Computing the Risk of Failures for High-Temperature Pressurized Pipelines

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Abstract

The methods for calculating the failures risks of the high-temperature pressurized pipelines are developed as the particulars of the generalized approaches for the risk assessment in stationary deterministic systems. It is considered the straight pipe as the necessary required part of pipelines, and it is proposed considering it under the internal and external pressures for the given temperature. The pipe's failures risks are measured by the gamma-percentile life. It is proposed the mathematical model representing the deterministic properties of the high-temperature pressurized pipe needed for calculating their failures risks. In this mathematical model it is took into account accumulating the irreversible strains and damages in the pipe's during operating due to the high-temperature creep, and this mathematical model is represented as the theory of creep initial-boundary-value problem and numerical solving of such problems is briefly discussed. It is proposed the general approximation of the high-temperature pressurized pipe's life depending on the internal and external operating pressures. It is considered the particular example of the high-temperature pressurized pipe made from the stainless austenitic steel, and its gamma-percentile life is computed. It is shown that the gamma-percentile life is the failures risks quantitative measure which really gives the most fully and correct characteristic of the failures risks for the high-temperature pressurized pipes during their operating, because the gamma-percentile life of the pipe is very sensitive to the value of required probability of operating without the failures.

Keywords

evaluating, risks, failure, pipelines, reliability

1. Introduction

Pressurized high-temperature pipelines are used basically in the steam boilers of high-capacity thermal power plants to provide the most fuel efficiency [1–3]; besides such pipes are used in different manufacturing which the high temperatures are needs like the polyethylene facilities [4] for example. It is naturally, the simultaneous existence of the high pressures and the high temperatures of the large amounts of chemical active mediums, like for example combusted gases and the superheated steam, creates the risks of ruptures of the pipelines with the highly

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dangerous aftermaths. Due to these circumstances, it is significantly necessary having the estimating about operating risks of the high-temperature pressurized pipelines in all stages of their life from their designing and manufacturing thru their operating and it requires solving the different kind problems [4–9]. So, computing the risk of failures for the high-temperature pressurized pipelines is in current interest problem which is needed to provide possibilities of developing the power and manufacturing equipment including decreasing the carbon emission per unit produced power due to increasing the fuel efficiency by increasing of the working parameters needed to intensification of the heat processes.

2. Related works

Existing of the high temperatures and the high pressures contribute to activation the complicated damaging mechanisms in the structural materials which are leading to fractures and to raptures before the expected life time and which are creating the potential risks of the different dangerousness during operating of the high-temperature pressurized pipelines. The most of the damages under the high temperatures and the mechanical loads are due to the high-temperature creep [7, 9, 10–12]. Exactly the creep is the main mechanism of damaging the high-temperature pressurized pipelines, although the fatigue [7, 11] or the corrosive processes [9] can have the additional significant influencing. Although, the creep phenomena are wide researched in general theoretically [13, 14] and in relations to different engineering applications including the pipelines [7, 9–12] the conventional approaches for the risk estimation about operating the pipelines under the creep are not existed at present, despite a lot of researches like [6, 7] are existed. Such problem state is due to the most of researchers like [6, 7] deal with the particularities of the problem about risks of failure the structures under the creep conditions suitable for the separate tasks. At the same time, the general approaches for the risks assessments in the deterministic systems were developed in the previous research [15]. Thus, the purpose of this research is in developing the methods for computing the risks of failures of the high-temperature pressurized pipelines, so that these methods will be the particulars of the general approaches previously developed in the research [15]. To realize the formulated above purpose the follows objectives will be accomplished:

- It will be proposed the schematization representing the main typical part of the high-temperature pressurized pipelines required for calculating their operational failures risks according with the generalized approaches [15]
- It will be proposed the mathematical model representing the deterministic properties of the main typical part of the high-temperature pressurized pipelines required for calculating their operational failures risks according with the generalized approaches [15]. Besides, the numerical analysis using the proposed models will be briefly discussed
- It will be considered the particular example of the main typical part of the high-temperature pressurized pipeline made from the stainless austenitic steel, and using the generalized approaches [15] the reliability indexes giving the failures risks quantitative estimations will be computed as well as their properties will be discussed.

It is necessary to note that using the generalized approaches [15] for solving the particular problem about computing the risks of failures of the high-temperature pressurized pipelines is the principal in this research.

3. Methodology of calculating the pipelines' operational failures risks

We will consider here the briefly propositions of the generalized approach [15] for risks estimating needed for understanding this research, as well as the pipelines schematizing which is suitable for their failures risks calculations.

3.1. The generalized approach for calculating the risks

Although, the generalized approaches for risks assessments was discussed in the research [15], it is necessary to give their briefly here taking into account the specific of the considered particular problem about calculating the failures risks of the high-temperature pressurized pipelines. To have the quantitative risks assessments in the researched system it is necessary to consider this system's state parameter(s), which can divide the normal and failure states. Such parameter in general case can be imagined as the abstract real value so that some of this value is corresponded to the normal, but others – to the failure states. Thus, risks assessments can be reduced in general to defining the probability of the event that the state parameter(s) having values corresponded to the failure states. In the considered particular case of calculating the failures risks of the high-temperature pressurized pipelines the state parameter dividing the normal and failure states are naturally the time t^* from beginning of operating to the moment of limiting state will occur. The failures of the pipelines are cannot be foresaw exactly, so they can be imagined as the probable events. Since the life of the pipelines divide theirs normal and failure states, it is suitable to represent the life as the random value corresponding with the probable nature of the failure. There are a lot of opportunities to represent the quantitative measure of the risk, but the more suitable in this research is the gamma-percentile life t_γ, which is the operating time during whose the limiting state (failure) will not be achieved with the probability value of the gamma in percentile. So, the gamma-percentile life can be found by resolving the follows understandable relation:

$$\int_0^{t_\gamma} f\left(t^*\right) dt^* = 1 - \frac{\gamma}{100}, \tag{1}$$

where $f\left(t^*\right)$ is the lifetime's density function.

Although, the life of the pipeline is considered as the random value, but this value have some deterministic properties. Really, it is clearly understood that increasing the operational temperature or the pressure will lead to decreasing at least the mean life of the pipelines. It is naturally that such deterministic properties of the pipelines have influencing on the probabilistic characteristics of theirs life and on the lifetime's density function as the results of this. To represent these deterministic properties of the pipelines it is suitable to represent their life as the deterministic function of some parameter p defining the operational conditions:

$$t^* = t^*\left(p\right). \tag{2}$$

The parameter p in the function (2) can represent the pressure or the temperature or other kind parameters which have the most significant influence on of the pipeline life.

Taking into account existing of the deterministic property (2) of the pipeline's life, the random value of this life can be imagined as the consequence of uncertainties of the operating conditions those not allow having the exact value of the p parameter in the function (2). The uncertainties of the operating conditions can be imagined as the random value of the parameter defining them, and the lifetime's density function needed for estimating the gamma-percentile life (1) can be found using the well-known result about density function of the random values produced by the given random value function (2):

$$f\left(t^*\right) = g\left(p\left(t^*\right)\right)\left|\frac{dp}{dt^*}\right|, \quad (3)$$

where $g(p)$ is the density function of the p value; $p\left(t^*\right)$ is the inverse function to the function (2)

All this discussed above including the formulas (1)–(3) are just the adaptation of the generalized approaches [15] for the considered particular case of calculating the failures risks for the pipelines. At the same time, it is possible existence more than one parameters defining the operating conditions of the pipelines, but this case was not considered in the research [15]. This case has no principal difficulties, because instead the function (2) of one variable we will have just the function of several variables:

$$t^* = t^*\left(p_1, p_2, \ldots, p_N\right), \quad (4)$$

where $p_1, p_2, \ldots, p_N$ are the parameters defining the operating conditions of the pipeline.

Taking into account the uncertainties of the operating conditions, we can imagine the function (4) as the function of the random variables. So, the lifetime's density function needed for estimating the gamma-percentile life (1) can be found using the well-known result about density function of the random values produced by the given function (4) of several random values:

$$f\left(t^*\right) = \frac{dF}{dt^*}, \quad F\left(t^*\right) = \int\int_{D\left(t^*\right)}\ldots\int g\left(p_1, p_2, \ldots, p_N\right) dp_1 dp_2 \ldots dp_N, \quad (5)$$

where $g\left(p_1, p_2, \ldots, p_N\right)$ is the density function of the parameters $p_1, p_1, \ldots, p_N$; $D\left(t^*\right)$ is the domain of the values $p_1, p_2, \ldots, p_N$ satisfying the condition $t^*\left(p_1, p_2, \ldots, p_N\right) \le t^*$.

Of course, the formula (5) is more cumbersome than the formula (3), but all difficulties of computing by the formula (5) are in technique only and are not principal, because integration can be performed numerically. It is necessary to note that density function $g(p)$ or $g\left(p_1, p_2, \ldots, p_N\right)$ of the parameters defining the operating conditions must be chosen on the basis of the expert estimations about the operational conditions of the pipelines taking into account the possible valued of the those parameters and using the known density functions corresponded to uniform, to triangle, to Weibull's or to other distributions how it was discussed in the research [15].

3.2. Pipelines schematizing suitable for failures risks calculations

The discussed above generalized approaches for risks assessment are reduced to defining the deterministic properties of the considered systems which in the particular case of failures risks calculating for pipelines are reduced to defining the life as the function of the parameter defining the operating conditions. We will consider further the main part of any pipelines reducing to the straight pipe under the internal and external pressures p_1 and p_2 as shown on the Figure 1. Of course, the pipelines are having not only the straight parts, but the different curvilinear parts and the edge parts like the flanges, as well as the intermediate supports and others. Besides, the straight parts of pipelines can have bending and can torqueing [16]. At the same time, considering all these features of the pipelines will make significantly cumbersome calculating the failures risks. Actually, it is sufficiently to consider only the internal and external pressures as the operating parameters of the straight pipe in this research as it is shown on the Figure 1, because it is the principal for all kind pipelines.

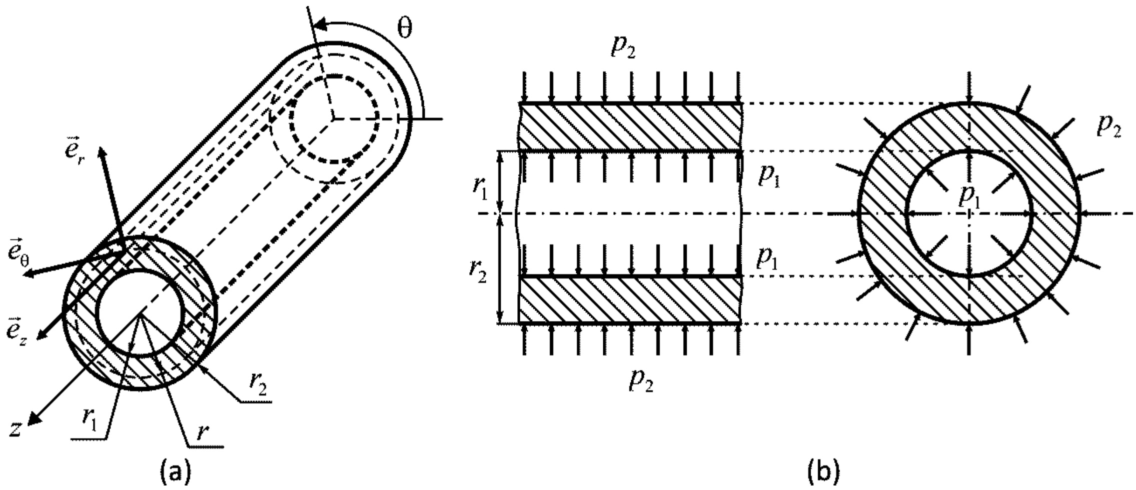


Figure 1: The fragment of the piper (a) and their cross-sections (b)

Taking into account the assumed schematization of the pipelines (Figure 1), we can consider the general presented deterministic properties (4) of the pipe predetermining their failures risks as the depending of the pipe's life from the internal and external pressures:

$$t^* = t^*(p_1, p_2), \qquad (6)$$

where p_1 is the internal and p_2 is the external pressure of the pipe (Figure 1).

The discussed above generalized approaches for risks assessment requires the given density functions of the parameters defining the operational conditions as was shown by the relation (5) and their particular case (3). According with the discussed above generalized recommendations, we will assume that the internal and external pressures satisfy the follows inequalities:

$$p_1^{\min} \le p_1 \le p_1^{\max},\ p_2^{\min} \le p_2 \le p_2^{\max}, \qquad (7)$$

where $p_1^{\min}$ and $p_1^{\max}$ are the minimal and maximal possible values of the internal pressure; $p_2^{\min}$ and $p_2^{\max}$ are the minimal and maximal possible values of the external pressure.

The assumptions (7) are the natural, because the pipeline's operating pressures have limits, but defining the bounds of such limits is the really difficult problems solving by using the expert estimations usually. When we have no any data about the operating pressures it is possible and suitable to assume that all values (7) are having the equal probabilities, so that the uniform distributions are assumed for the pressures random values inside the intervals defining by double inequalities (7).

3.3. Defining the straight pipes' life under the high-temperature creep

As was noted above, the high temperatures and the pressures lead to the creep consisting of the in reversible strains growth and cumulating the damages in the pipe's structural material during the time. Exactly these creeps' strains and these damages can lead to ruptures of the pipes during some time from the beginning of operation. So, estimating the life of the high-temperature pressurized pipes can be reduced to modelling of accumulation the creep strains and the damages in their structural material under the operational temperature and pressures. It is well-known, the velocities of creep strains growths and the damages accumulations under the given temperature are depended on the internal mechanical stresses. This requires considering the stress-strain state of the pipe taking into account the current creep strains to define the current velocities of the creep strains and the damages.

3.3.1. The mathematical model of the pipe's damaging due to creep

The stress strain-state of the pipe (Figure 1) under the accepted assumptions can be represented by the radial displacements and the radial and circumferential stresses depending on the radial coordinate r and the time t only:

$$u_r = u_r(r,t),\ \sigma_r = \sigma_r(r,t),\ \sigma_\theta = \sigma_\theta(r,t), \quad (8)$$

where u_r is the radial displacement; σ_r is the radial and σ_θ is the circumferential stresses.

The creep sand damages of the pipe's structural material under the stress-strain state (8) can be defined by the creep strains and by the Kachanov-Rabotnov damage parameter [17] for example, and all these must depend on the radial coordinate r and the time t only in this considered case:

$$c_r = c_r(r,t),\ c_\theta = c_\theta(r,t),\ \omega = \omega(r,t), \quad (9)$$

where c_r is the radial and c_θ is the circumferential creep strains; ω is the Kachanov-Rabotnov damage parameter.

Neglecting the inertia forces of the pipe, we can have the follows differential equations with the boundary conditions to define the stress-strain state of the pipe need to determine the velocities of creep strains and the damages:

$$-\frac{1}{E}\sigma_r + \frac{\nu}{E}\sigma_\theta + \frac{\partial u_r}{\partial r} - c_r = \alpha\Delta T,\ -\frac{1}{E}\sigma_\theta + \frac{\nu}{E}\sigma_r + \frac{u_r}{r} - c_\theta = \alpha\Delta T,\ \frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} = 0, \quad (10)$$

$$\sigma_r(r_1,t) = -p_1,\ \sigma_r(r_2,t) = -p_2, \quad (11)$$

where E is the Young's module, ν is the Poisson's ratio and α is the thermal expansion coefficient of the pipe's structural material; ΔT is the difference between the pipe's operating temperature and the temperature of the naturally unloaded state of this pipe; r_1 is internal and r_2 is external radii of the pipe (Figure 1).

The boundary-value problem (10), (11) must be complemented by the equations defining the creep strains' and the damage parameter's velocities, and these complementary equations can be proposed in the follows view:

$$\frac{\partial c_r}{\partial t}=\frac{3}{2}\frac{B_1\sigma_i^{m_1-1}}{(1-\omega)^{m_1}}\left(\frac{2}{3}\sigma_r-\frac{1}{3}\sigma_\theta\right),\ \frac{\partial c_\theta}{\partial t}=\frac{3}{2}\frac{B_1\sigma_i^{m_1-1}}{(1-\omega)^{m_1}}\left(\frac{2}{3}\sigma_\theta-\frac{1}{3}\sigma_r\right),\ \frac{\partial\omega}{\partial t}=B_2\left(\frac{\sigma_i}{1-\omega}\right)^{m_2}, \tag{12}$$

$$c_r(r,0)=0,\ c_\theta(r,0)=0,\ \omega(r,0)=0, \tag{13}$$

where B_1, m_1, B_2, m_2 are the parameters defining the creep strains' and the damage parameter's velocities for the given temperature; is the stress intensity defining as follows

$$\sigma_i=\frac{1}{\sqrt{2}}\sqrt{(\sigma_\theta-\sigma_r)^2+\sigma_r^2+\sigma_\theta^2}\ . \tag{14}$$

Using the mathematical model (8)–(14), we can define the life t^* of the pipe from the follows condition:

$$\exists r^*,t^*:\ r_1\le r^*\le r_2,t^*>0,\ \omega\left(r^*,r^*\right)=1, \tag{15}$$

where r^* the coordinate of the firstly formed macroscopic defect in the pipe's material.

The condition (15) defining the pipe's life is the consequence of sense of the used in equations (12) Kachanov-Rabotnov parameter. It is necessary to note, that the condition (15) corresponds to the time moment of forming the macroscopic visible defect, so the pipe can resist to the loads sometime after the time moment defined by the condition (15). At the same time, the possibilities of the pipe to resist the loading after the time moment defining by the condition (15) are significantly limited and the condition (15) really can be used to define the pipe's life approximately. Thus, finding the pipe's life is reduced to solving the initial-boundary-value-problem (10)-(13) before the condition (15) will be satisfied. To do this solving for the given values of the internal and external pressures in the boundary conditions (11) it will be possible to find the pipe's life corresponding to those pressures, so the initial-boundary-value-problem (10)-(13) with the condition (15) actually defines indirectly the function (6). So, we cannot have the exact analytical view of the function (6), but we can have the grid values of this function (6) corresponded to the preliminary chosen values of the pressures (7). At the same time, having the grid values of the function (6), it is possible to do calculating by the formulas (3) or (5) to find the lifetime density function and to use them in further calculating for finding the gamma-percentile life of the pipe from the condition (1).

3.3.2. Numerical engineering analysis of the pipe's damaging due to creep

Defining the grid value representing the function (6) needed for calculating the gamma-percentile life representing the quantitative measure of the failures risks of the pipelines requires solving a lot of the initial-boundary-value problems (10)–(13) with the condition (15) for the different sets of the internal and external pressures' values (7), that is to solving the sets of the considered pipe's engineering analysis problems. It is naturally, that solving the initial-boundary-value problem (10)-(13) can be possible only by using the numerical methods. So, the numerical engineering analysis of the pipe's life is fundamentally required for calculating their failures risks, and this is reduced to numerical solving the initial-boundary-value problem (10)-(13). Solving the initial-boundary-value problems of the theory of creep including their particular case (10)-(13) is actually the separate subject area and it is not reasonable for fundamentally discussing in this research, because such numerical solving is only one from the several steps needed for considered here calculating the failures risks of the pipelines. At the same time, some general notes about the numerical solving the initial-boundary-value problem (10)-(13) must be briefly discussed here all the same.

To discuss generally about the numerical methods for solving the initial-value problems of the theory of creep it is suitable to have the generalized formulation of such problems, so that the considered above problem (8)–(15) will be involved in them as the particular case. To have such generalized formulation of the theory of creep problems for some structural element it is suitable to consider this structural element as the infinite set of the points of the Euclidian space E, so that all of them can be distinguished between each other by the $\mathbf{r}$ coordinates' vector, which in the considered particular case (8)–(15) is reduced to the r (Figure 1) radial coordinate. The geometry of the considered structural element can be imagined by the domain Υ with the boundary υ, so that in the considered above particular case (8)–(15) about the pipe (Figure 1) all whose can be defined as:

$$\Upsilon = (r_1 < r) \cap (r < r_2), \;\; \upsilon = (r = r_1) \cup (r = r_2). \tag{16}$$

Further, it is suitable to introduce the vector $\mathbf{u}^{(1)}(\mathbf{r},t)$ representing the state of the considered structural element as the continuum, and the vector $\mathbf{u}^{(2)}(\mathbf{r},t)$ representing the irreversible creep strains as well as the different damage parameters. In the considered particular case (10)–(13) these introduced vectors can be fellows:

$$\mathbf{u}^{(1)} = (u_r \quad \sigma_r \quad \sigma_\theta)^T, \;\; \mathbf{u}^{(2)} = (\sigma_r \quad \sigma_\theta \quad \omega)^T, \tag{17}$$

where ^{T} is the transpose operation.

These introduced vectors $\mathbf{u}^{(1)}$ and $\mathbf{u}^{(2)}$ allow representing the differential equations for the wide kinds of the initial-boundary-value problems of the theory of creep in the follows view:

$$\mathbf{A}^{(1)}\left(\mathbf{u}^{(1)}\right) + \mathbf{A}^{(2)}\left(\mathbf{u}^{(2)}\right) = \mathbf{f}^{(1)} \; \forall P(\mathbf{r}) \in \Upsilon, \;\; \mathrm{B}^{(1)}\left(\mathbf{u}^{(1)}\right) = \mathbf{p}^{(1)} \; \forall P(\mathbf{r}) \in \upsilon, \tag{18}$$

$$\frac{\partial \mathbf{u}^{(2)}}{\partial t} = \mathbf{f}^{(2)}\left(\mathbf{u}^{(2)};\mathbf{u}^{(1)}\right), \;\; \mathbf{u}^{(2)}(\mathbf{r},0) = \mathbf{0} \;\; \forall P(\mathbf{r}) \in \Upsilon \cup \upsilon, \tag{19}$$

where $\mathbf{A}^{(1)}(\bullet)$, $\mathbf{A}^{(2)}(\bullet)$ are the linear operators and $\mathbf{f}^{(1)}$ is the vector representing the differential equations, but $\mathrm{B}^{(1)}(\bullet)$ and $\mathbf{p}^{(1)}$ are the linear operator and the given vector representing the boundary conditions all defining the considered structural element as the continuum; $P(\mathbf{r})$ is the point of the Euclidian space E associated with the vector $\mathbf{r}$; $\mathbf{f}^{(2)}\left(\mathbf{u}^{(2)};\mathbf{u}^{(1)}\right)$ is the given vector function defining the creep strains and the damage parameters velocities; $\mathbf{0}$ is the zero vector which has the required size.

The correspondence between the relations (18), (19) and (10)–(13) is evident taking into account relations (16), (17). We can imagine the relations (18) as the boundary-value problem relatively the $\mathbf{u}^{(1)}$ vector for the given $\mathbf{u}^{(2)}$ vector in each moment of the time. The numerical methods for solving the boundary-value problems like the finite differences, the Galerkin's methods and the finite elements methods are well-known and based on discretization the considered problem (18). So, applying any of noted above numerical methods for solving the boundary-value problem (18) will lead to so that instead the problem (18) we will have their discrete analogue:

$$\mathbf{A}_n^{(1)} \cdot \mathbf{u}_n^{(1)} + \mathbf{A}_n^{(2)} \cdot \mathbf{u}_n^{(2)} = \mathbf{f}_n^{(1)}, \tag{20}$$

where $\mathbf{u}_n^{(1)}$ and $\mathbf{u}_n^{(2)}$ are the vectors giving the discrete representations of the vectors $\mathbf{u}^{(1)}$ and $\mathbf{u}^{(2)}$ relatively; $\mathbf{A}_n^{(1)}$, $\mathbf{A}_n^{(2)}$ and $\mathbf{f}_n^{(1)}$ are some matrices and some vector giving the discrete representations of the boundary-value problem (18); n is the parameter defining the discretization like the count of the grid nodes, or the count of the trial functions, like the grid step or others similar.

The particular view of the discrete representation (20) is significantly dependent from the used numerical method and it will not be discussed here, because choice of such numerical method for analysis has no principal influencing on calculating the failures risks after this analysis will be accomplished. But it is important for us, the possibility of resolving the relation (20) in the follows view:

$$\mathbf{u}_n^{(1)} = \left(\mathbf{A}_n^{(1)}\right)^{-1} \cdot \left(\mathbf{f}_n^{(1)} - \mathbf{A}_n^{(2)} \cdot \mathbf{u}_n^{(2)}\right). \tag{21}$$

From the other side, the discrete form (20) means the possibilities of representing the initial-value problem (19) into the corresponding discrete form:

$$\frac{d\mathbf{u}_n^{(2)}}{dt} = \mathbf{f}_n^{(2)}\left(\mathbf{u}_n^{(2)};\mathbf{u}_n^{(1)}\right), \quad \mathbf{u}_n^{(2)}(0) = \mathbf{0}_n, \tag{22}$$

where $\mathbf{f}_n^{(2)}\left(\mathbf{u}_n^{(2)};\mathbf{u}_n^{(1)}\right)$ is the discrete representation of the differential equations (19); $\mathbf{0}_n$ is the zero vector with the size corresponded with the n parameter's value.

It is understandable, the relation (22) considering with the additional relation (21) represents the initial-value problem relatively the vector $\mathbf{u}_n^{(2)}$. So, solving the initial-value problem defined by means the (21), (22) relations by any well-known numerical methods like the Runge-Kutta for example or by others will allow find the vector $\mathbf{u}_n^{(2)}$, but using the relation (21) will allow find the vector $\mathbf{u}_n^{(1)}$ too. It is necessary to note, the integrating of the initial-value problem defining by the relations (21), (22) by the numerical methods actually requires solving the linear algebraic equations systems representing in the view (21) by means production on the inverse matrix at least one time on each step. For example, four orders Runge-Kutta method requires solving this linear system (21) by four times per integrating step. It is necessary to note also, the discrete representations (20), (22) of the primary considered problem (18), (19) require the big sizes of the vectors, and it is led to the big number of the linear algebraic equations (21) needed to be solved on each of the numerical integrating's steps.

3.4. Calculating the failures risks of the pipe

The failures risks of the pipelines are quantitative measured by the gamma-percentile life (1) and calculating of it requires defining the density function of the pipe's life. In its turn, defining the pipe's life density function can be performed by the generalized formula (5) or by the particular formula (3) and all these require the deterministic properties representation (6) as well as the density function for the internal and external pressures.

3.4.1. Approximating of deterministic properties

As was noted above, it is impossible having the exact analytical function (6) defining the deterministic properties of the considered pipe needed for calculating their life's density function, but we can have only the grid values representing this needed function (6). To have the grid values representing the needed function (6) we must choose some given values of the internal and external pressures (7):

$$p_1^{\min} \le p_{1(i)} \le p_1^{\max},\ i=1,2,\ldots,N_1, \tag{23}$$

$$p_2^{\min} \le p_{2(j)} \le p_2^{\max},\ j=1,2,\ldots,N_2, \tag{24}$$

where $p_{1(i)}$ and $p_{2(j)}$ are the some values of the internal and external pressures on the pipe; N_1 and N_2 are the counts of the internal and external pressured nodal values.

Solving the initial-boundary-value problem (10)–(13) with the additional condition (15) for all combinations of the internal (23) and external (24) pressures, we will have the grid values of the function (6) representing the pipe's life depending on the internal and external pressures:

$$t_{(ij)}^*,\ \ i=1,2,\ldots,N_1,\ j=1,2,\ldots,N_2, \tag{25}$$

where $t_{(ij)}^*$ is the pipe's life was found for both the internal pressure $p_{1(i)}$ and the external pressure $p_{2(j)}$ by the numerical solving of the initial-boundary-value problem (10)–(13) with the additional condition (15).

The grid values (23)–(25) allow having the approximation of the function (6) including on the analytical view. It is naturally that the suitable view of the analytical approximation of the function (6) is predefined by the properties inherent for the initial-boundary-value problem (10)-(13). To have such suitable view of the function (6) approximation we will consider the last equation (12) with the last initial condition (13) and additional condition (15) separately from other equations (10)-(13) for the given unchanging value of the stress intensity. Such consideration will allow having the follows:

$$\frac{d\omega}{dt} = B_2\left(\frac{\sigma_i}{1-\omega}\right)^{m_2},\ \ \omega(0)=0. \tag{26}$$

The condition (15) corresponded to the simplified initial-value problem (26) will have the view:

$$\omega\left(t^*\right)=1. \tag{27}$$

It is possible to divide the variables in the simplified differential equation (26):

$$\left(1-\omega\right)^{m_2} d\omega = B_2\sigma_i^{m_2} dt. \tag{28}$$

Integrating the relation (28) taking into account the initial condition (26) leads to the result:

$$1-\left(1-\omega\right)^{m_2+1} = B_2\left(m_2+1\right)\sigma_i^{m_2} t. \tag{29}$$

Condition (27) and the relation (29) allow finding the life depending from the stress intensity:

$$t^* = \frac{\sigma_i^{-m_2}}{B_2\left(m_2+1\right)}. \tag{30}$$

Relation (30) represent the deterministic property that increasing the stress intensity will lead to decreasing the pipe's life, due to the restriction $m_2 > 0$. From the other side, the stress intensity in the pipe is defined by the internal and external pressures, so for the constant external pressure for example we can reasonably assume that:

$$t^* = Bp_1^{-m}, \tag{31}$$

where B and m are the constant parameters for each given external pressure p_2

It is necessary to note that the B and m constants must be differ from the constant B_2 and m_2 because they represent the integrated deterministic property of the pipeline's life taking into account the changing on the damage velocity due to the creep strains accumulation during the time as defined by the initial-boundary-value problem (10)–(13). At the same time the view (31) is really reasonable due to the property (30) of existing in the initial-boundary-value problem (10)–(13). So, we will use the relation (31) as the foundation for approximation of the function (6) using the grid values (23)-(25), but we will imagine influencing of the external pressure by the dependence of the approximation's parameters:

$$t^* = B\left(p_2\right) p_1^{-m(p_2)}. \qquad (32)$$

To define the functions $B\left(p_2\right)$ and $m\left(p_2\right)$ involved in approximation (32) we will use the least square method. To do this, it is suitable to represent the approximation (32) in the follows view:

$$\lg t^* = \lg B\left(p_2\right) - m\left(p_2\right)\lg p_1. \qquad (33)$$

The discrepancies of the relation (33) on the grid values (23)–(25) can be defined in the view:

$$S_j = \sum_{i=1}^{N_1}\left(\lg B\left(p_{2(j)}\right) - m\left(p_{2(j)}\right)\lg p_{1(i)} - \lg t^*_{(ij)}\right)^2, \quad j = 1,2,\ldots,N_2. \qquad (34)$$

The least square method for the discrepancies (34) can be represented by the conditions:

$$\frac{\partial S_j}{\partial \lg B\left(p_{2(j)}\right)} = 0, \; \frac{\partial S_j}{\partial m\left(p_{2(j)}\right)} = 0, \quad j = 1,2,\ldots,N_2. \qquad (35)$$

Substituting relation (34) into conditions (35) will lead to the linear algebraic equations systems which will allow defining the values $\lg B\left(p_{2(j)}\right)$ and $m\left(p_{2(j)}\right)$, $j = 1,2,\ldots,N_2$:

$$\begin{cases} N_1 \lg B\left(p_{2(j)}\right) - \sum_{i=1}^{N_1}\left(\lg p_{1(i)}\right) m\left(p_{2(j)}\right) = \sum_{i=1}^{N_1} \lg t^*_{(ij)}, \\ \sum_{i=1}^{N_1}\left(\lg p_{1(i)}\right)\lg B\left(p_{2(j)}\right) - \sum_{i=1}^{N_1}\left(\lg^2 p_{1(i)}\right) m\left(p_{2(j)}\right) = \sum_{i=1}^{N_1} \lg p_{1(i)} \lg t^*_{(ij)}, \end{cases} \qquad (36)$$

where $j = 1,2,\ldots,N_2$.

Instead the exact analytical representations of the functions $B\left(p_2\right)$ and $m\left(p_2\right)$ it will be possible having the nodal values of these functions $\lg B\left(p_{2(j)}\right)$ and $m\left(p_{2(j)}\right)$, $j = 1,2,\ldots,N_2$, which will allow having the corresponding interpolations.

3.4.2. Calculating the pipe's life density function

The density function of the pipe's life needs to calculate the gamma-percentile life (1) representing the quantitative measure of the failure risks for this pipe. Calculating the density function of the pipe's life can be performed by the generalized formula (5), and it needs having the internal and external pressures density function. Taking into account the assumption that the internal and external pressures (7) are having the uniform densities distributions, we will have the follows density function:

$$g\left(p_1,p_2\right)=\begin{cases}\left(p_2^{\max}-p_2^{\min}\right)^{-1}\left(p_1^{\max}-p_1^{\min}\right)^{-1}\forall p_1,p_2\in\left(p_1^{\min}\le p_1\le p_1^{\max}\right)\cap\left(p_2^{\min}\le p_2\le p_2^{\max}\right),\\ 0\forall p_1,p_2\in\neg\left(\left(p_1^{\min}\le p_1\le p_1^{\max}\right)\cap\left(p_2^{\min}\le p_2\le p_2^{\max}\right)\right).\end{cases}\quad(37)$$

Due to the relation (37), the generalized formula (5) can be reduced to the follows:

$$f\left(t^*\right)=\frac{dF}{dt^*},\quad F\left(t^*\right)=\left(p_2^{\max}-p_2^{\min}\right)^{-1}\left(p_1^{\max}-p_1^{\min}\right)^{-1}\int_{p_1^{\min}}^{p_1^{\max}}\int_{p_2^{\min}}^{p_2^{\max}}D\left(p_1,p_2,t^*\right)dp_1dp_2\,,\quad(38)$$

$$D\left(p_1,p_2,t^*\right)=\begin{cases}1\quad\forall p_1,p_2,t^*:\ B\left(p_2\right)\cdot p_1^{-m\left(p_2\right)}\le t^*,\\ 0\quad\forall p_1,p_2,t^*:\ B\left(p_2\right)\cdot p_1^{-m\left(p_2\right)}>t^*.\end{cases}\quad(39)$$

Computing by the formulas (38), (39) is really cumbersome, but it has no any principal difficulties and can be performed by using the different kind of the mathematical software like the Scilab free open source software or similar to them other free open source or commercial software. In any case, the relations (38), (39) give as the opportunities for defining the density function of the pipe's life which will allow calculating the gamma-percentile life (1) for estimating the failures risks for researched pipe.

4. Results of calculating the pipeline's failure risks and their discussions

Application of the based on the generalized approaches [15] the developed and discussed above approach for calculating the high-temperature pressurized pipeline's failures risks will be considered further.

4.1. Calculating data and pipe's life results

As the example, we will consider the high-temperature pressurized pipe which is similar to the used in the steam boiler's superheaters. The sizes of the considered pipe are (Figure 1):

$$r_1=18\text{ mm},\quad r_2=20\text{ mm}.\quad(40)$$

We will consider the austenitic stainless steel 18Cr-8Ni type as the structural material of the pipe; the characteristics of such material at the 500°C can be chosen are follows:

$$E=1{,}62\cdot10^5\text{ MPa},\quad\nu=0{,}3,\quad\alpha=18{,}4\cdot10^{-6}\ 1/\text{K},\quad(41)$$

$$m_1=2{,}023,\ B_1=8{,}859\cdot10^{-13}\text{ MPa}^{-m_1}/\text{hour},\quad m_2=12{,}344,\ B_2=3{,}799\cdot10^{-35}\text{ MPa}^{-m_2}/\text{hour}.\quad(42)$$

The operating temperature of the pipe is chosen so that:

$$\Delta T=500°\text{C}.\quad(43)$$

We will assume that the possible values of the internal and external pressures (7) during the pipe's operating cannot be less or more on 10% from theirs expected values, so that:

$$p_1^{\min}=0{,}9\overline{p}_1,\ p_1^{\max}=1{,}1\overline{p}_1,\ p_2^{\min}=0{,}9\overline{p}_2,\ p_2^{\max}=1{,}1\overline{p}_2,\quad(44)$$

where $\overline{p}_1$ and $\overline{p}_2$ are the expected values of the internal and external pressures.

The expected values of the internal and external pressures will be assumes as follows:

$$\overline{p}_1=30\text{ MPa},\quad\overline{p}_2=10\text{ MPa}.\quad(45)$$

The assumed data (40)–(45) allow solving the initial-boundary-value problem (10)–(13) with the additional condition (15) representing the mathematical model of the high-temperature pressurized pipe. The well-known Galerkin's classical method was used for numerical solving the creep theory initial-boundary-value problem (10)–(13) which is useful especially for one dimension spatial domains. We will not discuss here the application of the Galerkin's method because it will be very cumbersome and this is irrelevant for the considered problem about the failures risks since any other method can be used for solving the initial-boundary-value problem (10)–(13). The results of numerical solving the creep theory initial-boundary-value problem (10)–(13) by the Galerkin's method is represented in the Table 1 and on the Figure 1 in which we can see the grid values (23)–(25) of the function (6) representing generally the deterministic properties of the considered pipe needed for its failures risks calculating. We can see that increasing the internal pressure leads to decreasing the life, but increasing the external pressure leads to increasing the life of the pipe. Such property is due to the internal pressure is bigger than the external pressure, so increasing the external pressure "helps" the pipe to hold the internal pressure.

Table 1

The grid values of the high-temperature pressurized pipe's life

Internal pressure (MPa)	The pipe's life (hours) for the given external pressure (MPa)				
	9,0	9,5	10,0	10,5	11,0
27	$9{,}22\cdot10^5$	$1{,}35\cdot10^6$	$1{,}98\cdot10^6$	$2{,}95\cdot10^6$	$4{,}45\cdot10^6$
28	$4{,}57\cdot10^5$	$6{,}57\cdot10^5$	$9{,}51\cdot10^5$	$1{,}39\cdot10^6$	$2{,}05\cdot10^6$
29	$2{,}31\cdot10^5$	$3{,}29\cdot10^5$	$4{,}70\cdot10^5$	$6{,}77\cdot10^5$	$9{,}80\cdot10^5$
30	$1{,}18\cdot10^5$	$1{,}67\cdot10^5$	$2{,}38\cdot10^5$	$3{,}39\cdot10^5$	$4{,}85\cdot10^5$
31	$5{,}81\cdot10^4$	$8{,}45\cdot10^4$	$1{,}21\cdot10^5$	$1{,}72\cdot10^5$	$2{,}45\cdot10^5$
32	$2{,}68\cdot10^4$	$4{,}03\cdot10^4$	$6{,}00\cdot10^4$	$8{,}71\cdot10^4$	$1{,}25\cdot10^5$
33	$1{,}29\cdot10^4$	$1{,}86\cdot10^4$	$2{,}77\cdot10^4$	$4{,}17\cdot10^4$	$6{,}18\cdot10^4$

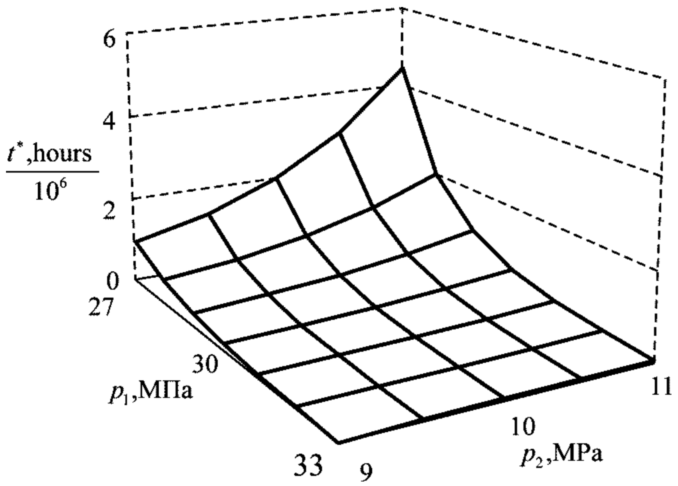


Figure 2: The pipe's life depending on the internal and external pressures

It is necessary to note, the obtained results (Figure 2) show that relatively small changes in the internal and external pressures can lead to significant changing of the pipe's life. Exactly such property will contribute to the failures during the pipe's operating due to natural existing of some changes the operating pressures.

4.2. Approximating the deterministic properties of the pipe

Having the grid values (Table 1) of the function (6) defining the deterministic properties of the high-temperature pressurized pipe required for calculating its failures risks, we can calculate the grid values of the functions $B(p_2)$ and $m(p_2)$ needed for the approximation (32) of these function (6) by using the least squares method (34)–(36). The obtained results for the grid values of the functions $B(p_2)$ and $m(p_2)$ needed for the approximation (32) of the pipe's life are presented in the Table 2 and on the Figure 3. We can see (Table 2 and Figure 3) in this particular case that the $m(p_2)$ exponent of the approximation (32) actually is practically not depended on the p_2 external pressure, but the coefficient $B(p_2)$ of the approximation (32) has noticeable increasing with increasing the p_2 external pressure.

It is necessary to note that using the approximation (32) is possible only due to their correspondence with the mathematical model (12) of accumulating the damages which are defined by the Kachanov-Rabotnov's parameter and it had shown by the relations (26)–(30). At the same time, for modelling the damages of the pipe due to creep it is possible to use other governing equations differ from the considered above equations (12). It naturally, those other governing equations can require other approximations of the pipe's life different from the used here approximation (32). However, it seems that the relation (31) is the more fundamental, so it seems it can be used for approximating the pipe's life even for the different kinds of the governing equations modelling the pipe's damages due to creep.

Table 2
The grid values of the parameters used in approximation of the pipe's life

External pressure $p_{2(j)}$ (MPa)	Approximation parameters	
	$B\left(p_{2(j)}\right)$ $\left(\text{MPa}^m \cdot \text{hour}\right)$	$m\left(p_{2(j)}\right)$
9,0	$2{,}20395 \cdot 10^{36}$	21,19818477
9,5	$2{,}35350 \cdot 10^{36}$	21,10819570
10,0	$2{,}62393 \cdot 10^{36}$	21,03011873
10,5	$3{,}83460 \cdot 10^{36}$	21,03127510
11,0	$9{,}06623 \cdot 10^{36}$	21,17383657

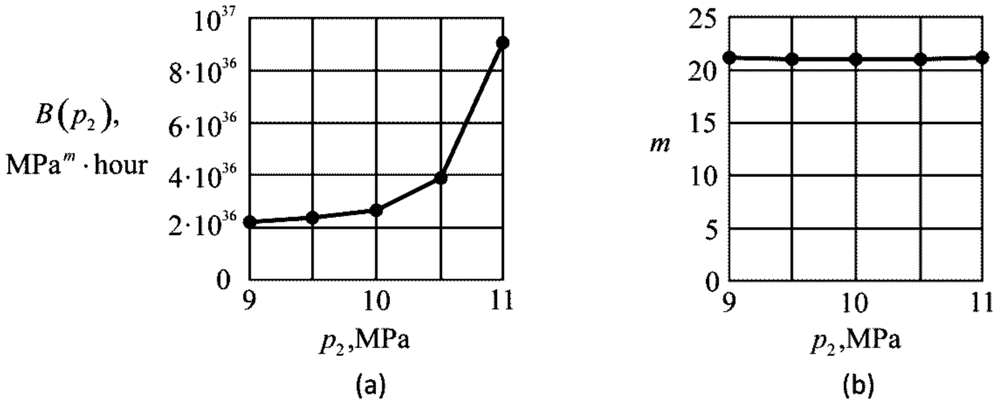


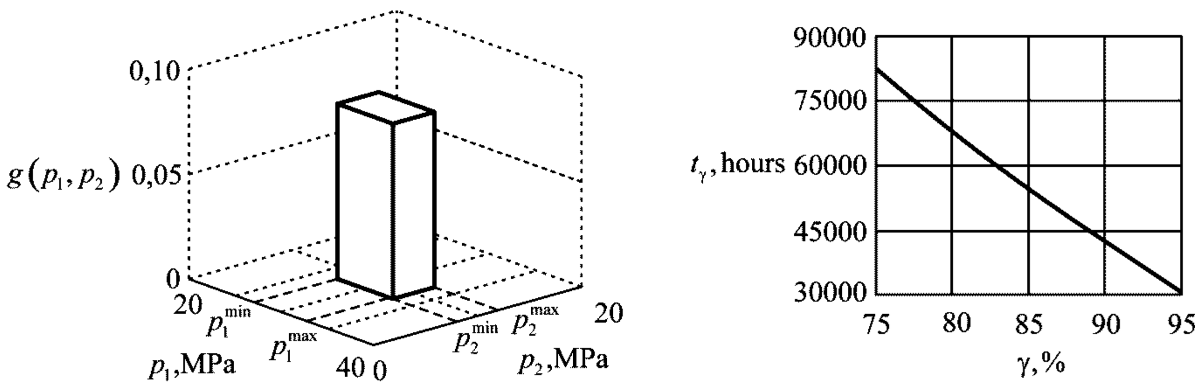
Figure 3: The grid values (markers) and the linear interpolation (lines) of the coefficient (a) and of the exponent (b) of the approximation of the pipe's life

Thus, having the results (Table 2 and Figure 3), it is possible to define the pipe's life for the given values of the internal and external pressures (7).

4.3. Results about the pipe's failures risks and theirs discussing

As was assumed above, the random values of the internal and external pressures during the pipe's operating are limited by the inequalities (7) and are had the uniform density function (37) inside the intervals corresponded to these inequalities (7). he density function (37) of the internal and external pressures for the used calculating data (44), (45) and the gamma-percentile life of the high-temperature pressurized pipe calculated using that density function by the formulas (38), (39) are shown on the Figure 4.

From the calculated particular results we can see (Figure 4) that the gamma-percentile life is the really generalized parameter which allows giving the most correct estimation about the failures risks of the high-temperature pressurized pipes as was assumed primary. Really, the gamma-percentile life of the pipe is very sensitive to the value of required probability of operating without the failures. So, if we can have the smaller probability of the pipe's operating without the failures then we can permit the more life for such pipe. At the same time, if the big probability of the pipe's operating without the failures is required then we must limit the life of this pipe, although it is exist the probability that this pipe will work further without failures some maybe even long time. This is very important for defining the lifetime of the high-temperature pressurized pipelines which are the critical structural elements like the steam boiler superheaters, the nuclear fuel claddings and similar elements which must provide the high probability of operating without any failures during the given time. So, the gamma-percentile life is the failures risks quantitative measure which really gives the most fully and correct characteristic of the failures risks for the high-temperature pressurized pipes during their operating.



(a) (b)

Figure 4: The density function of the internal and external pressures (a) and the gamma-percentile life (b) of the high-temperature pressurized pipe

The main cause of the obtained results (Figure 4) is in the sensitivity of the pipe's life to the internal and external pressure (Figure 2) about those was discussed above. So, the small reliability of the pipe can be established by the existing of their significant sensitivity of the life to the operating pressures even without defining the gamma-percentile life. This is very useful for designing the pipelines: the more reliable design is having the smaller sensitivity of the life to the operational pressures.

5. Conclusion

In this research the methods for calculating the failures risks of the high-temperature pressurized pipelines are developed, so that these developed methods are the particulars of the generalized approaches for the risk assessment in the stationary deterministic systems. Due to this research the follows results are developed.

It is proposed the schematization representing the main typical part of the high-temperature pressurized pipelines required for calculating their operational failures risks according with the generalized approaches. This schematization reduces to considering the straight pipes represent the necessary parts of the pipelines. It is proposed considering these straight pipes under the internal and external pressures under the given operating temperature because the main function of the pipe is necessarily in dividing the mediums with different pressures. Although, such schematization neglects some factors like bending and torque of the pipes, but it is not so cumbersome and it allows having the primary estimations about the pipe's failures risks need under the design stage while considering bending and torque can be for the more detailed researches. It is shown that the failures risks of the pipe can be fully defined by the gamma-percentile life, and to define it is necessary to have the dependence of the pipe's life on the operating internal and external pressures which represents the deterministic properties of the pipe.

It is proposed the mathematical model representing the deterministic properties of the main typical part of the high-temperature pressurized pipelines required for calculating their operational failures risks according with the generalized approaches. In this mathematical model it is took into account accumulating the irreversible strains and damages in the pipe's structural material during operating due to high-temperature creep, and this mathematical model is represented as the theory of creep initial-boundary-value problem. Such mathematical modelling the high-temperature pressurized pipe allow defining the life for the given sizes, material properties, and the operational loading needed for calculating their gamma-percentile life representing the failures risks. Numerical solving the theory of creep initial-boundary-value problems is briefly discussed. It is shown that such problems can be reduced to the initial-value (Cauchy) problem, building of which requires solving the linear algebraic equations systems. It is shown that choosing the methods for numerical solving the theory of creep initial-boundary-value problem is not principal for calculating the failures of the risks and any suitable numerical methods can be used to do this. It is proposed the general approximation of the high-temperature pressurized pipe's life depending on the internal and external operating pressures.

It is considered the particular example of the high-temperature pressurized straight pipel made from the stainless austenitic steel, and using the generalized approaches the gamma-percentile life giving the failures risks quantitative estimations is computed. It is shown that the gamma-percentile life is the failures risks quantitative measure which really gives the most fully and correct characteristic of the failures risks for the high-temperature pressurized pipes during their operating, because the gamma-percentile life of the pipe is very sensitive to the value of required probability of operating without the failures.

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