k-Limited Erasing Performed by Scattered Context Grammars

Jiří Techet

Department of Information Systems, Faculty of Information Technology,
Brno University of Technology, Božetěchova 2, 612 66 Brno, Czech Republic
techet@fit.vutbr.cz

Abstract. A scattered context grammar, G, erases nonterminals in a k-limited way, where $k \geq 1$, if for every sentence belonging to $G's$ language, there is a derivation such that in every sentential form, between every two symbols from which G derives non-empty strings, there is a string of no more than k nonterminals from which G derives empty words. This paper demonstrates that any scattered context grammar that erases nonterminals in this way can be converted to an equivalent scattered context grammar without any erasing productions while in general, this conversion is impossible.

Keywords: scattered context grammars, erasure of nonterminals

1 Introduction

This paper discusses scattered context grammars, which represent an important type of semi-parallel grammars (see [1–4, 6–8, 12, 13]). It concentrates its investigation on the role of erasing productions and the way they are applied in these grammars. While scattered context grammars with erasing productions characterize the family of recursively enumerable languages, the same grammars without erasing productions cannot generate any non-context-sensitive language (see [3, 4]). As a result, in general, we cannot convert any scattered context grammar with erasing productions to an equivalent scattered context grammar without these productions. In this paper, we demonstrate that this is always possible if the original grammar *erases its nonterminals in a k-limited way*, where k is a positive integer; for every sentence there is a derivation such that in every sentential form, between any two symbols from which the grammar derives non-empty strings, there is a string of no more than k nonterminals from which the grammar derives empty strings later in the derivation. Consequently, the scattered grammars that have erasing productions but apply them in a k-limited way are equivalent to the grammars that do not have erasing productions at all.

In [3] it was demonstrated that a language generated by propagating scattered context grammars is closed under restricted homomorphism. Note that our definition of k-limited erasing differs significantly from the way how symbols can be erased using restricted homomorphism. While in case of restricted homomorphism a language can be generated by a propagating scattered context grammar in case that at most k symbols are deleted between every two terminals in a sentence, in case of a scattered context grammar which erases its nonterminals in a k-limited way virtually unlimited number of symbols can be deleted between every two terminals in a sentence in case that during the derivation process between two non-erasable symbols there is a string of at most k erasable symbols. Therefore, the result presented in this paper represents a generalization of the previously published result.

2 Preliminaries

We assume that the reader is familiar with the language theory (see [5, 9–11]). For an alphabet, V, $\mathrm{card}(V)$ denotes the cardinality of V. V^* represents the free monoid generated by V under the operation of concatenation. The unit of V^* is denoted by ε. Set $V^+ = V^* - \{\varepsilon\}$. For $w \in V^*$, $|w|$ and $\mathrm{alph}(w)$ denote the length of w and the set of symbols occurring in w, respectively. For $L \subseteq V^*$, $\mathrm{alph}(L) = \{a : a \in \mathrm{alph}(w), w \in L\}$. Let $\mathrm{pos}(a_1 \dots a_i \dots a_n, i) = a_i$ for $1 \leq i \leq n$, $a_1 \dots a_n \in V^+$.

A *context-free grammar* (see [5]), a *CFG* for short, is a quadruple, $G = (V, T, P, S)$, where V is an alphabet, $T \subseteq V$, $S \in V - T$, and P is a finite set of productions such that each production has the form $A \rightarrow x$, where $A \in V - T$, $x \in V^*$. Let $\mathrm{lhs}(A \rightarrow x)$ and $\mathrm{rhs}(A \rightarrow x)$ denote A and x, respectively. If $A \rightarrow x \in P$, $u = rAs$, and $v = rxs$, where $r, s \in V^*$, then G makes a *derivation step* from u to v according to $A \rightarrow x$, symbolically written as $u \Rightarrow v\ [A \rightarrow x]$ in G or, simply, $u \Rightarrow v$. Let $\Rightarrow^+$ and $\Rightarrow^*$ denote the transitive closure of $\Rightarrow$ and the transitive-reflexive closure of $\Rightarrow$, respectively. The *language of* G is denoted by $L(G)$ and defined as $L(G) = \{x : x \in T^*, S \Rightarrow^* x\}$.

3 Definitions and Examples

A *scattered context grammar* (see [1–4, 6–8, 12, 13]), a *SCG* for short, is a quadruple, $G = (V, T, P, S)$, where V is an alphabet, $T \subseteq V$, $S \in V - T$, and P is a finite set of productions such that each production has the form $(A_1, \dots, A_n) \rightarrow (x_1, \dots, x_n)$, for some $n \geq 1$, where $A_i \in V - T$, $x_i \in V^*$, for $1 \leq i \leq n$. If every production $(A_1, \dots, A_n) \rightarrow (x_1, \dots, x_n) \in P$ satisfies $x_i \in V^+$ for all $1 \leq i \leq n$, G is a *propagating scattered context grammar*, a *PSCG* for short. If $(A_1, \dots, A_n) \rightarrow (x_1, \dots, x_n) \in P$, $u = u_1 A_1 u_2 \dots u_n A_n u_{n+1}$, and $v = u_1 x_1 u_2 \dots u_n x_n u_{n+1}$, where $u_i \in V^*$, $1 \leq i \leq n$, then G makes a *derivation step* from u to v according to $(A_1, \dots, A_n) \rightarrow (x_1, \dots, x_n)$, symbolically written as $u \Rightarrow v\ [(A_1, \dots, A_n) \rightarrow (x_1, \dots, x_n)]$ in G or, simply, $u \Rightarrow v$. Set $\pi((A_1, \dots, A_n) \rightarrow (x_1, \dots, x_n)) = |A_1 \dots A_n| = n$ and $\rho((A_1, \dots, A_n) \rightarrow (x_1, \dots, x_n)) = \{A_1 \rightarrow x_1, \dots, A_n \rightarrow x_n\}$. Let $\Rightarrow^+$ and $\Rightarrow^*$ denote the transitive closure of $\Rightarrow$ and the transitive-reflexive closure of $\Rightarrow$, respectively. The *language of* G is denoted by $L(G)$ and defined as $L(G) = \{x : x \in T^*, S \Rightarrow^* x\}$.

The *core grammar underlying a scattered context grammar*, $G = (V, T, P, S)$, is denoted by core(G) and defined as the context-free grammar core$(G) = (V, T, \mathrm{cf}(P), S)$ with $\mathrm{cf}(P) = \{B \to y : B \to y \in \rho(p)$ for some $p \in P\}$. Let $v = u_1A_1u_2A_2 \dots u_nA_nu_{n+1} \Rightarrow u_1x_1u_2x_2 \dots u_nx_nu_{n+1} = w\ [(A_1, \dots, A_n) \to (x_1, \dots, x_n)]$ in G. The *partial m-step context-free simulation* of this step by core(G) is denoted by $\mathrm{pcf}_m(v \Rightarrow w)$ and defined as core(G)*'s* m-step derivation of the form $u_1A_1u_2A_2 \dots u_nA_nu_{n+1} \Rightarrow u_1x_1u_2A_2 \dots u_nA_nu_{n+1} \Rightarrow \dots \Rightarrow u_1x_1u_2x_2 \dots u_mx_mu_{m+1}A_{m+1} \dots u_nA_nu_{n+1}$ where $m \leq n$. The *context-free simulation* is a special case of the partial m-step context-free simulation for $m = n$, denoted by $\mathrm{cf}(v \Rightarrow w)$. Let $v = v_1 \Rightarrow^* v_n = w$ be a derivation in G of the form $v_1 \Rightarrow v_2 \Rightarrow v_3 \Rightarrow \dots \Rightarrow v_n$. The context-free simulation of $v \Rightarrow^* w$ by core(G) is denoted as $\mathrm{cf}(v \Rightarrow^* w)$ and defined as $v_1 \Rightarrow^* v_2 \Rightarrow^* v_3 \Rightarrow^* \dots \Rightarrow^* v_n$ such that for all $1 \leq i \leq n-1$, $v_i \Rightarrow^* v_{i+1}$ in core(G) is the context-free simulation of $v_i \Rightarrow v_{i+1}$ in G. Let $S \Rightarrow^* x$ in G be of the form $S \Rightarrow^* uAv \Rightarrow^* x$. Let $\mathrm{cf}(S \Rightarrow^* x)$ in core(G) be the context-free simulation of $S \Rightarrow^* x$ in G. Let t be the derivation tree corresponding to $S \Rightarrow^* x$ in core(G) (regarding derivation trees and related notions, we use the terminology of [5]). Consider a subtree rooted at A in t. If the frontier of this subtree is ε, then G *erases* A in $S \Rightarrow^* uAv \Rightarrow^* x$, symbolically written as $\check{A}$, and if this frontier differs from ε, then G *does not erase* A during this derivation, symbolically written as $\hat{A}$. If $w = \hat{A}_1 \dots \hat{A}_n$ or $w = \check{A}_1 \dots \check{A}_n$, we write $\hat{w}$ or $\check{w}$, respectively. Let $G = (V, T, P, S)$ be a *SCG*, and let $k \geq 0$. G *erases its nonterminals in a k-limited way* if for every $y \in L(G)$ there exists a derivation $S \Rightarrow^* y$ such that every sentential form x of the derivation satisfies the following two properties:

1. Every $x = uAvBw$, $\hat{A}$, $\hat{B}$, $\check{v}$, satisfies $|v| \leq k$.
2. Every $x = uAw$, $\hat{A}$, satisfies: if $\check{u}$ or $\check{w}$, then $|u| \leq k$ or $|w| \leq k$, respectively.

Examples

1. Observe that the grammar $G_1 = (\{S, A, B, C, A', B', C', a, b, c, y\}, \{a, b, c, y\}, \{(S) \to (ABC), (A) \to (aAA'), (B) \to (bBB'), (C) \to (cCC'), (A, B, C) \to (y, y, y), (A', B', C') \to (y, y, y)\}, S)$ generates the language $L(G_1) = \{a^n y^{n+1} b^n y^{n+1} c^n y^{n+1} : n \geq 0\}$. Therefore, there does not exist any restricted homomorphism h such that $h(L(G_1)) = \{a^nb^nc^n : n \geq 0\}$. However, as demonstrated by the following example, there exists a scattered context grammar which erases its nonterminals in a k-limited way.
2. Observe that the grammar $G_2 = (\{S, A, B, C, A', B', C', a, b, c\}, \{a, b, c\}, \{(S) \to (ABC), (A) \to (aAA'), (B) \to (bBB'), (C) \to (cCC'), (A, B, C) \to (\varepsilon, \varepsilon, \varepsilon), (A', B', C') \to (\varepsilon, \varepsilon, \varepsilon)\}, S)$ generates the language $L(G_2) = \{a^nb^nc^n : n \geq 0\}$. As the derivation of any string $aa \dots aabb \dots bbcc \dots cc \in L(G_2)$ may be of the form

$$
\begin{aligned}
S &\Rightarrow ABC \Rightarrow^* aAA'bBB'cCC' \Rightarrow aAbBcC \\
&\Rightarrow^* aaAA'bbBB'ccCC' \Rightarrow aaAbbBccC \\
&\Rightarrow^* aa \dots aaAA'bb \dots bbBB'cc \dots ccCC' \\
&\Rightarrow aa \dots aaAbb \dots bbBcc \dots ccC \Rightarrow aa \dots aabb \dots bbcc \dots cc,
\end{aligned}
$$

the grammar erases its nonterminals in a 2-limited way.

3. Consider the grammar $G_3 = (\{S, A, B, A', B', a, b, c\}, \{a, b, c\}, \{(S) \to (AA), (A, A) \to (aA, A'A), (A, A) \to (B, B), (B, B) \to (bBc, B'B), (B, B) \to (\varepsilon, \varepsilon), (A', B') \to (\varepsilon, \varepsilon)\}, S)$. Observe that $L(G_3) = L(G_2)$. However, because the first part of every derivation has the form

$$S \Rightarrow AA \Rightarrow aAA'A \Rightarrow aaAA'A'A \Rightarrow^* aa \dots aAA'A' \dots A'A$$

and all A''s are deleted in the second part of the derivation, there does not exist any k such that G_3 erases its nonterminals in a k-limited way.

4 Results

The main result of this paper follows next.

Theorem 1. *For every SCG, G, which erases its nonterminals in a k-limited way there exists a PSCG, $\bar{G}$, such that $L(G) = L(\bar{G})$.*

Proof. Let $G = (V, T, P, S)$ be a *SCG* which erases its nonterminals in a k-limited way. For every $p = (A_1, \dots, A_i, \dots, A_n) \to (x_1, \dots, x_i, \dots, x_n) \in P$ let $\lfloor p, i \rfloor$ denote $A_i \to x_i$ for all $1 \le i \le n$. Let $\Psi = \{\lfloor p, i \rfloor : p \in P, 1 \le i \le \pi(p)\}$ and $\Psi' = \{\lfloor p, i \rfloor' : \lfloor p, i \rfloor \in \Psi\}$. Set $\bar{N}_1 = \{\langle x \rangle : x \in (V - T)^* \cup (V - T)^* T (V - T)^*, |x| \le 2k + 1\}$. For every $\langle x \rangle \in \bar{N}_1$ and $\lfloor p, i \rfloor \in \Psi$, define

$$\text{lhs-replace}(\langle x \rangle, \lfloor p, i \rfloor) = \{\langle x_1 \lfloor p, i \rfloor x_2 \rangle : x_1, x_2 \in V^*, x_1 \,\text{lhs}(\lfloor p, i \rfloor) x_2 = x\}.$$

Set $\bar{N}_2 = \{\langle x \rangle : \langle x \rangle = \text{lhs-replace}(\langle y \rangle, \lfloor p, i \rfloor), \langle y \rangle \in \bar{N}_1, \lfloor p, i \rfloor \in \Psi\}$. For every $\langle x \rangle \in \bar{N}_1$ and $\lfloor p, i \rfloor' \in \Psi'$, define

$$\text{insert}(\langle x \rangle, \lfloor p, i \rfloor') = \{\langle x_1 \lfloor p, i \rfloor' x_2 \rangle : x_1, x_2 \in V^*, x_1 x_2 = x\}.$$

Set $\bar{N}_2' = \{\langle x \rangle : \langle x \rangle = \text{insert}(\langle y \rangle, \lfloor p, i \rfloor'), \langle y \rangle \in \bar{N}_1, \lfloor p, i \rfloor' \in \Psi'\}$. For every $x = \langle x_1 \rangle \langle x_2 \rangle \dots \langle x_n \rangle \in (\bar{N}_1 \cup \bar{N}_2 \cup \bar{N}_2')^*$ for some $n \ge 1$, define

$$\text{join}(x) = x_1 x_2 \dots x_n.$$

For every $x \in \bar{N}_1 \cup \bar{N}_2 \cup \bar{N}_2'$, define

$$\text{split}(x) = \{y : x = \text{join}(y)\}.$$

Set $\bar{V} = T \cup \bar{N}_1 \cup \bar{N}_2 \cup \bar{N}_2' \cup \{\bar{S}\}$. Define the *PSCG*,

$$\bar{G} = (\bar{V}, T, \bar{P}, \bar{S}),$$

with $\bar{P}$ constructed as follows:

1. For every $p = (S) \to (x) \in P$, add $(\bar{S}) \to (\langle \lfloor p, 1 \rfloor \rangle)$ to $\bar{P}$;

2. For every $\langle x \rangle \in \bar{N}_1$, every $X \in \text{insert}(\langle x \rangle, \lfloor p, n \rfloor')$, where $p \in P$, $\pi(p) = n$, every $\langle y \rangle \in \bar{N}_1$, and every $Y \in \text{lhs-replace}(\langle y \rangle, \lfloor q, 1 \rfloor)$, where $q \in P$, add
   (a) $(X, \langle y \rangle) \to (\langle x \rangle, Y)$, and
   (b) $(\langle y \rangle, X) \to (Y, \langle x \rangle)$ to $\bar{P}$;
   (c) if $\langle x \rangle = \langle y \rangle$, add
   $(X) \to (Y)$ to $\bar{P}$;
   (d) $(X) \to (\langle x \rangle)$ to $\bar{P}$;
3. For every $\langle x \rangle \in \bar{N}_1$, every $X \in \text{insert}(\langle x \rangle, \lfloor p, i \rfloor')$, where $p \in P$, $i < \pi(p)$, every $\langle y \rangle \in \bar{N}_1$, and every $Y \in \text{lhs-replace}(\langle y \rangle, \lfloor p, i+1 \rfloor)$, where $q \in P$, add
   (a) $(X, \langle y \rangle) \to (\langle x \rangle, Y)$ to $\bar{P}$;
   (b) if $\langle x \rangle = \langle y \rangle$ and $\text{pos}(X, l) = \lfloor p, i \rfloor'$, $\text{pos}(Y, m) = \lfloor p, i+1 \rfloor'$, $l < m$, add
   $(X) \to (Y)$ to $\bar{P}$;
4. For every $\langle x_1 \lfloor p, i \rfloor x_2 \rangle \in \text{lhs-replace}(\langle x \rangle, \lfloor p, i \rfloor)$, $\langle x \rangle \in \bar{N}_1$, $\lfloor p, i \rfloor \in \Psi$, $x_1, x_2 \in V^*$, and every $Y \in \text{split}(x_1 \, \text{rhs}(\lfloor p, i \rfloor) \lfloor p, i \rfloor' x_2)$, add
   $(\langle x_1 \lfloor p, i \rfloor x_2 \rangle) \to (Y)$ to $\bar{P}$;
5. For every $a \in T$, add
   $(\langle a \rangle) \to (a)$ to $\bar{P}$.

Denote the set of productions introduced in step i of the construction by ${}_iP$, for $1 \le i \le 5$.

Let $S \Rightarrow^* y \Rightarrow^* w$ in $\bar{G}$, $w \in L(\bar{G})$, $y \in (\bar{N}_1 \cup \bar{N}_2 \cup \bar{N}_2')^*$, and let for every $\langle z \rangle \in \text{alph}(y)$ there exist

1. $A \in \text{alph}(z)$ such that $\hat{A}$ or
2. $\lfloor p, i \rfloor \in \text{alph}(z)$, $\lfloor p, i \rfloor \in \Psi$, $A = \text{lhs}(\lfloor p, i \rfloor)$ such that $\hat{A}$.

Then we write $\tilde{y}$.

Basic Idea. $\bar{G}$ simulates G by using nonterminals of the form $\langle \ldots \rangle$. In each nonterminal of this form, during every simulated derivation step, $\bar{G}$ records a substring of the corresponding current sentential form of G.

The rule constructed in (1) only initializes the simulation process. By rules introduced in (2) through (4), $\bar{G}$ simulates the application of a scattered context rule p from P in a left-to-right way. In greater detail, by using a rule of (2), $\bar{G}$ nondeterministically selects a scattered context rule p from P. Suppose that p consists of context-free rules $r_1, \ldots, r_{i-1}, r_i, \ldots, r_n$. By using rules of (3) and (4), $\bar{G}$ simulates the application of r_1 through r_n one by one. To explain this in greater detail, suppose that $\bar{G}$ has just completed the simulation of r_{i-1}. Then, to the right of this simulation, $\bar{G}$ selects $\text{lhs}(r_i)$ by using a rule of (3). That is, this selection is made inside of $\bar{G}$'s nonterminal in which the simulation of r_{i-1} has been performed or in one of the nonterminals appearing to the right of this nonterminal. After this selection, by using a rule of (4), $\bar{G}$ performs the replacement of the selected symbol $\text{lhs}(r_i)$ with $\text{rhs}(r_i)$.

If a terminal occurs inside of a nonterminal of $\bar{G}$, then a rule of (5) allows $\bar{G}$ to change this nonterminal to the terminal string contained in it.

Rigorous Proof. (Due to the requirements imposed on the length of this paper, the formal proofs of the following three lemmas are omitted.)

Lemma 1. *Every successful derivation in $\bar{G}$ can be expressed in the following way:*

$$\begin{aligned} \bar{S} &\Rightarrow_{\bar{G}} \langle \lfloor p, 1 \rfloor \rangle && [p_1] \\ &\Rightarrow^{*}_{\bar{G}} u && [\Phi] \\ &\Rightarrow^{+}_{\bar{G}} v && [\Theta], \end{aligned}$$

where $p_1 = (\bar{S}) \to (\lfloor p, 1 \rfloor) \in {}_1P$, $u = \langle a_1 \rangle \langle a_2 \rangle \dots \langle a_n \rangle$, $n \geq 1$, $a_1, a_2, \dots, a_n \in T$, $v = a_1 a_2 \dots a_n$, Φ and Θ are sequences of productions from $({}_2P \cup {}_3P \cup {}_4P)$, and ${}_5P$, respectively.

Lemma 2. *Let*

$$w_1 \in \mathrm{split}(u_1 \lfloor p, 1 \rfloor u_2 A_2 \dots u_n A_n u_{n+1}) \text{ and } \lambda = u_1 A_1 u_2 A_2 \dots u_n A_n u_{n+1},$$

$u_1, \dots, u_{n+1} \in V^*$, $A_1, \dots, A_n \in V - T$, $A_1 = \mathrm{lhs}(\lfloor p, 1 \rfloor)$, $p \in P$, and $\tilde{w}_1$; then, every partial h-step context-free simulation

$$\begin{aligned} \mathrm{pcf}_h(\lambda = {} & u_1 A_1 u_2 A_2 \dots u_n A_n u_{n+1} \\ &\Rightarrow_G u_1 x_1 u_2 x_2 \dots u_n x_n u_{n+1} \quad [p = (A_1, \dots, A_n) \to (x_1, \dots, x_n)]) \end{aligned}$$

of the form

$$\begin{aligned} & u_1 A_1 u_2 A_2 u_3 A_3 \dots u_n A_n u_{n+1} = \lambda \\ \Rightarrow_{\mathrm{core}(G)} \; & u_1 x_1 u_2 A_2 u_3 A_3 \dots u_n A_n u_{n+1} && [A_1 \to x_1] \\ \Rightarrow_{\mathrm{core}(G)} \; & u_1 x_1 u_2 x_2 u_3 A_3 \dots u_n A_n u_{n+1} && [A_2 \to x_2] \\ \Rightarrow^{h-2}_{\mathrm{core}(G)} \; & u_1 x_1 u_2 x_2 u_3 x_3 \dots u_h x_h u_{h+1} A_{h+1} \dots u_n A_n u_{n+1} \end{aligned}$$

is performed in $\mathrm{core}(G)$ *if and only if*

$$\begin{aligned} & w_1 \\ \Rightarrow_{\bar{G}} \; & w'_1 && [p^4_1] \\ \Rightarrow_{\bar{G}} \; & w_2 && [p^3_2] \\ \Rightarrow_{\bar{G}} \; & w'_2 && [p^4_2] \\ \Rightarrow^{2h-6}_{\bar{G}} \; & \vdots \\ \Rightarrow_{\bar{G}} \; & w_h && [p^3_h] \\ \Rightarrow_{\bar{G}} \; & w'_h && [p^4_h] \end{aligned}$$

is performed in $\bar{G}$, where $p^3_2, \dots, p^3_h \in {}_3P$, $p^4_1, \dots, p^4_h \in {}_4P$, and

$$\begin{aligned} & w'_1 \in \mathrm{split}(u_1 x_1 \lfloor p, 1 \rfloor' u_2 A_2 \dots u_n A_n u_{n+1}), \\ & w_2 \in \mathrm{split}(u_1 x_1 u_2 \lfloor p, 2 \rfloor \dots u_n A_n u_{n+1}), \\ & w'_2 \in \mathrm{split}(u_1 x_1 u_2 x_2 \lfloor p, 2 \rfloor' \dots u_n A_n u_{n+1}), \\ & \vdots \\ & w_h \in \mathrm{split}(u_1 x_1 u_2 x_2 \dots u_h \lfloor p, h \rfloor u_{h+1} A_{h+1} \dots u_n A_n u_{n+1}), \\ & w'_h \in \mathrm{split}(u_1 x_1 u_2 x_2 \dots u_h x_h \lfloor p, h \rfloor' u_{h+1} A_{h+1} \dots u_n A_n u_{n+1}); \end{aligned}$$

in addition, every $w \in \{w_2, \dots, w_h, w'_1, \dots, w'_h\}$ satisfies $\tilde{w}$.

Corollary 1. *The result from Lemma 2 holds for a context-free simulation.*

Let

$$\begin{aligned} & u_1 A_1 u_2 A_2 \dots u_n A_n u_{n+1} \\ \Rightarrow_G \ & u_1 x_1 u_2 x_2 \dots u_n x_n u_{n+1} \quad [p = (A_1, \dots, A_n) \to (x_1, \dots, x_n)]) \end{aligned}$$

for some $p \in P$. Then, $\rightharpoonup$ denotes the simulation of this derivation step in $\bar{G}$ as shown in Lemma 2 and Corollary 1. We write $w_1 \rightharpoonup w'_n$ $[p]$, or, shortly, $w_1 \rightharpoonup w'_n$. Therefore, $w_1 \Rightarrow_{\bar{G}}^{2n-1} w'_n$ from Lemma 2 is equivalent to $w_1 \rightharpoonup w'_n$.

Lemma 3. *Let $x_1 \in V^*$ and $\bar{x}'_1 \in \mathrm{split}(x'_{11} \lfloor p_1, 1 \rfloor x'_{12})$, where $x'_{11} \,\mathrm{lhs}(\lfloor p_1, 1 \rfloor) x'_{12} = x_1$, $\lfloor p, 1 \rfloor \in \Psi$, and $\bar{\bar{x}}'_1$; then every derivation*

$$\begin{aligned} & x_1 \\ \Rightarrow_G \ & x_2 \quad [p_1] \\ & \vdots \\ \Rightarrow_G \ & x_{m+1} \quad [p_m] \end{aligned}$$

is performed in G if and only if

$$\begin{aligned} & \bar{x}'_1 \\ \rightharpoonup \ & \bar{x}_1 \quad [p_1] \\ \Rightarrow_{\bar{G}} \ & \bar{x}'_2 \quad [p'_2] \\ \rightharpoonup \ & \bar{x}_2 \quad [p_2] \\ \Rightarrow_{\bar{G}} \ & \bar{x}'_3 \quad [p'_3] \\ & \vdots \\ \rightharpoonup \ & \bar{x}_m \quad [p_m] \\ \Rightarrow_{\bar{G}} \ & \bar{x}'_{m+1} \quad [p'_{m+1}] \end{aligned}$$

is performed in $\bar{G}$, where $x_2, \dots, x_{m+1} \in V^*$, $p_1, \dots, p_m \in P$, $p'_2, \dots, p'_{m+1} \in {}_2P$,

$$\begin{aligned} \bar{x}_i &\in \mathrm{split}(x_{i1} \lfloor p_i, \pi(p_i) \rfloor' x_{i2}), \\ \bar{x}'_j &\in \mathrm{split}(x'_{j1} \lfloor p_j, 1 \rfloor x'_{j2}), \end{aligned}$$

for all $1 \le i \le m$, $2 \le j \le m$, and

$$\bar{x}'_{m+1} \in \mathrm{split}(x'_{(m+1)1} \lfloor p_{m+1}, 1 \rfloor x'_{(m+1)2}) \text{ with } x_{m+1} \notin T^*,$$

or

$$\bar{x}'_{m+1} \in \mathrm{split}(x_{m+1}) \text{ with } x_{m+1} \in T^*,$$

where $x_{i1} x_{i2} = x_i$ for all $1 \le i \le m$, $x'_{j1} \,\mathrm{lhs}(\lfloor p_j, 1 \rfloor) x'_{j2} = x_j$ for all $2 \le j \le m+1$, and every $\bar{x} \in \{\bar{x}_1, \dots, \bar{x}_m, \bar{x}'_2, \dots, \bar{x}'_{m+1}\}$ satisfies $\bar{\bar{x}}$.

From Lemma 1,

$$\bar{S} \Rightarrow_{\bar{G}} \langle \lfloor p, 1 \rfloor \rangle.$$

As $\langle \lfloor p, 1 \rfloor \rangle \in \text{split}(\lfloor p, 1 \rfloor)$, $S = \text{lhs}(\lfloor p, 1 \rfloor)$, G's simulation as described in Lemma 3 can be performed, so

$$\langle \lfloor p, 1 \rfloor \rangle \Rightarrow^*_{\bar{G}} u\ [\Phi],$$

where Φ is a sequence of productions from ${}_2P \cup {}_3P \cup {}_4P$. If a successful derivation is simulated, then we obtain $u = \langle a_1 \rangle \langle a_2 \rangle \dots \langle a_n \rangle$, $n \geq 1$, $a_1, a_2, \dots, a_n \in T$. Finally, by the application of productions from ${}_5P$ we obtain

$$u \Rightarrow^+_{\bar{G}} v,$$

where $v = a_1 a_2 \dots a_n$. Therefore, for every SCG, G, which erases its nonterminals in a k-limited way there exists a $PSCG$, $\bar{G}$, such that $L(G) = L(\bar{G})$. □

This work was supported by GAČR grant 102/05/H050 and FRVŠ grant FR762/2007/G1.

References

1. H. Fernau. Scattered context grammars with regulation. *Annals of Bucharest Univ., Math.-Informatics Series*, 45(1):41–49, 1996.
2. J. Gonczarowski and M. K. Warmuth. Scattered versus context-sensitive rewriting. *Acta Informatica*, 27:81–95, 1989.
3. S. Greibach and J. E. Hopcroft. Scattered context grammars. *Journal of Computer and System Sciences*, 3:233–247, 1969.
4. A. Meduna. A trivial method of characterizing the family of recursively enumerable languages by scattered context grammars. *EATCS Bulletin*, 56:104–106, 1995.
5. A. Meduna. *Automata and Languages: Theory and Applications*. Springer-Verlag, London, 2000.
6. A. Meduna. Generative power of three-nonterminal scattered context grammars. *Theoretical Computer Science*, 237:625–631, 2000.
7. A. Meduna and J. Techet. Generation of sentences with their parses: the case of propagating scattered context grammars. *Acta Cybernetica*, 17:11–20, 2005.
8. D. Milgram and A Rosenfeld. A note on scattered context grammars. *Information Processing Letters*, 1:47–50, 1971.
9. G. E. Revesz. *Introduction to Formal Language Theory*. McGraw-Hill, New York, 1983.
10. G. Rozenberg and A. Salomaa, editors. *Handbook of Formal Languages*, volume 1 through 3. Springer-Verlag, Berlin, 1997.
11. A. Salomaa. *Formal Languages*. Academic Press, London, 1973.
12. G. Vaszil. On the descriptional complexity of some rewriting mechanisms regulated by context conditions. *Theoretical Computer Science*, 330:361–373, 2005.
13. V. Virkkunen. On scattered context grammars. *Acta Universitatis Ouluensis*, Series A, Mathematica 6:75–82, 1973.