More than just One Box

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Abstract. To support the representation of furniture assembly, we need an ontology to describe the shapes of integrated three-dimensional objects. There are few existing formal axiomatizations in this domain. MWorld Ontology, with nine modules, is a first order logic ontology proposed in this paper which allows the description of topological shapes composed of boxsets, boxes, surfaces, edges and points. We introduced boxset as the class for the shapes of integrated three-dimensional objects, *componentOf* as the proper parthood relation between the components and the whole, as well as *semicomplements* to capture the relationship between the disjoint components of the same whole. We proposed terms for coincident shapes in adjacent components: joint point, joint edge and joint surface. We reused CardWorld and BoxWorld, as an extension, we introduced *featureOf* as a new parthood relation to represent the relationship between substructure or basic shapes and their superstructure. As such, these theories can represent aspects of shapes with different dimensions following multidimensional and mereological pluralism approaches. All concepts and relationships are axiomatized and demonstrated with examples.

Keywords. Shape Ontology, Multidimensional, Integrated Objects, Component, Solid Physical Objects, Parthood, Topology, Mereological Pluralism

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Use Case Expecting parents Alice and Bob were looking for children's furniture for their soon to arrive baby, they logged into the SNM portal and made a request. A family whose children are now in university was offering a crib, but it was disassembled and had been stored in garage for years. Bob accepted the kind offer and wanted to give a try with assembling the bed. Following the instruction manual, he started with the headboard section. First step was to insert support bars into the headboard bottom cross and push down until firmly in place. Then he slid the headboard top cross onto the support bars. At last he inserted the wood dowels two at a time into ends of both headboard top and bottom crosses to connect with the two headboard posts.

1. Introduction

The above use case is drawn from Social Needs Marketplace (SNM)[16]. To support the representation of such an assembly process, as demonstrated in *Figure 1*, we of course need a process ontology to specify states, activities, and different orderings on the occurrences of activities. We also need an ontology to describe the different components of the three-dimensional physical object at each stage of the assembly process, as well

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as the shapes of these components. Within this paper, we will focus on an ontology to logically specify such components and their topological shapes.

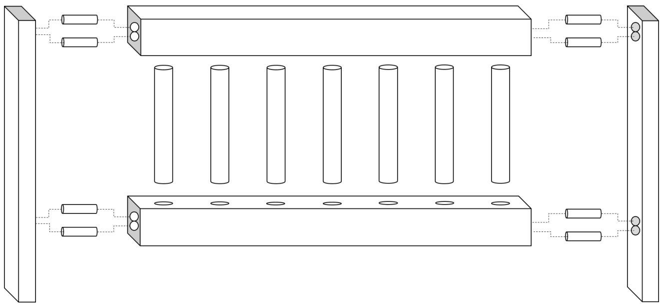

Figure 1. Simulated Instruction of Headboard Assembly

Following the principle of ontology lifecycle, we reuse two closely related existing first-order ontologies for surfaces and boxes, CardWorld[8] and BoxWorld[1][6]. There are four disjoint categories of entities within a model domain of the CardWorld and BoxWorld Ontologies: points, edges, surfaces, and boxes, corresponding to the shapes of zero-, one-, two-, and three-dimensional objects, respectively. For two-dimensional objects, CardWorld captures the relationship between points, edges, and surfaces. For three-dimensional objects, BoxWorld describes properties of a single box and its parts. The set of edges in a surface, the set of border edges, and the set of edges that meet at the same vertex, each forms a cyclic ordering. Both ontologies only use the notions of incidence and betweenness, rather than the Euclidean geometry as axiomatized by Hilbert and Tarski. And they have their basic ontological commitments based on a binary relation, part, which describes the incidence relations between different categories of objects.

But how about the shapes of objects that are composed by more than one box?

In this paper, we introduce MWorld Ontology, which is a first order logic ontology for shapes of integrated three-dimensional objects. Axioms of MWorld are categorized into nine modules. The relationships between these theories, the BoxWorld and the CardWorld Ontology is shown in *Figure 2* below. We also extended the BoxWorld with a *featureOf* parthood relation to represent the parthood relationship between a shape substructure and the whole, for example, holes and voids can be features of a box. There are three parthood relations in MWorld Ontology. We reuse *part* as the basic parthood relation between basic shapes of different dimensions in a weak tripartite incidence structure, an example is that edge is a part of some surface. In MWorld, we named *componentOf* as the proper parthood relationship between the composing box or boxset and composed boxset. Corresponding modules of these parthood ontologies are axiomatized and synonymous with the classic mereology.

2. Extension to BoxWorld

We introduce *featureOf* as the general parthood relation between shapes in different dimensions in BoxWorld. Following *Axiom (1)*, a feature in the BoxWorld is an enclosed

[1] `colore.oor.net/boxworld/boxworld.clif`

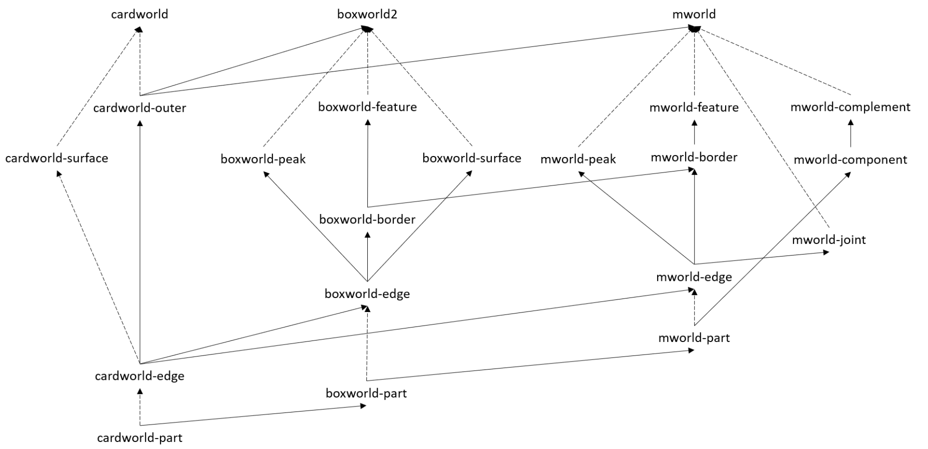

Figure 2. Relationships between the modules in the CardWorld, BoxWorld2 and MWorld Ontologies. Solid lines denote conservative extension and dashed lines denote nonconservative extension.

shape or its substructure. It can be a basic atomic shape in its dimension: a point in zero-dimension, an edge in one-dimension, a surface in two-dimension and a box in three-dimension. A feature can also be a hole, the shape of a void, and a corner of a table with three edges meet at the same vertex. *featureOf* is the parthood relation between feature and the whole. The extended BoxWorld has one new module $boxworld\_feature$ and one updated closure module $boxworld2$.

$$(\forall x,y) featureOf(x,y) \supset (\exists z) featureOf(z,y) \wedge \neg featureOf(z,x) \qquad (1)$$

f_1 to f_5 in $Figure 3$ below are examples of features of a cube.

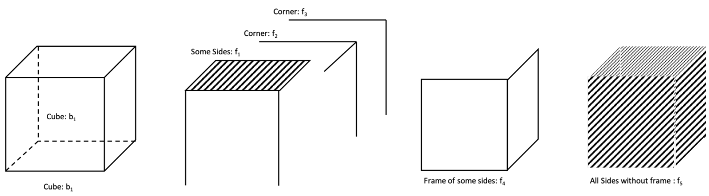

Figure 3. Examples of features to a cube

Linking back to our motivation scenario, Bob can now indicate the wear at the top of both posts, where most corners are already rounded and the straight corner features have disappeared, showing the signs of age.

3. MWorld

We need representation of a collection of connected three-dimensional shapes to describe the structure of furniture, for example the assembled children's bed. Thus, we introduce MWorld as an extension to BoxWorld that was able to represent shapes that are composed of multiple boxes. The axioms of MWorld are decomposed into nine modules: *mworld_part*, *mworld_edge*, *mworld_border*, *mworld_peak*, *mworld_component*, *mworld_joint*, *mworld_complement*, *mworld_feature* and *mworld*.

The module with basic ontological commitments *mworld_part* extends *boxworld_part* with the new class of boxset, elements of which are composed of boxes. The module *mworld_edge* is an adjustment from *boxworld_edge* with violating axioms removed, for instance, "Every border meets another unique border at a vertex." is no longer true in MWorld, as in *Figure 5(v)*, two borders meet at a joint edge. The modules *mworld_border* and *mworld_peak* are adjustments from *boxworld_border* and *boxworld_peak* with updated importing statements from MWorld. As in the updated BoxWorld above, *featureOf* in boxsets also represents incomplete shapes - it can be a hole in the boxset, a void, or a boxset with some surfaces missing. The sole axiom in the module *mworld* is a closure axiom, so that all objects are either points, edges, surfaces, boxes or boxsets.

3.1. Boxset and its components

In MWorld, we introduce a new parthood relation *componentOf*, which is a primitive relation to describe the proper parthood between the boxes or boxsets composing a boxset and the whole boxset, as axiomatized in *Axiom (2)* and *(3)* from *Figure 4*. *Axioms (4)* and *(5)* ensure that a boxset is composed by at least two components and every box in a boxset meets another distinct box in that boxset. *Axiom (6)* shows that two boxes or boxsets are semicomplements of each other when they are both components of the same boxset and they don't overlap in features or boxes.[2]

One thing to note is that boxsets are composed of boxes, but a box is not always part of a boxset. This nature is unlike the basic ontological commitments of CardWorld and BoxWorld, where the part relation forms a weak tripartite incidence structure over the three disjoint sorts of objects: every point is part of an edge, every edge is part of a surface, and every surface is part of a box.

3.2. Joints that coincide

A joint as defined in *Axiom (7)*, either coincides with a part of a component or is created to be coincident with the intersection between two components. We say that two shapes coincide when they are congruent and there is no distance between them. *Axiom (8)* makes sure that if there is a part of a component that coincides with a joint, then there must be another component of the same whole that meets this component at the joint.

Examples of how boxsets can be composed by boxes are listed in *Figure 5*. Examples *5(i)* to *5(iv)* show the scenarios of a joint point: *5(i)* shows that point p_1 of cone b_2 meets surface s_1 of cube b_1 at joint point j_{p1} which coincides with p_1; *5(ii)* shows that point p_1 of cube b_1 meets point/vertex p_2 of cone b_2 at joint point j_{p1} which coincides with p_1 and p_2; *5(iii)* shows that point p_1 of cone b_2 meets edge/boundary e_1 of cube b_1 at joint point j_{p1} which coincides with p_1; and *5(iv)* shows that edge/boundary e_1 of cube

[2]In Lattice Theory, the definition to semicomplement is: in a lattice L bounded below an element y is called complement of x if $x \wedge y = 0$; and L is said to be *semicomplemented* (SC) if each $x \in L$ (with $x \neq 1$ if 1 exists in L) admits at least one nonzero semicomplement.[17]

$$(\forall x,y) componentOf(x,y) \supset (Box(x) \vee Boxset(x)) \wedge Boxset(y) \tag{2}$$

$$(\forall x,y) componentOf(x,y) \supset (\exists z) componentOf(z,y) \wedge \neg componentOf(z,x) \tag{3}$$

$$(\forall b_1,x) componentOf(b_1,x) \supset (\exists b_2) componentOf(b_2,x) \wedge (b_1 \neq b_2) \tag{4}$$

$$\begin{aligned}&(\forall b_1,x) componentOf(b_1,x) \wedge Box(b_1) \supset \\ &(\exists b_2,j) componentOf(b_2,x) \wedge Box(b_2) \wedge (b_1 \neq b_2) \wedge meets(b_1,b_2,j)\end{aligned} \tag{5}$$

$$\begin{aligned}&(\forall x,y) semicomplements(x,y) \supset \neg componentOf(x,y) \wedge \neg componentOf(y,x) \wedge (x \neq y) \wedge \\ &((\neg\exists z) componentOf(z,x) \wedge componentOf(z,y)) \wedge \\ &((\exists b) Boxset(b) \wedge componentOf(x,b) \wedge componentOf(y,b))\end{aligned} \tag{6}$$

$$\begin{aligned}&(\forall j) Joint(j) \supset (\exists b_1,b_2,y_1,y_2) semicomplements(b_1,b_2) \wedge box(b_1) \wedge box(b_2) \wedge \\ &part(y_1,b_1) \wedge part(y_2,b_2) \wedge (coincides(j,y_1) \vee coincides(j,y_2) \vee \\ &(coincides(j,y_1) \wedge coincides(j,y_2)) \vee ((\exists i) intersection(i,y_1,y_2) \wedge coincides(j,i)))\end{aligned} \tag{7}$$

$$\begin{aligned}&(\forall b_1,x,j)(\exists y) componentOf(b_1,x) \wedge Joint(j) \wedge part(y,b_1) \wedge coincides(j,y) \supset \\ &(\exists b_2) componentOf(b_2,x) \wedge meets(b_1,b_2,j)\end{aligned} \tag{8}$$

Figure 4. Selected Axioms from $T_{component}$, T_{joint} and $T_{semicomplement}$ in MWorld Ontology

b_1 meets the surface s_1 of sphere b_2 at a new joint point j_{p1}. The shape of sphere is a box consisting of one sole surface, and it does not have any edge or point.

Example *5(v)* to *5(vii)* show the scenarios of a joint edge: *5(v)* shows that edge e_1 of cube b_1 meets edge e_2 of cube b_2 at joint edge j_{e1} which coincides with e_1 and e_2, similarly, we also have p_1 of b_1 and p_3 of b_2 coincide at j_{p1} and p_2 of b_1 and p_4 of b_2 coincide at j_{p2}; *5(vi)* shows that surface s_1 of cube b_1 meets surface s_2 of cube b_2 at a joint area j_{s1}, while the boundaries of j_{s1} coincide with e_1 and e_2 as j_{e1} and j_{e2} respectively, in addition, the four circled joint points of j_{s1} also coincide with all four vertices of both j_{e1} and j_{e2}; and *5(vii)* shows that surface s_1 of cube b_1 meets surface s_2 of cylinder b_2 at the new joint edge j_{e1}, and the two vertices of j_{e1}, which are also the interceptions between s_2 and e_1 and e_2 respectively, are also created as j_{p1} and j_{p2}.

Example *5(viii)* and *5(ix)* show the scenarios of a joint surface/area: in example *5(viii)*, two cubes are stacked together horizontally, where surface s_1 of cube b_1 meets surface s_2 of cube b_2 at the new joint area j_{s1}, and similarly as in *5(vi)*, boundaries/edges and vertices/points of j_{s1} coincide with the boundaries/edges and vertices/points of s_1 and s_2; last but not least, *5(ix)* shows that surface s_1 of cube b_1 meets surface s_2 of

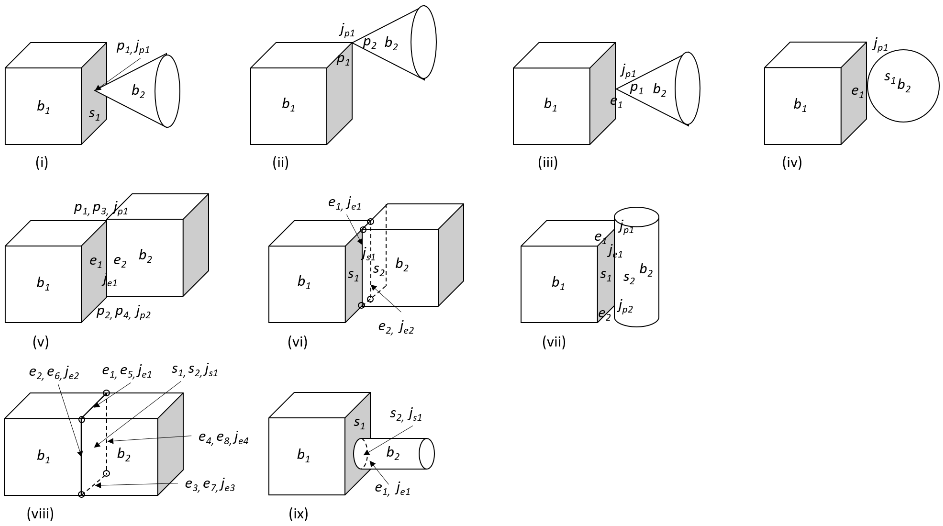

Figure 5. Examples of Joints

cylinder b_2 at joint area j_{s1} which coincides with s_2, and similarly, the boundary/edge e_1 of s_2 and b_2 is coincided with joint edge j_{e1}.

In a similar case as example *5(viii)*, if the two cubes are melted in with each other instead of stacked together, the surface s_1 of cube b_1 and the surface s_2 of cube b_2 will disappear with no new joint surface created. Same situation goes for the boundaries/edges and vertices/points that are melted in.

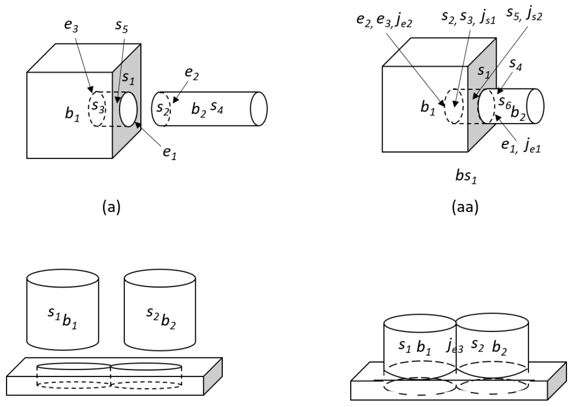

Figure 6. Illustrations of Headboard Assembly

Back to our story at the beginning of this paper, in the first step of assembling the headboard, Bob inserted the support bars into the headboard bottom cross-piece. *Figure 6(a)* simulates the scenario where the headboard bottom cross and one of the bars are placed separately. The assembled shape is demonstrated in *6(aa)*, where the middle joint edge j_{e1}, bottom joint edge j_{e2}, bottom joint area j_{s1} and round joint surface j_{s2} are specified with the original shapes they coincide with. Bob then slid the headboard top cross onto the support bars, similar to the example *5(ix)* where the joint area j_{s1} is on the bottom surface s_1 of the headboard top cross b_1, and coincides with the top surface s_2 of one of the support bars b_2. The representation of relationships in *6(aa)* are shown in *Axiom (9)* below.

$$
\begin{aligned}
&Boxset(bs_1)\\
&\wedge componentOf(b_1,bs_1)\wedge componentOf(b_2,bs_1)\wedge semicomplements(b_1,b_2)\\
&\wedge Box(b_1)\wedge Surface(s_3)\wedge Surface(s_5)\wedge Edge(e_1)\wedge Edge(e_3)\\
&\wedge featureOf(s_3,b_1)\wedge featureOf(s_5,b_1)\wedge featureOf(e_1,b_1)\wedge featureOf(e_3,b_1)\\
&\wedge Box(b_2)\wedge Surface(s_2)\wedge Surface(s_4)\wedge Edge(e_2)\\
&\wedge featureOf(s_2,b_2)\wedge featureOf(s_4,b_2)\wedge featureOf(e_2,b_2)\\
&\wedge JointEdge(j_{e1})\wedge coincides(j_{e1},e_1)\wedge featureOf(j_{e1},bs_1)\\
&\wedge JointEdge(j_{e2})\wedge coincides(j_{e2},e_2)\wedge coincides(j_{e2},e_3)\wedge featureOf(j_{e2},bs_1)\\
&\wedge JointSurface(j_{s1})\wedge coincides(j_{s1},s_2)\wedge coincides(j_{s1},s_3)\wedge featureOf(j_{s1},bs_1)\\
&\wedge JointSurface(j_{s2})\wedge coincides(j_{s2},s_5)\wedge featureOf(j_{s2},bs_1)\\
&\wedge Surface(s_6)\wedge featureOf(s_6,bs_1)
\end{aligned}
\tag{9}
$$

The last step Bob performed was inserting the wood dowels into the ends of crosses, two dowels at a time. This scenario is captured by *Figure 6(b)* and *6(bb)*, while the assembly of each individual dowel is similar with the first step featured in *Figure 6(aa)*, the adjacency of two dowels creates another joint edge marked as j_{e3}.

4. Shapes and Objects

For objects with shapes, we also have *pieceOf* from *SoPhOs*[15] as the parthood relation in shaped objects. The *pieceOf* relation is a defined relation:

$$(\forall x,y)pieceOf(x,y) \equiv (\exists f_1,f_2)bounds(f_1,x)\wedge bounds(f_2,y)\wedge featureOf(f_1,f_2) \tag{10}$$

where *bounds* is a primitive relation in *SoPhOs* captures the relationship between an enclosed or non-enclosed shape feature and the object that is partially or fully bounded by the shape feature. *featureOf* is a reflexive parthood relation in the extended BoxWorld Ontology as mentioned above in Section 2. One corresponding example is that the handle object of a coffee mug is a piece of the mug object. Of course, the *pieceOf* relation is able to describe shape identified parthood relationship of both atomic shaped objects and integrated shaped objects. For instance, the back piece of an assembled dining chair includes the back cushion and the upper piece of the wooden frame.

4.1. Shape Spatial Structure - MultiDimensional Occupy

Pieces share boundaries with the whole, while containment does not. Shape of a solid physical object encloses some physical space, it is represented with the multidimensional Occupy Ontology[7]. The parthood relationship of the physical spaces occupied by the shapes of physical entities is denoted by *containedIn*:

$$(\forall x, y)\, containedIn(x, y) \equiv (\exists r_1, r2)\, occupies(x, r_1) \land occupies(y, r_2) \land region\_part(r_1, r_2) \quad (11)$$

5. Relationship to Assembly Processes

We began the paper with a motivating scenario that described the assembly of a baby's crib. How is an ontology for assembly processes related to the shape ontology described within this paper?

Using the PSL Ontology as the underlying generic process ontology, the notion of state is represented by reified fluents. Intuitively, a change in state is captured by fluents that are either achieved or falsified by an activity occurrence. The prior relation is used to specify the fluents that are intuitively true prior to an activity occurrence and the holds relation specifies the fluents that are intuitively true after an activity occurrence. Furthermore, a fluent can only be changed by the occurrence of activities. Thus, if some fluent holds after an activity occurrence, but after an activity occurrence later along the branch it is false, then an activity must occur at some point between that changes the fluent. This also leads to the requirement that the fluents holding after an activity occurrence will be the same fluents that are prior to any successor occurrence, since there cannot be an activity occurring between them.

Using the methodology introduced by [1], we classify all possible activities in a domain by characterizing possible all changes in the domain. We translate a domain ontology to a domain state ontology. Activity occurrences correspond to mappings between models of the domain ontology. Finally, we classify activities with respect to possible changes.

For example, the mereology on material objects leads to a classification of material removal and addition activities. A mereology of components leads to activities that change the corresponding $componentOf$ fluent. Assembly activities achieve $componentOf$, while disassembly activities falsify the $componentOf$ fluent.

6. Previous Research

There are limited existing formal axiomatizations in first order logic for shape representations. Shape grammars[18] have been proposed as a way of modelling the shapes of objects; however, such approaches do not provide a logical theory, and hence do not support automated reasoning through deduction or model construction. We therefore focus only on ontological approaches. In [2], Aameri proposed the Shape Ontology as an extension to CardWorld and BoxWorld, with an extended module describing relationships between multiple boxes. In her Shape Ontology, she considers the entity that consists of multiple boxes as one dimension higher than the three-dimensional box which is a different approach than that of this paper. We took the perspective that the integration of multiple three-dimensional shapes is still three-dimensional shape, and we introduce features and joints to represent substructures and superstructure of boxes which allow description to more shape forms. The approach of multidimensional mereotopology are also adopted in GFO-Space theory[4] and CODIB[9], which deal with an arbitrary mereotopology instead of focusing on shapes of objects.

Our multidimensional approach falls into what is often called the family of 3D representation of physical objects, in which all of an object's parts exist at any point in time. This approach can also be seen in the continuants of BFO[3] and the endurants of DOLCE[12] upper ontologies, although these upper ontologies are based on a time-indexed version of mereological monism.

The term *component* is commonly adapted in works of mereology, the meaning we give is similar to the previous works, but the application domain is different. We follow the approach of mereological pluralism[14]. In one of the earliest works in the area, Winston[19] presented a taxonomy of part-whole relations, and included *component-integral object* as a parthood relationship but for abstract concepts like phonology to linguistics. Later, Odell[13] also included component as one of his six proposed kinds of aggregation relationships. However, neither Winston nor Odell provided axiomatizations of their different parthood relations. In more recent work, Keet[10] introduced a taxonomy as summarization of Odell's approach to types of part-whole relations[13], and also provided OWL axiomatizations of the taxonomy. Bittner and Donnelly[5] have also presented an axiomatization of *CmpOf* in biological ontology.

Koslicki lists material components and formal components as kinds of proper parts in her book[11]. She defines that material components are intuitively from which these wholes come into existence, and formal components act as a sort of recipe in specifying the range and configuration of material components eligible to compose a whole of this kind. Our concept of *componentOf* is more aligned with the definition of the formal component concept, but in the domain of shape representation.

7. Conclusion

There are few existing first order logic axiomatizations in describing shapes of integrated three-dimensional objects. With Bob's assembly scenario in the use case as an motivation, we proposed MWorld Ontology and reused existing topological shape ontologies CardWorld and BoxWorld. MWorld is a first order logic ontology with nine modules, which allows the description of topological shapes from zero-dimension to three-dimension, including boxsets, boxes, surfaces, edges and points. We named boxset as the class for the shapes of integrated three-dimensional objects, *componentOf* as the proper parthood relation between the composing components and the whole, as well as *semi-complements* as the relation between disjoint components of the same whole. We also proposed terms in MWorld for the coincident shapes in adjacent components: joint point, joint edge and joint surface. Furthermore, as an extension to BoxWorld, we introduced *featureOf* as a new general parthood relation to capture the relationship between substructure or basic shape and its superstructure. As such, these theories can represent aspects of the shapes with different dimensions following multidimensional and mereological pluralism approaches. Future researches to MWorld include discussions to convexity, granularity and shape orientation. In addition, to better follow the mereological pluralism approach, we suggest that CardWorld and BoxWorld be revised with a different terminology other than *part* for the parthood relationship between basic shape entities and the whole. What's more, with incorporation of process and motion ontologies, we can then describe Bob's full day of fun assembly.

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