Low Complexity Joint Super-Resolution Algorithm for Range Azimuth of TDM-MIMO LFMCW Radar

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ABSTRACT

Aiming to solve the problem that the joint range and azimuth super-resolution algorithm of vehicle millimeter wave radar is too complex to be implemented quickly, a low complexity joint super-resolution method based on direct selection of frequency domain data is presented. The algorithm first transforms the space-time range-domain joint data into frequency domain by fast Fourier transformation, and stores and processes the two-dimensional frequency domain data of the area of interest. Based on the equivalence between Fourier transformation and beam space transformation based on DFT transformation, the range-azimuth joint MUSIC super-resolution in frequency domain data is achieved, and the fast joint estimation of target information is completed. The orthogonality of frequency domain subspace and the theory of frequency domain beam dimension reduction super-resolution algorithm are deduced. The relationship between the resolution and estimation performance of distance and azimuth of the algorithm and signal-to-noise ratio is simulated. The simulation results show that the accuracy and resolution of the algorithm are much higher than traditional FFT, and the computational complexity of the algorithm is greatly reduced compared with traditional MUSIC.

Keywords

LFMCW, Joint distance-azimuth estimation, Frequency domain, Beam space, MUSIC

1. Introduction

Among the target parameter estimation algorithms of vehicle mounted radar, the estimation accuracy and resolution of the traditional Fast Fourier Transform (FFT) algorithm is insufficient. And super-resolution algorithms such as Multiple Signal Classification (MUSIC) algorithm and Estimating Signal Parameter via Rotational Invariance Technologies (ESPRIT) algorithm have high accuracy and resolution but huge computation.

The high efficiency and low complexity of space-time multi parameter joint super-resolution algorithm is an urgent problem to be solved at present, which can not be avoided in engineering. Bienvenu G[1] et al. proposed a high-resolution target bearing estimation method to improve accuracy and resolution and even the statistical stability[3]-[4].

Based on the joint super-resolution complexity of vehicle mounted radar, this paper proposes a range azimuth joint super-resolution method based on frequency domain beam dimensionality reduction. Based on the equivalence of range angle FFT and multi-dimensional Discrete Fourier Transform (DFT) beam space transformation of space-time data, the corresponding frequency domain data area of the target area is selected according to the prior information of the target, MUSIC joint super-resolution based on beam dimension reduction. The dimension of beam space data is greatly reduced, which makes it possible to realize joint super resolution engineering and realize fast joint estimation of multiple parameters of vehicle borne radar. Compared with the traditional FFT algorithm, the resolution of target parameter estimation is significantly improved, which is a fast joint super-resolution algorithm feasible in engineering.

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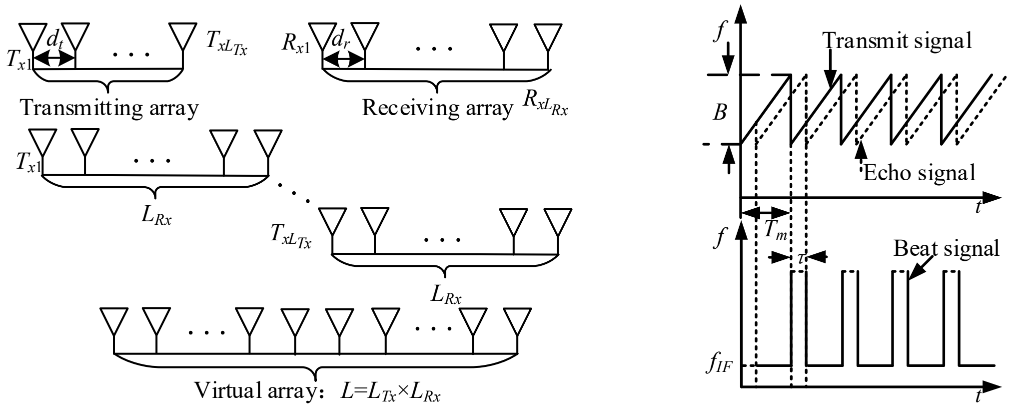
2. TDM-MIMO LFMCW Radar Signal Model

Set the number of transmitting antennas of Division Multiplexing Multiple Input Multiple Output (TDM-MIMO) radar as L_{TX}, and the number of receiving antenna elements as L_{RX}. The simplified array model is shown in Figure 1(a). Set the spacing of receiving antenna elements as $d_r = \lambda/2$, the spacing of transmitting antenna elements as d_t, and meet the requirements of $d_t = L_{RX} \times d_r$. Assume that the spacing between any receiving antenna l_{RX} and the first receiving antenna in the receiving antenna array is $d_{rlRX} = (l_{RX} - 1) d_r$.



(a)TDM-MIMO radar array model. (b) Sawtooth time-frequency diagram.

Figure 1. Radar model and signal time-frequency diagram.

The sawtooth Linear Frequency Modulated Continuous Wave (LFMCW) signal and echo signal is shown in Figure 1(b).

Under the above model, obtain TDM-MIMO LFMCW radar sawtooth beat signal model[5] $\boldsymbol{x}(t)$ as

$$\boldsymbol{x}(t) = A_{mp} e^{j2\pi\left[\left(\frac{2vf_c}{c} + \frac{2\mu R}{c} + \frac{2\mu v m T_m}{c}\right)t + \frac{2Rf_c}{c} + \frac{2f_c v m T_m}{c} + \frac{l d_r \sin\theta}{\lambda}\right]} + \boldsymbol{G}(t) \tag{1}$$

In the formula, A_{mp} represent the signal amplitudes, f_c represents the signal carrier frequency, $\mu = B/T_m$ represents the FM slope, where B represents the signal bandwidth, T_m represents the signal repetition period, $m = 0,1....,M-1$ represents the sequence number of repetition period, $l = 0,1....,L-1$ represents the array element sequence number of virtual array receiving antenna, $d_r = \lambda/2$ is the virtual antenna spacing. $\boldsymbol{G}(t)$ is additive white Gaussian noise.

Assume that the number of sampling points in each Chirp is N, and according to space-time equivalence, the guidance vectors for super-resolution of angle dimension and distance dimension are:

$$\boldsymbol{a}_R(R) = \left[1, e^{-j2\pi\frac{2\mu R}{cf_s}}, \cdots, e^{-j2\pi\frac{2\mu R}{cf_s}(N-1)}\right]^T \quad \boldsymbol{a}_\theta(\theta) = \left[1, e^{-j2\pi\frac{d\sin\theta}{\lambda}}, \cdots, e^{-j2\pi\frac{d\sin\theta}{\lambda}(L-1)}\right]^T \tag{2}$$

3. Beam space transformation in frequency domain based on time-frequency equivalence

The beam space dimension reduction super-resolution algorithm based on DFT transform is to obtain the beam space data by multiplying the original sampling data and the beam space conversion matrix. At the same time, the steering vector also reduces the dimension according to the beam selection. In this method, the multi-dimensional data are first FFT transformed, and only the frequency domain data corresponding to the parameter region of interest are stored; Only when the data directly selected in the frequency domain is equivalent to the beam space transformation data, the super-resolution of some frequency domain data is equivalent to the beam space reduced super-resolution of the original data.

According to reference [6] and space-time equivalence, the beam conversion matrix in the beam space array flow pattern of each dimension is defined. The beam transformation matrices defining the angle and range dimensions are $\boldsymbol{W}_{\theta}^{H} \in \mathbb{C}^{L_b \times L}, \boldsymbol{W}_{R}^{H} \in \mathbb{C}^{N_b \times N}$ respectively, and the number of effective beam selections in the angle and range dimensions are L_b, N_b respectively, that is, the reduced dimension data length. Then the l and n elements are

$$\boldsymbol{w}_{\theta}^{H} = \begin{bmatrix} 1 & e^{-jl\frac{2\pi}{\mathrm{L}_b}} & \cdots & e^{-j(L_b-1)l\frac{2\pi}{L_b}} \end{bmatrix} \quad \boldsymbol{w}_{\mathrm{R}}^{H} = \begin{bmatrix} 1 & e^{-jn\frac{2\pi}{N_b}} & ,\cdots, & e^{-j(N_b-1)n\frac{2\pi}{N_b}} \end{bmatrix} \tag{3}$$

In the formula, $l, 0 \le l \le (L_b - 1), L_b \le L$, when $L_b = L$, that is, $\boldsymbol{W}_{\theta}^{H}$ is the angle dimension full beam transformation matrix. $0 \le n \le (N_b - 1), N_b \le N$, when $N_b = N$, $\boldsymbol{W}_{\mathrm{R}}^{H}$ is the range dimension full beam transformation matrix.

Assuming that the matrices of one-dimensional Fourier transform are $\boldsymbol{F}_N$ and $\boldsymbol{F}_L$ respectively, where N、L respectively represent the length of the vector to be operated in the distance dimension and angle dimension, and data $\boldsymbol{X}_{NL}$ is the distance angle two-dimensional raw data of $N \times L$, then the frequency domain data matrix $\boldsymbol{Y}_{NL}$ obtained by two-dimensional Fourier transform is expressed as:

$$\boldsymbol{Y}_{NL} = \boldsymbol{F}_N \boldsymbol{X}_{NL} \boldsymbol{F}_L^T \tag{4}$$

From left to right, it is FFT for each column, from right to left, it is FFT for each row, and it is the same to do the left first and the right first. Two dimensional Fourier transforms are two one-dimensional Fourier transforms, and they are independent of order. The same is true for higher dimensional Fourier transforms. If $\boldsymbol{x}_1, \cdots, \boldsymbol{x}_L$ is the column vector of $\boldsymbol{X}_{NL}$, $\boldsymbol{X}_{NL}$ can be written as $\boldsymbol{X}_{NL} = \sum_{i}^{L} \boldsymbol{x}_i \boldsymbol{e}_i^T$ transformation, where $\boldsymbol{e}_i$ is the ith column of the identity matrix $\boldsymbol{I}_L$, so

$$\begin{aligned} vec(\boldsymbol{Y}_{NL}) &= vec(\boldsymbol{F}_N \boldsymbol{X}_{NL} \boldsymbol{F}_L^T) = vec\left\{ \boldsymbol{F}_N \left(\sum_{i=1}^{L} \boldsymbol{x}_i \boldsymbol{e}_i^T \right) \boldsymbol{F}_L^T \right\} = \sum_{i=1}^{L} vec(\boldsymbol{F}_N \boldsymbol{x}_i \boldsymbol{e}_i^T \boldsymbol{F}_L^T) = \sum_{i=1}^{L} vec\left\{ (\boldsymbol{F}_N \boldsymbol{x}_i)(\boldsymbol{F}_L \boldsymbol{e}_i)^T \right\} \\ &= \sum_{i=1}^{L} \boldsymbol{F}_L \boldsymbol{e}_i \otimes \boldsymbol{F}_N \boldsymbol{x}_i = (\boldsymbol{F}_L \otimes \boldsymbol{F}_N) \sum_{i=1}^{L} (\boldsymbol{e}_i \otimes \boldsymbol{x}_i) = (\boldsymbol{F}_L \otimes \boldsymbol{F}_N) vec(\boldsymbol{X}_{NL}) \end{aligned} \tag{5}$$

Where ⊗ represents Kronecker multiplication. Reference [7] defines the data matrix after two-dimensional beam space transformation as $\boldsymbol{Z}$, and

$$vec(\boldsymbol{Z}) = (\boldsymbol{W}_{\mathrm{R}}^{H} \otimes \boldsymbol{W}_{\theta}^{H}) vec(\boldsymbol{X}_{NL}) \tag{6}$$

From Eq. (5)and Eq.(6), it can be seen that the two-dimensional beam space transform is equivalent to the two-dimensional Fourier transform. Eq. (5)is another expression of two-dimensional FFT, and also has the same form as that of two-dimensional beam space transformation when all beams are taken; That is, two-dimensional FFT and two-dimensional beam space are equivalent in full beam. When the two dimensions of beam space dimensionality reduction are reduced to L_b and N_b respectively, it is also corresponding to the two-dimensional FFT data directly selecting data according to the linear correspondence of the beam. Therefore, it is equivalent to take 2D frequency domain data directly and reduce the dimension of 2D beam space.

The above proves the equivalence of beam space dimensionality reduction and direct selection of frequency domain data. For the beam space super-resolution algorithm, literature [6] has completed the one-dimensional MUSIC proof based on the beam space. The beam space based MUSIC algorithm can be expanded from one-dimensional to two-dimensional through equation (5). In the range azimuth two-dimensional joint estimation, equation (6) realizes the dimensionality reduction transformation from the original data matrix $\boldsymbol{X}_{NL}$ to the data matrix $\boldsymbol{Z}$, which is covariance estimation It provides a basis for reducing the computation of eigenvalue decomposition and subsequent peak searching operations. The beam space search guidance vector is defined as:

$$\boldsymbol{b}(\theta, \mathrm{R}) = \left(\boldsymbol{W}_{\mathrm{R}}^H \boldsymbol{a}_R\right) \otimes \left(\boldsymbol{W}_\theta^H \boldsymbol{a}_\theta\right) = \boldsymbol{b}_R \otimes \boldsymbol{b}_\theta \tag{7}$$

The covariance matrix is:

$$\boldsymbol{R}_{beam-2MUSIC} = \frac{1}{N_b L_b} \boldsymbol{Z}\boldsymbol{Z}^H \tag{8}$$

After eigenvalue decomposition, the noise subspace $\boldsymbol{U}_{\mathrm{n}-beam-2MUSIC}$ can be obtained, and the spectral peak search function is obtained as follows:

$$\mathrm{P}_{beam-2MUSIC} = \frac{1}{\boldsymbol{b}^H(\theta, \mathrm{R}) \boldsymbol{U}_{\mathrm{n}-beam-2MUSIC} \boldsymbol{U}_{n-beam-2MUSIC}^H \boldsymbol{b}(\theta, R)} \tag{9}$$

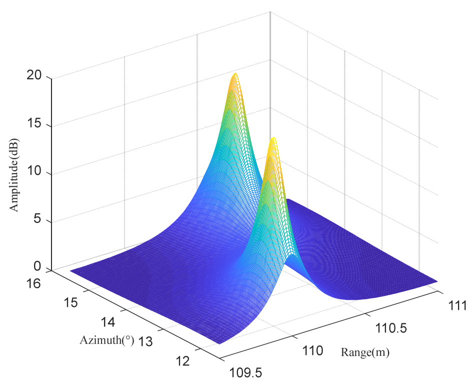
The spectral peak is obtained by searching the spectral peak of the above equation, and the corresponding information of the peak is the two-dimensional information of the range azimuth angle of the target. Compared with the traditional two-dimensional MUSIC algorithm, the MUSIC algorithm after the beam space reduces the data dimension in terms of covariance estimation, eigenvalue decomposition and spectral peak search, greatly reducing the calculation time.Based on the joint super-resolution in the frequency domain, the data storage pressure and computational complexity have been significantly reduced. According to existing research, the resolution and parameter estimation performance of the beam space dimension reduction algorithm under the condition of reasonable beam selection.

4. Simulation experiment

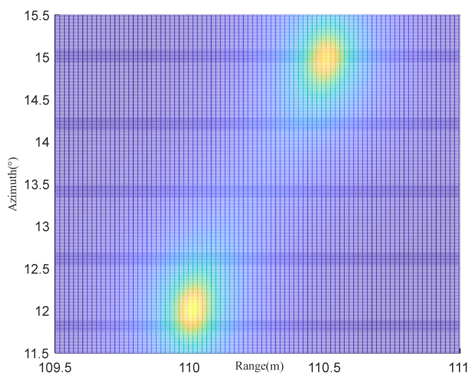
The conditions for simulation are as follows: The 24GHz millimeter wave TDM-MIMO radar platform transmits FMCW signals with a bandwidth of 150MHz. The period of a Chirp is 16 us, the number of snapshots is 300, the number of transmitting and receiving antennas is 2 and 15 respectively. The search points of azimuth dimension and distance dimension are $l_s = 89, n_s = 236$. The number of wave beams in angle dimension and distance dimension is $L_b = 8$ and $N_b = 13$ respectively. Set the parameters as Target1: (110 m, 12 °); Target2: the (110.5 m ,15 °).

4.1 Effective Estimation Diagram of Target Information

When plotting $SNR = 0$ dB , the range velocity dimension effectiveness estimation diagram of the range azimuth joint frequency domain beam reduction MUSIC algorithm is compared with the effectiveness estimation diagram of the traditional 2DFFT algorithm, as shown in Figure 2 and Figure 3 respectively.

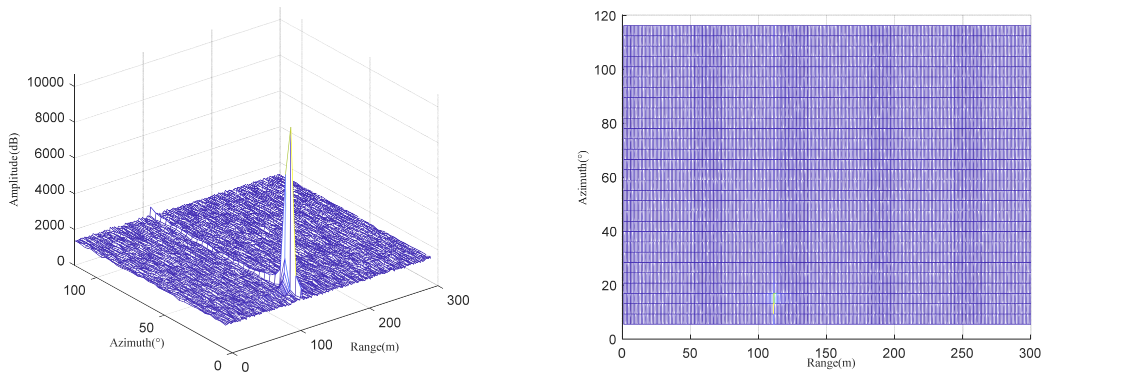


(a) The proposed algorithm spectral peak estimation



(b) Effective estimation of the proposed algorithm

Figure2. Spectral Peak Estimation and Effective Estimation of the proposed algorithm.



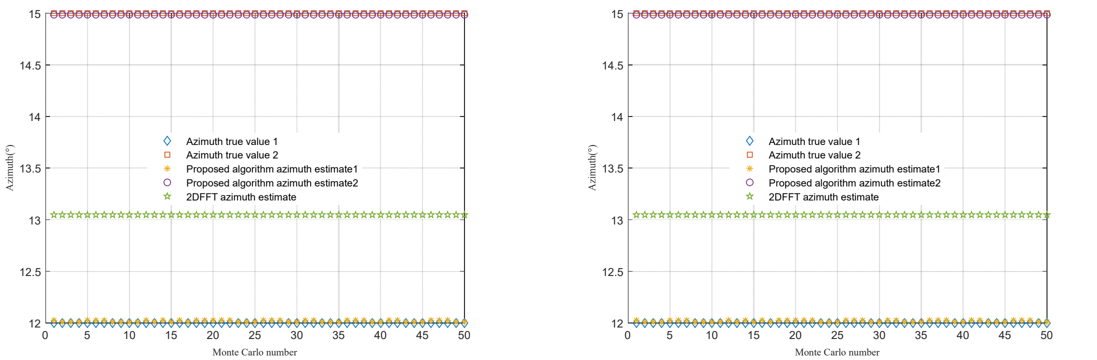
(a) 2DFFT algorithm joint information estimation spectral peak (b) 2DFFT algorithm joint information effective estimation

Figure3. Effectiveness estimation of joint range azimuth information of target using 2DFFT algorithm.

It can be seen from the observation in Figure 2 that the algorithm proposed in this paper can effectively realize the effective estimation of target information. The comparison between Figure 3 and Figure 2 shows that under this condition, 2DFFT cannot distinguish two targets. The algorithm proposed in this paper can achieve effective resolution of two targets and more accurate parameter estimation.

4.2 Target Information Distribution Map

Through 50 Monte Carlo experiments, parameters distribution diagram of the proposed algorithm in this paper and 2DFFT algorithm are compared as in Figure 4.



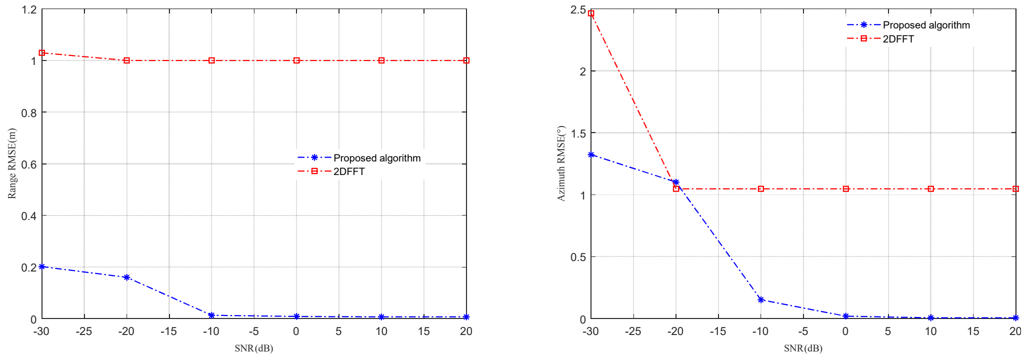
(a) Distance dimension target information distribution. (b) Azimuth dimension target information distribution.

Figure4. Joint estimation information distribution map of target range and azimuth information.

It can be seen from Figure 4 that under the current simulation conditions, the traditional 2DFFT algorithm cannot complete the target resolution of Target1 and Target2, while the algorithm proposed in this paper can successfully resolve two targets. The algorithm realizes two-dimensional super-resolution.

4.3 Performance Analysis - RMSE Statistics

Set the echo signal to noise ratio variation range of the signal as $-30\text{ dB}:10\text{ dB}:20\text{ dB}$, and the number of Monte Carlo is 50. The traditional 2DFFT algorithm and the proposed algorithm proposed in this paper are used to analyze the estimation error of Target1. The experimental results are as follows.



(a) Distance dimension RMSE. (b) Azimuth dimension RMSE.

Figure5. Simulation Experiment of Target Range and Azimuth Estimation Performance Comparison.

It can be seen from the simulation results in Figure 5 that under different signal-to-noise ratios, the proposed algorithm proposed in this paper has higher estimation accuracy than the traditional 2DFFT algorithm in terms of target range and angle dimensions, and its estimation performance improves with the increase of signal-to-noise ratio, and the range dimension estimation accuracy is slightly higher than the angle dimension estimation accuracy.

4.4 Complexity analysis

According to the above parameters, for the traditional 2DMUSIC algorithm, the data storage amount is $\mathcal{O}((NL))$ after data acquisition. The algorithm proposed in this paper can reduce the data storage amount to $\mathcal{O}((N_bL_b))$, which can reduce the data storage amount by $(30\times 200)/(8\times 13)=57.7$. Based on MUSIC joint super-resolution algorithm, the computational complexity includes $\mathcal{O}\left((NL)^3\right)$ of eigenvalue decomposition processing part and $\mathcal{O}\left((NL)\mathrm{n}l\right)$ of spectral peak search part. The total complexity of the algorithm is $\mathcal{O}\left((NL)^3+(NL)\mathrm{n}l\right)$. The complexity of the proposed algorithm, eigenvalue processing and spectral peak search is reduced to $\mathcal{O}\left((N_bL_b)^3\right)$ and $\mathcal{O}\left((N_bL_b)\mathrm{n}l\right)$ respectively, and t Total computational complexity reduced to 6.5×10^4. When the selected target area is small, the number of beams can also be smaller to further reduce the complexity.

5. Conclusion

In this paper, a range azimuth joint super-resolution algorithm based on frequency domain beam dimensionality reduction for vehicle borne radar is proposed. Compared with the traditional 2DFFT algorithm, the algorithm can effectively improve the resolution and estimation accuracy. Compared with the general MUSIC algorithm, the dimensionality reduction of the frequency domain space-time beam greatly reduces the data storage pressure and the scale of data processing, realizes the multi-level reduction of data storage and computational complexity, facilitates the realization of engineering applications, and provides effective technical support and reasonable solutions for the solution of the vehicle borne radar target accurate detection problem.

6. References

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