The Weak Completion Semantics

Emmanuelle-Anna Dietz Saldanha and Steffen Hölldobler and
Isabelly Lourêdo Rocha

International Center for Computational Logic, TU Dresden, 01062 Dresden, Germany
dietz@iccl.tu-dresden.de and sh@iccl.tu-dresden.de and isabellylr@gmail.com

Abstract This is a gentle introduction to the weak completion semantics, a novel cognitive theory which has been successfully applied to a number of human reasoning tasks. In this paper we do not focus on formalities but rather on principles and examples. The reader is assumed to be familiar with classical propositional logic and the suppression task.

1 Introduction

The weak completion semantics is a novel cognitive theory, which recently has outperformed twelve established cognitive theories on syllogistic reasoning [19, 23]. It is based on ideas first expressed in [27], viz. to encode knowledge as logic programs and, in particular, to use licenses for inferences when encoding conditionals, to make assumptions about the absense of abnormalities, to interprete programs under a three-valued (Kleene) logic [20], to compute a supported model for each program as least fixed point of an appropriate semantic operator, and to reason with respect to these least fixed points. But the weak completion semantics differs from the approach presented in [27] in that all concepts are formally specified, it is based on a different three-valued (Łukasiewics) logic [21],[1] all results are rigorously proven, and it has been extended in many different ways. In particular, the weak completion semantics has been applied to the suppression task [6], to the selection task [7, 9], to the belief bias effect [24], to reasoning about conditionals [3, 5, 9], to human spatial reasoning [4], to syllogistic reasoning [22, 23], and to contextual reasoning [10, 25]. Furthermore, there exists a connectionist encoding of the weak completion semantics based on the core method [8, 15, 17].

Modeling human reasoning tasks under the weak completion semantics is done in three stages. Firstly, the background knowledge is encoded as the weak completion of a logic program, i.e. a finite set of facts, rules, and assumptions. The program is specified with respect to certain principles, some of which have been identified in cognitive science and computational logic, others are new principles which need to be confirmed in future experiments. Secondly, a supported model for the weak completion of the program is computed. It turns out that under the Łukasiewicz logic this model is unique and can be obtained as the least fixed point of an appropriate semantic operator. Thirdly, reasoning is done with

[1] Alternatively, the three-valued logic S3 [26] could be applied as well.

respect to the unique supported model. This three-stage process is augmented by abduction if needed.

In this paper a gentle introduction to the weak completion semantics is provided. We will give an informal introduction into the three stages focussing on the suppression task in Sections 2 and 3 and on reasoning about indicative conditionals in Section 4. In each case, we will discuss how the programs, i.e. the sets of facts, rules, and assumptions are obtained, how they are weakly completed, how their unique supported models are generated, and how reasoning is performed with respect to these models. We will avoid formal definitions, theorems, and proofs; they can be found in [14] and the referenced technical papers. However, we assume the reader to be familiar with classical propositional logic.

2 Reasoning with respect to Least Models

2.1 Modus Ponens

Knowledge is encoded as positive *facts*, negative *assumptions*, and *rules*. Consider the statements *she has an essay to write* and *if she has an essay to write, then she will study late in the library* from the suppression task [1]. The first statement will be encoded in propositional logic as the fact $e \leftarrow \top$, where e denotes that she has an essay to write and $\top$ is a constant denoting truth. The second statement is a conditional which will be encoded as a *license for inferences* $\ell \leftarrow e \wedge \neg ab_1$ following [27], where ℓ denotes that she will study late in the library and ab_1 is an abnormality predicate. As in the given context nothing abnormal is known about the conditional, the assumption $ab_1 \leftarrow \bot$ is added, where $\bot$ is a constant denoting falsehood. This expression is called an assumption because – as illustrated later – it can be overriden if more knowledge becomes available. The given implications – a logic *program* – are *weakly completed* by adding the only-if-halves to obtain the set

$$\mathcal{K}_1 = \{e \leftrightarrow \top,\ \ell \leftrightarrow e \wedge \neg ab_1,\ ab_1 \leftrightarrow \bot\}.$$

The left- and the right-hand-sides of the equivalences are considered as *definiendum* and *definiens*, respectively. In particular, the propositional variables e, ℓ, and ab_1 are defined by $\top$, $e \wedge \neg ab_1$, and $\bot$, respectively. In other words, the set $\mathcal{K}_1$ is a set of definitions which encode the given background knowledge.

If a subject is asked *whether she will study late in the library*, then a *model* for this set is constructed. In a model, propositional variables are mapped to the truth values *true*, *false*, and *unknown* such that all equivalences occurring in $\mathcal{K}_1$ are simultaneously mapped to *true*. In fact, there is always a unique least model if a set like $\mathcal{K}_1$ is interpreted under the three-valued Łukasiewicz logic [16, 21],[2] whose truth tables are depicted in Table 1.

[2] This does not hold if a set is interpreted under Kleene logic [20]. For example, the equivalence $a \leftrightarrow b$ has two minimal models. In the first minimal model both, a and b, are mapped to *true*. In the second minimal model both, a and b, are mapped to *false*. The interpretation, where both, a and b, are mapped to *unknown* is not a model for $a \leftrightarrow b$.

	¬
⊤	⊥
U	U
⊥	⊥

∧	⊤	U	⊥
⊤	⊤	U	⊥
U	U	U	⊥
⊥	⊥	⊥	⊥

∨	⊤	U	⊥
⊤	⊤	⊤	⊤
U	⊤	U	U
⊥	⊤	U	⊥

←	⊤	U	⊥
⊤	⊤	U	⊥
U	U	⊤	⊤
⊥	⊥	U	⊤

↔	⊤	U	⊥
⊤	⊤	U	⊥
U	U	⊤	U
⊥	⊥	U	⊤

Table1. The truth tables of the Łukasiewicz logic, where *true*, *false*, and *unknown* are abbreviated by ⊤, ⊥, and U, respectively.

In the example, the model is constructed in two steps.[3] In the first step, $e \leftrightarrow \top$ and $ab_1 \leftrightarrow \bot$ are satisfied by the following mapping:

true	*false*
e	ab_1

In the second step, because the right-hand-side of the equivalence $\ell \leftrightarrow e \wedge \neg ab_1$ is evaluated to *true* under the given mapping, its left-hand-side ℓ must also be *true* and will be added to the model:

true	*false*
e	ab_1
ℓ	

The query *whether she will study late in the library* can now be answered positively given this model.

2.2 Alternative Arguments

If the statement *if she has a textbook to read, then she will study late in the library* is added to the example discussed in Section 2.1, then this statement will be encoded by the rule $\ell \leftarrow t \wedge \neg ab_2$ and the assumption $ab_2 \leftarrow \bot$, where t denotes that she has a textbook to read. Weakly completing the given implications we obtain the set

$$\mathcal{K}_2 = \{e \leftrightarrow \top,\ \ell \leftrightarrow (e \wedge \neg ab_1) \vee (t \wedge \neg ab_2),\ ab_1 \leftrightarrow \bot,\ ab_2 \leftrightarrow \bot\}.^4$$

If a subject is asked *whether she will study late in the library*, then a model for $\mathcal{K}_2$ is constructed as follows. In the first step, $e \leftrightarrow \top$, $ab_1 \leftrightarrow \bot$, and $ab_2 \leftrightarrow \bot$ are satisfied by the following mapping:

true	*false*
e	ab_1
	ab_2

[3] In [16, 27], a function is defined which computes this model.

[4] The set does not include the equivalence $t \leftrightarrow \bot$. In logic programming this equivalence is added under the *completion* semantics [2].

Because $e \wedge \neg ab_1$ is *true* under this mapping, so is the right-hand side of the equivalence $\ell \leftrightarrow (e \wedge \neg ab_1) \vee (t \wedge ab_2)$ and, consequently, ℓ must be true as well:

true	*false*
e	ab_1
	ab_2
ℓ	

The query *whether she will study late in the library* can now be answered positively given this model.

2.3 Additional Arguments

If the statement *if the library is open, then she will study late in the library* is added to the example discussed in Section 2.1, then this statement will be encoded by the rule $\ell \leftarrow o \wedge \neg ab_3$ and the assumption $ab_3 \leftarrow \bot$, where o denotes that the library is open. As argued in [27] a subject being confronted with the additional statement may become aware that not being open is an exception for the rule $\ell \leftarrow e \wedge ab_1$. This can be encoded by the rule $ab_1 \leftarrow \neg o$. Likewise, she may not go to the library without a reason and the only reason mentioned so far is writing an essay. Thus, not having an essay to write is an exception for the rule $\ell \leftarrow o \wedge \neg ab_3$. This can be encoded by adding the rule $ab_3 \leftarrow \neg e$. Weakly completing all implications we obtain the set

$$\mathcal{K}_3 = \{e \leftrightarrow \top,\ \ell \leftrightarrow (e \wedge \neg ab_1) \vee (o \wedge \neg ab_3),\ ab_1 \leftrightarrow \bot \vee \neg o,\ ab_3 \leftrightarrow \bot \vee \neg e\}.$$

The example shows how the intial assumption $ab_1 \leftarrow \bot$ is overriden by $ab_1 \leftarrow \neg o$. In $\mathcal{K}_3$ the definition of ab_1 is now $\bot \vee \neg o$ which is semantically equivalent to $\neg o$. Likewise $ab_3 \leftarrow \bot$ is overriden by $ab_3 \leftarrow \neg e$.

If a subject is asked *whether she will study late in the library*, then a model for $\mathcal{K}_3$ is constructed as follows. In the first step, $e \leftrightarrow \top$ is satisfied by the following mapping:

true	*false*
e	

Because the right-hand-side of the equivalence $ab_3 \leftrightarrow \bot \vee \neg e$ is mapped to *false*, ab_3 must be mapped to *false* as well:

true	*false*
e	
	ab_3

The remaining propositional variables ℓ, ab_1, and o are neither forced to be *true* nor *false* and, hence, remain *unknown*. The constructed mapping is a model for $\mathcal{K}_3$. As ℓ is not mapped to *true*, suppression is taking place.

2.4 The Denial of the Antecedent

Now suppose that in the example discussed in Section 2.1 the fact that *she has an essay to write* is replaced by *she does not have an essay to write*. This denial of the antecedent is encoded by $e \leftarrow \bot$ instead of $e \leftarrow \top$. Weakly completing the implications we obtain the set

$$\mathcal{K}_4 = \{e \leftrightarrow \bot,\ \ell \leftrightarrow e \wedge \neg ab_1,\ ab_1 \leftrightarrow \bot\}.$$

If a subject is asked *whether she will study late in the library*, then a model for $\mathcal{K}_4$ is constructed as follows. In the first step, $e \leftrightarrow \bot$ and $ab_1 \leftrightarrow \bot$ are satisfied by the following mapping:

true	*false*
	e
	ab_1

Under this mapping the right-hand-side of the equivalence $\ell \leftrightarrow e \wedge \neg ab_1$ is mapped to *false* and, consequently, ℓ will be mapped to *false* as well:

true	*false*
	e
	ab_1
	ℓ

The query *whether she will study late in the library* can now be answered negatively given this model.

The cases, where the denial of the antecedent is combined with alternative and additional arguments can be modelled in a similar way, but now the alternative argument leads to suppression [6].

3 Skeptical Abduction

3.1 The Affirmation of the Consequent

Consider the conditional *if she has an essay to write, then she will study late in the library*. As before, it is encoded by the rule $\ell \leftarrow e \wedge \neg ab_1$ and the assumption $ab_1 \leftarrow \bot$. Their weak completion is

$$\mathcal{K}_5 = \{\ell \leftrightarrow e \wedge \neg ab_1,\ ab_1 \leftrightarrow \bot\}.$$

As the least model of this set we obtain:

true	*false*
	ab_1

Under this model the propositional variables ℓ and e are mapped to *unknown*. Hence, if we observe that *she will study late in the library*, then this observation cannot be explained by this model. We propose to use *abduction* [13] in order to explain the observation. Because e is the only undefined propositional letter in this context, the set of *abducibles* is $\{e \leftarrow \top,\ e \leftarrow \bot\}$. The observation ℓ can be explained by selecting $e \leftarrow \top$ from the set of abducibles, weakly completing it to obtain $e \leftrightarrow \top$, and adding this equivalence to $\mathcal{K}_5$. Thus, we obtain $\mathcal{K}_1$ again and conclude that she has an essay to write.

3.2 Alternative Arguments and the Affirmation of the Consequent

Consider the conditionals *if she has an essay to write, then she will study late in the library* and *if she has a textbook to read, then she will study late in the library.* As in Section 2.2 they are encoded by two rules and two assumptions, which are weakly completed to obtain

$$\mathcal{K}_6 = \{\ell \leftrightarrow (e \wedge \neg ab_1) \vee (t \wedge \neg ab_2),\ ab_1 \leftrightarrow \bot,\ ab_2 \leftrightarrow \bot\}.$$

As the least model of this set we obtain:

true	*false*
	ab_1
	ab_2

Under this model the propositional variables ℓ, e, and t are mapped to *unknown.* Hence, if we observe that *she will study late in the library*, then this observation cannot be explained by this model. In order to explain the observation we consider the set $\{e \leftarrow \top,\ e \leftarrow \bot,\ t \leftarrow \top,\ t \leftarrow \bot\}$ of abducibles because e and t are undefined in $\mathcal{K}_6$. There are two minimal explanations, viz. $e \leftarrow \top$ and $t \leftarrow \top$. Both are weakly completed to obtain $e \leftrightarrow \top$ and $t \leftrightarrow \top$, and are added to $\mathcal{K}_6$ yielding $\mathcal{K}_2$ and

$$\mathcal{K}_7 = \{t \leftrightarrow \top,\ \ell \leftrightarrow (e \wedge \neg ab_1) \vee (t \wedge \neg ab_2),\ ab_1 \leftrightarrow \bot,\ ab_2 \leftrightarrow \bot\},$$

respectively. We can now construct the least models for $\mathcal{K}_2$ and $\mathcal{K}_7$:

true	*false*
e	ab_1
	ab_2
ℓ	

true	*false*
t	ab_1
	ab_2
ℓ	

Both models explain ℓ, but they give different reasons for it, viz. e and t. More formally, the literals ℓ, e, t, $\neg ab_1$, and $\neg ab_2$ follow *credulously* from the background knowledge $\mathcal{K}_6$ and the observation ℓ because for each of the literals there exists a minimal explanation such that the literal is true in the least model of the background knowledge and the explanation. But only the literals ℓ, $\neg ab_1$, and $\neg ab_2$ follow *skeptically* from the background knowledge $\mathcal{K}_6$ and the observation ℓ because all literals are true in the least models of the background knowledge and each minimal explanation. Hence, if a subject is asked whether *she will study late in the library* then a subject constructing only the first model and, thus, reasoning *credulously*, will answer positively. On the other hand, a subject constructing both models and, thus, reasoning *skeptically*, will not answer positively. As reported in [1] only 16% of the subjects answer positively. It appears that most subjects either reason credulously and construct only the second model or they reason skeptically.

4 Indicative Conditionals

In this section we will extend the weak completion semantics to evaluate indicative conditionals. In particular, we will consider obligation and factual conditionals. Consider the conditionals *if it rains, then the streets are wet* and *if it rains, then she takes her umbrella* taken from [9]. The conditionals have the same structure, but their semantics appears to be quite different.

4.1 Obligation Conditionals

The first conditional is an *obligation conditional* because its consequence is obligatory. We cannot easily imagine a case, where the condition *it rains* is *true* and its consequence *the streets are wet* is not. Moreover, the condition appears to be *necessary* as we cannot easily imagine a situation where the consequence is *true* and the condition is not. We may be able to imagine cases where a flooding or a tsunami has occurred, but we would expect that such an extraordinary event would have been mentioned in the context. We are also not reasoning about a specific street or a part of a street, where the sprinkler of a careless homeowner has sprinkled water on the street while watering the garden.

4.2 Factual Conditionals

The second conditional is a *factual conditional.* Its consequence is not obligatory. We can easily imagine the case, where the condition *it rains* is *true* and its consequence *she takes her umbrella* is *false.* She may have forgotten to take her umbrella or she has decided to take the car and does not need the umbrella. Moreover, the condition does not appear to be necessary as she may have taken the umbrella for many reasons like, for example, protecting her from sun. The condition is *sufficient.* The circumstance where the condition is *true* gives us adequate grounds to conclude that the consequence is true as well, but there is no necessity involved.

4.3 Encoding Obligation and Factual Conditionals

When we consider the two conditionals as background knowledge, then their different semantics should be reflected in different encodings. Following the principles developed in Section 2 we obtain

$$\mathcal{K}_8 = \{s \leftrightarrow r \wedge \neg ab_4,\ u \leftrightarrow r \wedge \neg ab_5,\ ab_4 \leftrightarrow \bot,\ ab_5 \leftrightarrow \bot\},$$

where s, r, and u denote that the *streets are wet*, *it rains*, and *she takes her umbrella*, respectively. Its least model is:

true	*false*
	ab_4
	ab_5

The propositional variables s, r, and u are *unknown*. Because r is undefined in $\mathcal{K}_8$, the set of abducibles contains $r \leftarrow \top$ and $r \leftarrow \bot$. Because the second conditional is a factual one, it should not necessarily be the case that r being *true* implies u being *true* as well. This can be prevented by adding $ab_5 \leftarrow \top$ to the set of abducibles because this fact can be used to override the assumption $ab_5 \leftarrow \bot$. Moreover, because the condition of the second conditional is sufficent but not necessary, observing u may not be explained by r being *true* but by some other reason. Hence, $u \leftarrow \top$ is also added to the set of abducibles. Alltogether, we obtain the set

$$\mathcal{A}_8 = \{r \leftarrow \top,\ r \leftarrow \bot,\ ab_5 \leftarrow \top,\ u \leftarrow \top\}$$

of abducibles for $\mathcal{K}_8$.

4.4 The Evaluation of Indicative Conditionals

Let *if* X *then* Y be a conditional, where the condition X and the consequence Y is a literal. We would like to evaluate the conditional with respect to some background knowledge. The background knowledge is represented by a finite set $\mathcal{K}$ of definitions and a finite set $\mathcal{A}$ of abducibles. As discussed in Section 2.1, each set of definitions has a unique least model; let $\mathcal{M}$ be this model. Considering the sets $\mathcal{K}_8$ and $\mathcal{A}_8$, then let $\mathcal{M}_8$ be the least model of $\mathcal{K}_8$, i.e. the mapping, where ab_4 and ab_5 are mapped to *false* and all other propositional letters occurring in the example are mapped to *unknown*.

Because $\mathcal{M}$ is a mapping assigning a truth value to each formula, we can simply write $\mathcal{M}(X)$ or $\mathcal{M}(Y)$ to obtain the truth values for the literals X and Y, respectively. The given conditional *if* X *then* Y shall be evaluated as follows:

1. If $\mathcal{M}(X)$ is *true*, then the conditional is assigned to $\mathcal{M}(Y)$.
2. If $\mathcal{M}(X)$ is *false*, then the conditional is assigned to *true*.
3. If $\mathcal{M}(X)$ is *unknown*, then the conditional is evaluated with respect to the skeptical consequences of $\mathcal{K}$ given $\mathcal{A}$ and considering X as an observation.

The first case is the standard one: The condition X of the conditional is *true* and, hence, the value of the conditional hinges on the value of the consequence Y. If Y is mapped to *true*, then the conditional is *true*; if Y is mapped to *unknown*, then the conditional is *unknown*; if Y is mapped to *false*, then the conditional is *false*.

The second case is also standard if conditionals are viewed from a purely logical point: if X is mapped to *false*, then the conditional is *true* independent of the value of the consequence Y. However, humans seem to treat conditionals whose condition is *false* different. In particular, the conditional may be viewed as a counterfactual. In this case, the background knowledge needs to be revised such that the condition becomes *true.* This case has been considered in [5], but it is beyond the scope of this introduction to discuss it here.

The third case is interesting: If the condition of a conditional is *unknown*, then we view the condition as an observation which needs to be explained. Moreover, we consider only skeptical consequences computed with respect to minimal explanations.

4.5 The Denial of the Consequent

As a first example consider the conditional *if the streets are not wet, then it did not rain* (*if* $\neg s$ *then* $\neg r$). Its condition $\neg s$ is *unknown* under $\mathcal{M}_8$. Applying abduction we find the only minimal explanation $r \leftarrow \bot$ for the observation $\neg s$. Together with the background knowledge $\mathcal{K}_8$ we obtain

$$\mathcal{K}_9 = \{s \leftrightarrow r \wedge \neg ab_4,\ u \leftrightarrow r \wedge \neg ab_5,\ ab_4 \leftrightarrow \bot,\ ab_5 \leftrightarrow \bot,\ r \leftrightarrow \bot\}.$$

Its least model is:

true	*false*
	ab_4
	ab_5
	r
	s
	u

It explains $\neg s$. Moreover, the consequence $\neg r$ of the conditional is mapped to *true* making the conditional *true* as expected.

As a second example consider the conditional *if she did not take her umbrella, then it did not rain* (*if* $\neg u$ *then* $\neg r$). Its condition $\neg u$ is *unknown* under $\mathcal{M}_8$. Applying abduction we find two minimal explanations for the observation $\neg u$, viz. $r \leftarrow \bot$ and $ab_5 \leftarrow \top$. Together with the background knowledge $\mathcal{K}_8$ we obtain $\mathcal{K}_9$ and

$$\mathcal{K}_{10} = \{s \leftrightarrow r \wedge \neg ab_4,\ u \leftrightarrow r \wedge \neg ab_5,\ ab_4 \leftrightarrow \bot,\ ab_5 \leftrightarrow \bot \vee \top\},$$

respectively. Their least models are:

true	*false*
	ab_4
	ab_5
	r
	s
	u

true	*false*
ab_5	ab_4
	u

Whereas the first explanation explains $\neg u$ by stating that it did not rain, the second explanations explains $\neg u$ by stating that the abnormality ab_5 is *true.*

She may have simply forgotten her umbrella when she left home. Whereas the first explanation entails that it did not rain, the background knowledge and the second explanation does neither entail r nor $\neg r$. Hence, $\neg r$ follows credulously, but not skeptically from the background knowledge and the observation $\neg u$. Because conditionals are evaluated skeptically, the conditional is evaluated to *unknown* as expected.

4.6 The Affirmation of the Consequent

As another example consider the conditional *if the streets are wet, then it rained* (*if* s *then* r). Its condition s is unknown under $\mathcal{M}_8$. Applying abduction we find the only minimal explanation $r \leftarrow \top$ for the observation s. Together with the background knowledge $\mathcal{K}_8$ we obtain:

$$\mathcal{K}_{11} = \{s \leftrightarrow r \wedge \neg ab_4,\ u \leftrightarrow r \wedge \neg ab_5,\ ab_4 \leftrightarrow \bot,\ ab_5 \leftrightarrow \bot,\ r \leftrightarrow \top\}.$$

Its least model is:

true	*false*
r	ab_4
	ab_5
s	
u	

It explains s. Moreover, the consequence r of the conditional is mapped to *true* making the conditional *true* as well.

As final example consider the conditional *if she took her umbrella, then it rained* (*if* u *then* r). Its condition u is again *unknown* under $\mathcal{M}_8$. Applying abduction we find two minimal explanations, viz. $r \leftarrow \top$ and $u \leftarrow \top$. Together with the background knowledge $\mathcal{K}_8$ we obtain $\mathcal{K}_{11}$ and

$$\mathcal{K}_{12} = \{s \leftrightarrow r \wedge \neg ab_4,\ u \leftrightarrow (r \wedge \neg ab_5) \vee \top,\ ab_4 \leftrightarrow \bot,\ ab_5 \leftrightarrow \bot, \},$$

respectively. Their least models are:

true	*false*
r	ab_4
	ab_5
s	
u	

true	*false*
u	ab_4
	ab_5

Whereas the first explanation explains u by stating that it rained, the second explanation explains u by stating that she took her umbrella for whatever reason. As before, r follows credulously but not skeptically. Hence, the conditional is evaluated to *unknown*. Skeptical reasoning yields the expected answer again, whereas a creduluous approach does not.

In [9] it is also shown that the approach adequately models the abstract as well as social version of the selection task [12,28]. The conditional *if there is the* *letter D on one side of the card, then there is the number 3 on the other side* is considered as a factual one with necessary condition, whereas the conditional *if a person is drinking beer, then the person must be over 19 years of age* is considered as an obligation with sufficient condition. Reasoning skeptically yields the adequate answers.

5 Conclusion

The weak completion semantics is a novel cognitive theory which has been applied to adequately model various human reasoning tasks. Background knowledge is encoded as a set of definitions based on the following principles:

- positive information is encoded as facts,
- negative information is encoded as assumptions,
- conditionals are encoded as licenses for inferences, and
- the only-if halves of definitions are added.

For each set of definitions a set of abducibles is constructed as follows:

- all facts and assumptions for the propositional letters which are undefined in the background knowledge are added,
- the abnormalities of factual conditionals are added as facts, and
- the conclusions of conditionals with sufficient condition are added as facts.

The background knowledge admits a least supported model under Łukasiewicz logic, which can be computed as the least fixed point of an appropriate semantic operator. Reasoning is performed with respect to the least supported model. If an observation is *unknown* under the least supported model, then skeptical abduction using minimal explanations is applied. There exists a connectionist realization.

The approach presented in this paper is restricted to propositional logic and does neither consider counterfactuals nor contextual abduction. These extensions are presented in [5, 10, 22, 23]. In particular, if the weak completion semantics is extended to first-order logic, then additonal principles are applied in the construction of the background knowledge like

- existential import and Gricean implicature,
- unknown generalization,
- search for alternative models,
- converse interpretation,
- blocking of conclusions by double negatives,
- negation by transformation,

but it is beyond the scope of this introduction to discuss these principles.

There are a variety of open problems and questions. For example, skeptical abduction is exponential [11, 18]. Hence, it is infeasible that humans reason skeptically if the reasoning episodes become larger. We hypothesize that humans generate some, but usually not all minimal explanations and reason skeptically with respect to them. Which explanations are generated? Are short or simple explanations preferred? Are more explanations generated if more time is available? Is the generation of explanations biased and, if so, how is it biased? Does attention play a role?

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