Building Intuitionistic Fuzzy Sets in Machine Learning

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Abstract

The construction of intuitionistic fuzzy sets is a difficult task. Some approaches have been proposed in the literature and they have been used successfully in some application domains. However, these approaches do barely take into account the representativeness of the data used to build the intuitionistic fuzzy set. In this paper, a new approach is proposed to build intuitionistic fuzzy sets (IFS). This approach is based on the use of a representativeness degree of the data. This approach enables to build an IFS with an intuitionistic fuzzy index that is a good indicator of the lack of knowledge associated with the data that make it a good approach to be used in a Machine learning setting.

Keywords

Intuitionistic fuzzy sets, Machine learning, lack of knowledge, representativeness.

1. Introduction

(Atanassov) Intuitionistic Fuzzy Sets (IFS) [1] have been proposed to represent a kind of lack of knowledge in the membership of elements to a set. This lack of knowledge is valued through the margin of hesitancy. Similarly, such a representation of a lack of knowledge exists with the Interval-Valued Fuzzy Sets that offer a representation close to the IFS one [2]. This capacity to associate such an information about the lack of knowledge to data is a powerful property of this representation model that find a natural application domain in Machine learning.

Indeed, in Machine learning the aim is to build a model from a given training set (the inductive reasoning approach) with the underlying hypothesis that this training set is completely representative of the whole universe of the data. However, it is well-known that such an induction process is often not applicable with great success in real-world problems: *"From the early days, theoreticians of machine learning have focused on the iid assumption, which states that the test cases are expected to come from the same distribution as the training examples. Unfortunately, this is not a realistic assumption in the real world.* [...] *As a practical consequence, the performance of today's best AI systems tends to take a hit when they go from the lab to the field."* [3].

Using IFS to tackled this problem in Machine learning could be a very promising approach and obtained some success till today [4, 5]. However, a weakness of the IFS approach is still the construction of the IFS from a dataset. Even if some approaches exists [6, 7], they have not been defined expressly to be used in a Machine learning setting.

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In this paper, we propose a new approach to construct IFS from a dataset that takes care of the specificity of the Machine learning setting. Firstly, in Section 2 a recall of existing approaches is done. Secondly, in Section 3, the proposed approach is introduced. Thirdly, in Section 4, an experimental study is presented to bring out some main properties of the proposed approach. Finally, a conclusion and some future work are presented.

2. Construction of intuitionistic fuzzy sets: existing approaches

In this section, we focus on existing approaches to build IFS. The following notations are used. Let $\mathcal{U} = \{u_1, \ldots, u_n\}$ be a discrete universe and let A be a subset of $\mathcal{U}$, and let $P = \{p_1, \ldots, p_n\}$ be a probability distribution on $\mathcal{U}$ with $0 \leq p_{i+1} < p_i \leq 1$ and $\sum_{i=1}^{n} p_i = 1$.

The question that should be answered is *"how to define the intuitionistic fuzzy set A of $\mathcal{U}$ from P?"* There does not exist a lot of approaches in the literature to answer this question, we present hereafter the 2 main ones. First of all we present the approach based on the definition of a mass assignment [7], and afterwards we present the approach by [6] based on the use of an intuitionistic fuzzy generator.

2.1. Basic recalls

Intuitionistic fuzzy sets have been introduced by Atanassov [1].

An IFS A of U is defined as: $A = \{(u, \mu_A(u), \nu_A(u)) \mid u \in U\}$ with $\mu_A : U \to [0, 1]$ and $\nu_A : U \to [0, 1]$ such that for all $u \in U$, $0 \leq \mu_A(u) + \nu_A(u) \leq 1$. The values $\mu_A(u)$ and $\nu_A(u)$ are, respectively, the membership degree and the non-membership degree of u to A.

The *margin of hesitancy* (or *intuitionistic fuzzy index*) of u to A is defined as $\pi_A(u) = 1 - (\mu_A(u) + \nu_A(u))$, it informs about the lack of knowledge about A when it exists. When the margin of hesitancy according to A is equal to zero for all u, that is to say when $\mu_A(u)+\nu_A(u) = 1, \forall u \in U$, then A is a Zadeh's fuzzy set.

2.2. The mass assignment approach

This approach has been introduced in [7] and detailed also in [8]. It has been used in Machine learning applications presented in [9] and [5].

2.2.1. Mass assignment

From a probability distribution P, a mass assignment m_A of a fuzzy set A of $\mathcal{U}$ can be defined thanks to the mass assignment theorem introduced in [10] that enables such a construction:

$$\begin{aligned} m_A(F_i) &= \mu_i - \mu_{i+1} \text{ for } i = 1, \ldots, n-1 \\ m_A(F_n) &= \mu_n \end{aligned}$$

where $F_i = \{u \in \mathcal{U} \mid p(u) \geq p_i\}$ and

$$\mu_i = |F_i| p_i + \sum_{j=i+1}^{n} (|F_j| - |F_{j-1}|) p_j.$$

2.2.2. Algorithm to build an IFS from a mass assignment

In [7], the building of an IFS $A = \{(u, \mu_A(u), \nu_A(u)) \,|\, u \in \mathcal{U}\}$ could be done from two independent probability distributions on $\mathcal{U}$: P^+ the probability distribution connected to the membership of the elements of $\mathcal{U}$ to the IFS A, and P^- the probability distribution connected to their non-membership to the IFS A. The process is done according to the following steps [7]:

1. On the one hand, the mass assignment m_A^+ is build from P^+ with the mass assignment theorem (see Section 2.2.1). As stated in [7]: $m_A^+(u)$ is the possibility that u has the value $m_A^+(u)$ and thus, it is considered that $m_A^+(u) = \mu_A(u) + \pi_A(u)$.
2. On the other hand, the mass assignment m_A^- is build from P^- with the mass assignment theorem. $m_A^-(u)$ is the possibility that u has the value $m_A^-(u)$ and it is considered that $m_A^-(u) = \nu_A(u) + \pi_A(u)$.
3. Finally, the aggregation of m_A^+ and m_A^- enables to obtain μ_A and ν_A taking into account that, for all $u \in \mathcal{U}$, $\mu_A(u) + \nu_A(u) + \pi_A(u) = 1$, it gives $m_A^+(u) + m_A^-(u) = 1 + \pi_A(u)$ and thus for all $u \in \mathcal{U}$, $\pi_A(u) = m_A^+(u) + m_A^-(u) - 1$ that leads to the determination of the values $\mu_A(u)$ and $\nu_A(u)$.

By means of the above process, the IFS $A \subseteq \mathcal{U}$ can be completely defined from P^+ and P^-.

2.2.3. Discussion

This approach to define the IFS A could have the drawback to produce a negative value for $\pi_A(u)$ as there is no guarantee that $m_A^+(u) + m_A^-(u) \geq 1$ for all $u \in \mathcal{U}$. Indeed, an IFS could not necessarily be build from any probability distributions and IFS could not be a convenient model in this case.

However, one solution when facing a negative value for $\pi_A(u)$ is to set it to 0, it is the solution that has been used in the experimental part of this paper.

In a Machine learning setting, this approach to define the IFS A is interesting because it does not need any hyper-parameter to be set.

2.3. The fuzzy generator approach

The fuzzy generator approach to build an IFS that is described in this section has been proposed in [6]. This approach has been mainly used in image segmentation [11] [4], or in clustering problems [12] to cite some examples of its use.

2.3.1. Intuitionistic fuzzy generator

The definition of an intuitionistic fuzzy generator (IFG) has been introduced in [6]: a function $\Phi : [0, 1] \to [0, 1]$ is called an intuitionistic fuzzy generator if $\Phi(x) \leq 1 - x$ for all $x \in [0, 1]$.

An example of such a generator is:

$$N(x) = \frac{1 - x}{1 + \lambda x}, \text{ with } \lambda > 0$$

where $N(0) = 1$ and $N(1) = 0$.

2.3.2. Construction of an IFS by means of an IFG

An IFS can be defined from a fuzzy set by means of an IFG [6]: let $A = \{(u, \mu_A(u)) \mid u \in \mathcal{U}\}$ be a fuzzy sets on $\mathcal{U}$, and let Φ be an intuitionistic fuzzy generator, then the set $\tilde{A} = \{(u, \mu_A(u), \Phi(\mu_A(u))) \mid u \in \mathcal{U}\}$ is an intuitionistic fuzzy set on $\mathcal{U}$.

2.3.3. Discussion

In [4] and in [12], the IFG presented in Section 2.3.1 is used to build the IFS, but another IFG can be used, see for instance [11].

In a Machine learning setting, this approach to build IFS is very promising, in particular in non-supervised Machine learning problems. However, the choice of at least one hyper-parameter, the IFG, should be done. Moreover, if the IFG given in Section 2.3.1 is used, it is also mandatory to choose a good value for λ.

3. A new approach to build an IFS

The proposed approach is dedicated to the construction of an IFS in a machine learning setting (but not limited to this kind of application domain). As a consequence, we consider that the information provided to build the IFS is not only probability distributions but a complete set of instances separated into two classes.

Let $\mathcal{X}$ be a universe of values and $\mathcal{U} = \{u_1, \ldots, u_n\} \subseteq \mathcal{X}$ be a discrete subset of $\mathcal{X}$, and let $\mathcal{C} = \{+, -\}$ be a set of classes over the elements of $\mathcal{X}$. For $i = 1, \ldots, n$, let n_i^+ be the number of instances of u_i that are associated with class $+$, and let n_i^- be the number of instances of u_i that are associated with class $-$. We denote by $n^+ = \sum_{i=1}^n n_i^+$ and $n^- = \sum_{i=1}^n n_i^-$ and we assume in the following that $n^+ \neq 0$ and $n^- \neq 0$. Moreover, we denote by $n_i = n_i^+ + n_i^-$, $\forall i = 1, \ldots, n$ and we assume that $n_i \neq 0, \forall i = 1, \ldots, n$.

In the following, a *training set* is the tuple $\langle \mathcal{U}, \mathcal{C}, (n_1^+, \ldots, n_n^+), (n_1^-, \ldots, n_n^-) \rangle$.

The set of values $n_1^+, \ldots, n_n^+$ (resp. $n_1^-, \ldots, n_n^-$) defines a probability distribution P^+ (resp. P^-) over $\mathcal{U}$ such that $P^+(u_i) = \frac{n_i^+}{n^+}$ (resp. $P^-(u_i) = \frac{n_i^-}{n^-}$).

These two probability distributions provide us with information about the elements of $\mathcal{U}$ and their membership (P^+) or non-membership (P^-) to a set $A \subseteq \mathcal{U}$ that we want to build.

Our approach aims at defining A as an intuitionistic fuzzy set of $\mathcal{U}$ according to the knowledge that is provided by a training set. This approach is composed of two main steps. First of all, from the training set two corresponding weighted distributions are built taking into account the representativeness of the training set. Secondly, the IFS A is built using these two weighted distributions. The process is detailed in the following.

3.1. Representativeness and weighted distributions

Given a training set $TS = \langle \mathcal{U}, \mathcal{C}, (n_1^+, \ldots, n_n^+), (n_1^-, \ldots, n_n^-) \rangle$, our approach aims at building an IFS A over $\mathcal{U}$. As usual in machine learning, the training set provides only a restricted view about $\mathcal{X}$, thus this brings out the question of its representativeness.

3.1.1. Representativeness degree

To highlight the representativeness of a training set, we introduce the use of a degree that should be set either by a user that knows the problem under study, or automatically by means of an objective decision process. This degree highlights how the training set can be considered as sufficiently representative to infer knowledge that could be generalised to $\mathcal{X}$.

The *representativeness degree* $\rho \in [0, 1]$ of the training set $\langle \mathcal{U}, \mathcal{C}, (n_1^+, \ldots, n_n^+), (n_1^-, \ldots, n_n^-) \rangle$ is such that:

- $\rho = 0$ when the training set is not representative of $\mathcal{X}$. In this case, the knowledge it provides are not usable for any elements of $\mathcal{X}$ not in $\mathcal{U}$.
- $\rho = 1$ when the training set is completely representative of $\mathcal{X}$. In this case, the knowledge it provides is completely usable for any elements of $\mathcal{X}$.
- the greater ρ, the more representative the training set.

The representativeness degree evaluates how much we could be confident in the fact that the probability distributions induced by $(n_i^+)_{i=1,..,n}$ and $(n_i^-)_{i=1,..,n}$ reflects the respective probability distributions on $\mathcal{X}$. This representativeness degree is either given by the user that appreciate how the knowledge provided by $\mathcal{U}$ could be generalised to $\mathcal{X}$, or it could be determined automatically (to be studied in future work).

3.1.2. Lack-of-knowledge degree

The representativeness degree is an information about the training set. It could be used to weight the information associated with any example $u \in \mathcal{U}$ provided in the training set.

Definition 1. *Let* $TS = \langle \mathcal{U}, \mathcal{C}, (n_1^+, \ldots, n_n^+), (n_1^-, \ldots, n_n^-) \rangle$ *be a training set of* $\mathcal{X}$*, and* $\rho \in [0, 1]$ *be the representativeness degree of* TS*. The* lack-of-knowledge degree *of* $u_i \in \mathcal{U}$ *is defined as* $l(u_i) = \rho * \frac{n_i}{n_{max}}$ *with* $n_{max} = \sup_{\{i=1,\ldots,n\}} n_i$*.*

The lack-of-knowledge degree takes into account not only the representativeness of the training set, but also, for a given property (membership or non-membership) the number of elements that possess this property. This is represented by the use of $n_{\max} = \sup_{\{i=1,\ldots,n\}} n_i$ to weight the value n_i. This degree enables us to take into account the representativeness of $\mathcal{U}$ to reduce the influence of an example used for the construction of the IFS A from P^+ and P^-.

It is easy to see that, for all $i = 1, \ldots, n$, $l(u_i) \in [0, 1]$ as $\rho \in [0, 1]$ and $0 \leq \frac{n_i}{n_{\max}} \leq 1$.

3.1.3. Weighted probability distributions

The lack-of-knowledge degree is used to weight the influence of the information related to an example u with regard to the global information brought out by the training set.

Definition 2. *Let* $TS = \langle \mathcal{U}, \mathcal{C}, (n_1^+, \ldots, n_n^+), (n_1^-, \ldots, n_n^-) \rangle$ *be a training set of* $\mathcal{X}$*, and* $\rho \in [0, 1]$ *be the representativeness degree of* TS*. The* weighted probability distributions over $\mathcal{U}$ related to the classes $\mathcal{C}$ *are defined* $\forall i = 1, \ldots, n$ *as:* $P_w^+(u_i) = l(u_i)\frac{n_i^+}{n_i}$ *and* $P_w^-(u_i) = l(u_i)\frac{n_i^-}{n_i}$*.*

It is easy to see that, for all $i = 1, \ldots, n$, as $l(u_i) \in [0, 1]$ and $\frac{n_i^+}{n_i} \in [0, 1]$ (resp. $\frac{n_i^-}{n_i} \in [0, 1]$), we have $P_w^+(u_i) \in [0, 1]$ (resp. $P_w^-(u_i) \in [0, 1]$).

3.2. Construction of an IFS

From a training set, associated with a representativeness degree, it is possible to define an intuitionistic fuzzy set taking into account the information related to the classes: class $+$ representing the information related to the membership of elements to A and class $-$ representing the information related to their non-membership to A.

Definition 3. *Let* $TS = \langle \mathcal{U}, \mathcal{C}, (n_1^+, \ldots, n_n^+), (n_1^-, \ldots, n_n^-) \rangle$ *be a training set of* $\mathcal{X}$*, and* $\rho \in [0, 1]$ *be the representativeness degree of* TS*. The* intuitionistic fuzzy set A *is defined as*

$$A = \{(u, P_w^+(u), P_w^-(u)) \mid u \in \mathcal{U}\}.$$

Indeed, it is easy to see that A is an IFS as we have for all $u_i \in \mathcal{U}$:

- $P_w^+(u_i) \in [0, 1]$ and $P_w^-(u_i) \in [0, 1]$ (see Section 3.1.3);
- $P_w^+(u_i) + P_w^-(u_i) = l(u_i)\frac{n_i^+}{n_i} + l(u_i)\frac{n_i^-}{n_i} = l(u_i)$ and $0 \leq l(u_i) \leq 1$ (see Section 3.1.2).

4. Experimental study

A preliminary experimental study is presented in this section to study the influence of the representativeness degree, and to compare the IFS built by means of each of the presented approaches.

The experiments are conducted on a training set proposed in [7]. The set $\mathcal{U}$ contains 10 elements associated with different sizes n_i. In [7], two probability distributions on $\mathcal{U}$ are given to represent the membership and the non-membership to the IFS A to build. In our experiments, these probability distributions have been used to generate the values $(n_1^+, \ldots, n_n^+)$, and $(n_1^-, \ldots, n_n^-)$ by considering that the total size of the set $+$ (resp. $-$) is 1000 elements. The resulting training set is presented in the 2 first columns in Table 1.

A global analysis of this training set leads us to consider that, among the elements u, there are some elements that are more representative than others. For instance, u_1 has only a size of 125 where u_5 is associated with a size of 351. Indeed, this highlights the fact that we could be less confident in the decision that could be drawn from this training set for u_1 than for u_5.

4.1. Comparison of approaches

In Table 1, we present the IFS built by each of the 3 approaches presented in this paper: our proposed approach with $\rho = 1$ (complete representativeness of the training set), the approach by [7], and the approach by [6] with $\lambda = 0.5$. Concerning this last approach, as it takes into account only one probability distribution, the given IFS is obtained with only the information related to $+$.

The three IFS built by these three approaches are very different. It should be seen that

- the IFS built by [6] does not provide a great hesitancy (intuitionistic indices π) unlike the other 2 approaches. In this approach, this hesitancy is related to the choice of λ, but it takes into account only one information from the training set: the probability distribution associated with the membership of the elements.

	Size		Proposed app. ($\rho = 1$)			Approach by [7]			Approach by [6] ($\lambda = 0.5$)		
	+	-	μ_A	ν_A	π_A	μ_S	ν_S	π_S	μ_B	ν_B	π_B
u1	0	125	0.0	0.356	0.644	0.0	0.994	0.006	0.0	1.0	0.0
u2	0	128	0.0	0.365	0.635	0.0	1.0	0.0	0.0	1.0	0.0
u3	34	117	0.097	0.333	0.57	0.038	0.796	0.166	0.034	0.934	0.032
u4	165	80	0.47	0.228	0.302	0.26	0.272	0.468	0.165	0.717	0.118
u5	301	50	0.858	0.142	0.0	0.5	0.0	0.5	0.301	0.537	0.162
u6	301	50	0.858	0.142	0.0	0.5	0.0	0.5	0.301	0.537	0.162
u7	165	80	0.47	0.228	0.302	0.26	0.272	0.468	0.165	0.717	0.118
u8	34	117	0.097	0.333	0.57	0.038	0.796	0.166	0.034	0.934	0.032
u9	0	128	0.0	0.365	0.635	0.0	1.0	0.0	0.0	1.0	0.0
u10	0	125	0.0	0.356	0.644	0.0	0.994	0.006	0.0	1.0	0.0

Table 1
Comparison of the 3 approaches.

- the IFS built by [7] provides an hesitancy, however, it does not take into account the representativeness of each element.
- the IFS built by the proposed approach highlights the importance of the information provided on each element. This leads to an hesitancy more important for u_1 than for u_5 for instance.

4.2. Influence of the representativeness degree

In order to study the influence of the representativeness degree, the proposed approach has been used with different values of this degree for the same training set. The training is similar to the one detailed in the previous section. The resulting IFS obtained with the approach used with increasing values of the representativeness degree ρ is presented in Table 2. This table should be completed with the result for $\rho = 1$ given in Table 1.

These results show the great influence of the representativeness degree in the resulting IFS, this influence can be highlighted by the values of the intuitionistic indices π of the resulting IFS: the greater the representativeness degree, the lower the values of the intuitionistic indices. This perfectly highlights the behaviour of the approach to build an IFS that is needed in a Machine learning setting: the intuitionistic fuzzy index is a representation of the lack of knowledge that is associated with the membership of an element u to the IFS A [13], therefore the representativeness of the training set induces the lack of knowledge that is associated with the data belonging to this training set.

A main question arises here on *"how to choose a good value for ρ?"*. This value could be set by the user if he/she has sufficient knowledge to evaluate the representativeness of the training set. Otherwise, ρ could also be set by means of an automatic approach, for instance, by considering the size of the training set with regard to the dimension of $\mathcal{X}$.

	Size		Proposed app. ($\rho = 0.1$)			Proposed app. ($\rho = 0.25$)			Proposed app. ($\rho = 0.75$)		
	+	-	$\mu_{0.1}$	$\nu_{0.1}$	$\pi_{0.1}$	$\mu_{0.25}$	$\nu_{0.25}$	$\pi_{0.25}$	$\mu_{0.75}$	$\nu_{0.75}$	$\pi_{0.75}$
u1	0	125	0.0	0.036	0.964	0.0	0.089	0.911	0.0	0.267	0.733
u2	0	128	0.0	0.036	0.964	0.0	0.091	0.909	0.0	0.274	0.726
u3	34	117	0.01	0.033	0.957	0.024	0.083	0.892	0.073	0.25	0.677
u4	165	80	0.047	0.023	0.93	0.118	0.057	0.825	0.353	0.171	0.476
u5	301	50	0.086	0.014	0.9	0.214	0.036	0.75	0.643	0.107	0.25
u6	301	50	0.086	0.014	0.9	0.214	0.036	0.75	0.643	0.107	0.25
u7	165	80	0.047	0.023	0.93	0.118	0.057	0.825	0.353	0.171	0.476
u8	34	117	0.01	0.033	0.957	0.024	0.083	0.892	0.073	0.25	0.677
u9	0	128	0.0	0.036	0.964	0.0	0.091	0.909	0.0	0.274	0.726
u10	0	125	0.0	0.036	0.964	0.0	0.089	0.911	0.0	0.267	0.733

Table 2
Results with various representativeness degrees.

5. Conclusion

In this paper, the problem of building intuitionistic fuzzy sets is tackled. After a survey of the two main existing approaches from the literature to build IFS from probability distributions, a new approach is proposed. The aim of this new approach is to be used in a Machine learning setting where it is important to take into account the representativeness of the training set.

The proposed approach is based on the use of a representativeness degree of the given training set and the determination of a lack-of-knowledge degree to weight the importance of each element of the training set. This approach enables to build an IFS where the intuitionistic fuzzy index is a good indicator of the lack of knowledge associated with the given training set.

In future work, even if a good choice could be to set ρ to 1, a first study to conduct will be to find a way to make the choice of a good value for ρ, for instance by means of some automatic ways to set this value. Secondly, the resulting IFS will be used in a complete Machine learning approach, to build a classification model from a given training set. Moreover, future work will also deepen the study on the proposed approach in term of convergence property and the complexity

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