A Mathematica Package for Graphing Equations and Inequalities in Non-Rectangular Coordinate Systems

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Abstract

Solving and plotting inequalities are crucial mathematical skills that have a significant impact on problem-solving in a variety of disciplines. These tools provide an effective way to represent and analyze numerical and geometric relationships, facilitating decision-making, planning, and understanding complex situations in everyday life and professional fields. This paper presents and describes a new package coded in the Wolfram Language of the Mathematica symbolic calculation system, `NRGraphics`, which allows plotting equations and inequalities in non-rectangular plane coordinate systems. The authors have developed this package to provide a tool for the teaching process of graphical representation of equations and inequalities in non-rectangular plane coordinate systems.

Keywords

Non-rectangular coordinates, Wolfram Language, equations, inequalities

1. Introduction

Solving inequalities constitutes an essential topic in mathematics with outstanding applications in many problems of theoretical and applied science [1, 2, 3, 4].

The symbolic calculation system Mathematica stands out on this topic among its competitors because it includes commands that solve inequalities that involve more than one variable [5]. In addition, it contains commands that allow obtaining the graphs of the regions of the plane in the Cartesian coordinate system, which are the solution set of inequalities of this type [6].

However, although there are many general-purpose or specific-purpose computer algebra systems [7], none have been found that allow obtaining the graphs of equations and inequalities in non-rectangular plane coordinate systems.

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2. Domain theory and previous work

The graphical representation of inequalities in the Cartesian coordinate system in the plane can be performed without significant complications with the `RegionPlot` command incorporated into Mathematica [6]. However, Mathematica does not include any command to graph inequalities in non-Cartesian coordinates in the plane.

There is a contribution from user Heike on the Mathematica Stack Exchange blog to graph equations in polar coordinates given implicitly [8]. From this contribution, the fundamental idea can be extracted to develop an algorithm to modify points on the plane graphed with Mathematica via transformations.

On the other hand, Mathematica incorporates the `PolarPlot` command. However, this command is limited to polar coordinates and explicit expressions of $\rho = f(\theta)$. On the other hand, our package is not limited to polar coordinates and can operate, in particular, with implicit expressions of the form $E(\rho, \theta) = 0$. The following example shows us the existing limitation.

Mathematica

Is impossible plot $r^2 - \cos(2\theta) = 0$ with `PolarPlot`.

```
In[1]:= PolarPlot[r^2-Cos[2*θ]==0,{r,0,1},{t,0,2*Pi}]
Out[1]= PolarPlot::nonopt
```

3. The package: `NRGraphics`

The `NRGraphics` package includes two commands: `ContourNonCartesianPlot` and `InequalityNonCartesianPlot`. The syntaxes of both commands are:

`ContourNonCartesianPlot[`$E(u,v), \{u, u_1, u_2\}, \{v, v_1, v_2\}$,*options*`]`

and

`InequalityNonCartesianPlot[`$R(u,v), \{u, u_1, u_2\}, \{v, v_1, v_2\}$,*options*`]`

$E(u,v)$ is an equation for u and v; $R(u,v)$ is an expression of inequalities and Boolean operators. All graphical results are displayed in the rectangle:

$$u_1 \le u \le u_2 \wedge v_1 \le v \le v_2 \,.$$

Both commands are similar to the `ContourPlot` and `RegionPlot` commands, respectively, that Mathematica incorporates.

This package is hosted at

https://www.wolframcloud.com/obj/b17ff7eb-c55e-42be-aae2-89347c935fc3

And the way to install it is explained in

https://support.wolfram.com/5648?src=mathematica

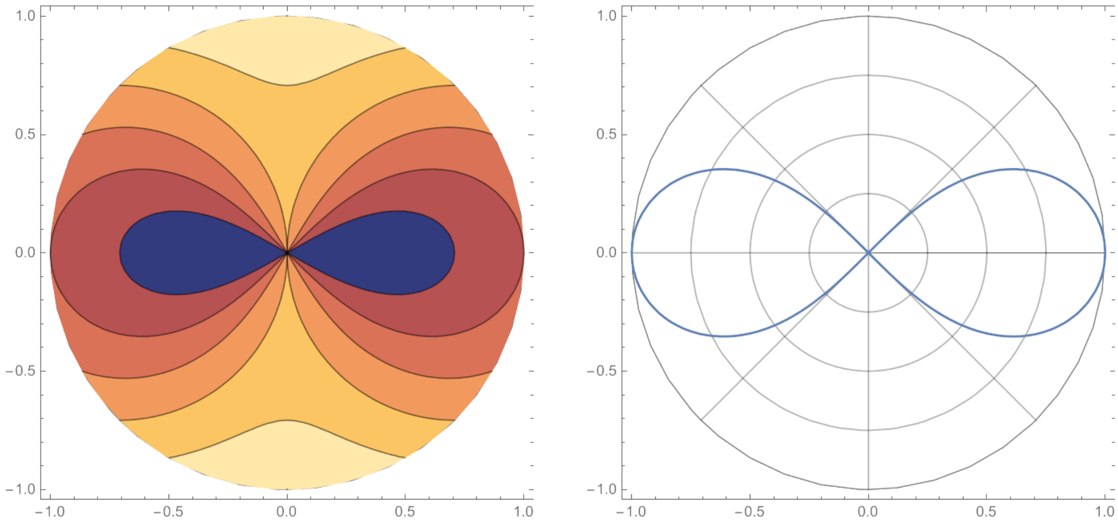

Figure 1: The contour plot of $r^2 - \cos(2t)$ (left) and plot of $r^2 - \cos(2t) = 0$ (right).

4. Results

First, we initialize the package:

Mathematica

Initialization of the `NRGraphics.m` package.

```
In[1]:= «NRGraphics.m
```

Below are examples of graphs obtained with the commands incorporated in the package and whose syntax was disclosed in the previous section.

Mathematica

Plot the contours of $r^2 - \cos(2t)$ in polar coordinates.

```
In[2]:= ContourNonCartesianPlot[r^2-Cos[2*t],{r,0,1},{t,0,2*Pi}]
```

Out[2]= *See Fig. 1 (left).*

Mathematica

Plot an equation, $r^2 - \cos(2t) = 0$, in polar coordinates.

```
In[3]:= ContourNonCartesianPlot[r^2-Cos[2*t]==0,{r,0,1},{t,0,2*Pi}]
```

Out[3]= *See Fig. 1 (right).*

Mathematica

Plot an equation, $r - 2(1 + \cos(t)) = 0$, in polar coordinates.

```
In[4]:= ContourNonCartesianPlot[r-2*(1+Cos[t])==0,{r,0,4},{t,0,2*Pi}]
```

Out[4]= *See Fig. 2 (left).*

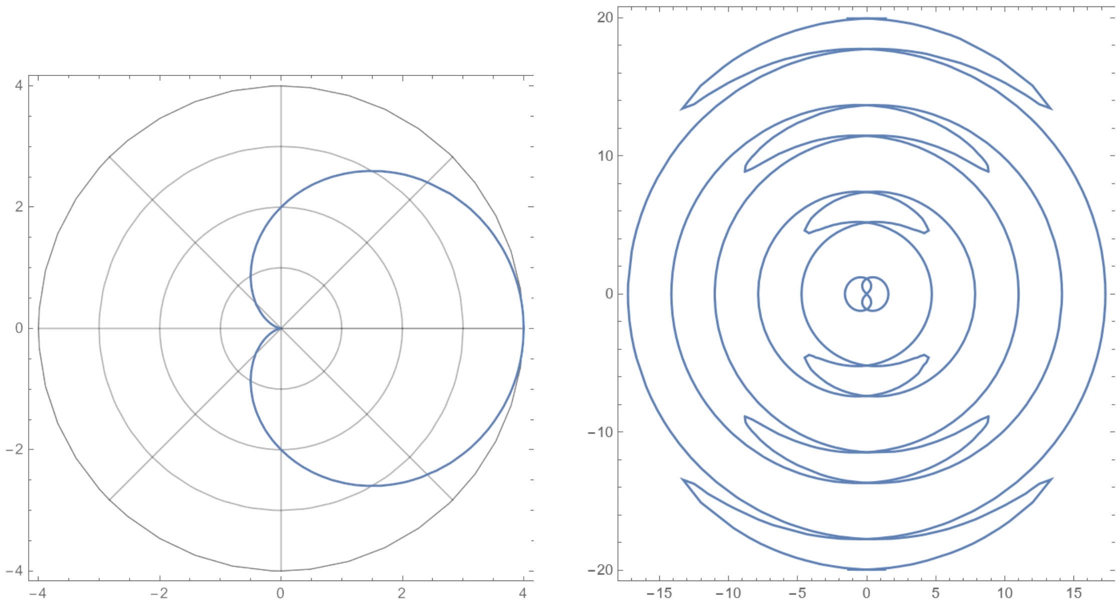

Figure 2: The plot of $r - 2(1 + \cos(t)) = 0$ (left) and plot of $t^2 - \left(\frac{3\pi}{4}\cos(r)\right) = 0$ (right).

Mathematica

Plot an equation, $t^2 - \left(\frac{3\pi}{4}\cos(r)\right) = 0$, in polar coordinates.

In[5]:=
```
ContourNonCartesianPlot[r^2-Cos[2*t]==0,{r,-20,20},
   {t,-2*Pi,2*Pi}]
```
Out[5]= *See Fig. 2 (right).*

Mathematica

Plot two equations, $r^2 - \cos(2t) = 0, r - (1 + \cos(t)) = 0$, in polar coordinates.

In[6]:=
```
ContourNonCartesianPlot[{r^2-Cos[2*t]==0,r-(1+Cos[t])==0},
   {r,0,2},{t,0,2*Pi}]
```
Out[6]= *See Fig. 3.*

Mathematica

Plot an inequality, $r - 2(1 + \cos(t)) \leq 0$, in polar coordinates.

In[7]:=
```
InequalityNonCartesianPlot[r^2-Cos[2*t]<=0,{r,0,1},{t,0,2*Pi}]
```
Out[7]= *See Fig. 4 (left).*

Mathematica

Plot an inequality, $r^2 - \cos(2t) < 0 \wedge r - (1 + \cos(t)) < 0$, in polar coordinates.

In[8]:=
```
InequalityNonCartesianPlot[r^2-Cos[2*t]<0&&
r-(1+Cos[t])<0,{r,0,2},{t,0,2*Pi}]
```

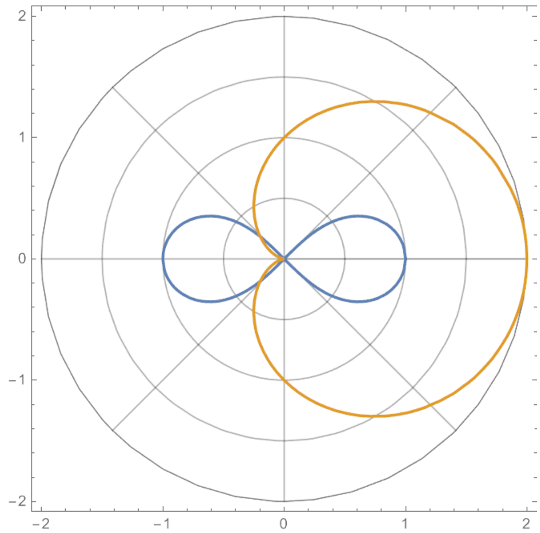

Figure 3: The plot of $r^2 - \cos(2t) = 0, r - (1 + \cos(t)) = 0$.

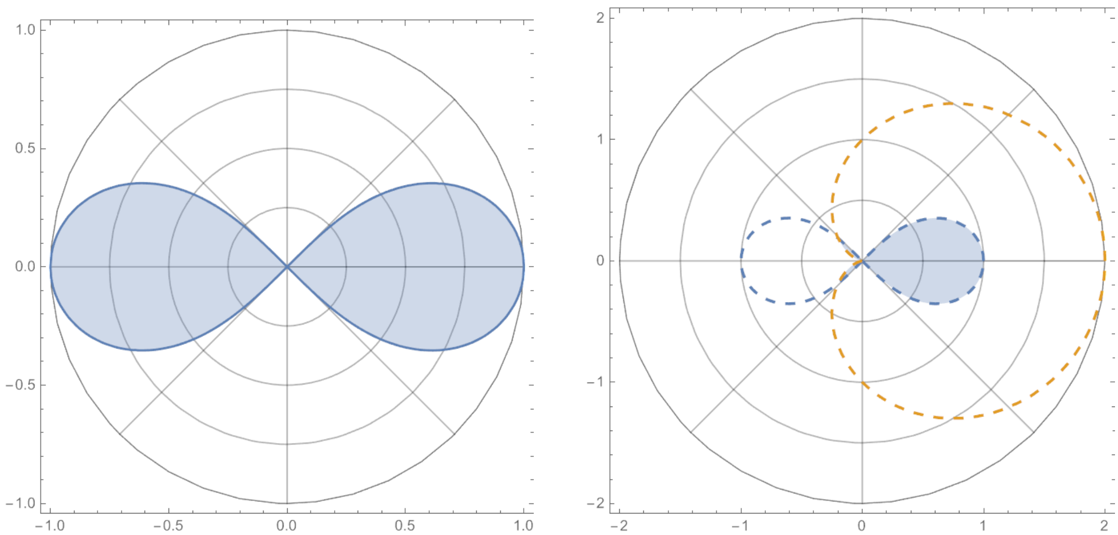

Figure 4: The plot of the inequality $r^2 - \cos(2t) \leq 0$. (left) and plot of the inequality $r^2 - \cos(2t) < 0 \wedge r - (1 + \cos(t)) < 0$ (right).

Out[8]= *See Fig. 4 (right).*

Mathematica

Definition of the omega number.

```
In[9]:= omega=2*NIntegrate[1/Sqrt[1-r^4],{r,0,1}]
Out[9]= 2.62206
```

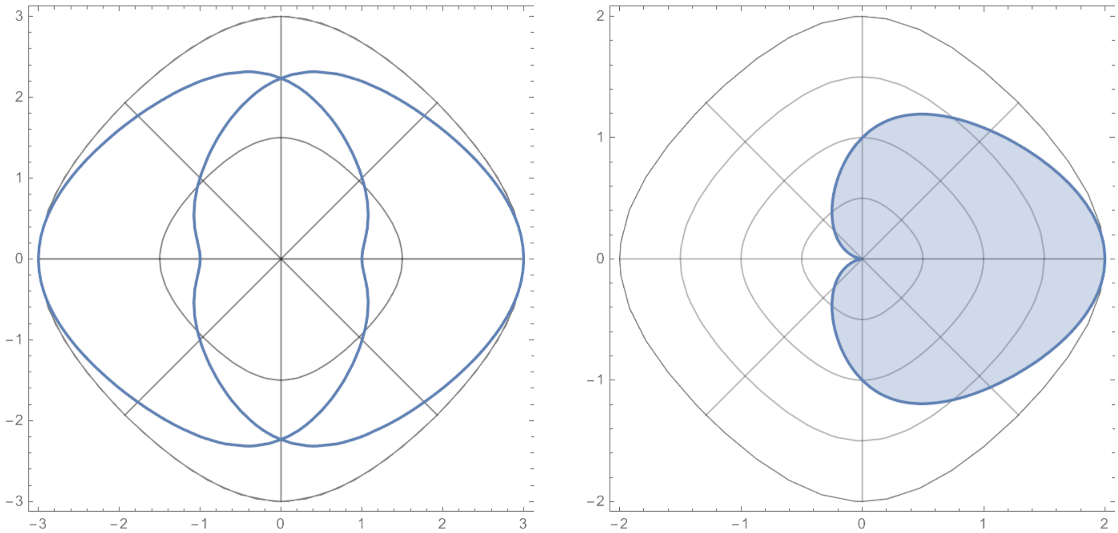

Figure 5: The plot of the inequality $4\,\mathrm{cosl}(v)+u^2-5=0$. (left) and plot of the inequality $u-(\mathrm{cosl}(v)+1)\leq 0$ (right).

Mathematica

Definition of the lemniscatic sine and cosine.

```
In[10]:= Sinl[x_]:=JacobiSN[x,-1]
         Cosl[x_]:=JacobiSN[omega/2-x,-1]
```

Mathematica

Definition of "lemniscatic polar" coordinates.

```
In[11]:= f=Function[{u,v},{u*Cosl[v],u*Sinl[v]}];
```

Mathematica

Plot an equality, $4\,\mathrm{cosl}(v)+u^2-5=0$, in "lemniscatic polar" coordinates.

```
In[12]:= ContourNonCartesianPlot[u^2-5+4Cosl[v]==0,{u,-3,3},
           {v,0,2*omega}]
```

Out[12]= *See Fig. 5 (left).*

Mathematica

Plot an inequality, $u-(\mathrm{cosl}(v)+1)\leq 0$, in "lemniscatic polar" coordinates.

```
In[13]:= InequalityNonCartesianPlot[u-(1+Cosl[v])<=0,{u,0,2},
           {v,0,2*omega}]
```

Out[13]= *See Fig. 5 (right).*

Next, the following problem will be solved: Find a transformation $T:\mathbb{R}^2\to\mathbb{R}^2$ from a plane

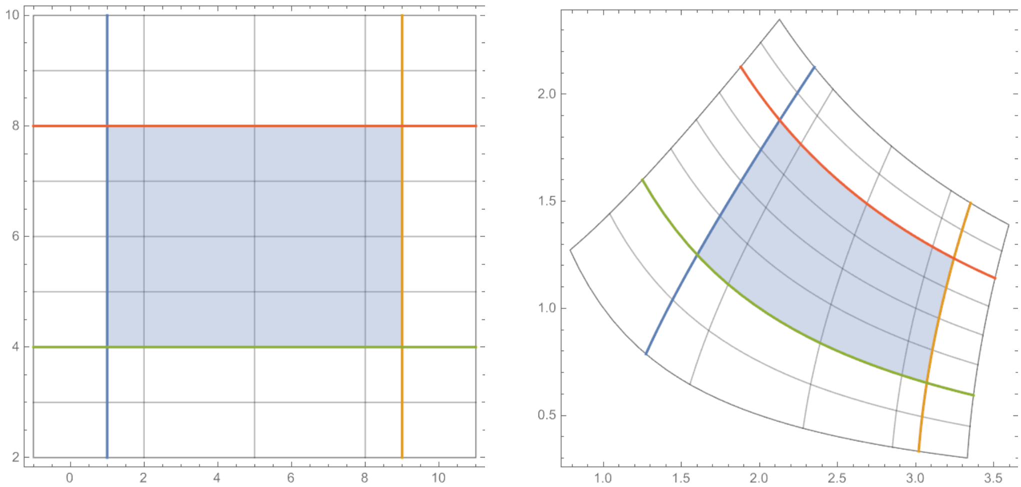

Figure 6: The region D, where $(u, v) = T^{-1}(x, y) = (x^2 - y^2, 2xy)$ (left), and the region $T(D)$, where $(x, y) = T(u, v)$ (right).

uv to a plane xy that transforms a region D of the uv plane in a region $T(D)$ of the xy plane, bounded by the graphs of $x^2 - y^2 = 9$, $x^2 - y^2 = 1$, $xy = 2$, $xy = 4$, for $x > 0$ [9].

The proposed solution to the problem is provided below.

Mathematica

Calculation of the coordinate functions $x = t_1(u, v), y = t_2(u, v)$, of the transformation T, from $(u, v) = (x^2 - y^2, 2xy)$.

```
In[14]:= Reduce[{u,v}=={x^2-y^2,2xy}&& x>0&& u>0&& v>0,{x,y}]
```

Out[14]= $v > 0\&\&u > 0\&\&x == \text{Root}[-v^2 - 4u\#1^2 + 4\#1^4\&, 2]\&\&y == v/(2x)$

Mathematica

Plotting the region $D : 1 \leq u \leq 9 \wedge 4 \leq v \leq 8$ in the uv plane.

```
In[15]:= InequalityNonCartesianPlot[1 ≤ u ≤ 9 ∧ 4 ≤ v ≤ 8,
           {u,-1,11},{v,2,10},Transformation->Function[{u,v},{u,v}]]
```

Out[15]= *See Fig. 6 (left).*

Mathematica

Defining the transformation T and plotting the region $T(D)$ in the xy plane.

```
In[16]:= T=Function[{u,v},{x=Root[-v^2-4*u*#1^2+4*#1^4&,2],v/(2*x)}];
         InequalityNonCartesianPlot[1 ≤ u ≤ 9 ∧ 4 ≤ v ≤ 8,
           {u,-1,11},{v,2,10},Transformation->T]
```

Out[17]= *See Fig. 6 (right).*

5. Conclusions and future work

In this paper, the new `NRGraphics.m` package is presented and described. The `NRGraphics.m` package includes two commands to obtain graphs of equations and inequalities in plane non-rectangular coordinate systems. The coding of the package has been done in Mathematica v.11.2.0.0, but it can run in later versions. All the results shown in this paper have been obtained on an HP laptop with an Intel(R) Core(TM) i3-3110M CPU @ 2.40 GHz, with a RAM of 4.00 GB, 64-bit Operating System, and $\times 64$ processor. The authors are committed to developing command packages linked to education, and a short-term objective is to improve the package so that it allows graphing equations in three-dimensional non-rectangular coordinate systems so that any subsequent results will be published by this means.

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