Design of a Smart ABN Device for Early Math Education

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***Abstract*—New methodologies are emerging in the current educational system as an alternative to traditional teaching, some of which are related to the area of basic mathematics for primary school students. One of them is the Open Calculation method Based on Numbers, ABN (Abierto Basado en Números). This method is based on showing what the meaning of the number is. Thus, the manipulation of objects is its base. The aim of this paper is to present the design and development of a smart device for teaching ABN method. An electronic device has been designed to facilitate the learning of the students in a simple and physical way of the basic operations, trying not to lose the essence of the ABN method. In addition, the device saves any interaction that the student has when performing an operation to allow showing analytics of the gathered data in the future.**

***Index Terms*—early math education, primary school, educational systems, ABN method, data gathering, technological tool**

I. Introduction

Technologies today are causing a great transformation, and education is one of the areas that has the greatest impact [1]. The way in which education is taught is constantly evolving, pursuing to improve its quality, and revolutionising the way in which the student obtains, processes, and interprets information.

An analysis of the current situation in the classroom reveals certain weaknesses in the field of mathematics, especially in some countries, such as Spain, Ukraine or Argentina, according to the results obtained in the evaluation of the educational system, PISA (Programme for International Student Assessment) [2]. This area is fundamental for the development of STEAM, Science, Technology, Engineering, Arts and Mathematics skills, which are currently the most demanded ones for future works [3], and even present ones, where there is already a lack of professionals to cover [4].

New ways of teaching mathematics have appeared in the current educational situation, among them, the ABN method, "Open Calculation method Based on Numbers" (Abierto Basado en Números). This method is undergoing a great development in a short time, becoming one of the alternatives to the conventional method known as CBC, "Closed Based on Cipher" (Cerrado Basado en Cifras) [5].

The main purpose of ABN method is to help children to know the meaning of the number [6]. This is typically worked with the manipulation of objects, especially using chopsticks. Thus, it is easy to use it in the classroom although it has some limitations. The student's progress cannot be followed to help him to improve in the future and to offer him a more individualised education.

There are already some web sites to support ABN, but they are not tangible. All the tools that exist are web applications. Therefore, the aim of this paper is to present the design and development of a smart device that allows the student to learn the basic operations in a simple and physical way using ABN. This smart device saves all the interaction that the student has with it during performing the operation so that the teacher can have a follow up of the child learning in the future.

The rest of the paper is organised as follows: Section II presents the principles on which the ABN method is based, Section III defines the requirements that the device must fulfil, Section IV describes the physical design for the implementation of the device, Section V presents the physical device appearance, and an example of an addition and add-subtraction; and, finally, Section VI concludes and presents the future work of this research.

II. The ABN method

ABN stands for Open Calculation method Based on Numbers, and it was born to help the children with the simple arithmetic expressions [7]. The resolution of operations with the traditional models prevents the adequate development of the mental arithmetic [8]. The ABN method ensures that the student does not learn the contents in a mechanical way, memorizes rules and works on the operations in a single way, but gives children the freedom to experiment by themselves and to make their own experiences the source that gives meaning to mathematics [6]. The student works with manipulative and motivating materials to achieve this.

The principles that support ABN method are based on the evidence of the MRE, "Realistic Mathematics Education", approach [9] and are as follows [10]:

- Principle of equality: all students can achieve an acceptable mathematical competence with the corresponding help, the existence of a "mathematical gen" is rejected.
- Principle of experience: manipulation of objects is essential for the child to build his own learning; experimentation is necessary to acquire the abstract concepts of mathematics.
- Principle of using whole numbers: when a transaction is complex it is divided into smaller whole numbers, never into meaningless units. The student always manipulates, operates, calculates and estimates based on numbers.
- Principle of transparency: all the steps and processes carried out can be visualized and the symbolic materials used help to reflect reality.
- Principle of adaptation to the individual rhythm of each subject: it is an open calculation method and does not follow any pattern, the students have flexibility to perform the operation.
- Principle of self-learning and self-control: there is the possibility of controlling the intermediate steps carried out, the student verifies the accuracy of what he is doing.

The following operations can be performed by means of ABN: addition, subtraction, multiplication, and division, but also "new operations": double addition, double subtraction, and addition-subtraction.

A. Characteristics of an operation in ABN

The order by which an operation is started is irrelevant in this method, ABN does not follow the strict order of the magnitudes that constitute the number [12]. In addition, the numbers can be decomposed into smaller parts to facilitate the operation.

The possibility of operating more than one order of magnitude simultaneously exists. The calculations in ABN can be done recursively in one direction or another, depending on the strategy of each student.

The students work a lot the "friends of 10" in this method, later extended to "friends of 100, 1000…". It consists of filling in quantities until a higher order is reached., e.g., having 10 units and replacing them with 1 ten.

Thus, two actions appear when performing operations: "group" and "ungroup". "Group" is mandatory if an order of magnitude is completed when performing an addition, e.g., 10 is reached. The set of 10 is grouped to a higher order of magnitude.

The action of "ungroup" appears with the subtractions. If there is an order of magnitude in one term of the operation and not in the other one, a higher order is opened in the last one, e.g., 1 ten is decomposed into 10 units.

B. Addition

The essence of addition is to remove an amount from one operand and add it to the other. Therefore, the solution will be reached when one of the two summands is zero. The children represent the terms with chopsticks, each group of chopsticks in a "tray" (see Fig. 1). Thus, they have two groups of chopsticks representing each number. The addition consists of passing chopsticks from one tray to another, until one is empty.

In Fig. 1, there is an example of how a student performs an addition step by step. The steps are detailed as follows:

- First, the child writes the terms of the addition in the tray and grid (Fig. 1.A).
- Then, the child removes 3 tens from the right tray. The amount that the student decides to "remove" or "put" must be placed in the left column of the grid. Thus, (s)he writes 30 and updates the terms of the sum, 87 + 8 (Fig. 1.B).
- After that, (s)he passes 3 units to be able to group 1 ten in the left tray with the 7 single units. The sum is updated, 90 + 5 (Fig. 1C).
- Finally, the student moves the remaining 5 units and now, (s)he has one empty tray. Therefore, the solution has been found: 95 (Fig. 1D).

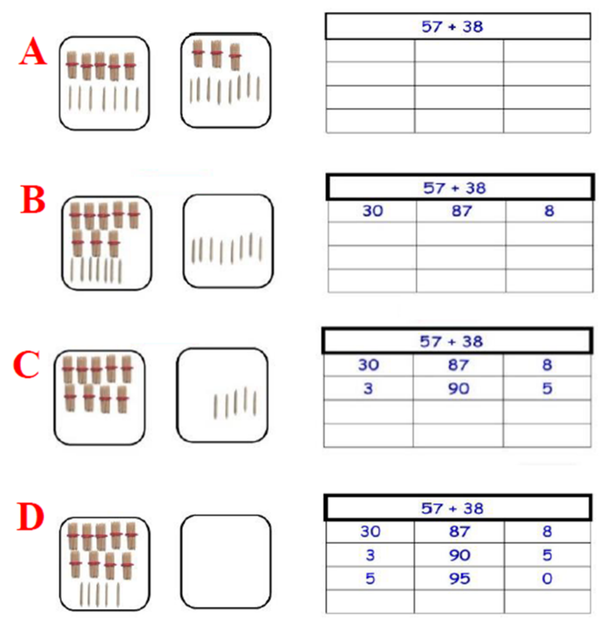

Fig. 1. Steps to calculate the addition of "57 + 38" using chopsticks and a grid in the ABN method.

C. Subtraction

The terms of the operation are represented in the same way in the case of subtraction. But in this case, the toothpicks that are removed from one tray are removed in the other as well. The operation ends when one tray is empty, just as in the addition.

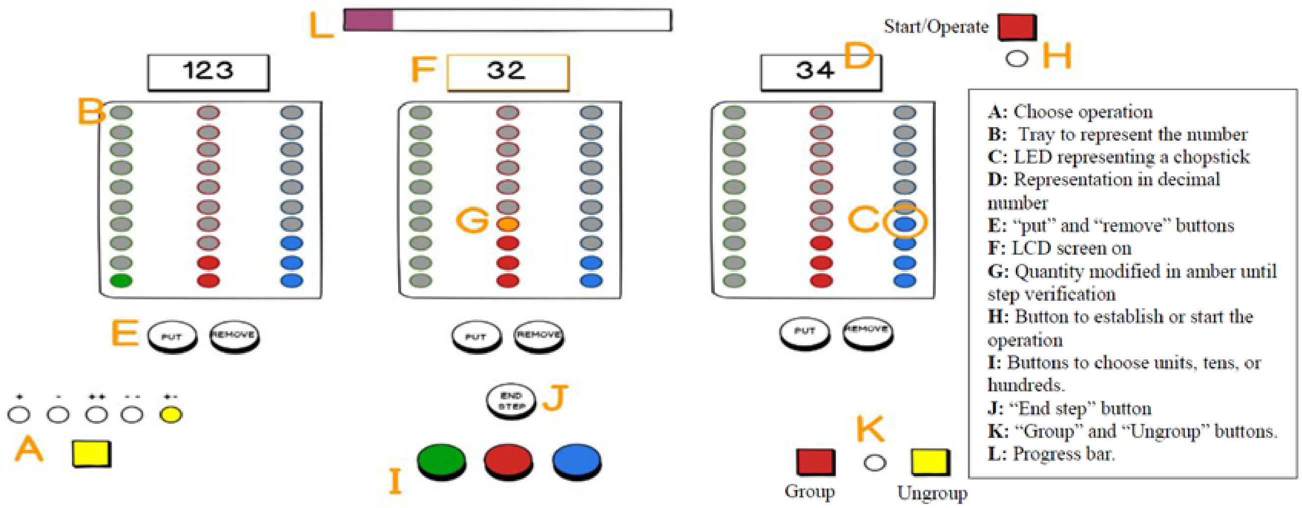

Fig. 2. Example in the design prototype showing an intermediate step of the add-subtraction 123 + 142 – 134

III. Smart device requirements

The smart device is designed as a tool to make it easier for students to learn basic operations. The essence of ABN cannot be lost. Therefore, tangible and motivating materials must be used to capture the student's attention.

The device must fulfil the following requirements:

- To allow the student to perform the following operations: addition, subtraction, double addition, double subtraction, and addition-subtraction.
- To help the student with the learning of the operations
- The user must be able to perform two- and three-digit operations.
- The number must be physically and visually represented.
- The complete interaction of the student with the device must be able to be saved.
- The teacher will be able to keep track of the operations performed by his students.

IV. Physical design

The physical smart device has been designed according to the following requirements.

- The following operations can be performed: addition, subtraction, double addition, double subtraction, and addition-subtraction. A strip of 5 LEDs will be used to choose which of the operations to perform (Fig. 2.A).
- A different LED will turn on when pressing the button (Fig. 2.A) to determine the chosen operation. The symbols next to each LED represent the operations that can be performed (see Table I). The button will move the LED that is illuminated one position on each pulse. The chosen operation will be the one that has the lit LED fixed.
- The traditional grid is equivalent to a set of three LED strips (Fig. 2.B) and each LED represents a toothpick (Fig. 2.C). Therefore, each grid is composed of a strip of ten LEDs that light up in blue to represent the units, a second strip of ten LEDs that light up in red representing the tens, and a last strip of ten LEDs that light up in green for the hundreds.
- Each grid has its own LCD display on which the number in decimal will be represented by the LEDs on the tray at any given time (Fig. 2.D).
- Each grid has two buttons at the bottom (Fig. 2.E). One is for "remove" and the other is for "put".
- The LCD screen of the tray on which something is being modified will remain illuminated (Fig. 2.F). This way the student knows where (s)he is at any moment. All screens turn on again at the end of the step.
- It also has a button to indicate when the initial operands are being set and when the operation begins (Fig. 2.H).
- In the centre is the "control unit" (Fig. 2.I). It consists of a button for the units, another one for the tens an a third one for the hundreds. Once the student has decided whether (s)he wants to put or remove from each of the grids, these buttons are the ones the child has to press to indicate whether (s)he wants to modify units, tens or hundreds.
- There is a button (Fig. 2.J) to indicate the end of the step. It must be pressed when the student finishes modifying the quantities. The LEDs of the grids appear in yellow until the moment of verifying if the changes made are correct (Fig. 2.G). If the button (Fig. 2.J) is pressed and the movements made are right, the new quantities of LEDs on are established with their corresponding colour. If there is an error, the LEDs remain on yellow.
- The concepts of "group" and "ungroup" are important in ABN, so two buttons will also be used to perform these actions (Fig. 2.K). An order of magnitude has been completed when one of the LED strips is fully lit. Thus, a LED located between these buttons will light up to

alert the user of this. If the “group” button is pressed, the ten LEDs will turn off and one LED of the higher order will light up. This action is mandatory, that is, the student cannot have a complete order and not “group” to a higher one. However, “ungroup” is a voluntary action and the user is not notified. The “ungroup” button should be pressed if the user needs it.
- Six LEDs will be placed on the top of the grids to indicate progress (Fig. 2.L). It has been considered important because in this way the teacher will be able to identify immediately if there are any students who need help. The progress is given by the number of steps in which the operation is made. For each step, a progress light turns on; if more steps than the “optimal” ones are performed,it lights up in a different color. The approach for this depends on the operation to be performed.

TABLE I
SYMBOLS AND THEIR OPERATION

Symbol	Operation
+	Addition
-	Subtraction
+ +	Double addition
- -	Double subtraction
+ -	Addition-subtraction

V. IMPLEMENTATION

The device has been implemented in compliance with all requirements, as shown in Fig. 3.

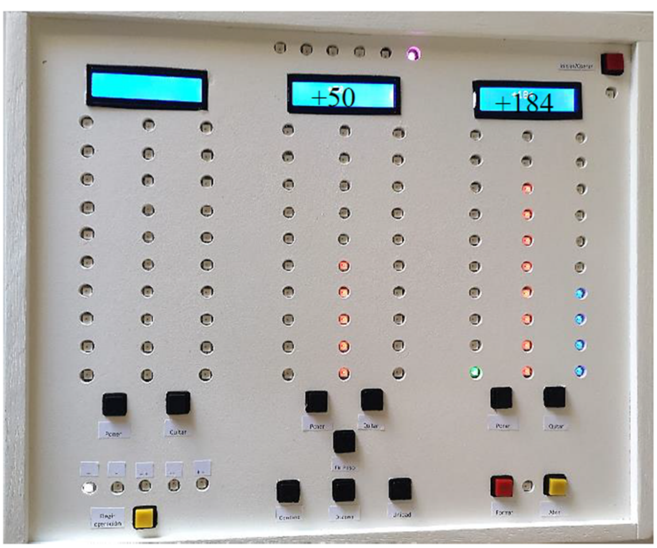

Fig. 3. Example of an intermediate step of the addition 51 + 183 by means of the smart ABN device. In the Figure, the number 50 is represented in the middle tray, and the number 184 is shown in the right tray.

An example of how the device works in an addition according to its flowchart is shown after its implementation to help understand it better (see Fig. 4.).

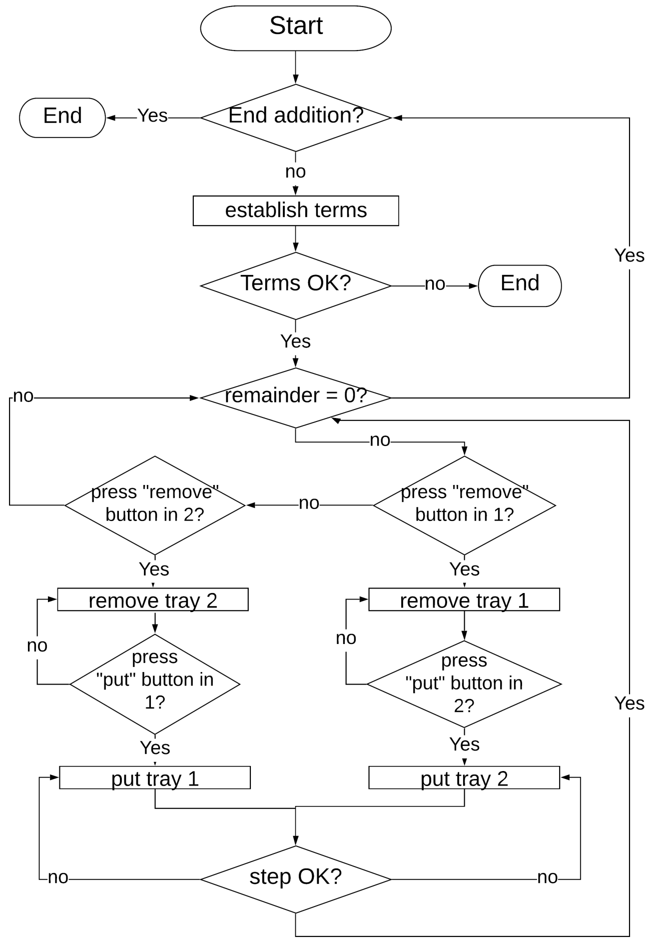

Fig. 4. Flowchart with the addition operation

A. Addition example

A student should perform the addition 51 + 183. In order to add two numbers, the student should follow the flowchart shown in Fig. 4. One possible solution implemented by the student could be as follows:

- First, as shown in Fig. 4, the terms of the operation are established. The child puts 5 tens, and 1 unit in the middle tray; and 1 hundred, 8 tens, and 3 units in the right one. The device changes to the “operate” mode once the operands are correct and it will continue in that mode while the remainder of the operation is different from 0.
- Then, the student decides to remove one unit from tray 2. Now there is only one option, (s)he must put that unit in tray 1. As the step is correct, the terms of the operation are updated to 50 + 184 (see Fig. 3).
- The student realizes that (s)he needs two tens to be able to group one hundred in tray 1, so for the next step the child decides to remove two tens from tray 2 and put them in tray 1. (S)he presses the group button, so the strip of ten LEDs turns off and a hundred LED is lit up.
- Finally, the student removes the remaining three tens from

tray 2 and (s)he puts them into tray 1. The step is correct, and the remainder is 0. Thus, the addition finishes (see Fig. 4).

B. Addition-subtraction example

Addition-subtraction is the union of an addition and a subtraction. If the operation is supposed to be A + B - C, A + B must be greater than C for the operation to be properly performed.

The following steps can be performed once the operation is in progress:

- If the student starts removing from tray 1, that is from C, (s)he will be able to remove from A, B or remove from A and B the sum of what (s)he has removed from C.
- If the student starts removing from B, the only option is to put that amount in A; since these are the terms that are between the "+" sign.
- If the student starts removing from tray A, the only option is to continue adding the corresponding amount in B.

This operation is best understood by stating a problem. For example, Olivia has 32€ and her uncle gives her 44€ . She spends 28€ . How much money does Olivia have left?

- First, the student sets the operands on the device: 32 + 44 – 28.
- (S)he decides that Olivia spends 20€ and (s)he removes 2 tens from C and B. Now the operands are 32 + 24 - 8.
- Then, the child decides that Olivia spends 6€ and (s)he removes 6 units from A. Now, the student must remove that amount from the other trays. In this case, (s)he decides to remove 4 units from B and 2 from C. The terms of the operation are: 30 + 20 - 2.
- The student only has 2 units in one tray. Thus, (s)he presses the "ungroup" button. The smart device will open the ten of the term that is greater, in this case, 30. Now, a red LED turns off and a strip of 10 blue LEDs lights up. The student can remove the 2 units from A and C. The terms are now: 28 + 20 - 0.
- The child already knows that Olivia has already spent the 28€ , but (s)he does not know what Olivia has in total. Therefore, (s)he decides to remove the 2 tens from B and put them in C. Finally, the smart device has the terms: 48 + 0 – 0 and the student knows that Olivia has 48€ left.

VI. Conclusions and Future Work

The ABN method works on acquiring the sense of the number from the earliest ages to understand the meaning of basic operations. In this process, it is essential to use a model that helps the students to make tangible something as abstract as the orders of units established by the decimal numbering system. Using chopsticks, the student is able to manipulate the concepts that the decimal system requires. Once the student acquires greater degree of abstraction, the toothpicks can be replaced by models with different figurative content. The smart device presented in this paper is within this model. The green LEDs represent the hundreds, red LEDs the tens and blue LEDs the units. This colour code is because teachers working with ABN identify with red rubber the tens and green one the hundreds.

The use of this smart device contributes to the development of the capacity of abstraction of the student, as it is a further step in the transition to the representation of the different orders of units in the decimal numbering system. At the same time, it simplifies the algorithm of the basic operations when replacing the chopsticks with the use of the smart device, maintaining the essence of the manipulative idea of "group" or "ungroup" the different orders of magnitude.

Once the smart device has been developed, the aim is to carry out an evaluation of the device with real users in a school that uses the ABN method in its classrooms. The intention is to use this smart device in the classes for several weeks and with different Primary Education courses.

In addition, a web application is going to be developed as a complement of the smart device to help the teacher. To do this, the application requires that the teachers can see the interaction of the student in real time, and they can send operations directly to each student's device. Also, the web application requires sequencing the contents by levels, saving the record of each class, and saving all the interaction of the students to observe their evolution.

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