Non-Rigid Morphological Image Registration*

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Abstract. A variational method to non rigid registration of multi-modal image data is presented. A suitable deformation will be determined via the minimization of a morphological, i.e., contrast invariant, matching functional along with an appropriate regularization energy.

1 Introduction

Various different image acquisition technologies such as computer tomography and magnetic resonance tomography and a variety of novel sources for images, such as functional MRI, 3D ultrasound or densiometric computer tomography (DXA) deliver a range of different type of images. Due to different body positioning, temporal difference of the image generation and differences in the measurement process the images frequently can not simply be overlayed. Indeed corresponding structures are situated at usually nonlinearly transformed positions. In case of intra-individual registration, the variability of the anatomy can not be described by a rigid transformation, since many structures like, e. g., the brain cortex may evolve very differently in the growing process. Frequently, if the image modality differs there is also no correlation of image intensities at corresponding positions. What still remains, at least partially, is the local image structure or "morphology" of corresponding objects.

In the context of image registration, one aims to correlate two images – a reference image R and a template image T – via an energy relaxation over a set of in general non-rigid spatial deformations.

Let us denote the reference image by $R : \Omega \to \mathbb{R}$ and the template image by $T : \Omega \to \mathbb{R}$. Here, both images are supposed to be defined on a bounded domain $\Omega \in \mathbb{R}^d$ for $d = 1, 2$ or 3 with Lipschitz boundary and satisfying the *cone condition* (cf. e. g. [1]). We ask for a deformation $\phi : \Omega \to \Omega$ such that $T \circ \phi$ is optimally correlated to R.

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The congruence of the shapes instead of the equality of the intensities is the main object of the registration approach presented here. At first, let us define the morphology $M[I]$ of an image I as the set of level sets of I:

$$M[I] := \{\mathcal{M}_c^I \,|\, c \in \mathbb{R}\}, \tag{1}$$

where $\mathcal{M}_c^I := \{x \in \Omega \,|\, I(x) = c\}$ is a single level set for the grey value c. I.e. $M[\gamma \circ I] = M[I]$ for any reparametrization $\gamma : \mathbb{R} \to \mathbb{R}$ of the grey values. Up to the orientation the morphology $M[I]$ can be identified with the normal map (Gauss map)

$$N_I : \Omega \to \mathbb{R}^d;\; x \mapsto \frac{\nabla I}{\|\nabla I\|}\,. \tag{2}$$

Let us call two images I_1 and I_2 morphologically equivalent if $M[I_1] = M[I_2]$. Morphological methods in image processing are characterized by an invariance with respect to the morphology [9]. Now, aiming for a morphological registration method, we will ask for a deformation $\phi : \Omega \to \Omega$ such that

$$M[T \circ \phi] = M[R]\,.$$

Thus, we set up a matching functional which locally measures the twist of the tangent spaces of the template image at the deformed position and the deformed reference image or the defect of the corresponding normal fields.

2 A morphological registration energy

In this section we will construct a suitable matching energy, which measures the defect of the morphology of the reference image R and the deformed template image T. Thus, with respect to the above identification of morphologies and normal fields we ask for a deformation ϕ such that

$$N_T \circ \phi \,||\, N_R^\phi\,, \tag{3}$$

where N_R^ϕ is the transformed normal of the reference image R on $\mathcal{T}_{\phi(x)}\phi(\mathcal{M}_{R(x)}^R)$ at position $\phi(x)$. From the transformation rule for the exterior vector product $D\phi\, u \wedge D\phi\, v = \mathrm{Cof}\, D\phi(u \wedge v)$ for all $v, w \in \mathcal{T}_x\mathcal{M}_{R(x)}^R$ one derives

$$N_R^\phi = \frac{\mathrm{Cof}\, D\phi\, N_R}{\|\mathrm{Cof}\, D\phi\, N_R\|}$$

where $\mathrm{Cof}\, A = \det A \cdot A^{-T}$ for invertible $A \in \mathbb{R}^{d,d}$. In a variational setting, optimality can be expressed in terms of energy minimization. We thus consider the following type of matching energy

$$E_m[\phi] := \int_\Omega g_0(\nabla T \circ \phi, \nabla R, \mathrm{Cof}\, D\phi)\, \mathrm{d}\mu. \tag{4}$$

where g_0 is a 0-homogenous extension of a function $g : S^{d-1} \times S^{d-1} \times \mathbb{R}^{d,d} \to \mathbb{R}^+$, i. e., $g_0(v, w, A) := 0$ if $v = 0$ or $w = 0$ and $g_0(v, w, A) := g(\frac{v}{\|v\|}, \frac{w}{\|w\|}, A)$ otherwise. As a first choice for the energy density g let us consider If we want to achieve invariance of the energy under non-monotone grey-value transformation, the following symmetry condition

$$g(v, w, A) = g(-v, w, A) = g(v, -w, A)\,. \tag{5}$$

has to be fulfilled. A useful class of matching functionals E_m is obtained choosing functions g which depend on the scalar product $v \cdot u$ or alternatively on $(\mathbb{1} - v \otimes v)u$ (where $\mathbb{1} - v \otimes v = (\delta_{ij} - v_i\, v_j)_{ij}$ denotes the projection of u onto the plane normal to v) for $u = \frac{Aw}{\|Aw\|}$ and $v, w \in S^{d-1}$, i. e.,

$$g(v, w, A) = \hat{g}\left((\mathbb{1} - v \otimes v)\frac{Aw}{\|Aw\|}\right)\,. \tag{6}$$

Let us remark that $\hat{g}((\mathbb{1} - v \otimes v)u)$ is convex in u, if $\hat{g}$ is convex. With respect to arbitrary grey value transformations mapping morphologically identical images onto each other, we might consider $\hat{g}(s) = \|s\|^\gamma$ for some $\gamma \geq 1$.

3 Regularization

Suppose a minimizing deformation ϕ of E_m is given. Then, obviously for any deformation ψ which exchanges the level sets $\mathcal{M}_c^R$ of the image R, the concatenation $\psi \circ \phi$ still is a minimizer. But ψ can be arbitrarily irregular. Hence, minimizing solely the matching energy is an ill-posed problem. Thus, we consider a regularized energy

$$E[\phi] = E_m[\phi] + E_{reg}[\phi]\,. \tag{7}$$

We interprete Ω as an isotropic elastic body and suppose that the regularization energy plays the role of an elastic energy while the matching energy can be regarded as an external potential contributing to the energy. Furthermore we suppose $\phi = \mathbb{1}$ to represent the stress free deformation.

$$E_{reg}[\phi] := \int_\Omega a\|D\phi\|_2^p + b\|\mathrm{Cof}\, D\phi\|_2^q + \Gamma(\det D\phi)\,\mathrm{d}\mu \tag{8}$$

with $\Gamma(D) \to \infty$ for $D \to 0, \infty$, e. g., $\Gamma(D) = \gamma D^2 - \delta \ln D$. In nonlinear elasticity such material laws have been proposed by Ogden and for $p = q = 2$ we obtain the Mooney-Rivlin model [3].

4 An existence result

Let us introduce a corresponding set of functions

$$\mathcal{I}(\Omega) := \left\{ I : \Omega \to \mathbb{R} \;\middle|\; I \in C^1(\bar{\Omega}), \exists\, \mathcal{D}_I \subset \Omega \text{ s. t. } \nabla I \neq 0 \text{ on } \Omega \setminus \mathcal{D}_I, \right.$$
$$\left. \mu(B_\epsilon(\mathcal{D}_I)) \stackrel{\epsilon \to 0}{\longrightarrow} 0 \right\}\,.$$

We then have the following result [5].

Theorem 1 (Existence of minimizing deformations) *Suppose* $d = 3$, $T, R \in \mathcal{I}(\Omega)$, *and consider the total energy for deformations* ϕ *in the set of admissible deformations*

$$\begin{aligned}\mathcal{A} := \{\phi : \Omega \to \Omega \mid \phi \in H^{1,p}(\Omega), \operatorname{Cof} D\phi \in L^q(\Omega), \\ \det D\phi \in L^r(\Omega), \det D\phi > 0 \ a.e.\ in\ \Omega, \phi = 1\!\!1 \ on\ \partial\Omega\}\end{aligned}$$

where $p, q > 3$ *and* $r > 1$. *Suppose* $W : \mathbb{R}^{3,3} \times \mathbb{R}^{3,3} \times \mathbb{R}^+ \to \mathbb{R}$ *is convex and there exist constants* $\beta, s \in \mathbb{R}$, $\beta > 0$, *and* $s > \frac{2q}{q-3}$ *such that*

$$W(A, C, D) \geq \beta\,(\|A\|_2^p + \|C\|_2^q + D^r + D^{-s}) \quad \forall A, C \in \mathbb{R}^{3,3},\ D \in \mathbb{R}^+ \tag{9}$$

Furthermore, assume that $g_0(v, w, A) = g(\frac{v}{\|v\|}, \frac{w}{\|w\|}, A)$, *for some function* $g : S^2 \times S^2 \times \mathbb{R}^{3,3} \to \mathbb{R}_0^+$, *which is continuous in* $\frac{v}{\|v\|}, \frac{w}{\|w\|}$, *convex in* A *and for a constant* $m < q$ *the estimate*

$$g(v, w, A) - g(u, w, A) \leq C_g \,\|v - u\|\, (1 + \|A\|_2^m)$$

holds for all $u, v, w \in S^2$ *and* $A \in \mathbb{R}^{3,3}$. *Then* $E[\cdot]$ *attains its minimum over all deformations* $\phi \in \mathcal{A}$ *and the minimizing deformation* ϕ *is a homeomorphism and in particular* $\det D\phi > 0$ *a.e. in* Ω.

Refer to [5] for functions g, for which the requirements of the theorem are fulfilled, an additional feature based energy and a description of the multiscale minimization algorithm, as well as further references.

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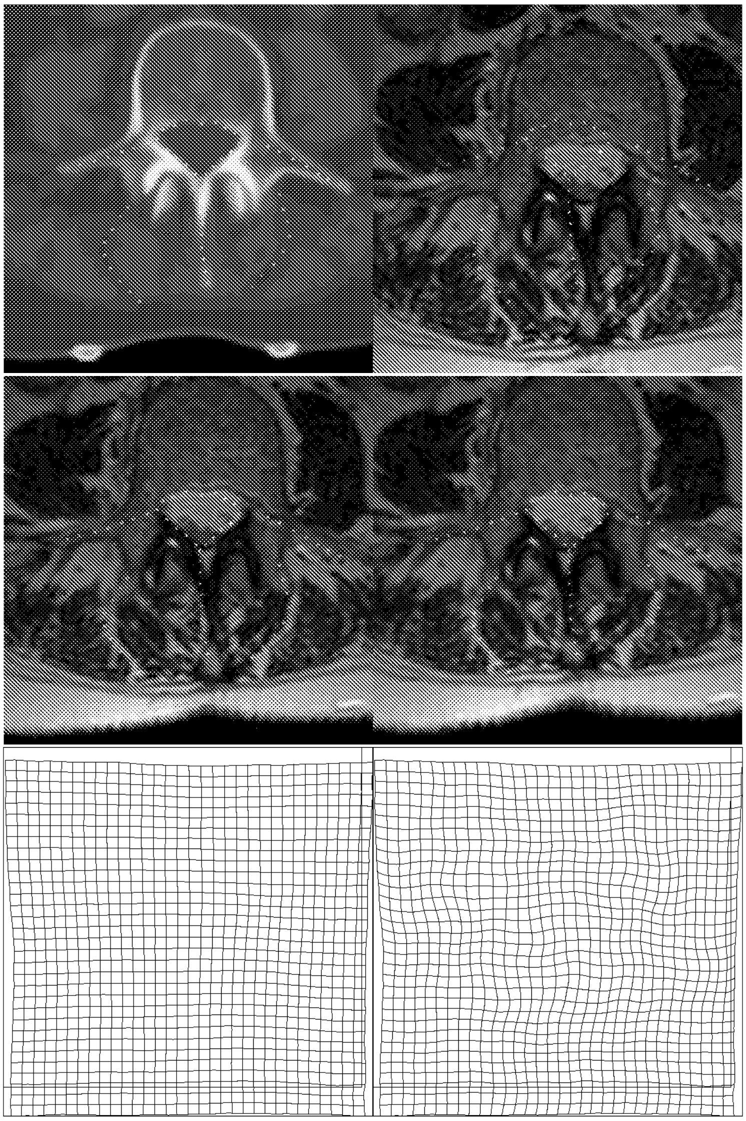

Fig. 1. Sectional morphological registration on a pair of MR and CT images of a human spine. Dotted lines mark certain features visible in the reference image. Top Left: reference, CT, Top Right: template, MR, with clearly visible misfit of structures marked by the dotted lines. Middle Left: deformed template $T \circ \phi_f$, where ϕ_f is the result of a feature based pre-registration [5]. Middle Right: deformed template $T \circ \phi$ after final registration where the dotted feature lines nicely coincide with the same features in the deformed template MR-image. All images have a resolution of 257^2. Additionally the deformations after the feature based registration resp. after the entire registration process are illustrated in the bottom row.