

Greedy Algorithm for Construction of Decision Trees for Tables with Many-Valued Decisions

Mohammad Azad¹, Igor Chikalov¹, Mikhail Moshkov¹, and Beata Zielosko^{1,2}

¹ Mathematical and Computer Sciences & Engineering Division
King Abdullah University of Science and Technology
Thuwal 23955-6900, Saudi Arabia

{mohammad.azad, igor.chikalov, mikhail.moshkov, beata.zielosko}@kaust.edu.sa

² Institute of Computer Science, University of Silesia
39, Będzińska St., 41-200 Sosnowiec, Poland

Abstract. In the paper, we study a greedy algorithm for construction of approximate decision trees. This algorithm is applicable to decision tables with many-valued decisions where each row is labeled with a set of decisions. For a given row, we should find a decision from the set attached to this row. We use an uncertainty measure which is the number of boundary subtables. We present also experimental results for data sets from UCI Machine Learning Repository for proposed approach and approach based on generalized decision.

Keywords: decision table with many-valued decisions, decision tree, greedy algorithm

1 Introduction

In the paper, we study decision trees for decision tables with many-valued decisions. We can meet such tables when we work with experimental or statistical data. In such data sets, we often have groups of rows with equal values of conditional attributes but, probably, different values of the decision attribute. In this case, instead of a group of rows, we can consider one row given by values of conditional attributes, and we attach to this row a set of decisions: either all decisions for rows from the group, or k the most frequent decisions for rows from the group [5]. As a result we obtain a decision table with many-valued decisions.

In the rough sets theory [9, 13] a generalized decision is often used to work with decision tables which have equal rows labeled with different decisions (inconsistent decision tables). In this case, we also work with the decision table with many-valued decisions. The set of decisions attached to equal rows is called the generalized decision for each of these equal rows [10, 11]. The usual way is to find for a given row its generalized decision. We will call this approach as approach based on generalized decision.

In our approach, we study a problem of finding of an arbitrary decision or one of the most frequent decisions from the set of decisions. Proposed approach was considered in [2, 7] for construction of tests (super-reducts), and in [3] for

construction of decision rules. In this paper, we propose this approach for construction of decision trees. It can be useful from the point of view of knowledge representation. If it is enough to find only one decision then a decision tree can have less nodes in comparison with a decision tree constructed using the generalized decision approach. In this case, proposed decision tree will be more understandable. Besides, if we consider a decision tree as a way for construction of rules then the number of rules for proposed approach can be less than the number of rules for approach based on generalized decision.

Decision tables with many-valued decisions can be considered as decision tables with an incomplete information because we don't know which decision should be chosen from the set of decisions. However, an incomplete information can be interpreted also in situations where instead of a single value of conditional attribute we have a subset of values of the attribute domain or missing values of attributes. Z. Pawlak [9] and E. Orłowska [8] proposed Non-deterministic Information System for dealing with an incomplete information. J. Quinlan in [12] presented extensions to ID3 algorithm to cope with unknown values of attributes. W. Cohen in [4] proposed algorithm for learning trees and rules for decision tables with many-valued attributes which are represented as "feature vectors". In [14] authors presented, for incomplete decision system, an algorithm for construction of decision trees and classification rule extraction, based on relationship between attributes.

In this paper, we present a greedy algorithm for construction of decision trees for decision tables with many-valued decisions. Theoretical results connected with a greedy algorithm for construction of approximate decision trees were presented in [5, 6]. To define the notion of approximate decision tree we use an uncertainty measure $B(T)$ for decision tables with many-valued decisions. We consider so-called α -decision trees where α is a real number such that $0 \leq \alpha < 1$. For a given row r of a table T , an α -decision tree localizes it in a subtable of T with uncertainty at most $\alpha B(T)$. If α equals 0, we have an exact decision tree for T .

We present an upper bound on the number of steps of the considered algorithm. From this bound it follows that, for an arbitrary natural t , the greedy algorithm has polynomial time complexity on tables which have at most t decisions in each set of decisions attached to rows.

In this paper, we consider only binary decision tables with many-valued decisions. However, the obtained results can be extended to the decision tables filled by numbers from the set $\{0, \dots, k-1\}$, where $k \geq 3$. One of the aims of this paper is to make some comparative study relative to depth and average depth of decision trees for proposed approach and approach based on generalized decision. We present experimental results for data sets from UCI ML Repository.

This paper consists of six sections. Section 2, contains main notions. In Sect. 3, we present a theorem for decision tables with at most t decisions in each set of decisions attached to rows. In Sect. 4, we study a greedy algorithm for α -decision tree construction. Section 5 contains experimental results, Sect. 6 – conclusions.

2 Main Notions

In this section, we consider definitions of notions corresponding to decision tables with many-valued decisions.

A *(binary) decision table with many-valued decisions* is a rectangular table T filled by numbers from the set $\{0, 1\}$. Columns of this table are labeled with attributes f_1, \dots, f_n . Rows of the table are pairwise different, and each row is labeled with a nonempty finite set of natural numbers (set of decisions). Note that each (binary) decision table with one-valued decisions can be interpreted also as a decision table with many-valued decisions. In such table, each row is labeled with a set of decisions which has one element.

We will say that T is a *degenerate* table if either T has no rows, or the intersection of sets of decisions attached to rows of T is nonempty.

A decision which belongs to the maximum number of sets of decisions attached to rows in T is called the *most common decision for T* . If we have more than one such decisions we choose the minimum one. If T is empty then 1 is the most common decision for T .

A table obtained from T by removal of some rows is called a *subtable* of T . A subtable T' of T is called a *boundary subtable* if T' is not degenerate but each proper subtable of T' is degenerate. We denote by $B(T)$ the number of boundary subtables of the table T . It is clear that T is a degenerate table if and only if $B(T) = 0$. The value $B(T)$ will be interpreted as *uncertainty* of T . An example of boundary subtables for decision table T_0 with many-valued decisions can be found in Fig. 1.

$$T_0 = \begin{array}{|c|c|c|c|c|} \hline & f_1 & f_2 & f_3 & d \\ \hline r_1 & 0 & 0 & 0 & \{1\} \\ r_2 & 0 & 1 & 1 & \{1, 2\} \\ r_3 & 1 & 0 & 1 & \{1, 3\} \\ r_4 & 1 & 1 & 0 & \{2, 3\} \\ r_5 & 0 & 0 & 1 & \{2\} \\ \hline \end{array}$$

$$T_1 = \begin{array}{|c|c|c|c|c|} \hline & f_1 & f_2 & f_3 & d \\ \hline r_2 & 0 & 1 & 1 & \{1, 2\} \\ r_3 & 1 & 0 & 1 & \{1, 3\} \\ r_4 & 1 & 1 & 0 & \{2, 3\} \\ \hline \end{array} \quad T_2 = \begin{array}{|c|c|c|c|c|} \hline & f_1 & f_2 & f_3 & d \\ \hline r_1 & 0 & 0 & 0 & \{1\} \\ r_4 & 1 & 1 & 0 & \{2, 3\} \\ \hline \end{array} \quad T_3 = \begin{array}{|c|c|c|c|c|} \hline & f_1 & f_2 & f_3 & d \\ \hline r_3 & 1 & 0 & 1 & \{1, 3\} \\ r_5 & 0 & 0 & 1 & \{2\} \\ \hline \end{array} \quad T_4 = \begin{array}{|c|c|c|c|c|} \hline & f_1 & f_2 & f_3 & d \\ \hline r_1 & 0 & 0 & 0 & \{1\} \\ r_5 & 0 & 0 & 1 & \{2\} \\ \hline \end{array}$$

Fig. 1. Boundary subtables T_1, T_2, T_3, T_4 of the decision table T_0

In the case of generalized decision approach we consider sets of decisions attached to rows of the table T_0 as values of the decision attribute (see Fig. 2). The number of boundary subtables for such approach is equal to 10. Each boundary subtable of the table T_0 has 2 rows (see Theorem 1).

Let $f_{i_1}, \dots, f_{i_m} \in \{f_1, \dots, f_n\}$ and $\delta_1, \dots, \delta_m \in \{0, 1\}$. We denote by

$$T(f_{i_1}, \delta_1) \dots (f_{i_m}, \delta_m)$$

the subtable of the table T which consists of all rows that at the intersection with columns f_{i_1}, \dots, f_{i_m} have numbers $\delta_1, \dots, \delta_m$ respectively.

$$T_0 = \begin{array}{c|ccc|c} & f_1 & f_2 & f_3 & d \\ \hline r_1 & 0 & 0 & 0 & \{1\} \\ r_2 & 0 & 1 & 1 & \{1, 2\} \\ r_3 & 1 & 0 & 1 & \{1, 3\} \\ r_4 & 1 & 1 & 0 & \{2, 3\} \\ r_5 & 0 & 0 & 1 & \{2\} \end{array} \Rightarrow \begin{array}{c|c} & d \\ \hline & 1 \\ & 2 \\ & 3 \\ & 4 \\ & 5 \end{array}$$

Fig. 2. Transformation of decision attribute from the table T_0 for generalized decision approach

A *decision tree over T* is a finite tree with root in which each terminal node is labeled with a decision (a natural number), and each nonterminal node is labeled with an attribute from the set $\{f_1, \dots, f_n\}$. Two edges start in each nonterminal node. These edges are labeled with 0 and 1 respectively.

Let Γ be a decision tree over T and v be a node of Γ . We attach to the node v a subtable $T(v)$ of the table T . If v is the root of Γ then $T(v) = T$. Otherwise, let nodes and edges in the path from the root to v be labeled with attributes f_{i_1}, \dots, f_{i_m} and numbers $\delta_1, \dots, \delta_m$ respectively. Then $T(v)$ is the subtable $T(f_{i_1}, \delta_1) \dots (f_{i_m}, \delta_m)$ of the table T .

It is clear that for any row r of T there exists exactly one terminal node v in Γ such that r belongs to $T(v)$. The decision attached to v will be considered as the result of the work of Γ on the row r .

Let α be a real number such that $0 \leq \alpha < 1$. We will say that Γ is an α -*decision tree for T* if for any terminal node v of Γ , the inequality $B(T(v)) \leq \alpha B(T)$ holds, and the node v is labeled with the most common decision for $T(v)$. An example of an exact decision tree constructed for the table T_0 can be found in Fig. 3, a 0.25-*decision tree* constructed for the table T_0 can be found in Fig. 5.

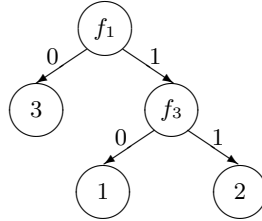


Fig. 3. Exact decision tree

We denote by $h(\Gamma)$ the *depth* of a decision tree Γ which is the maximum length of a path from the root to a terminal node. We denote by $h_\alpha(T)$ the minimum depth of an α -*decision tree* for the table T . We denote by $l(\bar{\delta})$ a *total path length* for an arbitrary row $\bar{\delta}$ of the table T . It is a length of the path from the root to a terminal node of Γ which accepts $\bar{\delta}$. An *average depth* of Γ is equal

to the total path length divided by $N(T)$ – the number of rows in decision table T .

3 Set of Tables $Tab(t)$

We denote by $Tab(t)$, where t is a natural number, the set of decision tables with many-valued decisions such that each row in the table has at most t decisions.

Theorem 1. [6] *Each boundary subtable of a table $T \in Tab(t)$ has at most $t + 1$ rows.*

Therefore, for tables from $Tab(t)$, there exists a polynomial algorithm for the computation of parameter $B(T)$. For example, for any decision table T with one-valued decisions (really, for any table from $Tab(1)$) the equality $B(T) = P(T)$ holds where $P(T)$ is the number of unordered pairs of rows of T with different decisions.

4 Greedy Algorithm U_α

To construct an α -decision tree for T , we first need to construct all boundary subtables of decision table T and to compute the value of $B(T)$.

Let α be a real number such that $0 \leq \alpha < 1$. Let T have n columns labeled with attributes f_1, \dots, f_n . We now describe an algorithm U_α which for a given decision table with many-valued decisions constructs an α -decision tree $U_\alpha(T)$ for the table T .

- *Step 1.* Construct a tree consisting of a single node labeled with the table T and proceed to the second step.
- Suppose $t \geq 1$ steps have been already made. The tree obtained at the step t will be denoted by G .
- *Step $(t + 1)$.* If no one node of the tree G is labeled with a table then we denote by $U_\alpha(T)$ the tree G . The work of the algorithm U_α is completed.
- Otherwise, we choose a node v in the tree G which is labeled with a subtable of the table T . Let the node v be labeled with the table T' .
If $B(T') \leq \alpha B(T)$ then instead of T' we mark the node v with the most common decision for T' and proceed to the step $(t + 2)$.
Let $B(T') > \alpha B(T)$. Then for $i = 1, \dots, n$, we compute the value

$$Q(f_i) = \max\{B(T'(f_i, 0)), B(T'(f_i, 1))\}.$$

Instead of T' we mark the node v with the attribute f_{i_0} where i_0 is the minimum i for which $Q(f_i)$ has the minimum value. For each $\delta \in \{0, 1\}$, we add to the tree G the node $v(\delta)$, mark this node with the subtable $T'(f_{i_0}, \delta)$, draw the edge from v to $v(\delta)$, and mark this edge with δ . Proceed to the step $(t + 2)$.

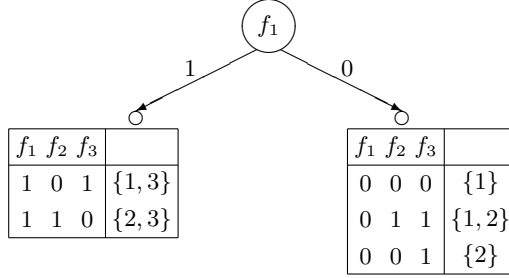


Fig. 4. Construction of 0.25-decision tree

Now we present an example of the greedy algorithm $U_{0.25}$ for construction of a 0.25-decision tree, for decision table T_0 . All boundary subtables of the decision table T_0 are depicted in Fig. 1.

The table T_0 is not degenerate, so for $i \in \{1, 2, 3\}$, we compute the value of $Q(f_i)$: $Q(f_1) = \max\{1, 0\} = 1$, $Q(f_2) = \max\{2, 0\} = 2$, $Q(f_3) = \max\{1, 1\} = 1$. We choose the attribute f_1 as a root of the 0.25-decision tree (see Fig. 4).

$B(T(f_1, 0)) = 1$ and $B(T(f_1, 1)) = 0$, so $B(T(f_1, 0)) \leq 0.25 \times B(T)$. We assign to 0.25-decision tree the most common decision for the subtable $T(f_1, 1)$, and the most common decision for the subtable $T(f_1, 0)$. The result of the algorithm $U_{0.25}$ work for the table T_0 is a 0.25-decision tree depicted in Fig. 5.

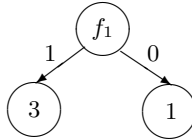


Fig. 5. 0.25-decision tree for the table T_0

Now, we present an upper bound on the number of steps of algorithm U_α .

Theorem 2. [6] *Let α be a real number such that $0 \leq \alpha < 1$, and T be a decision table with many-valued decisions. Then during the construction of the tree $U_\alpha(T)$ the algorithm U_α makes at most $2N(T) + 1$ steps where $N(T)$ is the number of rows in T .*

From this theorem it follows that for any natural t the algorithm U_α has a polynomial time complexity on the set $Tab(t)$.

Theorem 3. For any real α , $0 < \alpha < 1$, and for any nondegenerate decision table with many-valued decisions T

$$h(U_\alpha(T)) < h_0(T) \ln \frac{1}{\alpha} + 1.$$

5 Experimental Results

In this section, we present experimental results. First, we show how we constructed decision tables with many-valued decisions based on data sets from UCI Machine Learning Repository. We consider a number of decision tables from UCI Machine Learning Repository [1]. In some tables there were missing values. Each such value was replaced with the most common value of the corresponding attribute. Some decision tables contain conditional attributes that take unique value for each row. Such attributes were removed. In some tables there were equal rows with, possibly, different decisions. In this case each group of identical rows was replaced with a single row from the group which is labeled with the set of decisions attached to rows from the group.

Table 1. Characteristics of decision tables with many-valued decisions

Decision table	Rows	Attr	Spectrum						Removed attributes
			#1	#2	#3	#4	#5	#6	
balance-scale-1	125	3	45	50	30				left-weight
breast-cancer-1	193	8	169	24	0				tumor-size
breast-cancer-5	98	4	58	40					inv-nodes,node-caps,deg-malig, breast-quad,irradiat
cars-1	432	5	258	161	13				buying
flags-5	171	21	159	12					zone,language,religion,circles,sunstars
hayes-roth-data-1	39	3	22	13	4				marital_status
kr-vs-kp-5	1987	31	1564	423					katri,mulch,rimmx,skrxp,wknck
kr-vs-kp-4	2061	32	1652	409					katri,mulch,rimmx,wknck
lymphography-5	122	13	113	9					lymphatics,changes_in_node,changes_in_stru, special_forms,no_of_nodes_in
mushroom-5	4078	17	4048	30					odor,gill-size,stalk-root, stalk-surface-below-ring,habitat
nursery-4	240	4	97	96	47				parents,housing,finance,social
nursery-1	4320	7	2858	1460	2				parents
spect-test-1	164	21	161	3					F3
teeth-1	22	7	12	10					top incisors
teeth-5	14	3	6	3	0	5	0	2	bottom incisors,top canines,bottom canines, top premolars,bottom molars
tic-tac-toe-4	231	5	102	129					top-right-square,middle-middle-square, bottom-left-square,bottom-right-square
tic-tac-toe-3	449	6	300	149					middle-middle-square,bottom-left-square, bottom-right-square
zoo-data-5	42	11	36	6					feathers,backbone,breathes,legs,tail

To obtain rows which are labeled with sets containing more than one decision we removed from decision tables more conditional attributes. The information about such decision tables can be found in Table 1. This table contains name of initial table, number of rows (column “Rows”), number of attributes (column

“Attr”), spectrum of this table (column “Spectrum”), and names of removed attributes (column “Removed attributes”). Spectrum of decision table with many-valued decisions is a sequence $\#1, \#2, \dots$, where $\#i, i = 1, 2, \dots$, is the number of rows labeled with sets of decision with the cardinality equal to i .

Table 2 presents the depth of α -decision trees constructed by the algorithm U_α , for $\alpha \in \{0, 0.001, 0.01, 0.2, 0.5\}$.

Table 2. Depth of α -decision trees constructed by the algorithm U_α

Decision table	α					
	0.0	0.001	0.01	0.1	0.2	0.5
balance-scale-1	2	2	2	1	1	1
breast-cancer-1	6	4	3	2	1	1
breast-cancer-5	3	3	2	1	1	1
cars-1	5	3	2	1	1	1
flags-5	6	4	3	1	1	1
hayes-roth-data-1	2	2	2	1	1	1
kr-vs-kp-5	13	6	4	2	2	1
kr-vs-kp-4	12	6	4	2	2	1
lymphography-5	6	5	3	2	1	1
mushroom-5	7	3	2	1	1	1
nursery-4	2	2	1	1	1	1
nursery-1	7	3	2	1	1	1
spect-test-1	7	7	4	2	2	1
teeth-1	4	4	3	2	1	1
teeth-5	3	3	3	2	1	1
tic-tac-toe-4	5	4	3	2	1	1
tic-tac-toe-3	6	4	3	2	1	1
zoo-data-5	4	4	3	2	2	1

Table 3 presents the average depth of α -decision trees constructed by the algorithm U_α , for $\alpha \in \{0, 0.001, 0.01, 0.2, 0.5\}$.

Based on results presented in Table 2 and Table 3 we can see that depth and average depth of α -decision tree are decreasing when α is increasing, till $\alpha = 0.5$. The biggest value of depth of exact decision tree we can observe for decision table “kr-vs-kp-5” and “kr-vs-kp-4”. However, if we consider 0.1-decision tree for such decision tables, the depth is equal to 2.

To make some comparative study we also present results of experiments connected with depth and average depth of α -decision trees for approach based on generalized decision.

For decision tables described in Table 1 and $\alpha \in \{0, 0.001, 0.01, 0.2, 0.5\}$ we constructed α -decision trees by the greedy algorithm. Table 4 presents depth of constructed α -decision trees, Table 5 – the average depth of constructed α -decision trees, for approach based on generalized decision.

Table 3. The average depth of α -decision trees constructed by the algorithm U_α

Decision table	α					
	0.0	0.001	0.01	0.1	0.2	0.5
balance-scale-1	2.00	2.00	1.20	1.00	1.00	1.00
breast-cancer-1	3.72	3.05	2.23	1.35	1.00	1.00
breast-cancer-5	1.84	1.84	1.37	1.00	1.00	1.00
cars-1	2.90	2.63	2.00	1.00	1.00	1.00
flags-5	3.82	2.83	1.98	1.00	1.00	1.00
hayes-roth-data-1	1.74	1.74	1.74	1.00	1.00	1.00
kr-vs-kp-5	8.24	5.16	3.68	2.00	2.00	1.00
kr-vs-kp-4	8.13	5.24	3.69	2.00	1.58	1.00
lymphography-5	3.80	3.51	2.48	1.65	1.00	1.00
mushroom-5	2.78	2.20	1.61	1.00	1.00	1.00
nursery-4	1.33	1.33	1.00	1.00	1.00	1.00
nursery-1	2.82	2.00	1.67	1.00	1.00	1.00
spect-test-1	3.35	3.35	3.05	2.00	2.00	1.00
teeth-1	2.82	2.82	2.27	1.41	1.00	1.00
teeth-5	2.21	2.21	2.21	1.36	1.00	1.00
tic-tac-toe-4	3.00	2.90	2.34	1.34	1.00	1.00
tic-tac-toe-3	4.27	3.54	2.43	1.67	1.00	1.00
zoo-data-5	3.21	3.21	3.00	2.00	1.52	1.00

Table 4. Depth of α -decision trees – generalized decision approach

Decision table	α					
	0.0	0.001	0.01	0.1	0.2	0.5
balance-scale-1	3	3	2	1	1	1
breast-cancer-1	7	4	3	1	1	1
breast-cancer-5	4	3	2	1	1	1
cars-1	5	3	2	1	1	1
flags-5	6	4	2	1	1	1
hayes-roth-data-1	3	3	2	1	1	1
kr-vs-kp-5	14	6	4	2	2	1
kr-vs-kp-4	14	6	4	2	2	1
lymphography-5	7	5	3	2	1	1
mushroom-5	8	3	2	1	1	1
nursery-4	4	2	2	1	1	1
nursery-1	7	3	2	1	1	1
spect-test-1	10	9	4	2	2	1
teeth-1	4	4	3	2	1	1
teeth-5	3	3	3	2	1	1
tic-tac-toe-4	5	4	3	2	1	1
tic-tac-toe-3	6	4	3	2	1	1
zoo-data-5	7	7	4	2	2	1

Table 6, based on results from Tables 2 and 4, presents a comparison of the depth of α -decision trees for the proposed approach and the approach based on

Table 5. The average depth of α -decision trees – generalized decision approach

Decision table	α					
	0.0	0.001	0.01	0.1	0.2	0.5
balance-scale-1	3.00	2.76	2.00	1.00	1.00	1.00
breast-cancer-1	4.12	3.14	2.40	1.00	1.00	1.00
breast-cancer-5	2.60	2.19	1.40	1.00	1.00	1.00
cars-1	4.51	3.00	2.00	1.00	1.00	1.00
flags-5	3.91	2.83	1.88	1.00	1.00	1.00
hayes-roth-data-1	2.31	2.31	1.74	1.00	1.00	1.00
kr-vs-kp-5	9.49	5.33	3.89	2.00	2.00	1.00
kr-vs-kp-4	9.47	5.34	3.79	2.00	2.00	1.00
lymphography-5	4.36	3.57	2.44	1.65	1.00	1.00
mushroom-5	2.90	2.20	1.61	1.00	1.00	1.00
nursery-4	2.42	2.00	2.00	1.00	1.00	1.00
nursery-1	4.95	3.00	2.00	1.00	1.00	1.00
spect-test-1	4.34	4.31	3.83	2.00	2.00	1.00
teeth-1	2.82	2.82	2.27	1.41	1.00	1.00
teeth-5	2.21	2.21	2.21	1.36	1.00	1.00
tic-tac-toe-4	4.54	3.49	2.68	2.00	1.00	1.00
tic-tac-toe-3	5.28	3.69	2.60	1.70	1.00	1.00
zoo-data-5	4.10	4.10	3.19	2.00	2.00	1.00

Table 6. Comparison of depth of α -decision trees

Decision table	α					
	0.0	0.001	0.01	0.1	0.2	0.5
balance-scale-1	1.50	1.50	1.00	1.00	1.00	1.00
breast-cancer-1	1.17	1.00	1.00	0.50	1.00	1.00
breast-cancer-5	1.33	1.00	1.00	1.00	1.00	1.00
cars-1	1.00	1.00	1.00	1.00	1.00	1.00
flags-5	1.00	1.00	0.67	1.00	1.00	1.00
hayes-roth-data-1	1.50	1.50	1.00	1.00	1.00	1.00
kr-vs-kp-5	1.08	1.00	1.00	1.00	1.00	1.00
kr-vs-kp-4	1.17	1.00	1.00	1.00	1.00	1.00
lymphography-5	1.17	1.00	1.00	1.00	1.00	1.00
mushroom-5	1.14	1.00	1.00	1.00	1.00	1.00
nursery-4	2.00	1.00	2.00	1.00	1.00	1.00
nursery-1	1.00	1.00	1.00	1.00	1.00	1.00
spect-test-1	1.43	1.29	1.00	1.00	1.00	1.00
teeth-1	1.00	1.00	1.00	1.00	1.00	1.00
teeth-5	1.00	1.00	1.00	1.00	1.00	1.00
tic-tac-toe-4	1.00	1.00	1.00	1.00	1.00	1.00
tic-tac-toe-3	1.00	1.00	1.00	1.00	1.00	1.00
zoo-data-5	1.75	1.75	1.33	1.00	1.00	1.00

the generalized decision. Each input of this table is equal to the value of the

depth of an α -decision tree for the generalized decision approach divided by the value of the depth of an α -decision tree for the proposed approach.

Table 7, based on results from Tables 3 and 5, presents comparison of the average depth of α -decision trees for proposed approach and approach based on generalized decision. Each input of this table is equal to the value of average depth of α -decision tree for generalized decision approach divided by the value of average depth of α -decision tree for proposed approach.

The difference relative to depth and average depth of α -decision trees is noticeable for small values of α (see Table 6 and Table 7). In case of $\alpha \in \{0.2, 0.5\}$ values of depth and average depth of α -decision trees are almost the same. However, for Table 6, for decision table “nursery-4”, for $\alpha = 0.0$ and $\alpha = 0.01$, the depth of α -decision tree for proposed approach is 2 times shorter than for generalized decision approach. We can also find decision table (“breast-cancer-1” and $\alpha = 0.1$) where the depth of α -decision tree for proposed approach is longer (50%) than the depth of α -decision tree for generalized decision approach. Similar situation according to comparison of average depth, we can observe in Table 7.

Table 7. Comparison of the average depth of α -decision trees

Decision table	α					
	0.0	0.001	0.01	0.1	0.2	0.5
balance-scale-1	1.50	1.38	1.67	1.00	1.00	1.00
breast-cancer-1	1.11	1.03	1.08	0.74	1.00	1.00
breast-cancer-5	1.41	1.19	1.02	1.00	1.00	1.00
cars-1	1.56	1.14	1.00	1.00	1.00	1.00
flags-5	1.02	1.00	0.95	1.00	1.00	1.00
hayes-roth-data-1	1.33	1.33	1.00	1.00	1.00	1.00
kr-vs-kp-5	1.15	1.03	1.06	1.00	1.00	1.00
kr-vs-kp-4	1.16	1.02	1.03	1.00	1.27	1.00
lymphography-5	1.15	1.02	0.98	1.00	1.00	1.00
mushroom-5	1.04	1.00	1.00	1.00	1.00	1.00
nursery-4	1.82	1.50	2.00	1.00	1.00	1.00
nursery-1	1.76	1.50	1.20	1.00	1.00	1.00
spect-test-1	1.30	1.29	1.26	1.00	1.00	1.00
teeth-1	1.00	1.00	1.00	1.00	1.00	1.00
teeth-5	1.00	1.00	1.00	1.00	1.00	1.00
tic-tac-toe-4	1.51	1.20	1.15	1.49	1.00	1.00
tic-tac-toe-3	1.24	1.04	1.07	1.02	1.00	1.00
zoo-data-5	1.28	1.28	1.06	1.00	1.32	1.00

6 Conclusions

We presented the algorithm U_α , $0 \leq \alpha < 1$, for construction of α -decision trees, for decision tables with many-valued decisions. We showed that for an arbitrary

natural t , the considered algorithm has polynomial time complexity on tables which have at most t decisions in each set of decisions attached to rows. Based on results presented in Sect. 5, we can observe that greedy algorithm U_α constructs, for decision tables with many-valued decisions, α -decision trees with relatively short depth.

Based on results connected with comparison we can see that the depth and average depth of α -decision trees constructed in the framework of our approach (one decision from the set of decisions attached to row) is usually less than the depth and average depth of α -decision trees constructed in the framework of generalized decision approach (all decisions from the set of decisions attached to row). So, if it is enough to find only one decision, our approach can be used. However, it is necessary to find simpler uncertainty measures for greedy algorithm since the work with boundary subtables is too complicated.

References

1. Asuncion, A., Newman, D.J.: UCI Machine Learning Repository (<http://www.ics.uci.edu/~mllearn/>, 2007)
2. Azad, M., Chikalov, I., Moshkov, M., Zielosko, B.: Greedy algorithms for construction of approximate tests. *Fundam. Inform.* (2012) (to appear).
3. Chikalov, I., Zielosko, B.: Decision rules for decision tables with many-valued decisions. In Yao, J., Ramanna, S., Wang, G., Suraj, Z., eds.: *RSKT 2011*. Volume 6954 of LNCS., Springer (2011) 763–768
4. Cohen, W.W.: Learning trees and rules with set-valued features. In: *AAAI'96 - Volume 1*, AAAI Press (1996) 709–716
5. Moshkov, M., Zielosko, B.: *Combinatorial Machine Learning - A Rough Set Approach*. Springer, Heidelberg (2011)
6. Moshkov, M., Zielosko, B.: Construction of α -decision trees for tables with many-valued decisions. In Yao, J., Ramanna, S., Wang, G., Suraj, Z., eds.: *RSKT 2011*. Volume 6954 of LNCS., Springer (2011) 486–494
7. Moshkov, M., Zielosko, B.: Construction of tests for tables with many-valued decisions. In: *20th International Workshop Concurrency, Specification and Programming CS&P 2011*, September 28–30, Pultusk, Poland, Bialystok University of Technology (2011) 376–384
8. Orłowska, E., Pawlak, Z.: Representation of nondeterministic information. *Theor. Comput. Sci.* **29** (1984) 27–39
9. Pawlak, Z.: *Rough Sets - Theoretical Aspects of Reasoning about Data*. Kluwer Academic Publishers, Dordrecht (1991)
10. Pawlak, Z., Skowron, A.: Rough sets and boolean reasoning. *Inf. Sci.* **177**(1) (2007) 41–73
11. Pawlak, Z., Skowron, A.: Rudiments of rough sets. *Inf. Sci.* **177**(1) (2007) 3–27
12. Quinlan, J.R.: Induction of decision trees. *Mach. Learn.* (1986) 81–106
13. Skowron, A., Rauszer, C.: The discernibility matrices and functions in information systems. In: *Intelligent Decision Support. Handbook of Applications and Advances of the Rough Set Theory*. Kluwer Academic Publishers, Dordrecht (1992) 331–362
14. Yang, H., Xu, Z., Zhang, J., Cai, J.: A constructing method of decision tree and classification rule extraction for incomplete information system. In: *CASON '10*, IEEE Computer Society (2010) 49–52