B) for G_+ is maximally redundant, then any extension $B_* = B \cup m$, $m \notin B$, $m \in M$ implies that $(\mathrm{obj}(B_*), B_*)$ is not a good test for G_+. In general case, a set B is not closed for

DT (A, B), consequently, DT is not obligatory a formal concept. GMRT can be regarded as a special type of a concept [12].

To transform inferring GMRTs into an incremental process, we introduced two kinds of subtasks [13]:

1. For a set G_+, given a set of values B, where $B \subseteq M$, $\text{obj}(B) \neq \emptyset$, B is not included in any description of negative object, find all GMRTs $(\text{obj}(B_*), B_*)$ such that $B_* \subset B$;
2. For a set G_+, given a non-empty set of values $X \subseteq M$ such that $(\text{obj}(X), X)$ is not a test for positive objects, find all GMRTs $(\text{obj}(Y), Y)$ such that $X \subset Y$.

For solving these subtasks we need to form subcontexts of a given classification context. The following notions of object and value projections are developed to form subcontexts.

Definition 4. *The projection* $\text{proj}(d)$*,* $d \in D_+$ *is denoted by* $Z = \{z | \, z = \delta(g) \cap \delta(g_*) \neq \emptyset, g_* \in G_+$ *and* $(\text{obj}(z), z)$ *is a test for* $G_+\}$*,* $\delta(g) \in \text{proj}(d)$*.*

Definition 5. *The value projection* $\text{proj}(B)$ *on a given set* D_+ *is* $\text{proj}(B) = \{\delta(g) \mid B \subseteq \delta(g),\ g \in G_+\}$*.*

Let us consider four cases of incremental supervised learning GMRTs:

1. A new object becomes available over time.
2. Deleting an object from a classification context.
3. Adding a value (attribute) to a classification context.
4. Deleting a value (attribute) from a classification context.

In each case (stage of experiment in Sec.4) we obtain all the GMRTs in current $\mathbb{K}_{\pm}$.

2.1 Adding an object to $\mathbb{K}_{\pm}$

Suppose that each new object comes with the indication of its class membership. The following actions are necessary:

1. Checking whether it is possible to extend the extents of some existing GMRTs for the class to which a new object belongs (a class of positive objects, for certainty).
2. Inferring all GMRTs, such that their intents included into the new object description.
3. Checking the validity of GMRTs for negative objects, and, if it is necessary, modifying invalid GMRTs (test for negative objects is invalid if its intent is included in a new (positive) object description).

Thus the following cognitive acts are performed:

- Pattern recognition and generalization of knowledge (increasing the power of already existing inductive knowledge);

– Increasing knowledge (inferring new knowledge);
– Correcting knowledge (diagnostic reasoning).

The first act modifies already existing tests. The second act is reduced to subtask of the first kind. The third act can be reduced to subtasks both the first and second kinds. Both of them are solved by any algorithm of GMRTs inferring.

Let STGOOD_+ and STGOOD_- be the sets of all GMRT intents for positive and negative classes, respectively. Let $s \in \mathrm{STGOOD}_-$ and $Y = \mathrm{val}(s)$. If $Y \subseteq t_{\mathrm{new}}(+)$, where $t_{\mathrm{new}}(+)$ is the description of a new positive object, then s should be deleted from STGOOD_-.

For correcting the set of GMRTs for G_-, we have to find all $X \subseteq M, Y \subset X$ i.e. $\mathrm{obj}(X) \subset \mathrm{obj}(Y)$, and $(\mathrm{obj}(X), X)$ is a GMRT for G_-. Thus $\mathrm{obj}(Y)$ is a context for finding new tests for G_-.

We show that all new tests for G_- in this case are associated only with context $\mathrm{obj}(Y)$: $\mathrm{obj}(X) \subset \mathrm{obj}(Y) \leftrightarrow Y \subset X$. Assume that there exists a GMRT (with an intent Z) for G_- such that $\mathrm{obj}(Z) \not\subseteq \mathrm{obj}(Y)$. Then $\mathrm{obj}(Z)$ contains some objects not belonging to $\mathrm{obj}(Y)$ and Z will be included in some descriptions of objects not belonging to $\mathrm{obj}(Y)$ and, consequently, Z has been obtained at the previous steps of incremental learning algorithm.

2.2 Deleting an object from $\mathbb{K}_\pm$

Suppose that an object is deleted from $\mathbb{K}_\pm$. The following actions are necessary:

1. Selecting the set GMRTsub of all GMRTs containing this object in the extents.
2. Modifying tests of GMRTsub by removing object from their extents; in this connection, we observe that this modifying does not lead to loss of property 'to be test for corresponding elements of GMRTsub'.
3. After modifying a test in GMRTsub, we have the following possibilities. Let Y_* be the intent of a test in GMRTsub and $Y_* = \mathrm{val}(\mathrm{obj}(Y) \setminus i)$, where i is deleted object and $Y = \mathrm{val}(\mathrm{obj}(Y_*) \cup i)$. If $((\mathrm{obj}(Y) \setminus i)$ is included in the extent of an existing GMRTs, then this test $((\mathrm{obj}(Y) \setminus i), Y_*)$ has to be deleted; if $Y_* = Y$ and $((\mathrm{obj}(Y) \setminus i)$ is not included in the extent of any existing GMRT, then $((\mathrm{obj}(Y) \setminus i), Y_*)$ is a GMRT; if $Y_* \neq Y$, then $((\mathrm{obj}(Y) \setminus i), Y_*)$ is a new GMRT.

2.3 Adding a value (attribute) to $\mathbb{K}_\pm$

Suppose that a new value m_* is added to the set M of attributes. The task of finding all GMRTs, intents of which contain m_* is reduced to the problem of the second kind. The subcontext for this problem is the set of all objects whose descriptions contain m_*.

2.4 Deleting a value (attribute) from $\mathbb{K}_{\pm}$

Suppose that some value m is deleted from consideration. Let a GMRT (obj(X), X) be transformed into $(\mathrm{obj}(X \setminus m), X \setminus m)$. Then we have $((X \setminus m) \subseteq X) \leftrightarrow (\mathrm{obj}(X) \subseteq \mathrm{obj}(X \setminus m))$. Consider two possibilities: $\mathrm{obj}(X \setminus m) = \mathrm{obj}(X)$ and $\mathrm{obj}(X) \subset \mathrm{obj}(X \setminus m)$. In the first case, $(\mathrm{obj}(X \setminus m), X \setminus m)$ is GMRT. In the second case, $(\mathrm{obj}(X \setminus m), X \setminus m)$ is not a test. However, $\mathrm{obj}(X \setminus m)$ can contain extents of new GMRTs and these tests can be obtained by using subtasks of the first or second kind.

3 Dataset Description

33 female medical cadets were involved in our experiment. First dataset was formed at the moment of admission to academy (2009 year), and another was formed at the end of second year of learning (2011 year). The datasets are without missing values. The cadets are the same in both datasets. Classification attribute (dynamics of cadets' intellectual development) is based on analysis of measuring methods called Analogy, Cubes, Syllogisms, and Verbal memory, see, please, [18].

For each person, the difference of the estimates of each intellectual method has been calculated in two moments, taking into account the sign of the difference. Then these differences are summarized for all intellectual methods. If the sign of sum is positive (plus), the dynamics is considered to be positive, if the sign of sum is negative (minus) and its number is greater than 2, then the dynamics is considered to be negative. If the sum is equal to 0 or not greater 2, then the dynamics was considered to be neutral (zero). See, please, transformation of Dyn-column into Cl-column in Tab. 2. Within 33 medical cadets we obtained 5, 10, and 18 persons with neutral, negative, and positive dynamics, respectively.

Structural model attributes are based on MMPI questionnaire adopted by L.N. Sobchik [19]. Each attribute value from MMPI questionnaire is transformed to T-scale value using special questionnaire keys and K correction scale, see, please, [19] for the further information. After that T-scaled values are transformed to the scale with five values by means of rules given in Tab. 1. They respect L.N. Sobchik's representations of "normal" intervals. In Tab. 2 three abbreviations L, F, and K stands for Lie, Infrequency, and Defensiveness, respectively. They are validity scales. Ten other following scales are clinical: Hs (Hypochandriasis), D (Depression), Hy (Hysteria), Pd (Psychopathic Deviate), Mf (Masculinity/Feminity), Pa (Paranoia), Pt (Psychasthenia), Sc (Schizophrenia), Ma (Hypomania), and Si (Social Introversion).

For the further considerations, we include in training set only the persons with positive and negative dynamics of intellectual development.

Table 1. Interval scale for MMPI method

No	The meaning of intervals	Intervals' borders
1	Significantly below normal	≤ 30T
2	Below normal	$[31-44]$T
3	Normal	$[45-55]$T
4	Above normal	$[56-69]$T
5	Significantly above normal	≥ 70T

Table 2. Classification context

No	L	F	K	Hs	D	Hy	Pd	Mf	Pa	Pt	Sc	Ma	Si	Cl	Dyn
1	4	3	5	3	4	3	3	4	3	4	4	4	2	2	-7
2	4	4	5	3	4	3	3	3	2	4	4	4	2	2	-3
3	4	3	4	3	3	3	3	3	3	3	3	4	3	2	-3
4	5	4	5	3	4	3	4	2	4	3	4	3	3	2	-5
5	4	3	4	3	3	3	3	4	3	3	3	3	3	2	-4
6	3	3	4	3	3	3	3	3	3	3	3	4	2	2	-2
7	5	3	5	4	4	4	4	3	4	4	4	4	2	2	-7
8	4	3	4	3	3	3	3	4	3	3	3	3	2	2	-2
9	5	3	5	3	3	3	4	2	2	3	4	4	2	2	-2
10	4	3	4	3	2	2	3	2	2	3	3	4	3	2	-2
1	3	3	5	3	4	4	4	4	3	4	4	4	2	1	3
2	2	3	4	3	2	3	3	3	3	3	3	3	2	1	2
3	3	3	5	3	3	3	3	2	4	4	4	3	3	1	3
4	3	3	4	3	3	3	4	4	2	3	3	5	3	1	4
5	3	3	5	3	3	4	4	4	3	4	4	3	3	1	6
6	4	2	4	3	4	4	4	4	2	3	3	3	2	1	4
7	3	3	3	2	4	2	3	4	3	2	3	5	2	1	2
8	3	3	4	2	3	3	4	4	3	3	4	3	2	1	2
9	2	4	5	3	4	4	3	4	4	4	4	4	2	1	1
10	3	3	5	3	2	3	3	2	4	3	3	4	2	1	1
11	3	4	4	3	3	3	3	4	2	3	3	4	2	1	4
12	3	3	4	3	3	4	2	4	3	3	3	4	2	1	10
13	5	3	5	4	3	4	4	4	4	4	4	4	2	1	4
14	3	3	4	3	4	3	4	4	2	4	4	4	2	1	5
15	3	3	4	3	3	3	3	2	2	3	3	4	3	1	2
16	5	3	4	3	4	2	3	3	4	3	3	3	3	1	3
17	3	3	5	3	4	4	3	5	4	4	4	3	2	1	5
18	5	4	5	3	4	3	4	1	3	4	4	4	3	1	1
1	4	4	5	3	3	3	3	1	3	3	4	4	3	3	-1
2	3	4	4	4	3	4	4	3	4	3	4	5	3	3	-1
3	4	3	4	3	2	2	4	2	2	3	3	4	2	3	0
4	3	4	4	3	2	3	4	1	2	3	4	4	3	3	0
5	4	5	3	2	2	2	3	2	3	2	3	5	2	3	0

4 Experiments and discussion of results

The aim of modeling is to obtain GMRTs allowing to distinguish Class 1 and Class 2 of persons characterised by positive and negative dynamics of intellectual development, respectively. Intents of GMRTs have been regarded as logical rules determining the membership of persons to one or another class.

Recognizing the class membership for new persons not belonging to training set is performed as follows: If (and only if) description of a person contains a logical rule of only one class, then the person can be assigned to this class; if description of a person contains logical rules of both Class 1 and Class 2, then we have the case of contradiction; if description of a person does not contains any logical rules, then we have the case of uncertainty. In two last cases, it is necessary to continue learning by adding new persons' descriptions or to change the classification context.

Incremental learning of GMRTs is partitioned into several stages (see, please, Tab. 3) in accordance with expert reasoning. First seven stages were conducted without attributes Hs, D, Sc, and Si. Stage 1: training set contains 6 first persons of Class 1 and 6 first persons of Class 2. The result of Stage 1 is in Tab.4.

Table 3. Stages of Incremental Learning

Stage	Training sets		Tab. No
	Class 1	Class 2	
1	Persons 1-6	Persons 1-6	4
2	Pattern recognition		
3	Persons 1-6	Persons 1- 8	
4	Persons 1-6 and 8, 9, 13, 14,17	Persons 1-8	
5	Persons 1-6, and 8-11, 13-15, and 17	Persons 1-8	
6	Persons 1-17	Persons 1-8	5
7	Persons 1-17	Persons 1-8	5
8	Delete attribute Pt		
9	Add attribute Hs		
10	Delete 4th person from Class 2		

Stage 2 is a pattern recognition one; the control set contains persons 7 and 8 of Class 2 and persons 7 – 17 of Class 1. All persons of Class 2 and 5 persons (8, 9, 13, 14, 17) of Class 1 have been recognized correctly. Persons 10, 11, 15 of Class 1 have been recognized as persons of Class 2, and persons 7, 12, 16 of Class 1 have been assigned to neither of these classes. During Stage 4, rule (L=5,K=5,Pd=4,Pa=4) for Class 2 $\subset$ val(13) for person 13 of Class 1. This rule is deleted. During Stage 5, rule (Hy=3,Pd=3,Ma=4) for Class 2 is deleted (this rule $\subset$ val(11) for person 11 of Class 1). During Stage 6, two rules were absorbed

Table 4. Rules for Class 1 and Class 2 (Stage 1)

Rule No	L	F	K	Hy	Pd	Mf	Pa	Pt	Ma	Class	Persons
1					4	4				1	1,4,5,6
2	3	3	5					4		1	1,3,5
3	2	3	4	3	3	3	3	3	3	1	2
1	4			3	3					2	1,2,3,5
2				3	3				4	2	1,2,3,6
3		4	5	3						2	2,4

by Rule (Pd=3,Pa=4) and some new rules for Class 1 were obtained. Stage 7: correcting the rules for Class 2. The result is in Tab. 5.

Let us suppose that an expert decides to change one attribute in the model obtained in the previous stage. The problem is how to choose a candidate for deleting and then a candidate for adding. The most simple way is to do what an expert wish to see, however we can propose to an expert some more criteria to take into account. Let us imagine that we get some sets of GMRTs after deleting or adding an attribute. According to a definition of GTA we would recommend to maximize a total number of objects for a GMRT set (sum of rules' coverings) and minimize a total number of attributes for a GMRT set (sum of rules' lenghts). Minimizing a number of GMRTs can be one more criterion. An expert can choose only one criterion or combine some of them to be satisfied with the result obtained.

Step 8: deleting attribute Pt. This attribute is chosen after a short analysis of the GMRT sets (obtained without F, L, K e.t.c.) discussed above. The total attribute lengths of all GMRTs, and the total object coverings in the case of Pt deleting is 53, and 72, respectively. The comparison of such numbers is not very useful. We formed and compared the average attribute lengths (per one rule) and the average object coverings (per one rule), e.g. 2.94, and, 4 for this case, respectively. As a result, Rule (L=3,F=3,Pt=4) is deleted, and attribute Pt is deleted from Rule 14 in Tab.5.

Step 9: adding attribute Hs. This choice is explained by one main criterion – a number of rules. In this case one gets 17 rules, i.e. this number is even decreased in comparison with previous stage. In other cases the number of rules is the same (adding Sc), and bigger (25 and 19 when we add D and Si, respectively). As a result of stage 9, we add Hs in Rules 1,3,4 for Class 1, and Rules 2,7,8,11,12 for Class 2. One new Rule (Hs=3,Hy=4) for Class 1 is obtained, and two Rules 3,15 are deleted.

The results obtained allow to characterized the persons of Class 1 and Class 2 psychologically: Class 2 (negative dynamics) is characterized by the MMPI profiles similar to "indepth" profiles and Class 1 (positive dynamics) is characterized by the MMPI profiles similar to "harmonious" profiles and profiles similar to "convex" profiles (by Sobchik definition, [19]). However our expert

Table 5. Rules for Class 1 (Stage 6) and Class 2 (Stage 7)

Rule No	L	F	K	Hy	Pd	Mf	Pa	Pt	Ma	Class	Persons
1					4	4				1	1,4,5,6,8,13,14
2	2				3					1	2,9
3				4					3	1	5,6,17
4				4		4				1	1,5,6,9,12,13
5	3	3							3	1	3,5,8,17
6	3	3						4		1	1,3,5,14,17
7			4				2			1	4,6,11
8	3	3		3	3	2				1	3,10,15
9		4			3	4			4	1	9,11
10	3					4				1	1,4,5,7,8,11,12,14
11	3	3	5							1	1,3,5,10,17
12					3		4			1	3,9,10,17
13		3		2	3					1	7,16
14		3	4		3	3		3	3	1	2,16
15	3	3		4						1	1,5,12,17
1	4			3	3					2	1,2,3,5,8
2						3			4	2	2,3,6,7
3		4	5	3						2	2,4
4		3		3	3		3		4	2	1,3,6

recommended us to check rule's structure without 4th object in Class 2, which seems to be suspicious. The object has good description (psychological portrait) but bad results only in 4 questionnaires for evaluating intellectual development at second year of learning. Class labelling seems to be a mistake.

Step 10: deleting object 4 from Class 2. This classification context modification deeply changes the GMRTs set, but the rules number decrease to 16. For example deleting object 7, which also seems to be labelled by a mistake, leads to increasing the rules number to 18.

In the paper, we take into account only four user's criteria for adding (deleting) an attribute as follows: expert's preferences, total object coverings in extents, number of rules, and total lengths of attributes in intents. However one can try also to use such criteria as concepts stability, number of rules per one object, and many others. Another interesting problem is a choice of intervals to scale a data given in T-values into more expert-oriented ones. Sobchik's scales from Tab. 1 can be useful for cross-investigation comparisons but not so useful for pattern recognition and data mining purposes.

If $\mathbb{K}$ is given for one time period, we can use also another approach of $\mathbb{K}$ dynamics exploration. It is associated with concept stability, please, see definition, for example in [1]. An application of this approach to investigation of students difficulties during learning in high school is given in [7]. Stability shows how much the group depends on some of particular students. Intents of formal concepts are described by marks' on courses. Potential object removing should not change seriously well-studied (worse-studied) learning courses. An extensional stability index is proposed in this paper in a dual manner.

This static approach of $\mathbb{K}$ dynamics exploration for measuring potential object (attribute) removing is also completed in [7] by a dynamic mappings approach in two different time periods (G is not changing). However the problem setting (adding or removing attributes in $\mathbb{K}$) in this paper is different from our problem setting (four cases of $\mathbb{K}_{\pm}$ modification).

5 Conclusion

Four situations of GMRTs modeling (adding/deleting and an object or an attibute) in dynamic context are given in the paper. An application to modeling dynamics of cadets intellectual development using GMRTs is developed. This approach allows us to work with cadet dataset in a dynamically changing way. Step-by-step expert decisions about modification of classification rules can be implemented on-the-fly. This approach can be useful to academy psychologists, lecturers, and administrators for analysing dynamics of cadets intellectual development.

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An Ontology-Based Model of Technical Documentation Fuzzy Structuring

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Abstract. The article is concerned with the method for structuring the electronic archive of technical documentation on the basis of the domain-specific ontology. The ontology formal model, the technical document model, and the algorithm for clustering electronic archive content that has its origins in the modified fcm-method are presented. The authors are pioneered in offering the formalization of the measure of distance between ontological representations of the archive technical documents on the basis of hierarchy transformation complexities comparison. Different types of semantic relations between ontology concepts should be taken into account. Thus, the article considers the experimental results of the subset of the electronic archive technical documentation of the large project organization.

Keywords: ontology, clustering, technical document, fuzzy model, graph

Introduction

A modern large project organization possesses a sizable electronic archive of design and engineering documentation and engineering documentation. Its greater part is represented in unstructured text files. In actual truth, such an electronic archive contains the totally experience and knowledges of a great number of highly trained specialists that have been developing and designing complex systems over many years. In case of expanding the electronic archive, difficulties related to document analysis on the basis of predetermined properties ensure. Also skills of semantic processing of a great number of technical documentation and intimate knowledges of the subject area are required for persons who involved in complex technical systems designing. As a result, the important experience of previous developments fixed in electronic archives often becomes non-demanded. Thus, R&D cycle runtime increases.

The solution of the specified problem can be based on the use of intelligent methods and algorithms of text documents analysis in order to create the navigation structure of the technical documentation electronic archive. The paper [1] suggests using ontologies in intelligent document analysis.

Evaluating the specific character of project knowledges leads to the necessity of forming the project organization ontology with the special structure including features of a project process in the form of a subject area concept system, relations between these concepts, and interpretation functions. In such a manner the electronic archive should possesses properties of an intelligent system. At the moment mathematical methods and algorithms providing the means for structuring an electronic archive of technical documentation with consideration for its content and the specific character of a project organization subject area are not available.

Consequently, currently central problems include development of models, methods and algorithms for construction of the navigation structure of the technical documentation electronic archive on the basis of domain-specific clustering of partially formalized information resources.

In Section 1, the authors decribe the formal model of electronic archive ontology structure. Section 2 considers a technical document as an electronic archive resource and presents the ontological model. In its turn, Section 3 proposes the algorithm for ontology-oriented indexing of technical documents. The measure of distance in the context of ontology relating to the level of designing standards is formalized in Section 4. Section 5 offers some experimental results.

1 The structural model of an electronic archive ontology

A subject area of complex system designing places some constraints on the structure of an applied ontology. The rigid binding to standards and systems life cycle models applied at different stages of designing implies the necessity of forming the ontology that consists of a lot of levels, as indicated by 1.

Formally, the electronic archive ontology consists of two applied ontologies and may be written as the equation 1:

$$O = \langle O^D, O^{LC}, R_A \rangle, \tag{1}$$

where O^D is a subject area ontology component, O^{LC} is an ontology of designing systems life cycles, R_A is a unidirectional association relation between the ontology components. Let us consider the electronic archive ontology components in more detail (1).

In this way, let us write the domain-specific ontology as the following sequence:

$$O^D = \langle C, W, R^D, F^D \rangle,$$

where C is the set of electronic archive concepts that makes up a bulk of a conceptual apparatus of an automated system designing, $W = W^S \cup W^P$ is a set of subject area concepts, here W^S is a set of concepts on the level of standards, W^P is a set of concepts on the project level, R^D is a set of relations. Symbolically,

$$R^D = \{R_G^D, R_C^D, R_A^D\},$$

where R_G^D is anti-symmetric, transitive, irreflexive binary generalization relationship ('subclass_of'), R_C^D is a binary transitive composition relation ('part_of'), R_A^D is a binary relationship of unidirectional association.

The set of concepts C is defined by the following equation:

$$C = \left(C^{S_1} \cup C^{S_2} \cup \ldots C^{S_k}\right) \cup C^P,$$

where $C^{S_i}, i = \overline{1,k}$ is the set of subject area concepts for the standards of the i^{th} group, C^P is the set of subject area concepts extracting from the technical documentation of projects realized.

The set of interpreting functions is denoted as follows:

$$F^D = \{F_{WC^P}^D, F_{C^PC^S}^D\},$$

here $F_{WC^P}^D : \{W\} \to \{C^P\}$ is a function correlating a set of terms and a set of subject area concepts, $F_{C^PC^S}^D : \{C^P\} \to \{C^S\}$ is an interpretation function of the set of concepts allowing to go to the level of concepts defined in standards.

The ontology on a life cycle as a sequence component (eq. 1) consists of three sets and is denoted by the following equation:

$$O^{LC} = \langle M^{LC}, St^{LC}, R^{LC} \rangle,$$

here M^{LC} is a set of models of designing systems life cycles, St^{LC} is a set of life cycle stages.

Definition 1. *Terminological environment of concepts is the set of terms (layers) from the electronic archive technical documentation of projects realized.*

According to the paper [1], a semantic distance between the concept and terms in the technical document should be defined on the basis of the semantic relation idea. The idea encloses the use of 'distance' between words.

The semantic coefficient of the relation between the concept and the term (the semantic distance) is defined by the following equation:

$$S\left(c_i^{P(S)}, w_j\right) = \frac{\sum_{occur\left(c_i^{P(S)}, w_j\right)} \frac{1}{exp(sentence \cdot (paragraph+1))}}{num\left(occur\left(c_i^{P(S)}, w_j\right)\right)} \cdot \\ \cdot \frac{num\left(paragraph - cooccur\left(c_i^{P(S)}, w_j\right)\right)}{num\left(totalparagraph\right)},$$

here $c_i^{P(S)}, w_j$ is the i^{th} concept on the level of projects (standards) of the ontology and the j^{th} term, *sentence* is the distance expressed in the form of the

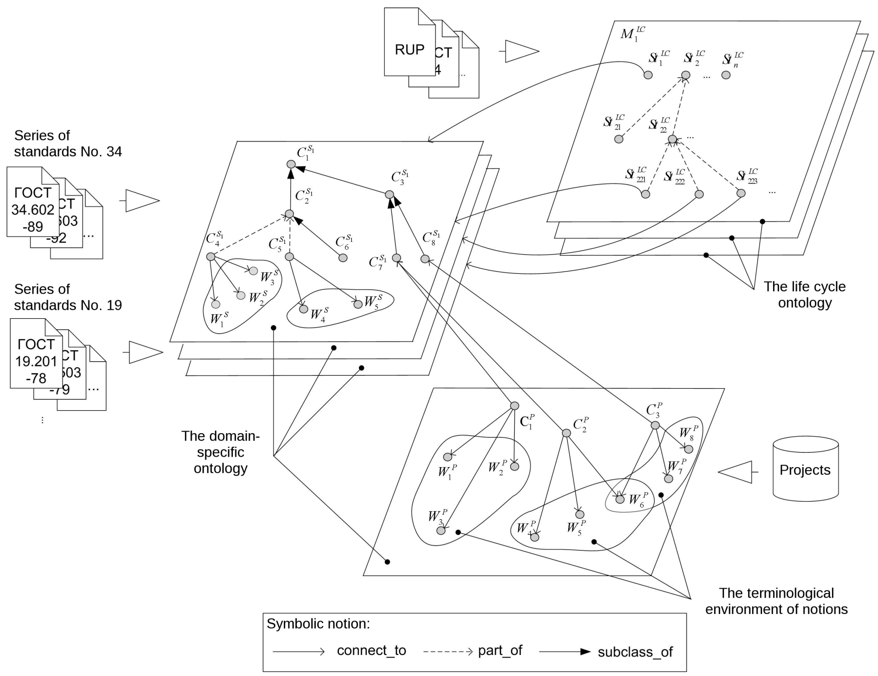

Fig. 1. The structure of the electronic archive applied ontology.

number of sentences between the concept and the term, *paragraph* is a distance expressed in the form of the number of paragraphs between the concept and the term, $num\left(paragraph - cooccur\left(c_i^{P(S)}, w_j\right)\right)$ is the number of paragraphs where coocurrence $c_i^{P(S)}$ and w_j exist, $num\left(occur\left(c_i^{P(S)}, w_j\right)\right)$ is the number of rencontres between $c_i^{P(S)}$ and w_j, $num\,(totalparagraph)$ is the number of paragraphs in the document.

After defining semantic distances between the concept and the document terms, its necessary to define the subset of terms that are appreciably semantically close to the concept. In case of defining the terminological environment, according to the paper [2], the hypothesis of λ-compactness that leans up the λ-distance, taking into account a normalized distance d between terms and the characteristics of a local density of terms τ about these elements.

If the semantic distances between all the pairs of terms with the terminological environment are defined, the graph connecting all terms can be plotted. After that, the most long edge (the graph diameter D) should be defined. Consider two terms w_i and w_j and denote the length of the edge connecting them (the semantic distance) as $\alpha\,(w_i, w_j)$. We obtain the normalized distance between terms $d = \frac{\alpha}{D}$.

Further, let us find the shortest edge between the ones adjusted to the edge (w_i, w_j). Its length is denoted by β_{min}. The ration between the lengths of adjusted intervals is denoted by $\tau^* = \frac{\alpha}{\beta_{min}}$. In order to normalize this value, let us find the largest value τ_{max} in the entire graph. The value $\tau = \frac{\tau^*}{\tau_{max}}$ is a normalized characteristic of a set local density nonhomogeneity about the ontology terms w_i and w_j. $\lambda = f(\tau, d)$ is a λ-distance between the terms w_i and w_j. According to the paper [2], the use of $\lambda = \tau^2 \cdot d$ as such a distance measure is suggested.

In order to define the terminological environment of the ontology concept on the level of realized projects, it is necessary to mark such an edge (w_i, w_j) that can be a boundary between terms related to the ontology concept and terms that are not included in the terminological environment of the concept. With the use of λ-KRAB algorithm, the final criteria characterizing the quality of such a disjunction of terms is denoted by the following equation:

$$F = h^4 \tau^2 d \to max,$$

where $h = 2 \cdot \frac{m^+}{m} \cdot \frac{m^-}{m}$, is the equinumerosity criteria of the specified classes of terms. Here m^+ is the number of terms included on the terminological environment of the concept, m^- is the number of other ones.

Thus, with the use of the λ-compactness hypophysis, the subset of terms that is included in the terminological environment of the concerned concept is defined.

Every terminological environment W_k of the concept $C_k^{P(S)}$ can be denoted by the following equation

$$\{(w_{1k}, f_{1k}), (w_{2k}, f_{2k}), \ldots, (w_{ik}, f_{ik}), \ldots, (w_{lk}, f_{lk})\},$$

here w_{ik} is i^{th} term k^{th} ontology concept, l_k is the total amount of term associated with the the k^{th} concept, f_{ik} is a normalized semantic weight of the i^{th} term in the terminological environment of the k^{th} concept (normalized semantic distance between the term and the concept in the context of the one ontology environment).

2 The ontology model of the technical document as an electronic archive resource

A technical document in the context of an electronic archive is considered is an information resource. Any one of technical documents can be considered as a container of partially structured information. On the one hand, we deal with a natural language text, but on the other hand, a technical document is proper structured. The structure is defined in different standards.

We compare a frequency of occurrence of terms in one technical document with a frequency of occurrence of the same terms in the whole set of documents. It is originally conceived that the terms are not valuable if the frequency of terms in the document analyzed is far in excess of the frequency in the whole set of documents. Symbolically, such a dependence can be denoted as follows:

$$f_i = tfidf_i = tf_i \cdot log\left(\frac{N}{df\left(w_i\right)}\right),$$

here $tfidf_i$ is a relative importance of the term w_i in a document, tf_i is a normalized frequency of term w_i occurrence, N is a number of documents, $df\left(w_i\right)$ is a number of documents containing a term w_i.

An ontological model of a technical document is such a document representation that corresponds to the applied ontology state of an electronic archive. By [3], it follows that the notion of electronic document passport including a semantic index can be an analog of such a model.

A section of a technical document can be shown as follows:

$$s_i^d = \langle ch_{s_i^d}, C^P_{s_i^d}, C^S_{s_i^d} \rangle,$$

where s_i^d is the i^{th} section of a technical document d, $ch_{s_i^d}$ is a unique name of the i^{th} section of a technical document d, $C^P_{s_i^d}, C^S_{s_i^d}$ is a subset of subject area concepts, defined in the context of the i^{th} section of a technical document d.

Let us denote the j^{th} term of the i^{th} section of a technical document d by $w_j^{s_i^d}$, than a set of terms of the i^{th} section of a technical document d can be defined as:

$$W_{s_i^d} = \{w_1^{s_i^d}, w_2^{s_i^d}, \ldots, w_{l_{s_i^d}}^{s_i^d}\},$$

where $l_{s_i^d}$ is a number of terms of the i^{th} section of a technical document d.

With the use of an interpretation function of the ontology $F^D_{WC^P} : \{W\} \to \{C^P\}$ on the stage of technical document indexing, we obtain the ontological representation of the document section:

$$oV^d_{s_i^d} = \langle ch_{s_i^d}, C^P_{s_i^d}, C^S_{s_i^d} \rangle, C^P_{s_i^d} \subseteq C^P, C^S_{s_i^d} \subseteq C^S \mid_{St_k^{LC}} .$$

$C^S_{s_i^d} \subseteq C^S \mid_{St_k^{LC}}$ means that the ontological representation of the document includes only ontology concepts of a subset C^S (on the level of standards using in automated systems designing) that correspond to the k^{th} stage of designing St_k^{LC}.

With the use of function $F^D_{C^P C^S} : \{C^P\} \to \{C^S\}$, we can get the final representation of a technical document section that considers the state on an electronic archive applied ontology:

$$\overline{oV^d_{s_i^d}} = \langle ch_{s_i^d}, \{C^P_{s_i^d} \cup C^S_{s_i^d}\} \rangle, C^P_{s_i^d} \subseteq C^P, C^S_{s_i^d} \subseteq C^S \mid_{St_k^{LC}} .$$

A formal ontology model of a technical document can be defined as follows:

$$oV^d = \langle S^d, \{C^P_d \cup C^S_d\} \rangle,$$

The two main parts can be marked in the above equation: a structural one (S^d) and a conceptual one ($\{C_d^P \cup C_d^S\}$) in the context of realized projects of the archive and standards applied in the process of automated system designing with regard to the stage of a life cycle.

3 Ontology-oriented indexing of technical documents

The ontology indexing of a technical document has in its basis the following function:

$$F_{oV^d} : s_i^d \rightarrow oV^d_{s_i^d},$$

here s_i^d is the i^{th} section of a technical document d, $oV^d_{s_i^d}$ is an ontological representation of the i^{th} section of a technical document d.

Notice that the method of computing a normalized weight of a term $w_j^{s_i^d}$ in the i^{th} section of a technical document d has in its basis the following equation:

$$f_j^{s_i^d} = 1 + log\left(tf_{w_j^{s_i^d}}\right) \cdot log\left(\frac{N}{dt}\right) \cdot \frac{1}{\sqrt{tf^2_{w_1^{s_i^d}} + tf^2_{w_2^{s_i^d}} + \ldots + tf^2_{w_n^{s_i^d}}}}, 1 \leq j \leq n,$$

here $f_j^{s_i^d}$ is a normalized weight of a term $w_j^{s_i^d}$ in the i^{th} section of a technical document d, $tf_{w_j^{s_i^d}}$ is a term $w_j^{s_i^d}$ frequency of occurrence, N is the total amount of documents, dt is a number of documents including a term $w_j^{s_i^d}$, n is a number of terms in the j^{th} section of a technical document d.

Definition 2. *A degree of manifestation of an electronic archive ontology concept is a degree of conjunction between a terminological environment and a set of concepts of a technical document fragment subject to the condition that a terminological environment includes terms that are semantically close to the concept.*

Computing the degrees of manifestation of ontology concepts for every section of a technical document is performed with the use of the apparatus of fuzzy irrelevance [4]. Fuzzy irrelevance between a set W (a set of ontology terms on the level of projects (standards) included in the terminological environment of concept) and a set $C^{P(S)}$ (a set of concepts of an applied ontology on the level of projects (standards)) denoted by $\tilde{\Gamma} = \left(W, C^{P(S)}, \tilde{O}\right)$ where W and $C^{P(S)}$ are crisp sets, $\tilde{O}$ is a fuzzy set in $W \times C^{P(S)}$. A set W is a domain of a function, a set $C^{P(S)}$ is a range of a function, and $\tilde{O}$ is a fuzzy graph of a fuzzy relevance.

The crisp relevance $\Gamma = \left(W, C^{P(S)}, O\right)$ with a chart O as a carrier of a fuzzy chart $\tilde{O}$ is called the carrier of fuzzy relevance $\tilde{\Gamma} = \left(W, C^{P(S)}, \tilde{O}\right)$. In the context of an ontology, a chart O defines parts of unidirectional associations R_A^D between a project concepts and terms in an ontology.

In order to find the meaning of concept domination, the method comparing the terminological environment of every concept in the ontology of a subject area ontology on the project level with the text analyzed. Let us remark that the minimal fragment of a text analyzed is a sentence and a maximal one is the whole document, as in different fragments of the text different concepts of the subject area are layed an emphasis on [5].

The algorithm of computing a degree of dominance of a concept in the text fragment consists of the following steps:

Step 1. Defining the maximal degree of manifestation of ontology concepts in the text fragment of a technical document d:

$$\mu_{\hat{fr_p^d}}\left(c^{P(S)}\right) = max_c\left(\mu_{fr_p^d}\left(c^{P(S)}\right)\right).$$

Step 2. Defining the mean of a degree of manifestation of ontology concepts without the concept with the maximum degree of manifestation (defined at the previous step):

$$\mu_{\tilde{fr_p^d}}\left(c^{P(S)}\right) = \frac{1}{n-1}\sum_{i=1}^{n-1}\mu_{fr_p^d}\left(\hat{c}_i^{P(S)}\right),$$

where $\hat{c}_i^{P(S)} \in c^{P(S)} - c_{max}^{P(S)}$, $c_{max}^{P(S)} = argmax_{c^{P(S)}}\left(\mu_{fr_p^d}\left(c^{P(S)}\right)\right)$, n is a number of concepts with a non-zero degree of manifestation for a text fragment fr_p^d.

Step 3. Defining a degree of manifestation of a concept in a text fragment fr_p^d:

$$\Delta_{fr_p^d}\left(c^{P(S)}\right) = \mu_{\hat{fr_p^d}}\left(c^{P(S)}\right) - \mu_{\tilde{fr_p^d}}\left(c^{P(S)}\right). \quad (2)$$

The equation 2 defines a quality of selection of a text fragment in a technical document in order to constrain the subject area concept that is fixed in an electronic archive ontology.

Having applied the ontology interpretation function $F_{WC^P}^D : \{W\} \to \{C^P\}$, we obtain an initial ontological representation of each segment. The representation consists of initial sets of concepts on the levels of projects and standards that require correction.

The results of the experiments with extracting text fragments on the basis of the genetic optimization show that averages 30% of concepts add up to 70% of the total degree of manifestation of all the concept of the text fragment.

The final step of forming the ontological representation of a technical document is the use of interpreting function $F_{C^P C^S}^D : \{C^P\} \to \{C^S\}$ that allows to specify a set of concepts on the level of standards resting on the subset of ontology concepts found in a technical document. The concepts correspond to the realized projects.

In case of realizing the above procedures, we get the final ontological representation for every i^{th} section of a technical document.

4 The ontological measure of distance between documents

Let us consider the formal measure of distance between documents in the context of ontology concepts relating to the level of designing standards. Every ontological representation can be illustrated in a form of a tree (a hierarchy) of subject area concepts. Such an hierarchy can be defined by finding a minimal tree including all concepts from the ontological representation [2].

The Levenshtein distance between hierarchies can be defined on the basis of computing an edit operation cost that should be found for each type of a semantic relation. Thus, an edit operation for a generalization relation is denoted by $\phi_{S_i}\left(R_G^D\right)$ and a 'part_of' one is denoted by $\phi_{S_i}\left(R_C^D\right)$. S_i shows belonging the value of an edit operation to the the i^{th} group of standards. Actually, in case of clustering, an edit operation is defined as a weight of a certain relation. The weight value lies in the range between 0 and 1 and have different values within the framework of every group of standards.

The total edit distance between the hierarchies is defined as the following equation:

$$\tau_{oV}^{*} = max_i \left(\sum_{s=1}^{m} \phi_{S_i}\left(R_G^D\right)_s + \sum_{l=1}^{n} \phi_{S_i}\left(R_C^D\right)_l \right),$$

where i is a group of standards number, s is an adding generalization relation number, l is an adding 'part_of' relation number. The total edit distance can be computed as a maximum one from all edit distance defined for every group of standards.

A normalization coefficient T_{oV} is defined on the basis of all semantic relation of a generalized hierarchy. Thus, a measure of distance between ontological representations of technical documents can be defined as follows:

$$\parallel oV^{d_1} - oV^{d_2} \parallel = \frac{\tau_{oV}^{*}}{T_{oV}}.$$

In order to create the navigation structure in the form of a nested set of clusters of technical documents, it is necessary to solve the problem of setting the weights of semantic relations between ontology concepts on the level of standards. As noted above, weight coefficients are defined as $\phi_{S_i}\left(R_G^D\right)$ and $\phi_{S_i}\left(R_C^D\right)$ for a generalization relation and 'part_of' relations respectively.

In view of the fact that the specified relations are used in the ontology concepts for different groups of standards, let us suppose that their optimal values for each group (in the context of their concept hierarchies) are generally different. Let us formulate the principle of the best value for weight coefficients of ontology semantic relations.

Let $\{oV^d\}^*$ be a set of ontological relations of documents included in the model sampling (the expert division of documents between classes). The following equation is true:

$$\{oV^d\}^* \subset \{oV^d\},$$

where $\{oV^d\}$ is a full set of ontological representation of electronic archive technical documents. The ontology is defined by the equation (1). On the level of standards, the generalization and 'part_of' relations are defined on the basis of concepts with corresponding weight coefficients $\phi_{S_i}\left(R_G^D\right)$ and $\phi_{S_i}\left(R_C^D\right)$, where S_i is the i^{th} group of designing standards used in ontology creation.

A set $\{oV^d\}^*$ consists of two subsets $\{oV^d\}_+^* \cup \{oV^d\}_-^*$ that correspond to the expert division of documents between two predetermined classes. The optimization problem of weight coefficients of semantic relations consists of finding such a set of coefficients as follows:

$$\{\langle\phi_{S_1}^*\left(R_G^D\right),\phi_{S_1}^*\left(R_C^D\right)\rangle,\langle\phi_{S_2}^*\left(R_G^D\right),\phi_{S_2}^*\left(R_C^D\right)\rangle,\ldots,\langle\phi_{S_n}^*\left(R_G^D\right),\phi_{S_n}^*\left(R_C^D\right)\rangle\}.$$

The clustering coefficient defined by the equation 3 should be as low as possible.

$$F^* = \frac{max\left(\bar{K_+}+\bar{K_-},\hat{K_+}+\hat{K_-}\right)}{N} \rightarrow min \tag{3}$$

where $\bar{K_-}$ and $\hat{K_-}$ are sets of absent documents respectively in the first and the second clusters, $\bar{K_+}$ $\hat{K_+}$ are sets of redundant documents respectively in the first and the second clusters, N is the number of documents.

5 The analysis of computational experiments result on the basis of FRPC JSC 'RPA 'Mars' electronic archive documentation

In case of analysis of computational experiments result on the basis of the documentation of FRPC JSC 'RPA 'Mars' electronic archive, the domain-specific ontology was used. The ontology consists of two series of standards used at the enterprise:

1. GOST 34. Information technologies. Open systems interconnections. (It consists of 108 ontology concepts at the level of standards).
2. GOST 19. Unified system for design documentation. (It consists of 111 ontology concepts at the level of standards).

The ontology level appropriate to the realized projects is based on the selection of FRPC JSC 'RPA 'Mars' electronic archive documentation that includes 5017 technical documents. The level consists of 81 concepts and 10078 unique terms comprising the terminological environment of concepts.

Thus, the domain-specific ontology consists of 300 concepts. They include 219 concepts at the level of standards used at the enterprise and 81 concepts and 10078 unique terms at the level of realized projects.

The expert of FRPC JSC 'RPA 'Mars' prepared the selection involving 5017 technical documents and grouped into two main sections:

– the section based on the documentation type that consists of 52 groups (GOST 2.601, 2.602, 2.102, 2.701 3.1201);

– the section based on work sectors that consists of 28 groups (products discussed in documents).

In order to perform the experiment of quality evaluation of structuring FRPC JSC 'RPA 'Mars' electronic archive documentation, the index containing both ontological and traditional representations of technical documents (set of 'termin-frequency' pairs) was used. Further, the indices were structured with the use of different variants and subsequent quality evaluation according to the following list:

– structuring the traditional representations of technical documents with the use of Oracle Text tools;
– structuring the traditional representations of technical documents with the use of the modified FCM-algorithm of clustering;
– structuring the ontological representations of technical documents with the use of the modified FCM-algorithm of clustering;
– structuring the ontological representations of technical documents with the use of the modified FCM-algorithm of clustering with regard to the life cycle models of the designing system.

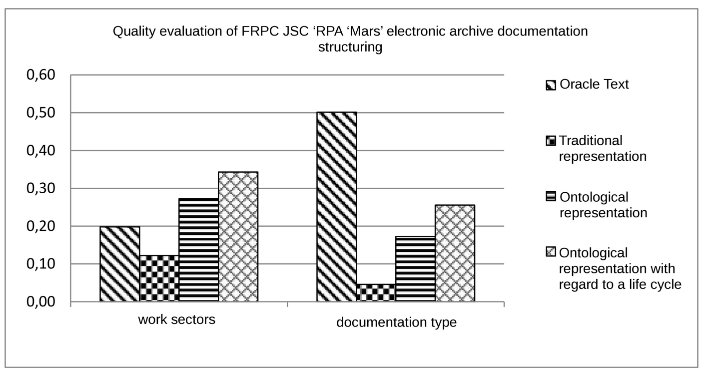

Fig. 2. Quality evaluation of FRPC JSC 'RPA 'Mars' electronic archive documentation structuring.

As indicated by Fig. 2, the most appropriate values of the evaluation function for ontological results with regard to the life cycle models of the designing system were obtained in case of structuring the technical documentation selection in work sectors as it performs structuring in individual documents content. In case of structuring according to the document type, Oracle Text outperforms the others.

The function of documentation structuring with the use of Oracle Text is based on the clustering algorithm considering a frequency of term occurrence in documents. The algorithm works well in case of structuring in accordance with the document type when Oracle Text gives the best results. The modified FCM-algorithm of clustering ontological representations of technical documents with regard to the life cycle models of the designing system provides structuring of highest quality in accordance with work sectors with regard to the content.

Conclusion

The computational experiments show that the results of structuring the ontological representations of technical documents with regard to the life cycle models of the designing system is 40% better than results structuring with the use of Oracle Text. The time spending on indexing and structuring processes of technical documentation ontological representations is, on the average, 7% less than the total time spending on indexing and structuring processes of technical documentation traditional representations. The ontological approach to indexing and structuring technical documentation makes possible structuring the electronic archive for less time. As this takes place, the most time spending is related to the process of documentation indexing.

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Maximum Dynamic Flow Finding Task with the Given Vitality Degree

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Abstract. This paper is devoted to the task of the maximum flow finding with nonzero lower flow bounds taking into account given vitality degree. Transportation network with the flow is considered in fuzzy conditions due to the fuzzy character of the network's parameters. Arcs of the network are assigned by the fuzzy arc capacities and nonzero lower flow bounds, vitality parameters and crisp transit times. All network's parameters can vary over time, therefore, it allows to consider network as dynamic one. The vitality parameter assigned to the arcs means ability of its objects to be resistant to weather conditions, traffic accidents and save and restore objects themselves, arc capacities of the network's sections in case of damage. The nonzero lower flow bounds are used to assess economic reliability of the transportation. Such methods can be applied in the real railways, roads and air roads solving the task of the optimal cargo transportation.

Keywords: Fuzzy dynamic graph, fuzzy nonzero lower flow bound, fuzzy vitality degree.

1 Introduction

The flow tasks [1] considered during the study of transportation networks are relevant due to their wide practical application, in particular, when finding the maximum amount of traffic between selected nodes on the road map, determining the routes of the optimal cost.

Important sphere of researches is dynamic networks [2-4], that take into account transit times along the arcs and don't assume instant flow distribution along the arcs. Another significant tool is considering dependence of arc capacities and lower flow bounds on flow departure time [5] and operating with fully dynamic networks instead of stationary-dynamic ones [6], using the notions of the time-expanded graphs [7-8].

Flow problems are connected with uncertainty of some kind, as changes in environment, measurement errors influence such network parameters, as arc flow bounds and vitality parameters. Therefore, we propose to consider these tasks in fuzzy conditions and we turn to the fuzzy graphs for solving such problems.

Vitality parameter [9-10] peculiar to arcs of the network usually isn't taken into account while studying networks. Its conventional definition was introduced by the authors H. Frank and I. Frisch in [11] as sensitivity of the network to damages.

However, vitality applied to the networks is ability of its objects and links among them to be resistant to weather conditions, traffic accidents and its combinations, and save and restore (fully or partially) objects themselves and their connections, arc capacities of the network's sections in case of damage. Nowadays, vitality of the network isn't taken into account, while railways and roads include the complex objects, such as stations, distillation ways, culverts, wagon, passenger and cargo managements. Sometimes network's parameters can be set qualitatively. Thus, one can set the notion "vitality degree" considering the roads and railways. In this case "vitality degree" is considered as probability of trouble-free operation of the road section and some subjective value, such as importance and reliability, etc.

Other words this paper presents method of the maximum flow finding with nonzero lower flow bounds in fuzzy dynamic network with given vitality degree.

The paper is structured as follows. In the Section 2 we give basic definitions and rules. Section 3 presents the proposed method. Section 4 provides numerical example illustrating the main steps of the proposed method. Section 5 is conclusion and future work.

2 Definitions and Rules

The proposed approach is based on the following notion of vitality.

Fuzzy directed path $\tilde{P}(x_i, x_m)$ of the graph $\tilde{G} = (X, \tilde{A})$ is a sequence of fuzzy directed arcs from the node x_i to the node x_m:

$\tilde{P}(x_i, x_m) =< \mu_{\tilde{A}} < x_i, x_j > / < x_i, x_j >>,$

$\mu_{\tilde{A}} < x_j, x_k > / < x_j, x_k >>, ..., \mu_{\tilde{A}} < x_l, x_m > / < x_l, x_m >>.$

Conjunctive durability of the path $\mu(\tilde{P}(x_i, x_m))$ is defined as

$$\mu(\tilde{P}(x_i, x_m)) = \underset{<x_\alpha, x_\beta> \in \tilde{P}(x_i, x_m)}{\&} \mu_{\tilde{A}} < x_\alpha, x_\beta > .$$

Fuzzy directed path $\tilde{P}(x_i, x_m)$ is called a simple path between vertices x_i and x_m if its part is not a path between the same vertices.

Vertex *y* is called a fuzzy accessible from the vertex x in the graph $\tilde{G} = (X, \tilde{A})$ if the fuzzy directed path from the node *x* to the node *y* exists.

The accessible degree of the node *y* from the node *x*, *(x≠y)* is defined by the following expression:

$$\gamma(x, y) = \max_{\alpha}(\mu(\tilde{P}_\alpha(x, y)), \alpha = 1, 2, ..., p,$$

where *p* is the number of various simple directed paths from vertex *x* to vertex *y*.

We consider the degree of fuzzy graph vitality as a degree of strong connection [10, 11], so it will be defined by the following expression:

$$V(\tilde{G}) = \underset{x_i \in X}{\&} \underset{x_j \in X}{\&} \gamma(x_i, x_j).$$

It means that there is a route between each pair of the graph vertices with a conjunctive strength not less than value *V*.

Let us introduce basic rules and definitions underlying this method.

Rule 1 of turning from the time-expanded fuzzy graph to the fuzzy graph without lower flow bounds [12]

Turn to the fuzzy graph $\tilde{G}_p^* = (X_p^*, \tilde{A}_p^*)$ from $\tilde{G}_p = (X_p, \tilde{A}_p)$. Introduce the artificial source s^* and sink t^* and arcs connecting the node-time pair $(t, \forall\theta \in \mathrm{T})$ and $(s, \forall\theta \in \mathrm{T})$ with $\tilde{u}^*(t, s, \forall\theta \in T, \forall\theta \in T) = \infty$, $\tilde{l}^*(t, s, \forall\theta \in T, \forall\theta \in T) = \tilde{0}$, $\tilde{v}^*(t, s, \forall\theta \in T, \forall\theta \in T) = \tilde{1}$. in the graph $\tilde{G}_p$. For arcs with $\tilde{l}(x_i, x_j, \theta, \vartheta) \neq \tilde{0}$: 1) reduce $\tilde{u}(x_i, x_j, \theta, \vartheta)$ to $\tilde{u}^*(x_i, x_j, \theta, \vartheta) = \tilde{u}(x_i, x_j, \theta, \vartheta) - \tilde{l}(x_i, x_j, \theta, \vartheta)$, $\tilde{l}(x_i, x_j, \theta, \vartheta)$ to $\tilde{0}$, $\nu^*(x_i, x_j, \theta, \vartheta) = \nu(x_i, x_j, \theta, \vartheta)$. 2) Introduce the arcs connecting s^* with (x_j, ϑ), and the arcs connecting t^* with (x_i, θ) with $\tilde{u}^*(s^*, x_j, \forall, \vartheta) = \tilde{u}^*(x_i, t, \theta, \forall) = \tilde{l}(x_i, x_j, \theta, \vartheta)$ zero lower fuzzy flow bounds $\tilde{l}^*(s^*, x_j, \forall, \vartheta) = \tilde{l}^*(x_i, t, \theta, \forall) = \tilde{0}$, $\tilde{v}^*(x_i, x_j, \theta, \vartheta) = \tilde{v}(x_i, x_j, \theta, \vartheta)$.

Definition 1 of the fuzzy residual network of the time-expanded graph.

Fuzzy residual network $\tilde{G}_p^{*\mu} = (X_p^{*\mu}, \tilde{A}_p^{*\mu})$ is the network without lower flow bounds $\tilde{G}_p^* = (X_p^*, \tilde{A}_p^*)$, which is constructed according to the following rules: if

$$\begin{cases} \tilde{\xi}^*(x_i, x_j, \theta, \vartheta) < \tilde{u}^*(x_i, x_j, \theta, \vartheta), \\ \tilde{v}^*(x_i, x_j, \theta, \vartheta) \geq \tilde{v}_{req}, \end{cases}$$

,then include the corresponding arc from $(x_i^{*\mu}, \theta)$ to $(x_j^{*\mu}, \vartheta)$ in $\tilde{G}_p^{*\mu}$ with $\tilde{u}^{*\mu}(x_i, x_j, \theta, \vartheta) = \tilde{u}^*(x_i, x_j, \theta, \vartheta) - \tilde{\xi}^*(x_i, x_j, \theta, \vartheta)$ and $\tau^{*\mu}(x_i, x_j, \theta, \vartheta) = \tau^*(x_i, x_j, \theta, \vartheta)$.

If

$$\begin{cases} \tilde{\xi}^*(x_i, x_j, \vartheta, \theta) > \tilde{0}, \\ \tilde{v}(x_i, x_j, \theta, \vartheta) \geq \tilde{v}_{req}. \end{cases}$$

Then include the corresponding arc from $(x_j^{*\mu}, \vartheta)$ to $(x_i^{*\mu}, \theta)$ in $\tilde{G}_p^{*\mu}$ with $\tilde{u}^{*\mu}(x_j, x_i, \vartheta, \theta) = \tilde{\xi}^*(x_i, x_j, \theta, \vartheta)$ and $\tau^{*\mu}(x_j, x_i, \vartheta, \theta) = -\tau^*(x_i, x_j, \theta, \vartheta)$.

Rule 2 of transition from the time-expanded fuzzy graph without lower flow bounds with the found maximum flow to the graph with the feasible flow

Turn to the graph $\tilde{G}_p$ from the graph $\tilde{G}_p^*$ as following: reject artificial nodes and arcs, connecting them with other nodes. The feasible flow vector $\tilde{\xi} = (\tilde{\xi}(x_i, x_j, \theta, \vartheta))$ of the value $\tilde{\sigma}$ is defined as: $\tilde{\xi}(x_i, x_j, \theta, \vartheta) = \tilde{\xi}^*(x_i, x_j, \theta, \vartheta) + \tilde{l}(x_i, x_j, \theta, \vartheta)$, where $\tilde{\xi}^*(x_i, x_j, \theta, \vartheta)$ – the flows, going along the arcs of the graph $\tilde{G}_p^*$ after deleting all artificial nodes and connecting arcs.

Rule 3 of the fuzzy residual network constructing with the feasible flow vector for all arcs, if $\tilde{\xi}(x_i, x_j, \vartheta, \theta) < \tilde{u}(x_i, x_j, \vartheta, \theta)$, then include the corresponding arc

(x_i^μ, θ) from the node-time pair to the node-time pair (x_j^μ, ϑ) in $\tilde{G}_p^\mu(\tilde{\xi})$ with arc capacity $\tilde{u}^\mu(x_i, x_j, \theta, \vartheta) = \tilde{u}(x_i, x_j, \theta, \vartheta) - \tilde{\xi}(x_i, x_j, \theta, \vartheta)$ and transit time $\tau^\mu(x_i, x_j, \theta, \vartheta) = \tau(x_i, x_j, \theta, \vartheta)$. For all arcs, if $\tilde{\xi}(x_i, x_j, \vartheta, \theta) > \tilde{l}(x_i, x_j, \vartheta, \theta)$, then include the corresponding arc, going from the node-time pair (x_j^μ, ϑ) to the node-time pair (x_i^μ, θ) in $\tilde{G}_p^\mu(\tilde{\xi})$ with arc capacity $\tilde{u}^\mu(x_j, x_i, \vartheta, \theta) = \tilde{\xi}(x_i, x_j, \theta, \vartheta) - \tilde{l}(x_i, x_j, \theta, \vartheta)$ and transit time $\tau^\mu(x_j, x_i, \vartheta, \theta) = -\tau(x_i, x_j, \theta, \vartheta)$.

Therefore, the proposed method of the maximum flow finding with nonzero lower flow bounds in fuzzy dynamic network consists in the maximum flow finding in the network without lower flow bounds. We turn to the time-expanded fuzzy graph and consequently to the graph without lower flow bounds for it and try to find the maximum flow in the graph. Based on the formulated rules and definitions, turn to the maximum flow finding with nonzero lower flow bounds in dynamic network in terms of partial uncertainty.

3 Presented Method of the Maximum Flow Finding Task with Nonzero Lower Flow Bounds in the Fuzzy Dynamic Network

Let us introduce the task of the maximum flow finding with nonzero lower flow bounds in dynamic network in terms of partial uncertainty and given vitality degree, represented by the model (1)-(6).

$$\textit{Maximize } \tilde{v}(p) \tag{1}$$

$$\sum_{\theta=0}^{p}\left(\sum_{x_j \in \Gamma(x_i)} \tilde{\xi}_{ij}(\theta) - \sum_{x_j \in \Gamma^{-1}(x_i)} \tilde{\xi}_{ji}(\theta - \tau_{ji}(\theta))\right) = \tilde{v}(p), x_i = s, \tag{2}$$

$$\sum_{\theta=0}^{p}\left(\sum_{x_j \in \Gamma(x_i)} \tilde{\xi}_{ij}(\theta) - \sum_{x_j \in \Gamma^{-1}(x_i)} \tilde{\xi}_{ji}(\theta - \tau_{ji}(\theta))\right) = \tilde{0}, x_i \neq s, t; \theta \in T, \tag{3}$$

$$\sum_{\theta=0}^{p}\left(\sum_{x_j \in \Gamma(x_i)} \tilde{\xi}_{ij}(\theta) - \sum_{x_j \in \Gamma^{-1}(x_i)} \tilde{\xi}_{ji}(\theta - \tau_{ji}(\theta))\right) = -\tilde{v}(p), x_i = t, \tag{4}$$

$$\tilde{l}_{ij}(\theta) \le \tilde{\xi}_{ij}(\theta) \le \tilde{u}_{ij}(\theta), \theta + \tau_{ij}(\theta) \le p, \theta \in T, \tag{5}$$

$$\tilde{v}_{ij}(\theta) \ge \tilde{v}_{req}, s(\theta) + \tau_{st}(\theta) \le p, \theta \in T. \tag{6}$$

Step 1. Go to the time-expanded fuzzy static graph $\tilde{G}_p$ from the given fuzzy dynamic graph $\tilde{G}$.

Step 2. Turn to the graph $\tilde{G}_p^* = (X_p^*, \tilde{A}_p^*)$ according to the *rule 1*.

Step 3. Build a fuzzy residual network $\tilde{G}_p^{*\mu}$ due to the ***definition 1.***

Step 4. Search the augmenting shortest path (in terms of the number of arcs) $\tilde{P}_p^{*\mu}$ from the artificial source s^* to the artificial sink t^* in the constructed fuzzy residual network according to the breadth-first-search.

4.1 Go to the step 5 if the augmenting path $\tilde{P}_p^{*\mu}$ is found.

4.2 The flow value $\tilde{\phi}^* < \sum\limits_{\tilde{l}(x_j,x_i,\vartheta,\theta)\neq\tilde{0}} \tilde{l}(x_i,x_j,\theta,\vartheta)$ is obtained, which is the maximum flow in $\tilde{G}_p^*$, if the path is failed to find. Exit.

Step 5. Pass the minimum from the arc capacities $\tilde{\delta}_p^{*\mu} = \min[\tilde{u}(\tilde{P}_p^{*\mu})]$, $\tilde{u}(\tilde{P}_p^{*\mu}) = \min[\tilde{u}^{*\mu}(x_i,x_j,\theta,\vartheta)$, $(x_i,\theta),(x_j,\vartheta) \in \tilde{P}_p^{*\mu}$ along this path $\tilde{P}_p^{*\mu}$.

Step 6. Update the fuzzy flow values in the graph $\tilde{G}_p^*$: replace the fuzzy flow $\tilde{\xi}^*(x_j,x_i,\theta,\vartheta)$ along the corresponding arcs going from (x_j^*,θ) to (x_i^*,ϑ) from $\tilde{G}_p^*$ by $\tilde{\xi}^*(x_j,x_i,\theta,\vartheta) - \tilde{\delta}_p^{*\mu}$ for arcs connecting node-time pair $(x_i^{*\mu},\vartheta)$ with $(x_j^{*\mu},\theta)$ in $\tilde{G}_p^{*\mu}$, such as $((x_i^{*\mu},\vartheta),(x_j^{*\mu},\theta)) \notin \tilde{A}_p^*$, $((x_i^{*\mu},\vartheta),(x_j^{*\mu},\theta)) \in \tilde{A}_p^{*\mu}$ and replace the fuzzy flow $\tilde{\xi}^*(x_i,x_j,\theta,\vartheta)$ along the arcs going from (x_i^*,θ) to (x_j^*,ϑ) from $\tilde{G}_p^*$ by $\tilde{\xi}^*(x_i,x_j,\theta,\vartheta) + \tilde{\delta}_p^{*\mu}$ for arcs connecting node-time pair $(x_i^{*\mu},\theta)$ with $(x_j^{*\mu},\vartheta)$ in $\tilde{G}_p^{*\mu}$, such as $((x_i^{*\mu},\theta),(x_j^{*\mu},\vartheta)) \in \tilde{A}_p^*$, $((x_i^{*\mu},\theta),(x_j^{*\mu},\vartheta)) \in \tilde{A}_p^{*\mu}$. Replace $\tilde{\xi}^*(x_i,x_j,\theta,\vartheta)$ by $\tilde{\xi}^*(x_i,x_j,\theta,\vartheta) + \tilde{\delta}_p^{*\mu}\tilde{P}_p^{*\mu}$.

Step 7. Compare flow value $\tilde{\xi}^*(x_i,x_j,\theta,\vartheta) + \tilde{\delta}_p^{*\mu}\tilde{P}_p^{*\mu}$ and $\sum\limits_{\tilde{l}(x_j,x_i,\vartheta,\theta)\neq\tilde{0}} \tilde{l}(x_i,x_j,\theta,\vartheta)$:

7.1. If the flow value $\tilde{\xi}^*(x_i,x_j,\theta,\vartheta) + \tilde{\delta}_p^{*\mu}\tilde{P}_p^{*\mu}$ is less than $\sum\limits_{\tilde{l}(x_j,x_i,\vartheta,\theta)\neq\tilde{0}} \tilde{l}(x_i,x_j,\theta,\vartheta)$, go to the **step 3**.

7.2. If the flow value $\tilde{\xi}^*(x_i,x_j,\theta,\vartheta) + \tilde{\delta}_p^{*\mu}\tilde{P}_p^{*\mu}$ is equal to $\sum\limits_{\tilde{l}(x_j,x_i,\vartheta,\theta)\neq\tilde{0}} \tilde{l}(x_i,x_j,\theta,\vartheta)$, turn to the graph $\tilde{G}_p$ from the graph $\tilde{G}_p^*$ according to the *rule 2*. Go to the **step 8**.

Step 8. Construct the residual network $\tilde{G}_p^{\mu}(\tilde{\xi})$ according to the rule 3.

Step 9. Define the shortest path $\tilde{P}_p^{\mu}$ $\tilde{G}_p^{\mu}(\tilde{\xi})$.

(I) Go to the **step 10** if the augmenting path $\tilde{P}_p^{\mu}$ is found.

(II) The maximum flow $\tilde{\xi}(x_i,x_j,\theta,\vartheta) + \tilde{\delta}_p^{\mu}\tilde{P}_p^{\mu} = \tilde{v}(p)$ in $\tilde{G}_p(\tilde{\xi})$ is found if the path is failed to find, then the maximum flow in "time-expanded" static fuzzy graph can be found at the **step 12**.

Step 10. Pass the flow value $\tilde{\delta}_p^{\mu} = \min[\tilde{u}(\tilde{P}_p^{\mu})]$, $\tilde{u}(\tilde{P}_p^{\mu}) = \min[\tilde{u}^{\mu}(x_i, x_j, \theta, \vartheta)$, $(x_i, \theta), (x_j, \vartheta) \in \tilde{P}_p^{\mu}$ along the found path.

Step 11. Update the flow values in the graph $\tilde{G}_p(\tilde{\xi})$.

Step 12. Turn to the initial dynamic graph $\tilde{G}$ as follows: reject the artificial nodes s', t' and arcs, connecting them with other nods.

4 Numerical Example

Let us describe the proposed algorithm. For example, assume that the original fuzzy dynamic network is shown in Fig. 1. It is necessary to find the maximum flow in the initial dynamic graph with the given vitality degree no less than $0,\tilde{7}$ and represent the result in the form of the triangular number.

Fuzzy upper flow bounds $\tilde{u}_{ij}$, depending on the flow departure time θ are shown in the Table I. Fuzzy lower flow bounds $\tilde{l}_{ij}$, depending on the flow departure time θ are shown in the Table II. Time parameters τ_{ij} depending on the flow departure time θ are shown in the Table III. Fuzzy vitality parameters $\tilde{v}_{ij}$, depending on the flow departure time θ are shown in the Table IV.

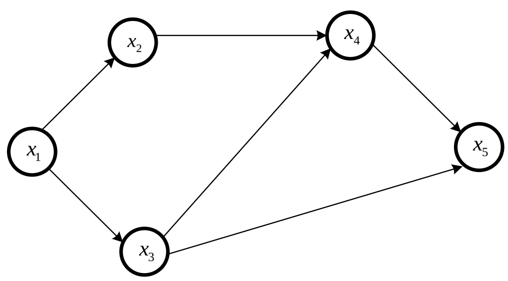

Fig. 1. Initial dynamic graph $\tilde{G}$

Construct time-expanded graph, as shown in Fig. 2.

Turn to the graph without lower flow bounds and find the augmenting paths for the graph in Fig. 3: $\tilde{P}_1^{*\mu} = s^*, (x_5, 2), (x_1, 0), t^*$ with $\tilde{7}$ flow units, $\tilde{P}_2^{*\mu} = s^*, (x_2, 1), (x_3, 2), (x_5, 3), (x_1, 0), t^*$ with $\tilde{3}$ flow units, $\tilde{P}_3^{*\mu} = s^*, (x_2, 1), (x_3, 2), (x_5, 3), (x_1, 0), (x_4, 1), t^*$ with $\tilde{7}$ flow units.

We obtain graph with the maximum flow in Fig. 4. Therefore, the task has a solution and we turn to the initial time-expanded graph with the feasible flow in Fig.

5. Finding the augmenting paths and pushing the flows among them, we obtain graph with the maximum flow in Fig. 6.

TABLE I. FUZZY UPPER FLOW BOUNDS $\tilde{u}_{ij}$, DEPENDING ON THE FLOW DEPARTURE TIME θ

Arcs of the graph	**Fuzzy upper flow bounds $\tilde{u}_{ij}$ at the time periods θ, time units.**			
	0	***1***	***2***	***3***
(x_1, x_2)	$\tilde{25}$	$\tilde{20}$	$\tilde{25}$	$\tilde{40}$
(x_1, x_4)	$\tilde{10}$	$\tilde{20}$	$\tilde{25}$	$\tilde{25}$
(x_1, x_5)	$\tilde{18}$	$\tilde{18}$	$\tilde{30}$	$\tilde{35}$
(x_2, x_3)	$\tilde{35}$	$\tilde{30}$	$\tilde{35}$	$\tilde{18}$
(x_3, x_4)	$\tilde{15}$	$\tilde{27}$	$\tilde{33}$	$\tilde{25}$
(x_3, x_5)	$\tilde{55}$	$\tilde{45}$	$\tilde{40}$	$\tilde{55}$
(x_4, x_5)	$\tilde{20}$	$\tilde{20}$	$\tilde{18}$	$\tilde{28}$

TABLE II. FUZZY LOWER FLOW BOUNDS $\tilde{l}_{ij}$, DEPENDING ON THE FLOW DEPARTURE TIME θ.

Arcs of the graph	**Fuzzy lower flow bounds $\tilde{l}_{ij}$ at the time periods θ, time units.**			
	0	***1***	***2***	***3***
(x_1, x_2)	$\tilde{10}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
(x_1, x_4)	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
(x_1, x_5)	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{20}$
(x_2, x_3)	$\tilde{6}$	$\tilde{0}$	$\tilde{15}$	$\tilde{0}$
(x_3, x_4)	$\tilde{0}$	$\tilde{8}$	$\tilde{0}$	$\tilde{0}$
(x_3, x_5)	$\tilde{25}$	$\tilde{15}$	$\tilde{0}$	$\tilde{0}$
(x_4, x_5)	$\tilde{0}$	$\tilde{5}$	$\tilde{0}$	$\tilde{10}$

TABLE III. TIME PARAMETERS τ_{ij} DEPENDING ON THE FLOW DEPARTURE TIME θ

Arcs of the graph	**Time parameters τ_{ij} at time periods θ, time units.**			
	0	***1***	***2***	***3***
(x_1, x_2)	1	1	1	2
(x_1, x_4)	1	3	2	2
(x_1, x_5)	4	4	1	1
(x_2, x_3)	4	1	1	1
(x_3, x_4)	1	1	2	2
(x_3, x_5)	2	2	1	1
(x_4, x_5)	5	4	1	3

TABLE IV. FUZZY VITALITY PARAMETERS v_{ij}, DEPENDING ON THE FLOW DEPARTURE TIME θ

Arcs of the graph	**Fuzzy vitality parameters $\tilde{v}_{ij}$ at time periods θ, vitality units**			
	0	***1***	***2***	***3***
(x_1, x_2)	$0,\tilde{8}$	$0,\tilde{4}$	$0,\tilde{6}$	$0,\tilde{5}$
(x_1, x_4)	$0,\tilde{7}$	$0,\tilde{2}$	$0,\tilde{8}$	$0,\tilde{9}$
(x_1, x_5)	$0,\tilde{4}$	$0,\tilde{8}$	$0,\tilde{6}$	$0,\tilde{3}$
(x_2, x_3)	$0,\tilde{7}$	$0,\tilde{8}$	$0,\tilde{7}$	$0,\tilde{4}$
(x_3, x_4)	$0,\tilde{7}$	$0,\tilde{9}$	$0,\tilde{7}$	$0,\tilde{6}$
(x_3, x_5)	$0,\tilde{3}$	$0,\tilde{4}$	$0,\tilde{7}$	$0,\tilde{4}$
(x_4, x_5)	$0,\tilde{8}$	$0,\tilde{3}$	$0,\tilde{3}$	$0,\tilde{4}$

The maximum flow in the initial graph with the vitality degree no less than $0,\tilde{7}$ is $2\tilde{5}+1\tilde{0}=3\tilde{5}$ flow units.

Let us define deviation borders of the obtained fuzzy number "near $3\tilde{5}$".

Since the calculations with fuzzy numbers are cumbersome and result in strong blurring of the resulting number's borders, we suggest to operate fuzzy numbers according to the method, described in [8]. In this case we will operate the central values of fuzzy numbers, blurring the result at the final step and presenting it as a triangular the number.

Therefore, deviation borders of the obtained fuzzy number "near $3\tilde{5}$" corresponded to the maximum flow in the graph $\tilde{G}$ are calculated according to the basic values of arc capacities in Fig. 7.

The detected result is between two adjacent basic values of the arc capacities: $3\tilde{1}$ with the left deviation $l_1^L = 8$, right deviation – $l_1^R = 7$ and $4\tilde{4}$ with the left deviation $l_2^L = 9$, right deviation $-l_2^R = 10$. We obtain deviations : $l_1^L \approx 8$, $l_1^R \approx 7$.

Therefore, the maximum flow in the fuzzy dynamic graph with the given vitality degree no less than $0,\tilde{7}$ can be represented by fuzzy triangular number (27, 35,42) units.

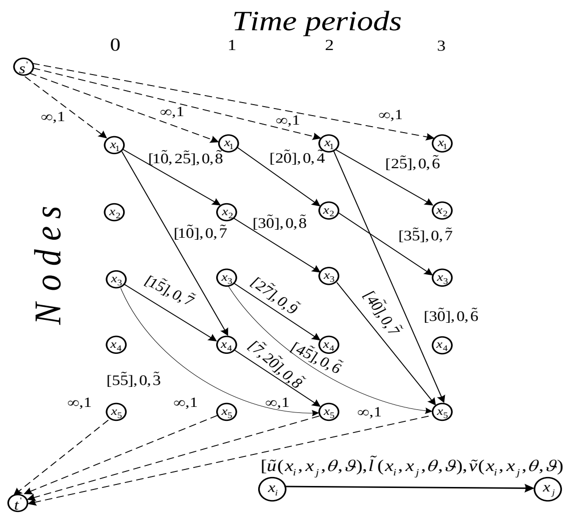

Fig. 2. Time-expanded graph $\tilde{G}_p$

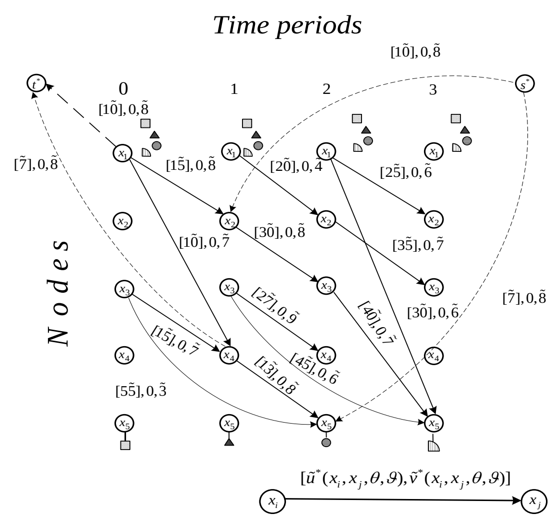

Fig. 3. $\tilde{G}_p^*$ – Time-expanded graph without lower flow bounds $\tilde{G}$

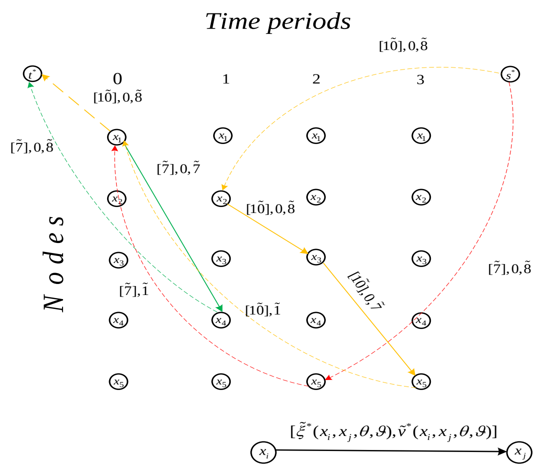

Fig. 4. Graph $\tilde{G}_p^*$ with the maximum flow

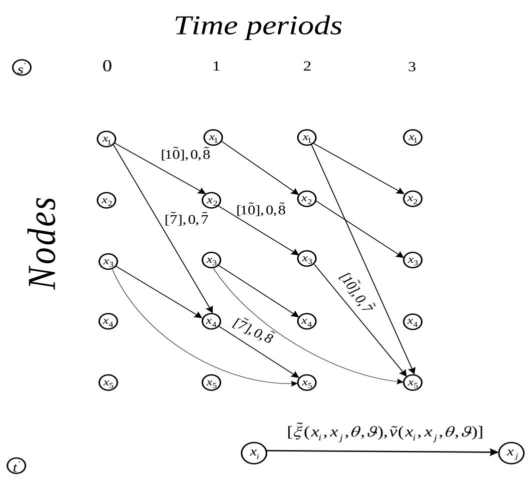

Fig. 5. Graph $\tilde{G}_p$ with the feasible flow

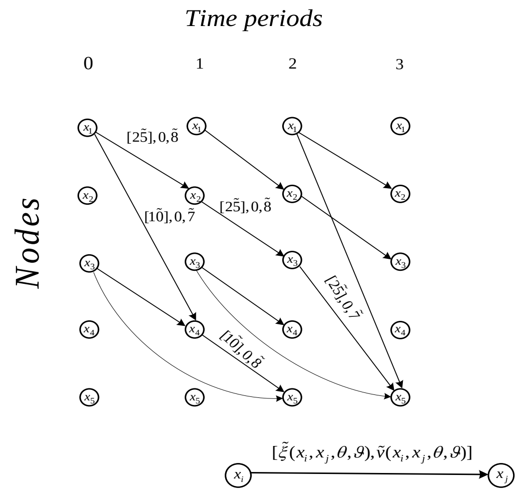

Fig. 6. Graph $\tilde{G}_p$ with the maximum flow

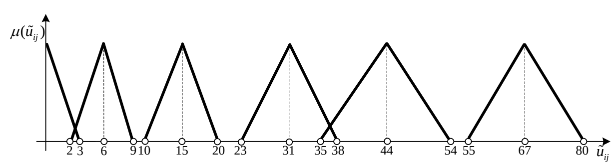

Fig. 7. Membership functions of the basic values of arc capacities of the network $\tilde{G}$

5 Conclusion and Future Work

Paper presents proposed algorithm of the maximum flow finding with nonzero lower flow bounds and vitality degrees in the fuzzy dynamic network with the required vitality degree based on the formulated definitions and rules. The considered network is represented as fuzzy graph with parameters, depending on the flow departure time and varying over time. Given lower flow bounds are used for assessing economic reliability of transportation. Given vitality degree reflects ability of its objects to be resistant to weather conditions, traffic accidents and save and restore objects themselves, arc capacities of the network's sections in case of damage. The proposed method has important practical value in transportation implementing on the real types of roads. In the future works we will propose methods of increasing the vitality degree in fuzzy dynamic networks.

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Clustering Techniques Versus Binary Thresholding for Detection of Signal Tracks in Ionograms

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Abstract. An ionogram is a display of the data produced by an ionosonde. It is a graph of the virtual height of the ionosphere plotted against frequency. In addition to "useful signal", an ionogram almost always contains noise of different nature, a so called background noise. That is why the signal filtering task becomes so important. There are two groups of methods to this end. The first group features methods of computer vision for image processing, namely, different filters and image binarization. The second group includes adapted clustering methods. In this paper, we show how several methods work for filtering "useful signal" from noise and emissions.

Keywords: ionograms, image filtering, image processing, similarity measures

1 Introduction

The data of radio sounding is necessary for enhancement of over-the-horizon radar systems, systems of shortwave communication, as well as for solution or many problems in radiophysics and geophysics [1].

Usually, the results obtained by an ionosonde are represented by means of ionograms[2]. An ionogram of oblique radio sounding of the ionosphere shows a dependence of the amplitude of the received signal from the frequency f of soudning and the group delay time τ[3].

Due to multipath shortwave propagation in the ionosphere, an ionogram contains tracks of different signal modes. In addition to the useful signal, there is a noise of different nature in ionogram images. In Fig. 1, one can see the mode of the signal's track (a sloped body in the bottom left part of the ionogram), background noise, and concentrated noise, i.e. vertical stripes[1].

When we work with ionograms one of the most important problem is to filter the useful signal from the noise. There are several types of useful signals. In fact,

[1] The data of ionograms shown in the paper are available at `https://drive.google.com/open?id=0Bxdto9RRxaqMY2pCYUI4eWR0T1U`. More comprehensive datasets are available from the second co-author by request.

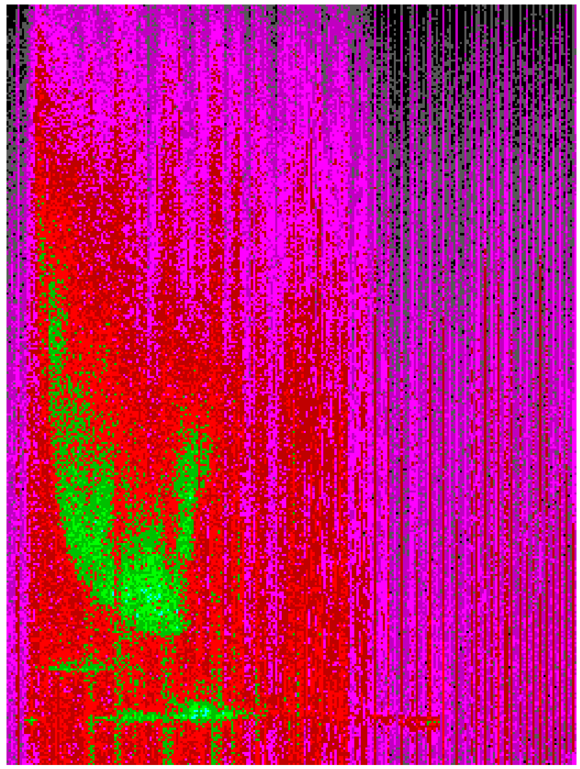

Fig. 1. Ionogram example

we have a problem similar to automatic classification or clusterization depending on the availability of training (labeled) data.

The rest of the paper is organised as follows. In Section 2, we consider signal segmentation using image processing methods. In Section 3, we use machine learning methods for the same purposes. We treat an input image as a dataset with each pixel as a separate element and then cluster it. In Section 4, we try to exploit the best of these methods to create our final algorithm. In the conclusion, we discuss shortly relevant techniques and problems for future work.

We should note that when we tested our methods, we tried several configurations for our models (sometimes enumerating parameters' values by grid search). Of course, there may be better configurations of parameters in a particular case.

2 Detection of signal tracks by image processing methods

In this approach, we consider an ionogram as an image. We need to filter out the noise and isolate the signal track of an input ionogram. We have tested two filters for image filtering: the median filter and the filter given by the matrix below.

$$Ker = \begin{pmatrix} 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}$$

In the next example, we show the original image and the results of application of two filters to the image and its binarization by thresholding.

Image binarization is the way to define the class of each pixel as signal/background by thresholding. That is we set the threshold value of brightness and apply it to all the pixels; the pixels with brightness higher than this threshold belong to the first class, and the remaining ones belong to the second. In Fig. 2, the images of the original ionogram are shown in three color model. And, in the remaining figures, for illustration we use only one color model.

It is clear that filtering with Ker matrix is able to better keep signal's shape and eliminate the noise in comparison with the median filter.

3 Detection of signal tracks by machine learning methods

Another approach is based on the ionogram representation in form of triples $\langle x, y, V \rangle$ for each original pixel, where x and y are pixel's coordinates and V is the value of the pixel brightness. After such transformaiton we try to do clusterization. We hypothesise that signal's pixels should belong to a separate cluster. This approach is similar to the well-known image segmentation methods that one can find, for example, in this book [4].

After clustering we again represent the results as an image. We replace the value of brightness of each input pixel by its cluster label. These three methods from scikit-learn machine learning environment [5] have been applied:

1. K-Means
2. DBscan [6]
3. Mean shift [7]

The last two methods have been chosen since they do not need to know the number of clusters in advance; moreover, according to locality hypotheis they can capture both similarity in signal/noise values and spatial closeness in axes x-y (in fact, f-τ).

Dbscan have worked rather good visually. Main disadvantage of this method is a necessity to configure its parameters separately for each image. In Fig. 4,

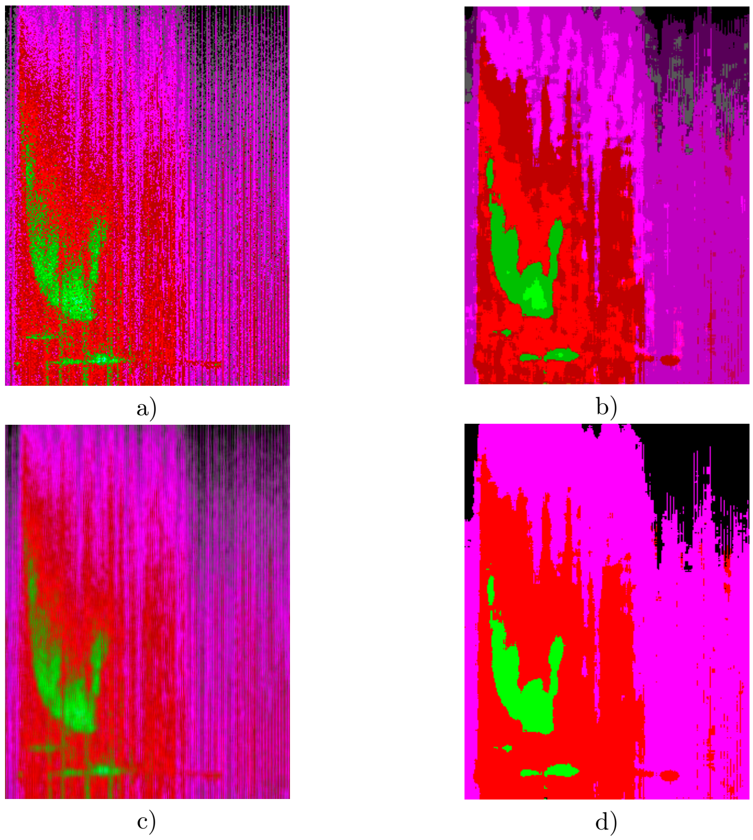

Fig. 2. Ionograms: a) the original image, b) preprocessing by the median filter, c) filtering with matrix $Kers$, d) binarization

you can find the results of processing of the original ionogram given in Fig. 3 by DBscan with $\varepsilon = 4$ (the neighbourhood size), $N = 100$ (the number of points within the neighbourhood).

Coordinates are scaled in the way below:

$$x_{new} = \frac{x_{old}}{\max(x_{old})}, \ y_{new} = \frac{y_{old}}{\max(y_{old})} \tag{1}$$

Next example launched with $\varepsilon = 1$, $N = 50$ and with following coordinate transformation:

$$x_{new} = \frac{x_{old}}{\max(x_{old})} \cdot 10, \ y_{new} = \frac{y_{old}}{\max(y_{old})} \cdot 10 \tag{2}$$

In the figures above, machine learning methods have been applied to the original image. However, we should note that we get better results if we first applied filtering and then clustering.

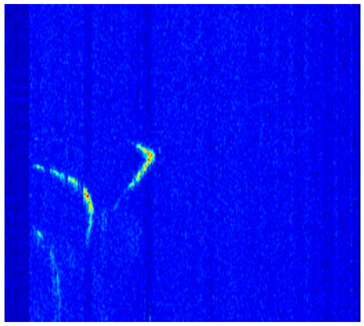

Fig. 3. Original image

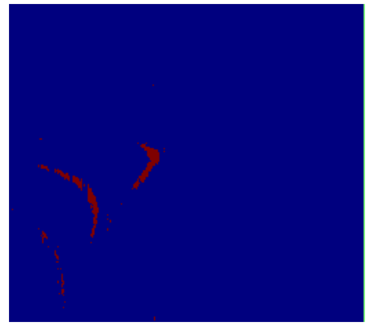

Fig. 4. DBscan results

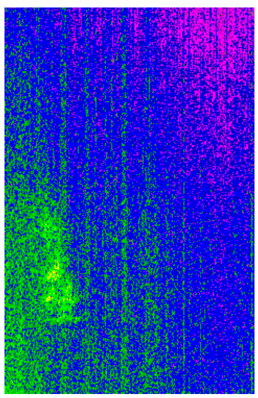

Fig. 5. Original image

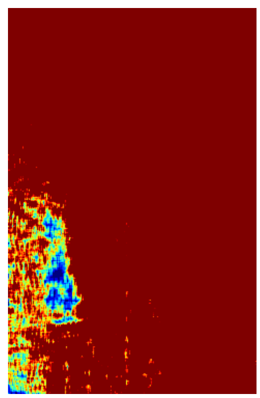

Fig. 6. Filtered image

Fig. 7. Mean shift results

It turns out that the most appropriate method for this task is MEAN SHIFT, applied after image filtering. The Python implementation of MEAN SHIFT allows us to choose the Parzen's window size automatically for each image. It depends on distance between objects; we have used 70th percentile of all pairwise distances. This property of MEAN SHIFT is much more suitable in comparison to DBSCAN since DBSCAN needs individual options for each image. Another advantage of MEAN SHIFT is its speed. Here we have also used coordinates transformation from Eq. 2.

4 Conclusion

This paper presents the first steps of comparison of image processing and machine learning techniques for signal detection in ionograms. Both groups of methods are suitable for noise filtering and isolation of the original (important) signal. We have compared several methods of computer vision and machine learning for this problem. It seems that Mean shift works better than its two competitors in the conducted comparison. In the future we plan to apply deep learning methods for better signal detection based on a large set of ionograms. The usage of autoencoder for automatic clustering of signal types is an attractive opportunity as well. Other image segmentation techniques that are widely used in computer vision community are highly relevant as well.

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Query Answering over Fact Bases for Fuzzy Interval Ontologies

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Abstract. Temporal ontologies contain events that are concepts and roles with references to temporal intervals. Therefore, a temporal ontology induces the interval ontology. We consider fuzzy interval ontologies written in a fuzzy Boolean extension of Allen's interval logic. Syntactically, the extended logic **ELA** is the set of all Boolean combinations of propositional variables and sentences of Allen's interval logic. Semantics of **ELA** is defined using fuzzy interpretations of propositional variables and atomic sentences of Allen's logic. An interval ontology in **ELA** is a finite set **ELA** sentences (formulas). A fact is an estimate of a formula i.e. an expression of the form $r \le \varphi \le s$ where φ is a **ELA** formula and $0 \le r \le s \le 1$. A fact base for an interval ontology is a finite set of facts with formulas from the ontology. We present a method of finding answers to queries addressed to fact bases for fuzzy interval ontologies. The method uses analytical tableaux.

Keywords: knowledge representation, ontologies, fuzzy ontologies, temporal logics, Allen's interval logic, query answering

1 Introduction

Temporal ontologies contain events that are concepts and roles with references to temporal intervals. Therefore, a temporal ontology induces the interval ontology. Consider an example.

Example 1. Suppose, we should define the structure of the concept *Agent* in some ontology for a multi-agent system. Then we may write declarations such as

Agent[*Name*: *String*, *Carry_out*: *Action*(*)],

Action[*Name*: *String*, *Interval*: (*Integer,Integer*), *Procedure*: *Program*].

The terms *Agent*(*Name*=*rob*07) and *Agent*(*Name*=*rob*07)**.***Carry_out***.***Interval* denote the robot *rob*07 and the temporal intervals of the actions carrying out by *rob*07. Let the robot *rob*07 is able to carry out the actions *a*, *b* and *c*, i.e. *Agent*(*Name*=*rob*07)**.***Carry_out* = {*a*, *b*, *c*}. These actions spend certain time. Thus, temporal intervals *A, B* and *C* are associated with the actions.

Suppose, there is the following knowledge about the intervals:

(1) If *p* is true then there is no time point at which both actions *a* and *b* are carried out;

(2) If q is true then the action b is carried out only when the action c is carried out. Consider the question:

(3) What Allen's relations are impossible between the C and A if both conditions p and q take place?

In Allen's interval logic (see [1, 2]) with implication, the statements (1) and (2) can be written as the interval ontology $\boldsymbol{O} = \{p \rightarrow A\ \boldsymbol{bb^*}B,\ q \rightarrow B\ \boldsymbol{edfs}\ C\}$(see further). The query (3) is written as $?x - p \wedge q \rightarrow C\, {-x}\, A$.

(*End of Example 1.*)

In Allen's interval logic **LA**, there are 7 basic relations between intervals: ***e*** (equals), ***b*** (before), ***m*** (meets), ***o*** (overlaps), ***f*** (finishes), ***s*** (starts), ***d*** (during). (See Table 1 for interpretation of these relations, where A^{-} and A^{+} denote the left and the right ends of the interval A). Let tr($A\ \theta\ B$) be the set of inequalities characterized of the basic Allen's relation θ (see the third column of Table 1). For example, tr($A\ \boldsymbol{f}\ B$) = $\{A^{-} > B^{-}, A^{+} \geq B^{+}, B^{+} \geq A^{+}\}$.

Table 1. Basic relations of Allen's interval logic

Interval relation	Illustration	Inequalities for the ends of intervals
$A\ \boldsymbol{b}\ B$	|===A===| |===B===|	$B^{-} > A^{+}$
$A\ \boldsymbol{m}\ B$	|===A===|=====B=====|	$A^{+} \geq B^{-},\ B^{-} \geq A^{+}$
$A\ \boldsymbol{o}\ B$	|===A===| |=====B=====|	$B^{-} > A^{-}, A^{+} > B^{-}, B^{+} > A^{+}$
$A\ \boldsymbol{d}\ B$	|===A===| |=====B=====|	$A^{-} > B^{-}, B^{+} > A^{+}$
$A\ \boldsymbol{s}\ B$	|===A===| |=====B=====|	$A^{-} \geq B^{-}, B^{-} \geq A^{-}, B^{+} > A^{+}$
$A\ \boldsymbol{f}\ B$	|===A===| |=====B=====|	$A^{-} > B^{-}, A^{+} \geq B^{+}, B^{+} \geq A^{+}$
$A\ \boldsymbol{e}\ B$	|=====A=====| |=====B=====|	$A^{-} \geq B^{-},\ B^{-} \geq A^{-}, A^{+} \geq B^{+}, A^{+} \geq B^{+}$

The inverted relations are marked by asterisks: ***b**** (after), ***m**** (met-by), ***o**** (overlapped-by), ***f**** (finished-by), ***s**** (started-by), ***d**** (contains); so, $A\ \alpha^* B \Leftrightarrow B\ \alpha\ A$.

Let $\Omega_0 = \{\boldsymbol{e}, \boldsymbol{b}, \boldsymbol{m}, \boldsymbol{o}, \boldsymbol{f}, \boldsymbol{s}, \boldsymbol{d}\}$ and $\Omega = \Omega_0 \cup \{\boldsymbol{b^*}, \boldsymbol{m^*}, \boldsymbol{o^*}, \boldsymbol{f^*}, \boldsymbol{s^*}, \boldsymbol{d^*}\} = \{\boldsymbol{e}, \boldsymbol{b}, \boldsymbol{m}, \boldsymbol{o}, \boldsymbol{f}, \boldsymbol{s}, \boldsymbol{d}, \boldsymbol{b^*}, \boldsymbol{m^*}, \boldsymbol{o^*}, \boldsymbol{f^*}, \boldsymbol{s^*}, \boldsymbol{d^*}\}$.

A *sentence* (*formula*) of **LA** is an expression of the form $A\ \omega\ B$ where ω is any subset of the set Ω and A, B are interval variables. If $\omega = \{\alpha\}$, then instead of $A\{\alpha\}B$ we write simply $A\ \alpha\ B$. If $\omega = \{\alpha_1, \alpha_2, \ldots, \alpha_k\}$ then we write $A\ \alpha_1\alpha_2\ldots\alpha_k\ B$ instead of $A\{\alpha_1, \alpha_2, \ldots, \alpha_k\}B$. By definition, the formula $A\ \alpha_1\alpha_2\ldots\alpha_k\ B$ is true if it is true at least one formula $A\ \alpha_i\ B$ ($1 \le i \le k$). The sentences of the form $A\ \alpha\ B$ with $\alpha \in \Omega_0$ are called *atomic*.

The fuzzy Boolean extension **ELA** Allen's interval logic is defined as follows.

SYNTAX of **ELA**:

• propositional variables are **ELA** formulas;

• every **LA** formula belongs to **ELA**, i.e. **LA** ⊆ **ELA**;

• if φ and ψ are **ELA** formulas then $\sim\varphi$, $\varphi \wedge \psi$ and $\varphi \vee \psi$ are also **ELA** formulas, and

$\varphi \to \psi$ is **ELA** formula considered as shorthand for $\sim\varphi \vee \psi$.

An (*interval*) *ontology* is a finite set of **ELA** formulas. Let P(***O***) be the set of all propositional variables entering the formulas from ***O***, and A(***O***) be the set of all atomic sentences entering the formulas from ***O***. Let B(***O***) = ∪{tr(β) | β ∈ A(***O***)}. For example, if ***O*** = {A ***o*** $B \to (B$ ***mf*** $C) \wedge p$, $q \to A$ ***s*** C, C ***o****A} then P(***O***) = {p, q} and A(***O***) = {B ***m*** C, B ***f*** C, A ***s*** C, A ***o*** C}, and B(***O***) = {$B^+ \ge C^-$, $C^- \ge B^+$, $B^- > C^-$, $B^+ \ge C^+$, $C^+ \ge B^+$, $A^- \ge C^-$, $C^- \ge A^-$, $C^+ > A^+$, $C^- > A^-$, $A^+ > C^-$, $C^+ > A^+$}.

SEMANTICS of **ELA** is defined using fuzzy interpretations.

A *fuzzy interpretation* of an ontology ***O*** is any function "..." from P(***O***) ∪ B(***O***) to $[0,1] = \{x \mid 0 \le x \le 1\}$ with the following constraints:

(a) If $X < Y$ and $Y \le X$ belong to B(***O***) then "$X < Y$" + "$Y \le X$" = 1;

(b) If $X = Y$, $X < Y$ and $Y < X$ belong to B(***O***) then "$X{=}Y$"+ max{"$X < Y$", "$Y < X$"}=1;

(c) If $X < Y$, $Y < Z$ and $X < Z$ belong to B(***O***) then "$X < Z$" ≥ min{"$X < Y$" ,"$Y < Z$"}, and the similar constraints which are obtained by replacing signs "<" by signs "≤" or "=".

We expand the function "..." to A(***O***) by "$A\ \theta\ B$"= min{"V" | $V \in$ tr($A\ \theta\ B$)}. Further, we expand "..." to formulas by the usual rules of Zadeh's fuzzy logic: "~ φ" = 1 –"φ", "φ ∧ ψ" = min{"φ","ψ"}, "φ ∨ ψ" = max{"φ","ψ"} [3].

Let r and s be numbers from [0,1] and φ be a **ELA** formula. Expressions of the forms $\varphi > r$, $\varphi \ge r$, $\varphi < r$ and $\varphi \le r$ are called *estimates* of the formula φ, and expressions of the form $r \le \varphi \le s$ (where $0 \le r \le s \le 1$) are called *bilateral estimates* of φ. The estimates are interpreted naturally. Let "..." be any interpretation of the ontology {φ}. Then "$\varphi > r$" $\Leftrightarrow_{\text{df}}$ "φ" $> r$, "$\varphi \ge r$" $\Leftrightarrow_{\text{df}}$ "φ" $\ge r$, "$\varphi < r$" $\Leftrightarrow_{\text{df}}$ "φ" $< r$, "$\varphi \le r$" $\Leftrightarrow_{\text{df}}$ "φ" $\le r$, "$r \le \varphi \le s$" $\Leftrightarrow_{\text{df}}$ $r \le$ "φ" $\le s$.

The set **EST** of all estimates for **ELA** formulas can be considered as a crisp logic with fuzzy interpretations. As every logic, **EST** has the relation '|=" of *logical consequence*. Let ***E*** ⊆ **EST** and σ ∈ **EST.** We state ***E*** |= σ when there is no fuzzy interpretation "...": ***E*** → [0,1] such that all estimates from ***E*** are true but the estimate σ is false.

We consider estimates with the relation "≤" as *facts*. For any interval ontology ***O*** = {$\varphi_1, \varphi_2, \dots, \varphi_n$} ($\varphi_i \in$ **ELA**), any set ***Fb*** = {$r_1 \le \varphi_1 \le s_1$, $r_2 \le \varphi_2 \le s_2, \dots, r_n \le \varphi_n \le s_n$} ($0 \le r_i, s_i \le 1$) of bilateral estimates is called a *fact base* for the ontology ***O***.

We can query a fact base and get the appropriate answers. Let $\psi = \psi[x_1, x_2, \dots, x_n]$ be an ELA formula in which some of its Allen's connectives are replaced with variables $x_1, x_2, \dots, x_n$ whose values are in Ω. A *query* is an expression of the form

$$? \ (x_1, x_2, \dots, x_m) - \psi[x_1, x_2, \dots, x_m], \quad (1.1)$$

where $\psi = \psi[x_1, x_2,\ldots, x_n]$ is an **ELA** formula in which some of its Allen's connectives are replaced with variables $x_1, x_2,\ldots, x_n$ whose values are in Ω. (For example, the expression $?(x_1, x_2) - (p \vee A$ ***bs*** $B) \rightarrow B\ x_1$***od*** $C \wedge \sim A\ x_2\ D$ is a query.)

The *answer* to query (1.1), addressed to the fact base ***Kb,*** is the set of all tuples $(g, h; \alpha_1, \alpha_2,\ldots, \alpha_m)$ with $\alpha_i \in \Omega$ and $g, h \in [0,1]$ such that ***Kb*** $\models g \leq \psi[\alpha_1, \alpha_2,\ldots, \alpha_m] \leq h$ with maximal g and minimal h. So, we have $g = \max\{r \mid$ ***Kb*** $\models r \leq \psi[\alpha_1, \alpha_2,\ldots, \alpha_m]\}$ and $g = \min\{s \mid$ ***Kb*** $\models \psi[\alpha_1, \alpha_2,\ldots, \alpha_m] \leq s\}$.

Remarks. 1) It is easy to prove that the maximum and the minimum exist. 2) Since "$\varphi \leq r$" $\Leftrightarrow$ "φ" $\leq r \Leftrightarrow 1-$"$\varphi$" $\geq 1 - r \Leftrightarrow$ "$\sim \varphi$" $\geq r \Leftrightarrow$ "$\sim \varphi \geq 1- r$" and "$r\leq \varphi\leq s$" $\Leftrightarrow r \leq$ "φ" $\leq s \Leftrightarrow r\leq$ "φ", "φ" $\leq s \Leftrightarrow$ "$\varphi \geq r$", "$\sim \varphi \geq 1- s$", then any fact base with bilateral estimates is equivalent to a fact base with lower estimates i.e. of the form $\varphi_i \geq r_i$. We will consider further only fact bases with lower estimates.

Example 3. Consider the ontology ***O*** from Example 1 as a fuzzy ontology with the fact base ***Fb*** = $\{p \rightarrow A$ ***bb*****B** ≥ 0.6, $q \rightarrow B$ ***edfs*** $C \geq 0.9\}$. In the next section we show that the set $\{(0.6,$ ***d***$), (0.6,$ ***e***$), (0.6,$ ***f***$), \{(0.6,$ ***s***$)\}$ is the answer to the query $?x - p \wedge q \rightarrow \sim C\ x\ A$.

(*End of Example 3*.)

Generally, we can associate with any fuzzy logic the crisp logic of estimates whose sentences are expressions of the form $r \leq \varphi \leq s$ where φ are formulas of the fuzzy logic and $0 \leq r \leq s \leq 1$. Umberto Straccia have studied a fuzzy description logic which are the logics of estimates for description logics [4, 5]. The logic of estimates for propositional logic was considered in [6] where the method of query answering over fact bases was described.

In the paper, we present the method (based on analytical tableaux [6]) for finding the answers to queries addressed a fact base for an interval ontology.

2 Finding Answers to Queries Addressed to a Fact Base

The method of analytical tableaux can be applied to the problem of finding answers to queries addressed to fact bases for fuzzy interval ontologies. We show, by example, how to do this.

Example 3. Consider again the interval ontology ***O*** and the its fact base from Example 2: ***Fb*** = $\{p \rightarrow A$ ***bb*****B** ≥ 0.6, $q \rightarrow B$ ***edfs*** $C \geq 0.9\}$. In Fig.1, it is shown the deduction tree constructed step by step from ***Fb*** and the estimate $p \wedge q \rightarrow \sim C\ x\ A < g$ which is corresponded to the body of the query $?x - p \wedge q \rightarrow \sim C\ x\ A$.

Constructing the deduction tree, we start with the initial branch containing the formulas $p \rightarrow A$ ***bb*****B** ≥ 0.6, $q \rightarrow B$ ***edfs*** $C \geq 0.9$. At the first step we apply the rule from Table 2 in the fourth row and second column (denote by T2(4,2) this rule) to the formula $p \rightarrow A$ ***bb*****B** ≥ 0.6 and we put the label "[1]" on the right of the formula. As a result of the application of the rule T2(4,2), the "fork" with the estimates $p \leq 0.4$ and A ***bb*****B** ≥ 0.6 are added to the initial branch and the label "1:" is put on the left of each of the estimates. At the step 2, the rule T2(4,2) is applied to $q \rightarrow B$ ***edfs*** $C \geq 0.9$. As a result, the "fork" with the estimates $q \leq 0.1$ and B ***edfs*** $C \geq 0.9$ are added to each of two current branches. At the step 8, the rule T8(1,2) is applied to the estimates $q \leq$

0.1 and $q > 1 - g$. As a result, we get the inequality $g \le 0.9$ that means the estimates $q \le 0.1$ and $q \ge 1 - g$ are inconsistent (and therefore, the first branch is inconsistent) if and if $g \le 0.9$. At step 9, the rule T8(1,2) is applied to the estimates $p \le 0.4$ and $p > 1 - g$. As a result, we obtain that the second branch is inconsistent if and only if $g \le 0.6$. At step 10, the rule T8(1,2) is applied to the estimates $q \le 0.1$ and $q \ge 1 - g$. As a result, we obtain that the third branch is inconsistent if and only if $g \le 0.9$. Thus, the first, second and third branch are inconsistent if and only if $g \le \min\{0.9, 0.6, 0.9\} = 0.6$.

At step 12, the rule T4(1,1) is applied to the estimates *B* ***edfs*** *C* ≥ 0.9 and *B* ***edfs*** *C* ≥ 0.9, and as result, the estimate *A* ***bb*dfmm*oo*s*** *C* ≥ 0.6 is obtained. Indeed, using Table 4 which is a fragment of the Allen's table of compositions (see [2]), we have

bb* ∘ ***edfs*** = ***b*** ∘ ***e*** ∪ ***b*** ∘ ***d*** ∪ ***b*** ∘ ***f*** ∪ ***b*** ∘ ***s*** ∪ ***b**** ∘ ***e*** ∪ ***b**** ∘ ***d*** ∪ ***b**** ∘ ***f*** ∪ ***b**** ∘ ***s*** =
b ∪ ***bdmos*** ∪ ***bdmos*** ∪ ***b*** ∪ ***b**** ∪ ***b*dfm*o**** ∪ ***b**** ∪ ***b*dfm*o**** =
bb*dfmm*oo*s.

At step 13, the rule T4(1,3) is applied to the estimate *A* ***bb*dfmm*oo*s*** *C* ≥ 0.6, and we have *C* ***b*bd*f*m*mo*os**** *A* ≥ 0.6. At step 14, the substitution {*x* := ***defs***, *g* := 0.6} is applied to the estimate *C* –*x* *A* < *g*, and we have *C* ***b*bd*f*m*mo*os**** *A* < 0.6. Finally, at step15 we obtain the contradiction: *C* ***b*bd*f*m*mo*os**** *A* ≥ 0.6 and *C* ***b*bd*f*m*mo*os**** *A* < 0.6. From the substitution we obtain the following answer to the query ?*x* – *p* ∧ *q* → ~ *C x A*: {(0.6, ***d***), (0.6, ***e***), (0.6, ***f***), {(0.6, ***s***)}.

(*End of Example 3*.)

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                         p ∧ q → ~ C x A < g       [3]
                         ..............................
                         p → A bb*B ≥ 0.6          [1]
                         q → B edfs C ≥ 0.9        [2]

          1: p ≤ 0.4   [9]                          1: A bb*B ≥ 0.6   [12]

2: q ≤ 0.1  [8]      2: B edfs C ≥ 0.9     2: q ≤ 0.1   [10]     2: B edfs C ≥ 0.9  [12]
...............................................................................................
3: p ∧ q >1– g  [4]  3: p ∧ q >1– g  [5]   3: p ∧ q >1– g  [6]   3: p ∧ q >1– g  [7]
3: ~ C x A < g       3: ~ C x A < g        3: ~ C x A < g        3: ~ C x A < g  [11]
4: p >1– g           5: p >1– g  [9]       6: p >1– g            7: p >1– g
4: q >1– g  [8]      5: q >1– g            6: q >1– g  [10]      7: q < 1– g
8: g ≤ 0.9  X        9: g ≤ 0.6  X         10: g ≤ 0.9  X        11: C –x A < g
                                                               {x := defs, g := 0.6} [14]
                                                  12: A bb*dfmm*oo*s C ≥ 0.6  [13]
                                               13: C b*bd*f*m*mo*os* A ≥ 0.6   [15]
                                               14: C b*bd*f*m*mo*os* A < 0.6   [15]
                                               15: X
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Fig. 1. Deduction tree for Example 2

Remark. In Example 2, the Tables 2, 3 and 4 were used to construct the deduction tree in Fig. 1. Generally, the Tables 5, 6, 7 may be needed. The inference rules entering all these tables are formed a complete system for query answering over fact bases for ontologies written in the language **ELA**.

Table 2. Inference rules for propositional connectives

$\sim\varphi > t$ ——— $\varphi < 1-t$	$\sim\varphi \geq t$ ——— $\varphi \leq 1-t$	$\sim\varphi < t$ ——— $\varphi > 1-t$	$\sim\varphi \leq t$ ——— $\varphi \geq 1-t$
$\varphi \wedge \psi > t$ ——— $\varphi > t$ $\psi > t$	$\varphi \wedge \psi \geq t$ ——— $\varphi > t$ $\psi > t$	$\varphi \wedge \psi < t$ ——— $\varphi < t \mid \psi < t$	$\varphi \wedge \psi \leq t$ ——— $\varphi \leq t \mid \psi \leq t$
$\varphi \vee \psi > t$ ——— $\varphi > t \mid \psi > t$	$\varphi \vee \psi \geq t$ ——— $\varphi \geq t \mid \psi \geq t$	$\varphi \vee \psi < t$ ——— $\varphi < t$ $\psi < t$	$\varphi \vee \psi \leq t$ ——— $\varphi \leq t$ $\psi \leq t$
$\varphi \rightarrow \psi > t$ ——— $\varphi < 1-t \mid \psi > t$	$\varphi \rightarrow \psi \geq t$ ------------------ $\varphi \leq 1-t \mid \psi \geq t$	$\varphi \rightarrow \psi < t$ ----------- $\varphi > 1-t$ $\psi < t$	$\varphi \rightarrow \psi \leq t$ ----------- $\varphi \geq 1-t$ $\psi \leq t$

Table 3. Fragment of Allen's table of compositions

	b	***d***	***f***	***s***
b	***b***	***bdmos***	***b***	***b***
b*	Ω	***b*dfm*o****	***b****	***b*dfm*o****

Table 4 . Inference rules with the composite relations ω and ρ

$A\ \omega\ B \geq r \quad B\ \rho\ C \geq s$ ——— $A\ \omega \circ \rho\ C \geq \min\{r, s\}$	$A\ \omega\ B \leq r \quad B\ \rho\ C \leq s$ ——— $A\ \omega \circ \rho\ C \leq \max\{r, s\}$	$A\ \omega\ B \geq r$ ——— $B\ \omega^* A \geq r$	$A\ \omega\ B \leq r$ ——— $B\ \omega^* A \leq r$
$A\ \omega\ B \geq r \quad A\ \rho\ B \geq s$ ——— $A\ \omega \cap B \geq \min\{r, s\}$	$A\ \omega\ B \leq r \quad A\ \rho\ B \leq s$ ——— $A\ \omega \cap B \leq \max\{r, s\}$	$A\ \alpha\omega\ B \geq r$ ——— $A\ \alpha\ B \geq r \mid A\ \omega\ B \geq r$	$A\ \alpha\omega\ B \leq r$ ——— $A\ \alpha\ B \leq r$ $A\ \omega\ B \leq r$

Table 5. Inference rules for modification of estimates

$(X \geq A^+) \geq t$ ——— $(X > A^-) \geq t$	$(X \geq A^+) > t$ ——— $(X > A^-) > t$	$(X \geq A^+) \leq t$ ——— $(X > A^-) \leq t$	$(X \geq A^+) < t$ ——— $(X > A^-) < t$

$(A^- \geq X) \geq t$ -------------- $(A^- \geq X) \geq t$	$(A^- \geq X) < t$ -------------- $(A^- \geq X) < t$	$(A^- \geq X) \leq t$ -------------- $(A^- \geq X) \leq t$	$(A^- \geq X) < t$ -------------- $(A^- \geq X) < t$
$X \in \{B^+, B^-\}$			

Table 6. Inference rules for Allen's connectives

$A\ \boldsymbol{b}\ B > t$ ―――――― $(B^- \geq A^+) > t$	$A\ \boldsymbol{b}\ B \geq t$ ―――――― $(B^- \geq A^+) \geq t$	$A\ \boldsymbol{b}\ B < t$ ―――――― $(B^- \geq A^+) < t$	$A\ \boldsymbol{b}\ B < t$ --------------- $(B^- \geq A^+) < t$
$A\ \boldsymbol{m}\ B > t$ ―――――― $(A^+ \geq B^-) > t$ $(B^- \geq A^+) > t$	$A\ \boldsymbol{m}\ B \geq t$ ―――――― $(A^+ \geq B^-) \geq t$ $(B^- \geq A^+) > t$	$A\ \boldsymbol{m}\ B < t$ ―――――― $(A^+ \geq B^-) < t \mid$ $(B^- \geq A^+) < t$	$A\ \boldsymbol{m}\ B \leq t$ ―――――― $(A^+ \geq B^-)\ \ t \mid$ $(B^- \geq A^+) < t$
$A\ \boldsymbol{o}\ B > t$ ―――――― $(B^- > A^-) > t$ $(A^+ > B^-) > t$ $(A^+ < B^+) > t$	$A\ \boldsymbol{o}\ B \geq t$ ―――――― $(B^- > A^-) \geq t$ $(A^+ > B^-) \geq t$ $(A^+ < B^+) \geq t$	$A\ \boldsymbol{o}\ B < t$ ―――――― $(B^- > A^-) < t \mid$ $(A^+ > B^-) < t \mid$ $(A^+ < B^+) < t$	$A\ \boldsymbol{o}\ B \leq t$ ―――――― $(B^- > A^-) \leq t \mid$ $(A^+ > B^-) \leq t \mid$ $(A^+ < B^+) \leq t$
$A\ \boldsymbol{f}\ B > t$ ―――――― $(A^- > B^-) > t$ $(A^+ \geq B^+) > t$ $(A^+ \geq B^+) > t$	$A\ \boldsymbol{f}\ B \geq t$ ―――――― $(A^- > B^-) \geq t$ $(A^+ \geq B^+) \geq t$ $(B^+ \geq A^+) > t$	$A\ \boldsymbol{f}\ B < t$ ―――――― $(A^- > B^-) < t \mid$ $(A^+ \geq B^+) < t \mid$ $(A^+ \geq B^+) < t$	$A\ \boldsymbol{f}\ B \leq t$ ―――――― $(A^- > B^-) \leq t \mid$ $(A^+ \geq B^+) \leq t \mid$ $(A^+ \geq B^+) < t$
$A\ \boldsymbol{s}\ B > t$ ―――――― $(A^- > B^-) > t$ $(B^+ \geq A^+) > t$ $(B^+ > A^+) > t$	$A\ \boldsymbol{s}\ B \geq t$ ―――――― $(A^- > B^-) \geq t$ $(B^+ \geq A^+) \geq t$ $(B^+ > A^+) > t$	$A\ \boldsymbol{s}\ B < t$ ―――――― $(A^- \geq B^-) < t \mid$ $(B^+ \geq A^+) < t \mid$ $(B^+ > A^+) < t$	$A\ \boldsymbol{s}\ B \leq t$ ―――――― $(A^- \geq B^-) \leq t \mid$ $(B^+ \geq A^+) \leq t \mid$ $(B^+ > A^+) \leq t$
$A\ \boldsymbol{d}\ B > t$ ―――――― $(A^- > B^-) > t$ $(B^+ < A^+) > t$	$A\ \boldsymbol{d}\ B \geq t$ ―――――― $(A^- > B^-) \geq t$ $(B^+ < A^+) \geq t$	$A\ \boldsymbol{d}\ B < t$ ―――――― $(A^- > B^-) < t \mid$ $(B^+ < A^+) < t$	$A\ \boldsymbol{d}\ B \leq t$ ―――――― $(A^- > B^-) \leq t \mid$ $(B^+ < A^+) \leq t$
$A\ \boldsymbol{e}\ B > t$ ―――――― $(B^- \geq A^-) > t$ $(A^- \geq B^-) > t$ $(B^+ \geq A^+) > t$ $(A^+ \geq B^+) > t$	$A\ \boldsymbol{e}\ B > t$ ―――――― $(B^- \geq A^-) \geq t$ $(A^- \geq B^-) \geq t$ $(B^+ \geq A^+) \geq t$ $(A^+ \geq B^+) \geq t$	$A\ \boldsymbol{e}\ B < t$ ―――――― $(B^- \geq A^-) < t \mid$ $(A^- \geq B^-) < t \mid$ $(B^+ \geq A^+) < t \mid$ $(A^+ \geq B^+) < t$	$A\ \boldsymbol{e}\ B \leq t$ ―――――― $(B^- \geq A^-) \leq t \mid$ $(A^- \geq B^-) \leq t \mid$ $(B^+ \geq A^+) \leq t \mid$ $(A^+ \geq B^+) \leq t$

Table 7. Inference rules for contrary pairs (where $V \in \{X \geq Y, X > Y\}$)

$\dfrac{p < x \quad p \geq t}{x \leq t}$	$\dfrac{p > x \quad p \leq t}{x \geq t}$	$\dfrac{(V) < x \quad (V) \geq t}{x \leq t}$	$\dfrac{(V) > x \quad (V) \leq t}{x \geq t}$
$V \in \{X \geq Y, X > Y\})$			

3 Conclusion

We have defined the fuzzy Boolean extension of Allen's interval logic and considered ontologies written in the extension. Fact bases for such ontologies consist of bilateral estimates for formulas from the ontologies. We have considered the problem of query answering over fact bases. For decision of this problem the analytical tableaux method was applied.

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Motion Control in Mechanical Transport Systems with Fuzzy Given Distance and Time

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Abstract. We investigate traffic management in the mechanical transport system. A major factor in the management of the MTS is search of the optimal route. Therefore, this article is described the routing in the MTS taking into account the inaccuracies and uncertainties of transportation options. We have developed a routing algorithm that takes into account the temporal fuzzy nature of the variables. We have illustrated the example of the developed routing algorithm.

Keywords. Routing algorithm, mechanical transport systems, fuzzy temporal graph

1. Introduction

Mechanical transport system (MTS) is a class of transport systems using conveyors for moving cargo [1]. Conveyors form a network, where nodes are switches direction. The switch is a mechanical device that directs the load unit from one conveyor output to one input of the adjacent conveyors [2]. Example of such systems is the MTS delivery of luggage at airports. The total number of conveyors and switches in these systems can be quite large, which suggests several options for each cargo transport unit. The main element in the management of MTS is a routing, which enables us to construct the optimal path taking into account various parameters.

2. Formulation of the problem

We have following problem taking into account the changing reality and inaccuracies of incoming information. It is necessary to build a set of optimal routes for each of the MTS units. Information about received routes is stored in each node of MTS. And for determination of optimal route is used as a time parameter, and the parameter range. These parameters are presented in fuzzy form. The initial data are given on the MTS by expert to analyze the system.

$$L^* = \min_{\hat{t}} \min_{w(\tilde{t})} \{ (\tilde{w}(\tilde{t}) < s_m, x_i >), \left(\tilde{w}(\tilde{t}) < x_i, x_j >\right), \left(\tilde{w}(\tilde{t}) < x_j, r_m >\right) \} \tag{1}$$

3. Routing Algorithm in MTS under fuzzy given distance and time

There are various ways to implement the routing of mechanical transport systems taking into account only the distance parameter, such as Ford's algorithm, Floyd' algorithm and etc [3]. We consider the algorithm that contains both parameters. The parameters are presented in fuzzy form. For the decision of problem, it is advisable to use the apparatus of the theory of graphs, namely based on fuzzy temporal graph.

Fuzzy temporal graph is a triple $\tilde{G} = (X, \tilde{U}_t, T)$, where *X* is set of vertices of the number of vertices c $|X| = \mathrm{n}, \mathrm{T} = \{1,2,\dots,\mathrm{N}\}$is the set of natural numbers, determining (discrete) time; $U_t = \{< \mu_t(x_i, x_j) | (x_i, x_j) >\}$ is fuzzy set of edges, where $x_i, x_j \in X, \mu_t(x_i, x_j) \in [0,1]$ is the value of the membership function μ_t for the edge (x_i, x_j) at time $t \in T$, and at different times for the same edge (x_i, x_j) values of the membership function (in general) different. Vertex x_j is fuzzy adjacent vertex x_i on the moment of time $t \in T$, if the condition $\mu_t(x_i, x_j) > 0$ [4].

Step 1. To form the matrix $\tilde{\mathrm{D}}^0$ (dimension N×N, where N is number of vertices in the graph). Each element i, j of the matrix $\tilde{\mathrm{d}}_{\mathrm{ij}}^0(\tilde{t})$ determines the length of the shortest arc leading from vertex i to vertex j. In the absence of such an arc put in $\tilde{\mathrm{d}}_{\mathrm{ij}}^0(\tilde{t}) = \infty$.

Step 2. Here $\tilde{D}^m$ denotes the dimension of the matrix $m \times m$ with $\tilde{\mathrm{d}}_{\mathrm{ij}}^m(\tilde{t}), m = \overline{1, m-1}$. To determine successively the elements of the matrix $\tilde{D}^m$ from elements of the matrix of $\tilde{D}^{m-1}$ for m, taking values $1, 2, \dots, N$:

$$\tilde{d}_{mj}^m(\tilde{t}) = \min_{i=\overline{1,m-1}} \{\tilde{d}_{mi}^0(\tilde{t}) + \tilde{d}_{ij}^{m-1}(\tilde{t})\} \; (j = \overline{1, m-1}) \tag{2}$$

$$\tilde{d}_{im}^m(\tilde{t}) = \min_{j=\overline{1,m-1}} \{\tilde{d}_{ij}^{m-1}(\tilde{t}) + \tilde{d}_{jm}^0(\tilde{t})\} \; (i = \overline{1, m-1}) \tag{3}$$

$$\tilde{d}_{ij}^m(\tilde{t}) = \min\{\tilde{d}_{im}^m(\tilde{t}) + \tilde{d}_{mj}^m(\tilde{t}), \tilde{d}_{ij}^{m-1}(\tilde{t})\} \; (i, j = \overline{1, m-1}) \tag{4}$$

Moreover, for all i and put m

$$\tilde{d}_{ii}^m(\tilde{t}) = 0 \tag{5}$$

As a result of the algorithm in the beginning is searched for the minimum distance, and then made to minimize the time.

Step 3. After the algorithm to produce defuzzification [5].

4. Example of illustration the routing algorithm in MTS with fuzzy given parameters and temporal dependence

You need to find the shortest routes to all nodes. Data are presented in fuzzy form. In round brackets is the distance, in square brackets indicates the time. Figure 1 shows example of MTS.

The initial matrix of distance and time as follows (equation (6)):

$$\widetilde{D}^0(\tilde{t}) = \begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & (0,0,0)[0,0,0] & (3,4,6)[1,3,5] & (4,5,6)[2,3,4] & - & - & - \\ 2 & - & (0,0,0)[0,0,0] & - & - & (2,5,7)[3,5,6] & - \\ 3 & - & - & (0,0,0)[0,0,0] & - & - & (5,6,7)[2,3,5] \\ 4 & (2,4,5)[3,4,5] & - & - & (0,0,0)[0,0,0] & - & - \\ 5 & - & (2,5,7)[3,5,6] & - & - & (0,0,0)[0,0,0] & (4,6,8)[2,3,4] \\ 6 & - & - & - & (3,4,7)[1,3,4] & (4,6,8)[2,3,4] & (0,0,0)[0,0,0] \end{array} \tag{6}$$

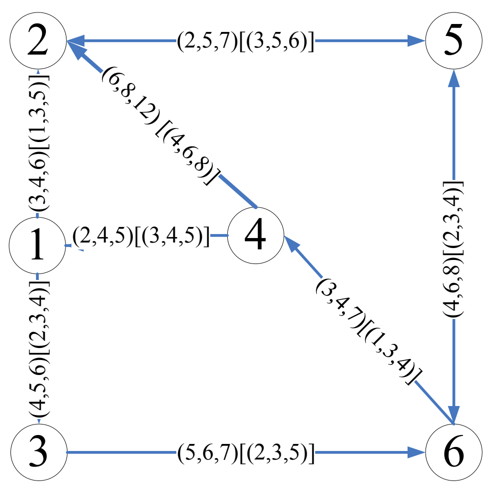

Fig. 1 – Illustration example of MTS scheme

Using the formula (5) of the algorithm we get equation (7).

$$\tilde{d}_{11}^{1}(\tilde{t}) = 0 \tag{7}$$

So we get the matrix $\widetilde{D}^1(\tilde{t})$ below

$$\widetilde{D}^1(\tilde{t}) = \begin{array}{c|c} & 1 \\ \hline 1 & (0,0,0)[0,0,0] \end{array} \tag{8}$$

Then the next phase for the elements use the formula (2) at the beginning, we obtain

$$\tilde{d}_{21}^{2}(\tilde{t}) = \min\{\tilde{d}_{21}^{0}(\tilde{t}) + \tilde{d}_{11}^{1}(\tilde{t})\} = \min\{\infty + (0,0,0)[0,0,0]\} = \infty \tag{9}$$

Consequently, instead of element $\tilde{d}_{21}^{2}(\tilde{t})$ in the matrix $\widetilde{D}^{2}(\tilde{t})$ we have element "–".
The next step we use the formula (3):

$$\begin{aligned}\tilde{d}_{12}^{2}(\tilde{t}) = \min\{\tilde{d}_{11}^{1}(\tilde{t}) + \tilde{d}_{12}^{0}(\tilde{t})\} &= \min\{(0,0,0)[0,0,0] + (3,4,6)[1,3,5]\} \\ &= (3,4,6)[1,3,5](route\ from\ 1\ to\ 2)\end{aligned} \tag{10}$$

Then we use formula (5)

$$\tilde{d}_{11}^{2}(\tilde{t}) = 0, \tilde{d}_{22}^{2}(\tilde{t}) = 0 \tag{11}$$

Fill matrix $\widetilde{D}^{2}(\tilde{t})$

$$\widetilde{D}^{2}(\tilde{t}) = \begin{array}{ccc} & 1 & 2 \\ 1 & (0,0,0)[0,0,0] & (3,4,6)[1,3,5] \\ 2 & - & (0,0,0)[0,0,0] \end{array} \tag{12}$$

We turn to the calculation of the matrix $\widetilde{D}^{3}(\tilde{t})$. Similarly, the use early in the formula (2):

$$\begin{aligned}\tilde{d}_{31}^{3}(\tilde{t}) &= \min\{\tilde{d}_{31}^{0}(\tilde{t}) + \tilde{d}_{11}^{2}(\tilde{t}); \tilde{d}_{32}^{0}(\tilde{t}) + \tilde{d}_{21}^{2}(\tilde{t})\} \\ &= \min\{\infty + (0,0,0)[0,0,0]; \infty + \infty\} = \infty\end{aligned} \tag{13}$$

$$\begin{aligned}\tilde{d}_{32}^{3}(\tilde{t}) &= \min\{\tilde{d}_{31}^{0}(\tilde{t}) + \tilde{d}_{12}^{2}(\tilde{t}); \tilde{d}_{32}^{0}(\tilde{t}) + \tilde{d}_{22}^{2}(\tilde{t})\} \\ &= \min\{\infty + (3,4,6)[1,3,5]; \infty + (0,0,0)[0,0,0]\} = \infty\end{aligned} \tag{14}$$

Using equation (3) we obtain the following values of the elements of the matrix:

$$\begin{aligned}\tilde{d}_{13}^{3}(\tilde{t}) &= \min\{\tilde{d}_{11}^{2}(\tilde{t}) + \tilde{d}_{13}^{0}(\tilde{t}); \tilde{d}_{12}^{2}(\tilde{t}) + \tilde{d}_{23}^{0}(\tilde{t})\} \\ &= \min\{(0,0,0)[0,0,0] + \infty; (3,4,6)[1,3,5] + \infty\} = \infty\end{aligned} \tag{15}$$

$$\begin{aligned}\tilde{d}_{23}^{3}(\tilde{t}) &= \min\{\tilde{d}_{21}^{2}(\tilde{t}) + \tilde{d}_{13}^{0}(\tilde{t}); \tilde{d}_{22}^{2}(\tilde{t}) + \tilde{d}_{23}^{0}(\tilde{t})\} \\ &= \min\{\infty + \infty; (0,0,0)[0,0,0] + \infty\} = \infty\end{aligned} \tag{16}$$

We recalculate values of the matrix by the formula (4) with the new values obtained in the previous step.

$$\begin{aligned}\tilde{d}_{12}^{3}(\tilde{t}) = \min\{\tilde{d}_{12}^{2}(\tilde{t}); \tilde{d}_{13}^{3}(\tilde{t}) + \tilde{d}_{32}^{3}(\tilde{t})\} &= \min\{(3,4,6)[1,3,5]; \infty + \infty\} \\ &= (3,4,6)[1,3,5](route\ (1,2))\end{aligned} \quad (17)$$

$$\tilde{d}_{21}^{3}(\tilde{t}) = \min\{\tilde{d}_{21}^{2}(\tilde{t}); \tilde{d}_{23}^{3}(\tilde{t}) + \tilde{d}_{31}^{3}(\tilde{t})\} = \min\{\infty; \infty + \infty\} = \infty \quad (18)$$

For further calculation of matrix elements we use the formula (5):

$$\tilde{d}_{11}^{3}(\tilde{t}) = 0, \tilde{d}_{22}^{3}(\tilde{t}) = 0, \tilde{d}_{33}^{3}(\tilde{t}) = 0 \quad (19)$$

Fill the matrix $\widetilde{D}^{3}(\tilde{t})$

$$\widetilde{D}^{3}(\tilde{t}) = \begin{array}{cccc} & 1 & 2 & 3 \\ 1 & (0,0,0)[0,0,0] & (3,4,6)[1,3,5] & (4,5,6)[2,3,4] \\ 2 & - & (0,0,0)[0,0,0] & - \\ 3 & - & - & (0,0,0)[0,0,0] \end{array} \quad (20)$$

We repeat similar iterations for the calculation of the matrix $\widetilde{D}^{4}(\tilde{t})$. Similarly, we use early in the formula (2):

$$\begin{aligned}\tilde{d}_{41}^{4}(\tilde{t}) &= \min\{\tilde{d}_{41}^{0}(\tilde{t}) + \tilde{d}_{11}^{3}(\tilde{t}); \tilde{d}_{42}^{0}(\tilde{t}) + \tilde{d}_{21}^{3}(\tilde{t}); \tilde{d}_{43}^{0}(\tilde{t}) + \tilde{d}_{31}^{3}(\tilde{t})\} \\ &= \min\{(2,4,5)[3,4,5] + (0,0,0)[0,0,0]; \infty + \infty; \infty + \infty\} \\ &= (2,4,5)[3,4,5]\ (route\ (4,1))\end{aligned} \quad (21)$$

$$\begin{aligned}\tilde{d}_{42}^{4}(\tilde{t}) = \min\{\tilde{d}_{41}^{0}(\tilde{t}) + \tilde{d}_{12}^{3}(\tilde{t}); \tilde{d}_{42}^{0}(\tilde{t}) + \tilde{d}_{22}^{3}(\tilde{t}); \tilde{d}_{43}^{0}(\tilde{t}) + \tilde{d}_{32}^{3}(\tilde{t})\} \\ = \min\{(2,4,5)[3,4,5] + (3,4,6)[1,3,5]; \infty \\ + (0,0,0)[0,0,0]; \infty + \infty\} \\ = (5,8,11)[4,7,10](route\ (4,1) \rightarrow (1,2))\end{aligned} \quad (22)$$

$$\begin{aligned}\tilde{d}_{43}^{4}(\tilde{t}) = \min\{\tilde{d}_{41}^{0}(\tilde{t}) + \tilde{d}_{13}^{3}(\tilde{t}); \tilde{d}_{42}^{0}(\tilde{t}) + \tilde{d}_{23}^{3}(\tilde{t}); \tilde{d}_{43}^{0}(\tilde{t}) + \tilde{d}_{33}^{3}(\tilde{t})\} \\ = \min\{(2,4,5)[3,4,5] + (4,5,6)[2,3,4]; \infty + \infty; \infty \\ + (0,0,0)[0,0,0]\} = (6,9,11)[5,7,9](route\ (4,1) \\ \rightarrow (1,3))\end{aligned} \quad (23)$$

Using equation (3), we obtain the following values of the elements of the matrix:

$$\begin{aligned}\tilde{d}_{14}^{4}(\tilde{t}) &= \min\{\tilde{d}_{11}^{3}(\tilde{t}) + \tilde{d}_{14}^{0}(\tilde{t}); \tilde{d}_{12}^{3}(\tilde{t}) + \tilde{d}_{24}^{0}(\tilde{t}); \tilde{d}_{13}^{3}(\tilde{t}) + \tilde{d}_{34}^{0}(\tilde{t})\} \\ &= \min\{(0,0,0)[0,0,0] + \infty; (3,4,6)[1,3,5] + \infty; \infty + \infty\} \\ &= \infty\end{aligned} \tag{24}$$

$$\begin{aligned}\tilde{d}_{24}^{4}(\tilde{t}) &= \min\{\tilde{d}_{21}^{3}(\tilde{t}) + \tilde{d}_{14}^{0}(\tilde{t}); \tilde{d}_{22}^{3}(\tilde{t}) + \tilde{d}_{24}^{0}(\tilde{t}); \tilde{d}_{23}^{3}(\tilde{t}) + \tilde{d}_{34}^{0}(\tilde{t})\} \\ &= \min\{\infty + \infty; (0,0,0)[0,0,0] + \infty; \infty + \infty\} = \infty\end{aligned} \tag{25}$$

$$\begin{aligned}\tilde{d}_{34}^{4}(\tilde{t}) &= \min\{\tilde{d}_{31}^{3}(\tilde{t}) + \tilde{d}_{14}^{0}(\tilde{t}); \tilde{d}_{32}^{3}(\tilde{t}) + \tilde{d}_{24}^{0}(\tilde{t}); \tilde{d}_{33}^{3}(\tilde{t}) + \tilde{d}_{34}^{0}(\tilde{t})\} \\ &= \min\{\infty + \infty; \infty + \infty; (0,0,0)[0,0,0] + \infty\} = \infty\end{aligned} \tag{26}$$

We recalculate values of the matrix by the formula (4) with the new values obtained in the previous step.

$$\begin{aligned}\tilde{d}_{12}^{4}(\tilde{t}) &= \min\{\tilde{d}_{12}^{3}(\tilde{t}); \tilde{d}_{14}^{4}(\tilde{t}) + \tilde{d}_{42}^{4}(\tilde{t})\} \\ &= \min\{(3,4,6)[1,3,5]; \infty + (5,8,11)[4,7,10]\} \\ &= (3,4,6)[1,3,5](route\ (1,2))\end{aligned} \tag{27}$$

$$\begin{aligned}\tilde{d}_{13}^{4}(\tilde{t}) &= \min\{\tilde{d}_{13}^{3}(\tilde{t}); \tilde{d}_{14}^{3}(\tilde{t}) + \tilde{d}_{43}^{3}(\tilde{t})\} \\ &= \min\{(4,5,6)[2,3,4]; \infty + (6,9,11)[5,7,9]\} \\ &= (4,5,6)[2,3,4](route\ (1,3))\end{aligned} \tag{28}$$

$$\begin{aligned}\tilde{d}_{21}^{4}(\tilde{t}) &= \min\{\tilde{d}_{21}^{3}(\tilde{t}); \tilde{d}_{24}^{3}(\tilde{t}) + \tilde{d}_{41}^{3}(\tilde{t})\} = \min\{\infty; \infty + (2,4,5)[3,4,5]\} \\ &= \infty\end{aligned} \tag{29}$$

$$\begin{aligned}\tilde{d}_{23}^{4}(\tilde{t}) &= \min\{\tilde{d}_{23}^{3}(\tilde{t}); \tilde{d}_{24}^{3}(\tilde{t}) + \tilde{d}_{43}^{3}(\tilde{t})\} = \min\{\infty; \infty + (6,9,11)[5,7,9]\} \\ &= \infty\end{aligned} \tag{30}$$

For further calculation of matrix elements we use the formula (5):

$$\tilde{d}_{11}^{4}(\tilde{t}) = 0, \tilde{d}_{22}^{4}(\tilde{t}) = 0, \tilde{d}_{33}^{4}(\tilde{t}) = 0, \tilde{d}_{44}^{4}(\tilde{t}) = 0 \tag{31}$$

Fill the matrix $\widetilde{D}^{4}(\tilde{t})$

$$\widetilde{D}^4(\tilde{t}) = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & (0,0,0)[0,0,0] & (3,4,6)[1,3,5] & (4,5,6)[2,3,4] & - \\ 2 & - & (0,0,0)[0,0,0] & - & - \\ 3 & - & - & (0,0,0)[0,0,0] & - \\ 4 & (2,4,5)[3,4,5] & (5,8,11)[4,7,10] & (6,9,11)[5,7,9] & (0,0,0)[0,0,0] \end{array} \quad (32)$$

As a result, similar calculations obtain intermediate matrix of routes $\widetilde{D}^5(\tilde{t})$.

$$\widetilde{D}^5(\tilde{t}) = \begin{array}{c|cc} & 1 & 2 \\ \hline 1 & (0,0,0)[0,0,0] & (3,4,6)[1,3,5] \\ 2 & - & (0,0,0)[0,0,0] \\ 3 & - & - \\ 4 & (2,4,5)[3,4,5] & (5,8,11)[4,7,10] \\ 5 & - & (2,5,7)[3,5,6] \end{array}$$

$$\begin{array}{c|ccc} & 3 & 4 & 5 \\ \hline 1 & (4,5,6)[2,3,4] & - & (5,9,13)[4,8,11] \\ 2 & - & - & (2,5,7)[3,5,6] \\ 3 & (0,0,0)[0,0,0] & - & - \\ 4 & (6,9,11)[5,7,9] & (0,0,0)[0,0,0] & (7,13,18)[7,12,16] \\ 5 & - & - & (0,0,0)[0,0,0] \end{array} \quad (33)$$

Using the $\widetilde{D}^5(\tilde{t})$ intermediate matrix performs calculations and obtain the resulting matrix routes.

$$\widetilde{D}^6(\tilde{t}) = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & (0,0,0)[0,0,0] & (3,4,6)[1,3,5] & (4,5,6)[2,3,4] \\ 2 & (11,19,27)[9,15,19] & (0,0,0)[0,0,0] & (15,24,33)[11,18,23] \\ 3 & (10,14,19)[6,10,14] & (11,17,22)[7,11,15] & (0,0,0)[0,0,0] \\ 4 & (2,4,5)[3,4,5] & (5,8,11)[4,7,10] & (6,9,11)[5,7,9] \\ 5 & (9,11,20)[6,10,13] & (2,5,7)[3,5,6] & (13,19,26)[8,13,17] \\ 6 & (5,8,12)[4,7,9] & (6,11,15)[5,8,10] & (19,13,18)[6,10,13] \end{array}$$

$$\begin{array}{c|ccc} & 4 & 5 & 6 \\ \hline 1 & (12,15,20)[5,9,13] & (5,9,13)[4,8,11] & (9,11,13)[4,6,9] \\ 2 & (9,15,22)[6,11,14] & (2,5,7)[3,5,6] & (6,11,15)]6,8,10] \\ 3 & (8,10,14)[3,6,9] & (9,12,15)[4,6,9] & (5,6,7)[2,3,5] \\ 4 & (0,0,0)[0,0,0] & (7,13,18)[7,12,16] & (11,15,18)[7,10,14] \\ 5 & (7,10,15)[3,6,8] & (0,0,0)[0,0,0] & (4,6,8)[2,3,4] \\ 6 & (3,4,7)[6,10,13] & (4,6,8)[2,3,4] & (0,0,0)[0,0,0] \end{array} \quad (34)$$

5. Conclusion

The specific management tool of MTS is routing. The proposed routing algorithm is applicable to any mechanical transport system. The use of the described routing method gives the best effect for MTS, which are operated in the unstable conditions for major changes of cargo flow intensity. This case of temporal dependence reflects the real environmental situation and its cost.

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Towards the Notion of Formal Concept of Objects with Attributes in a Sentence

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Abstract. We consider simple and complex objects in stationary and non-stationary cases and their usage in a sentence. An object is described by a conjunction and/or disjunction of attributes in case compatibility and incompatibility of the original attributes should be taken into account. For this setting, we consider the structure of a formal concept and the structure of the sets of objects and attributes together with operations similar to those of reasoning. The relation of the resulting model to FCA is considered.

Keywords: Formal Concept Analysis, attributes compatibility, attributes incompatibility

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