The synchronization of Kuramoto oscillator networks: forecasting financial index critical points

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Abstract. Organizations can be modeled as complex systems. Here we consider complex economic organizations such as the stock exchange and show that the Kuramoto equation is a suitable model to forecast maxima or minima of the financial signals.

Keywords: Kuramoto model, non linear oscillators, financial indexes

1 Introduction

In this paper we give an interpretation of the internal dynamics of economic and financial markets in terms of non linear interdependent oscillator networks and use it to obtain forecasts of maxima and minima. To this end propose the Kuramoto model (KM) of nonlinear oscillators [1], as a novel approach to forecast the indicators of the market evolution (the “indexes”).Parameters from the Hilbert and Fourier analysis of the index are imposed (as natural frequencies, couplings, initial conditions) to the KM to allow the order parameter to mimic the index phase behaviour as a consequence of the oscillators’ attempt to get synchronized with the index. In the past, Dal’Maso Peron [2] has constructed a correlation matrix from stock returns according to a varying time window. This matrix can be read as an adjacency matrix, whose entries are the couplings of the KM. Peron and colleagues show that a financial crisis appears when synchronization pushes markets towards a common behavior that reinforce itself again and again [7]. In [3] Junior and Franca, following Sornette and Johansen [4, 5] use a damped oscillator to realize a local approximation to the market index before crashes measuring the interdependences among markets. Yalamova [6] also proposes a theoretical model of synchronization of trading activities and suggests that crashes occur when all traders evolve towards a single trading rule. The self-organization of traders may produce criticality and crashes: a group of agents forms the nodes of a network, each node is modeled as a phase oscillator and the whole network is described by the famous Kuramoto equations [1]:

$$\vartheta_i' = \Omega_i + 1/N \ast\sum_k C \ast sin(\vartheta_k - \vartheta_i) \;,\; i,k = 1,2, \ldots N_{osc}$$

Vassilieva [10] uses oscillators in a way that allows learning; authors claim that a network of oscillators can learn to memorize and recall many noisy patterns changing the natural frequencies, rather than changing the coupling strengths as in neural nets. Finally, Preis and Stanley [8] study the amplitude of stocks volume and price fluctuations to find regularities close to the “switching points” (local minima/maxima) of the index. Differently from the above Authors, we focus on the phase of the index in order to use the capability of the Kuramoto model, somehow “learning” the original phase oscillations of the signal. Our goals are to mimic the phase behavior of the financial signal and forecast its major maxima and minima.

2 Formal analysis

Now we show how to obtain the basic Kuramoto parameters form the time-series. Starting from the (normalized) index $y(t)$ of real values:

$$y(t) = (1/N) \ast \sum_i y_i(t) \qquad i = 1,2, \ldots N$$

considering the Fourier series and the analytical signal:

$$y(t)= (1/N) \ast Re\{ \sum^{\infty}_{k=-\infty} Y_k \ast e^{j\omega_k t} \}$$

$$y(t)= (1/N) \ast Re\{2 \ast y^+(t) \} \text{, where: } y^+(t) \approx a \ast e^{j(\omega t+\varphi(t))} \text{ if } \omega > \mathrm{d}(\varphi(t))/\mathrm{d}t$$

$$Re\,\{2/N\, a_i \ast e^{j(\omega i t + \varphi i(t))}\} = Re\,\{1/N\, (\sum^{\infty}_{k=-\infty} Y^i_k \ast e^{j\omega i_k t})\} \text{ ,for } i = 1, 2, ..N$$

Deriving with respect to time and given $\omega_i t + \varphi_i(t) = \vartheta_i(t)$

$$1/N\, a_i \vartheta_i' = Y^i_o/N + 1/N \sum_k R(t) \ast \omega^i_k \ast Y^i_k \ast sin(\pi + \Psi - \vartheta_i(t))$$

Therefore the index $y(t)$ is:

$$y' \approx \sum_i (Y^i_o/N + 1/N \sum_k R(t) \ast \omega^i_k \ast Y^i_k)$$

3 The simulation results

In order to evaluate the goodness of the accordance between $R(t)$ and the index the phase synchronization parameter (*PS*) was used [9]: $PS = | \langle e^{j(\varphi_x(t) - \varphi_z(t))} \rangle |$ where $x(t)$ and $z(t)$ are the signals to be evaluated. $PS = 1$ means complete synchronization (*i.e. every critical points is exactly forecasted*), $PS = 0$ complete incoherence (*i.e. no forecast is correct*). In Figures 1, 2, 3 are shown the simulation results. While the amplitude forecast is poor, the major critical points of the index are clearly indicated and appear synchronized with the order parameter, while the other critical point have less influence.

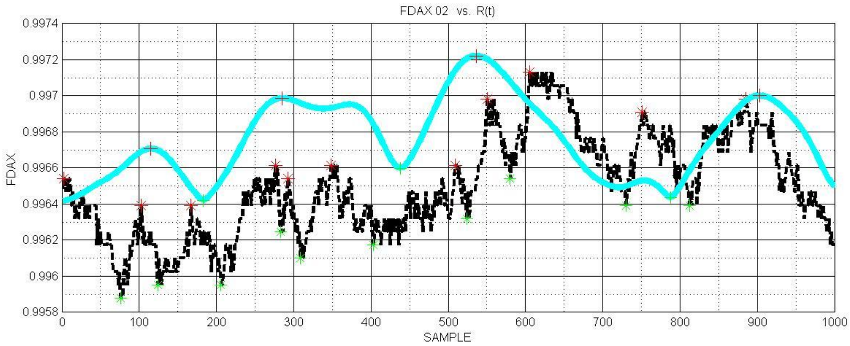
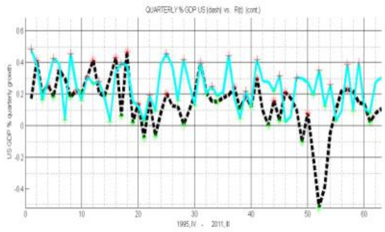


Fig. 1 German FDAX index (2002), 1000 data. The forecast *R*(t)$_*$*const* is blue, FDAX is dashed black. Major critical points are indicated as + (forecasts) e * (actual data). *PS* = 0.42 shows a very good accordance between the two curves. In this simulation 100 oscillators were used.



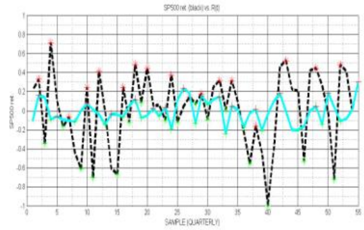


Fig. 264 data of the US quarterly GDP index1995 to 201164 were forecasted. The forecast *R*(t)$_*$const is blue, US GDP is dashed black. *PS* = 0.22 shows a good accordance between the two curves. In this simulation 13 oscillators were used. **Fig. 3** Standard and Poor 500 index 2010 (black dashed) normalized returns, 54 quarterly data. *PS* = 0.19; 13 oscillators used.

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