Algorithm for Direction Finding Using Spinning and Omnidirectional Antennas that Uses All Available Information

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Abstract

We propose a new algorithm for direction finding using a spinning Direction Finding (DF) antenna and an omnidirectional antenna. In the proposed algorithm we use all information about when a target is received using either antennas, whereas in literature only situation when the signal is received using the spinning DF antenna and the signal with the spinning DF antenna is stronger than with the signal received with the omnidirectional antenna. Simulated results show that the proposed algorithm is more accurate in direction finding and target positioning than the algorithm found in literature.

Keywords

Direction-of-Arrival, Spinning DF Antenna, Direction Finding, Electronic Intelligence, DOA

1. Introduction

Finding direction of a radar transmission is an important task of Electronic Intelligence (ELINT) systems [1]. The spinning, wide bandwidth antenna remains the most cost-effective technique for finding the Direction-of-Arrival (DoA) of a signal from emitters [2]. The basic principle is that the DoA of a signal is estimated based on a spinning antenna. In a simplest ideal case, the emitter is emitting isotropic Carrier Wave (CW) signal. In that case the DoA of emitter can be determined by taking the direction where the spinning Direction Finding (DF) antenna's mainlobe is directed when the strongest signal is received. In more complex situation the emitter is also making a scanning pattern and has a finite Beamwidth (BW). To determine whether the receiver antenna's mainlobe is directed towards the emitter or the emitter mainlobe towards the receiver an omnidirectional antenna is used. Figure 1 show possible antenna gain patterns. If the received amplitude of the directional antenna is smaller than from the omnidirectional antenna it is known that the pulse source direction is outside the main lobe of the spinning DF antenna.

In [3], the direction estimation in based on choosing the region where the received signal on the directional antenna has higher Pulse Amplitude (PA) than with the omnidirectional antenna (thick blue line in Figure 1). The transmitted signal is assumed to have a constant amplitude for which both amplitude and complex amplitude situations are evaluated. Article [3] compares several estimation methods for the direction estimation when the receiving antenna pattern is known and the transmitted pulses have a constant complex amplitude.
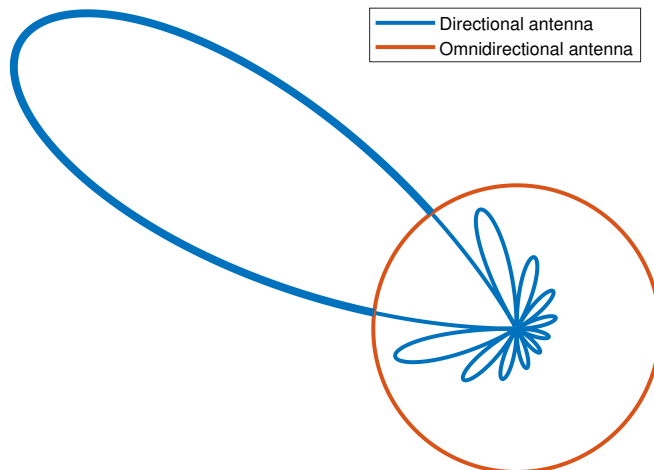
Figure 1: Example antenna gains for directional and omnidirectional antennas

In [4] algorithms for detecting multiple DoAs from targets are discussed. The downfall of the algorithms in [4] is that the antenna pattern is assumed to be Gaussian-shaped, which means that only the mainlobe is used and it is assumed to be a Gaussian. Same assumption is made also in [5]. This means that only pulses that are received stronger with the spinning DF antenna (thick line in Figure 1) are used, and all other information is neglected.

The assumption of a constant transmission amplitude does not usually hold as radar systems incorporate scanning patterns and the transmission direction changes. The constant amplitude assumption is valid approximation only if the transmission BW is high and the radar scanning speed is low compared to the receiver spinning DF antenna BW.

In this paper, we propose an algorithm that uses information from all received pulses and does not assume anything about the transmission pattern. Rest of this paper is organized as follows. Section 2 describes the models used in this paper. Section 3 shows how the probability density function (pdf) of DoA can be determined using all information. In Section 4 we show how the mean and variance of the estimates are computed. Section 5 contains example applications of the proposed algorithm and Section 6 concludes the article and discusses future work.

2. Models for the system

We assume here that the emitter is a pulsed radar, and we can associate pulses from a single radar together. Received pulses can be divided into three groups based on the PAs

1. Pulse is received with both antennas
2. Pulse is received only with the spinning DF antenna
3. Pulse is received only with the omnidirectional antenna

We model the PA received with the spinning DF antenna using model

$$A_r(t) = f_r\left(\theta(t)\right) f_t\left(t\right) + \varepsilon_r, \tag{1}$$

where $A_r(t)$ is the received amplitude, $f_r(\theta(t))$ is the receiver gain at time t to the direction of the transmitter, $f_t(t)$ is the field strength of the transmitted signal at the receiver at time t, and ε is a noise term, which is assumed to be zero mean normal with variance σ_r^2. We assume that the receiver antenna pattern $f_r(\theta(t))$ is known. We do not make any assumptions of the shape of transmission field strength $f_t(t)$. Transmission field strength can have complex shapes especially when the transmitter is an Active Electronically Scanned Array (AESA) radar. In AESA radars, the scanning pattern can be steered electronically from one direction to other via phase control practically without delay [6].

For the omnidirectional receiver, the antenna gain is constant g_o and the received PA is:

$$A_o(t) = g_o f_t(t) + \varepsilon_o, \tag{2}$$

where $\varepsilon_o \sim \mathrm{N}(0, \sigma_o^2)$. We assume that both antennas are connected to a multichannel receiver so that the sensitivity for both channels is same and we use same threshold A_{limit} for detecting a pulse for both antennas.

The model is more general than those found in [3]. In [3], the transmitter power and thus $f_t(t)$ is assumed constant and only values of $P(t) > A_o(t)$ are used. However, we make a simplification compared to [3] in that we consider signal absolute amplitude and not complex amplitude.

3. Probability of DoAs

When we have detected pulse with both antennas at time t_i we can solve the unknown field strength $f_t(t)$ from (2) and substitute it into (1) to get:

$$A_o(t_i) = f_r(\phi - \theta_i)\left(\frac{A_o(t_i) - \varepsilon_o}{g_o}\right) + \varepsilon_r, \tag{3}$$

where ϕ is the direction of the target in geographic coordinates. After this we compute the likelihood for target being at angle ϕ using the normal pdf

$$p_{ro}(A_r(t_i), A_o(t_i)|\phi) = p_{\mathrm{N}}\left(A_o(t_i)\middle|\frac{f_r(\phi - \theta_i)A_o(t)}{g_o}, \left(\frac{f_r(\phi - \theta_i)}{g_o}\right)^2 \sigma_o^2 + \sigma_r^2\right) \tag{4}$$

$$= \frac{1}{\sqrt{\left(\frac{f_r(\phi - \theta_i)}{g_o}\right)^2 \sigma_o^2 + \sigma_r^2}} e^{\frac{\left(A_o(t_i) - \frac{f_r(\phi - \theta_i)A_o(t)}{g_o}\right)^2}{2\left(\frac{f_r(\phi - \theta_i)}{g_o}\right)^2 \sigma_o^2 + 2\sigma_r^2}}, \tag{5}$$

where subscript ro refers to the case where pulse was received with both antennas.

If the pulse is received only with the directional antenna, we can deduct that the signal source is in the direction where the gain of the spinning DF antenna is higher than the gain of the omnidirectional antenna and the amplitude of the received PA is below the detection threshold of the omnidirectional antenna. This area corresponds to the area marked with thick blue line in Figure 1. Because the transmitting gain (and power) is unknown we cannot make as accurate evaluation of the DoA as in the first case. However, as A_{limit} is the threshold for detecting a pulse and we neglect the noise in the inequality we get

$$A_{\mathrm{limit}} \geq g_o f_t(t) \Rightarrow \tag{6}$$

$$f_t\left(t_i\right) \leq \frac{A_{\text{limit}}}{g_o}. \tag{7}$$

If we had a known value of $f_t\left(t_i\right)$ and (1) the likelihood is

$$p(A_r(t_i))|\phi, f_t\left(t_i\right)) = p_{\text{N}}\left(A_r(t_i)|f_t\left(t_i\right)f_r\left(\phi\right),\sigma_r^2\right). \tag{8}$$

By integrating the possible values of $f_t\left(t\right)$ using (7) we get

$$p_r(A_r(t_i))|\phi) = \int_0^{\frac{A_{\text{limit}}}{g_o}} p_{\text{N}}\left(A_r(t_i)|f_t\left(t_i\right)f_r\left(\phi\right),\sigma_r^2\right) \mathrm{d}f_t\left(t_i\right), \tag{9}$$

where subscript r refers to the case where the pulse was received only with the spinning DF antenna. This integral can be computed using the cumulative distribution function (cdf) of the normal distribution.

It is also possible that a pulse is detected only with the omnidirectional antenna. In this case, we get inequality

$$f_t\left(t_i\right) \leq \frac{A_{\text{limit}}}{f_r\left(\phi\right)}. \tag{10}$$

This can be used then with (2) to obtain likelihood as above:

$$p_o(A_r(t_i))|\phi) = \int_0^{\frac{A_{\text{limit}}}{\phi}} p_{\text{N}}\left(A_o(t_i)|f_t\left(t_i\right)g_o,\sigma_r^2\right) \mathrm{d}f_t\left(t_i\right), \tag{11}$$

where subscript o refers to that the pulse was detected only with the omnidirectional antenna.

When we combine all measurements we get likelihood for DoA ϕ

$$p(A_r, A_o|\phi) = \prod_i p_{ro}(A_r(t_i), A_o(t_i)|\phi) \prod_j p_r(A_r(t_j)|\phi) \prod_k p_o(A_o(t_k)|\phi) \tag{12}$$

and to get the pdf we use uninformative prior and normalize using Bayes' rule to get

$$p(\phi|A_r, A_o) = \frac{p(A_r, A_o|\phi)}{\int_{\phi=0}^{360} p(A_r, A_o|\phi)\mathrm{d}\phi}. \tag{13}$$

In practice, the computation of the probabilities above encounter often numerical errors and instead of using likelihoods the computations should be done using log-likelihoods until the normalization.

4. DoA and variance determination from the pdf

The pdf of DoAs is not always practical and it is often required to be transformed to a mean and variance of DoA in degrees. To obtain the mean of DoA first a weighted Cartesian mean μ is computed [7]
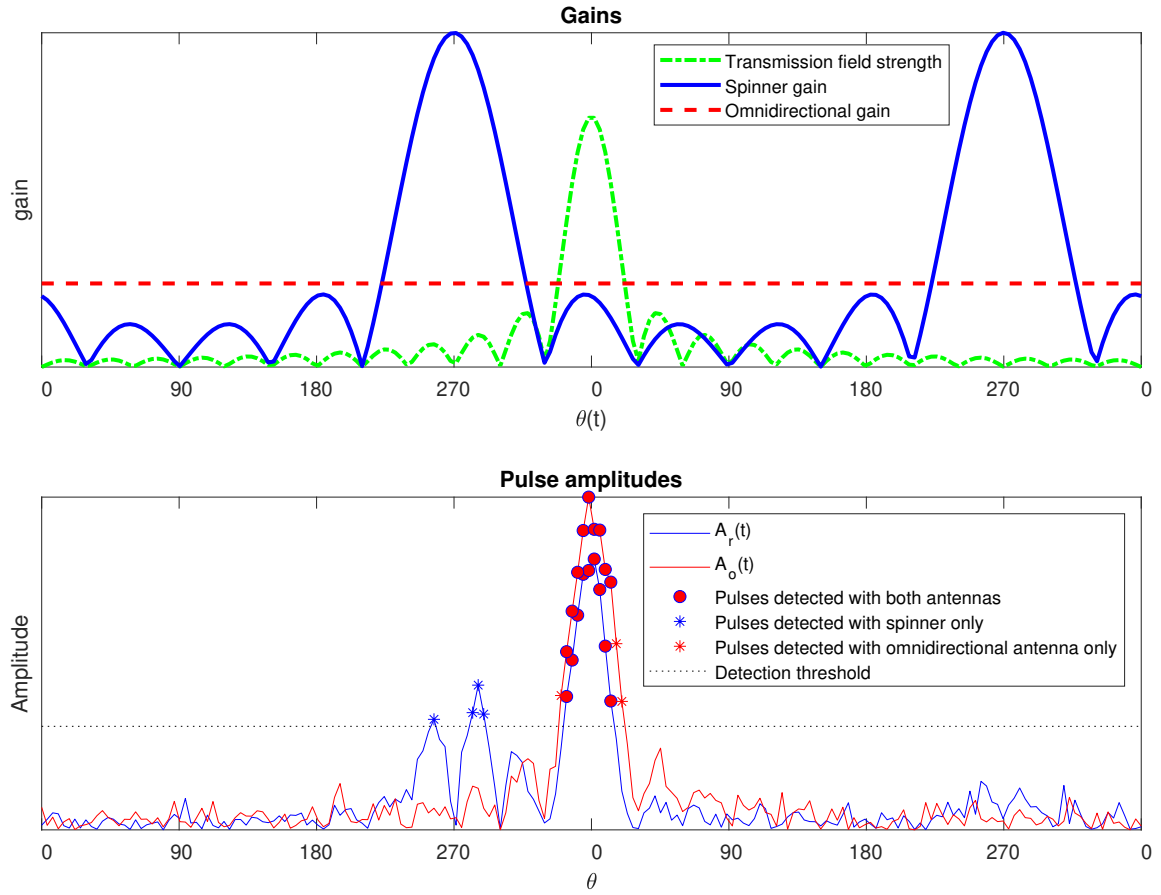
$$\mu_c = \int_{\phi=0}^{360} \begin{bmatrix} p(\phi)\sin\phi \\ p(\phi)\cos\phi \end{bmatrix} \tag{14}$$

and the mean of DoA μ_ϕ is

$$\mu_\phi = \angle\mu_c = \text{atan2}(\mu_{c,1}, \mu_{c,2}), \tag{15}$$

where atan2 is the four-quadrant inverse tangent. After the mean is solved the variance for the target direction is obtained by

$$\sigma_\phi^2 = \int_{-180}^{180} p\left(\phi + \mu_\phi \bmod 360\right)\phi^2\mathrm{d}\phi. \tag{16}$$

Figure 2: Receiver gain, transmission strength and amplitudes of in the first example

5. Examples

In this section, we show examples of the proposed algorithm. The antenna gain and transmission field strengths are simulated. The noise is simulated from a normal distribution with standard deviation 0.1 and the detection threshold for a pulse is 1.

In the first example radar is located at 270° from the receiver. The radar is making a circular scanning pattern so that it's mainlobe is directed towards the receiver when receiver has direction 0°. The DoA of radar pulses is solved using pulses received of two revolutions of the spinning DF antenna. In this situation the radar mainlobe is directed towards the receiver when the receivers spinning DF antenna's first sidelobe is directed towards the radar. Top plot of Figure 2 shows the gains as the function of spinning DF antenna angle. In the bottom plot the noisy signals and detected pulses are shown. All pulses that are received with both antennas come from the first sidelobe. Then there are few pulses detected with spinning DF antenna's mainlobe that come from sidelobes of the transmitter and few pulses detected with the omnidirectional antenna only.

In real world situations the spinning antenna could be rotating 1200 degrees in second and
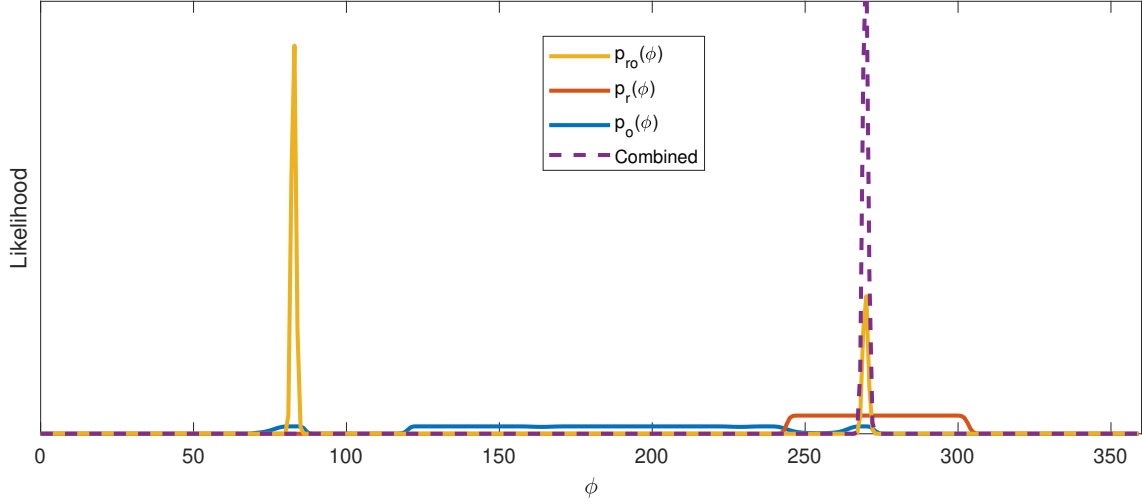
Figure 3: Likelihoods of different directions of the target radar

Table 1
Mean and standard deviation of DoA computed from different pulses

	μ	std
Spinning & omnidirectional	78°	90°
Spinning	274°	17°
Omnidirectional only	182°	50°
Combined	269°	1°

in our simulation there are 100 pulses for 720 degrees. This would make the pulse repetition frequency of the transmitting radar 167 Hz, which is a low value. We have chosen this small number of pulses to show how the proposed algorithm can perform in difficult situations with a small number of received pulses.

Figure 3 shows the likelihoods computed from the received pulses. Because all pulses received with both antennas are in the first sidelobe and the spinning DF antenna has two almost identical sidelobes the likelihood p_{ro} has two peaks and due to the noise in measurements the wrong likelihood peak is the stronger one. Algorithms in [3, 4, 5] assume that the pulses received are from the mainlobe and the omnidirectional antenna is used to discard pulses that are not and thus in this case those algorithms would not provide DoA estimates.

Likelihood computed from pulses received only with the spinning DF antenna is directed approximately to the correct direction but has much larger variance than the peaks that are computed with both antennas. But when these two are combined we get an accurate peak in the correct direction. In this example, the likelihood computed using only omnidirectional antenna does not provide more information to other measurements.

The mean and standard deviation estimates computed using different data are given in Table 1. The estimates were computed using a 1° resolution. Results show how the combined estimate is much more accurate than any of estimates computed alone. Results also show that all standard deviations are reasonable and can be used to determine the goodness of the DoA.
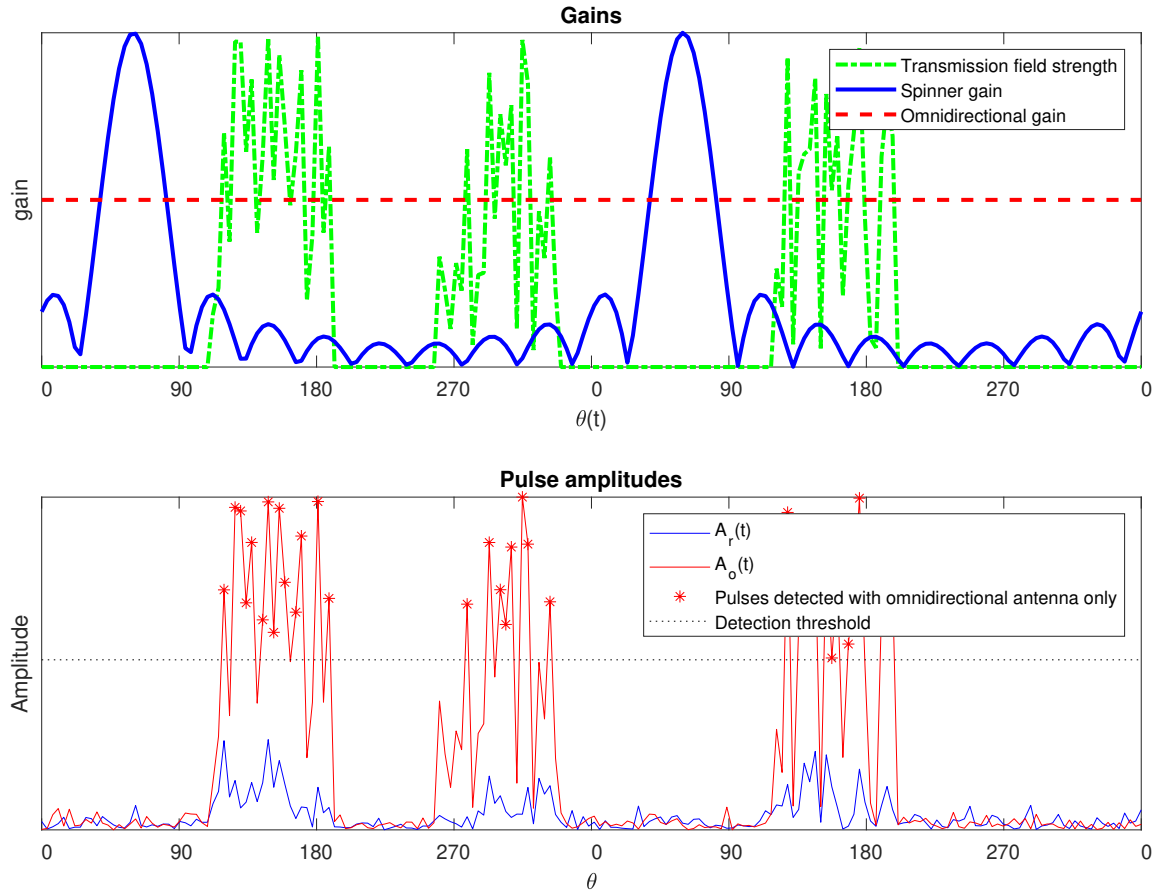
Figure 4: Gain, signal strength and amplitudes of in the second example

The second example shows how the proposed method can be used to determine direction even without any pulses detected with the spinning DF antenna. Algorithms in literature [2, 3, 4, 5] do not work as there are no pulses received with spinning DF antenna and with omnidirectional antenna simultaneously. This example also shows how the method is independent of the transmitter pattern. Figure 4 shows the gain and amplitudes. This time radar emits pulses with random amplitudes in blocks. These blocks are coming at times when the receiver antenna is directed away from the radar so all received pulses are received with the omnidirectional antenna, which contain no directional information in itself. However, as the pulses must have been emitted when the spinning DF antenna is pointing away from the radar, we still get a likelihood shown in Figure 5. The radar was simulated to direction $60°$ and the estimated direction is $45°$ with standard deviation $18°$.

In our third example, we show how the number of received samples affect to the direction-finding accuracy. The simulated situation provides a lot of information for DoA finding task. The situation is simulated 1000 times with varying the number of pulses and the exact time when the mainlobe of a slowly scanning radar is directed approximately towards the receiver. Figure 6 shows an example of the situation.
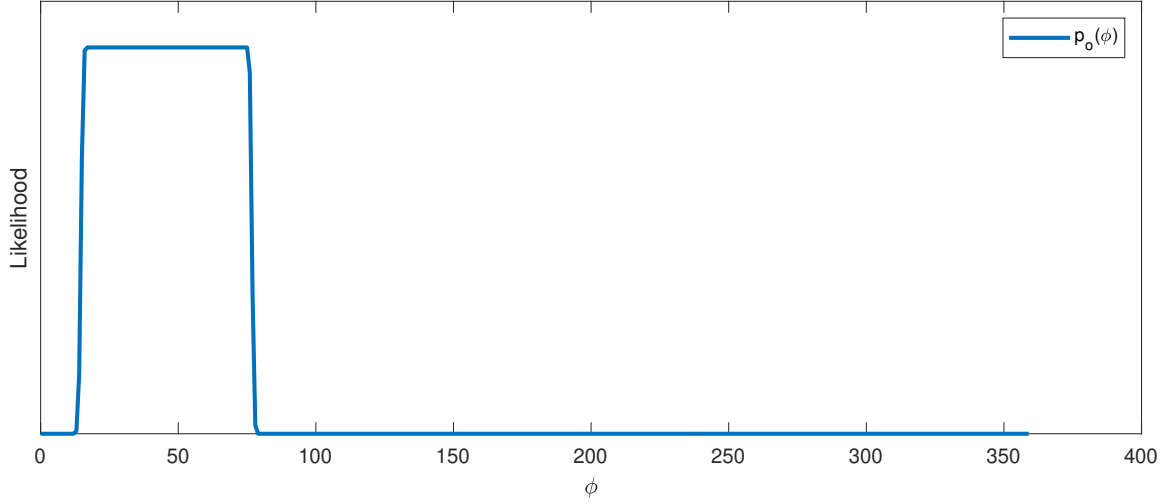
**Figure 5:** Likelihood of DoA in the second example

The average DoA estimation error as the function of total pulses is shown in Figure 7. In this test, we can see that the estimation accuracy using all information is better than using only pulses that are received with both antennas when number of pulses is less than 13 and with 7 pulses the average DoA error with all information is less than 1 degree, while the error with using only information when both antennas receive the pulse is more than 8 degrees. In these results the DoA estimation with omnidirectional antenna only is more accurate than the estimation with spinning antenna only. This can be explained looking to Fig. 6 where one can see that there are only a few pulses received with the directional antenna only compared to the pulses received with the omnidirectional antenna.

6. Conclusions and future work

In this paper, we studied how to use all available information from a spinning DF antenna and an omnidirectional antenna to determine the DoA of a set of pulses. We showed in examples that the proposed method can determine the DoA of a signal source and the use of all information. The use of all information gives estimates better accuracy and more importantly helps to determine estimate DoA in difficult situations where a traditional DoA estimation methods would not have provided any information about the DoA of the radar pulse.

In this paper, all data was simulated. In future, the algorithm should be tested with real data and radiation patterns of antennas. The algorithm could also be extended to be able to take into account imperfections in the radiation pattern of the real-world omnidirectional antennas, that is, to use a radiation pattern of the omnidirectional antenna instead of a constant g_o. Furthermore, the use of radiation patterns could be extended to take into account the elevation angle and the algorithm could incorporate the complex amplitude instead of the absolute amplitude.
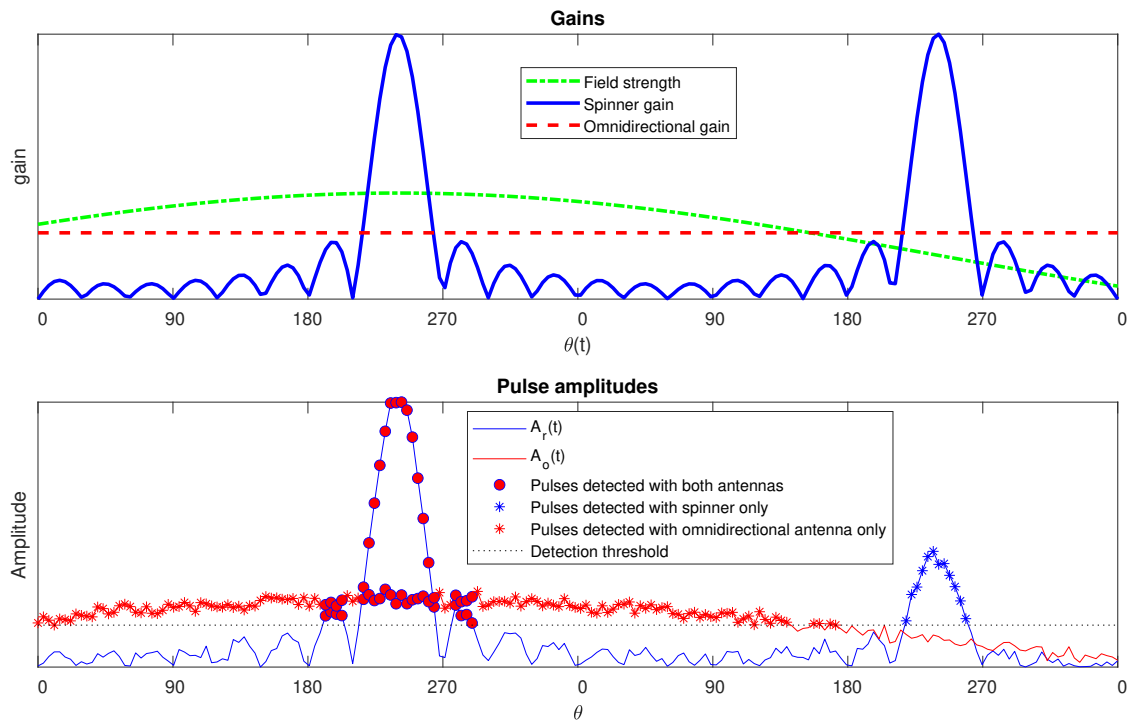
Figure 6: Gain, signal strength and amplitudes of one simulation in the third example
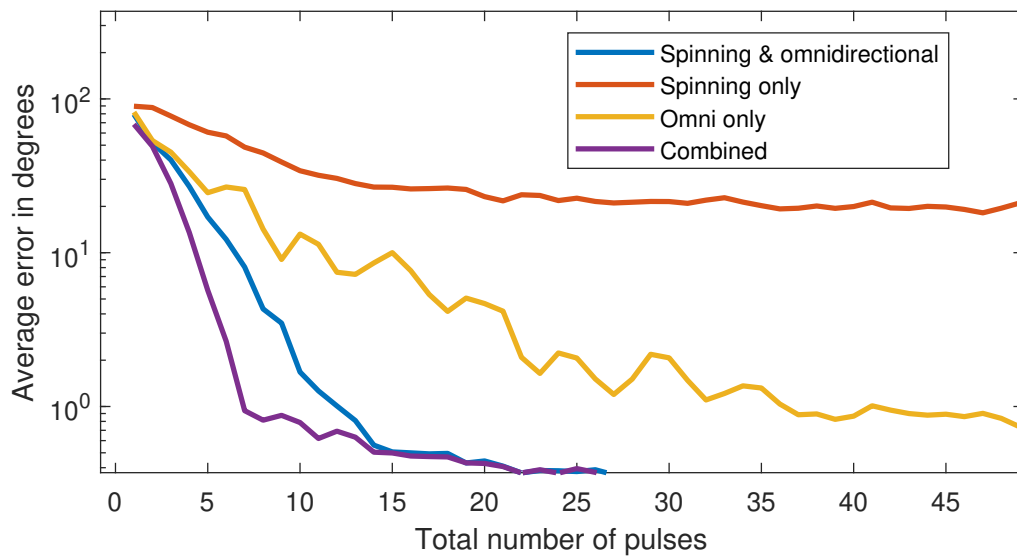
Figure 7: Average angle estimation error as function of number of pulses

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