The Project of Information System for Students Knowledge Evaluation

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Abstract

The paper provides an analysis of using the bimatrix game algorithm in designing an information system for students' knowledge evaluation. The information system function tools are based on the theory of matrix games resting upon analysis of the result of a conflict of two players – a teacher and a student whose interests were opposite to some extent. Comparison of the obtained results of solving matrix games with results of solving a bimatrix game confirms the possibility of justifying not only quantitative results on the players' average wins but also their qualitative behavior.

Keywords 1

Bimatrix game algorithm, information system, students, knowledge evaluation

1 Introduction

Development of a student's knowledge evaluation information system is based on the matrix game theory, particularly on the part thereof that is closely connected to the linear programming and came to be known as matrix game. A matrix game provides for availability of two players, a series of rounds in each of which both of the players shall move. The result is to be announced based on these moves: a tie or a winner. For mathematic analysis of the game, it is necessary to determine the winner and the quantity parameters of the win for each pair of moves. This shall be recorded in the payment matrix.

2 Latest achievements review

Andreas Breiter, Emese Stauke consider school institutions to be an essential component of the knowledge control system, which contributes to interested parties' access to the information on teaching and studying, as a confirmation of its success.

While developing a knowledge control information system at school, as per the authors' opinion, it is necessary to determine informational needs of users, the relevant data, and to establish the feedback. The authors have proposed approaches to support of decision taking in school control with use of normalized testing results.

The specifics of the knowledge level evaluation information system proposed by the authors and used in Hungary consist in the possibility of arranging the feedback on various levels for various target groups [1–3]. Zlatović, M., Balaban, I., & Hutinski, Ž. [4] have proposed a model of a knowledge evaluation adaptive online system and checked the efficiency of its implementation.

The analyzed model contributes to knowledge evaluation, it is formed as a chain of evaluation taking place during a certain period. As per the authors' opinion, the proposed system provides the possibility of: introducing new topics of the course for each iteration of the evaluation, making repeated evaluation of the course topics, evaluating with account taken of all marks already received, arranging a feedback from students, consulting during preparation for next evaluations [5,6].

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The researchers note that approbation of the proposed system confirms its efficiency in usage of the accumulative evaluation model.

Thus, they have proposed a model of an online evaluation of knowledge consisting of a series of marks linearly combined in a chain-like structure. This approach is aimed at orientation towards continuous knowledge level improvement.

After each automatic evaluation, a student receives a notice on the level of each educational target achievement, as well as tasks to be completed for successful further evaluation and repeated evaluation of previous educational topics.

Thus, usage of the developed system enabling a combination of different types of questions, adaptation between evaluations and individual feedback after evaluation for each educational target, contributes to increasing the level of students' knowledge[7,8].

The function tools of the system allow imitating the traditional evaluation system, and receiving a positive mark requires completing 50% of tasks.

Maier, Wolf und Randler [9] have analyzed the importance of feedback proposing to arrange it with use of computer multilevel tests, when their utility is acknowledged.

Researchers Adebayo, E. I. & Wokocha [10] convince that an information system for an educational institution control requires continuous support after implementation. They have studied industries where knowledge control information systems were created for maintaining the competitive advantage [11,12].

Mukhneri Mukhtar, Sudarmi Sudarmi, Mochamad Wahyudi and Burmansah [13] have completed a research for determining requirements to a knowledge control information system in a higher school having analyzed the experience of using information technology for creation of the mentioned systems.

Janis Grundspenkis [14] has generalized the experience of developing a knowledge evaluation information system, particularly a system of knowledge evaluation procedures automation.

The author notes that the knowledge level evaluation can be automated in accordance with Bloom taxonomy with selecting a format replies and/or solutions submitted by students. The researcher justifies usage of concept cards for knowledge evaluation and analyzes the specifics of the function tools of the developed adaptive intellectual knowledge evaluation system.

Mohamed AF Ragab, Amr Arisha [15] note that the effectiveness of knowledge management is determined by the possibility of evaluating knowledge based on the constructions of individual knowledge. The information on the conceptualization of individual knowledge and the characteristics of knowledge carriers has been analyzed.

The proposed information system for evaluating knowledge from several points of view, using a multidimensional system of theoretically based indicators, which creates an Individual Knowledge Index. The index is calculated using a unique mathematical formula that combines multi-criteria decision analysis methods to consolidate results.

The implementation of the technology allows to fully automate the evaluation process and helps to solve parametric multiplicity and arithmetic complexity. The authors proposed a complex integrated system of individual assessment of knowledge, in which knowledge is considered as a personal and humanistic concept [16].

3 Statement of basic material

The information system function tools are based on the theory of matrix games resting upon analysis of the result of a conflict of two players – a teacher and a student whose interests were opposite to some extent.

For development of an information system function algorithm, let us examine a conflict between two players C A and D B. Player C may follow his own strategies that we give in the following tuple: $\{C_1, C_2, \dots, C_m\}$. Player D selects one of the strategies that we give in the following tuple: $\{D_1, D_2, \dots, D_n\}$.

Both the players are participants of the game with known rules and a combinatory selection receives the reward. In the case when player C inclines to strategy C_i and player D to strategy D_j, we can present win C as c_{ij}. The win of player D shall be d_{ij}. In this case $c_{ij} \neq d_{ij}$, each of the players expects to get a reward.

Player A can adopt two strategies A_1 or A_2 to trust the student or to check his statements on the studied theoretical sections of the game theory discipline. The second player B (the student) has two strategies too. B_1 - to prepare a theoretical course for examination or not to prepare. The teacher may check or not check the student's theoretical knowledge directly during the examination. Let's assume that the value of win both for the teacher and for the student is conditionally equal to $a > 0$ units. Therefore, payment matrixes of the players look as follows:

$$C_A = \begin{bmatrix} -a & a \\ 0 & 0 \end{bmatrix}, \ C_B = \begin{bmatrix} 0 & -a \\ 0 & a \end{bmatrix}.$$

If the teacher is sure that the student has not mastered the theoretical course, he would check the student's knowledge (location in payment matrixes (1,2)). The student would study the course if he is sure that the teacher would check his knowledge (location (1,1)). And the other way round, the teacher does not need to check the student if he has mastered the course (location (2,1). Finally, the student would not master the course if he is sure that the teacher would not check his knowledge (location (2,2)).

Model examples of a bimatrix game solution in mixed strategies.

Let us consider the features of a conceptual bimatrix game if the players' payment matrixes are:

$$C_A = \begin{bmatrix} r & r+s \\ s & 0 \end{bmatrix}, \qquad C_B = \begin{bmatrix} t & s \\ t+s & 0 \end{bmatrix}.$$

Let us have $X_A = [p_1, p_2] = [p, 1-p]$, $X_B = [q_1, q_2] = [q, 1-q]$. In this case, the players' wins average values will be:

$$M_A(X_A, X_B) = \sum_{i=1}^{2} \sum_{j=1}^{2} a_{ij} p_i q_j = (r + s - 2qs)p + sq,$$

$$M_B(X_A, X_B) = \sum_{i=1}^{2} \sum_{j=1}^{2} b_{ij} p_i q_j = (s + t - 2ps)q + sp.$$

The game equilibrium description system (1) looks as follows:

$$\begin{cases} (r + s - 2qs)(1 - p) \le 0, \\ (2qs - r - s)p \le 0, \\ (s + t - 2ps)(1 - q) \le 0, \\ (2ps - t - s)q \le 0. \end{cases} \qquad (2)$$

Solving the system we obtain frequencies of using strategies by the players and their respective wins:

$$X_A = \left[\frac{t+s}{2s}, \frac{s-t}{2s}\right], \qquad \upsilon_x^A = M_A(X_A, X_B) = \frac{r+s}{2},$$

$$X_B = \left[\frac{r+s}{2s}, \frac{s-r}{2s}\right], \qquad \upsilon_x^B = M_B(X_A, X_B) = \frac{s+t}{2}.$$

The existence of equilibrium in bimatrix game pure strategies does not exclude the existence of equilibrium in mixed strategies.

Let the added utility of joint using computers of the same hardware platform be higher than the utility of acquiring the known platform, i.e. $s > r$ and $s > t$.

In this case, we the first player's feedback

$$p=\begin{cases} 0, & q<\dfrac{t+s}{2s}, \\ [0,1], & q=\dfrac{t+s}{2s}, \\ 1, & q>\dfrac{t+s}{2s}. \end{cases}$$

The second player would have the reaction

$$q=\begin{cases} 0, & p<\dfrac{r+s}{2s}, \\ [0,1], & p=\dfrac{r+s}{2s}, \\ 1, & p>\dfrac{r+s}{2s}. \end{cases}$$

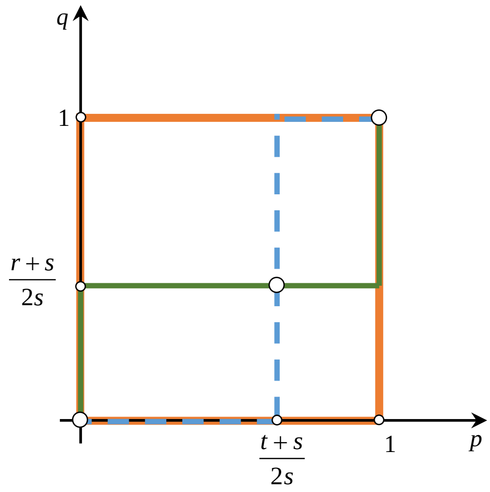

Fig. 1 Thee equilibrium points. Two points in pure strategies and one point in mixed ones

We make a graphic representation of the players' reaction on square $\{p,q \mid p\in[0,1], q\in[0,1]\}$ (Fig. 1). Fig. 1 shows the available three equilibrium points two of which are realized in pure strategies (A_1, B_2) with game values $\upsilon_1^A = r+s$ and $\upsilon_1^B = s$ i (A_2, B_1) with values $\upsilon_2^A = s$ and $\upsilon_2^B = t+s$ plus one point $\left(\dfrac{t+s}{2s}, \dfrac{r+s}{2s}\right)$ in mixed strategies for the first player with frequency vector $X_A=\left[\dfrac{t+s}{2s}, \dfrac{s-t}{2s}\right]$ and value $\upsilon_x^A=\dfrac{r+s}{2}$ and for the second one - $X_B=\left[\dfrac{r+s}{2s}, \dfrac{s-r}{2s}\right]$, $\upsilon_x^B=\dfrac{s+t}{2}$.

No. 2. Let us consider example No. 2 of a conceptual bimatrix game.

Payment matrixes of the players will be:

$$C_A=\begin{bmatrix} -a & a \\ 0 & 0 \end{bmatrix},\ C_B=\begin{bmatrix} 0 & -a \\ 0 & a \end{bmatrix}.$$

Let it be $X_A=[p_1,p_2]=[p,1-p]$, $X_B=[q_1,q_2]=[q,1-q]$. In this case, the average values of the players' wins will be:

$$M_A(X_A,X_B)=\sum_{i=1}^{2}\sum_{j=1}^{2} a_{ij}p_iq_j = ap(1-2q),$$

$$M_B(X_A,X_B)=\sum_{i=1}^{2}\sum_{j=1}^{2} b_{ij}p_iq_j = a(1-q)(1-2p).$$

The game equilibrium description system (1) looks as follows:

$$\begin{cases} a(1-p)(1-2q)\le 0, \\ ap(1-2q)\le 0, \\ a(1-q)(1-2p)\le 0, \\ aq(1-2p)\le 0. \end{cases}$$

Having solved the system, we obtain the frequencies of using strategies by the players and their respective wins:

$$X_A=\left[\frac{1}{2},\frac{1}{2}\right],\qquad \upsilon_x^A=M_A\left(X_A,X_B\right)=0,$$
$$X_B=\left[\frac{1}{2},\frac{1}{2}\right],\qquad \upsilon_x^B=M_B\left(X_A,X_B\right)=0.$$

We have the first player's feedback

$$p=\begin{cases} 0, & q<\frac{1}{2}, \\ [0,1], & q=\frac{1}{2}, \\ 1, & q>\frac{1}{2}. \end{cases}$$

The second player will have the following reaction:

$$q=\begin{cases} 0, & p<\frac{1}{2}, \\ [0,1], & p=\frac{1}{2}, \\ 1, & p>\frac{1}{2}. \end{cases}$$

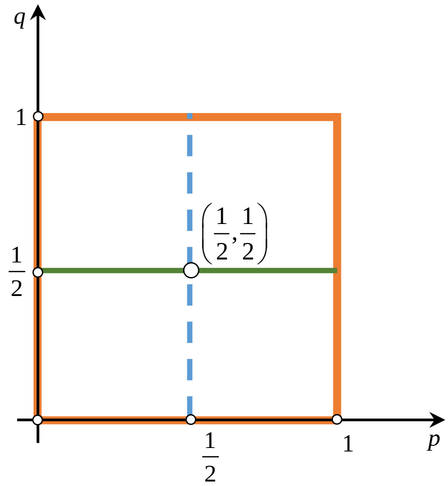

Fig. 2 One equilibrium point in mixed strategies.

Let's represent graphically the players' reaction on square $\{p,q\mid p\in[0,1],q\in[0,1]\}$ Fig. 2, which shows the presence of equilibrium point $\left(\frac{1}{2},\frac{1}{2}\right)$ in mixed strategies for the first player with frequency vector $X_A=\left[\frac{1}{2},\frac{1}{2}\right]$ and value $\upsilon_x^A=0$ and for the second player - $X_B=\left[\frac{1}{2},\frac{1}{2}\right]$, $\upsilon_x^B=0$.

No. 3 The bimatrix game is given by payment matrixes

$$C_A=\begin{bmatrix} 8 & 8 & 5 \\ 8 & 1 & 8 \\ 2 & 6 & 6 \end{bmatrix},\ C_B=\begin{bmatrix} 4 & 7 & 2 \\ 9 & 5 & 1 \\ 1 & 4 & 8 \end{bmatrix}.$$

Let the first and the second players' mixed solutions frequency vectors be:

$$X_A=[p_1,p_2,p_3]=[p_1,p_2,1-p_1-p_2],\ X_B=[q_1,q_2,q_3]=[q_1,q_2,1-q_1-q_2].$$

In this case, the game value for player A:

$$\upsilon_x^A = M_A\left(X_A, X_B\right) = \sum_{i=1}^{3}\sum_{j=1}^{3} a_{ij} p_i q_j = (7q_1 + 3q_2 - 1)p_1 + (4q_1 - 7q_2 + 2)p_2 - 4q_1 + 6,$$

and for player B:

$$\upsilon_x^B = M_B\left(X_A, X_B\right) = \sum_{i=1}^{3}\sum_{j=1}^{3} b_{ij} p_i q_j = (9p_1 + 15p_2 - 7)q_1 + (9p_1 + 8p_2 - 4)q_2 - 6p_1 - 7p_2 + 8 \cdot$$

For calculation of the problem solution, we compose a system of conditions (1)

$$\begin{cases} (7q_1 + 3q_2 - 1)p_1 + (4q_1 - 7q_2 + 2)p_2 \geq 0, \\ (7q_1 + 3q_2 - 1)(p_1 - 1) + (4q_1 - 7q_2 + 2)p_2 \geq 0, \\ (7q_1 + 3q_2 - 1)p_1 + (4q_1 - 7q_2 + 2)(p_2 - 1) \geq 0, \\ (9p_1 + 15p_2 - 7)q_1 + (9p_1 + 8p_2 - 4)q_2 \geq 0, \\ (9p_1 + 15p_2 - 7)(q_1 - 1) + (9p_1 + 8p_2 - 4)q_2 \geq 0, \\ (9p_1 + 15p_2 - 7)q_1 + (9p_1 + 8p_2 - 4)(q_2 - 1) \geq 0. \end{cases} \quad (4)$$

The solution of (4) is:

$$X_A = \left[\frac{4}{63}, \frac{3}{7}, \frac{32}{63}\right] \approx \left[0.06, 0.43, 0.51\right] \Rightarrow \left[6\%, 43\%, 51\%\right],\ \upsilon_A^x = \frac{362}{61} \approx 5{,}93,$$

$$X_B = \left[\frac{1}{61}, \frac{18}{61}, \frac{42}{61}\right] \approx \left[0.02, 0.29, 0.69\right] \Rightarrow \left[2\%, 29\%, 69\%\right],\ \upsilon_B^x = \frac{97}{21} \approx 4{,}62.$$

Note. The game considered has another two equilibrium points in mixed strategies:

$$C_A = \begin{bmatrix} 8 & 8 & 5 \\ 8 & 1 & 8 \\ 2 & 6 & 6 \end{bmatrix}, \qquad C_B = \begin{bmatrix} 4 & 7 & 2 \\ 9 & 5 & 1 \\ 1 & 4 & 8 \end{bmatrix},$$

$$(A_1, B_2) \Rightarrow \begin{bmatrix} X_{A_1} = [1,0,0] \\ X_{B_2} = [0,1,0] \end{bmatrix},\ (A_2, B_1) \Rightarrow \begin{bmatrix} X_{A_2} = [0,1,0] \\ X_{B_1} = [1,0,0] \end{bmatrix}.$$

Each point gives its win to the players. With account taken of the presence of equilibrium in the mixed strategies, we need to ground our selection of either of the equilibrium points – the game has equilibrium both in pure and in mixed strategies (when one doesn't know where to look first).

When an information system is designed, an algorithm is used, which interprets a bimatrix game as two matrix ones.

Bimatrix games are used when it is necessary to ground the choice of the optimum players' behavior. For making a thorough analysis, we conditionally divide a bimatrix game into two matrix ones.

Let us consider a bimatrix game and divide it into two matrix antagonistic games with zero sums (Fig. 2). In turn, we'll consider and solve each of them dually, as a primal and then as a dual linear optimization problem.

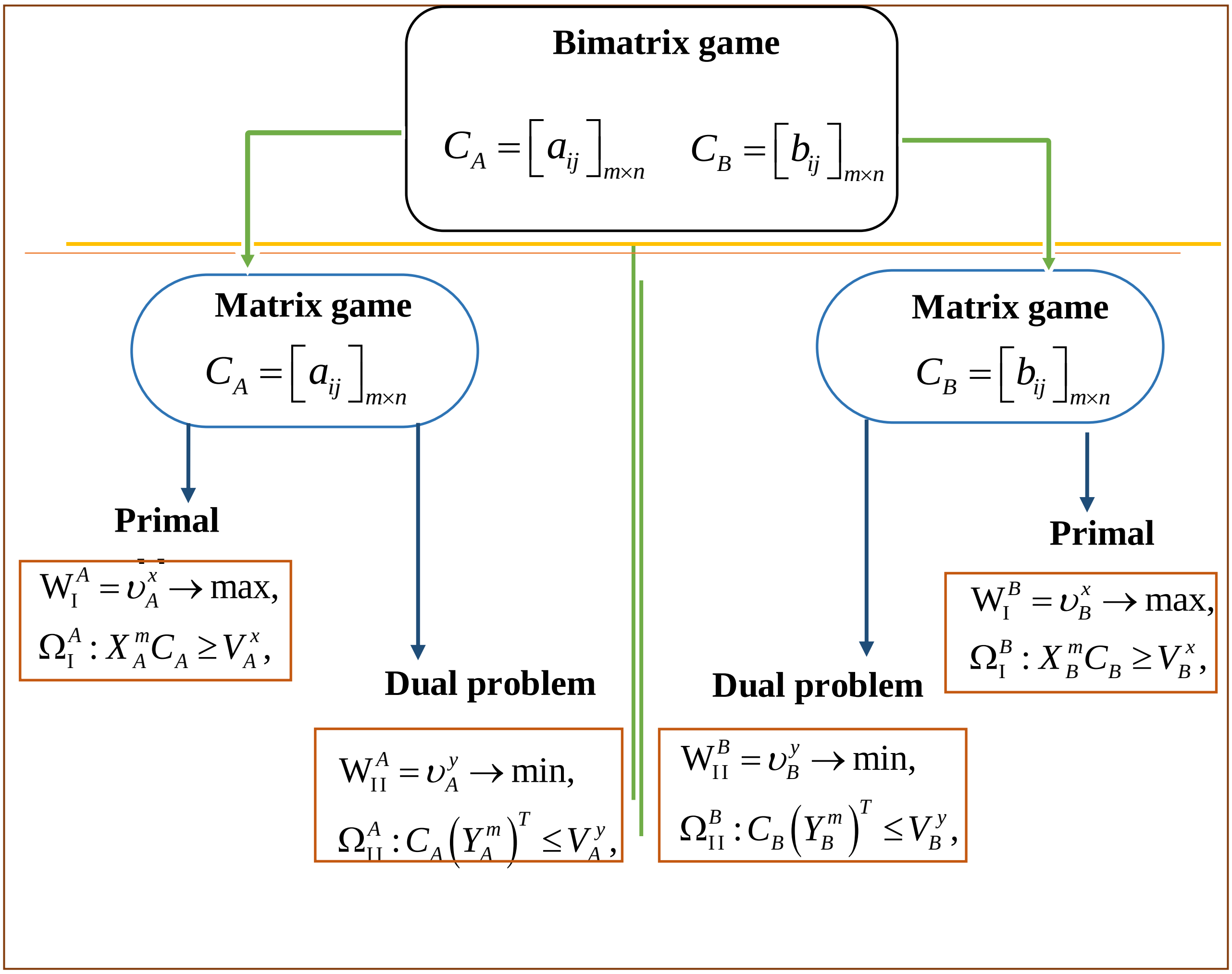

Figure 3: Bimatrix game consideration diagram

Thus, we have considered a bimatrix game given by payment matrixes C_A, C_B of two players A and B.

$$C_A=\left[a_{ij}\right]_{n\times n},\ C_B=\left[b_{ij}\right]_{n\times n}.$$

First, let's consider the matrix game for the payment matrix of player A. We find the lower $\alpha_A=\max\min C_A$ and the upper value of the game $\beta_A=\min\max C_A$. We'll solve the matrix game by reducing it to linear optimization problems. The primal problem is recorded as follows:

$$\begin{aligned}&W_I^A=\upsilon_A^x\to\max,\\&\Omega_I^A: X_A^m C_A\ge V_A^x,\end{aligned}\qquad(5)$$

where $X_A^m=\left[x_1^A,\ x_2^A,\dots,x_n^A\right]$ is a vector of frequencies used by the first player A for application of his own strategies $\left\{A_1,A_2,\dots,A_n\right\}$,($x_i^A\in\left[0,1\right]$, $i=1,2,\dots,n$, $x_1^A+x_2^A+\dots+x_n^A=1$)

$V_A^x=\left[\upsilon_A^x\right]_{n\times1}$ - the right-hand members column vector, υ_A^x - the game value.

For convenience of calculation, we reduce problem (5) to the standard linear optimization problem form

$$\mathrm{W}_{\mathrm{I}}^{*A} = x_1^{*A} + x_2^{*A} + \ldots + x_n^{*A} \to \min,$$
$$\Omega_{\mathrm{I}}^{*A} : X_A^{*m} C_A \geq 1,$$
$$x_i^{*A} = \frac{x_i^A}{\upsilon_A^x}, \quad i = 1, 2, \ldots, n.$$

where $X_A^{*m} = \left[x_1^{*A},\ x_2^{*A}, \ldots, x_n^{*A} \right]$ - normative frequency vector, ($x_1^{*A} + x_2^{*A} + \ldots + x_n^{*A} = \dfrac{1}{\upsilon_A^x}$)

In the result of calculating the primal linear optimization problem, we obtain frequency vector $X_A^m = \left[x_1^A,\ x_2^A, \ldots, x_n^A \right]$ and game value υ_A^x.

The following problem will be a dual linear optimization problem to (5):

$$\mathrm{W}_{\mathrm{II}}^{A} = \upsilon_A^y \to \min,$$
$$\Omega_{\mathrm{II}}^{A} : C_A \left(Y_A^m \right)^T \leq V_A^y, \tag{6}$$

where $Y_A^m = \left[y_1^A,\ y_2^A, \ldots, y_n^A \right]$ is a vector of frequencies used by the second player B for applying his own strategies $\left\{ B_1, B_2, \ldots, B_n \right\}$, ($y_i^A \in [0,1]$, $i = 1, 2, \ldots, n$, $y_1^A + y_2^A + \ldots + y_n^A = 1$)

$V_A^y = \left[\upsilon_A^y \right]_{n \times 1}$ - the right-hand members column vector, υ_A^y - game value.

We reduce problem (6) to the standard presentation form

$$\mathrm{W}_{\mathrm{II}}^{*A} = y_1^A + y_2^A + \ldots + y_n^A \to \min,$$
$$\Omega_{\mathrm{II}}^{*A} : C_A \left(Y_A^{*m} \right)^T \geq 1, \tag{7}$$
$$y_i^{*A} = \frac{y_i^A}{\upsilon_A^y}, \quad i = 1, 2, \ldots, n,$$

where $Y_A^{*m} = \left[y_1^{*A},\ y_2^{*A}, \ldots, y_n^{*A} \right]$ - is a normalized frequency vector. ($y_1^A + y_2^A + \ldots + y_n^A = \dfrac{1}{\upsilon_A^y}$)

The calculation of problem (7) has resulted in frequency vector $Y_A^m = \left[y_1^A,\ y_2^A, \ldots, y_n^A \right]$ and value υ_A^y. We proceed to consideration of a matrix game for payment matrix C_B of the second player B. Let’s determine the game value by calculation of the lower $\alpha_B = \max \min C_B$ and the upper game value $\beta_B = \min \max C_B$. We solve the matrix game by reducing it to linear optimization problems. The primal problem for the second player payment matrix is recorded as follows:

$$\mathrm{W}_{\mathrm{I}}^{B} = \upsilon_B^x \to \max,$$
$$\Omega_{\mathrm{I}}^{B} : X_B^m C_B \geq V_B^x, \tag{8}$$

where $X_B^m = \left[x_1^B,\ x_2^B, \ldots, x_n^B \right]$ is a vector of frequencies used by the second player B for applying his own strategies $\{B_1, B_2, \ldots, B_n\}$, ($x_i^B \in [0,1]$, $i = 1, 2, \ldots, n$, $x_1^B + x_2^B + \ldots + x_n^B = 1$)

$V_B^x = \left[\upsilon_B^x \right]_{n\times 1}$ - the right-hand member column vector, υ_B^x - the game value.

For convenience of calculation, we reduce problem (8) to the standard linear optimization problem form

$$\begin{aligned} &\mathrm{W}_{\mathrm{I}}^{*B} = x_1^{*B} + x_2^{*B} + \ldots + x_n^{*B} \to \min, \\ &\Omega_{\mathrm{I}}^{*B} : X_B^{*m} C_B \ge 1, \\ &x_i^{*B} = \frac{x_i^B}{\upsilon_B^x}, \quad i = 1, 2, \ldots, n. \end{aligned} \qquad (9)$$

where $X_B^{*m} = \left[x_1^{*B},\ x_2^{*B}, \ldots, x_n^{*B} \right]$ - is a normalized frequency vector, ($x_1^{*B} + x_2^{*B} + \ldots + x_n^{*B} = \dfrac{1}{\upsilon_B^x}$)

The primal linear optimization problem calculation resulted in frequency vector $X_B^m = \left[x_1^B,\ x_2^B, \ldots, x_n^B \right]$ and the game value υ_B^x.

The following problem will be a dual linear optimization problem to (8)

$$\begin{aligned} &\mathrm{W}_{\mathrm{II}}^{B} = \upsilon_B^y \to \min, \\ &\Omega_{\mathrm{II}}^{B} : C_B \left(Y_B^m \right)^T \le V_B^y, \end{aligned} \qquad (10)$$

where $Y_B^m = \left[y_1^B,\ y_2^B, \ldots, y_n^B \right]$ is a vector of frequencies used by the second player B for applying his own strategies $\{B_1, B_2, \ldots, B_n\}$, ($y_i^B \in [0,1]$, $i = 1, 2, \ldots, n$, $y_1^B + y_2^B + \ldots + y_n^B = 1$)

$V_B^y = \left[\upsilon_B^y \right]_{n\times 1}$ - the right-hand members column vector, υ_B^y - the game value.

We reduce (10) to the standard form of a linear optimization problem presentation

$$\begin{aligned} &\mathrm{W}_{\mathrm{II}}^{*B} = y_1^B + y_2^B + \ldots + y_n^B \to \min, \\ &\Omega_{\mathrm{II}}^{*B} : C_B \left(Y_B^{*m} \right)^T \ge 1, \\ &y_i^{*B} = \frac{y_i^B}{\upsilon_B^y}, \quad i = 1, 2, \ldots, n, \end{aligned} \qquad (11)$$

where $Y_B^{*m} = \left[y_1^{*B},\ y_2^{*B}, \ldots, y_n^{*B} \right]$ - is a normalized frequency vector. ($y_1^B + y_2^B + \ldots + y_n^B = \dfrac{1}{\upsilon_B^y}$)

The calculation of problem (11) resulted in vector $Y_B^{*m} = \left[y_1^A,\ y_2^A, \ldots, y_n^A \right]$ and value υ_B^y.

Comparing the obtained results of matrix games solution with solution of the bimatrix game, we can state about the possibility of grounding both the quantitative results on the players' average wins and their qualitative behavior. The bimatrix game of two players are given by matrixes

$$C_A = \begin{bmatrix} 2 & 5 & 6 \\ 6 & 7 & 1 \\ 6 & 3 & 6 \end{bmatrix} C_B = \begin{bmatrix} 3 & 7 & 8 \\ 7 & 8 & 1 \\ 8 & 4 & 4 \end{bmatrix}$$

Therefore, the solution for the first player A is $X_A = \begin{bmatrix} \frac{28}{61}, \frac{4}{61}, \frac{29}{61} \end{bmatrix} \approx [0.46, 0.07, 0.47] \Rightarrow [46\%, 7\%, 47\%]$, i.e. $\upsilon_A^x = \frac{108}{23} \approx 4,70$. The solution for the second player B will be $X_B = \begin{bmatrix} \frac{5}{23}, \frac{10}{23}, \frac{8}{23} \end{bmatrix} \approx [0.22, 0.43, 0.35] \Rightarrow [22\%, 43\%, 35\%]$, i.e. $\upsilon_B^x = \frac{344}{61} \approx 5,64$

4 Conclusions

The optimum mixed solution of the first player $X_A = \begin{bmatrix} \frac{28}{61}, \frac{4}{61}, \frac{29}{61} \end{bmatrix}$ in the bimatrix game coincides with mixed solution $X_B^m = \begin{bmatrix} \frac{28}{61}, \frac{4}{61}, \frac{29}{61} \end{bmatrix}$ in the matrix game given by payment matrix C_B of the second player B. Thus, the optimum behavior of the first player is a result of solving the primal optimization problem for the second player, i.e. the first player will not take account of his own payment matrix – all the "attention" is paid to the payment matrix of the second player. But at the same time, the value of reward $\upsilon_A^x = \frac{108}{23} \approx 4,70$ is a result of solving dual optimization problem $\upsilon_A^y = \frac{108}{23} \approx 4,70$ for the matrix game given by his own payment matrix C_A. Using the indicated algorithm, we can state that the mixed strategy of the second player $X_B = \begin{bmatrix} \frac{5}{23}, \frac{10}{23}, \frac{8}{23} \end{bmatrix}$ coincides with calculation of dual optimization problem $Y_A^m = \begin{bmatrix} \frac{5}{23}, \frac{10}{23}, \frac{8}{23} \end{bmatrix}$ for the first player. Therefore, the optimum behavior of the second player is fully conditional on payment matrix C_A of the first player – the values of own wins are ignored. The value of reward $\upsilon_B^x = \frac{344}{61} \approx 5,64$ is equal to the value of bimatrix game $\upsilon_B^y = \frac{344}{61} \approx 5,64$ of the dual problem for the second player.

Thus, it is reasonable, in designing the information system of students' knowledge evaluation, to use an algorithm that is based on solving two separate matrix games. A player shall find the win of the other player.

5 References

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