Systematic Review: State of Knowledge on Learning Difficulties and Teaching Strategies in Linear Algebra

Alethia Piñón Jiménez[1] and Diana Margarita Córdova Esparza[2]

1 Facultad de Informática, Universidad Autónoma de Querétaro, Av. de las Ciencias S/N, Juriquilla, Querétaro 76230, México
2 Facultad de Informática, Universidad Autónoma de Querétaro, Av. de las Ciencias S/N, Juriquilla, Querétaro 76230, México

Abstract

This paper presents a systematic review focusing on diagnosing learning difficulties and implementing didactic strategies in linear algebra. We aim to deepen the understanding of this topic over the past decade. Our research, guided by four questions, analyzed 78 articles, ultimately including 37 in this review. We based our search strategy on the PRISM protocol and used specific indicators. Our findings indicate that most authors in this review primarily use the APOE theory and genetic decomposition for formal diagnosis of learning problems. This approach helps build knowledge frameworks, especially in vector spaces and linear transformations. A key finding is the prevalent use of digital technology in both the models and strategies proposed in these studies. This review highlights opportunities for future research in diagnosing learning problems and developing innovative, technology-integrated strategies in education.

Keywords

Education, didactic strategy, linear algebra

1. Introduction

In the field of education, teaching and learning mathematics often presents significant challenges for teachers. These challenges include covering the subject's content within the allotted time and addressing the diverse learning difficulties that students face. Additionally, teachers must develop effective teaching strategies to enhance learning outcomes in mathematics.

Each researcher in this field brings their unique perspective, knowledge, and experience to analyze the state of knowledge on teaching and learning mathematics. Despite these efforts, learning problems in linear algebra, especially in abstract topics like vector spaces and linear transformations, persist (31).

This paper aims to conduct a systematic review to better understand how learning difficulties in linear algebra are formally diagnosed and what teaching strategies are being implemented. The importance of this review becomes evident when considering that linear algebra is a fundamental subject in science and engineering courses. It contributes significantly to developing students' logical, heuristic, and algorithmic thinking skills by using linear models to predict and control system behaviors.

Therefore, this review will analyze current knowledge on diagnosing student learning problems in linear algebra and the recent implementation of didactic strategies to improve teaching and learning in this field.

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alethia463@gmail.com (A. Piñón-Jiménez); diana.cordova@uaq.mx (D. M. Córdova-Esparza)
0000-0003-0326-4741 (A. Piñón-Jiménez); 0000-0002-5657-7752 (D. M. Córdova-Esparza)

2. Method

Our search strategy used the PRISM (Preferred Reporting Items for Systematic Reviews and Meta-Analyses) protocol as a reference and followed specific indicators. We guided our research with four key questions:

1. What are the main factors influencing learning problems in linear algebra?
2. Which learning theories have been applied to formally diagnose these learning problems and design teaching strategies for linear algebra?
3. What are the developed thematic strategies for linear algebra, and do they share any common characteristics?
4. What were the sizes of the groups used to validate the formal diagnoses or as pilot groups for implementing teaching strategies?

To address our research questions and achieve the study's objective, we conducted a systematic literature review. This method is known for systematically integrating empirical results related to a specific research problem (34). We developed our research methodology in four distinct stages, which we detail in the subsequent paragraphs.

Stage 1: Setting Inclusion and Exclusion Criteria for Research Studies

In this first stage, we established specific criteria for including and excluding studies in our research. For inclusion, we focused on research articles, excluding other document types like theses and book chapters. We considered articles published from 2013 to 2022, ensuring the research was no more than 10 years old. Additionally, we included studies written in Spanish, English, or Portuguese. The final inclusion criterion was that the articles must be related to teaching or learning linear algebra; we excluded articles on topics outside this specific educational area.

For exclusion, we omitted any articles that did not meet all our inclusion criteria. This also included articles that were duplicates in our study.

Stage 2: Developing the Search Strategy

In this stage, we executed our search strategy across various databases, yielding 71 articles for analysis. Our search criteria varied depending on the database to maximize results (see Figure 1). We selected databases that showed the highest number of relevant results for our topic. The databases and their respective search formulas were:

1. ERIC: Using the formula (“Education”) AND (“Linear Algebra”), we obtained 9 articles.
2. Scielo and DOAJ: We used (“Education”) AND (“Linear Algebra”) and (“Education”) AND (“Linear Algebra”), obtaining 8 and 30 articles, respectively.
3. Redalyc: With the formula (“Education”) AND (“Linear Algebra”), we found 8 articles.
4. Science Direct: We used (Teaching OR Learning) AND (“Linear Algebra”), leading to 9 articles.
5. Dialnet: The formulas (Teaching OR Learning) AND (“Linear Algebra”) and (Didactics) AND (“Linear Algebra”) resulted in 7 articles.

Additionally, we identified 49 articles through references. After applying our exclusion criteria, 7 of these were ultimately included in our study.

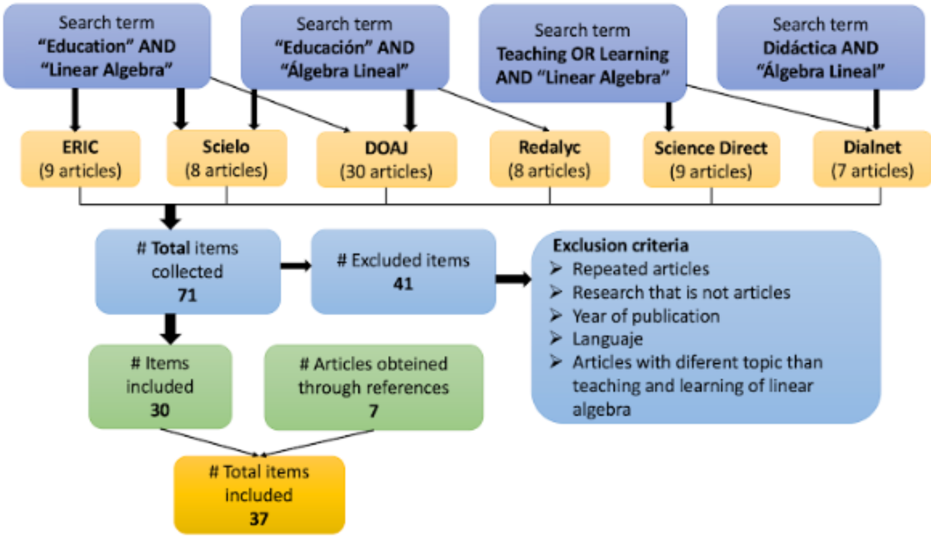

Figure 1: Overview of the Information Search and Data Collection Process. This flowchart details the search terms used across various databases, the number of articles retrieved from each, and the filtering process leading to the final selection of articles included in the review. It also outlines the exclusion criteria applied and the total number of articles analyzed.

Stage 3: Information Purification

In this stage, we conducted an initial review of the 78 articles gathered from the databases and references mentioned earlier. The purpose of this review was to assess each article's relevance to our research objectives. We rejected 34 articles during this process because they did not provide relevant data for our systematic review analysis or contribute to answering our research questions. Consequently, 37 articles were selected and included in our review.

Stage 4: Data Coding and Analysis

In this final stage, we analyzed the data based on specific categories. This structured approach helped us to thoroughly examine and understand the findings. The categories we focused on were:

1. Factors influencing learning problems in linear algebra.
2. Learning theories applied for diagnosing learning problems or implementing teaching strategies in linear algebra.
3. Thematic contents within the subject of linear algebra that were the focus of the research.
4. Strategies implemented in teaching linear algebra.
5. Sizes of the samples used for validation or implementation in pilot tests.

This categorization facilitated a comprehensive analysis of the collected data, aligning it closely with our research objectives.

3. RESULTS

Factors Influencing Linear Algebra Learning Problems

The factors identified as influencing learning problems in linear algebra are varied, as observed in the systematic review of the research. Despite this diversity, there is a notable consistency in the findings. This is apparent when we see that several factors recur across multiple studies. In some instances, more than one factor is repeated between different investigations, as detailed in Table 1. This repetition underscores the commonalities in challenges faced by learners in linear algebra.

Table 1

Influential Factors in Linear Algebra Learning Problems Identified in Scholarly Research

This table compiles pivotal studies on linear algebra, listing the year of publication, authors, article title, and the predominant factor influencing learning difficulties as identified in each piece of research.

Year of publication	Authors	Title article	Predominant Factor
2013	Nishizawa et al.	Increasing Reality and Educational Merits of a Virtual Game	Abstract
2013	Parraguez	The role of the body in the construction of the concept of Vector Space	Abstract
2013	Rosso & Barros	A taxonomy of errors in learning vector spaces	Abstract, Language, Various representations
2014	Birinci et al.	University students' solution processes in systems of linear equation	Abstract, Prior knowledge, Axiomatic
2014	Ramírez-Sandoval et al.	Coordination of semiotic representation records in the use of linear transformations in the plane	Various representations
2014	Salgado & Trigueros Gaisman	A teaching experience of values, vectors and eigenspaces based on APOE theory	Abstract
2015	Trigueros Gaisman et al.	Constructions and mental mechanisms for learning the matrix theorem associated with linear transformation	Abstract
2016	Berman & Shvartsman	Definitions are important: the case of linear algebra	Formalism
2016	Marins & Pereira	Advanced mathematical thinking manifested in tasks involving linear transformations	Formalism, Abstract
2017	Beltrán et al.	A Teaching Proposal for the Study of Eigenvectors and Eigenvalues.	Abstract, Formalism
2017	Costa & Rossignoli	Teaching linear algebra in an engineering school: Methodological and didactic aspects	Abstract, Without connection with other subjects, Language
2018	Pierri	From Practical to Theorical Thinking: The Impact of the Role-Play Activity.	Abstract, Formalism
2019	Álvarez-Macea & Costa	Teaching Linear Algebra in engineering courses: an analysis of the process of mathematical modeling within the framework of the Anthropological theory of didactics	Epistemological component, didactic schemes, Language

2019	Aytekin & Kiymaz	Teaching Linear Algebra Supported by GeoGebra Visualization Environment	Abstract, procedures memorization, lack of vinculation
2019	Gallo et al.	Interpretation of linear transformations in the plane using GeoGebra	Formalism, Language, Various representations
2019	García-Hurtado et al.	Linear algebra learning focused on plausible reasoning in engineering programs	Formalism
2019	Xavier et al.	Teaching-Learning of Matrices in the civil Engineering Course	Abstract, Formalism, Prior knowledge
2020	Parraguez	Construction of the meanings of vector space operations through linearly independent/dependent sets	Abstract, Formalism
2020	Pizarro	A Didactic Sequence for Teaching Linear Transformation: Unification of Methods and Problems, Modeling and Explanation of Learning	Concept application conditions
2021	Cárcamo et al.	Hypothetical learning trajectories: an example in a linear algebra course	Abstract
2021	Kariadinata	Students Reflective Abstraction Ability on Linear Algebra Problem Solving and Relationship with Prerequisite Knowledge.	Abstract
2021	Silva et al.	Creation and uses of LineAlg application as a learning object in basic education	Formalism
2021	Wibawa et al.	Learning Effectiveness Through Video Presentations and WhatsApp Group (WAG) in the Pandemic Time Covid-19	Abstract, Demonstrations, Large number of operations between variables

In our systematic review, we found that the most significant factor affecting linear algebra learning, as identified by various authors, is the subject's level of abstraction and formalism (see Figure 2). The high level of abstraction required by linear algebra itself poses a challenge for students, demanding a substantial degree of abstract thinking for proper understanding (37). As for formalism, it stems from the way linear algebra is presented, studied, and learned in the literature, which heavily relies on the formalism of mathematical language (25).

Other key aspects impacting linear algebra learning difficulties include students' challenges in differentiating between a concept and its various representations (29) and the use of diverse languages when discussing vector spaces and linear transformations (8). Additionally, the connection to the teacher's training emerges as a notable factor. If a teacher has a background in mathematics or a related field, the issue often lies in not having the foundational structures in place. Conversely, for engineering educators, the challenge is often linking the relevance and applicability of linear algebra concepts to their specific field (28).

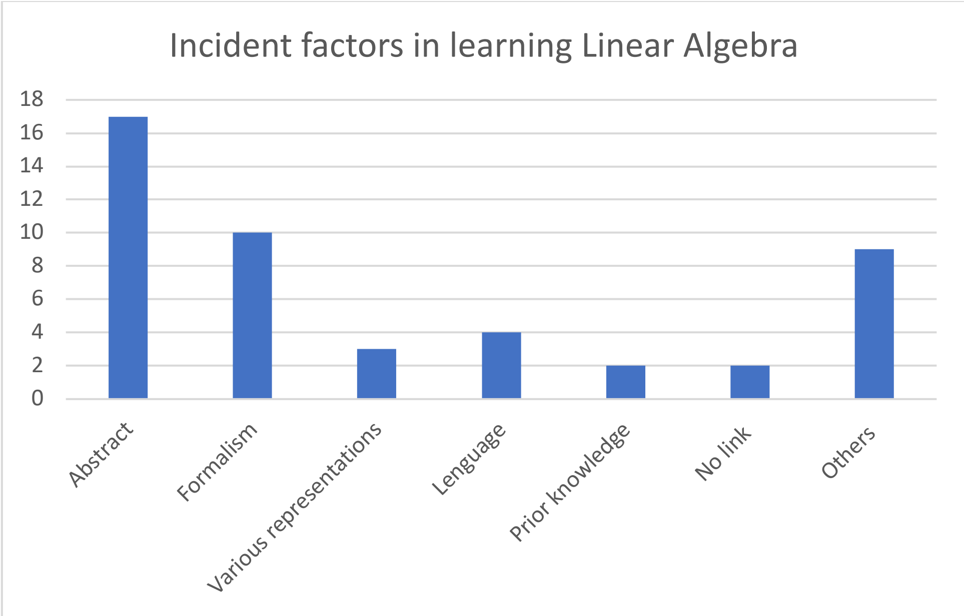

Figure 2: Prevalence of Factors Impacting Learning in Linear Algebra
This bar chart illustrates the frequency of various factors that influence the learning of linear algebra, as identified in the reviewed research, including abstraction, formalism, language, and others.

Learning Theories Applied in Linear Algebra

This section highlights the learning theories applied in diagnosing learning difficulties and implementing didactic strategies in linear algebra. It also covers the tools used in the various research projects analyzed. Additionally, we provide information about the countries where each study was conducted, as detailed in Table 2.

Table 2
Learning Theories and Research Tools Utilized in Linear Algebra Studies.
This table enumerates the studies included in the systematic review, outlining the applied learning theories, the research tools used, and the countries where the studies were conducted.

Autor y Año	País	Applied learning theory	Tool used
(Parraguez, 2013)	Chile	APOE	Semi-structured interview
(Rosso & Barros, 2013)	Argentina	Theory of didactic situations and constructivism	Problems situations
(Parraguez & Uzuriaga, 2014)	Chile	APOE	Questionnaire and interviews
(Ramírez-Sandoval et al., 2014)	México	Theory of semiotic represetations	Interview with sequence of 5 activities
(Salgado & Trigueros Gaisman, 2014)	México	APOE	Questionary and semi-structured interview
(Trigueros Gaisman et al., 2015)	Chile	APOE	Questionary and semi-structured interview

(Murillo & Beltrán, 2016)	Spain	APOE	RGB color system
(González & Roa, 2017)	Colombia	APOE	Internalization of concrete actions
(Roa-Fuentes & Parraguez, 2017)	Chile and Colombia	APOE	Questionary
(Costa, 2018)	Argentina	Anthropological Theory of the Didactic	Study and research activity
(Karrer, 2018)	Brazil	Theory of semiotic represetations	Using GeoGebra
(Rodríguez et al., 2018)	Chile	APOE	Questionnaire and interviews
(Álvarez-Macea & Costa, 2019)	Colombia	Anthropological Theory of the Didactic	Study and research activity
(Gallo et al., 2019)	Argentina	Theory of semiotic represetations	Series of computer activities using GeoGebra software
(Parraguez, 2020)	Chile	APOE	Written questionnaire
(Fortuny & Fuentealba, 2021)	Spain	Realistic mathematics education	Guía escrita, archivos de audio y video, entrevistas con algunos estudiantes.
(Betancur et al., 2022)	Colombia	APOE	Questionary and semi-structured interview

In the systematic review, the APOE theory emerges as the most frequently applied learning theory in the research works analyzed (see Figure 3). This theory has been predominantly used to diagnose learning problems in linear algebra more accurately and deeply. It employs genetic decomposition to develop mental schemes or structures that aid students in constructing knowledge about specific concepts (30).

Regarding the theory of semiotic representations, the reviewed studies have utilized it to support didactic strategies. These strategies involve varying representations of concepts, often enhanced by computational tools for better graphic representation (16). The anthropological theory of didacticism was applied to identify students' learning difficulties in linear algebra and to back didactic strategies using modeling, incorporating technology such as mobile devices and software (7).

The theory of didactic situations was employed to categorize common errors in learning the topic of vector spaces (32). Additionally, the column labeled "others" in Figure 3 includes various theories like the theory of didactic proposal situations and realistic mathematical education (10). These theories have been instrumental in supporting didactic proposals for teaching linear algebra.

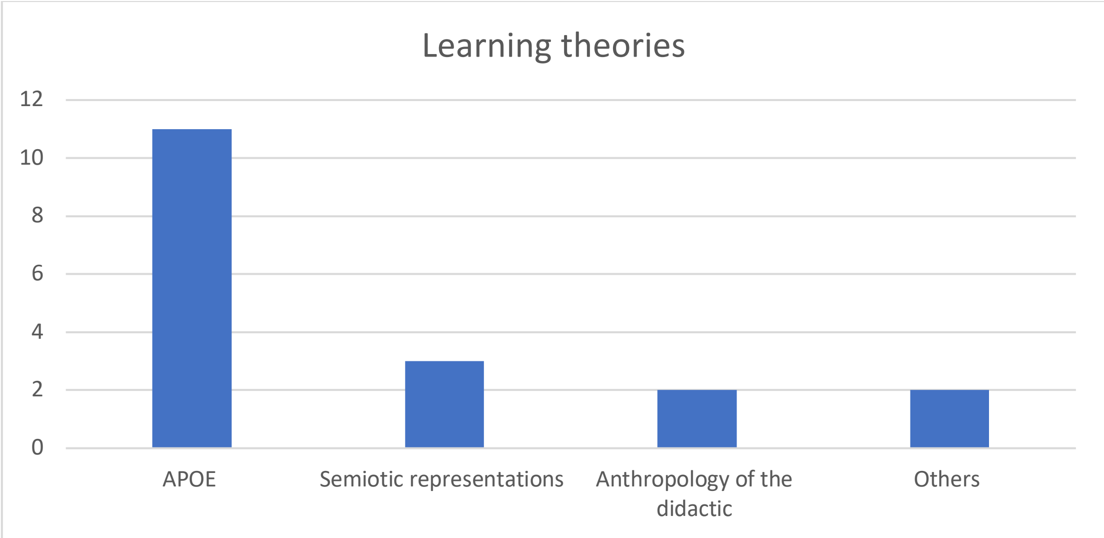

Figure 3: Distribution of Learning Theories in Reviewed Research. This figure illustrates the prevalence of different learning theories as applied in the research works reviewed. The APOE theory leads in application, followed by semiotic representations, the anthropological theory of the didactic, and other various theories.

In the systematic review, we noted the tools employed for conducting research. Prominent among these are questionnaires and interviews, particularly in studies implementing the APOE theory. The GeoGebra software stands out, along with the use of study guides on virtual platforms and a variety of activities grounded in learning theories.

It is also worth noting the global reach of research in the field of linear algebra education. Chile emerges as a leader in research production within Latin America. However, countries outside the American continent, such as Spain, Turkey, and Indonesia, also contribute significantly. This underscores the universal relevance of the challenges in teaching and learning linear algebra, indicating that these difficulties are common in classrooms worldwide, irrespective of location.

Regarding teaching strategies for linear algebra, the review also examined the specific subject topics that have been the focus of research and the sample sizes used in these studies (refer to Table 3).

Table 3

Overview of Teaching Models or Strategies, Topics, and Sample Sizes in Linear Algebra Research. This table details the teaching models or strategies applied to linear algebra topics, specifying the topics addressed and the sample sizes involved in each study.

Author and year of publication	Model or strategy	Topics	Sample size
(Nishizawa et al., 2013)	Digital technology	Vectors in 3D	40 students
(Yildiz Ulus, 2013)	Digital technology	Eigenvectors and eigenvalues	Not implementation
(Salgado & Trigueros Gaisman, 2014)	APOE-based activities	Eigenvectors and eigenvalues	34 students on average per semester
(Petrov et al., 2015)	Digital technology	Matrices and determinants, Vector spaces, Eigenvectors and eigenvalues	37 students

(Gabriel Vergara et al., 2016)	Digital technology	Systems of linear equations, Matrices, Eigenvectors and eigenvalues	35 teachers and 5 students
(Murillo & Beltrán, 2016)	Digital technology.	Vector spaces	Not implementation
(Torres et al., 2016)	Digital technology	Systems of linear equations, Vector spaces, Matrices, Linear transformations, Eigenvectors and eigenvalues	Not implementation
(Costa &Rossignoli, 2017)	Digital technology	Not specified	Voluntaries 295 students
(Meneu et al., 2017)	Activities	Eigenvectors and eigenvalues	Not implementation
(Costa, 2018)	Digital technology	Linear algebra with physics	50 students
(Karrer, 2018)	Digital technology	Linear transformations	2 students
(Kartika et al., 2018)	Digital technology	Vectors 3D	69 students
(Pierri, 2018)	Digital technology	Systems of linear equations, Matrices, Vector spaces	70 students
(Aytekin & Kiymaz, 2019)	Digital technology	Vector spaces	4 students
(Gallo et al., 2019)	Digital technology	Linear transformations	Not implementation
(García-Hurtado et al., 2019)	Mathematical modeling	System of linear equations, Matrices and determinants, Vectors, Vector spaces	36 students
(Villalobos & Ríos, 2019)	Digital technology	Vector operations	40 students
(Xavier et al., 2019)	Activities	Matrices	Not implementation
(Nissa et al., 2020)	Problem-based learning	Systems of linear equations, Matrices	21 students and 21 control group
(Pizarro, 2020)	Didactic engineering and Mathematical modeling	Linear transformations	17 students
(Fortuny & Fuentealba, 2021)	Hypothetical learning trajectories	Vector spaces	7 students
(Silva et al., 2021)	Digital technology	Matrices, systems of linear equations	Not implementation
(Wibawa et al., 2021)	Digital technology	Vector spaces	14 students

This review reveals a strong emphasis on the use of digital technology in teaching the topics discussed, with the specific tools and elements varying according to the research aims (Figure 4). For instance, there is a focus on utilizing various mathematical software (35), knowledge management platforms (26), web-based learning tools (17), virtual games (21), and virtual evidence portfolios (36).

The systematic and thorough diagnosis of mental structures that underpin the understanding of vector space concepts, linked to the design of proposed activities, was distinctly noted in the study by (33). However, a common thread across many studies is that topics of higher complexity and abstraction are most frequently addressed, both in diagnostic processes and in methodological proposals for teaching and learning.

Notably, studies targeting instruction within the domain of engineering, particularly mathematical modeling, are prominent (13). This aligns with the practical application requirements characteristic of engineering curriculums.

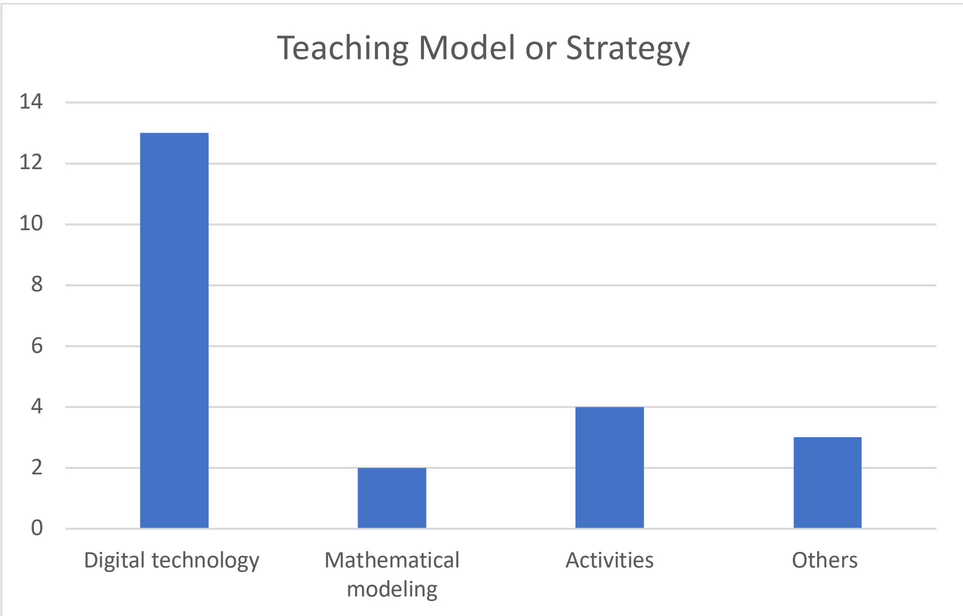

Figure 4: Frequency of Different Teaching Models or Strategies Used
This bar graph illustrates the frequency with which various teaching models or strategies are applied in linear algebra education, showcasing a predominant use of digital technology, followed by mathematical modeling, diverse learning activities, and other strategies.

Research Focus on Linear Algebra Topics
The systematic review of research works revealed that most teaching strategies and diagnostic efforts in linear algebra are focused on the more abstract concepts. Vector spaces (24), linear transformations (30), and matrices are the topics most frequently addressed. Less commonly, but still noteworthy, are studies on systems of linear equations (6) and eigenvalues and eigenvectors (3). These findings align with the goal of the research: to develop tools that mitigate the factors impacting the teaching and learning of complex linear algebra topics (4).

4. Discussion and Conclusions from the Systematic Review

The systematic review has led to several important conclusions regarding the factors that hinder students' learning of linear algebra. High levels of abstraction (23), unfamiliar formalism (18), language barriers (1), multiple representations of mathematical objects (12), lack of prior knowledge (40), and weak connections in learning (18) are significant challenges. Additionally, the complexity of new definitions, the quantity of operations between variables, and the subject's epistemological and axiomatic characteristics are noted as less frequent but still impactful factors.

In terms of learning theories, the review underscores the APOE theory as the predominant framework for in-depth research on learning difficulties in linear algebra. The theory's popularity suggests it effectively uncovers and addresses students' mental structures during knowledge construction, as highlighted by Rodriguez et al. (31). Despite this, the APOE theory's main application is in diagnosis, with other theories more commonly used to explore the results of various teaching and learning strategies, except in the work of Salgado and Trigueros (33). This review reveals a gap: the direct link between systematic diagnosis and strategy application is often absent. This could be due to educational institutions' urgent need to produce quick results, relying on authors' experience and conceptual understanding to design their approaches.

Digital technology's role is consistently significant in the research on teaching and learning strategies. Mathematical software applications (16), (20), (41), web-based learning tools—especially relevant during the COVID-19 pandemic for remote education (39), and virtual games (38) are some examples that reflect the growing, irreversible trend of digital integration in education. The main research focus in terms of content includes vector spaces (15) and linear transformations (14), likely due to their complex and abstract nature requiring a deep understanding.

Regarding sample sizes for statistical analysis in the reviewed studies, they ranged from 2 to 295 participants, with variations in application time and students' nationalities. This indicates a need for further research with larger populations, leveraging digital technology for more extensive validation and evaluation. The reviewed research, regardless of its focus, often bases some methodological aspects on the authors' experiences, their conceptual understanding, and sometimes the influence of a research community. The effectiveness of proposed solutions is most significantly validated by the experiences of those who implement them.

Therefore, future research should aim to enhance the authors' experiences and perspectives by developing methodologies that better connect with research communities and employing digital technology. This approach could allow a broader student population to engage with and benefit from the proposed methodologies in this review.

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