The Method of Construction of the Law of Safety Management of Critical Infrastructure Objects Under the Conditions of External Uncontrolled Influences

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Abstract

The intensive development of information technologies with a high degree of autonomy requires the development of autonomous management systems for optimal management. This issue is especially acute for critical infrastructure objects that have been proven to be affected by extreme external factors and impacts. It is proposed to consider the management process as management in a system with incomplete a priori information about the managed process. The process of managing which changes as information is accumulated and is used to improve the operation of the entire security system in general. An identification approach to the synthesis of indirect adaptive control is used, which consists in specifying the model of the object during the control process. On the basis of the refined model, a safety control signal of the object is produced. The model of the object needs clarification because the system is constantly affected by external uncontrolled influences. The difference between the proposed method and the existing ones is that the proposed method is to build a robust control system that allows you to compensate for unknown disturbances with a certain accuracy in the required time. At the same time, by appropriate selection of the parameters of the closed system, it is possible to make the error and time values sufficiently small.

The simulation of the operation of the security management system was carried out, the results of which proved that the quality of transient processes does not depend on disturbances that affect both the nature of the behavior of the solution of the differential equation describing the critical infrastructure object and its structure. Transient processes, first of all, depend on the initial conditions of the object model and parameters of the control system. And this means that if in the process of designing the security system, the initial conditions are correctly set and the changes (including uncontrolled) of the parameters of the system's functioning are properly monitored, it is possible to ensure the stable and safe operation of the facility over time..

Keywords

Object of critical infrastructure, object model, robust system, disturbance transitional process

1. Introduction

Modern society is characterized by the intensive development of information technologies with a high degree of autonomy. This issue is particularly acute for critical infrastructure facilities that operate

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under the influence of extreme factors and impacts. One of the factors of ensuring the safety of critical infrastructure facilities is optimal facility management. Therefore, the management process should be considered as management in a system with incomplete a priori information about the managed process, which changes as information accumulates and is used to improve the operation of the entire safety system in general. Using an identification approach to the synthesis of adaptive control (indirect adaptive control), which consists in refining the model of the object during the control process and producing a control signal based on the obtained model. Setting even such a management goal as ensuring the stability of a closed system does not allow the direct use of purely descriptive models and requires the development of special methods for parameter estimation (model selection). On the other hand, the mentioned goal of management leaves wide freedom to choose the identification algorithm. This circumstance can explain the existing diversity of evaluation algorithms that use one or another a priori information about the object and disturbance and one or another function of the inconsistency of the object and the model. At the same time, it is quite difficult to compare the quality of work of different laws of adaptive control, synthesized even under conditions of the same a priori information.

A natural way to strengthen the goal of adaptive management is to set an optimal task or the task of ensuring a guaranteed result for a given quality indicator. We consider the mentioned tasks as providing the adaptive regulator with the same control quality as when controlling an object with known parameters. Therefore, modeling the process of ensuring the safety of management of critical infrastructure objects under the conditions of external uncontrolled influences is a very relevant scientific task.3

2. Review of literary sources.

The development of modern society requires intensive development of information technologies with a high degree of autonomy. This issue is particularly acute in the management of critical infrastructure facilities that operate under the influence of extreme factors. A general method for solving such problems for stationary linear objects and asymptotic quality indicators was proposed in [1]. The main idea of the method uses the concept of multiple estimation of parameters [2] and consists in choosing at each moment of time the estimate that minimizes the optimal (or guaranteed) value of the quality indicator as a function of the object parameters, on a set of parameters consistent with observations and with a priori information about the system. In [3-4], this idea is independently used in the problem of synthesis of adaptive robust control with a given guaranteed result.

Robust means management under conditions of additive and multiplicative limited disturbances. Many works investigate evaluation algorithms that allow solving the task of synthesizing adaptive robust regulators with guaranteed results in the "strong" sense" and demonstrate the influence of a given quality indicator and a priori information about the system on the choice of an evaluation algorithm.

In the class of problems of adaptive and robust management, there are many methods and approaches to their solution. They are mainly based on the assumption that we know exactly the structure of the critical infrastructure management object, that is, the order of the system of differential equations that describe the features of the management object is known [5-7, 24]. At the same time, only parametric and external actions directed at the research object are unknown. Note that, as of now, there are quite a few works devoted to the study of the problems of managing critical infrastructure objects with unknown orders [8, 10-12]. Moreover, in these works, problems of controlling linear stationary objects with unknown but constant orders of the numerator and denominator of the transfer function, which describe mathematical models in the simplest stationary cases, are considered. However, as evidenced by works [9, 13-15, 24], the research of mathematical models that describe processes in systems where there are disturbances capable of influencing not only changes in the object's parameters, but also its order, deserve attention. In works [16-18, 20], an approach to the organization of management of objects of critical infrastructure where there are non-linear non-stationary objects with unknown parameters that are subjected to external and parametric uncontrolled influences is investigated. Moreover, these perturbations affect the order of the object in an unknown way. The solution to this problem is based on the application of a robust algorithm that allows compensating this class of uncertainties with a given accuracy in finite time. Actually, this is the key to the organization of countering the consequences of unauthorized impacts on critical infrastructure objects, which are constantly exposed to threats of unauthorized external and parametric uncontrolled impacts.

3. Formulation of the problem

The purpose of the work is to implement a method of constructing a law of safety management of critical infrastructure objects under conditions of external uncontrolled influences. Definition of a continuous control law for the critical infrastructure management object, which ensures the limitation of all signals in a closed system and the fulfillment of the target condition of ensuring the safety of the critical infrastructure object.

4. The main section

4.1. Mathematical definition of a scientific task

Let us have a non-stationary nonlinear control object, the dynamic processes of which are described by a differential equation of the form:

$$\begin{aligned} &Q_t(p,t)x(t) = V(p,t)\sigma(x,t)u(t) + K(p,t)\Phi(x,t)\xi(t) + W(p,t)w(t), \\ &p_i x(0) = x_i, \quad i = [1,n] \end{aligned} \tag{1}$$

where x(t) is an adjustable parameter, u(t) is a controlling influence, w(t) is an uncontrolled disturbance. For example, in the control system of the main circuit of the heat exchanger of the power unit of the thermal power plant, there may be temperature, or pressure or steam consumption at the output of the power unit; u(t) – consumption of fuel supplied to the power unit; w(t) is the fluctuation of the energy carrier supplied to the heating system of the heat exchanger, x(t) is the moisture concentration at the outlet of the heat exchanger.

We will use the following notations:

$$\begin{aligned} &Q\ (p,t) = q_n(t)p^n + q_{n-1}(t)p^{n-1} + \dots + q_0(t), \\ &V\ (p,t) = v_n(t)p^n + v_{n-1}(t)p^{n-1} + \dots + v_0(t), \\ &K\ (p,t) = k_n(t)p^n + k_{n-1}(t)p^{n-1} + \dots + k_0(t), \\ &W\ (p,t) = w_n(t)p^n + w_{n-1}(t)p^{n-1} + \dots + w_0(t), \end{aligned} \tag{2}$$

Here Q(p,t), V(p,t), K(p,t), W(p,t) are linear nonstationary differential operators with unknown parameters, σ(x,t) is a scalar function, ξ(t)∈R^n is a vector function, Φ(x,t)∈R^(1×n) is a matrix function. For example, Φ(x,t) and σ(x,t) in the mathematical model of the ship's movement determine the nonlinearity that depends on the maneuvering angle x(t), and ξ(t) are the unknown non-stationary parameters of the nonlinearity Φ(x,t). In addition, note that x_i∈R are unknown initial conditions.

Obviously, such an equation can be obtained for a wide class of mechanical, electromechanical and other technical systems using special methods of parameterization and coordinate transformation or linearization. The quality of transient processes at the output will be determined by the reference model:

$$Q_m(p)x_m(t) = k_m v(t). \tag{3}$$

Here, x_m (t) is the output of the reference model, v(t) is the delayed system effect, Q_m (p) is a known linear normalized differential operator with constant coefficients, k_m is a known high-frequency amplification factor [19, 21-25].

Assumption 1. It will be assumed that the following conditions are met:

- Polynomials Q(p,t), V(p,t), K(p,t), W(p,t), their order degQ(p,t)≤n, degV(p,t)≤m, degK(p,t)≤n, degW(p,t)≤n and negative degree η=n-m≥1 are unknown. The coefficients of the operators Q(p,t), V(p,t), K(p,t), W(p,t) are bounded functions, and the non-zero coefficients at higher powers of Q(p,t) and V(p,t) are positive functions.
- The coefficients of the operators Q(p,t), V(p,t), K(p,t), W(p,t) and the vector function ξ(t) depend on the vector of unknown parameters ω∈M, where M is a known bounded set.

- It is known that $\eta_u \geq \eta$ is the upper limit of the relative power of η. The order of the operator Q_m (p)-η_u, k_m>0.
- The operator V(p,t) is stable, and for an arbitrary fixed instant of time t, the polynomial V(p,t) is a Hurwitz polynomial, where λ is a complex variable of the Laplace transform. The polynomial Qm (p) is a hurwitz polynomial.
- The elements Φi (x,t), i=(1,n)¯of the matrix function Φ(x,t) are unknown and satisfy the global Lipschitz condition on x(t), smooth functions bounded on t; ξ(t) is an unknown vector whose components are smooth bounded functions. The nonlinearity σ(x,t) is known, and σ(x,t)>0 for any x(t)∈R and t.
- System influence v(t) and disturbance w(t) are bounded functions.
- In the system, the equations are not available for measurements of derivative signals x(t), u(t), and v(t).

It follows from assumption 1 that the dynamic order of object (1) is unknown. At the same time, the object can change as a result of the influence of parametric disturbances on it. For example, if q_n (t)=0 and q(n-1) (t)≠0, then deg Q(p,t)=n-1, if qn (t)=q(n-1) (t)=0 and q(n-2) (t)≠0, then deg Q(p,t)=n-2, etc. Similarly for the operator V(p,t): if vm (t)=0 and v_(m-1) (t) ≠0, then degV(p,t)=m-1, if v_m (t)=v(m-1) (t)=0 and v(m-2) (t)≠0, then deg V(p,t)=m-2, etc. The requirement to know the signs of the zero coefficients at higher degrees of the operators Q(p,t) and V(p,t) (condition A) and the function σ(x,t) (condition E) is related to the knowledge sign of the high-frequency gain of the object (1). This allows for negative feedback in the control system. The goal of management is to find a continuous regulation law for the critical infrastructure management object, which ensures the limitation of all signals in a closed system and the fulfillment of the target condition:

$$|\varepsilon(t)| = |x\ (t) - x_m(t)| < \delta, \tag{4}$$

for a finite time T for all ω∈M, where δ>0 is a sufficiently small number.

4.2. The method of determining the continuous regulation law

Let's decompose the operators V(p,t) and Q(p,t) into components

$$\begin{aligned} Q(p,t) &= Q_0(p) + \Delta Q(p,t), \\ V(p,t) &= V_0(p) + \Delta V(p,t). \end{aligned} \tag{5}$$

Here ΔV(p,t)=c_01^T (t) [1,p,…,p^(n¯-2)]^T, c_{01} (t) is a vector given by the coefficients of the operator V(p,t)-V_0 (p) and such that the second relation in (4) is always fulfilled, V_0 (p) is an arbitrary stationary linear differential operator of degree n¯-η_u and the polynomial V0 (λ) is a covariant, n¯is the upper limit of the order of the operator Q(p,t). Note that the value of n¯is necessary only for the justification of the structure of the closed control system, and not for its implementation. Regarding the structure of ΔV(p,t), we can say that :

- if m<n¯-ηu, then degΔV(p,t)=n¯-ηu;
- if m=n¯-ηu, then degΔV(p,t)≤n¯-ηu;
- if m>n¯-ηu, then degΔV(p,t)=m,

that is, there is always a vector c_01 that ensures the validity of the expansion of the operator V(p,t). In one case, it has all non-zero components, in the other, the corresponding number of zero components. Next, Q0 (p) is an arbitrary linear stationary differential operator such that the polynomial Q0 (λ) is a Hurwitz polynomial and deg 〖Q0 (p)〗 =n¯. Then the operator ΔQ(p,t) is the difference Q(p,t)-Q_0(p), and degQ(p,t)≤n¯, i.e.:

- if degQ(p,t)<deg 〖Q0 (p)〗 , then degΔQ(p,t)=deg 〖Q0 (p)〗 ;
- if degQ(p,t)=deg 〖Q0 (p)〗 , then degΔQ(p,t)≤n¯-1.

It is this representation that allows solving the formulated problem, which differs from the known methods of parametrizing the equation [1]. Due to the arbitrariness of the operators V0 (p) and Q0 (p), we choose them in such a way that:

$(Q_0(p))/(V_0(p))=Q_m(p)$.

Then, taking into account relations (1), (2) and (4), we write down the tracking error $\varepsilon(t)=x(t)-x_m(t)$ in the following way:

$$Q_m(p)\varepsilon(t)=\sigma(x,t)u(t)+\varphi(t), \tag{6}$$

Where $\varphi(t)=V/(V0^{\wedge}(p))\,[\Delta V(p,t)\sigma(x,t)u(t)-\Delta Q(p,t)x(t)+K(p,t)\Phi(x,t)\xi(t)+W(p,t)w(t)-k\_m\,v(t)]$.

To regulate the object (1), we set the control law:

$$u(t)=\frac{h(t)}{\sigma(x,t)},\quad h(t)=\alpha T(p)\bar{v}(t). \tag{7}$$

Let's transform into the error equation (5):

$$Q_m(p)\varepsilon(t)=T(p)(\alpha v(t)+\varphi_1(t)+\alpha\Delta(t)),\quad \varphi_1=\frac{\varphi(t)}{T(t)}. \tag{8}$$

Here α>0, T(t) is a linear differential operator of degree η_u such that the polynomial T(λ) is a Hurtz polynomial, and the roots of the polynomial T(λ) are the roots of the transfer function of the closed system; $\bar{\upsilon}(t)$ – evaluation of the signal υ(t), υ(t) – auxiliary control influence (formation of the functions $\bar{\upsilon}(t)$ and υ(t) will be described in more detail below); $\Delta(t)=\bar{\upsilon}(t)-\upsilon(t)$ is the estimation error of the signal υ(t). Let's introduce an additional (auxiliary) contour:

$$Q_m(p)\bar{\varepsilon}(t)=\beta T(p)v(t),\quad \beta>0. \tag{9}$$

Considering equations (7) and (8), we write the discord equation $\xi(t)=\varepsilon(t)-\bar{\varepsilon}(t)$:

$$Q_m(p)\xi(t)=T(p)(\alpha\bar{\Delta}(t)+\varphi_1(t)), \tag{10}$$

Where $\varphi_1(t)=(\alpha-\beta)v(t)+\dfrac{\varphi(t)}{T(t)}$ a new perturbation function, which includes a priori functional and parametric uncertainty,); $\bar{\Delta}(t)=\bar{\upsilon}(t)-\upsilon(t)$ is the estimation error of the signal υ(t).

Let us define the law of the auxiliary control influence υ(t) in the form:

$$v(t)=-\frac{Q_m(p)\xi(t)}{\beta T(t)}=-\frac{\varphi_1(t)}{\beta}. \tag{11}$$

Solving equation (10) with respect to the variable υ(t), we obtain that

$\upsilon(t)=-\varphi(t)/(\alpha\cdot T(p))$.

Let's substitute the last result in relation (7). Then the equation of the closed system with respect to the tracking error can be written in the form:

$$Q_m(p)\varepsilon(t)=T(p)\alpha\bar{\Delta}(t). \tag{12}$$

To estimate η_u derivatives of the signal υ(t), we will use the approach proposed in [4].

$$\dot{v}(t)=G_0v(t)+D_0(\bar{v}(t)-v(t)),\quad \bar{v}(t)=Lv(t). \tag{13}$$

Here $\vartheta(t)\in R^{\wedge}(\eta_u)$, $D_0=-[d1\,\mu^{\wedge}(-1),d2\,\mu^{\wedge}(-2),\ldots,d(\eta u)\,\mu^{\wedge}(-\eta u)]^{\wedge}T$, and d1,d2,…,d(η_u) are chosen from the condition of the density of the matrix:

$$G=G_0-DL,\quad G_0=\begin{bmatrix}0 & I_{\eta u-1}\\ 0 & 0\end{bmatrix}. \tag{14}$$

I(ηu-1) is a unit matrix of order ηu-1, D=[d1,d2,...,d(ηu)]^T, μ is a sufficiently small value, L=[1,0,...,0]. Using the filter (12) allows you to estimate η_u of the derivatives of the signal υ(t) and, thereby, to implement relation (6).

To assess the accuracy of observations, we will additionally consider the vector of deviations

$\bar{\tau}(t)=\Gamma^{\wedge}(-1)\,(\vartheta(t)-\theta(t))$,

where

$\Gamma=\mathrm{diag}\,\{\mu^{\wedge}(\eta u-1),\mu^{\wedge}(\eta u-2),\ldots,\mu,1\},\theta(t)=[\upsilon(t),\dot{\upsilon}(t),\ldots,\upsilon^{\wedge}(\eta u)\,(t)]$.

Differentiating $\bar{\tau}(t)$ with respect to t taking into account (12), we obtain

$\dot{\bar{\tau}}(t)=\mu^{\wedge}(-1)\,G\bar{\tau}(t)+\bar{b}\bar{\upsilon}^{\wedge}((\eta\_u+1))\,(t),\bar{\Delta}(t)=\mu^{\wedge}(\eta\_u-1)\,L\bar{\tau}(t),\bar{b}=[0,\ldots,0,1]^{\wedge}T$.

We perform the transformation in the equation Δ̅(t)=μ^(ηu-1) Lτ̅(t) relative to Δ̅(t). Then:

$$\dot{\tau}(t) = G\tau(t) + b\dot{v}(t), \quad \bar{\Delta}(t) = \mu^{\eta u-1} L\tau(t). \tag{15}$$

Here:

τi (t)=τ̅i (t)-μ^(1+i-ηu) υ^((i-1)) (t),i=(2,ηu)

τ1 (t)=τ̅1 (t), b=[μ^(2-η),0,…,0]^T.

The last two equations are equivalent with respect to the variables τ1 (t)=τ̅1 (t), since they are different vector-matrix forms of writing the same equation

(p^(ηu)+d1 μ^(-1) p^(ηu-1)+⋯+d(ηu) μ^(-ηu)) τ̅1 (t)=p^(ηu) υ(t).

Considering equations (12) and (13), the tracking error equation (11) takes the form:

$$\dot{\varsigma}(t) = A_m \varsigma(t) + \alpha\ \mu^{\eta u-1} \bar{b} g \Delta(t), \quad \varepsilon(t) = L\varsigma(t). \tag{16}$$

where ε(t)∈R^(ηu), A_m∈R^($\eta_u \times \eta_u$) is a matrix in the form of Frobenius with the characteristic polynomial Q_m (λ), Δ(t)=[η1(t),η1(t),...,η1^(ηu) (t)], g is a vector composed of coefficients of the polynomial T(λ) [21-24].

4.3. Results of simulation of the control law built according to the proposed method

Consider a non-linear non-stationary critical infrastructure management object of the form (1)

$$\begin{aligned} &\left[q_4(t)p^4 + q_3(t)p^3 + q_2(t)p^2 + q_1(t) + q_0(t)\right]x(t) = \left[v_1(t)p + v_0(t)\right]\sigma(x,t)u(t) + \\ &+\left[\bar{k}_4(t)p^4 + \bar{k}_3(t)p^3 + \bar{k}_2(t)p^2 + \bar{k}_1(t) + \bar{k}_0(t)\right]\Phi(x,t)\xi(t) + \\ &+\left[w_4(t)p^4 + w_3(t)p^3 + w_2(t)p^2 + w_1(t) + w_0(t)\right]w(t). \end{aligned} \tag{17}$$

The uncertainty class M is given by the inequalities:

0≤q4≤5, 0≤q3≤15, 0,5≤q2≤20, -10≤q1≤10, -10≤q0≤10,

0≤v1 (t)≤3, 0.5≤v0 (t)≤3,

-4≤k̅i (t)≤4, -5≤wi (t)≤5,-2≤ξi (t)≤2,i=(1,n)̅,|w(t)|≤10

We will assume that the polynomial v1 (t) p^ +v0 (t) is stable and the polynomial L{v1 (t) p^ +v_0(t)} is Hurwitz for any fixed instant of time t; L{·} is the Laplace transform operator; upper estimate relative to the degree η of the control object (a) η̅u=4;

σ(x,t)=1+|x(t)|+|x(t)|^2 sin^2 2t.

The remaining parameters in equation (a) are unknown. Analyzing the uncertainty class M, it can be stated that in the process of operation (a) the order of the characteristic polynomial Q(p,t) can take values from the set {2,3,4}, and the polynomial V(p,t) 0 and 1. Since η̅u=4, then deg 〖Q_m (p)=4〗. Then the equations of the reference model are given in the form (p+1)^4 x_m (t)=v(t), km=1,v(t)=2+sin0,5t+2 sint+P1, P1 – rectangular pulses with an amplitude of 2, a period of 4 and a duration of 2s. Let's choose β=50. The distribution of zeros of the transfer function of the closed system (11) is determined by the operator : T(p)=1/100 p^4+2p^3+200p^2+200p+100.

Then, in accordance with relations (8) and (10), the equation of the auxiliary control influence and the auxiliary circuit will be written in the form:

$$v(t) = \frac{(1+p)^4}{0{,}01p^4 + 2p^3 + 200p^2 + 200p + 100}\xi(t), \tag{18}$$

$$\varepsilon(t) = \frac{0{,}01p^4 + 2p^3 + 200p^2 + 200p + 100}{(1+p)^4} v(t), \tag{19}$$

Here ξ(t)=ε(t)-ε ̅(t), ε(t)=x(t)-x_m (t) misalignment and tracking errors, respectively. Using the observation equation (12), we obtain estimates of υ^((i)) (t), i=(0,4) ̅:

ϑ1(t)=-ϑ2(t)-d1 μ^(-1) (ϑ1 (t)-υ(t)),
ϑ2(t)=-ϑ3(t)-d_2 μ^(-1) (ϑ2 (t)-υ(t)),
ϑ3(t)=-ϑ4(t)-d_3 μ^(-1) (ϑ3 (t)-υ(t)),
ϑ4(t)=-d4μ^(-1) (ϑ4 (t)-υ(t)),

where D=[d1,d2,d3,d4]=[20,150,500,625]^T, μ= 〖10〗 ^(-2). When α=50 is set, the control law, in accordance with equation (6), will be written in the form:

$$u(t) = \frac{h(t)}{1+|x(t)|+|x(t)|^2 \sin^2(2t)}, \tag{20}$$

$$h(t) = 50\left[0{,}01\dot{v}_4 + 2v_4(t) + 200v_3(t) + 200v_2(t) + 100v_1(t)\right]. \tag{21}$$

We will assume that the initial conditions in the system are zero. At the same time, in equation (a), the nonlinearities are specified in the following way:

$$\begin{aligned} &\Phi(x,t) = \left[\ln^4(1+|x|), \ln^4(1+|x|), \cos 3x + 2\sin x, \ln^4(1+|x|) + x^2\ \right], \\ &\xi(t) = \left[0{,}5\sin t, \cos 2t, 1+\sin t, 2\right]^T \end{aligned} \tag{22}$$

and external disturbances to the object of critical infrastructure act according to the law:

$$w(t) = 1{,}5 + 1{,}5\sin 1{,}5t + \cos(0{,}8t + \frac{\pi}{3}) + P_1(t-0{,}5). \tag{23}$$

Then the law of managing the security system of critical infrastructure objects under the conditions of external uncontrolled influences built according to the given methodology in a graphic form will have the form depicted in Fig. 1. It should be noted that Fig. 1 shows the control law for external disturbances, which are simulated by rectangular pulses with an amplitude of 1 (in relative units), a period of 15 s and a duration of 5, 8, and 12 s, respectively.

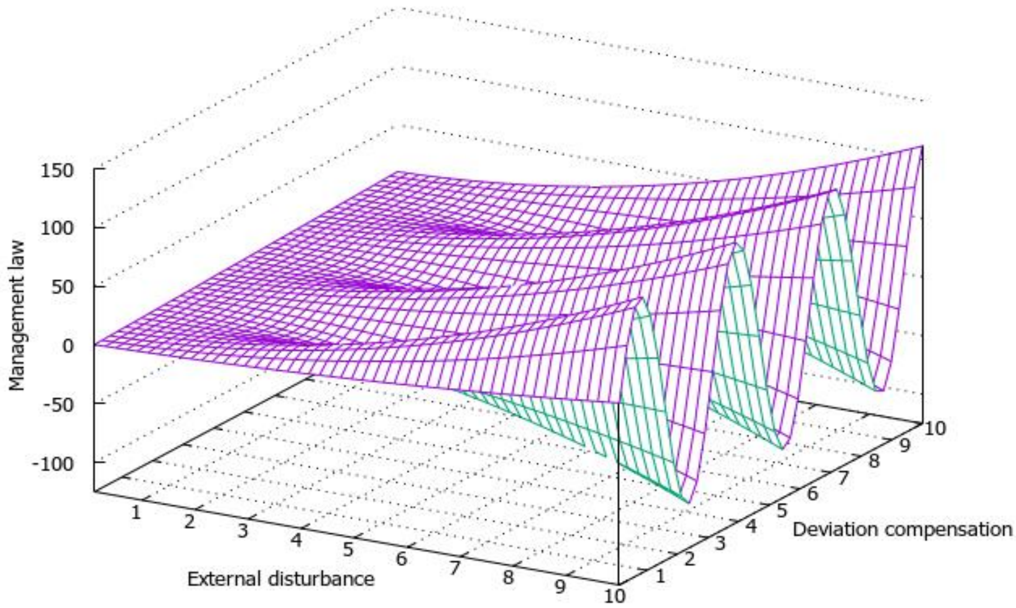

Figure 1: The law on the management of the security system of critical infrastructure objects, the first option of external disturbances. (first option).

The law of managing the safety system of critical infrastructure objects under the conditions of external uncontrolled influences, built according to the given methodology, in graphic form will have the form depicted in Fig. 2. The simulation was carried out under external disturbances, which are simulated by rectangular pulses with an amplitude of 2 (in relative units), a period of 15 s and a duration of 7, 9, and 15 s, respectively (the second option).

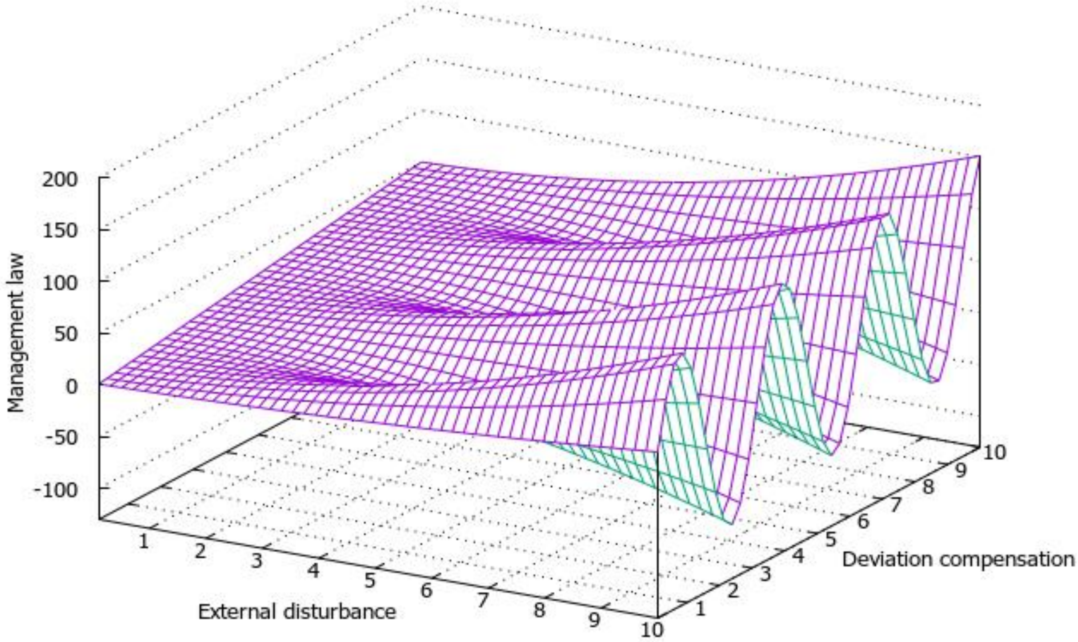

Figure 2: The law on the management of the security system of critical infrastructure objects, the first version of external disturbances (the second version)

The law of managing the security system of critical infrastructure objects under the conditions of external uncontrolled influences, built according to the given methodology, in graphic form will have the form depicted in Fig. 3. The simulation was carried out under external disturbances, which are simulated by rectangular pulses with an amplitude of 3 (in relative units), a period of 15 s and a duration of 9, 11, and 17 s, respectively (the second option).

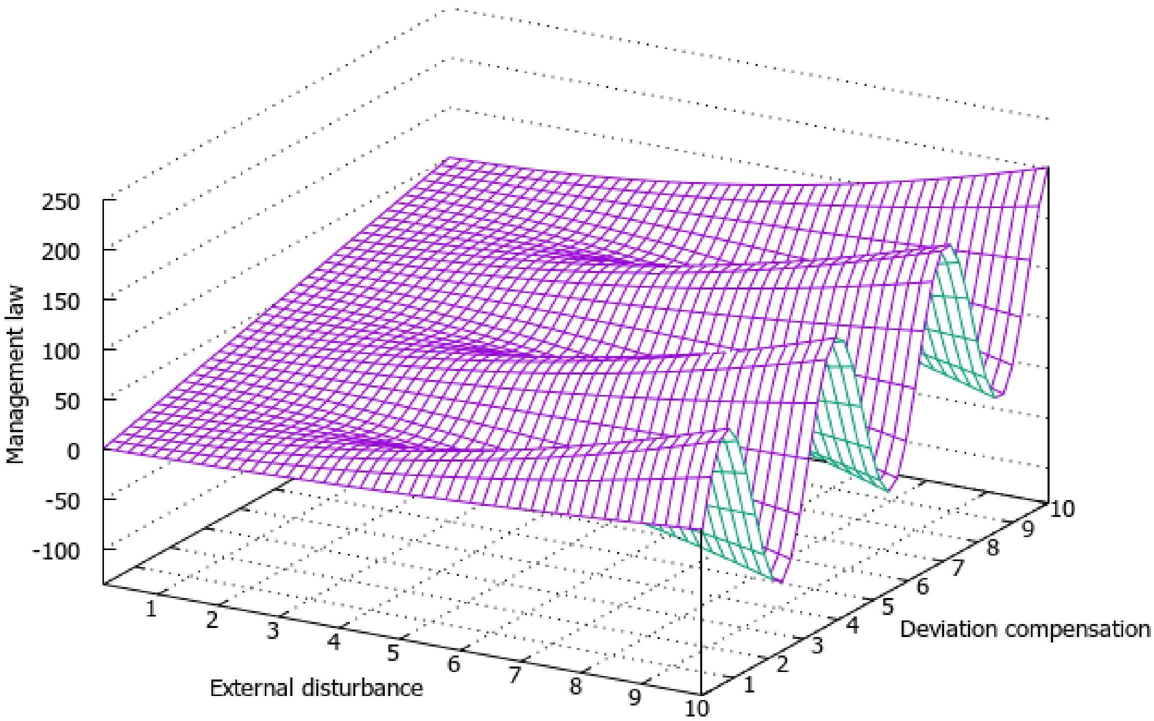

Figure 3: Fig. 3. The law on the management of the security system of critical infrastructure objects, the first version of external disturbances (the third version)

Figures 1- 3 show that disturbances that affect the structure of object (a) do not significantly affect the results of transient processes. The results, first of all, depend on the initial conditions of the object itself, the values of μ in relation (12), α and β in formulas (6), (8) and (10) and the matrix D in relation (12).

5. Conclusion

For critical infrastructure objects, a method of constructing a law of safety management of critical infrastructure objects under the conditions of external uncontrolled influences is proposed. Namely, the method of building a robust control system that allows you to compensate for uncontrolled external influences and parametric uncertainty. The difference of the proposed method lies in the fact that the robust control system allows to compensate for unknown disturbances with a certain accuracy in the required time. At the same time, by appropriate selection of the parameters of the closed system, it is possible to make the error and time sufficiently small.

The simulation results proved that the quality of transient processes does not depend on disturbances that affect both the nature of the behavior of the solution of the differential equation describing the critical infrastructure object and its structure. Transient processes, first of all, depend on the initial conditions of the object model and parameters of the control system. And this means that if in the process of designing the security system, the initial conditions are correctly set and the changes (including uncontrolled) of the parameters of the system's functioning are properly monitored, it is possible to ensure the stable and safe operation of the facility over time.

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