Mathematical Modeling and Processing of High Resolution Rhythmocardio Signal Based on a Vector of Stationary and Stationary Related Random Sequences

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Abstract.
Work is about substantiation of mathematical model of high resolution rhythmocardio signal in the form of a vector of stationary and stationary related random sequences. Investigated the structure of probabilistic characteristics of this model for analysis of cardiac rhythm in modern cardiodiagnostic system. Based on a new mathematical model of vector rhythmocardiosignals,was developed methods for statistical evaluation of their spectral-correlation characteristics, which are used as diagnostic features in automated diagnostic systems for functional diagnostics of the heart condition and adaptive regulatory mechanisms of the human body
Keywords: mathematical model, cardiac rhythm, vector of stationary and stationary related random sequences.

1. Introduction

Analysis of the heart rhythm makes it possible to evaluate not only the state of the cardiovascular system, but also the state of the adaptive capacity of the whole human body. Most modern systems for automated cardiac rhythm analysis are based on statistical analysis of rhythmocardio signal, which is an ordered set of durations of R-R intervals in a registered electrocardiogram [1-8]. However, this approach is uninformative, because the R-R intervals reflect only the change in the duration of the cardiac cycles and not the totality of the time intervals between single-phase values of the electrocardio signal for all its phases.

In [9, 10], a new approach to the analysis of cardiac rhythm on the basis of high resolution rhythmocardio signal was developed. As noted in these works, the classical rhythmocardio signal is embedded in the high resolution rhythmocardio signal, which is the basis for increasing the level of informativeness of the analysis of cardiac rhythm in modern computer systems of functional diagnostics of the human heart state.

In [9, 10], it is justified to use a vector of random variables as a mathematical model of high resolution rhythmocardio signal. But, this model is a relatively bad mathematical model of high resolution rhythmocardio signal, since it does not allow to study its temporal dynamics. To take into account the temporal dynamics of the high resolution rhythmocardio signal, it is necessary to use a mathematical apparatus of the theory of random sequences, namely, to consider it as a vector of random sequences.

1.1. Setting objectives

In this work, we will develop a mathematical model of high resolution rhythmocardio signal in the

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form of a vector of stationary and stationary related random sequences. Let's write down the structure of probabilistic characteristics of this model for the analysis of cardiac rhythm in modern cardiac diagnostics systems.

Based on this model, we develop methods for statistical analysis of the rhythmocardiosignal with increased resolution in the framework of the spectral correlation theory of random processes

2. Research results

In the most general form stochastic models that takes into account the dynamics of changes high resolution rhythmocardio signal is vector $\Xi_L(\omega', m) = \left\{ T_l(\omega', m), \omega' \in \Omega', l = \overline{1,L}, m \in Z \right\}$ random sequences. In this vector, the index m indicates the cycle number of the electrocardio signal, and the index l indicates the reference number of the electrocardio signal within its cycle. The number L of intervals per cycle of the electrocardio signal determines the resolution of the rhythmocardio signal and sets the number of phases in the cycle of the electrocardio signal that can be separated by methods of segmentation and detection in solving the problem of automatic formation of the rhythm cardio signal from the electrocardio signal.

Justify of probabilistic characteristics of the vector $\Xi_L(\omega', m)$ random sequences. One of the simplest stochastic models that takes into account the dynamics of high resolution of rhythmocardio signal is the vector $\Xi_L(\omega', m) = \left\{ T_l(\omega', m), \omega' \in \Omega', l = \overline{1,L}, m \in Z \right\}$ stationary and stationary related random sequences. First of all, note that the vector $\Xi_L(\omega', m)$ stationary and stationary related random sequences, in the particular case, if its components are stationary sequences with independent values, that is, white noises given on the set of integers, is a known model of high resolution rhythmocardio signal in the form of a vector of random variables, which was developed in [9, 10]. But in practice, the hypothesis of the independence or non-correlation of the time intervals between single-phase values of the electrocardio signal is not true, requiring a stochastic dependence between the rhytocardiogram intervals with higher resolution, and hence the use of a more complex and more general mathematical model as a vector $\Xi_L(\omega', m)$ stationary and stationary related random sequences.

The defining property of a vector $\Xi_L(\omega', m)$ of stationary and stationary related random sequences is the invariance of its family of distribution functions to time shifts by an arbitrary integer $k \in Z$. For any distribution function $F_{p_{T_{l_1}...T_{l_p}}}(x_1,...,x_p, m_1,...,m_p)$ order p ($p \in N$) from the family of vector $\Xi_L(\omega', m)$ distribution functions of stationary and stationary related random sequences there must be such equality:

$$F_{p_{T_{l_1}...T_{l_p}}}(x_1,...,x_p, m_1,...,m_p) = F_{p_{T_{l_1}...T_{l_p}}}(x_1,...,x_p, m_1+k,...,m_p+k),$$

$$x_1,...,x_p \in R, m_1,...,m_p \in Z, l_1,...,l_p \in \overline{1,L}, k \in Z. \quad (1)$$

Distribution function $F_{p_{T_{l_1}...T_{l_p}}}(x_1,...,x_p, m_1,...,m_p)$ in the case, when $l_1 = l_2 = ... = l_p = l$ is a distribution function $F_{p_{T_l}}(x_1,...,x_p, m_1,...,m_p)$ l- stationary components $T_l(\omega', m)$ of vector $\Xi_L(\omega', m)$ - so it must to be p-dimensional auto-function of distribution for stationary random sequence $T_l(\omega', m)$, that describing the time distances between single-phase electrocardiogram for it l-phase. So, if $p = 1$, then we will have one-dimensional $F_{1_{T_l}}(x, m)$ auto-function distribution of stationary random sequence $T_l(\omega', m)$.

In the case where equality $l_1 = l_2 = ... = l_p = l$ is not executed then the distribution function $F_{p_{T_{l_1}...T_{l_p}}}(x_1,..,x_p,m_1,..,m_p)$ is a p-dimensional compatible distribution function for several (at least two) stationary components of a vector $\Xi_L(\omega',m)$, what describing the time distances between single-phase intervals of an electrocardio signal generally for its various phases.

The distribution functions family of vector $\Xi_L(\omega',m)$ of stationary and stationary related random sequences most fully describes its probabilistic structure, however, methods for statistically estimating the distribution function $F_{p_{T_{l_1}...T_{l_p}}}(x_1,..,x_p,m_1,..,m_p)$ have too high computational complexity for their practical use in the computer diagnostic systems of the functional state of the cardiovascular system of the human body. We can use not just vector distribution functions $\Xi_L(\omega',m)$ but we can use momentary functions by $s=\sum_{j=1}^{p}s_j$ order, which, if they are also invariant to time offsets (offsets by argument m).

So, if there is a mixed initial momentary function $c_{s_{T_{l_1}...T_{l_p}}}(m_1,..,m_p)$ order $s=\sum_{j=1}^{p}s_j$ vector`s $\Xi_L(\omega',m)$ stationary and stationary related random sequences, then it has equality:

$$c_{s_{T_{l_1}...T_{l_p}}}(m_1,..,m_p)=M\left\{T_{l_1}^{s_1}(\omega',m_1)...\cdot T_{l_p}^{s_p}(\omega',m_p)\right\}=c_{s_{T_{l_1}...T_{l_p}}}(m_1+k,..,m_p+k),$$

$$m_1,..,m_p\in Z, l_1,..,l_p\in\{\overline{1,L}\}, k\in Z. \qquad (2)$$

If there is a mixed central momentary function $r_{s_{T_{l_1}...T_{l_p}}}(m_1,..,m_p)$ order $s=\sum_{j=1}^{p}s_j$ vector`s $\Xi_L(\omega',m)$ stationary and stationary related random sequences, then it has equality:

$$r_{s_{T_{l_1}...T_{l_p}}}(m_1,..,m_p)=M\left\{\left(T_{l_1}(\omega',m_1)-c_{1_{T_{l_1}}}\right)^{s_1}...\cdot\left(T_{l_p}(\omega',m_p)-c_{1_{T_{l_p}}}\right)^{s_p}\right\}=$$

$$=r_{s_{T_{l_1}...T_{l_p}}}(m_1+k,..,m_p+k),$$

$$m_1,..,m_p\in Z, l_1,..,l_p\in\{\overline{1,L}\}, k\in Z. \qquad (3)$$

Where $\left\{c_{1_{T_{l_1}}},..,c_{1_{T_{l_p}}}\right\}$ is the set of first-order initial moments (mathematical expectations) of stationary random sequences from the set $\{T_{l_1}(\omega',m),..,T_{l_p}(\omega',m)\}$.

In practice, for analysis of high resolution rhythmocardio signal, it is reasonable to use mixed high-order momentary functions, namely, mixed second-order initial momentary functions - covariance functions and mixed second-order central momentary functions - correlation functions. In this case, the initial second-order momentary functions for the vector $\Xi_L(\omega',m)$ stationary and stationary related random sequences are presented as a matrix of covariance functions:

$$C_T = \begin{bmatrix} c_{2_{T_1T_1}}(m_1,m_2) & c_{2_{T_1T_2}}(m_1,m_2) & \cdots & c_{2_{T_1T_p}}(m_1,m_2) \\ c_{2_{T_2T_1}}(m_1,m_2) & c_{2_{T_2T_2}}(m_1,m_2) & \cdots & c_{2_{T_2T_p}}(m_1,m_2) \\ \vdots & \vdots & \cdots & \vdots \\ c_{2_{T_pT_1}}(m_1,m_2) & c_{2_{T_pT_2}}(m_1,m_2) & \cdots & c_{2_{T_pT_p}}(m_1,m_2) \end{bmatrix}, \quad (4)$$

which can be presented more compactly as:

$$C_T = \left[c_{2_{T_{l_1}T_{l_2}}}(m_1,m_2), l_1, l_2 = \overline{1 \cdot L} \right], \quad (5)$$

where each of its elements is a covariance function $c_{s_{T_{l_1}T_{l_2}}}(m_1,m_2)$, which is given as:

$$c_{2_{T_{l_1}T_{l_2}}}(m_1,m_2) = M\left\{ T_{l_1}(\omega', m_1) T_{l_2}(\omega', m_2) \right\}, m_1, m_2 Z, l_1, l_2 \left\{ \overline{1 \cdot L} \right\}. \quad (6)$$

Vector`s components $\Xi_L(\omega', m)$ random sequences are stationary and stationary related sequences, then their covariance functions are functions of only one integer argument u, which is equal to $u = m_1 - m_2$. Therefore, the covariance matrix of this random vector can be represented as follows:

$$C_T = \left[c_{2_{T_{l_1}T_{l_2}}}(u), l_1, l_2 = \overline{1 \cdot L} \right], \quad (7)$$

where each of its elements is a covariance function $c_{2_{T_{l_1}T_{l_2}}}(u)$, which is equal to:

$$c_{2_{T_{l_1}T_{l_2}}}(u) = c_{2_{T_{l_1}T_{l_2}}}(m_1 - m_2), u, m_1, m_2 Z, l_1, l_2 \left\{ \overline{1 \cdot L} \right\}. \quad (8)$$

Provided that $l_1 = l_2 = l$, the covariance function $c_{s_{T_l T_l}}(u)$ is an auto-covariance function l-stationary components $T_l(\omega', m)$ of vector $\Xi_L(\omega', m)$, which describes the time distances between single-phase intervals of electrocardiogram for l-phase. If $l_1 \neq l_2$, that means the covariance function $c_{2_{T_{l_1}T_{l_2}}}(u)$ is the mutual covariance function for two stationary components of a vector $\Xi_L(\omega', m)$, they describe the time distances between single-phase intervals of electrocardiogram l_1 and l_2-phase.

Mixed central second-order momentary functions for a vector $\Xi_L(\omega', m)$ stationary and stationary related random sequences are presented as a matrix of correlation functions:

$$R_T = \begin{bmatrix} r_{2_{T_1T_1}}(m_1,m_2) & r_{2_{T_1T_2}}(m_1,m_2) & \cdots & r_{2_{T_1T_p}}(m_1,m_2) \\ r_{2_{T_2T_1}}(m_1,m_2) & r_{2_{T_2T_2}}(m_1,m_2) & \cdots & r_{2_{T_2T_p}}(m_1,m_2) \\ \vdots & \vdots & \cdots & \vdots \\ r_{2_{T_pT_1}}(m_1,m_2) & r_{2_{T_pT_2}}(m_1,m_2) & \cdots & r_{2_{T_pT_p}}(m_1,m_2) \end{bmatrix}, \quad (9)$$

which can be presented more compactly as:

$$R_T = \left[r_{2_{T_{l_1} T_{l_2}}} (m_1, m_2), l_1, l_2 = \overline{1,L} \right], \tag{10}$$

where each of its elements is a correlation function $r_{s_{T_{l_1} T_{l_2}}} (m_1, m_2)$, which is given as:

$$c_{2_{T_{l_1} T_{l_2}}} (m_1, m_2) = M\left\{ \left(T_{l_1}(\omega', m_1) - c_{1_{T_{l_1}}} \right) \left(T_{l_2}(\omega', m_2) - c_{1_{T_{l_2}}} \right) \right\}, \tag{11}$$

$$m_1, m_2 \in Z, l_1, l_2 \in \left\{ \overline{1,L} \right\}$$

The components of the vector $\Xi_L(\omega', m)$ random sequences are stationary and stationary related sequences, their correlation functions are functions of only one integer argument u, which is equal to $u = m_1 - m_2$. This correlation matrix of this random vector can be represented as:

$$R_T = \left[r_{2_{T_{l_1} T_{l_2}}} (u), l_1, l_2 = \overline{1,L} \right], \tag{12}$$

where each of its elements is a correlation function $r_{2_{T_{l_1} T_{l_2}}} (u)$, which is equal to:

$$r_{2_{T_{l_1} T_{l_2}}} (u) = r_{2_{T_{l_1} T_{l_2}}} (m_1 - m_2), u, m_1, m_2 Z, l_1, l_2 \left\{ \overline{1,L} \right\}. \tag{13}$$

Provided that $l_1 = l_2 = l$, correlation function $r_{2_{T_l T_l}} (u)$ is auto-correlation function l-stationary components $T_l(\omega', m)$ of vector $\Xi_L(\omega', m)$, which describes the time distances between single-phase intervals of electrocardiogram for l-phase. If $l_1 \neq l_2$, then the correlation function $r_{2_{T_{l_1} T_{l_2}}} (u)$ is a mutual correlation function for two stationary components of a vector $\Xi_L(\omega', m)$, description of time distances between single-phase intervals of electrocardiogram l_1 and l_2 -phase.

Figures 1-4 show the results of statistical processing of the high resolution rhythmocardio signal, by statistical evaluation of its corresponding probability characteristics.

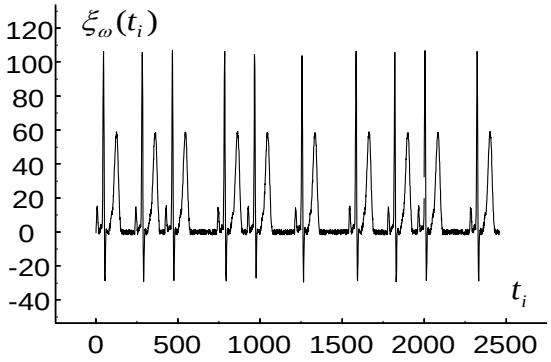

Figure 1: Several cycles of the investigated electrocardio signal

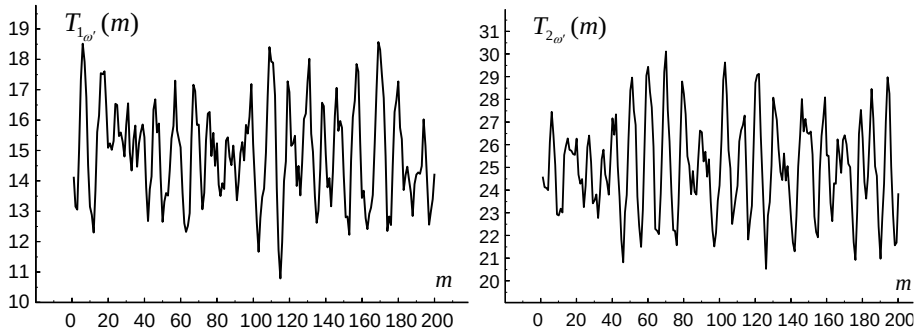

Figure 2: Schedule of $T_{1_{\omega'}}(m)$, $T_{2_{\omega'}}(m)$ realizations of the first component $T_1(\omega', m)$ and second component $T_2(\omega', m)$of the vector rhythmocardiogram, that describes duration: a) P-intervals of electrocardio signal; b) R - intervals of electrocardio signal

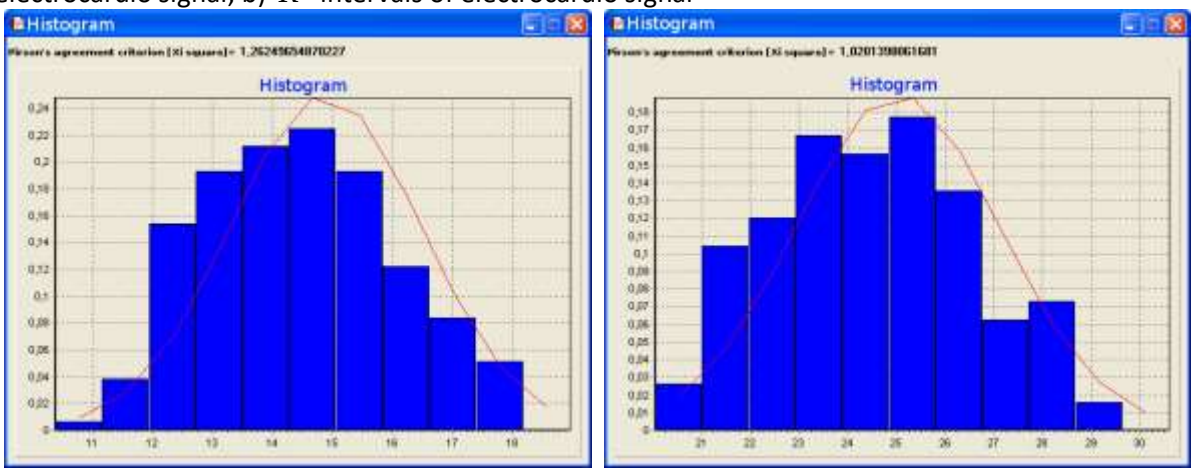

Figure 3: Histograms of $T_{1_{\omega'}}(m)$, $T_{2_{\omega'}}(m)$ realizations of the first component $T_1(\omega', m)$ and second component $T_2(\omega', m)$, of the vector rhythmocardiogram describing the duration accordingly: a) P - intervals of electrocardio signal; b) R - intervals of electrocardio signal

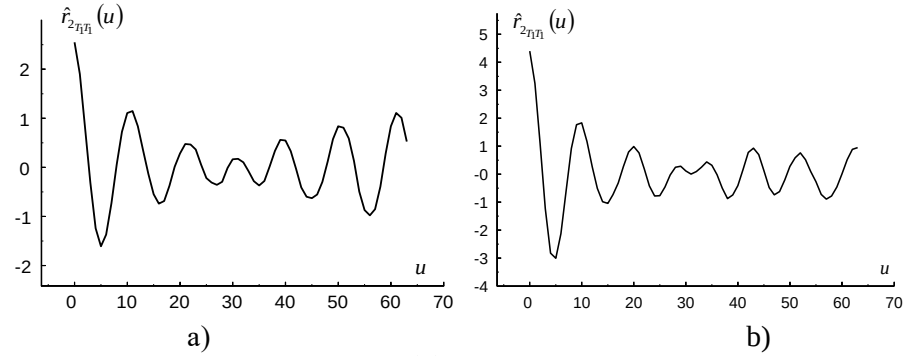

Figure 4: Schedule of implementation $\hat{r}_{2_{T_1T_1}}(u)$ statistical estimates of autocorrelation functions $r_{2_{T_1T_1}}(u)$ ($l_1 = l_2 = 1$) first component $T_1(\omega', m)$ and second component $T_2(\omega', m)$, what describing the duration accordingly: a) P - intervals of electrocardio signal; b) R - intervals of electrocardio signal.

3. Conclusions

The mathematical model of high resolution rhythmocardio signal in the form of a vector of stationary and stationary related random sequences is substantiated. The structure of probabilistic characteristics of this model for analysis of cardiac rhythm in modern cardiodiagnostic systems is investigated. Unlike the existing model of high resolution rhythmocardio signal in the form of a vector of random variables, new model take into account the temporal dynamics of the high resolution rhythmocardio signal, which is the basis for increasing the level of informativeness of the analysis of cardiac rhythm in modern computer systems of functional diagnostics. Based on a new mathematical model of a rhythmocardiosignal with increased resolution, a statistical estimation of its probabilistic characteristics is carried out within the framework of the spectral correlation theory of random processes.

In the future scientific researches it is planned to justify the choice of the minimum number of diagnostic features necessary for carrying out the diagnosis in the analysis of heart rhythm on the basis of the obtained statistical estimates.

4. References

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