On Concept Forgetting in Description Logics with Qualified Number Restrictions*

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Forgetting refers to an ontology engineering technique that seeks to produce new ontologies from existing ones using only a subset of their signature while preserving all logical consequences up to the names in the subset. This can be done by eliminating from the original ontology a set of concept and role names in such a way that all logical consequences are preserved up to the names in the remaining signature. The ontology produced by forgetting, namely the *forgetting solution*, can be seen as a *view* of the original ontologies. In traditional databases, a view is a subset of the database, whereas in ontologies, a view is more than a subset; it may contain not only axioms contained in the ontology, but also those entailed by the ontology (implicitly contained in the ontology). Forgetting has a number of potential applications such as ontology reuse, versioning, alignment, merging, debugging, repair, and logical difference computation [1,12,16,3,14,5,2,17,2].

Forgetting can be defined as the dual of *uniform interpolation* [15] or model-theoretically as *semantic forgetting* [5,20,4]. The two notions differ in the sense that uniform interpolation preserves all *logical consequences* up to certain signatures while semantic forgetting preserves *semantic equivalence* up to certain signatures. The results of semantic forgetting (the *semantic solutions*), are in general stronger than those of uniform interpolation (the *uniform interpolants*). This means that semantic solutions always entail uniform interpolants, but the converse does not hold. Uniform interpolants are always expressed in the source logic, while semantic solutions are often not expressible in the source logic, and may require the target language to be extended.

Practical methods for computing uniform interpolants include the method implemented in the LETHE system [7,8,9,11], and the method developed by [13]. LETHE handles $\mathcal{ALC}$, $\mathcal{ALCH}$, $\mathcal{SIF}$, $\mathcal{SHQ}$-TBoxes, and $\mathcal{ALC}$ with ABoxes. The method of [13] handles $\mathcal{ALC}$-TBoxes. Practical methods for computing semantic solutions of forgetting have been developed, implemented and evaluated in work of [18,19]. These methods are based on non-trivial generalisations of Ackermann's Lemma, and attempt to eliminate concept and role names from ontologies expressible in the description logic $\mathcal{ALCOIH}(\nabla, \sqcap)$.

This paper introduces a practical method for computing solutions of concept forgetting in description logics with qualified number restrictions. While allowing more problems to be solved, admitting qualified number restrictions significantly increases the difficulty of the problem. Our method handles in particular

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$\mathcal{ALCOQ}$-ontologies and the extension with the universal role, role negation, role conjunction and role disjunction, which means that it can handle expressive description logics that cannot be handled by other methods at present. The method is terminating, sound, but incomplete for $\mathcal{ALCOQ}(\neg, \sqcap, \sqcup)$-ontologies. When it succeeds, the method returns a uniform interpolant in $\mathcal{ALCOQ}(\neg, \sqcap, \sqcup)$. The results of an evaluation with a prototype implementation shows that the method is computationally feasible and is able to find a uniform interpolant in more than 90% of the test cases taken from a large corpus of biomedical ontologies. In only 13.2% of these cases the uniform interpolant was also a semantic solution. The prototype, along with the test ontologies and their statistical information, can be downloaded/found at `http://www.cs.man.ac.uk/~schmidt/sf-fame/`.

At present, practical methods for forgetting in description logics with qualified number restrictions are the resolution-based approach of the LETHE system [9,6,10], which can perform concept forgetting in the description logic $\mathcal{SHQ}$. An empirical comparison between LETHE and the prototype on the $\mathcal{ALCQH}$-fragments of a corpus of biomedical ontologies can be found in [21]. Our method admits the universal role, role negation, role conjunction and role disjunction in the language. An advantage of this is that solutions computed by the prototype are in general stronger than those computed by LETHE. Often, a stronger solution means a better one. For example, the solution of forgetting the concept name $\{\mathsf{Male}\}$ from the ontology

$$\{\mathsf{A} \sqsubseteq {\geq}2\mathsf{hasSon}.\mathsf{Male},\ \mathsf{A} \sqsubseteq {\geq}3\mathsf{hasDaughter}.\neg\mathsf{Male},\\ \mathsf{hasSon} \sqsubseteq \mathsf{hasChild},\ \mathsf{hasDaughter} \sqsubseteq \mathsf{hasChild}\}$$

computed by LETHE is

$$\{\mathsf{A} \sqsubseteq {\geq}2\mathsf{hasSon}.\top,\ \mathsf{A} \sqsubseteq {\geq}3\mathsf{hasDaughter}.\top,\\ \mathsf{hasSon} \sqsubseteq \mathsf{hasChild},\ \mathsf{hasDaughter} \sqsubseteq \mathsf{hasChild}\},$$

while the solution of the prototype includes an additional axiom

$$\mathsf{A} \sqsubseteq {\geq}5(\mathsf{hasSon} \sqcup \mathsf{hasDaughter}).\top,$$

where role disjunction is used. Upon the solution of LETHE, if we further forget the role names hasSon and $\mathsf{hasDaughter}$, the uniform interpolant is $\{\mathsf{A} \sqsubseteq {\geq}3\mathsf{hasChild}.\top\}$, while upon the intermediary solution of the prototype, the solution is $\{\mathsf{A} \sqsubseteq {\geq}5\mathsf{hasChild}.\top\}$, which is stronger and closer to the fact: A has at least 5 children. This shows an advantage of our method where extra expressivity allows intermediary information ($\mathsf{A} \sqsubseteq {\geq}5(\mathsf{hasSon} \sqcup \mathsf{hasDaughter}).\top$) to be captured which produces a better solution.

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