Meaningful Learning of University Students About the Normal Curvature of the Implicitly Given Hyperbolic Paraboloid Based on GeoGebra

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Abstract

Achieving significant learning in teaching Geometry in three-dimensional space to undergraduate students is a challenge for the teacher since it always requires a very high level of abstraction in students to visualize mathematical objects, which is often very difficult. difficult. But in recent years, as a result of the pandemic, the use of virtual tools has accelerated, developing skills of both teachers and students in the management of technological resources for teaching mathematics that help strengthen meaningful learning; One of these tools is GeoGebra, which enhances the teaching of geometry in the classroom by providing students with a graphic visualization from three different points of view and allows them to develop their creativity and autonomy in the search for knowledge. This article aims to analyze the normal curvature at all points of the hyperbolic paraboloid given implicitly from the shape operator, answering the questions of undergraduate students, specifically in the students of the Differential Geometry course of the mathematics specialty, given that Generally, in the existing literature of the course, only the normal curvature at the origin is calculated. To achieve this, first two tangent vector fields were inferred to the hyperbolic paraboloid given implicitly, then the shape operator and the normal curvature were calculated at all points of the hyperbolic paraboloid given implicitly and finally the deduced formulas were implemented in GeoGebra using the The use of some specific commands allowed us to obtain a real-time graphical visualization of the analysis of the normal curvature at all points of the saddle, given implicitly, from the shape operator. It may be possible to present examples of the calculation and graphic display of the normal curvature at some points on the surface other than the origin, presenting a contribution to the understanding of the topic.

Keywords

Normal curvature, hyperbolic paraboloid, GeoGebra, meaningful learning

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1. Introduction

Learning geometry improves the development of deductive thinking and reasoning by enhancing students' visualization and spatial ability [1], but achieving meaningful learning in teaching Geometry in three-dimensional space in undergraduate students is everything. This is a challenge for the teacher since it always requires a very high level of abstraction in students to visualize mathematical objects that are often difficult to achieve. Technological advances and the acceleration of virtuality in recent years have allowed the use of Information and Communications Technologies (ICT) in the teaching-learning process in geometry, generating more significant and relevant learning in students [2].

One of the technological tools that is becoming an almost essential ICT resource in classrooms, especially in the teaching of geometry, is GeoGebra, given the ease that the user can have to start using this tool due to its handling. almost intuitive and above all to its continuous development, which means that each of the new versions that appear offers unique options that further increase its power and effectiveness [3].

As recent advances in research in mathematics education point out, teaching enhanced by technology can have significant potential, showing itself to be an essential support to overcome some logistical obstacles such as the large number of students per teacher, the reduced number of class hours available, the heterogeneity of students' mathematical training [4], numerous studies show that the use of GeoGebra software improves mathematical problem-solving skills by promoting collaborative work, socio-student integration and improving the climate of class [5].

This research arises from the study of a traditional undergraduate differential geometry course where the normal curvature of a surface is defined, in the direction of a unit tangent vector, as the scalar product of the shape operator in said vector by the vector, especially the specific case. of the surface known as the saddle (hyperbolic paraboloid), analyzing, as is common in existing literature, the normal curvature only at the origin of coordinates. To carry out this study in class, GeoGebra was used, constituting a teaching practice that can facilitate visualization of mathematical objects by students, allowing to generate more significant learning and a better understanding of the geometric properties of the hyperbolic paraboloid and naturally arising the need to elucidate the existence of specific vectors in the direction of which the primary curvatures come to be the maximum and minimum values of the normal curvature.

Given what has been described, the research question arises: Will it be possible to analyze the normal curvature at all points of the hyperbolic paraboloid given implicitly from the shape operator using GeoGebra?

This paper aims to analyze the normal curvature at all points of the hyperbolic paraboloid given implicitly from the shape operator and its visualization in GeoGebra.

2. Calculation of normal curvature at all points of the implicitly given hyperbolic paraboloid

2.1. Deduction of two vector fields tangent to the implicitly given hyperbolic paraboloid

To deduce two vector fields tangent to the implicitly given hyperbolic paraboloid, the following theorem is established and proven.

Theorem 1. *Let $M : g(x, y, z) = c$ a surface in $\mathbb{R}^3$ and suppose that there exists a neighborhood of $\overline{p} \in M$ in which*

1. $\frac{\partial g}{\partial x}(\overline{p}) \neq 0$ *then $T_{\overline{p}}(M)$ which is the tangent space of M in $\overline{p}$ is generated by* $\vec{V}(\bar{p}) = \left(-\frac{\partial g}{\partial z}(\bar{p}), 0, \frac{\partial g}{\partial x}(\bar{p})\right), \vec{W}(\bar{p}) = \left(-\frac{\partial g}{\partial y}(\bar{p}), \frac{\partial g}{\partial x}(\bar{p}), 0\right)$.
2. $\frac{\partial g}{\partial y}(\overline{p}) \neq 0$ *then $T_{\overline{p}}(M)$ is generated by* $\vec{V}(\bar{p}) = \left(0, -\frac{\partial g}{\partial z}(\bar{p}), \frac{\partial g}{\partial y}(\bar{p})\right), \vec{W}(\bar{p}) = \left(\frac{\partial g}{\partial y}(\bar{p}), -\frac{\partial g}{\partial x}(\bar{p}), 0\right)$.
3. $\frac{\partial g}{\partial z}(\overline{p}) \neq 0$ *then $T_{\overline{p}}(M)$ is generated by* $\vec{V}(\bar{p}) = \left(0, \frac{\partial g}{\partial z}(\bar{p}), -\frac{\partial g}{\partial y}(\bar{p})\right), \vec{W}(\bar{p}) = \left(\frac{\partial g}{\partial z}(\bar{p}), 0, -\frac{\partial g}{\partial x}(\bar{p})\right)$.

It should be understood that if the hypothesis fails in any of the items then any of the other two can be used.

Proof. According to [1] since the other two proofs are similar. Let $\vec{v} \in T_p(M)$, according to Lemma 3.8 [6] $(\nabla g)(\bar{p}).\vec{v} = 0$.This is

$$\left(\frac{\partial g}{\partial x}(\bar{p}), \frac{\partial g}{\partial y}(\bar{p}), \frac{\partial g}{\partial z}(\bar{p})\right) \cdot (v_1, v_2, v_3) = 0 .$$

or in equivalent form

$$\frac{\partial g}{\partial x}(\bar{p})\, v_1 + \frac{\partial g}{\partial y}(\bar{p})\, v_2 + \frac{\partial g}{\partial z}(\bar{p})\, v_3 = 0$$

and since $\frac{\partial g}{\partial x}(\bar{p}) \neq 0$, it is possible to write

$$v_1 = \frac{1}{\frac{\partial g}{\partial x}(\bar{p})}\left(-\frac{\partial g}{\partial z}(\bar{p})\, v_3 - \frac{\partial g}{\partial y}(\bar{p})\, v_2\right) .$$

Whereby

$$\vec{v} = \left(\frac{1}{\frac{\partial g}{\partial x}(\bar{p})}\left(-\frac{\partial g}{\partial z}(\bar{p})\, v_3 - \frac{\partial g}{\partial y}(\bar{p})\, v_2\right), v_2, v_3\right)$$

$$\vec{v} = \frac{1}{\frac{\partial g}{\partial x}(\bar{p})}\left(-\frac{\partial g}{\partial z}(\bar{p})\, v_3 - \frac{\partial g}{\partial y}(\bar{p})\, v_2, \frac{\partial g}{\partial x}(\bar{p})\, v_2, \frac{\partial g}{\partial x}(\bar{p})\, v_3\right) .$$

even more

$$\vec{v} = \frac{v_3}{\frac{\partial g}{\partial x}(\bar{p})}\left(-\frac{\partial g}{\partial z}(\bar{p}), 0, \frac{\partial g}{\partial x}(\bar{p})\right) + \frac{v_2}{\frac{\partial g}{\partial x}(\bar{p})}\left(-\frac{\partial g}{\partial y}(\bar{p}), \frac{\partial g}{\partial x}(\bar{p}), 0\right).$$

Which indicates that v can be written as a linear combination of

$$\vec{V}(\bar{p}) = \left(-\frac{\partial g}{\partial z}(\bar{p}), 0, \frac{\partial g}{\partial x}(\bar{p})\right), \quad \vec{W}(\bar{p}) = \left(-\frac{\partial g}{\partial y}(\bar{p}), \frac{\partial g}{\partial x}(\bar{p}), 0\right).$$

Therefore, $V(\bar{p})$ and $W(\bar{p})$ generate $T_p(M)$. □

2.2. Calculation of the shape operator at all points of the hyperbolic paraboloid given implicitly

The shape operator of a surface

According to [6], if $\overline{p}$ is a point of M, then for every tangent vector $\vec{v}$ to M in $\overline{p}$, let

$$S_p(\vec{v}) = -\nabla_v U$$

where U is a unit normal vector field in a neighborhood of $\overline{p}$ of M. S_p is called the form operator of M on $\overline{p}$. At each point $\overline{p}$ of $M \subset \mathbb{R}^3$, the shape operator is a linear operator

$$S_p : T_p(M) \to T_p(M)$$

in the tangent plane of M at $\overline{p}$.

Normal curvature

According to [6], let $\overrightarrow{u}$ be a unit vector tangent to $M \subset \mathbb{R}^3$ at a point $\overline{p}$. Then, the number

$$k(\vec{u}) = S(\vec{u}) \cdot \vec{u}$$

is called the normal curvature of M in the direction of $\vec{u}$.

To calculate the shape operator at all points of the hyperbolic paraboloid given implicitly, it will be taken into account that if $\vec{v}$ is a nonzero vector (not necessarily of unit length) then a unit vector in the direction of $\vec{v}$ will be $\overrightarrow{u} = \frac{\vec{v}}{\|\vec{v}\|}$ so that

$$k(\vec{v}) = S\left(\frac{\vec{v}}{\|\vec{v}\|}\right).\frac{\vec{v}}{\|\vec{v}\|} = \frac{1}{\|\vec{v}\|^2} S(\vec{v}).\vec{v}$$

then

$$k(\vec{v}) = \frac{S(\vec{v}).\vec{v}}{\vec{v}.\vec{v}}$$

Theorem 2. *Given the hyperbolic paraboliode $M : z = xy$ and a point $p = (p_1, p_2, p_3) \in M$ then the direction in which maximum curvature occurs is*

$$\overrightarrow{v_1} = \left(\sqrt{\frac{p_1^2+1}{p_1^2+p_2^2+2}}, \sqrt{\frac{p_2^2+1}{p_1^2+p_2^2+2}}, p_1\sqrt{\frac{p_2^2+1}{p_1^2+p_2^2+2}} + p_2\sqrt{\frac{p_1^2+1}{p_1^2+p_2^2+2}}\right)$$

and the direction in which the minimum curvature occurs is

$$\overrightarrow{v_2} = \left(\sqrt{\frac{p_1^2+1}{p_1^2+p_2^2+2}}, -\sqrt{\frac{p_2^2+1}{p_1^2+p_2^2+2}}, -p_1\sqrt{\frac{p_2^2+1}{p_1^2+p_2^2+2}} + p_2\sqrt{\frac{p_1^2+1}{p_1^2+p_2^2+2}}\right)$$

the main curvatures are given by

$$k_1 = \frac{\sqrt{p_1^2+1}\sqrt{p_2^2+1}}{\sqrt{p_1^2+p_2^2+1}\left(1+p_1^2+p_2^2+p_1^2p_2^2+p_1p_2\sqrt{1+p_1^2}\sqrt{1+p_2^2}\right)}$$

$$k_2 = -\frac{\sqrt{p_1^2+1}\sqrt{p_2^2+1}}{\sqrt{p_1^2+p_2^2+1}\left(1+p_1^2+p_2^2+p_1^2p_2^2-p_1p_2\sqrt{1+p_1^2}\sqrt{1+p_2^2}\right)}$$

The Gaussian and mean curvature are given by

$$K = k_1k_2 = -\frac{1}{\left(1+p_1^2+p_2^2\right)^2}$$

$$H = \frac{k_1+k_2}{2} = -\frac{p_1p_2}{\left(1+p_1^2+p_2^2\right)^{\frac{3}{2}}}$$

3. Calculation and visualization of the shape operator at all points of the implicitly given hyperbolic paraboloid with GeoGebra

Learning geometry using technological tools improves meaningful learning in students, that is, in the search for knowledge about the object of study, the connections between its elements, and also activates and motivates students towards the study and understanding of the topic in question [7], this stimulates students' interest in learning geometry, leading them to ask questions about the behavior of mathematical objects in conditions other than those presented in the classroom, thus creating autonomous learning. In this work, GeoGebra will be used for all its advantages since it has an intuitive user interface that facilitates the creation and manipulation of mathematical objects. It is free mathematical software designed for use at all educational levels, created in 2001 at the University of Salzburg by Markus Hohenwarter later used at Atlantis University. Its open source (GNU GPL) uses the Java platform, guaranteeing its portability to Windows, Linux, Solaris or Mac OS spreadsheets [8]. In this sense, GeoGebra

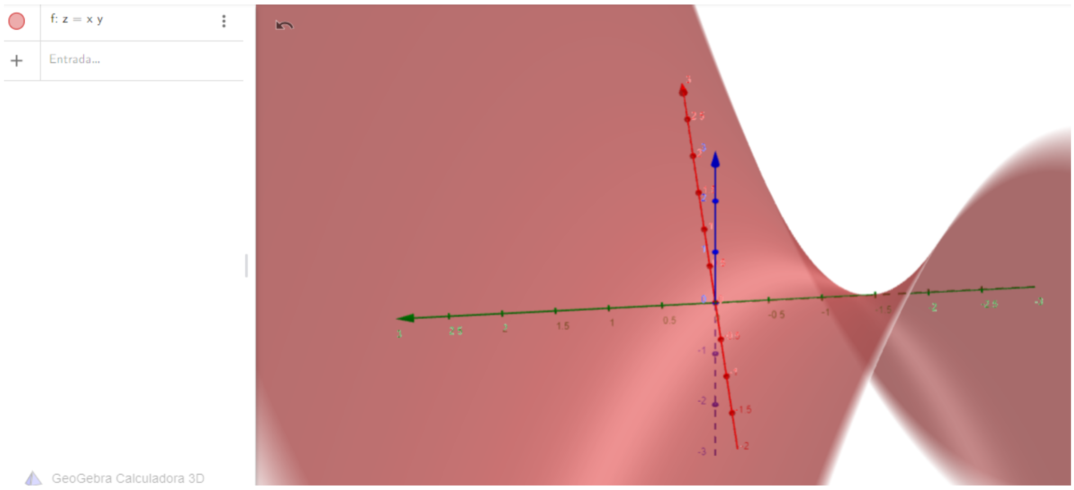

Figure 1: Hyperbolic paraboliode graph in GeoGebra.

is a very attractive tool for teaching and learning geometry. Stimulate and develop students' creativity by discovering, recognizing, identifying, seeking new relationships and generating new knowledge. Although there are also other tools such as Derive, Maple, Winplot for free use that can be presented as an alternative to using Geogebra.

Below we will show the steps to follow in the calculation and visualization of the shape operator at all points of the hyperbolic paraboloid given implicitly using the GeoGebra graphing calculator.

1 Start the GeoGebra software and enter the expression $z = xy$ in the command line, the graph of the surface will immediately appear in the graphic view window, this can be seen in Figure1

2 Next we generate 2 sliders u and v with minimum and maximum values of -5 and 5 respectively. To define a point that moves on the surface we write $P = (u, v, uv)$ on the command line. GeoGebra will immediately show us in its graphic view a point on the surface that will move on it as we move the sliders that define the variables u and v. Which can be seen in Figure 2.

3 Then in P we calculate the vectors v_1 and v_2, using the formulas shown in Theorem 2, in whose directions are the maximum and minimum curvatures respectively, which can be seen in Figure 3.

4 Next in P, the planes p_1 and p_2 that cut the hyperbolic paraboloid in the directions where its curvature is minimum and maximum are calculated, as well as their intersection curves and k_1 and k_2. which are the values of the minimum and maximum curvature respectively. Which can be seen in Figure 4 and Figure 5.

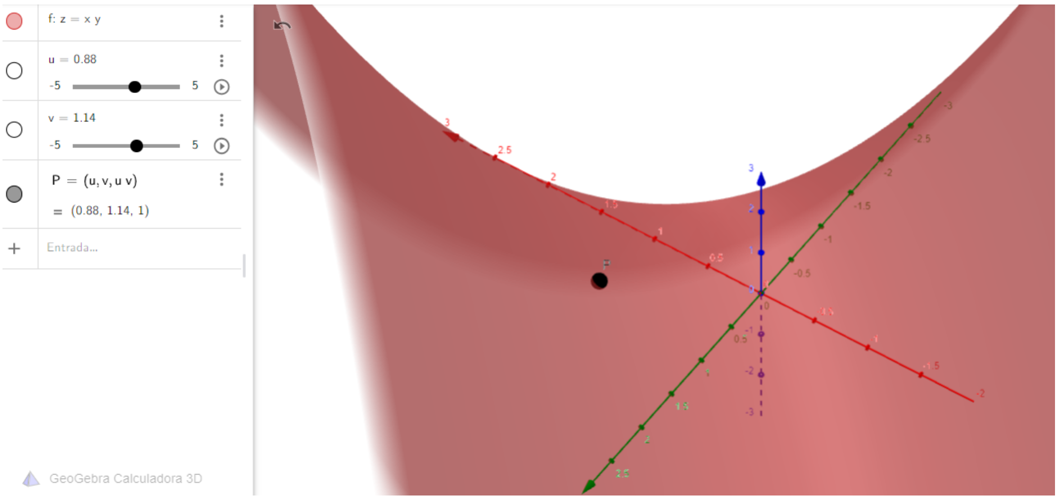

Figure 2: Graph of a point on the hyperbolic paraboliode the use of sliders allows the point to move on the surface.

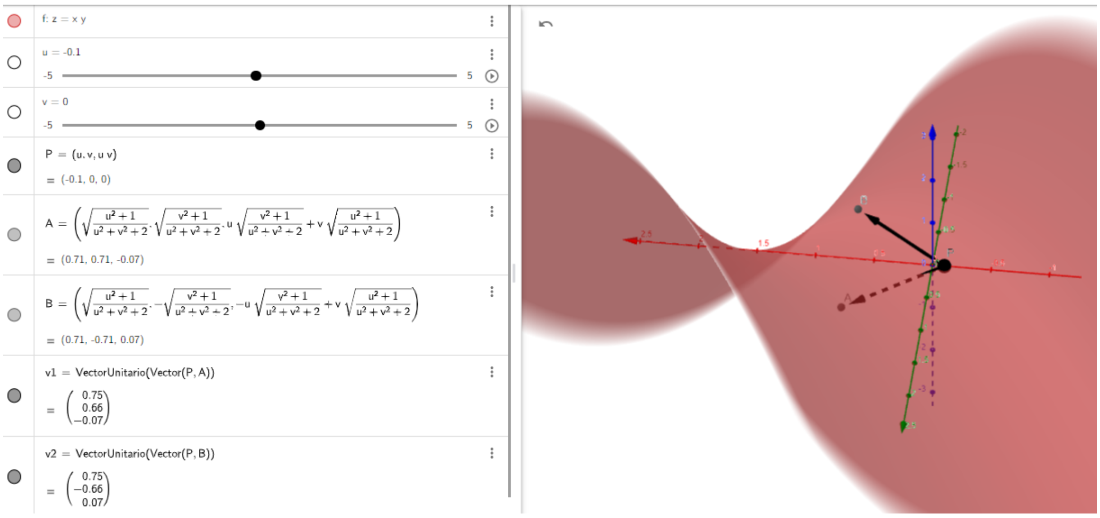

Figure 3: Graph of vectors v_1 and v_2 in whose directions are the maximum and minimum curvatures respectively at a point P of the hyperbolic paraboliode.

4. Conclusion

Through the use of the GeoGebra tool, it is demonstrated that the student is not only capable of understanding the geometric characteristics of mathematical objects, which often require a higher level of abstraction to be able to generate a mental representation that in many cases is very difficult for them. , but graphic visualization goes further since they begin to wonder if these characteristics are the same in other environments or conditions, thus generating autonomous knowledge. This is how students' concern arises about the study of the normal curvature at all points of the hyperbolic paraboloid given implicitly by the shape operator

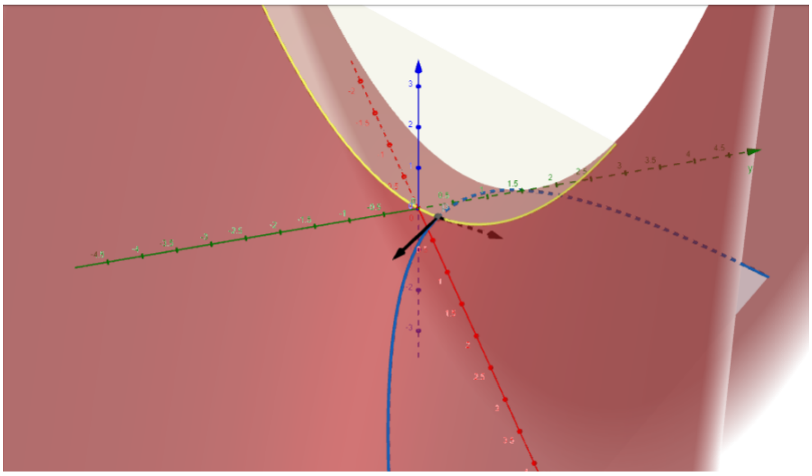

Figure 4: Graph of the planes p_1 and p_2 that cut the hyperbolic paraboloid in the directions where its curvature is minimum and maximum, as well as their intersection curves.

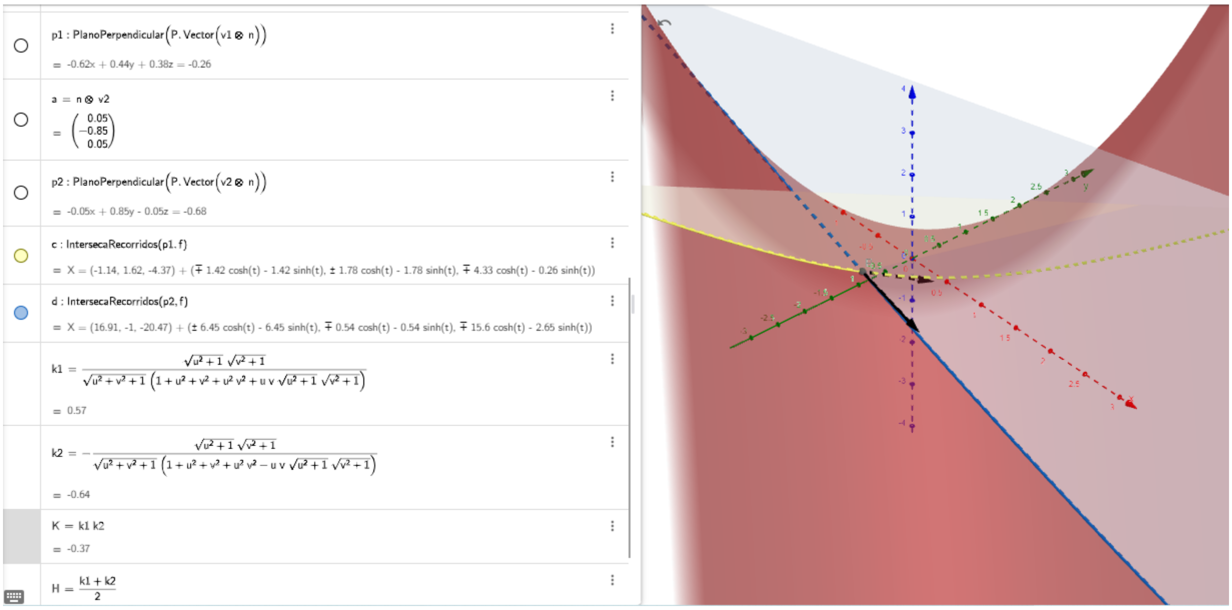

Figure 5: Calculation of the planes p_1 and p_2 that cut the hyperbolic paraboloid in the directions where its curvature is minimum k_1 and maximum k_2 respectively, as well as the Gaussian curvature K and average H in GeoGebra.

of the hyperbolic paraboloid and not only at the origin of coordinates as is usual. Through this experience the students implicitly deduced two vector fields tangent to the hyperbolic paraboloid given that when evaluated at any point belonging to said surface they generate the respective tangent space at said point, the shape operator was calculated at all points of the given hyperbolic paraboloid implicitly the shape and normal curvature were calculated at all points of the hyperbolic paraboloid given implicitly by the shape operator and the principal curvatures; as well as the mean and Gaussian curvatures, presenting a graphic visualization of this with the help of the GeoGebra tool.

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