Mathematical Model Building for COVID-19 Diseases Data in European Countries

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Abstract

The paper deals with the problem of mathematical model building for COVID-19 diseases data. The literature analysis showed that a number of models already exist for these purposes. In this paper, the authors pay attention to the use of regression analysis methods to describe statistical data. For data on new cases of diseases in Ukraine, Poland and Italy, a comparative analysis of the use of regression models based on polynomials of the 5th, 7th and 10th order, mathematical model building in a sliding window, as well as a segmented regression model was carried out. During the use of the segmented regression model, additional optimization of the switching point abscissa was performed. The choice of the best model was performed according to the criterion of the minimum standard deviation. The research results can be used in process of solving the problems of predicting the spread of COVID-19 in the different countries.

Keywords 1

Statistical data processing, regression analysis, model building, COVID-19.

1. Introduction

In December 2019, there was an outbreak of pneumonia in Wuhan 2019 – 2020, as a result of which the COVID-19 virus strain was detected for the first time.

By the first ten days of June 2020, the pandemic had affected 188 countries, more than 7 cases of infection were detected in the world and 411,000 people died.

Now we have more than 37 million cases of infection, more than 1 million people died and more than 28 million people already recovered (Table 1).

The COVID-19 pandemic completely changed our life. It is profoundly affecting how people engage with one another across industries and geographies. Physical distancing and other quarantine measures have shifted activities once considered critical to have in person to digital and remote channels.

Despite the quarantine measures and the fact that the peak of coronavirus cases passed in the world (in accordance with statistical prediction), a decreasing the number of detected cases has not been observed for a sufficiently long period [1].

Ukraine occupies the 25-th position in total country list, but recently there is a significant increasing the COVID-19 total cases and new deaths in our country.

It's very important to build mathematical models for COVID-19 spreading and these models must be adequate and powerful.

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Table 1.
Report coronavirus cases for the top five countries in terms of the number of cases detected on 12 October 2020

#	Country, Other	Total Cases	New Cases	Total Deaths	New Deaths	Total Recovered
1	World	37 679 079	+339 008	1 078 570	+3 965	26 124 315
2	USA	7 792 816	+37 289	214 985	+250	3 075 077
3	India	7 120 538	+66 732	109 150	+816	6 149 535
4	Brazil	5 094 979	+12 342	150 488	+290	4 526 393
5	Russia	1 305 093	+13 406	22 594	+123	1 019 905
6	Colombia	911 317	+ 8 570	27 834	+174	789 787

2. Literature review and problem statement

There are many scientific works devoted to mathematical models for COVID-19 spreading and there are many different models in use, which can describe epidemiological models of COVID-19.

A key limitation in our understanding of the COVID-19 pandemic is that we do not know the true number of infections. Instead, we only know of infections that have been confirmed by a test. However, because many infected people never get tested, we know that confirmed cases are only a fraction of true infections.

In the papers [2, 3], scenarios is described for the first time for the spreading the coronavirus epidemic in Moscow and it is shown that, with the introduced quarantine measures, the epidemic is expected to stretch for more than a year, and in the case of more severe measures, it will be possible to suppress the epidemic and significantly reduce the number of deaths. However, the population will not produce group immunity, and the population remains vulnerable to repeated pandemic.

The mathematical models for spreading the COVID-19 coronavirus epidemic in China were built by groups of Chinese scientists [4 – 6]. This mathematical model is based on the SEIR structure taking into account passenger flows, the impact of quarantine measures and the incubation period.

The SIR (Susceptible, Infected, and Recovered) model is the basic model for describing the spread of infectious diseases and was proposed in the 1920s by the Scottish pidemiologists Anderson Kermak and William McKendrick. According to the SIR, the population is divided into three groups: susceptible (S), infected (I), and recovered (R).

The SEIR model without vital dynamics has a form in the case of closed population with no births or deaths:

$$\begin{aligned} \frac{dS}{dt} &= -\frac{RSI}{N}, \\ \frac{dE}{dt} &= \frac{\beta SI}{N} - \sigma E, \\ \frac{dI}{dt} &= \sigma E - \gamma I, \\ \frac{dR}{dt} &= \gamma I, \end{aligned} \tag{1}$$

where $N = S + E + I + R$ is the total population, S is susceptible people, E is exposed people, I is infectious, R is recovered, β, σ, γ are unknown coefficients.

Such type of models were used by Imperial College London (ICL), The Institute for Health Metrics and Evaluation (IHME), Youyang Gu (YYG) and The London School of Hygiene & Tropical Medicine (LSHTM) for mathematical processing a daily new infections in the United States [5].

Fig. 1 shows the mean estimates of the true number of daily new infections in the United States from 4 models mentioned above.

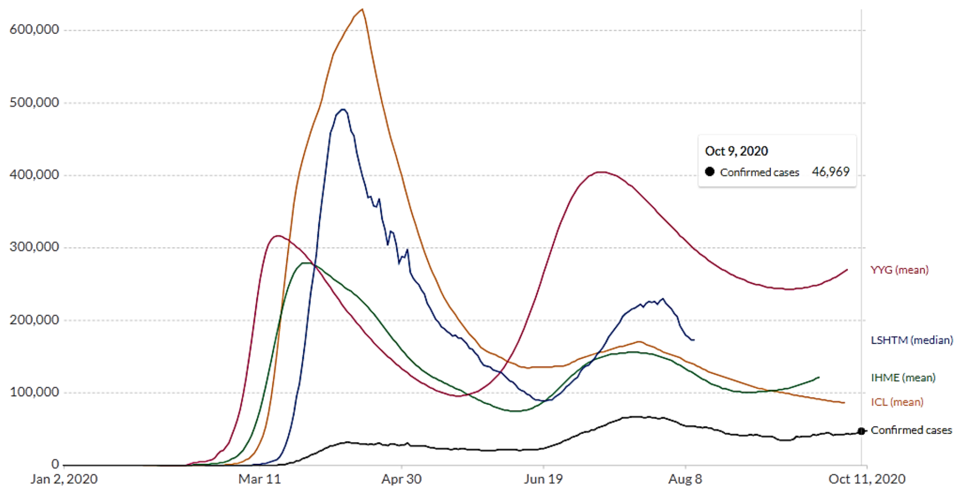

Figure 1: The mean estimates of the true number of daily new infections in the United States from ICL, IHME, YYG and LSHTM models

We can see that estimates significantly differ in data used [5] and assumptions made.

Two things are clear from this chart: all four models agree that true infections far outnumber confirmed cases. But the models disagree by how much, and how infections have changed over time.

In the article [6] authors propose following model to approximate data from the distribution of COVID-19 in Cuba:

$$M(t) = A\frac{1-e^{-F(t)}}{1+e^{-F(t)}},$$

where

$$F(t) = \sum_{i=1}^{n} a_i t^i, a_0 = 0.$$

It's so called half-logistic curve of growth with polynomial variable transfer.

Such model gave good results in the COVID-19 spreading data analysis in Bulgaria and especially for predicting the expected initial saturation level at an early prediction stage.

In paper [7] authors propose to use exponential half-logistic distribution with cumulative distribution function.

Models in [6] and [7] show good accuracy of approximation in the specific section – at the end of the considered time interval.

Another example of COVID-19 data processing is shown in [8]. Authors discussed the model based only on the daily fatalities number, and for this purpose an R^2 score based error metric is used.

Numerical examples with approximation of the step function by sigmoidal logistic functions are presented in [9].

All of mentioned methods generally have a disadvantage associated with the complexity of mathematical calculations. This paper will consider simpler methods for building mathematical models for COVID-19 data.

In general, any system for building mathematical models should combine the principles of artificial intelligence [10], and simultaneously must have the adaptability properties [11].

One of the ways to achieve high accuracy and flexibility during mathematical models building can be the usage of the theory of regression analysis [12, 13]. This theory was qualitatively used in other areas of knowledge: geography [14], during analysis of aero-material consumption [15], during assessment of the quality of navigation equipment [16, 17], for models building for nonlinear dynamical objects [18], for models building of reliability parameters [19, 20], etc.

It should be noted that the statistical data processing is one of the ways to make reliable and timely decisions [21, 22]. The use of reliable and at the same time uncomplicated data processing algorithms improves the efficiency of the system.

This paper is based on modern methods of regression analysis, which are described in detail in [23 – 25]. The aim of the paper is to build simple and accurate model for daily COVID-19 disease data for different countries.

Let us perform a mathematical statement of the problem.

Assume that for a set of two-dimensional data (x_i, y_i), where x_i is day number, y_i is new COVID-19 cases quantity, there is a set of approximation functions $\widehat{y_i} = f_m(x_i, a_{k,m})$, where $a_{k,m}$ is a vector of k parameters for the m-th approximation function, m is a number of approximation functions. For function f_m, standard deviation σ between statistical data y_i and evaluation $\widehat{y_i}$ can be estimated.

The best mathematical model is selected based on the following criterion

$$k = inf\left(s \in \mathbf{N}\ \forall\ j: \sigma(f_s(x_i, a_{k,s}) \leq)\sigma(f_j(x_i, a_{k,j})\right). \quad (2)$$

3. Statistical data analysis and mathematical model building

The initial data for mathematical models building are data on daily new cases of COVID-19 disease for Ukraine, Poland and Italy. These data are shown in Fig. 2 – 4.

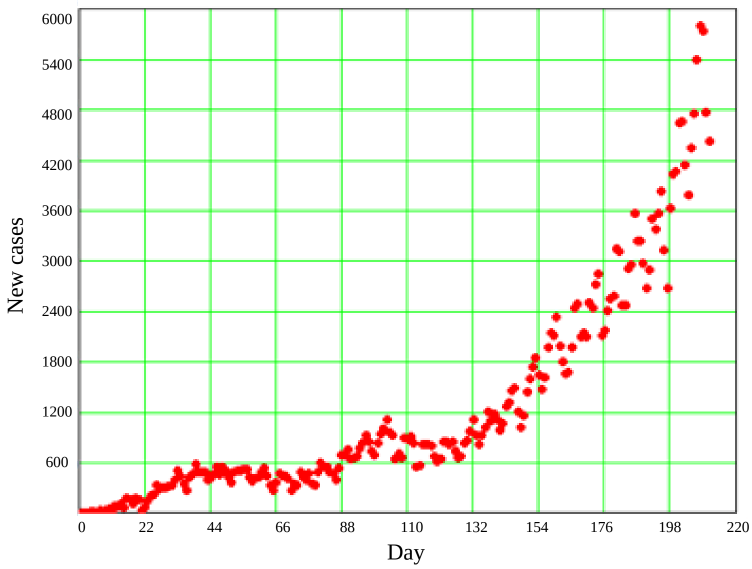

Figure 2: New cases of COVID-19 disease for Ukraine (from March 15 to October 12)

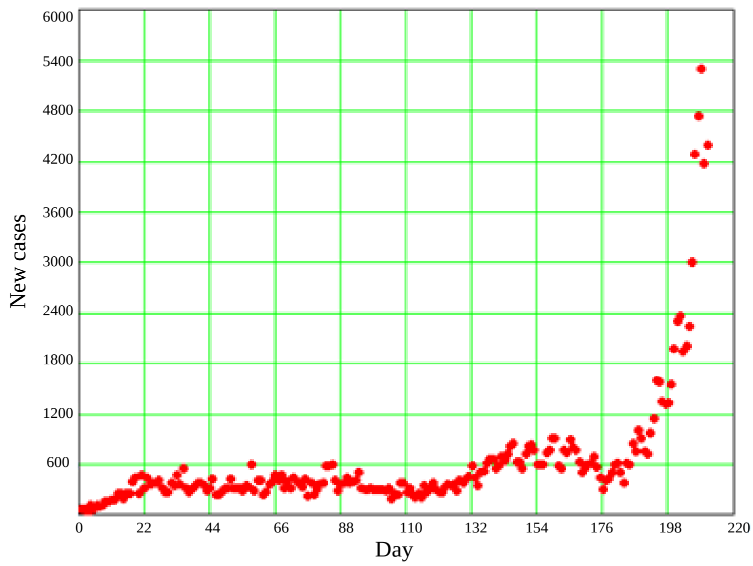

Figure 3: New cases of COVID-19 disease for Poland (from March 15 to October 12)

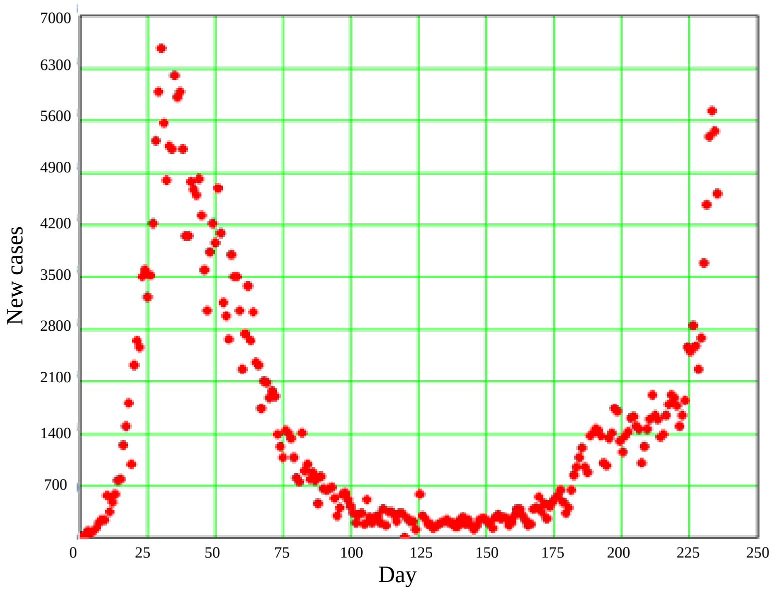

Figure 4: New cases of COVID-19 disease for Italy (from February 21 to October 12)

Let us make comparative analysis of usage different regression models. First, we will analyze data for Ukraine.

1. Regression models based on polynomials of the 5th, 7th and 10th order.

To find the mathematical equations for such models, the ordinary least squares method was used. In this case for polynomial of the 5th order, two options was calculated: 1) ordinary and 2) with zero point containing. For polynomial of the 7th order calculation was performed only for case of zero point containing.

As a result, the following models were obtained

$$f_1(x) = -135 + 24.6x - 0.427x^2 + 3.89 \cdot 10^{-3}x^3 - 1.68 \cdot 10^{-5}x^4 + 3.78 \cdot 10^{-8}x^5. \quad (3)$$

$$f_2(x) = 13.465x - 0.145x^2 + 8.59 \cdot 10^{-4}x^3 - 2.59 \cdot 10^{-6}x^4 + 1.32 \cdot 10^{-8}x^5. \quad (4)$$

$$f_3(x) = 13.28x - 0.41x^2 + 0.015x^3 - 2.75 \cdot 10^{-4}x^4 + 2.38 \cdot 10^{-6}x^5 - \\ -9.62 \cdot 10^{-9}x^6 + 1.48 \cdot 10^{-11}x^7. \quad (5)$$

$$f_4(x) = -133 + 78.4x - 11.7x^2 + 0.793x^3 - 0.027x^4 + 5.02 \cdot 10^{-6}x^5 - \\ -5.69 \cdot 10^{-6}x^6 + 3.96 \cdot 10^{-8}x^7 - 1.65 \cdot 10^{-10}x^8 + 3.78 \cdot 10^{-13}x^9 - 3.67 \cdot 10^{-16}x^{10}. \quad (6)$$

The results of approximation using four models are shown in Fig. 5. As can be seen from the graphs, models (3), (4) and (5) have approximately the same character. Model (6) has a drawback in the form of the presence of a maximum point in the final approximation section, which will affect on the forecasting accuracy.

The free coefficients of models (3) and (6) are equal –135 and –133, respectively, which does not correspond to the physical nature of the observation.

To compare the accuracy of the approximation, standard deviations were calculated (Table 2).

Table 2.
Standard deviations different approximation models

Approximation models	Standard deviation
Model (3)	240.426
Model (4)	240.839
Model (5)	238.7
Model (6)	216.732

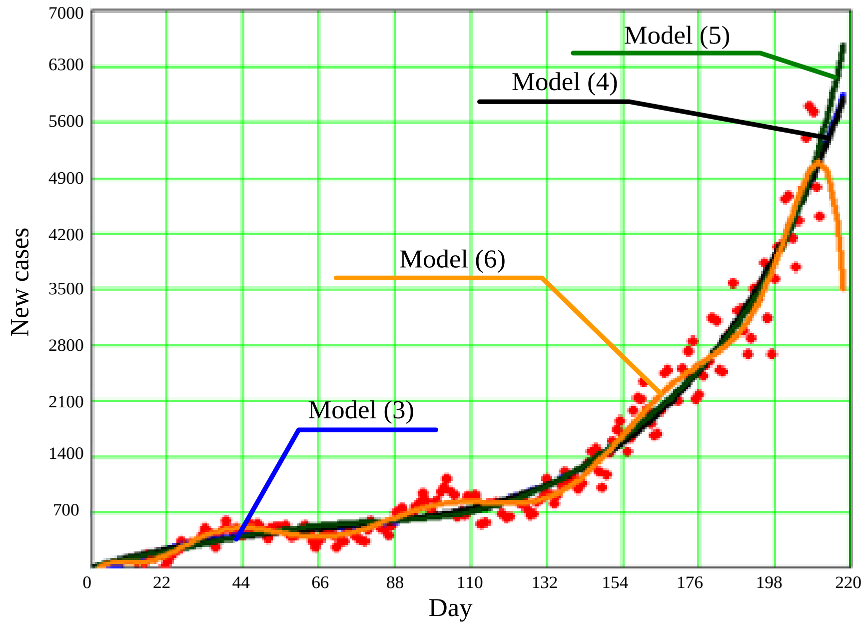

Figure 5: The results of approximation using models (3) – (6)

The standard deviation for model (6) is minimal, but the forecasting quality does not correspond to the trend of data changes. Therefore, model (5) is more preferable.

Let us compare the forecasting quality of different models by reducing the sample size. We will predict the number of new cases 5 and 10 days ahead. That is, for data before September 1, we will predict the values of new cases on September 5 and September 10 etc. The obtained values were compared with the true values and the relative forecasting error was found. The calculation results are shown in Table 3 and 4.

Table 3.
Comparative analysis of relative forecasting errors for 5 days ahead (from September 5 to October 10)

Type of polynomial	Date 5.09	Date 10.09	Date 15.09	Date 20.09	Date 25.09	Date 30.09	Date 5.10	Date 10.10
5th order	2.8	21.2	2.1	6.8	0.7	9.4	0.7	21.6
5th order (with zero)	4.3	19.6	1.4	6.3	1.2	9.6	0.5	22
7th order	31.7	3.4	31.8	21.2	15.4	20.3	8	19.4
10th order	2.1	7.16	6.1	0.14	2.5	15	10.5	25.6

Table 4.
Comparative analysis of relative forecasting errors for 10 days ahead (from September 5 to October 10)

Type of polynomial	Date 5.09	Date 10.09	Date 15.09	Date 20.09	Date 25.09	Date 30.09	Date 5.10	Date 10.10
5th order	2.2	24.1	24.8	9.6	3.9	4.5	3	30.3
5th order (with zero)	1.1	21	22.2	8.5	4.6	5.3	2.5	30.6
7th order	68	50.8	25.7	63.7	54.4	32.3	21.3	41.9
10th order	140.3	61.3	182.8	1.3	18.1	17.8	27	60.7

As can be seen from Tables 3 and 4, polynomials of 5th and 10th orders have the smallest error in forecasting for 5 days ahead. When forecasting 10 days ahead, the 10th order polynomial has a large error value. The 7th order polynomial in both cases has the largest error.

To analyze forecasting errors, we recalculate models (3) – (6) for data before September 1. As a result, we obtain the equations

$$f_1(x) = -134 + 26.9x - 0.625x^2 + 8.684 \cdot 10^{-3}x^3 - 5.96 \cdot 10^{-5}x^4 + 1.64 \cdot 10^{-7}x^5. \quad (7)$$

$$f_2(x) = 11.6x - 0.096x^2 + 1.04 \cdot 10^{-3}x^3 - 1.08 \cdot 10^{-5}x^4 + 5.05 \cdot 10^{-8}x^5. \quad (8)$$

$$f_3(x) = -36.4x + 4.92x^2 - 0.18x^3 + 3.11 \cdot 10^{-3}x^4 - 2.74 \cdot 10^{-5}x^5 + \\ +1.19 \cdot 10^{-7}x^6 - 2.04 \cdot 10^{-10}x^7. \quad (9)$$

$$f_4(x) = -127 + 76.9x - 11.8x^2 + 0.815x^3 - 0.028x^4 + 5.5 \cdot 10^{-4}x^5 - \\ -6.54 \cdot 10^{-6}x^6 + 4.85 \cdot 10^{-8}x^7 - 2.2 \cdot 10^{-10}x^8 + 5.64 \cdot 10^{-13}x^9 - 1.69 \cdot 10^{-16}x^{10}. \quad (10)$$

The calculation results of the relative forecasting error for models (7) – (10) for data from September 1 to October 12 are shown in Fig. 6.

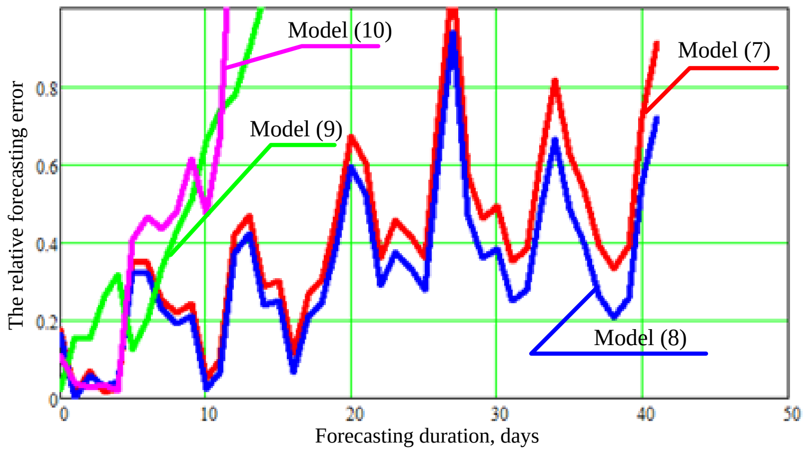

Figure 6: Forecasting errors for models (7) – (10)

So the best model in terms of long-term forecasting is model (8), that is, 5th order polynomial containing zero point.

2. Regression model in a sliding window.

In this case, a 5th order polynomial in a sliding window was used as an approximating function. At the first stage of building the model, the best width of the sliding window was calculated, which in this case was equal to 75 days.

The mathematical model was found using the ordinary least squares method. The result is the equation

$$f_5(x) = (2.73 \cdot 10^6 - 79650x + 921.5x^2 - 5.29x^3 + 0.015x^4 - 1.7 \cdot 10^{-5}x^5)h(x-b), \quad (11)$$

where $h(x)$ is Heaviside step function, b is time moment of sliding window beginning.

The result of approximation using sliding window model is shown in Fig. 7.

Visual analysis of the graphs shows approximately the same trend with the result of the approximation based on the 5th order polynomial (containing the zero point).

The standard deviation for this model is 352.2.

Let us compare the forecasting quality by reducing the sample size. We will predict the number of new cases 5 and 10 days ahead. The calculation results are shown in Table 5.

Table 5.

Comparative analysis of relative forecasting errors for 5 and 10 days ahead (from September 5 to October 10)

Forecasting duration	Date 5.09	Date 10.09	Date 15.09	Date 20.09	Date 25.09	Date 30.09	Date 5.10	Date 10.10
5 days ahead	7.4	27.5	14.9	1.9	6.3	3.8	4.4	11.5
10 days ahead	63.4	17.8	45.6	22.3	16.1	13.3	19.3	24.3

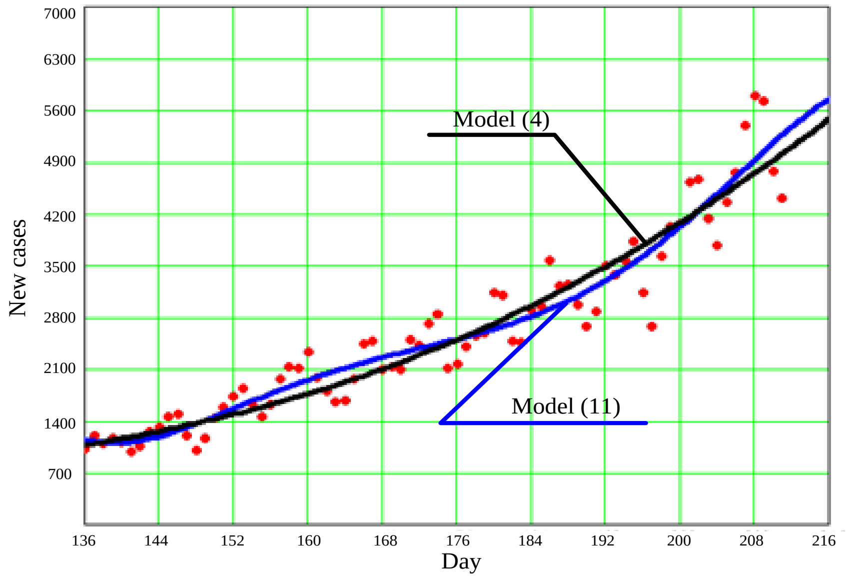

Figure 7: The results of approximation using sliding window model

Analysis of the data from Table 5 shows satisfactory results for the case of forecasting for 5 days ahead.

3. Segmented regression.

To select a segmented regression model, a visual analysis of the geometric structure of data in Fig. 2 can be previously made. In the simplest case, we can use a linear-polynomial model, when a linear function is used in the first section, and a polynomial in the second.

For such a case, the approximation function can be written as follows

$$f_6(x) = a + bx + c(x - x_{sw})h(x - x_{sw}) + d(x - x_{sw})^2 h(x - x_{sw}), \tag{12}$$

where a, b, c, d are unknown coefficients, x_{sw} is a switching point abscissa.

To find the unknown coefficients, the ordinary least squares method is used.

To find the abscissa of the switching point, additional optimization must be performed [26]. Optimization is carried out in the following sequence:

– the data is approximated by formula (12) for several options of the values of the switching point abscissa,

– for each option, the standard deviation is calculated,

– the obtained dependence of standard deviations on the value of the abscissa of the switching point is approximated by a parabola of the second degree,

– the optimum (minimum) of the parabola is found, this minimum corresponds to the optimal abscissa of the switching point.

As a result of calculations, the optimal value of abscissa of the switching point was obtained $x_{sw} = 104$. For this value, equation (12) will be as follows

$$f_6(x) = 32.685 + 7.065x - 12.11(x - 104)h(x - 104) + 0.412(x - 104)^2 h(x - 104). \tag{13}$$

The result of approximation using segmented regression is shown in Fig. 8. The standard deviation for this model is 234.2. The calculation results of forecasting quality are shown in Table 6.

Table 6.

Comparative analysis of relative forecasting errors for 5 and 10 days ahead (from September 5 to October 10)

Forecasting duration	Date 5.09	Date 10.09	Date 15.09	Date 20.09	Date 25.09	Date 30.09	Date 5.10	Date 10.10
5 days ahead	6.5	15.2	4.9	11.1	16	5	7.4	21
10 days ahead	5.4	15.3	14.4	13.4	14	16	10.3	23.4

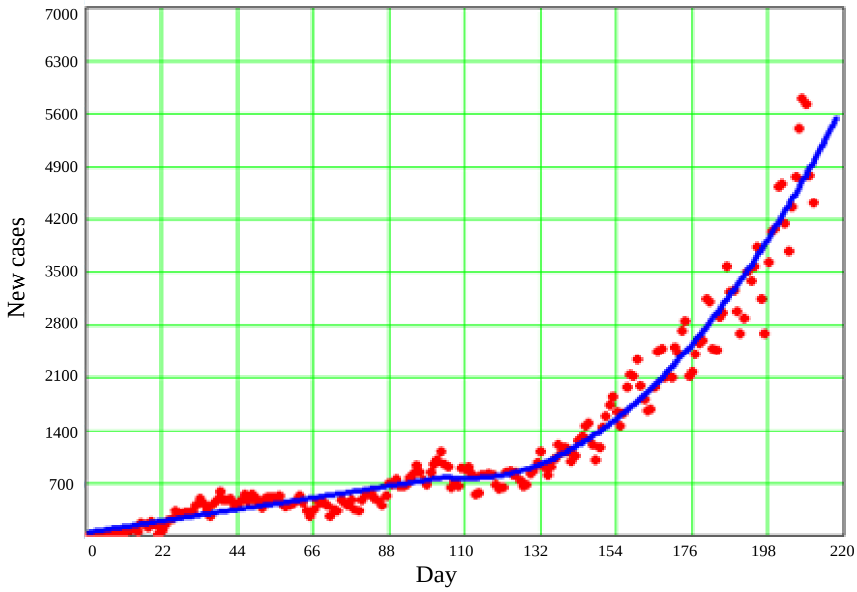

Figure 8: The results of approximation using model (13)

Analysis of the data from Table 5 shows satisfactory results for the case of forecasting for 5 and 10 days ahead. Relative forecasting errors does not exceed 21 % for 5 days ahead and 23.4 % for 10 days ahead.

The resulting model is the most preferable from the point of view of both forecasting properties and taking into account the geometric structure of the initial data.

Let us perform similar calculations for the data on the COVID-19 diseases in Poland, presented in Fig. 3.

As a result, the following models were obtained

$$f_1(x) = -377 + 84.2x - 2.834x^2 + 38 \cdot 10^{-3}x^3 - 2.21 \cdot 10^{-4}x^4 + 4.59 \cdot 10^{-7}x^5. \quad (14)$$

$$f_2(x) = 53.035x - 2.047x^2 + 0.03x^3 - 1.815 \cdot 10^{-4}x^4 - 3.905 \cdot 10^{-7}x^5. \quad (15)$$

$$f_3(x) = 44.1x - 2.65x^2 + 0.08x^3 - 1.24 \cdot 10^{-3}x^4 + 1.007 \cdot 10^{-5}x^5 - \\ -4.044 \cdot 10^{-8}x^6 + 6.337 \cdot 10^{-11}x^7. \quad (16)$$

$$f_4(x) = 124 - 44x + 8.175x^2 - 0.458x^3 + 0.013x^4 - 2.02 \cdot 10^{-4}x^5 + \\ +1.94 \cdot 10^{-6}x^6 - 1.15 \cdot 10^{-8}x^7 + 4.1 \cdot 10^{-11}x^8 - 8.085 \cdot 10^{-14}x^9 + 6.79 \cdot 10^{-17}x^{10}. \quad (17)$$

$$f_6(x) = 182.35 + 2.505x + 1.75(x - 187)h(x - 187) + 7.161(x - 187)^2h(x - 187). \quad (18)$$

The results of approximation using models (14) – (18) are shown in Fig. 9 and 10. To compare the accuracy of the approximation, standard deviations were calculated (Table 7).

Table 7.
Standard deviations different approximation models

Approximation models	Standard deviation
Model (14)	266.774
Model (15)	273.029
Model (16)	183.218
Model (17)	179.751
Model (18)	205.676

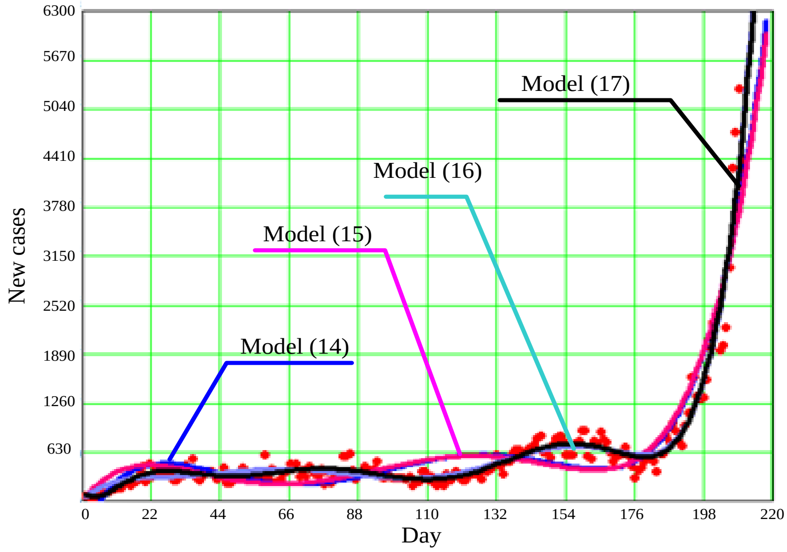

Figure 9: The results of approximation using models (14) – (17)

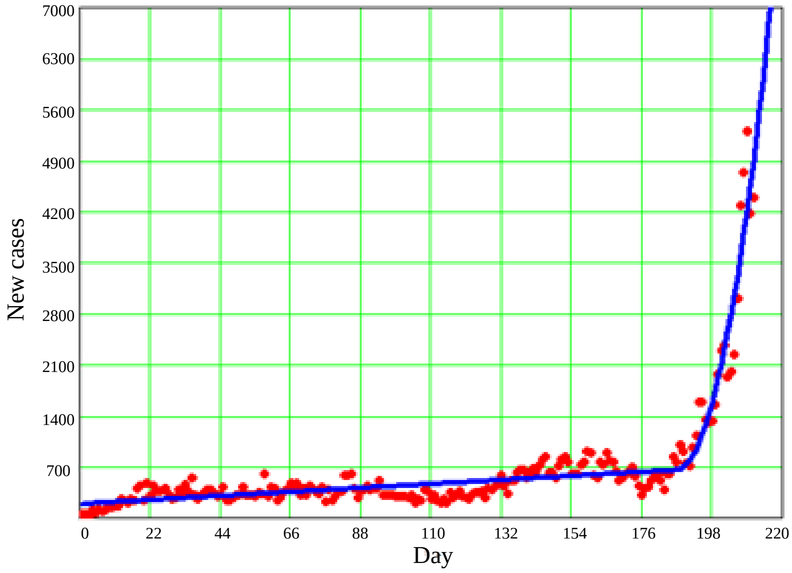

Figure 10: The results of approximation using model (18)

So for the data on diseases in Poland, the most preferable model according to the criterion of the minimum standard deviation is regression model using a polynomial of the 10th order.

Let us perform similar calculations for the data on the COVID-19 diseases in Poland, presented in Fig. 4.

As a result, the following models were obtained

$$f_1(x) = -2629 + 491x - 11.6x^2 + 0.107x^3 - 4.38 \cdot 10^{-4}x^4 + 6.685 \cdot 10^{-7}x^5. \quad (19)$$

$$f_2(x) = 295x - 7.183x^2 + 0.065x^3 - 2.588 \cdot 10^{-4}x^4 + 3.89 \cdot 10^{-7}x^5. \quad (20)$$

$$f_3(x) = 73.74x + 7.72x^2 - 0.288x^3 + 3.696 \cdot 10^{-3}x^4 - 2.248 \cdot 10^{-5}x^5 + \\ +6.593 \cdot 10^{-8}x^6 - 7.508 \cdot 10^{-11}x^7. \quad (21)$$

$$f_4(x) = 1371 - 656x + 77.2x^2 - 2.999x^3 + 0.06x^4 - 7.082 \cdot 10^{-4}x^5 + \\ +5.35 \cdot 10^{-6}x^6 - 2.62 \cdot 10^{-8}x^7 + 8.06 \cdot 10^{-11}x^8 - 1.43 \cdot 10^{-13}x^9 + 1.115 \cdot 10^{-16}x^{10}. \quad (22)$$

The results of approximation using models (19) – (22) are shown in Fig. 11.

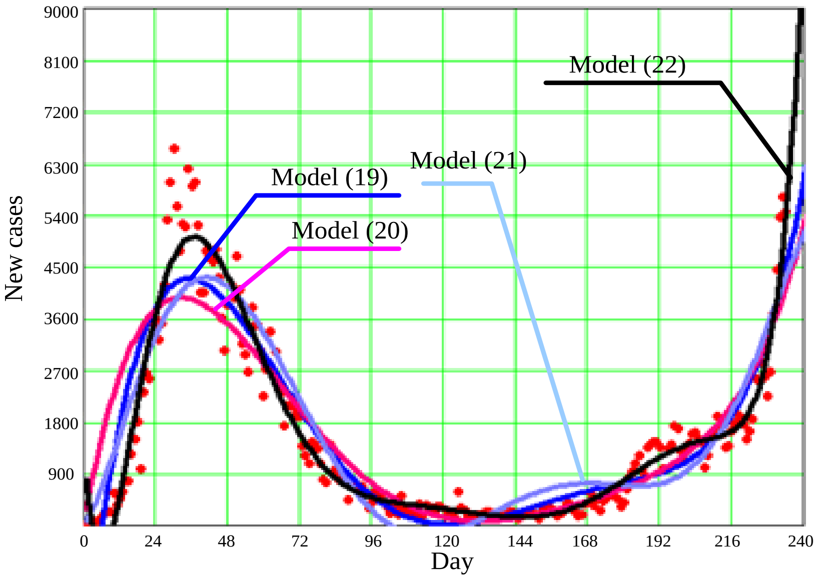

Figure 11: The results of approximation using models (19) – (22)

The most preferable model for the data on diseases in Italy according to the criterion of the minimum standard deviation is regression model using a polynomial of the 10th order.

4. Conclusion

The paper deals with the problem of mathematical model building for COVID-19 diseases data. For data on new cases of diseases in Ukraine, Poland and Italy, a comparative analysis of the use of regression models based on polynomials of the 5th, 7th and 10th order, mathematical model building in a sliding window, as well as a segmented regression model was carried out. For the data on diseases in Ukraine, the most preferable model is regression model using a polynomial of the 5th order. For the data on diseases in Poland and Italy, the most preferable model is regression model using a polynomial of the 10th order. The segmented regression model is the most preferable from the point of view of both forecasting properties and taking into account the geometric structure of the initial data.

The research results can be used in process of solving the problems of predicting the spread of COVID-19 in the different countries.

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