An Analysis of Final Grades in a Mathematics Course for the Enhancement of Upcoming Engineering Students' Academic Performance by Using Multivariate Statistical Techniques.

Alberto Hananel[1,2], Sandra Loaiza[1] and Jaime Collantes[3]

[1]Santo Toribio de Mogrovejo Catholic University, Av. San Josemaria Escriva de Balaguer 855, Chiclayo, 14012, Peru
[2]University of Granada, Av. del Hospicio, s/n, Granada, Granada, 18010, Spain
[3]Pedro Ruiz Gallo National University, Juan XXIII 391, Lambayeque, 14013, Peru

Abstract

The objective of this work is to show the importance of the use of Multivariate Statistics in the field of Research, particularly in Teaching Innovation. With the required mathematical support, it was proven that it is possible to improve the grading system semiannually providing feedback on the evaluation instruments and the weightings used in the formulas to obtain the final average of a previous subject, in which unsatisfactory results were obtained, using different methods of multivariate statistical analysis such as multiple linear regression, multivariate normal distribution, the principal components method and factorial analysis. The results obtained suggest that it is preferable to condense classic evaluations such as exams and assignments, in learning outcomes by units, and that it is necessary to incorporate a different evaluation, which allows assessing punctual attendance to classes, since, although usually this variable is not part of the evaluation, it was found to be a key piece in the approval of a subject. Finally, a Chernoff face analysis was carried out for the comparison of the final results obtained in the "MATLAB" 2018-II and "Numerical Methods" 2019-I subjects, taught at the School of Environmental Civil Engineering of the participant university, concluding that the implementation of this proposal, with which the approval of the entire second group of students was achieved, allows the optimization of academic performance through a methodology that is aimed at any teacher who wishes to replicate the same.

Keywords

Chernoff faces, multivariate statistics, teaching innovation, academic performance

CISETC 2023: International Congress on Education and Technology in Sciences, December 04–06, 2023, Zacatecas, Mexico.

ahananel@ugr.es (A. Hananel); sloayza@usat.edu.pe (S. Loaiza); lcollantes@unprg.edu.pe (J. Collantes)
0000-0003-4113-8623 (A. Hananel); 0000-0002-0311-1692 (S. Loaiza); 0000-0001-9262-9399 (J. Collantes)

1. Introduction

One of the most important problems of the Peruvian higher education system in the area of Engineering is the low approval rate of students in Mathematics, which added to what is established in article 102 of the Peruvian University Law 30220: “the disapproval of the same subject three times gives rise to the student being temporarily separated from the university for one year, being able to enroll only in this subject for the following academic semester, and proceeding to definitive withdrawal if the student fails for the fourth time”, causes, among other things, an increase in the cost of academic credit, which is reflected as a latent difficulty, especially for the families of those university students with said academic problems and limited resources, causing greater family expenses, expenses for public universities. and private, greater investment in human and technological resources for the Peruvian state, as well as the “waste” of university places in Engineering majors, which ultimately has an impact on student dropouts from the educational system.

Without downplaying the lack of interest or prerequisites of some students to face these subjects, in addition to other factors, Marsh pointed out in 1980 [1] that the understanding of a subject is influenced by who teaches it, especially when evaluating mathematical skills. In fact, three decades later, it was proven that teaching-evaluation practices in the classroom can have many nuances, but they fundamentally depend on the intervention of the teacher [2]. Consequently, we are convinced that strengthening teaching capabilities in formative assessment is very important because theoretically it contributes to achieving progress in students, improving their learning since it is one of the valuable strategies when developing competencies [3]. This suggests stopping seeing the evaluation as a grade, choosing which evaluations should be formative and which summative and reviewing the formulas to obtain the final average more carefully, which is why it is essential that teachers, who use these formulas, before starting a new academic semester can provide feedback on the weights used in their formulation, modifying their current evaluation tool to better measure the student’s progression towards the desired results: research to innovate teaching.

This research was based on the final results of the subject “MATLAB” 2018-II, obtained by students from Santo Toribio de Mogrovejo Catholic University (USAT) School of Civil and Environmental Engineering. Firstly, Matrix and Vector Algebra was used, finding some patterns in students who passed the course with the minimum passing grade, and in turn it allowed us to characterize the partial grades of the students with the best averages in the subject, finding a mathematical basis of the importance of the “puntuality” in class variable, and at the same time highlighting the importance of the “responsibility” variable, suggesting the need to establish a different evaluation instrument that not only measures the classic qualified practices, partial exams and group work, but also allows the performance or achievement obtained class by class to be measured with some score and that is also evident in the weight of a new evaluation instrument to be built. Secondly, using the multivariate normal function, we investigated whether the chosen students follow a usual distribution or not to determine how many students could have atypical behavior, in order not to bias the investigation, since some pattern was found in the first weeks of evaluations it would be possible to detect, above all, students with low performance with whom some methodological strategies could be used so that, above all, they do not abandon the subject in the first weeks. Third, a review of the multiple linear regression was performed to determine if there was significance with the proposed evaluation model and if it is possible to discard some of the components of the typical evaluation system using the variable selection method. Fourthly, factorial analysis was applied to obtain the number of ideal principal components in order to improve the explanation of the statistical model and the interpretation of blocks of variables and individuals that can be associated to reduce the dimensionality of the problem due to the high number of the variables chosen by the researcher to evaluate the students. Finally, a Chernoff face analysis was carried out as a graphic representation of the data in order to find intuitive characteristics that represent variables that are very difficult to understand by themselves.

This research aims to highlight the importance of the use of multivariate statistics in the research area, particularly in teaching innovation, designing a methodology to optimize the academic performance of a group of students by providing feedback consistent with the results obtained in the previous academic semester, since one of the proposed objectives was to find relationships between students passing, failing, high and low scores and a set of proposed evaluations using different multivariate statistical analysis techniques, as well as improving the typical grading system by providing feedback. of the evaluation instrument and the respective weights used in the formulas to obtain the final grade of the subject, contrasting the final grades obtained between the subjects "MATLAB" 2018-II (without teaching innovation) and "Numericla Methods" 2019-I (with teaching innovation).

Finally, it is worth mentioning that the purpose of this unpublished research is to motivate teachers to use this methodology to corroborate their final results and optimize the academic achievement of learning Sciences and Humanities according to this methodology. It is important to note that this research has the disadvantage of having been carried out in a single academic semester and prior to the pandemic, however, it has had the advantage that 100 % of the students subjected to the application of the aforementioned methodology passed the subject, fueling interest in continuing to replicate it, especially with the advances that are currently available given the virtuality, hoping that the use of quantitative methods such as the one worked on becomes a solution alternative solution to the problem of low academic performance of some students, especially in universities like ours whose minimum passing grade for a subject is 14 in vigesimal base.

2. Theoretical Framework

Statistics is an important research tool that aims to describe and seek the appropriate approach to problems based on inferences about a set of data that allow us to model a reality. In fact, research in higher education tends to produce a large amount of data, so the use of some statistical methods for its processing, analysis and interpretation plays a transcendent role in this scenario, evidencing the need for teachers to know the contents. of Statistics in a factual way to solve the problems of their own educational reality. However, despite the fact that this reality can be applied to a wide variety of subjects in which Mathematics for Engineering fits, and despite the fact that the majority of its teachers take Statistics subjects in their curriculum, it is unfortunate to observe that do not use it for educational research [4]. This new context requires that teachers continually train and show positive attitudes in the use of these innovative means, especially for evaluation. Although in recent decades' enormous progress has been made in the incorporation of educational technology, the training of teachers who use it results in a slower process. Currently, some research has been carried out in the educational field, which has resulted in new approaches that require the use of digital processing tools and methodologies such as what is commonly known as Big Data [5]. However, this methodology dates back to the late nineties when a way to represent a large amount of information was shown using a diagram of faces and asymmetries [6], However, due to the lack of adequate software at that time, it was limited like most Multivariate Statistics methods [7], and consequently it would not have been possible to adapt it to evaluate student performance. Yang's principal components research [8] rescues its added value with respect to variable selection techniques, which he mentions can be applied to mathematical formulas, especially to averages.

Of all the statistical methods, there is one that allows large amounts of data to be graphically compared, known as Chernoff faces, which helps researchers detect patterns, clusters and correlations. In this method, each of the person's facial features is represented by a data set of a particular variable. There are some studies like Morris's [9], that demonstrate the effectiveness of this method in experimental analysis, but that have not been specific in educational research to understand how adequate feedback on evaluations and grades could help identify some variables that influence passing a subject, which in our specific case It is limited to Mathematics for Engineering. Not much research has been carried out in this field, however, among the most notable is a research on attitudes towards mathematics among Engineering students from Venezuelan autonomous universities that used Multivariate Statistics, carrying out a factorial analysis of the principal components that determined a coherent structure made up of three groups: tastes, difficulty and usefulness [10]. In the same year, a study was carried out on academic performance using the multivariate statistical technique of logistic regression, concluding that one of its significant variables is the way of obtaining the average grade of the students, highlighting the importance of not failing a subject in this first stage of the degree since it could condition the student's subsequent academic results [11]. This justifies our concern to prevent engineering students from failing a Mathematics subject that is always present in the first years of their degree.

Many of the multivariate analysis techniques for these data have been illustrated in the exhaustive study carried out by Pérez [12]. Sabiote [13] conducted a case study in a Science subject to analyze the correlational-predictive importance of the influence of class attendance on university academic performance. Ávila [14] conducted a study in which it was determined that the inclusion of mathematical software turns out to be an indispensable tool for the development of thinking skills and the improvement of learning.

From 2012 to the present, some works such as Guzmán's [15] have put predictive and explanatory models based on multivariate statistics to find variables that may have an influence on university academic performance, especially in Mexico, however in Peru there are no investigations of this type despite the fact that university authorities continually express their concern to avoid student dropout in the first year of the degree due to the disapproval of core subjects such as Mathematics, a subject that from our point of view should not cause problems for those who study Engineering due to its presumed affinity, but which nevertheless our daily work in the classroom, even more so after the pandemic, reflects another reality.

3. Methodology and Description of the Experience

The study population with which we worked has been made up of all the students of the School of Environmental Civil Engineering enrolled in the subject MATLAB 2018-II (35), whose scatter graph of all the qualifiers according to all their evaluations is given by Figure 1.

Figure 1: Matrix Plot of the results in the different student evaluations "MATLAB" 2018-II.

3.1. MATLAB 2018-II Sample Experiments: Matrix and Vector Algebra

3.1.1. Patterns of students who passed with 14

For the spectral analysis by eigenvalues, two students who passed the subject with symmetry characteristics were considered, where one of them obtained 14 in the average of exams and the other student 14 in the average of group work, with the remaining grades being failing. Their respective eigenvalues were calculated in each of the groups, which, resorting to spectral decomposition, turned out to have the same eigenvectors {[0,65, 0,75}; [0,75, 0,65]} (grades between 13 and 15) for different eigenvalues {23,27, 4,97}. This means that between the 65 % and the 75 % of students who pass the subject with 14. They do it by not achieving notes between 14 and 14,49 but obtaining approximately 13,33. Statistically this means that the majority of students who pass the subject do this "with the fair".

3.1.2. Importance of punctual attendance to classes

Two students with the best averages in the subject were considered for this test, and considering three variables: exam average, group work average and percentage of punctual attendance to the subject. Being F and V the grade vectors of the students with the best averages in the subject, a relationship was established between them, for which their joint projections were calculated $L_F \approx 31{,}6158, \quad L_V \approx 31{,}6115, being \quad \cos(\theta) = \frac{FV}{L_F L_V} \; and \quad obtaining \quad \theta \approx 0{,}30°$

This means that results among these students are very similar in terms of a generally progressive performance that is projected to obtain the highest averages. Projection of a randomly approved student was found υ on $F\left(\frac{vF}{L_v^2}v\right)$ obtaining the vector [17,40, 18,64, 103,59]. This means that a student who wants to obtain the highest averages in the subject should strive to achieve 17 as an exam average and 18 in the group work average, but at the same time he or she must attend the class punctually to 100 % of all classes. This suggests that to obtain the highest grade in the subject, in addition to obtaining high grades, one always attends classes.

3.1.3. Responsibility and punctual attendance lead to approval

Three students were considered who obtained the best averages and let $x1$, $x2$ and $x3$ be vectors containing the average of exams, the average of group work and the percentage of attendance to the subject, respectively. The correlation between the exam grades and the semi-sums of the presentation grades and the percentage of punctual class attendance was calculated $\frac{1}{2}x_2 + \frac{1}{2}x_3$.

We proceeded to see how related their proportion is, for this the correlation coefficient was obtained, which turned out to -0,7178 (partially good correlation), which means that the probability is good that a good student of the subject has passed given that he obtained good results in his group work and almost always attended classes. This suggests having reached a pattern, which indicates that obtaining good grades in group work and regularly attending classes directly influences obtaining good results in the exams, and this also leads to their approval.

3.1.4. Need for a different evaluation grade

A symmetric matrix of order 2 called S was considered, whose information contains a qualifier higher than the 20th average grade on the main diagonal (11) and the minimum passing grade (14) with respect to the measurement of two students being $X = [x_1, x_2]$ exam notes and group work, x_1, x_2, respectively and with a mean $\overline{X}$,which were measured in the absence of one more grade to estimate the minimum grade required. The solid ellipsoid of certainty was constructed $(X-\overline{X})' S^{-1} (X-\overline{X}) \leq 1$ order to observe the possibility of stratification of sectors. It was determined that the graph obtainedwas not precisely an ellipsoid of certainty but a hyperboloid, which suggests that when students pass with the minimum grade 14 It is not known exactly if “they deserved to approve”, because 70 % of real learning of the subject was not obtained, but the third evaluation could have been decisive in obtaining a high grade that allowed the student to “mathematically” pass according to the formula for obtaining the average of the subject with at least 13,5, but where it is clear that the students of the subject “MATLAB” 2018-II manage to pass it but with the minimum grade. This suggests that the weight of the final average assigned to a new item should not be very high to avoid the approval of students who do not reach the minimum required competencies (they should have at most between 15 % and 17,5 % of the "weight" of the subject).

3.1.5. Appropriate weight for a qualifier of an evaluation not considered

Let $A(x, y)$ be a function that considers the influence of punctual class attendance on the approval of students who present qualifications as in the previous section such that A is given by $0{,}5(x + y + 8)$ si $0 < x < 11$ and $0 < y < 14$, and 0 in any other case. This function weights the previous notes by adding up to a maximum 4/3 knitted (1,33) according to the weights assigned to both the exam grade and the group work grade and the weight of a new evaluation that would be considered a product of punctual class attendance. To do this, we proceeded to find the probability that a student who obtains a grade greater than 8 in the average of exams (in particular 14), obtains as an average of group work a grade less than 10 with which he fails the subject, since It was analyzed that if a student passes the exams with 14, there should be a third evaluation that allows the group work grade to not have much influence on the disapproval of the subject in the event of some unfortunate slip. To achieve this task, a conditional probability was calculated $P\ (x > 8/y < 10)$ getting 0,11. This means that the probability that a student achieves an average grade in the exams greater than 8 but fails the group work with a grade less than 10 is very low and equal to 0,11. Therefore, it is necessary to consider in such cases a new evaluation that assesses punctual attendance at classes with some weighting of at least the 11 % of the course weight, which is in accordance with the previous section. This note will henceforth be called "individual work average".

3.2. Population experiments: Multivariate Normal Distribution

One of the rules of the syllabus of the subject was considered, which is quite frequent in the preparation of the same by teachers in the area of Mathematics for Engineering, with which the majority of students obtain passing grades according to a normality, which mentioned that "the grade for each group work in any unit is the semi-sum of the report grade with the support grade, the latter will be given randomly to any of the students in the group. If the support does not exist, the grade of the report will be considered only". The bivariate normal population was considered $X = [x_1, x_2]$ with means $\mu = [\mu_1, \mu_2]$, respectively. Let x_1, be the students who obtained "zero" (minimum grade) as average of the supports of the group works ($\mu_1 = 0$), and let x_2 be with $\mu_2 = 2$, students who obtained at least "two" of "ten" (maximum grade) in such evaluation. The bivariate normal density function was considered:

$$f(x_1, x_2) =$$

$$\frac{1}{2\pi\sqrt{\sigma_{11}\sigma_{22}(1-\rho_{12}^2)}} e^{-\frac{1}{2(1-\rho_{12}^2)}\left[\left(\frac{x_1-\mu_1}{\sqrt{\sigma_{11}}}\right)^2+\left(\frac{x_2-\mu_2}{\sqrt{\sigma_{22}}}\right)^2-2\rho_{12}\left(\frac{x_1-\mu_1}{\sqrt{\sigma_{11}}}\right)\left(\frac{x_2-\mu_2}{\sqrt{\sigma_{22}}}\right)\right]}$$

The variances and covariances were obtained $\sigma_{11} = 2$, $\sigma_{22} = 1$ and $\rho_{12} = 0.5$. The constant density contours were determined with the 50% of probability, which throught the calculation of the eigenvalues 2,37, 0,63 determined the values ρ_{12} = 0,5, θ = 60∘, RM = 0,90, Rm = 1,74, those that allowed us to construct the graph of the normality ellipsoid where the atypical behavior of students who present grades lower than 2 in their group work is observed. To determine precisely the number of students who follow this distribution, the calculation of the marginal distributions (semi-sum and semi-difference of the maximum and minimum grades in these supports) was carried out for the population vector $V = [V_1 V_2]$ obtaining the joint

density $V \sim N\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1,10355 & 0,25 \\ 0,25 & 0,396447 \end{bmatrix}\right)$ from which it was concluded that V

followed a normal distribution and it was observed that the students who did not follow a normal distribution (0.25) are approximately a quarter of the population, which is approximately the number of disapproved students with very low grades (between 0 and 2) in the assignments. group, this suggests an approximate of 8 students who completely failed the subject without the slightest concern about any of such activities.

The results suggest that a random student who shows interest in supporting his group work has a high probability of passing the subject. Coincidentally, according to the calculations carried out to obtain the ellipsoid, the students who failed in the subject turned out to be those people who also obtained grades no higher than 2 in support of their work, with which it can be statistically inferred that those who do not pay interest in performing in an acceptable manner in supporting their group work, nor do they have a high probability of passing the subject.

3.3. Sample population estimates: Multiple linear regression models

Let y, x_1, x_2, x_3, be the final average, the exam average, the group work average, and the percentage of punctual attendance to the subject with respect to the study population "MATLAB" 2018-II.

The coefficients were estimated according to the model $\mu_{y,x1,x2,x3} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$, getting $\beta_0 = -0{,}0286762$, $\beta_1 = 0{,}710764$, $\beta_2 = 0{,}266286$ and $\beta_3 = 0{,}166012$ generating a multiple linear regression model of the form:

$$Final_{Average} = -0{,}0286762 + 0{,}710764x_1 + 0{,}266286x_2 + 0{,}166012x_3,$$

observing that the weight of the grades obtained in the exams had the greatest influence on the final average, followed by the evaluations of the group work and with a non-negligible and significant weight in the percentage of class attendance, which means that the students who attend classes punctually were favored with at least 3 points compared to the average. To validate the significance of these variables in the way of obtaining the final average mentioned in the syllable (for removal otherwise), a variable selection technique was applied, which showed that none should be discarded, verifying that the evaluated variables must be maintained. This means the evaluations should be weighted with approximately the following weights: More than double and less than triple (β_1/β_2) of weight to the exams than to the assignments, and some different evaluation activity must be implemented in the classroom that motivates class attendance, which should represent approximately a quarter of the weight of the exam grade (β_3/β_1), in addition to considering an evaluation that does not depend on exams, assignments, or evaluations assigned in the classroom, which has an impact on at least one point of the final average of some activity to counteract the impact of the greater weight of the evaluation of the exams (this is reflected in the coefficient $\beta_0 = -0{,}0286762$), possibly with some grade coming from some voluntary assessed activity in class. This would lead to considering an average of the form:

Exam 1: 55 % of 35 %, making a total of 19 % to the weight of the first unit exam,
Exam 2: 55 % of 25 %, making a total of 1 % to the weight of the second unit exam,
Exam 3: 55 % of 15 %, making a total of 8 % to the weight of the second unit exam,
Exam 4: 55 % of 25 %, making a total of 17 % to the weight of the third unit exam,
which proposes assigning 61 % from weight to exams, 22,5 % to works and 15,25 % to any other activity leaving a margin of 1,25 % (up to two and a half points average on the scale of 20), so considering similar weights, but more "friendly" to the obtained, the formula for obtaining the final average of the subject was determined:

$$Final_{Average} = 0{,}6(\text{Unit exams}) + 0{,}25(\text{Group work}) + 0{,}15(\text{Individual works}). \quad (1)$$

3.4. Factorial Analysis of the Model

The grades of the 10 most outstanding students according to average in the subject were considered. In order to work with homogeneous information, the database was standardized; subsequently, a factorial analysis was developed which included the Scree test (figure 2), the rotated component matrix using the varimax method (figure 3), and an exhaustive analysis to obtain the main components.

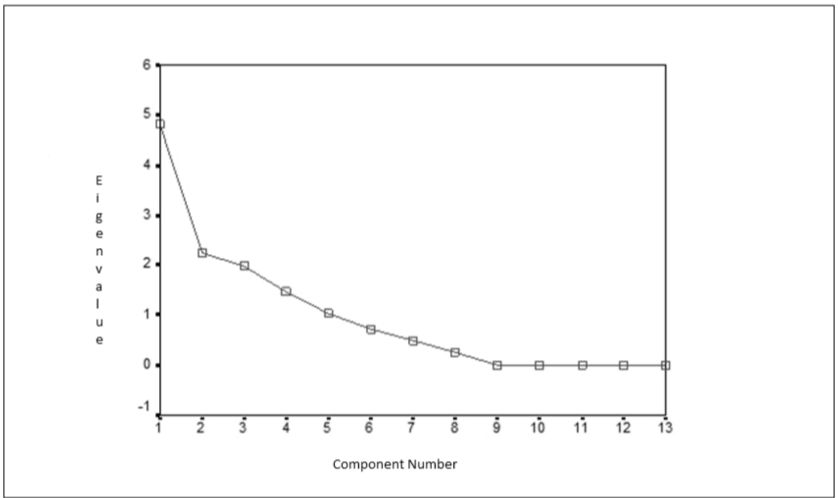


Figure 2: The Scree plot for students enrolled in the subject "MATLAB 2018-II": 2 main components were obtained with the possibility of including a third.

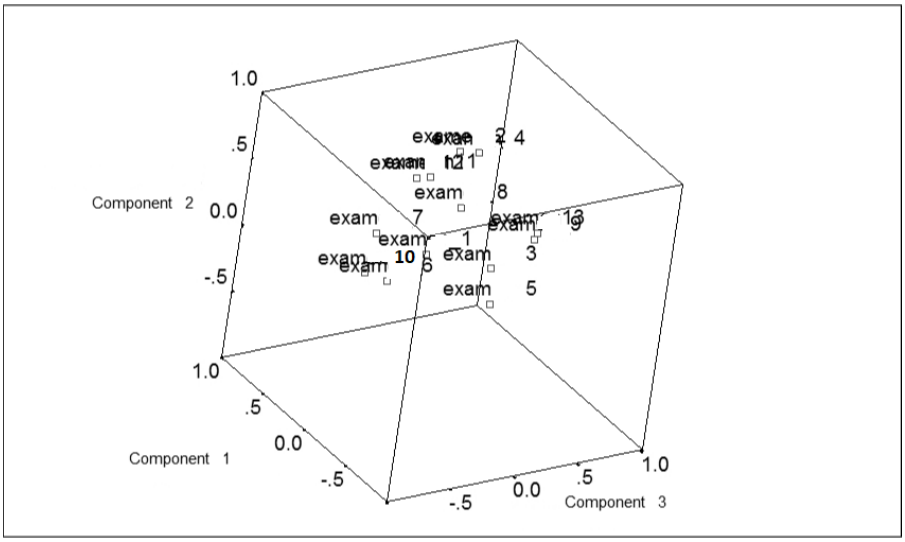


Figure 3: Graph of the principal components matrix for students enrolled in the subject MATLAB 2018-II in a rotated space: 2 components were obtained with the possibility of alignment towards a third.

4. Results and Discussion

4.1. Regarding Factorial Analysis

The 10 students with the highest averages in the MATLAB 2018-II subject showed similarities with respect to the grades they obtained in their evaluations, however only 8 of them passed. These evaluations were concentrated in the form of blocks (Block 1: evaluations 1, 2, 4, 8; block 2: evaluations 7, 9, 11, 12, 13; block 3: evaluations 3, 6, 10) as seen in Figure 4. These blocks show that the highest averages of the subject were achieved by those students who obtained good grades from the beginning, but also by those who, regardless of the initial results, strived to achieve passing grades from the middle of the subject until it ended. It can also be seen that evaluation 5 was of little importance (approximately at the beginning of the second unit), since they were possibly evaluated with a new and more difficult topic, but that at those instances of the course the students still did not give up. because they know they have more opportunities to improve their grades.

All approved students have maintained regularity (almost always 14) with respect to all their evaluations, however it was interesting to know some patterns. According to Multivariate Statistics, the edge of the factor circle is reserved for the individual with the best score, in this case average, which, as seen in figure 5 is awarded to student 1, followed by students 4, 5, 6 and away from students 8 and 9 who maintained a similar academic performance throughout the subject (almost always obtaining 14 in all evaluations). It is observed that students number 1 and 4 (the students with the best averages) have been very attentive to passing evaluations 7, 9, 11, 12, 13 (the last exams). Student 7 secured his approval by obtaining excellent grades in the midterm exams and in the work grade (evaluations 3, 6, 10), while student 2 obtained a passing grade thanks to the fact that he passed with a higher grade the evaluations that turned out to be more difficult for their other classmates (evaluations 2 and 11). This means that obtaining very good grades at the beginning and end of the course can be decisive for approval. Mention that, as you can see, students 3 and 10 are not in said graph, which means that they failed (only 8 students passed).

It can also be seen how none of the students worried about satisfactorily performing the first evaluation of the subject (the so-called "Diagnostic evaluation") because it is an evaluation that has no impact on the grade, as demonstrated by this graph, but from it academic performance improves.

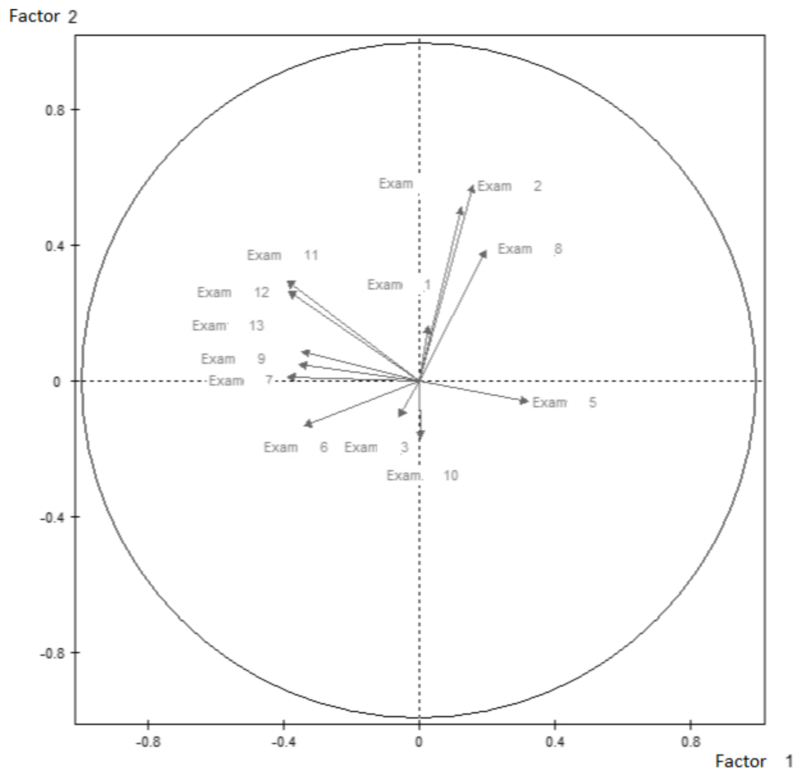


Figure 4: Principal Components Graph in relation to the 10 students with the highest averages in the MATLAB 2018-II subject.

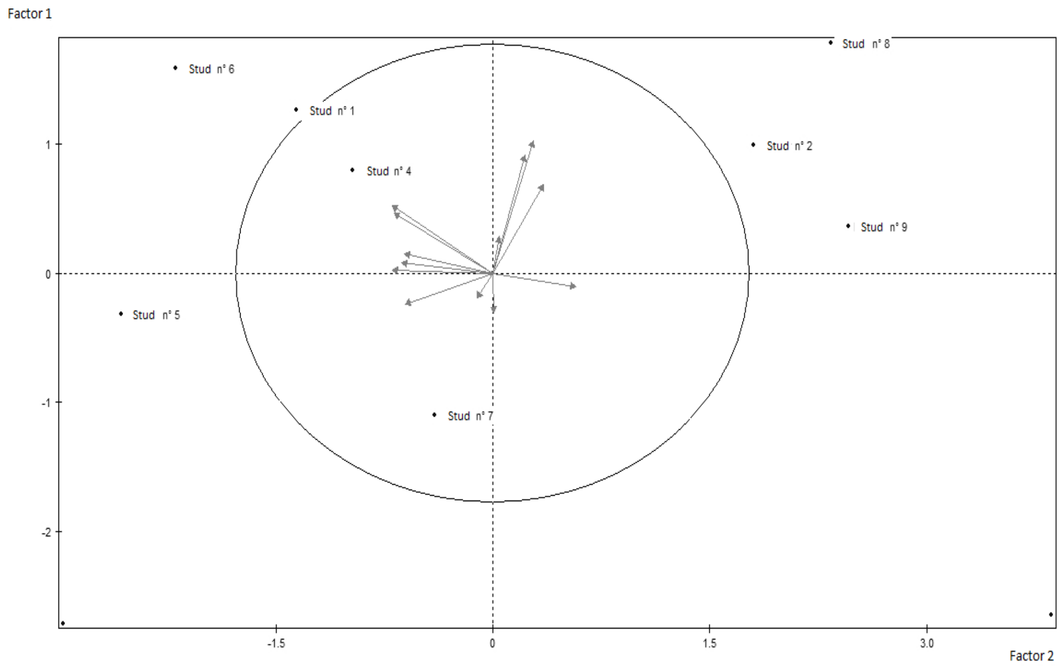


Figure 5: Principal Components Graph in relation to the 10 students with the highest averages in the MATLAB 2018-II subject (students and evaluations).

4.2. Analysis of results “MATLAB 2018-II” according to Chernoff faces

Of the 35 students enrolled in the MATLAB 2018-II course, only 8 passed, 19 disapproved, 6 withdrew, and 2 were disqualified. This same reality is evident in the Chernoff graph (figure 6 (a)), where you can see sad, angry or disappointed faces, and you can see that the students who passed the subject, except for one, share similar characteristics between the proportionality of the width of their nose with respect to their smile (happy). This means that punctual class attendance can be a variable to consider for passing the subject, in the same way that asymmetries of these traits end up being the lowest averages. Among the features that attract attention are the width of the face, the separation of the eyes, the angle formed by the eyebrows and the radius or roundness of the ears. These features turn out to be, contrasting with the real base of results and their respective nomenclatures in the syllable, the grades for qualified practices, the grade for the partial exam 1, the grade for assignments, and the grade for the partial exam, respectively. This confirms and complements what was stated in the previous section, which means that this type of evaluation should continue to be maintained in the form of obtaining the average but, suggesting it be smaller, given that there are other features that also turn out to be important correspondingly. to other evaluations (possibly with less weight) but that can allow the student to fail the subject if they are careless in the “main evaluations”. Therefore, it is considered that a different note should be considered to better organize the activities. Many activities in the evaluation may make it difficult for the teacher to pay due attention to each student due to the number of items to be measured. It is preferable to condense them.

4.3. Improvement in the form of evaluation for “Numerical Methods 2019-I”

Adapting the equation (1) and taking as a recommendation what was previously described by Chernoff’s analysis with “MATLAB” 2018-II, it was decided to implement the proposal for the subject “Numerical Methods 2019-I” under a new form of evaluation:

1. Unit Exams = Learning Outcomes 1 and 2 (RA1,RA2).
2. Individual assignments=Learning outcome 3 (RA3).
3. Group work=Learning outcome 4 (RA4).

proposing the “Formula to obtain the final grade of the subject (NF)”:

$$NF = RA1 * 0{,}35 + RA2 * 0{,}25 + RA3 * 0{,}15 + RA4 * 0{,}25,$$

and in order to take advantage of the two and a half points indicated by the statistical analysis for another type of evaluation, this clause was considered in the syllable “Voluntary participation in the classroom will be graded by adding one point to each unit exam in the respective unit in which they are applied (maximum four participations per unit)” .

4.4. Results obtained with the implementation

As can be seen by contrasting the results obtained in the course of “Numerical methods” 2019-I with the reality where the analysis of results by Multivariate Statistics was not implemented “MATLAB” 2018-II, the Chernoff faces of the students the graph represents (see figure 6 (b)) They now turn out to be all smiling and good-looking. It is observed as a more striking fact than had been previously noted in this manuscript, that there is a close relationship between students who do attend classes punctually and those who pass the subject (see features “nose width” and “mouth length”). Indeed, and comparing with the final report of the subject “Numerical methods” 2019-I, of the 12 students enrolled all passed with the grades seen in Table 1.

Tabla 1
Learning results, percentage of punctual attendance and final average of students enrolled in the subject “Numerical methods” 2019-I.

STUDENT	RA1	RA2	RA3	RA4	AP (%)	AVERAGE
E01	19.50	14.00	13.00	14.00	**84.40**	15/20
E02	20.00	6.00	12.00	10.00	**81.30**	14/20
E03	19.00	20.00	20.00	17.00	**100.00**	19/20
E04	3.00	20.00	14.00	15.00	**93.80**	14/20
E05	17.50	19.00	15.00	8.50	**100.00**	15/20
E06	6.00	18.50	12.00	7.00	**93.80**	14/20
E0/	11.50	17.50	13.00	7.00	**100.00**	14/20
E08	18.50	20.00	20.00	7.00	**90.60**	14/20
E09	12.50	10.00	18.00	7.50	**90.60**	16/20
E10	16.00	15.00	12.00	10.00	**96.90**	15/20
E11	19.00	20.00	20.00	16.00	**84.40**	18/20
E12	16.00	10.00	16.00	7.00	**96.90**	14/20

5. Conclusions

1. It is possible to improve our grading system by providing feedback on the evaluation instruments used with some of the groups in our charge, especially where not so satisfactory results are obtained, and asking ourselves if such instruments and their respective weights assigned for the calculation of the final averages have been the most appropriate. In summary, continuous feedback on assessment instruments and their appropriateness is supported by educational theories such as constructivism, experiential learning, social learning theory and the formative assessment approach, and can be an effective strategy to improve the system of qualifications and promote meaningful and lasting learning.

2. In order not to guess, it is possible to analyze the results using Multivariate Statistics, and based on this, change the instruments or calibrate them with the respective mathematical support, which benefit the student interested in learning. Bentler's article, despite being from 1987, supports the results obtained here since factorial analysis has allowed us to evaluate and validate this model in the educational field, this is a great reference for how multivariate techniques are used [16].

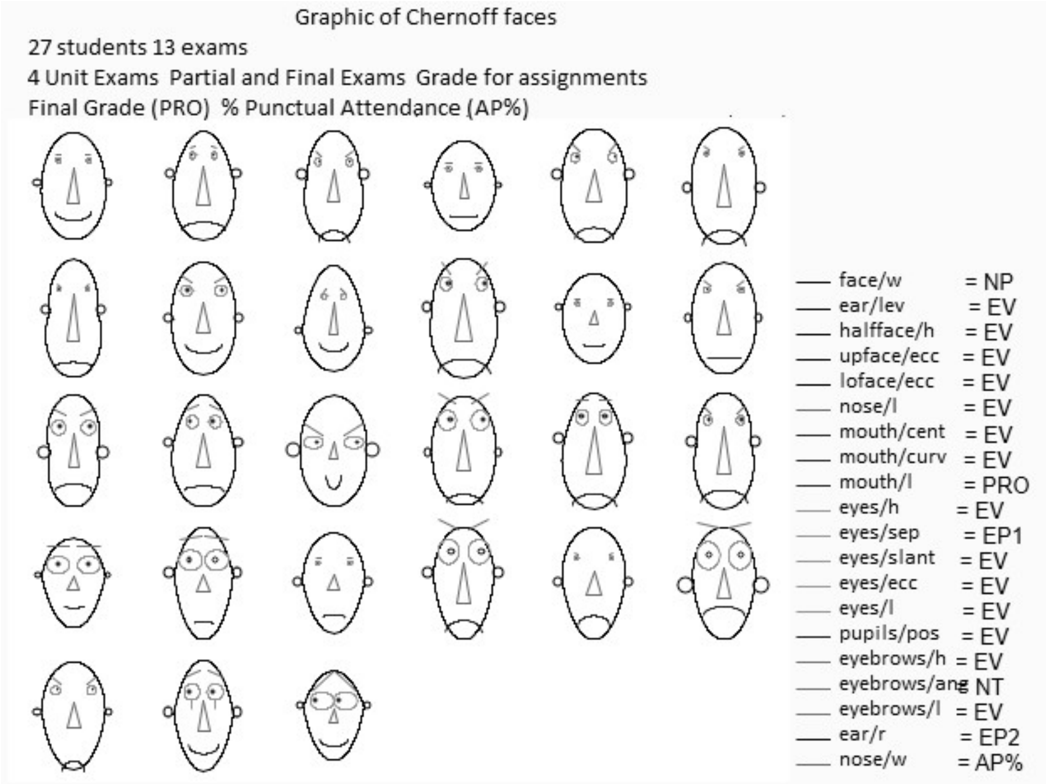
3. The majority of students who pass the subjects do so with a minimum grade, influenced by the grades for presentation and support of work, but from those who pass with high grades, it has been learned that punctual attendance at classes is a non-dismissable variable, done in 2019, the Universidad del Pacífico developed work to evaluate quality levels focusing on a set of indicators that conclude that students' punctual attendance at classes is one of these ([17]), Therefore, it is concluded that the need to incorporate an evaluation grade other than exams and assignments must be met, which, although they are instruments that should not be removed from the evaluation system, alternatives should be sought that value punctual attendance at classes with some weighting of at least ten percent of the final average (2 points in Peru).

4. Chernoff face analysis is non-trivial in showing us that considering many summative activities can reduce the attention due to each student by the number of items to be measured. It is preferable to condense all of these into unit-based learning outcomes.

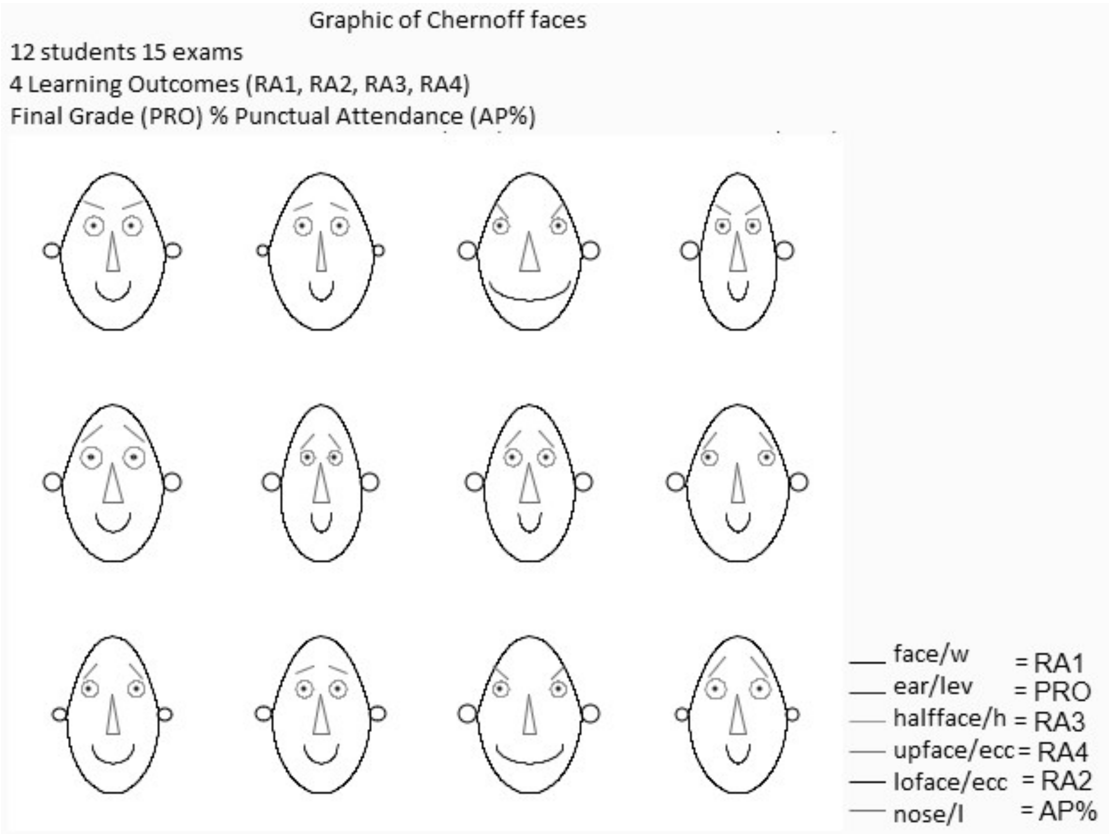
5. It is not necessary to have the same subject semester by semester to be able to make estimates that optimize the academic performance of another group of students, since the data usually behave through a normal distribution. It is suggested to use this teaching innovation proposal in order to validate the results and observe if the same is true for subjects other than Mathematics. At the same time, the great task for future research could be to explore the dimensionality of the study variables in spaces that are more difficult to measure such as "punctual assistance" and its implication in virtual environments.

Acknowledgements

We would like to thank Dr. Miguel Rodríguez from the Department of Applied Mathematics of UGR for his fruitful discussion to this research, which has been developed thanks to the facilities granted to the corresponding author by the university during the pandemic for the development of some international activities that required such work under USAT affiliation.



(a) Chernoff face graph of evaluations results of the subject "MATLAB 2018-II".



(b) Chernoff face graph of evaluations results of the subject "Numerical methods" 2019-I.

Figure 6: Contrast between the final results obtained in the subject "MATLAB" 2018-II (without implementation of the proposal) and the subject "Numerical methods" 2019-I (with implementation of the proposal).

References

[1] H. W. Marsh, The influence of student, course, and instructor characteristics in evaluations of university teaching, American Educational Research Journal 17 (1980) 219–237. https://doi.org/10.2307/1162484

[2] Y. Chávez Ruiz, F. Martínez Rizo, Evaluar para aprender: hacer más compleja la tarea a los alumnos, Educación matemática 30 (2018) 211–246. DOI: 10.24844/EM3003.09

[3] W. H. Bizarro Flores, P. J. Paucar Miranda, E. Chambi-Mescco, Avaliação formativa: uma revisão sistemática dos estudos em sala de aula, Horizontes Revista de Investigación en Ciencias de la Educación 5 (2021) 872–891.

[4] https://doi.org/10.33996/revistahorizontes.v5i19.244

[5] N. F. Medina Martínez, Las variables complejas en investigaciones pedagógicas. Apuntes universitarios, Revista de Investigación 2 (2015). https://doi.org/10.17162/au.v0i2.244

[6] V. B. Villares, M. J. F. Sánchez, Percepciones y uso de los medios digitales en educación artística: un estudio descriptivo, Innoeduca. International Journal of Technology and Educational Innovation 8 (2022) 75–90.

[7] https://doi.org/10.24310/innoeduca.2022.v8i1.12069

[8] B. Flury, H. Riedwyl, Graphical representation of multivariate data by means of asymmetrical faces, Journal of the American Statistical Association 76 (1981) 757–765. https://doi.org/10.2307/2287565

[9] E. Olmos, El rendimiento estudiantil: una metodología para su medición, Revista Economía 13 (1997) 7–26.

[10] H. Yangxin, Principal components estimate of regression coefficient in the multivariate linear models [j], CHINESE JOURNAL OF ENGINEERING MATHEMATICS 2 (1994). https://dx.doi.org/10.37624/IJERT/13.7.2020.1616-1622

[11] C. J. Morris, D. S. Ebert, P. L. Rheingans, Experimental analysis of the effectiveness of features in chernoff faces, in: 28th AIPR Workshop: 3D Visualization for Data Exploration and Decision Making, volume 3905, International Society for Optics and Photonics, 2000, pp. 12–17. DOI:10.1117/12.384865

[12] Y. Álvarez, M. Ruiz Soler, Actitudes hacia las matemáticas en estudiantes de ingeniería en universidades autónomas venezolanas, Revista de Pedagogía 31 (2010) 225–249.

[13] M. del Carmen Ibarra, J. C. Michalus, Análisis del rendimiento académico mediante un modelo logit, Revista Ingeniería Industrial 9 (2010).

[14] C. Pérez López, Técnicas de análisis multivariante de datos, Aplicaciones con SPSS, Madrid, Universidad Complutense de Madrid (2004) 121–154.

[15] C. R. Sabiote, L. H. Torres, Análisis correlacional-predictivo de la influencia de la asistencia a clase en el rendimiento académico universitario. estudio de caso en una asignatura, Profesorado. Revista de Currículum y Formación de Profesorado 13 (2009) 1–13.

[16] M. C. Ávila, E. D. Chourio, L. C. Carniel, Z. Á. Vargas, El software matemático como herramienta para el desarrollo de habilidades del pensamiento y mejoramiento del aprendizaje de las matemáticas, Revista Electrónica Actualidades Investigativas en Educación 7 (2007) 10–11. https://doi.org/10.15517/aie.v7i2.9264

[17] M. P. Guzmán Brito, Modelos predictivos y explicativos del rendimiento académico univer- sitario: caso de una institución privada en México, Ph.D. thesis, Universidad Complutense de Madrid, 2012.

[18] P. M. Bentler, C.-P. Chou, Practical issues in structural modeling, Sociological methods & research 16 (1987) 78–117.

[19] https://doi.org/10.1177/0049124187016001004

[20] R. M. Jorgechagua Saavedra, Análisis costo-efectividad de la inversión en educación superior universitaria en la generación de I+D+I: caso UNSCH 2018 (2019).