Mathematical Modeling Covid-19 Wave Structure of Distribution

Kateryna Molodetska[a] and Yuriy Tymonin[a]

[a] *Polissia National University, Staryi Blvd., 7, Zhytomyr, 10008, Ukraine*

Abstract

For mathematical modeling of the spread of the Covid-19 epidemic, a wave structure, which represents a complex flow of epidemic events in the form of a set of simple epidemic flows (epidemic waves) is considered. To represent the wave structure we need to decompose a complex flow of epidemic events, given by statistics, into elementary flows. Mathematical models of the wave structure of the epidemic are represented by a set of elementary epidemic flows (waves) shifted along the time axis and different applications in the values of the parameters. Application of Covid-19 propagation waves allows not only to describe the basic concepts of the epidemic quantitatively but also build a reliable forecast of the spread of the epidemic. An important consequence of the Covid-19 wave pattern is the possibility of conducting a comparative parametric analysis of specific wave patterns of epidemic spread. Based on the results of the analysis we can assess the results of the epidemic control. The calculations of wave structures have been made for two European countries – Ukraine, Italy as well as the world leaders in the distribution of Covid-19 – the United States, Brazil and Russia.

Keywords 1

Covid-19 propagation waves; mathematical modeling; data approximation; parametric analysis; epidemic waves

1. Introduction

Studying of the mechanisms of the spread of epidemics is an important way to control diseases, along with finding new drugs, vaccination and preventive measures [1-5]. The reduction in damage from the coronavirus epidemic is connected with the use of methods and tools of mathematical modeling of the spread of Covid-19. Modeling allows making quantitative calculations, comparative analysis, and forecasting of temporary descriptions of major categories of the epidemic, such as the number of people who have become ill, recovered, and died [6-9]. Covid-19 distribution models are required to be adequate to the descriptions of the basic concepts. They also must comply with statistical data.

The SIR model developed by A. Kermak and W. McKendrick in 1927-1933 is widely used to describe the epidemic [10]. The SIR model is based on a scheme of the epidemic transition of the number of individuals from one category to another: susceptible (S) become infected (I), then they recover (R). The SIR model is represented by a system of first-order differential equations that describe the time dependences of variables that reflect basic concepts [11].

The class of SIR models that implements the concept of epidemic transition has gained wide popularity, development and in addition to the SIR model also contains its variations: SIRS, SEIR, SIS, MSEIR [11]. The experience of applying SIR class models to mathematical modeling of Covid-19 spread [11-13] has shown incomplete correspondence of calculations of basic variables to statistical data.

IDDM'2020: 3rd International Conference on Informatics & Data-Driven Medicine, November 19–21, 2020, Växjö, Sweden

EMAIL: kmolodetska@gmail.com (K. Molodetska); YTimonin45@gmail.com (Y. Tymonin)
ORCID: 0000-0001-9864-2463 (K. Molodetska); 0000-0002-0179-5226 (Y. Tymonin)

CEUR Workshop Proceedings (CEUR-WS.org)

It was proved in the analytical review [14] that any predictions based on the models of the SIR class and its derivatives cannot be considered correct, and certain coincidences of the predicted data can be random.

Insufficient accuracy of SIR class models necessitates new approaches to mathematical modeling of the spread of Covid-19. The nature of the statistical data on the spread of the Covid-19 coronavirus epidemic shows that they have a high similarity to the wave process.

1. In [15] it is noted that the representative of the WHO European Bureau Oleg Storozhenko claimed that the world is facing the second wave of coronavirus infection. At the same time, in some countries, the third wave has begun.
2. To characterize the spread of Covid-19, highly visual and informative representations in the form of epidemic waves are used. However, this "wave" representation places increased demands on the Covid-19 propagation models.

The spread of Covid-19 is a complex regular stream of epidemic events that reflect the number of events happening at each subsequent moment – a day. Cases of infection, recovery, and deaths are considered as epidemic events. Primary statistical information about regular streams of events is given in the form of time series.

Epidemic streams have an important distinctive feature, meaning that the stream starts at zero before the outbreak of the epidemic begins and ends at zero after the outbreak ends. This feature gives a convex character the descriptions of epidemic flows.

The real stream of epidemic events can be defined as a complex, compositional stream made up of some elementary streams. The complexity of the epidemic flow prevents both analysis and projections.

Analytical work requires determining the wave structure of a complex flow, i.e. it is composed of elementary streams. Therefore, there is an urgent problem, which consists in decomposing a complex stream of epidemic events into elementary streams. However, the solution of these problems comes up against the problems of mathematical modeling of simple and complex epidemic flows and their interaction.

The study aims to develop new mathematical models of simple and complex epidemic flows, which will enable formalization of the processes of Covid-19 spread, as well as the exploration of these mathematical models in order to develop and implement effective anti-epidemic measures. To achieve this aim, such tasks must be completed:

- analyse possible ways of mathematical formalisation of epidemiological events;
- develop a model of elementary epidemiological flow;
- propose an approach to formalising a complex epidemiological flow based on simple flows;
- conduct computational experiments with the proposed models based on statistical data from different countries.

2. Models and methods

2.1. Approximation problem solving method

Let's consider a problem that aims at decomposing a complex stream of epidemic events into elementary ones. Such tasks are solved according to the following stages:

- implementation of the approximation of time series, the result of which should be an analytical model of a complex flow;
- defining an analytical model of a simple, elementary stream;
- solving the problem of decomposing a complex epidemic stream into elementary streams.

Since the construction of a complex flow model by approximating time series is very problematic, this task must be solved according to the "bottom-top" scheme, by defining the models of the elementary flow with their subsequent superposition.

The analytical model of a simple flow is a continuous convex function, bell-shaped in the foreseeable time interval, i.e. the function starts and ends with a null value. Besides, we introduce an additional requirement that expands the properties of the model and consists of the asymmetry of the bell shape. Asymmetry is manifested in the fact that the growth and decay of the function have different rates. This requirement expands the capabilities of flow modeling, but excludes the use of well-known symmetric bell-shaped functions, for example, probability distribution functions. Therefore, there arises the problem of constructing a continuous, asymmetrically bell-shaped function and using it as a model of an elementary epidemic flow.

2.2. Construction of the epidemic elementary flow model

As a mathematical basis for the model of an elementary epidemic flow, we use the functions of limited growth, which have proven their effectiveness in the models of conflict interaction [16-18]. Assuming that these models can be used to describe different epidemic events, we will restrict ourselves to cases of infection.

Then, the number of infected individuals $X(t)$ can be set by the function of limited growth, as a solution to a nonlinear differential equation of the 2nd order [18]:

$$a_2 X(t)\frac{d^2X(t)}{dt^2} + \left(1 + a_1 X(t)\right)\frac{dX(t)}{dt} + \left(a_0 X(t) - \varphi\right)\vartheta X^{\theta}(t) = 0, \qquad (1)$$

where φ is the growth rate; $\{a_0; a_1; a_2\}$ – phenomenological coefficients, which are considered as parameters of the epidemic: a_0 – an indicator of the number of susceptible to infection; a_1 – coefficient of flow asymmetry; a_2 – a level of susceptibility of population to the virus; ϑ, θ – multiplication coefficients.

To represent the model of an elementary flow, we restrict ourselves to nonlinear differential equations of the 1st order for $a_2 \approx 0$ and $\theta = 1$, $\vartheta = 1$. Assuming that the epidemic flow is described by the rate of infection, we present the expression for the flow using the derivative from (1)

$$x(t) = \frac{dX(t)}{dt} = \frac{\varphi - a_0 X(t)}{1 + a_1 X(t)} X(t), \qquad (2)$$

where phenomenological coefficients have the following meaning: a_1 – coefficient of flow asymmetry; a_0 – an indicator of the number of susceptible to infection.

Equation (2) can be considered as a generalized representation of the logistic Verhulst equation, to which it is reduced at $a_1 = 0$. Discrete functions of the number of infected people are described by the expression [18]

$$X_{k+1} = \left(1 + \frac{\varphi - a_0 X_k}{1 + a_1 X_k}\right) X_k. \qquad (3)$$

Following (2), discrete flow functions can be determined through the increment in the number of infected $x_{k+1} = X_{k+1} - X_k$.

We can say that the functions of the elementary epidemic flow describe a simple epidemic wave. In Figure 1 shows asymmetric epidemic waves represented by the functions of elementary flows and calculated according to expression (3). Calculations were carried out for one value of growth indicators $\varphi = 0{,}2$ and different values of the phenomenological coefficient $a_1 = \{0; 0{,}2; 0{,}5; 1; 2\}$. In a particular case, when $a_1 = 0$, i.e. for the logistic model, the wave degenerates into a symmetric one.

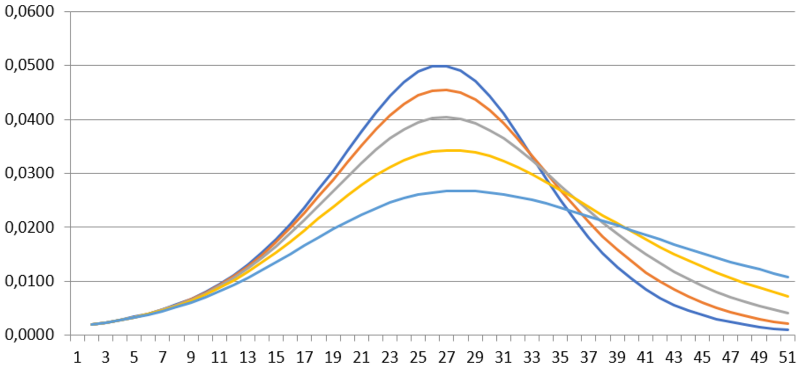

Figure 1: Graphs of bell-shaped epidemic waves with pronounced asymmetry

2.3. Construction of a complex epidemic stream model

To build a complex flow model by superposition of elementary flow models, it is required to determine the permissible operations that can be performed with simple flow models. The algebra of simple epidemic flows contains the following additive algebraic operations:

- addition of a stream with a constant (shift of the function along the value axis)
  $$x(t) = f(t, \varphi, a_0, a_1, x(0)),\ y(t) = f(t, \varphi, a_0, a_1, x(0)) + c,$$
- multiplying the flow by a constant (scaling)
  $$x(t) = f(t, \varphi, a_0, a_1, x(0)),\ y(t) = f(t, \varphi, a_0, a_1, x(0)) \cdot c,$$
- time shift of the flow
  $$x(t) = f(t, \varphi, a_0, a_1, x(0)),\ y(t) = f(T + t, \varphi, a_0, a_1, x(0)).$$

These operations do not change the parameter values. When adding the streams, we get a complex stream $z(t) = x(t) + y(t)$, where $x(t) = f(t, \varphi, a_0, a_1)$ and $y(t) = f(t, \varphi, b_0, b_1)$. When building a model of a complex flow by superposition, all kinds of operations are used.

3. Experiments

Complex flow wave models were formed by the superposition of sequential selection of epidemic waves given by elementary flow models. The calculations of the models of the elementary flow of the wave pattern were carried out using the discrete flow function (3). The approximation error was estimated by the deviation of the complex flow function from the time series data. To estimate the approximation error, the relative indicator *MAPE* was used. The average absolute error calculated by the formula $\delta x = \frac{1}{N}\sum_N \frac{|\overline{x}_k - x_k|}{\overline{x}_k}$, where $\overline{x}_k$ are the values of statistical data.

To calculate the wave models of epidemic flows, two European countries were selected – Italy and Ukraine, as well as the world leaders in the spread of the epidemic – the United States, Brazil and Russia. We should consider that, at the same time, another wave is superimposed on the wave picture of the epidemic, which has its own parameters for different countries. This wave, the last in the considered time interval, mainly determines the forecast of the epidemic development.

The wave structures of the epidemic in different countries have a different wave pattern of the spread of Covid-19. The parametric analysis shows the specific nature of the development of the epidemic in different countries. Parameter values can be associated with preventive and curative measures.

3.1. Wave pattern of Covid-19 spread in Italy

The spread of Covid-19 in the time interval from 1.03 to 31.08 2020 was considered [19]. Figure 2 shows a waveform representation of the spread of Covid-19 in Italy:

- statistical data (curve 1);
- the curve of the general, complex flow (curve 2);
- four curves of elementary streams (epidemic waves, curves 3).

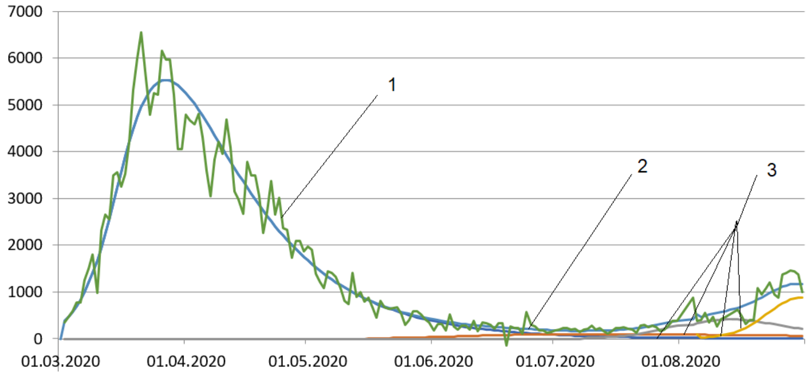

Figure 2: Epidemic waves of Covid-19 spread in Italy

The error in approximating the statistical data of the complex flow curve does not exceed 20%. The large value of the error can be explained by the large scatter of the initial data. If we reduce the spread of data by averaging them, then the error can be reduced to 10%. The figure shows a well-defined bell-shaped epidemic stream of the first wave. Also, the general flow contains a small second wave and a noticeable third wave.

Table 1 shows the parameters of the waves of the general epidemic stream, in which the first wave dominates.

Table 1

Parameters of flows in Italy

Wave parameters	1st wave	2nd wave	3rd wave	4th wave
Growth rate φ	0,236	0,11	0,17	0,25
$10^6 \cdot a_1$	20	5000	80	70
$10^6 \cdot a_0$	0,98	9,2	11	10
Wave peak	5531	99	413	
Wave peak date	27.03.2020	10.07.2020	13.08.2020	

From Table 1, the following differences between the parameters of the third and the first waves can be noticed:

- decrease in the rate of infection φ;
- increase in the coefficient of asymmetry a_1;
- increase in the rate of susceptible to infection a_0;
- decrease in peak values of waves.

In general, the wave character of the general flow indicates a decrease in the level of the epidemic in August-September.

3.2. Wave picture of the spread of Covid-19 in Ukraine

The spread of Covid-19 in the time interval from 1.04 to 10.09 2020 was considered [20, 21]. Figure 3 shows the wave representations of the spread of Covid-19 in Ukraine: the curve of the total, complex flow (curve 1) and 5 curves of elementary flows (epidemic waves, curves 2).

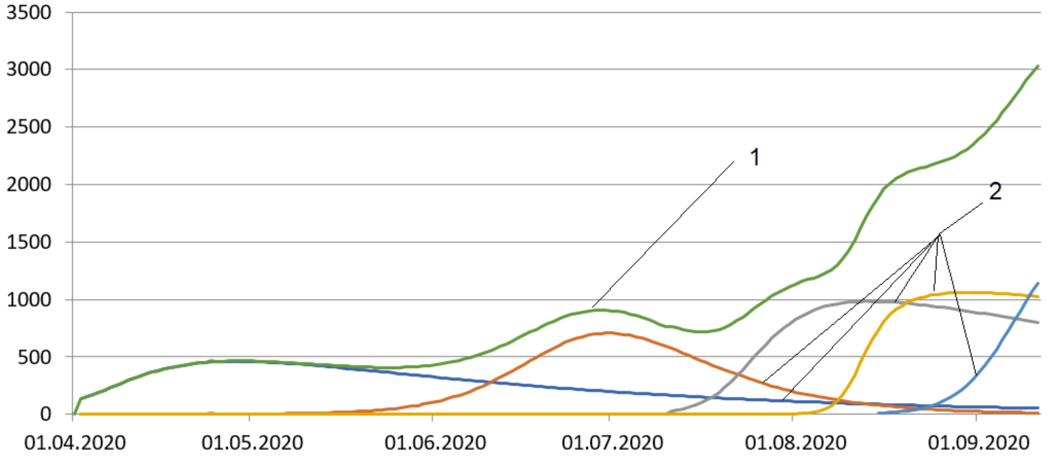

Figure 3: Epidemic waves of the spread of Covid-19 in Ukraine

The total flow contains the sequence of four waves with growing peaks, as a result of which there is an increase in the total flow of the epidemic. Table 2 shows the parameters of the four waves of the total flow.

Table 2

Flow parameters in Ukraine

Wave	1st wave	2nd wave	3rd wave	4th wave	5th wave

parameters					
Growth rate φ	0,2	0,13	0,3	0,55	0,26
$10^6 \cdot a_1$	230	30	200	500	600
$10^6 \cdot a_0$	4,7	4,2	2,7	36,7	130
Wave peak	462	706	982	938	
Wave peak date	29.04.2020	30.06.2020	13.08.2020	19.08.2020	

Table 2 demonstrates the following differences in wave parameters:

- increase in the rate of infection φ;
- close values of the asymmetry coefficient a_1;
- increase in the rate of susceptible to infection a_0;
- increase in peak values of waves.

In general, the wave nature of the general flow indicates an increase in the epidemic.

3.3. Wave pattern of Covid-19 spread in Russia

Russia ranks third in the world ranking of the spread of Covid-19 [22]. The spread of Covid-19 in the time interval from 1.04 to 31.08 2020 was considered. In Figure 4 shows the wave representations of the spread of Covid-19 in Russia, including:

- total flow curve (curve 1);
- three curves of elementary streams (curve 2).

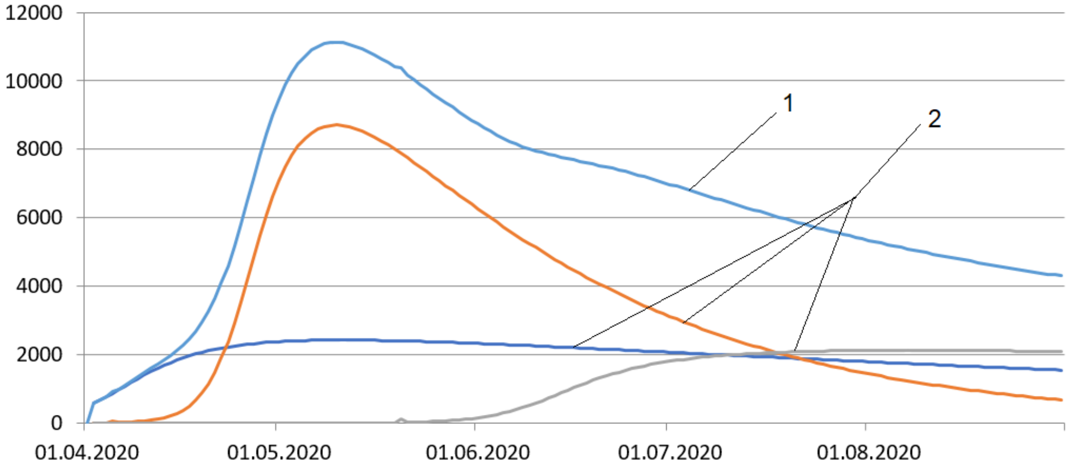

Figure 4: Epidemic waves of the spread of Covid-19 in Russia

The total stream contains a sequence of three waves, where the first and third waves having similar peaks. The wave picture shows the dominance of the second wave and, in general, there is a decrease in the overall flow of the epidemic.

Table 3 shows the parameters of the waves of the general flow, in which the second wave dominates.

Table 3

Wave parameters in Russia

Wave parameters	1st wave	2nd wave	3rd wave
Growth rate φ	0,36	0,25	0,21
$10^6 \cdot a_1$	24	77	80
$10^6 \cdot a_0$	0,65	0,41	0,2
Wave peak	2422	8708	2121
Wave peak date	30.06.2020	10.05.2020	13.08.2020

From Table 3, the following differences in wave parameters follow:

- decrease in the rate of infection φ;
- increase in the coefficient of asymmetry a_1;
- increase in the coefficient a_0;

- decrease in peak values of waves.

In general, the wave character of the general flow indicates a decrease in the epidemic.

3.4. Wave Pattern of Covid-19 spread in Brazil

Brazil ranks third in the global ranking of Covid-19 spread [23]. The spread of Covid-19 in the time interval from 1.05 to 31.08 2020 was considered. Figure 5 shows the wave representations of the spread of Covid-19 in Brazil:

- the curve of the total flow (curve 1)
- 8 curves of elementary flows (curves 2).

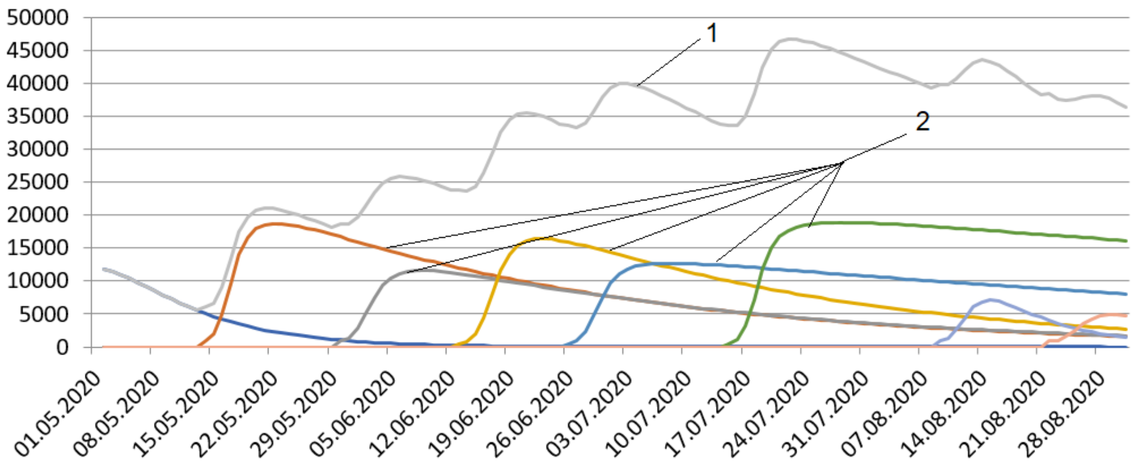

Figure 5: Epidemic waves of the spread of Covid-19 in Brazil

The total stream contains a sequence of three waves, with the first and second waves having similar peaks. The wave picture is dominated by the second wave and, in general, there is a decrease in the overall flow of the epidemic.

Table 4 shows the parameters of the waves of the total flow. A large number of epidemic waves and their similarity should be noted.

Table 4

Parameters of waves in Brazil

Wave parameters	1st wave	2nd wave	3rd wave	4th wave	5th wave	6th wave	7th wave	8th wave
Growth rate φ	0,5	2,1	1,5	1,7	1,9	2,3	1,5	1,0
$10^6 \cdot a_1$	16	90	100	80	130	110	120	100
$10^6 \cdot a_0$	2,0	2,3	2,5	2,3	1,3	0,65	15	12,5
Wave peak	11776	18706	11590	16420	12666	938,29	7070	4988
Wave peak date	2.05.20	29.05.20	9.06.20	23.06.20	8.07.20	28.07.20	15.08.20	29.08.20

Table 4 shows the following differences in wave parameters:

- increase in the rate of infection φ;
- close, in general, values of the coefficient of asymmetry a_1;;
- increase in the rate of susceptible to infection a_0;
- increase in peak values of waves.

In general, the wave nature of the general flow indicates a decrease in the epidemic.

3.5. Wave Pattern of COVID-19 Spread in the United States

The data about the spread of Covid-19 in the time interval from 03.15 to 31.08.2020 was considered [24]. According to it, the United States is the world leader in the number of Covid-19 cases. Figure 6 shows the wave representations of the spread of Covid-19 in the United States:

- a total flow curve (curve 1);
- 3 elementary flow curves (curves 2).

The total flow contains a sequence of three waves, with the second wave dominating in the wave picture and, in general, a decrease in the total flow of the epidemic. The first and third waves have similar peaks.

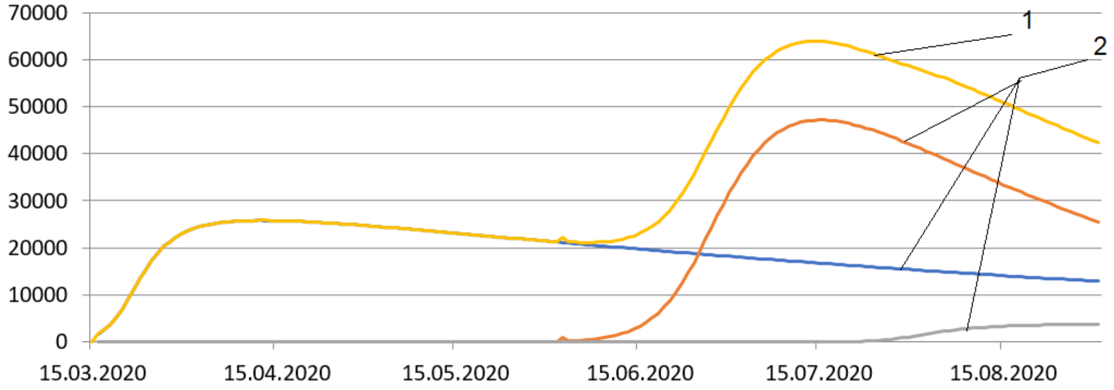

Figure 6: Epidemic waves of the spread of Covid-19 in the United States

Table 5 shows the parameters of the waves of the total flow, in which the first and second waves dominate.

Table 5

Wave parameters in the USA

Wave parameters	1st wave	2nd wave	3rd wave
Growth rate φ	0,54	0,25	0,35
$10^6 \cdot a_1$	17	3	80
$10^6 \cdot a_0$	0,098	0,063	0,03
Wave peak	25838	47140	3853
Wave peak date	13.04.2020	15.07.2020	31.08.2020

Table 5 demonstrates the following differences in wave parameters:

- - decrease in the rate of infection φ;
- - increase in the coefficient of asymmetry a_1;
- - decrease in the coefficient a_0;
- - decrease in peak values of waves.

In general, the wave character of the general flow indicates a decrease in the epidemic.

4. Conclusions

For qualitative characteristics, the spread of Covid-19, the concept of “wave” is used, which has increased visualization. The prevalence of Covid-19 in the world continues to grow, and the next outbreak of the epidemic is considered as a “second wave”. The quantitative wave representation of the epidemic requires formalization of the process of the spread of Covid-19. The epidemic process is a complex stream of epidemic events, such as the cases of infection, recovery, and death.

For the first time in order to analyze the spread of Covid-19, it was proposed to use a wave structure, which represents a complex stream of epidemic events in the form of a set of simple epidemic streams (epidemic waves). To represent the wave structure we had to decompose a complex stream of epidemic events, given by statistical information, into elementary streams.

The derivative of the restricted growth function is used as an analytical model of the elementary epidemic flow. Elementary epidemic stream (epidemic wave) has an asymmetric bell-shaped appearance. Asymmetry reflects the fact that the rate of rise and fall of the wave is different. The asymmetry property fundamentally distinguishes the model of an elementary epidemic flow from the well-known symmetric bell-shaped functions, in particular, those used for the description of the probability distribution.

The wave structure of the epidemic is represented by a set of elementary epidemic flows (waves) shifted along the time axis which differs in the values of the parameters. The wave structure is determined by the sequential selection of waves and the values of their parameters. Summing up the values of the elementary epidemic flows, we can obtain an analytical description of the complex flow of epidemic events. The flow can be considered as a solution to the approximation problem for the given statistical information.

We have also carried out the approximation calculations for two European countries – Ukraine, Italy as well as the world leaders in the spread of Covid-19: the United States, Brazil and Russia. The wave structures of the epidemic in different countries have a different wave pattern of the spread of Covid-19. The parametric analysis shows the specific nature of the development of the epidemic in different countries. Parameter values can be closely connected with preventive and curative measures.

In general, wave models of the epidemic have visibility and enhanced capabilities for analyzing and predicting the spread of Covid-19, which indicates the feasibility of further research in this sphere.

5. References

[1] J. Yang, and Y. Zhang. Epidemic spreading of evolving community structure. Chaos, Solitons & Fractals 140 (2020) 110101. doi: 10.1016/j.chaos.2020.110101.

[2] Z. Xie, et al. Spatial and temporal differentiation of COVID-19 epidemic spread in mainland China and its influencing factors. Science of The Total Environment 744 (2020) 140929. doi: 10.1016/j.scitotenv.2020.140929.

[3] B. Hu, et al. First, second and potential third generation spreads of the COVID-19 epidemic in mainland China: an early exploratory study incorporating location-based service data of mobile devices. International Journal of Infectious Diseases 96 (2020) 489-495. doi: 10.1016/j.ijid.2020.05.048.

[4] J. Chen, M. Hu, and M., Li. Traffic-driven epidemic spreading dynamics with heterogeneous infection rates. Chaos, Solitons & Fractals 132 (2020) 109577. doi: 10.1016/j.chaos.2019.109577.

[5] C. Coll, and E. Sánchez. Epidemic spreading by indirect transmission in a compartmental farm. Applied Mathematics and Computation 386 (2020) 125473. doi: 10.1016/j.amc.2020.125473.

[6] D. Wu, et al. Impact of inter-layer hopping on epidemic spreading in a multilayer network. Communications in Nonlinear Science and Numerical Simulation 90 (2020) 105403. doi: 10.1016/j.cnsns.2020.105403.

[7] H. Huang, Y. Chen, and Y. Ma. Modeling the competitive diffusions of rumor and knowledge and the impacts on epidemic spreading. Applied Mathematics and Computation 388 (2021) 125536. doi: 10.1016/j.amc.2020.125536.

[8] S. Chen, et al. Buying time for an effective epidemic response: The impact of a public holiday for outbreak control on COVID-19 epidemic spread. Engineering (2020) doi: 10.1016/j.eng.2020.07.018.

[9] D. Han, et al. How the individuals' risk aversion affect the epidemic spreading. Applied Mathematics and Computation 369 (2020) 124894. doi: 10.1016/j.amc.2019.124894.

[10] H. Weiss, The SIR model and the Foundations of Public Health. MATerials MATemàtics 2013 (3) (2013) 1–17.

[11] A. Comunian, R. Gaburro, and M. Giudici. Inversion of a SIR-based model: A critical analysis about the application to COVID-19 epidemic. Physica D: Nonlinear Phenomena 413 (2020) 132674. doi: 10.1016/j.physd.2020.132674.

[12] H. Hu, and X. Zou. Traveling waves of a diffusive SIR epidemic model with general nonlinear incidence and infinitely distributed latency but without demography. Nonlinear Analysis: Real World Applications 58 (2021) 103224. doi: 10.1016/j.nonrwa.2020.103224.

[13] I. Cooper, A. Mondal, and C. Antonopoulos. A SIR model assumption for the spread of COVID-19 in different communities. Chaos, Solitons & Fractals 139 (2020) 110057. doi: 10.1016/j.chaos.2020.110057.

[14] Forsite Covid-19: transition to the coronavirus pandemic extinction phase, 2020 (in Ukrainian). URL: http://wdc.org.ua/uk/covid19-attenuation.

[15] The second wave of coronavirus has begun in the world, and some countries are already experiencing a third – WHO, 2020 (in Russian). URL: https://strana.ua/news/283859-v-mire-nachalas-vtoraja-volna-covid-19-voz.html.

[16] K. Molodetska, Y. Tymonin, and I. Melnychuk. The conceptual model of information confrontation of virtual communities in social networking services. International Journal of Electrical and Computer Engineering (IJECE) 10(1) (2020) 1043-1052. doi: 10.11591/ijece.v10i1.pp1043-1052.

[17] K. Molodetska, and Y. Tymonin. System-dynamic models of destructive informational influence in social networking services. International Journal of 3D Printing Technologies and Digital Industry 3(2) (2019) 137-146.

[18] R. Hryshchuk, K. Molodetska, and Y. Tymonin. Modelling of conflict interaction of virtual communities in social networking services on an example of anti-vaccination movement. In: Int. Workshop on Conflict Management in Global Information Networks 2588 (2020) 250-264. URL: http://ceur-ws.org/Vol-2588/paper21.pdf

[19] Distribution COVID-19 in Italy (2020). URL: https://ru.wikipedia.org/wiki/.

[20] Distribution COVID-19 in Ukraine (2020). URL: https://ru.wikipedia.org/wiki/

[21] System for monitoring the spread of the coronavirus epidemic (2020). URL: https://covid19.rnbo.gov.ua/

[22] Distribution COVID-19 in Russia (2020).URL: https://ru.wikipedia.org/wiki/

[23] Distribution COVID-19 in Brasil (2020). URL: https://ru.wikipedia.org/wiki/

[24] Distribution COVID-19 in USA (2020). URL: https://ru.wikipedia.org/wiki/