

A Multi Agent Systems framework for integrating environmental parameters in the design of shell structures

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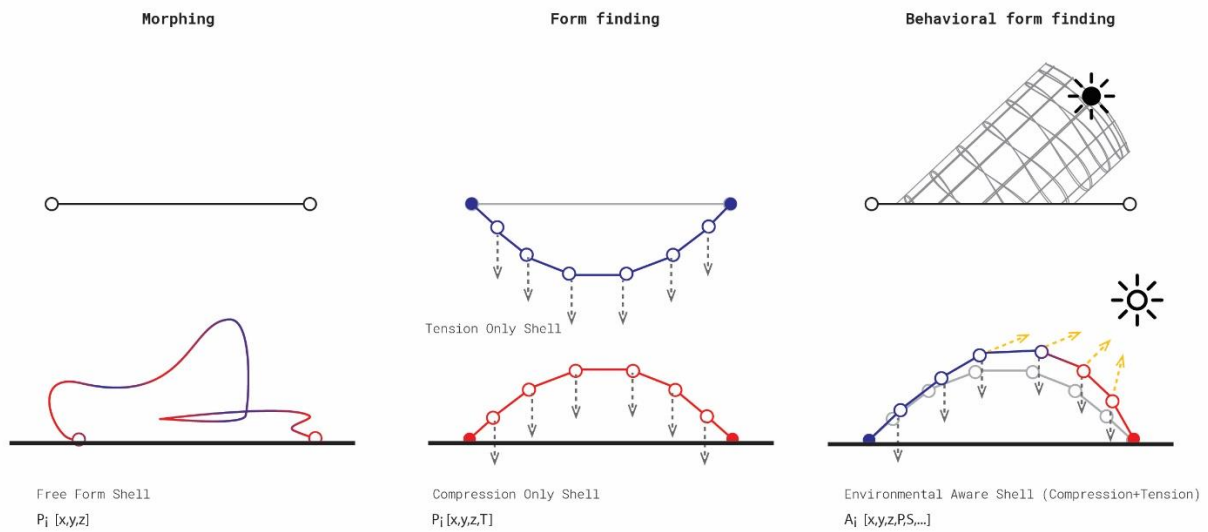


Figure 1 Illustration showing diagrammatically three different approaches towards form finding of shells. On the left. The resulting shape is the outcome of free form morphing, in the middle the shape is the outcome of applied (physical) forces and on the right the shape is the resultant of both physical and virtual (solar) forces.

1. Introduction

Design approaches based on self-organization and concepts of Multi Agent Systems (MAS) are becoming increasingly relevant in the field of architectural design. A good example is the application of agent based modelling and simulation techniques for design purposes, which can help reduce the complexity of building design (Groenewolt et al., 2018, Schwinn and Menges, 2015). The research builds upon concepts of MAS and object-oriented programming and addresses questions related to design exploration and optimization. Apart from suggesting an alternative design paradigm that expands the solution space of possible design solutions, a MAS framework can be employed in order to implement integrative planning processes, in which parameters relating to both architecture and engineering disciplines can be accounted for in the early design stage (Anumba et al., 2001). Instead of each discipline in the Architecture, Engineering and Construction (AEC) developing independent design solutions which are often times difficult to bring together for producing a coherent building design, agent based modelling offers the opportunity to combine them in an integrated loop. In such an approach each solution can be updated dynamically based on the circumstances and specific parameters of the project at stake (Macal and North, 2009).

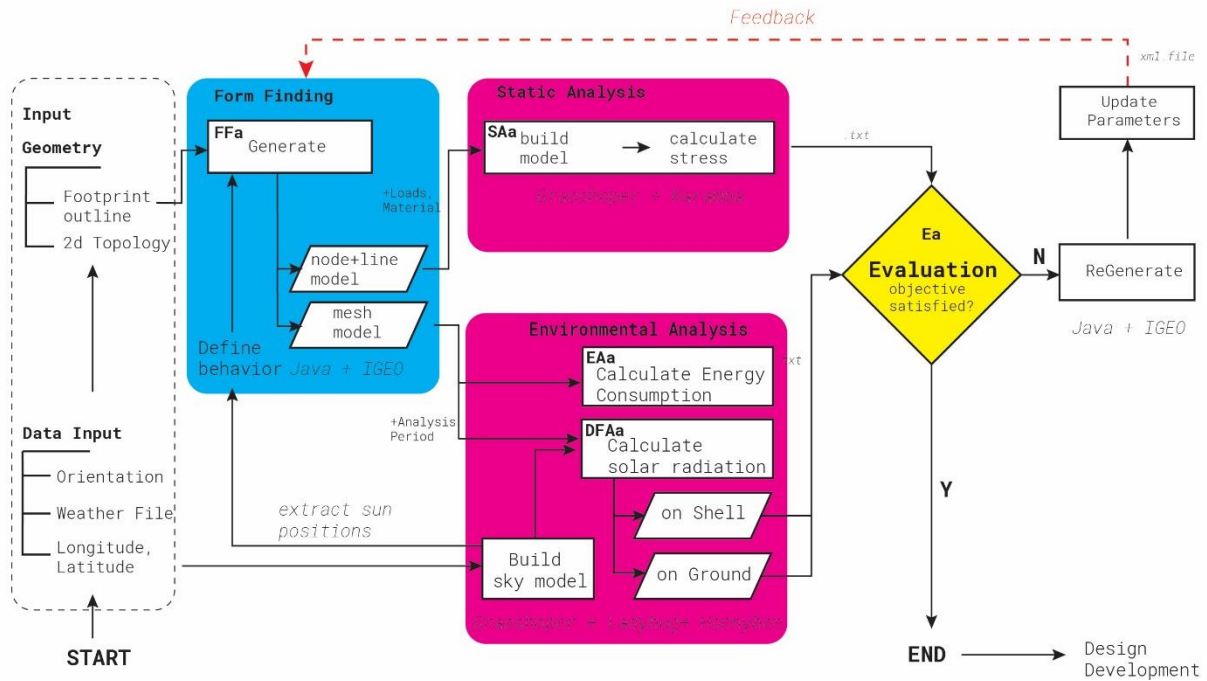


Figure 2 Flowchart diagram of the proposed behavioral form finding workflow which graphically illustrates the inputs and outputs for each agent class.

2. Background

A number of innovative physical form finding techniques were developed independently in the 20th century by practitioners such as A. Gaudi, H. Isler, F. Candela and F. Otto (Adriaenssens et al., 2014). These techniques were empirical and were driven by the motivation to create open plan spaces with large spans that were conditioned by economic and material constraints. Despite providing a more intuitive way to design structures, due to their complexity but also due to the prevalence of analytic methods for structural design, these empirical methods remained largely unexplored until recently. An increasing number of researchers working on the intersection of design and computing have started revisiting such methods from a computational perspective in an attempt to enable architects deal with hard design problems that include engineering and fabrication constraints in a more rigorous way (Kolarevic, 2004, Gramazio and Kohler, 2014).

In the last two decades a number of computational based approaches have been developed for exploring architectural form based on the concepts of form finding and optimization (Adriaenssens et al., 2014, Lachauer et al., 2010), evolutionary computation and behavioral design (Menges, 2007) as well as rule based models (Fricker et al., 2007). Kilian, inspired by A. Gaudi hanging chain models developed one of the first digital form finding tools (Kilian, 2006). The tool was based on the hanging chain principle which was introduced by Hooke in the 17th century and demonstrated how fabrication schemas can be linked to real time form finding simulation. Piker has introduced a particle physics engine for simulating structures based on the combination of Dynamic Relaxation and the co-rotational formulation of Finite Elements Methods (Piker, 2013). Rippmann and Block introduced an interactive form finding tool based for compression only vault design which is based on graphic statics. The tool is based

on Thrust Network Analysis (TNA), a method which generates possible 3d shell geometries by combining projective geometry, duality theory and linear optimization (Block and Ochsendorf, 2007).

In the field of Multi Agent Systems and Agent Based Modelling (ABM) there have been developed a number of design tools inspired by complex adaptive systems and emergent behaviours observed in nature (Bonabeau et al., 1999). These tools are driven by environmental conditions and allow behavioural modelling but have mostly focused on specific agent models such as the “boids” developed by C. Reynolds (Reynolds, 1987). Additionally, although in many disciplines MAS has been used for optimization processes in architectural design, they have been mainly used for generating designs.

More recently there has been a significant effort towards developing integrated design approaches which narrow the gap between modelling and analysis by using data to drive the design (Gerber and Lin, 2013). Yet in most cases the rationalization is happening after a design is generated and thus more research is necessary to develop tools which use local relationships and analytical data that generate models that are pre-rationalized, the aforementioned approaches have pushed the boundaries of integrated architectural design and generative design respectively. However, the former approaches have mainly emphasized in the integration of geometric and structural design (boundary condition, supports, loads etc.) but are not considering environmental parameters such as the location and/or position of the sun in the form finding process (Kilian, 2014), while the latter ones have yet to develop agent models specific to the AEC, which are relevant in the contemporary practice (Pantazis and Gerber, 2018).

3. Methodology

In this work we present the extension of a computational framework for architectural geometry, in which agents represent building elements (Pantazis and Gerber, 2018). The framework has been tested previously by the authors for developing façade designs, where the agents represented façade panels and their placing was conditioned by environmental parameters (Gerber et al., 2017). In this work the same framework is applied for the design of shell structures by incorporating environmental parameters such as the annual solar path in the form finding process. The aim is to extend existing computational form finding approaches (Rippmann et al., 2012, Piker, 2013) by introducing behaviours which allow the integration of daylight as a shaping force apart from typical forces such as gravity and tension.

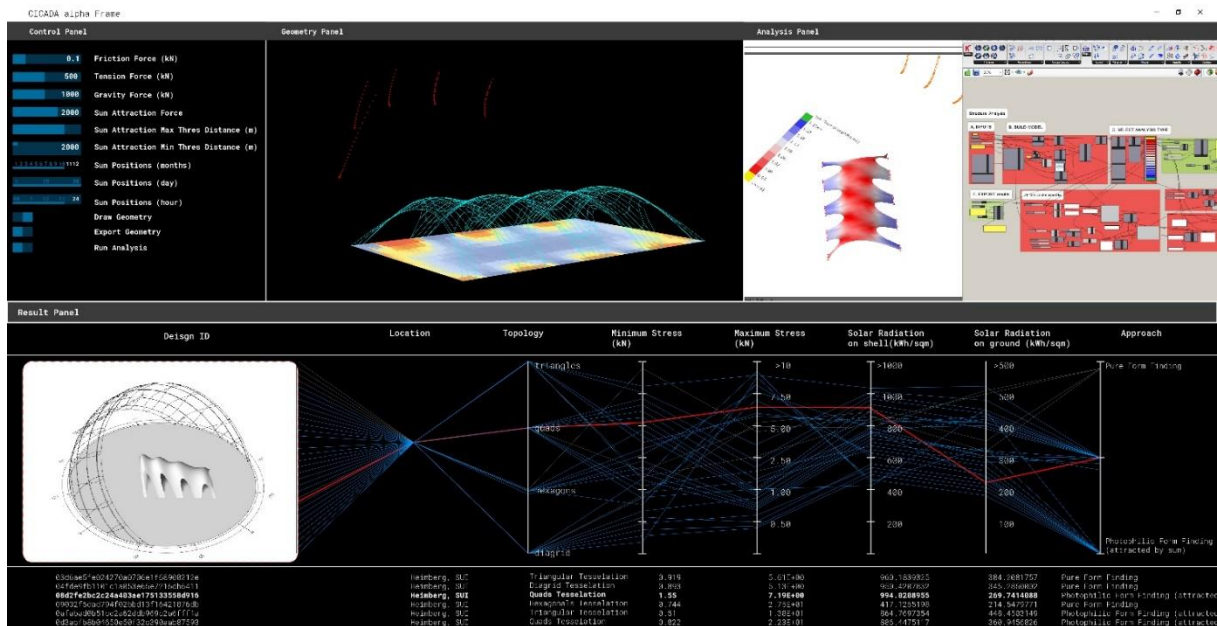


Figure 3 Graphical User Interface of the alpha version of the tool. On the top left side (control panel) are all the input parameters, in the middle is the geometry viewport and on the left is the window where we call Rhinoceros 3d and Grasshopper. In the bottom all analytical results are presented in a parallel line plot along the geometric design.

Behaviours which relate to the orientation of the site and the related solar path are described, namely a photophobic and photophilic one, in the next section along with the developed agent classes. These behaviours can be adjusted in real time in order to steer form finding away from purely form found shapes. In Figure 2 the workflow of the design approach is illustrated graphically showing the types of agents (colour of box indicates the type) and data exchanged. The designer retains control over local behaviours among the agents as well as a number of global parameters such as the initial topology of the geometry, the support condition and the environment of the agents (location, orientation). In return once the designer runs the system, different global configurations are emerging based on the behaviours and the adjustment of the initial conditions. To ensure that the design process is integrative, domain specific data and the results of external analysis (stress and solar radiation analysis) can be used as input for agent behaviours and/or can be visualized so that the designer can evaluate the design alternatives based on design performance data.

Each generated design is analysed a) geometrically (different types of meshing) b) environmentally (total annual thermal energy consumption and solar radiation) and c) structurally (Von Mises Stress analysis and displacement). The analytical results are used to evaluate the effect of different behaviours on the generated outcomes and to apply a corresponding weight on the. Figure 3 shows the developed Graphical User Interface (GUI) which allows the designer to visualize the analytical results using parallel line plots (Bostock et al., 2011) in order to help develop intuition of the trade-offs between different performance metrics (Clevenger et al., 2013).

2.1 Steering structural design of shells via applying environmental behaviours

Four different agent classes are implemented from the MAS framework (Pantazis and Gerber, 2018) namely: a generative agent class, two specialist agents classes and one evaluation agent class. A generative agent class, namely a form finding agent (FFa) is implemented which combines properties of a (physics) particle simulation such as position, velocity, gravity and tension forces with forces relating to the position of the sun in specific timestamps (solar path). Additional classes include, one specialist agent class which is tasked with the structural analysis (Structural Analysis agent) and a second specialist agent class, which is tasked with the energy analysis (Energy Analysis agent) of the generated shells. The proposed methodology is based on the following assumptions: the generative agents are represented as particles that are interconnected to represent a mesh surface. Each connection among the agents is modelled as linear elastic spring with variable stiffness which is controlled by a tension force.

Additional forces are applied on each Form Finding agent (FFa), instead of only modelling loads which are typical in existing form finding methods (i.e. gravity, dead loads and tension). Such “virtual” loads, include for instance a solar force which is modelled based on location and orientation. In this early stage of the development, two basic behaviors are implemented and tested namely a photophilic and photophobic one. The behaviors are assigned to the FFas and are used as a tool to augment the purely form found shapes. The hypothesis is that agents can attain new equilibrium states whereby applying iteratively different weighting value to the selected behavior a generated shell can be optimized not only for weight but also based on its environmental performance (i.e. increase amount of daylight availability, decrease annual energy consumption).

A photophilic behavior is defined as one where a number of positions on the solar path of a specific location exert a force on the form finding agents which steers them towards those positions, when the agents are within a distance threshold. On the contrary a photophobic behavior is defined as one where specific positions on the solar path are exerting a force that steers the agents away from these positions. For instance, depending on the longitude/latitude and orientation of the structure, the designer may assign a photophilic behavior to the positions of the sun during the morning hours which increase daylight during the operating hours of the building and a photophobic behavior to the positions of the sun during the afternoon hours which significantly increase heat gain and might negatively the total energy consumption. The photophilic/photophobic behaviors are expressed as forces on the FFa and to ensure that the behavior is not leading to undesired results, the solar attraction force is scaled according to the number of the attractor points, the distance of the point to the structure and a weight $pb(w)$ as seen in the equation below.

$$F = \frac{\text{SunAttractionForce}}{N(\text{pt})} * D * pb(w)$$

In order to be able, to evaluate the impact of a photophilic behavior (pb) on the design performance a weight w is applied to it. The weighting factors is correlated with the collected analytical results with the following heuristic function:

$$pb(w) = \frac{tE(i, BaseCase) + tR(i, BaseCase)}{disp(t)} - \sum (Pnlty(i))$$

The heuristic function uses the total energy consumption (tE), the total Radiation (tR) underneath the structure and the maximum displacement in order to calculate the value of the weight for the behavior. The heuristic function is used to adjust the value of the force according to the desired result. The closer to the desired analytical results a solution is compared to a base case (i.e. the purely form found case), the higher the weight of the photophilic behavior becomes.

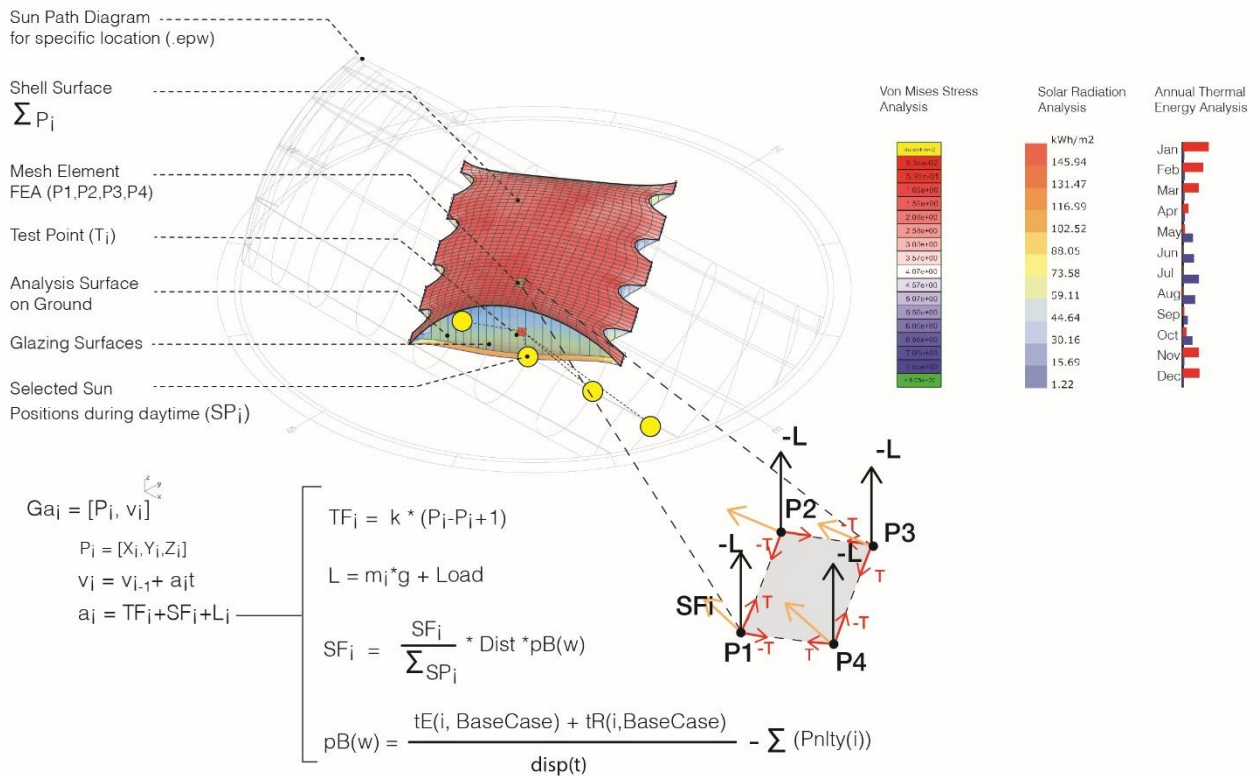
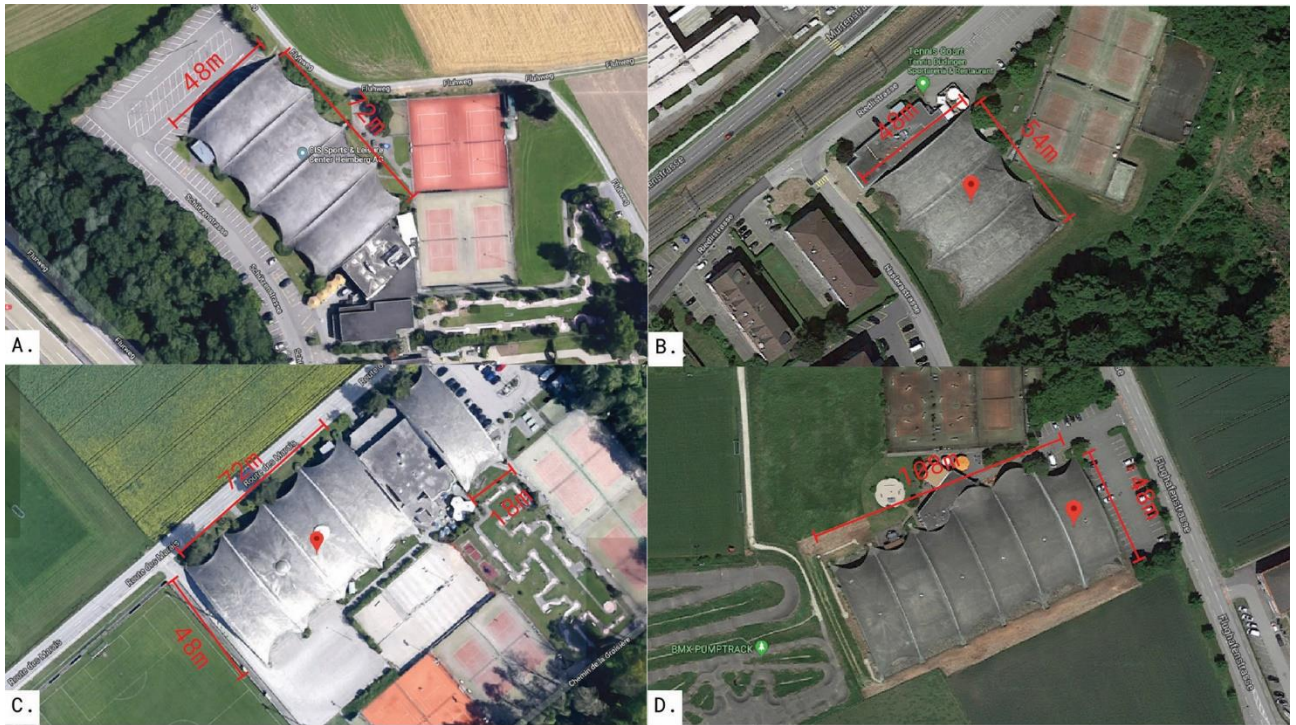


Figure 4 Experimental Design set up that describes, the environment, design parameters and the heuristic function that couples the form finding with the photophilic behavior and three different types of analysis (structural, radiation and thermal energy analysis)

A penalty is added to reduce the weight in the case that a point is moved to an undesired location. Using this heuristic function we run the system where the designer can interactively change the value of the behavior based on the assessment of the analytical results and the geometry. Once she sets a value for the behavior the systems runs for a specified amount of iterations in order to generate design alternatives which satisfy the predefined performance targets (Figure 4).



Location	Program	Span (m)	Length (m)	Type	Material	Coordinates
A. Heimberg, Berne , CH	Sports Center	48.00	72.00	Thin Shell	Reinforced concrete	46° 47' 33.77" N 7° 35' 43.33" E
B. Düdingen, Fribourg, CH	Sports Center	48.00	54.00	Thin Shell	Reinforced concrete	46° 51' 20.35" N 7° 11' 56.88" E
C. La Tène, Neuchâtel, CH	Sports Center	48.00	72.00	Thin Shell	Reinforced concrete	47° 01' 25.21" N 7° 01' 13.20" E
D. Grenchen, Solothurn, CH	Sports Center	48.00	108.00	Thin Shell	Reinforced concrete	47° 10' 57.20" N 7° 24' 32.25" E

Figure 5 Four different shell structures designed by Isler and constructed in Switzerland (1978-1988)

4. Experimental Design

An experimental design is developed using an existing thin shell concrete structure design by H. Isler to apply and test the proposed methodology and show how such an approach can lead to quantifiable results. Isler used fabric and physical form finding to design a number of structures (Chilton and Chuang, 2017). One of the most widely applied design from Isler is that of tennis and sports halls that he has built in various locations in Switzerland (Figure 5). In this case study, a tennis hall which was built in 1978 in Heimberg, a small town in Switzerland is revisited. The thin shell structure has a span of 48 meters, a length of 72.00m and is supported in 10 points. It is made out of 100mm thick reinforced concrete, has a footprint of approx. 3000 sqm and is still being used as a sports hall. Isler developed different fabric models for one bay (48x18m) to test how different design parameters such as the fabric density and mesh orientation affect the form of the shell. A design was finally selected, scaled for 1:1 construction and multiplied according to site requirements.

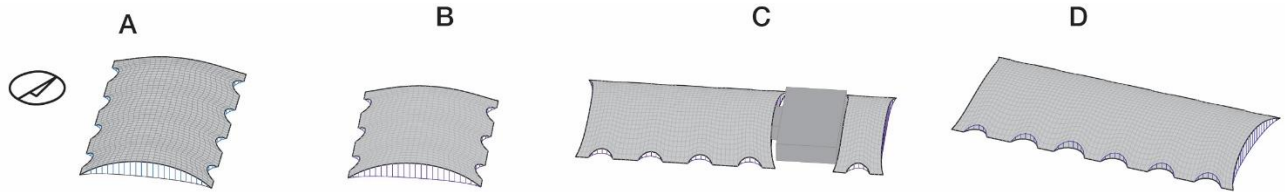
The design served as an archetype for three more shell structures that were designed and built in the next decade. The shell structures are all located in Switzerland have exactly the same span but their overall length and orientation vary. The material of all structures is untreated concrete, which was cast on top of 50 mm insulation Styrofoam panels, while the rest of the envelope is single panel curtain walls.

3.1 Design Process

Although little information is publicly available for the detailed geometry of the Heimberg shell, by accessing information about the shell via an online database (www.structurae.com), one can get the basic design parameters, simulate the structure and generate a 3d model using Rhinoceros 3d and the Kangaroo Particle Physics Solver. As a first step after generating the 3d models is to simulate the existing structure and analyze it structurally. To do so we modelled the concrete material and analyzed the structure using Karamba, a finite element analysis software geared towards interactive use in the visual scripting editor Grasshopper (Preisinger and Heimrath, 2014). The same process is applied to all four structures that were designed based on the same model. Apart from modelling the structure parametrically using the Kangaroo Particle Physics Solver, an agent based model is developed in order to explore more design alternatives. The established MAS framework is used for evaluating environmental parameters in parallel to form finding. The design process can be summarized as follows:

1. Definition of typical parameters in structural design such as: F , the outline of the provided footprint in the form a polyline with n corners; P , as well as determine the number and type of support conditions $S(n,t)$, the material stiffness (E), material properties (G), and loads (L).
2. Determine the max amount of agents and the topology of their connections. This is the discretization of the given input outline and sets the initial geometric configuration of the mesh surface for the form finding. In this case we developed 4 different topological variations, namely: orthogonal topology, triangular, hexagonal and rhomboidal, but selected the rectangular one for clarity purposes.
3. Provide: the location's Longitude and Latitude (LL) of well as the orientation (O) of the structure, orientation (N,S,E,W), and the creation of a weather data (.epw file). Additionally the designer provides a generic use of the space (i.e. School, Office, Gym)
4. Develop an environmental behavior for the agent, i.e. photophilic or photophobic behavior depending on a design objective. The behavior can be simple and straightforward such as get attracted (steer towards) by selected sun positions to allow direct sunlight in the morning.
5. Apply external physical loading (self-weight, dead load) on the surface and external virtual loading (i.e. sun attraction) to derive the shape of the shell. The stiffness, weight and level of attraction of nodes are adjusted to find the equilibrium state. This is the main difference between the suggested approach and Isler's physical modelling, or computational form finding approaches. At each time step the position of the agents is updated not only based on the summation of gravity forces but also on additional ones that act upon it. By introducing specific positions of the sun as "virtual forces" we directly link environmental parameters with form finding, which can be adjusted by providing "weights" for each force. In this step, the designer specifies the duration of the form finding process and the "weight" of the photophilic or photophobic behavior.
6. Once a global equilibrium is reached, and the velocity of each agent is close to 0, a NURBS geometry is generated based on the optimal force distribution, which can be directly exported to Rhinoceros 3d for further analysis

7. The generated geometry is automatically passed for structural analysis and environmental analysis using Grasshopper in conjunction with environmental simulation and analysis software which were used to model the existing structures (Preisinger and Heimrath, 2014, Roudsari et al., 2014). The results are collected and are used to inform the weight of the environmental behavior.
8. The process is repeated iteratively until the design objectives are met or the user stops the process



Information	Case A	Case B	Case C	Case D
Location (Switzerland)	Heimberg, Berne	Düdingen, Fribourg	La Tène, Neuchâtel	Grenchen, Solothurn
Span (m)	48	48	48	48
Length (m)	72	54	72	108
footprint Area (sqm)	3079.00	2266.00	3777.00	4670.00
shell area (sqm)	3253	2398	4005	4941
Structural Analysis				
max displacement [cm]	1.38E+00	1.44E+00	1.33E+00	2.02E+00
min Stress [kN/cm ²]	1.27E+00	1.25E+00	2.82E-01	1.21E+00
max Stress [kN/cm ²]	8.37E+00	8.40E+00	1.00E+01	8.70E+00
Solar Radiation Analysis				
Annual solar Radiation on shell (kwh/m ²)	1131400.00	1403500.0000	1403500.00	1699900.00
Annual solar Radiation below shell (kwh/m ²)	418107.5499	298344.4512	726504.195	401665.1759
Annual solar Radiation on shell /sqm	367.4569665	585.2793995	350.4369538	344.0396681
Annual solar Radiation below shell /sqm	135.7932933	131.6612759	192.3495353	86.00967364
Energy Analysis				
Annual Total Thermal Energy (kW/h)	786400.1646	439080.4149	767175.4635	780620.7278
Annual Cooling Energy Needed (kW/h)	843.452221	1005.380435	28815.24791	832.444715
Annual Heating Energy Nedeed (kW/h)	777960.7124	429030.0344	738360.2156	772300.283
Annual Daylight Factor(%)	95.898399	94.843977	95.5	96.682832
Total Thermal Energy per area (kW/h/sqm)	255.4076533	193.7689386	203.1176763	167.1564728

Figure 6 Table with all the analytical results of the 4 different case study shell structures

Specialist agents classes are implemented for each of the performed analyses which communicate with the FFA, namely: a Structural Analysis agent (SAa) which calculates displacement and Von Mises Stress, an Energy Analysis agent (EAa) which accounts for the total thermal energy required annually for the structure and a Daylight Factor Analysis (DFAa) which accounts for the amount of sunlight on and beneath the shell structure. The input for SAa is material specifications (Concrete, 4000 ksi) section thickness, loads and support conditions. The input for the EAa and DFAa in order to be able to run the energy analysis are the following: 1) coordinates of the structure, 2) a corresponding weather file (.epw), 3) the orientation of the structure (N,S,W,E), 4) the program of the space, 5) the programmatic schedule of the space and 6) the type of glazing and some basic material properties (diffuse color, reflectance).

3.2 Results & Analysis

In Figure 6 we tabularize and compare the analytical results for each of the original Isler structures. The results indicate that the structures structurally perform identically with small differences which are due to the difference in size. However, their environmental performance is varying quite significantly depending on the case.

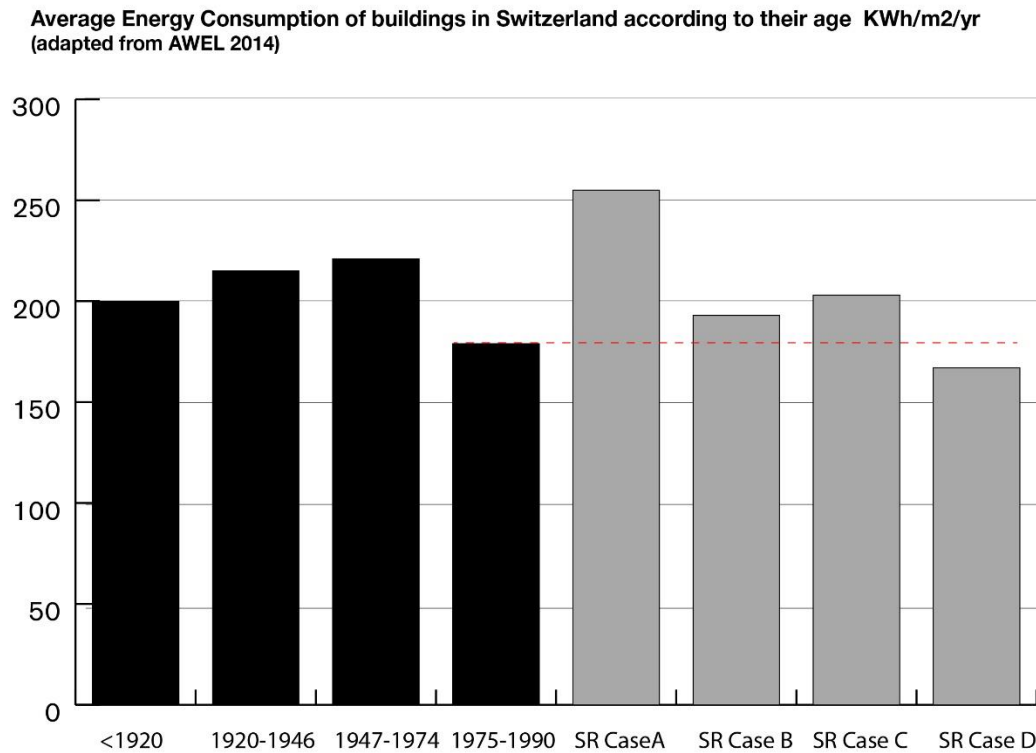


Figure 7 Comparison of simulated results (SR) with the average energy consumption of buildings in Switzerland according to the age of the Swiss building stock (adapted from AWEL 2014) KWh/m²

The annual energy consumption for instance Case D, which is the longest structure and is also oriented along a South East/North West orientation axis has the lowest average Daylight Factor. Due to the fact that the performed simulations are not based on detailed 3d models, in order to validate our results we compare them against the results of a survey from the Chair of Ecological System Design at ETH Zurich. This survey catalogs the embodied environmental impact of building stock in Switzerland since the 1920's (Ostermeyer et al., 2018). The survey lists the range of average energy consumption of buildings according their age. In Figure 7 we plot the simulated results compared to results of the survey which measures the annual energy consumption per square meter. The standard deviation between the simulated results and ones coming from the survey is calculated $\sigma = 33.7$ kWh/sqm.

In Figure 8 we use a parallel line plot to compare Case A with a subset of the behaviorally form found shapes. A photophilic behavior is applied to the structure and the system is run iteratively in order to generate new shell shapes, that perform better than the base cases in terms of Annual Thermal Energy Consumption, Daylight Factor Analysis, Displacement and Stress. The blue lines indicate the values of the four existing case studies while the red ones show design alternatives of Case Study A, after applying a photophilic behavior.

Over time the system generates alternatives that decrease the average annual consumption by 12%, increase the daylight factor analysis by 9% and increase the average solar radiation underneath the structure by 102% with regards to the base case. The amount of max displacement is increased yet is maintained within allowable thresholds (<5cm).

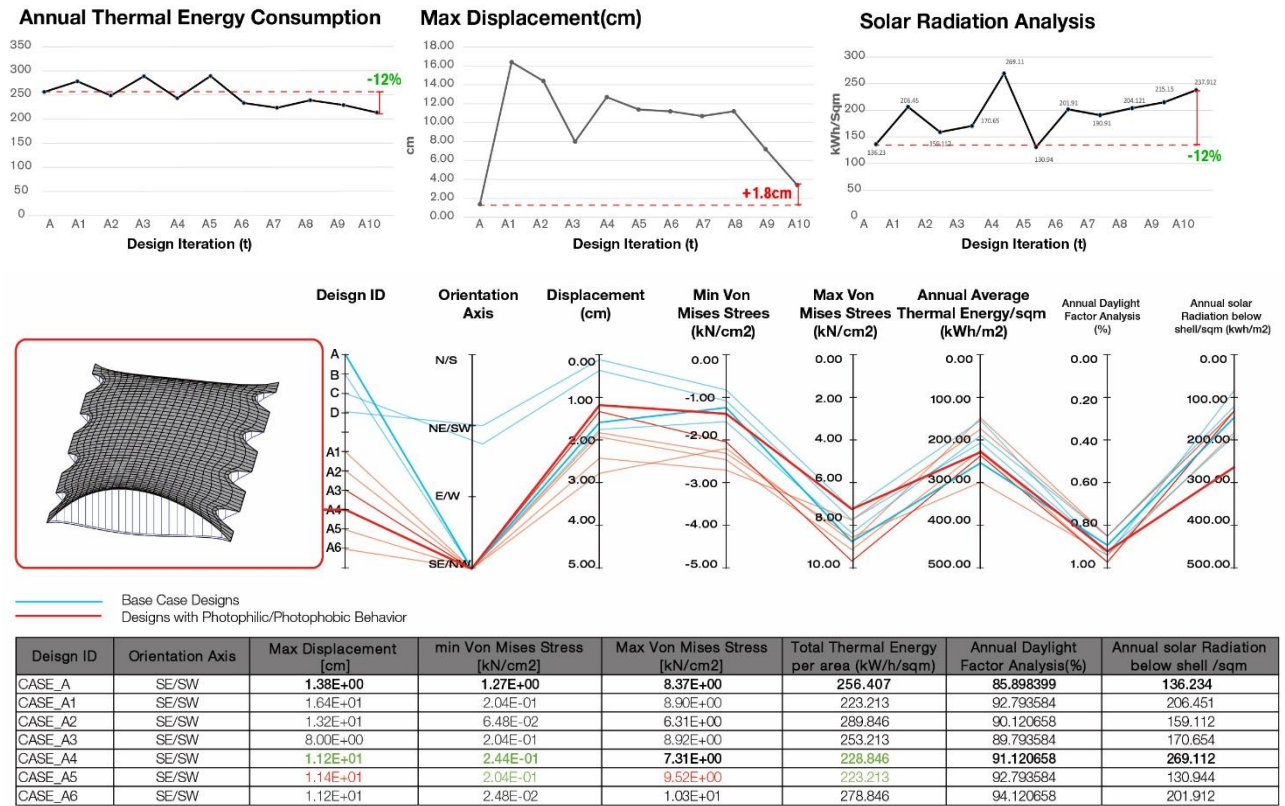


Figure 8 A subset of design alternatives presented to the designer. The base case designs are marked with the blue line, while the behavioral ones are marked with red. Highlighted is a design alternative that meets the design objectives based on the available analytical metrics

4 Discussion

This work presents the application of a MAS framework for the design of shell structures which are pre-rationalized for a set of environmental objectives. The aim is to explore design alternatives that provide the same amount of light levels underneath the structure independent of their location and orientation but without affecting the structural integrity of the shells. Initial results show that by implementing a photophilic/photophobic behaviour design alternative can be generated that satisfy both structural and environmental performance targets. Unlike conventional ways of employing performance-based design approaches, the main concern of the research at this point is not solely an increase in efficiency or speed but rather in proposing an alternative form finding method which is not based on purely analytical design methods. The framework is focused in the early design stage and therefore at this stage the structural analysis is coarser and therefore is not accounting for shell bucking and creep induced failure. The

reason for this is that at an early design stage the importance lies more in quickly exploring global shapes and easily evaluating to what level they meet a set of performance objectives. Once the solution space of possible design alternatives is reduced, more rigorous and detailed analysis can be performed on more refined and detailed geometries.

Despite the fact that we are interested in generating designs that perform within a specified range the focus of the work at this point is to test if a formal design method based upon MAS can extend the designers ability to explore larger solution spaces and lead to solutions, which would not be attainable conventional design and building methods.

5 References

- ADRIAENSSENS, S., BLOCK, P., VEENENDAAL, D. & WILLIAMS, C. 2014. *Shell Structures for Architecture: Form Finding and Optimization*, London and New York, Taylor & Francis - Routledge.
- ANUMBA, C., UGWU, O., NEWNHAM, L. & THORPE, A. 2001. A multi-agent system for distributed collaborative design. *Logistics Information Management*, 14, 355-367.
- BLOCK, P. & OCHSENDORF, J. 2007. Thrust network analysis: A new methodology for three-dimensional equilibrium. *International Association for Shell and Spatial Structures*, 155, 167.
- BONABEAU, E., DORIGO, M. & THERAULAZ, G. 1999. *Swarm intelligence: from natural to artificial systems*, New York, USA, Oxford university press.
- BOSTOCK, M., OGIEVETSKY, V. & HEER, J. 2011. D³ data-driven documents. *IEEE transactions on visualization and computer graphics*, 17, 2301-2309.
- CHILTON, J. & CHUANG, C.-C. 2017. Rooted in nature: aesthetics, geometry and structure in the shells of Heinz Isler. *Nexus Network Journal*, 19, 763-785.
- CLEVENGER, C. M., HAYMAKER, J. R. & EHRICH, A. 2013. Design exploration assessment methodology: testing the guidance of design processes. *Journal of Engineering Design*, 24, 165-184.
- FRICKER, P., HOVESTADT, L., BRAACH, M., DILLENBURGER, B., DOHMEN, P., RÜDENAUER, K., LEMMERZAHN, S. & LEHNERER, A. 2007. *Organised Complexity*, Frankfurt am Main, Germany, 25th eCAADe Conference Proceedings.
- GERBER, D. J. & LIN, S.-H. E. 2013. Designing in complexity: Simulation, integration, and multidisciplinary design optimization for architecture. *Simulation*, 1-24.
- GERBER, D. J., PANTAZIS, E. & WANG, A. 2017. A multi-agent approach for performance based architecture: Design exploring geometry, user, and environmental agencies in façades. *Automation in Construction*, 76, 45-58.
- GRAMAZIO, F. & KOHLER, M. 2014. Authoring Robotic Processes. *Made by Robots*, 229, 136.
- GROENEWOLT, A., SCHWINN, T., NGUYEN, L. & MENGES, A. 2018. An interactive agent-based framework for materialization-informed architectural design. *Swarm Intelligence*, 12, 155-186.
- KILIAN, A. 2006. Design innovation through constraint modeling. *International journal of architectural computing*, 4, 87-105.
- KILIAN, A. 2014. Steering Form. In: ADRIAENSSENS, S., BLOCK, P., VEENENDAAL, D. & WILLIAMS, C. (eds.) *Shell Structures for Architecture: Form Finding and Optimization*. London and New York: Taylor & Francis - Routledge.

- KOLAREVIC, B. 2004. *Architecture in the digital age: design and manufacturing*, Taylor & Francis.
- LACHAUER, L., RIPPMANN, M. & BLOCK, P. 2010. Form Finding to Fabrication: A digital design process for masonry vaults. *Proceedings of the International Association for Shell and Spatial Structures (IASS) Symposium*.
- MACAL, C. M. & NORTH, M. J. Agent-based modeling and simulation. Winter simulation conference, 2009. Winter Simulation Conference, 86-98.
- MENGES, A. Computational morphogenesis. Proceedings for 3rd International ASCAAD Conference, 2007 Alexandria, Egypt. 725-744.
- OSTERMEYER, Y., NÄGELI, C., HEEREN, N. & WALLBAUM, H. 2018. Building inventory and refurbishment scenario database development for Switzerland. *Journal of Industrial Ecology*, 22, 629-642.
- PANTAZIS, E. & GERBER, D. 2018. A framework for generating and evaluating façade designs using a multi-agent system approach. *International Journal of Architectural Computing (IJAC)*, 16, 248-270.
- PIKER, D. 2013. Kangaroo: form finding with computational physics. *Architectural Design*, 83, 136-137.
- PREISINGER, C. & HEIMRATH, M. 2014. Karamba—A toolkit for parametric structural design. *Structural Engineering International*, 24, 217-221.
- REYNOLDS, C. W. 1987. Flocks, herds and schools: A distributed behavioral model. *ACM SIGGRAPH computer graphics*, 21, 25-34.
- RIPPMANN, M., LACHAUER, L. & BLOCK, P. 2012. Interactive vault design. *International Journal of Space Structures*, 27, 219-230.
- ROUDSARI, M., PAK, M. & SMITH, A. 2014. Ladybug: A Parametric Environmental Plugin for Grasshopper to Help Designers Create an Environmentally-Conscious Design.
- SCHWINN, T. & MENGES, A. 2015. Fabrication Agency: Landesgartenschau Exhibition Hall. *Architectural Design*, 85, 92-99.