Linking Exploration Systems with Local Logics over Information Systems

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Abstract. In the paper, we discuss an extension of exploration systems introduced by Andrzej Ehrenfeucht and Grzegorz Rozenberg. The extension is defined by adding an interpretation of nodes and edges in zoom structure of exploration system. The interpretation is based on the concepts, namely local logic and logic infomorphism, from the notion of information flow by Jon Barwise and Jerry Seligman. This extension makes it possible, in particular, to give a natural interpretation of reaction systems in exploration systems as tools for controlling attention in reasoning about the perceived situation in the physical world.

Key words: reaction system, zoom structure, exploration system, information system, local logic, infomorphism, logic infomorphism

1 Introduction

In the paper, we present a preliminary discussion about possible links between the exploration systems and the information flow approach.

The original motivation behind reaction systems (mostly taken from [1] and [2]) was to model interactions between biochemical reactions in the living cells. Therefore, the formal notion of reaction reflects the basic intuition behind biochemical reactions. A biochemical reaction can take place if in a given state all of its reactants are present and none of its inhibitors is present. When a reaction takes place, it creates its products.

Zoom structures were introduced to integrate structure of a depository of knowledge of a discipline of science (*e.g.*, biology) in the context of reasoning about the perceived situation related to reaction systems in the physical world. A discipline of knowledge must be structured and the integrating structure here is a well-founded partial order which is well suited to represent a hierarchical structure of knowledge. Exploration systems combine the zoom structures with the reaction systems that are "running within" zoom structures (see, *e.g.*, [3, 4]).

We propose to extend exploration systems by adding interpretation of nodes and edges of zoom structures. The interpretation of nodes of zoom structures, in the form of labels of nodes, is defined by local logics (related to information systems) [5]; the labels of edges in the zoom structure are interpreted as logic infomorphisms between local logics labeling nodes linked by edges. Through local logic it is possible to address a notion of reasoning with respect to local knowledge. Logic infomprphisms can be treated as abstract representations of communications between local logics because each of two local logics linked by a logic infomorphism has some knowledge about facts derivable by the second one. It is possible to treat exploration system as a distritbuted basis for reasoning about the perceived situation related to biochemical processes running in the physical world.

The content of the paper is organized as follows. In Sect. 2 we present the basic concepts of reaction systems. Rudiments of the information flow approach are included in Sect. 3. The zoom structures, exploration systems, and their extension are discussed in Sect. 4.1.

2 Reaction Systems

In this section we recall some basic notions concerning reaction systems (mostly taken from [1] and [2]). The original motivation behind reaction systems was to model interactions between biochemical reactions in the living cells. This leads to the following definitions.

Definition 1. *A* reaction *is a triplet* $a = (R_a, I_a, P_a)$*, where* R_a, I_a, P_a *are finite nonempty sets with* $R_a \cap I_a = \emptyset$*. If* S *is a set such that* $R_a, I_a, P_a \subseteq S$*, then* a *is a reaction in* S*.*

The sets R_a, I_a, P_a, are called the reactant set of a, the inhibitor set of a, and the product set of a, respectively. Clearly, since R_a, I_a are disjoint and nonempty, then if a is a reaction over S, then $|S| \geq 2$. We will use $rac(S)$ to denote the set of all reactions over S.

The enabling of a (biochemical) reaction in the given state of a biochemical system and the resulting state transformation are defined as follows.

Definition 2. *Let* T *be a finite set*

- *Let* a *be a reaction. Then* a *is* enabled by T*, denoted by* $en_a(T)$*, if* $R_a \subseteq T$ *and* $I_a \cap T = \emptyset$*. The result of* a *on* T*, denoted by* $res_a(T)$*, is defined by:* $res_a(T) = P_a$ *if* $en_a(T)$ *and* $res_a(T) = \emptyset$*, otherwise.*
- *Let* A *be a finite set of reactions. The* result of A on T, *denoted by* $res_A(T)$*, is defined by:* $res_A(T) = \bigcup_{a \in A} res_a(T)$*.*

The intuition behind a finite set T is that of a state of a biochemical system, *i.e.*, a set of biochemical entities present in the current biochemical environment. Thus a single reaction a is enabled by state T if T separates R_a from I_a, *i.e.*, $R_a \subseteq T$ and $I_a \cap T = \emptyset$. When a is enabled by T, then its result on T is just P_a. For a set A of reactions, its result on T is cumulative, *i.e.*, it is the union of the results of all individual reactions from A. Since reactions which are not enabled by T do not contribute to the result of A on T, $res_A(T)$ can be defined by

$$res_A(T) = \bigcup \{res_a(T) | a \in A \text{ and } en_a(T)\}.$$

Now the central notion of a reaction system is defined as follows.

Definition 3. *A* reaction system *is an ordered pair* $\mathcal{A} = (S, A)$*, where* S *is a finite set such that* $|S| \geq 2$ *and* $A \subseteq rac(S)$ *is a nonempty set of reactions in* S*.*

Thus a reaction system is basically a finite set of reactions over a set S, which is called the *background set of* $\mathcal{A}$ and its elements are called *entities*. The *result function* of $\mathcal{A}$, $res_{\mathcal{A}} : 2^S \longrightarrow 2^S$ is defined by $res_{\mathcal{A}} = res_A$.

The behaviour of a reaction system (which results from the interactions between its reactions) is determined by its dynamic processes which are formally defined as follows.

Definition 4. *Let* $\mathcal{A} = (S, A)$ *be a reaction system and let* $n \geq 1$ *be an integer. An (n-step)* interactive process in A *is a pair* $\pi = (\gamma, \delta)$ *of finite sequences such that* $\gamma = C_0, \ldots, C_n$ *and* $\delta = D_0, \ldots, D_n$*, where* $C_0, \ldots, C_n, D_0, \ldots, D_n \subseteq S$*, and* $D_i = res_{\mathcal{A}}(D_{i-1} \cup C_{i-1})$ *for all* $i \in \{1, \ldots, n\}$*.*

The sequence γ is the *context sequence of* π and the sequence δ is the *result sequence of* π. Then, the sequence $\tau = W_0, W_1, \ldots, W_n$ defined by $W_i = C_i \cup D_i$ for all $i \in \{0, \ldots, n\}$ is the *state sequence of* π with $W_0 = C_0$ called the *initial state of* π (and of τ). If $C_i \subseteq D_i$ for all $i \in \{1, \ldots, n\}$, then we say that π (and τ) is context-independent. Note that we can assume then that $C_i = \emptyset$ for all $i \in \{1, \ldots, n\}$ without changing the state sequence.

Thus, an interactive process begins in the initial state $W_0 = C_0 \cup D_0$. The reactions from A enabled by W_0 produce the result D_1 which together with C_1 forms the successor state $W_1 = C_1 \cup D_1$. The iteration of this procedure determines π: for each $i \in \{0, \ldots, n-1\}$, the successor of state W_i is $W_{i+1} = C_{i+1} \cup D_{i+1}$, where $D_{i+1} = res_{\mathcal{A}}(W_i)$.

The context sequence formalizes the intuition that, in general, a reaction system is not a closed system and so its behavior is influenced by its environment. Note that a context-independent state sequence is determined by its initial state W_0 and the number of steps (n). In general, for an n-step interactive process π of $\mathcal{A}$, π is determined by its context sequence and n.

Also, in a context-independent state sequence $\tau = W_0, \ldots, W_i, W_{i+1}, \ldots, W_n$, during the transition from W_i to W_{i+1} all entities from $W_i - res_{\mathcal{A}}(W_i)$ vanish. This reflects the assumption of *no permanency*: an entity from a current state vanishes unless it is produced/sustained by A. Clearly, if π is not context-independent, then an entity from a current state W_i can be also sustained (thrown in) by the context (C_{i+1}). This feature is also a major difference with standard models of concurrent systems such as Petri nets (see, *e.g.*, [6]).

3 Barwise and Seligman's logic for distributed system

In [5], the formal counterpart of information available to different sources/agents, including their prior knowledge, is captured through the notion of classification; a classification specifies an agent's information and knowledge regarding which object satisfies which properties or is of which type. The formal definition is given as follows.

Definition 5. *A classification* $A = \langle Tok(A), Typ(A), \models_A \rangle$ *consists of*
(i) a set, $Tok(A)$*, of objects to be classified, called tokens of* A*,*
(ii) a set, $Typ(A)$*, of properties used to classify the tokens, called the types of* A*, and*
(iii) a binary relation, $\models_A$*, between* $Tok(A)$ *and* $Typ(A)$*.*

If $a \models_A \alpha$, then a is said to be of type α in A. That is, $\models_A$ basically specifies which token is of which type. Following the literature of rough sets [7, 8], the notion of classification, presented in [5], can be viewed as a special kind of information system, which is a tuple $(U, \mathcal{A}, \{V_a\}_{a\in\mathcal{A}}, \{f_a\}_{a\in\mathcal{A}})$ consisting of respectively sets of objects, attributes, a set of values for each attribute, and a set of functions for each attribute specifying which object satisfies which attribute with what value. In the context of classification, U is basically $Tok(A)$, $\{(a, v) : a \in \mathcal{A}, v \in V_a\}$ is $Typ(A)$, and for $u \in U$, $f_a(u) = v$ can be associated with $u \models_A (a, v)$ for each $(a, v) \in Typ(A)$.

Now the notion of infomorphism, defined below, represents relationship between classifications, and provides a way of moving information back and forth between them.

Definition 6. *Let* $A = \langle Tok(A), Typ(A), \models_A \rangle$ *and* $B = \langle Tok(B), Typ(B), \models_B \rangle$ *be two classifications. An infomorphism* $f : A \rightleftarrows B$ *from* A *to* B *is a contravariant pair of functions* $f = (\hat{f}, \check{f})$ *such that* $\hat{f} : Typ(A) \mapsto Typ(B)$ *and* $\check{f} : Tok(B) \mapsto Tok(A)$ *satisfying the following fundamental property of infomorphisms.*
$\check{f}(b) \models_A \alpha$ *iff* $b \models_B \hat{f}(\alpha)$ *for each* $b \in Tok(B)$ *and* $\alpha \in Typ(A)$*.*

The notion of an *interpretation*, sometimes also called a *translation* of one language into another, is an example of infomorphism between classifications. There are two aspects of an interpretation; one is to do with tokens (structures), and the other is to do with types (sentences). An interpretation $\mathcal{I} : L_1 \rightleftarrows L_2$ of languages L_1 into L_2 does two things. At the level of types, it associates with every sentence α of L_1, a sentence $\mathcal{I}(\alpha)$ of L_2, its translation. At the level of tokens, it associates with every structure M for L_2, a structure $\mathcal{I}(M)$ for L_1. The relation that $\mathcal{I}(M) \models_{L_1} \alpha$ iff $M \models_{L_2} \mathcal{I}(\alpha)$, presents that what $\mathcal{I}(\alpha)$ says about the structure M is equivalent to what α says about the structure $\mathcal{I}(M)$.

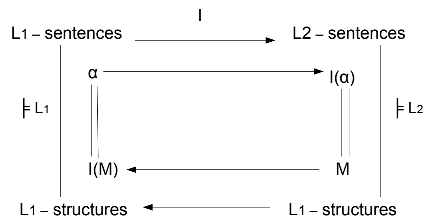

Fig. 1. Interpretation: a translation of one language to another

Definition 7. *Given the infomorphisms $f : A \rightleftarrows B$ and $g : B \rightleftarrows C$, the composition*
$gf : A \rightleftarrows C$ of f and g is the infomorphism defined by $\hat{gf} = \hat{g}\hat{f}$ and $\check{gf} = \check{f}\check{g}$.

Given a classification of information, often it is found that some tokens are identical with respect to some types, and distinct with respect to the rest. The example, as given in [5], might render a better understanding in this regard.

> *My copy of today's edition of the local newspaper bears much in common with that of my next door neighbour. If mine has a picture of President Clinton on page 2, so does hers. If mine has three sections, so does hers. . . . Mine has orange juice spilled on it, hers does not. Hers has the crossword puzzle solved, mine does not.*

In the theory of classification, this aspect is captured by the following notions of invariant and quotient classification.

Definition 8. *Given a classification A, an invariant is a pair $I = (\Sigma, R)$ consisting of a set $\Sigma \subseteq Typ(A)$ of types of A and a binary relation R between tokens of A such that if aRb, then for each $\alpha \in \Sigma$, $a \models_A \alpha$ if and only if $b \models_A \alpha$.*

In the above definition though R needs not to be an equivalence relation, in the further considerations R is considered to be the smallest equivalence relation containing the concerned relation.

Definition 9. *Let $I = (\Sigma, R)$ be an invariant on the classification A with respect to an equivalence relation R. The quotient of A by I, denoted as A/I, is the classification with types Σ, whose tokens are the R-equivalence classes of tokens of A, and with $[a]_R \models_{A/I} \alpha$ if and only if $a \models_A \alpha$.*

One can notice that the notion of invariance, as defined in [5], also corresponds to the notion of indiscernibility in the context of rough set literature. In an information system, given by the tuple $(U, \mathcal{A}, \{V_a\}_{a\in\mathcal{A}}, \{f_a\}_{a\in\mathcal{A}})$, two objects x, y of U are said to be indiscernible (i.e., $xIND(A)y$) if $f_a(x)$ and $f_a(y)$ receive the same value from V_a for any $a \in \mathcal{A}$. Moreover, the notion of sequent, defined below, also has a counterpart in rough set literature. A sequent can be viewed as a non-deterministic decision rule, i.e., relation between two (finite) sets of descriptors (e.g. (a, v) for $a \in \mathcal{A}$ and $v \in V_a$) describing the available data of the information system.

Example 1. For a given information system $\mathbb{A}$=$(U, \mathcal{A}, \{V_a\}_{a\in\mathcal{A}}, \{f_a\}_{a\in\mathcal{A}})$ and the indiscernibility relation $IND(A)$ one can define two classifications $Cl(\mathbb{A}) = (U, \Sigma, \models_{\mathbb{A}})$ and $Cl(\mathbb{A}/IND(A))$ $=(U/IND(A), \Sigma, \models_{\mathbb{A}/IND(A)})$, where Σ is a subset of $Type(A)$ (cf. below Def. 5), $x \models_{\mathbb{A}} \alpha$ denotes that x satisfies α, and $[x]_{IND(A)} \models_{\mathbb{A}/IND(A)} \alpha$ means that $x \models_{\mathbb{A}} \alpha$ [7, 8]. One can easily check that these two classifications can be linked by infomorphisms $(id, g) : Cl(\mathbb{A}) \rightleftarrows Cl(\mathbb{A}/IND(A))$, where id is the identity on Σ and g assigns to $[x]_{IND(A)}$ any object from $[x]_{IND(A)}$ and $(id, h) : Cl(\mathbb{A}/IND(A)) \rightleftarrows Cl(\mathbb{A})$, where id is the identity on Σ and $h(x) = [x]_{IND(A)}$ for $x \in U$.

□

As pointed out in [5],

> *one way to think about information flow in a distributed system is in terms of a 'theory' of the system, that is, a set of known laws that describes the system.*

Based on this general notion of classification, the notion of sequent or notion of consequence of a deductive logic is captured as follows.

As a classification, say $(Tok(A), Typ(A), \models_A)$, specifies a perspective about the properties of the $Tok(A)$ we may call the classification as *classification of* A considering A to refer to that particular perspective.

Definition 10. *Let* $cl(A) = (Tok(A), Typ(A), \models_A)$ *be a classification of* A.
(i) For any $\Gamma, \Delta \subseteq Typ(A)$, $\langle \Gamma, \Delta \rangle$ *is considered to be a sequent of* $Typ(A)$.
(ii) A sequent $\langle \Gamma, \Delta \rangle$ *is a partition of* $\Sigma' \subseteq Typ(A)$ *if* $\Gamma \cup \Delta = \Sigma'$ *and* $\Gamma \cap \Delta = \phi$.
(iii) A binary relation $\vdash$ *between subsets of* $Typ(A)$ *is called a (Gentzen) consequence relation.*
(iv) A theory $T = (\Sigma, \vdash)$ *is a pair, where* $\Sigma \subseteq Typ(A)$ *and* $\vdash$ *is a consequence relation on* Σ.
(v) A constraint of the theory T *is a sequent* $\langle \Gamma, \Delta \rangle$ *such that* $\Gamma \vdash \Delta$.
(vi) A token a *of* $Tok(A)$ *satisfies* $\langle \Gamma, \Delta \rangle$ *provided that if* a *is of type* α *for every* $\alpha \in \Gamma$, *then* a *is of type* β *for some* $\beta \in \Delta$. *A token not satisfying a sequent is called a counterexample to the sequent.*
(vii) The theory $T(cl(A)) = (Typ(A), \vdash_A)$ *generated by* $cl(A)$ *is the theory whose constraints are the set of sequents satisfied by every token of* $Tok(A)$.
(viii) A theory whose constraints are satisfied by every token of the classification is called a complete theory.

Here it is to be noted that sequents are all possible pairs of sets of types, and some of them come under the consequence relation. Usually, some natural conditions are imposed on the set of sequents if one would like to consider it as a theory. Below we present such conditions in the definition of the regular theory.

Definition 11. *A theory $T = (\Sigma, \vdash)$ is regular if it satisfies the following properties viz., identity, weakening, and global cut for all types α, and all set $\Gamma, \Gamma', \Delta, \Delta'$, $\Sigma', \Sigma_0, \Sigma_1$ of types.*
Identity $\alpha \vdash \alpha$
Weakening *If $\Gamma \vdash \Delta$, then $\Gamma, \Gamma' \vdash \Delta, \Delta'$.*
Global cut *If $\Gamma, \Sigma_0 \vdash \Delta, \Sigma_1$ for each partition $\langle \Sigma_0, \Sigma_1 \rangle$ of Σ', then $\Gamma \vdash \Delta$.*

Proposition 1. *The theory $T(cl(A)) = (Typ(A), \vdash_A)$ generated by the classification $cl(A)$ of A is a regular theory.*

Proposition 2. *Any regular theory $T = (\Sigma, \vdash)$ satisfies the following condition.*
Finite cut: If $\Gamma, \alpha \vdash \Delta$ and $\Gamma \vdash \Delta, \alpha$, then $\Gamma \vdash \Delta$.

Definition 12. *Given two theories $T_1 = (Typ(T_1), \vdash_{T_1})$ and $T_2 = (Typ(T_2), \vdash_{T_2})$, a (regular theory) interpretation $f : T_1 \mapsto T_2$ is a function from $Typ(T_1)$ to $Typ(T_2)$ such that for each $\Gamma, \Delta \subseteq Typ(T_1)$ if $\Gamma \vdash_{T_1} \Delta$, then $f(\Gamma) \vdash_{T_2} f(\Delta)$.*

The notion of local logic puts the idea of a classification together with that of a regular theory. Moreover, introducing a notion of normal tokens it models resonable but unsound inferences.

Definition 13. *A local logic $\mathcal{L} = (Tok(\mathcal{L}), Typ(\mathcal{L}), \models_{\mathcal{L}}, \vdash_{\mathcal{L}}, N_{\mathcal{L}})$ consists of*
(i) a classification $cl(\mathcal{L}) = (Tok(\mathcal{L}), Typ(\mathcal{L}), \models_{\mathcal{L}})$,
(ii) a regular theory $Th(\mathcal{L}) = (Typ(\mathcal{L}), \vdash_{\mathcal{L}})$, and
(iii) a subset $N_{\mathcal{L}} \subseteq Tok(\mathcal{L})$, called the normal tokens of $\mathcal{L}$, which satisfies all the constraints of $Th(\mathcal{L})$.

Definition 14. *A logic infomormhism $f : \mathcal{L}_1 \rightleftarrows \mathcal{L}_2$ consists of a contravariant pair $f = (\hat{f}, \check{f})$ of functions such that*
(i) $f : cl(\mathcal{L}_1) \rightleftarrows cl(\mathcal{L}_2)$ is an infomorphism of classifications,
(ii) $\hat{f} : Th(\mathcal{L}_1) \mapsto Th(\mathcal{L}_2)$ is a theory interpretation, and
(iii) $\check{f}(N_{\mathcal{L}_2}) \subseteq N_{\mathcal{L}_1}$.

It can be observed that through these notions of classification, local logic, and logic infomorphism the target of the authors [5] was to formalize respectively an individual's information base, logical reasoning base, and flow of information from one individual to another in the process of decision making.

4 Exploration Systems and Their Extension Grounded in Local Logics over Information Systems

In this section, we consider exploration systems which combine zoom structures with reaction systems "running within" zoom structures (see, *e.g.*, [3, 4]). The original intuition and motivation was that a zoom structure is the integrating structure of a depository of knowledge of a discipline of science (*e.g.*, biology). A discipline of knowledge must be structured and the integrating structure here is a well-founded partial order which is well suited to represent a hierarchical structure of knowledge (as, *e.g.*, is the case in biology).

Formally zoom structures are defined as follows (we consider irreflexive partial orders; recall that a partial order is well-founded if every walk against its edges is finite).

Definition 15. *A* zoom structure *is a 6-tuple* $\mathcal{Z} = (D, E, \Gamma, \Delta, \{D_i\}_{i \in \Gamma}, \{E_j\}_{j \in \Delta})$, *where*

(i) D *is a non-empty set,*
(ii) $E \subseteq D \times D$ *is such that the* E^+ *(*i.e.*, the transitive closure of* E*) is a well-founded partial order,*
(iii) Γ, Δ *are finite sets,*
(iv) $\{D_i\}_{i \in \Gamma}$ *is a partition of* D *(into non-empty sets), and*
(v) $\{E_j\}_{j \in \Delta}$ *is a partition of* E *(into non-empty sets).*

Obviously, $\mathcal{Z}$ can be also seen as a node- and edge-labelled graph, where D is its set of nodes labelled by elements of Γ, and E is its set of edges labelled by elements of Δ.

Data structures for implementing large sets of data are often hierarchical: in accessing specific data one usually performs a series of zoom operations each of which leads from a topic to its "subtopic." This is reflected in the basic notion of an inzoom of $\mathcal{Z}$.

Definition 16. *Let* $\mathcal{Z} = (D, E, \Gamma, \Delta, \{D_i\}_{i \in \Gamma}, \{E_j\}_{j \in \Delta})$ *be a zoom structure. An* inzoom *of* $\mathcal{Z}$ *is a finite sequence* $x = x_1, x_2, \ldots, x_n$ *such that* $n \geq 2$, $x_i \in D$ *for* $i \in \{1, \ldots, n\}$, *and, for each* $i \in \{2, \ldots, n\}$, $(x_i, x_{i-1}) \in E$.

The set of inzooms of $\mathcal{Z}$ is denoted by $INZOOM(\mathcal{Z})$.

Thus an inzoom represents a "reverse walk" in $\mathcal{Z}$, *i.e.*, a walk through nodes such that each single step goes against an edge of E. In the framework of zoom structures, inzooms (rather than nodes) are basic units for reasoning about and the usage of zoom structures.

While a zoom structure represents the *static* integrating structure of a depository of knowledge, the dynamic processes of exploring depositories of knowledge are represented by reaction systems "embedded" (rooted) in zoom structures. The embedding of a reaction system in a zoom structure is realized by requiring that the background of the reaction system consists of inzooms of the zoom structure.

Definition 17. *Let* $\mathcal{Z}$ *be a zoom structure. A reaction system* $\mathcal{A} = (S, A)$ *is* rooted in $\mathcal{Z}$ *if* $S \subseteq INZOOM(\mathcal{Z})$.

Recall that a reaction system $\mathcal{A} = (S, A)$ specifies, through the result function $res_{\mathcal{A}}$, a set-theoretical transformation of the set of subsets of its background set S (hence on the states of $\mathcal{A}$). (When one allows processes of $\mathcal{A}$ to be more general than context-independent, then more general transformations are considered.) The background set can be any set and if we choose it to be a set of zooms of

$\mathcal{Z}$, then we root $\mathcal{A}$ in $\mathcal{Z}$, " allowing" $\mathcal{A}$ to explore (the knowledge deposited in) $\mathcal{Z}$.

This leads to the notion of an exploration system.

Definition 18. *An* exploration system *is an ordered pair* $\mathcal{E} = (\mathcal{Z}, \mathcal{F})$*, where* $\mathcal{Z}$ *is an extended zoom structure and* $\mathcal{F}$ *is a family of reaction systems rooted in* $\mathcal{Z}$*.*

In the original definition (see [3]) $\mathcal{Z}$ is a construct more general than a zoom structure. However, for the purpose of our discussion it suffices to assume here that $\mathcal{Z}$ is a zoom structure.

Exploration systems can be used for reasoning about perceived situation in the physical world. Note that objects in D do not have to belong to the ground level of hierarchical modeling obtained by sensory based perception of reality. They can be constructs of higher level of the hierarchical modeling for perception based reasoning about the currently perceived situation. Moreover, edges in E can be interpreted as links representing possible relevant interactions between objects from D. This means that results of interactions can be used in perception based reasoning about the currently perceived situation.

Example 2. Let $\mathcal{Z} = (D, E, \Gamma, \Delta, \{D_i\}_{i\in\Gamma}, \{E_j\}_{j\in\Delta})$ be a zoom structure such that $D = \{x_1, x_2, \ldots, x_{10}\}$, $\Gamma = \{1, 2, 3\}$, $\Delta = \{4, 5, 6\}$, $D_1 = \{x_1, x_2, x_3\}$, $D_2 = \{x_4, x_5, x_6, x_7\}$, $D_3 = \{x_8, x_9, x_{10}\}$, $E = E_4 \cup E_5 \cup E_6$, $E_4 = \{(x_5, x_7), (x_8, x_{10}), (x_1, x_4), (x_3, x_5)\}$, $E_5 = \{(x_6, x_7), (x_4, x_7), (x_2, x_3)\}$, and $E_6 = \{(x_5, x_{10}), (x_1, x_3), (x_1, x_2), (x_3, x_8), (x_9, x_{10})\}$ (see Figure 2). It is illustrated in Figure 2.

Let $\mathcal{A} = (S, A)$ be a reaction system rooted in $\mathcal{Z}$ such that $S = \{(x_3, x_2, x_1), (x_3, x_1), (x_3, x_2), (x_2, x_1), (x_7, x_4), (x_{10}, x_8)\}$ and A contains the reaction $a = (R_a, I_a, P_a)$ with $R_a = \{(x_3, x_2, x_1), (x_2, x_1)\}$, $I_a = \{(x_{10}, x_8)\}$, and $P_a = \{(x_7, x_4)\}$. This gives an example of a reaction system rooted in a zoom structure.

4.1 Exploration Systems Grounded in the Space of Information Systems

We propose to extend exploration systems by adding interpretation of nodes and edges of zoom structures. The interpretation of nodes of zoom structures is given in the form of labels of nodes defined by local logics (related to information systems) [5] and interpretation of edges in the zoom structure are logic infomorphisms between local logics labeling nodes linked by the edges.

Having such a framework, following the information flow approach [5] one can construct local logics for individual agents as well as local logic representing the whole network of local logics. However, such a global logic will be very complex what makes it hardly possible to derive efficiently conclusions of the basis of such a local logic. Moreover, due to the cumulation of uncertainties the reasoning on the basis of such a logic may be not satisfactory.

Instead of this we propose to construct local logics only for some fragments of zoom structures which are relevant for the perceived situation. Namely, we propose to construct local logics corresponding to subnetworks defined by the

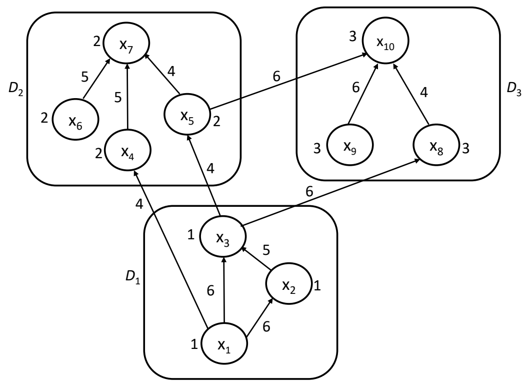

Fig. 2. Zoom structure from Example 2.

partition of nodes given by a zoom structure (representing subdomains of knowledge). It should be noted that the partition blocks can be further restricted by using reactants of relevant reactions from exploration system. In the consequence, the fragments of networks for which it is necessary to construct locals logic representing them is substantially reduced. The aim is to make the reasoning process efficient and leading to conclusions on the perceived situation. The products of reactions from the considered exploration system are used as pointers indicating relevant fragments of zoom structure. These fragments are used in further steps of reasoning on the basis of local logics toward understanding the perceived situation.

There is one more extension we propose to the zoom structure defined above. This is specified by a selection function making it possible to select, from the family of reaction systems given in the considered exploration system, a relevant reaction system, for the next step of reasoning on the basis of the current information on the currently perceived situation. We assume here that this information is represented, in particular, in a distinguished nodes (called sensory nodes) of extended zoom structure, where information systems and corresponding to them local logics are labeling nodes.

5 Conclusions

We presented a preliminary discussion about extension of exploration systems defined in [3, 4]. In the full version of the paper we plan to give more details about this extension and its possible applications.

In our further research, we plan to consider the exploration systems as dynamic complex networks with the structures changing by the control mechanisms responsible for the behavior of exploration systems. The control of an agent, using a given exploration system interacting with the environment, is aiming to satisfy the 'needs' of the agent. It should be noted that the needs may change with time. One may ask how such complex exploration systems may be constructed and modified with time. Here, we would like to point to two special strategies following two kinds of judgments used for making changes in the current exploration system. The first one is based on aggregation of information systems labeling nodes of zoom structures of these exploration systems and consequently the local logics corresponding to them. The aggregations are such as operations of join of information systems with some relevant constraints. These constraints are used to filter Cartesian products of sets of objects in the joint information systems to obtain relevant computational building blocks (granules) for describing the perceived situation, *e.g.*, the ones which are used for approximation of complex vague concepts responsible for triggering action or plans (see, *e.g.*, [9]). The second kind of strategies is based on the ability of agents to create the so called complex granules making it possible to extend the fragments of the physical world, perceived by agents, to the new fragments localized in the scope of these complex granules (see, *e.g.*, [10–12]). More detailed discussion on the issues related to dynamic behavior of exploration systems will be included in our next papers.

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