Rewriting $\mathcal{ALCHIQ}$ to Disjunctive Existential Rules (Extended Abstract)*

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Among the many approaches towards efficient reasoning in description logics (DLs), consequence-preserving translations of DL theories into rule languages are among the most prominent. Table 1 gives an overview of works in this area. The natural strength of rule reasoners is their good scalability towards large sets of facts. Indeed, all of the rewritings in Table 1 are independent of the given facts (ABox), which can be used unchanged for reasoning with the rule-based theory. In general, we can describe this idea of rewriting as follows:

Definition 1. *Consider fragments $\mathcal{L}_1$ and $\mathcal{L}_2$ of first-order logic that include ground facts. An $\mathcal{L}_2$-theory $\mathcal{T}_2$ is a* fact-entailment preserving rewriting *(or simply* rewriting *in our context) of an $\mathcal{L}_1$-theory $\mathcal{T}_1$ if, for every set $\mathcal{F}$ of ground facts and every ground fact φ over the signature of $\mathcal{T}_1$, we have $\mathcal{T}_1, \mathcal{F} \models \varphi$ iff $\mathcal{T}_2, \mathcal{F} \models \varphi$. If such a rewriting can always be computed, then $\mathcal{L}_1$ is (effectively, fact-entailment preserving)* rewritable *to $\mathcal{L}_2$.*

All of the works in Table 1 establish rewritability in this sense for the given fragments, and in particular preserve entailments of all role (i.e., binary) atoms. If only entailments of class (i.e., unary) atoms are of interest, then all uses of $\mathcal{ALC}$ in the table can also be replaced by $\mathcal{S}$ – and by $\mathcal{SR}$ at the cost of exponentially larger rewritings – based on common preprocessing techniques [14]. Entailment of (complex) role atoms is also a special case of regular path query answering, which can be solved by combined methods that we do not discuss here [16].

While the works in Table 1 rely on diverse rewriting methods, they must all obey some complexity-related constraints. In particular, if $\mathcal{L}_1$ is rewritable to $\mathcal{L}_2$, then $\mathcal{L}_2$'s data complexity for fact entailment subsumes that of $\mathcal{L}_1$. Hence, Horn-DLs (P-complete in data) are rewritable to Datalog, while non-Horn DLs (co-NP-complete in data) supposedly are not.

Combined complexity is also a limiting factor. It is ExpTime-complete for Datalog and co-NExpTime-complete for Datalog$^{\vee}$, but drops to P and NP, respectively, if rule sizes are bounded († in Table 1). This explains why †-rewritings from (N)ExpTime-complete DLs to Datalog$^{(\vee)}$ must produce exponentially large rule sets. The alternative is to allow polynomially growing rule

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sizes (and especially predicate arities). The last two lines in Table 1 are rewritings that use constant rule sets and rewrite TBoxes to facts in (sub)polynomial time. This works for $\mathcal{EL}^{++}$ due to its P-complexity, and for Horn-$\mathcal{ALC}$ due to the high expressive power of Datalog$^{\exists}$ theories, even when these are constant.

Work	**Source**	**Target**	**Size**
[9]	$\mathcal{ALCHIQ}$	Datalog$^{\vee}$	exp. †
[8]	Horn-$\mathcal{SHIQ}$	Datalog	exp. †
[15]	$\mathcal{SHIQ}b_s$	Datalog$^{\vee}$	exp. †
[3]	$\mathcal{SHI}$	Datalog$^{\vee}$	exp. †
[4]	Horn-$\mathcal{ALCHOIQ}$	Datalog	exp. †
[6]	Horn-$\mathcal{SHIQ}$	Datalog	exp. †
	Horn-$\mathcal{SRIQ}$	Datalog	2exp.†
[14]	Horn-$\mathcal{ALCHOIQ}$	Datalog	poly.
[1]	$\mathcal{ALCHIO}$	Datalog$^{\vee}$	poly.
[11]	$\mathcal{EL}^{++}$	Datalog	poly. †
[5]	Horn-$\mathcal{ALC}$	Datalog$^{\exists}$	poly. †

†: rules of bounded size independent on input

Table 1. From DLs to rule languages, where Datalog$^{\vee}$ and Datalog$^{\exists}$ denote Datalog extended with disjunctions and existential quantifiers

Given this rich body of research in rule rewritings, there is a surprising shortage of rewriting-based reasoners. KAON2 uses the exponential $\mathcal{SHIQ}$ rewriting of [9], and DReW the polynomial $\mathcal{EL}^{++}$ rewriting of [11] – both systems are discontinued. Rule reasoners, in contrast, have been thriving in recent years, and scalable systems exist both for Datalog$^{\vee}$ (e.g., answer set programming engines [12]) and for Datalog$^{\exists}$ (e.g., engines for existential rules [2,13,17]).

A possible explanation is that known rewritings still suffer from many shortcomings. Indeed, exponentially large rule sets (Table 1, lines 1–7) and rule sizes in the order of the ontology (lines 8–9) both impair practical performance, whereas static rewritings (lines 10–11) are often better implemented in dedicated "consequence-based" reasoners rather than relying on general-purpose rule reasoners [10]. Also, Table 1 highlights that many DLs are not supported by any polynomial rewriting.

In our work [7], we therefore study polynomial rewritings for hitherto unsupported DLs. The result are two new rewriting approaches for the DL $\mathcal{ALCHIQ}$: a rewriting to Datalog$^{\vee}$ that (unavoidably) requires unbounded (but still linear) rule sizes, and a rewriting to Datalog$^{\vee\exists}$ that achieves bounded rule sizes. Datalog with existential quantifiers is co-re-complete for fact entailment, but we show that our rewriting leads to rule sets for which entailment can be decided with standard algorithms, while still matching the original DL's co-NP data complexity. Both results are new, and also illustrate the potential advantage of considering rules with existential quantifiers as a target for rewriting.

Finally, we also consider Horn-$\mathcal{ALCHIQ}$, the disjunction-free fragment of $\mathcal{ALCHIQ}$. Whereas our rewriting of $\mathcal{ALCHIQ}$ to Datalog$^{\vee}$ and Datalog$^{\vee\exists}$, respectively, could be applied here, it produces rule sets that always contain disjunctions. We therefore combine several known results to obtain alternative, disjunction-free rewritings for this case.

The details of our rule-based rewritings are published at the 29th International Joint Conference on Artificial Intelligence (IJCAI 2020) [7].

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